The Reasoner, 2008, 2 (11), pp. 8-9, ISSN 1757-0522

Indispensability Argument and Set Theory

The Quinean indispensability argument, as put by Mark Colyvan (2001: *The Indispensability of Mathematics*, Oxford University Press, 192 pp., p.1): "... mathematical entities are indispensable to our best physical theories and therefore share the ontological status of scientific entities."

Of course, one may take several different positions with respect to the ontological status of scientific entities such as, for example, quarks (quarks can't be observed even in principle). Do quarks "really exist", or are they only a (currently successful) theoretical construct used by physicists in their models? Perhaps, the "least committed" position could be the formalist one: let us define the "real existence" of some scientific entity as its invariance in *future* scientific theories. If quarks will be retained as a construct in our best future physical theories, then one may think of quarks as "really existing". Even from such a very formalistic point of view, the Quinean argument seems quite reasonable. Indeed, if some mathematical entity is indispensable to our best physical theories, then shouldn't we believe, that this entity "exists" in the same sense as quarks are believed to exist?

However, imagine two mathematical entities E1 and E2, such that the existence of E1 contradicts the existence of E2. Can both of such entities be indispensable to our best physical theories? As an example, let us consider two well-known versions of set theory:

ZFC, i.e. ZF+AC, where ZF stands for Zermelo-Fraenkel axioms, and AC is the Axiom of Choice, see Thomas Jech (2006: *Set Theory*, Springer, 772 pp., Chapter 1);

ZF+AD, where AD is the so-called Axiom of Determinacy, see Akihiro Kanamori (2003: *The Higher Infinite: Large Cardinals in Set Theory from Their Beginnings*, Springer, 564 pp., Chapter 6).

AD contradicts AC, hence, these theories cannot be used together. Currently, ZFC is almost generally acknowledged as the formal basis for theoretical mathematics. If ZF+AD would be used instead of ZFC, then we would have a slightly different theoretical mathematics. Worse, or better than the actual one? Who knows... But: as a basis for the *applied* mathematics, ZFC and ZF+AD can be used equally well! All the mathematical inferences, currently necessary for physical theories, can be performed in ZF, i.e. in ZFC and in ZF+AD as well. Then, which of both set theories is indispensable to our best physical theories - ZFC, or ZF+AD?

May one believe that some of the proper ZFC inferences (i.e. inferences involving AC that can't be performed in ZF alone) could, some time in the future, be applied in physical theories? But so could proper ZF+AD inferences as well!

Would you say now that this is nothing new? That with the non-Euclidean geometries we have exactly the same situation: there are several geometries contradicting each other, but all of them are indispensable to our best physical theories? Indeed, the Euclidean geometry and non-Euclidean geometries are now become special cases of a more general theory that inspired Einstein's general relativity theory - the so-called Riemannian geometry, see Peter Petersen (2006: *Riemannian Geometry*, Springer, 408 pp.).

And with set theories we have the same situation! As a set theory, ZF+AD is much more powerful than ZFC. According to a theorem proved by W. Hugh Woodin, ZF+AD can be "embedded" into a powerful extension of ZFC, obtained by adding one of the so-called large cardinal axioms ("There are infinitely many Woodin cardinals"), and conversely, this powerful extension of ZFC can be "embedded" into ZF+AD. See Kanamori (2003: Theorem 32.16). Should this mean, as in the case of non-Euclidean geometries, that set theories ZFC and ZF+AD are *both* indispensable to our best physical theories?

But then, how about the most fundamental mathematical entity - the famous unique "world of sets" to which we ought to have ontological commitment and that must be studied in set theory as the only structure worth of consideration? And in which the famous Continuum Hypothesis must be either true, or false, independently of the ability of human mathematicians to decide this? Which of the axioms - AC, or AD is true in this "world"? Most set theorists accept AC, and reject AD, i.e. for them, AC is true in the "world of sets", and AD is false. Applying to set theory the above-mentioned formalistic explanation of the existence of quarks, we could say: if, for a long time in the future, set theorists will continue their believing in AC, then one may think of a unique "world of sets" as existing in the same sense as quarks are believed to exist.

But, as we see, this is only a "light-weight" opinion that can't be justified by the Quinean indispensability argument! And, when it can't, then "what is the fuss about?" (as put by a prominent logician).

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