TOWARDS M-MATRIX

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0.1 PREFACE

This book belongs to a series of online books summarizing the recent state Topological Geometrodynamics (TGD) and its applications. TGD can be regarded as a unified theory of fundamental interactions but is not the kind of unified theory as so called GUTs constructed by graduate students at seventies and eighties using detailed recipes for how to reduce everything to group theory. Nowadays this activity has been completely computerized and it probably takes only a few hours to print out the predictions of this kind of unified theory as an article in the desired format. TGD is something different and I am not ashamed to confess that I have devoted the last 37 years of my life to this enterprise and am still unable to write The Rules.

If I remember correctly, I got the basic idea of Topological Geometrodynamics (TGD) during autumn 1977, perhaps it was October. What I realized was that the representability of physical space-times as 4-dimensional surfaces of some higher-dimensional space-time obtained by replacing the points of Minkowski space with some very small compact internal space could resolve the conceptual difficulties of general relativity related to the definition of the notion of energy. This belief was too optimistic and only with the advent of what I call zero energy ontology the understanding of the notion of Poincare invariance has become satisfactory. This required also the understanding of the relationship to General Relativity.

It soon became clear that the approach leads to a generalization of the notion of space-time with particles being represented by space-time surfaces with finite size so that TGD could be also seen as a generalization of the string model. Much later it became clear that this generalization is consistent with conformal invariance only if space-time is 4-dimensional and the Minkowski space factor of imbedding space is 4-dimensional. During last year it became clear that 4-D Minkowski space and 4-D complex projective space CP_2 are completely unique in the sense that they allow twistor space with Kähler structure.

It took some time to discover that also the geometrization of also gauge interactions and elementary particle quantum numbers could be possible in this framework: it took two years to find the unique internal space (CP_2) providing this geometrization involving also the realization that family replication phenomenon for fermions has a natural topological explanation in TGD framework and that the symmetries of the standard model symmetries are much more profound than pragmatic TOE builders have believed them to be. If TGD is correct, main stream particle physics chose the wrong track leading to the recent deep crisis when people decided that quarks and leptons belong to same multiplet of the gauge group implying instability of proton.

There have been also longstanding problems.

- Gravitational energy is well-defined in cosmological models but is not conserved. Hence the conservation of the inertial energy does not seem to be consistent with the Equivalence Principle. Furthermore, the imbeddings of Robertson-Walker cosmologies turned out to be vacuum extremals with respect to the inertial energy. About 25 years was needed to realize that the sign of the inertial energy can be also negative and in cosmological scales the density of inertial energy vanishes: physically acceptable universes are creatable from vacuum. Eventually this led to the notion of zero energy ontology (ZEO) which deviates dramatically from the standard ontology being however consistent with the crossing symmetry of quantum field theories. In this framework the quantum numbers are assigned with zero energy states located at the boundaries of so called causal diamonds defined as intersections of future and past directed light-cones. The notion of energy-momentum becomes length scale dependent since one has a scale hierarchy for causal diamonds. This allows to understand the non-conservation of energy as apparent.

  Equivalence Principle as it is expressed by Einstein's equations follows from Poincare invariance once it is realized that GRT space-time is obtained from the many-sheeted space-time of TGD by lumping together the space-time sheets to a regionof Minkowski space and endowing it with an effective metric given as a sum of Minkowski metric and deviations of the metrices of space-time sheets from Minkowski metric. Similar description relates classical gauge potentials identified as components of induced spinor connection to Yang-Mills gauge potentials in GRT space-time. Various topological inhomogenities below resolution scale identified as particles are described using energy momentum tensor and gauge currents.

- From the beginning it was clear that the theory predicts the presence of long ranged classical electro-weak and color gauge fields and that these fields necessarily accompany classical electromagnetic fields.

  It took about 26 years to gain the maturity to admit the obvious: these fields are classical correlates for long range color and weak interactions assignable to dark matter. The only possible conclusion is that TGD physics is a fractal consisting of an entire hierarchy of fractal copies of standard model physics. Also the understanding of electro-weak massivation and screening of weak charges has been a long standing problem, and 32 years was needed to discover that what I call weak form of electric-magnetic duality gives a satisfactory solution of the problem and provides also surprisingly powerful insights to the mathematical structure of quantum TGD.

  The latest development was the realization that the well- definedness of electromagnetic charge as quantum number for the modes of the induced spinors field requires that the CP_2 projection of the region in which they are non-vanishing carries vanishing W boson field and is 2-D. This implies in the generic case their localization to 2-D surfaces: string world sheets and possibly also partonic 2-surfaces. This localization applies to all modes except covariantly constant right handed neutrino generating supersymmetry and mplies that string model in 4-D space-time is part of TGD. Localization is possible only for Kähler-Dirac assigned with Kähler action defining the dynamics of space-time surfaces. One must however leave open the question whether W field might vanish for the space-time of GRT if related to many-sheeted space-time in the proposed manner even when they do not vanish for space-time sheets.

I started the serious attempts to construct quantum TGD after my thesis around 1982. The original optimistic hope was that path integral formalism or canonical quantization might be enough to construct the quantum theory but the first discovery made already during first year of TGD was that these formalisms might be useless due to the extreme non-linearity and enormous vacuum degeneracy of the theory. This turned out to be the case.

- It took some years to discover that the only working approach is based on the generalization of Einstein's program. Quantum physics involves the geometrization of the infinite-dimensional "world of classical worlds" (WCW) identified as 3-dimensional surfaces. Still few years had to pass before I understood that general coordinate invariance leads to a more or less unique solution of the problem and in positive energyontology implies that space-time surfaces are analogous to Bohr orbits. This in positive energy ontology in which space-like 3-surface is basic object. It is not clear whether Bohr orbitology is necessary also in ZEO in which space-time surfaces connect space-like 3-surfaces at the light-like boundaries of causal diamond CD obtained as intersection of future and past directed light-cones (with CP_2 factor included). The reason is that the pair of 3-surfaces replaces the boundary conditions at single 3-surface involving also time derivatives. If one assumes Bohr orbitology then strong correlations between the 3-surfaces at the ends of CD follow. Still a couple of years and I discovered that quantum states of the Universe can be identified as classical spinor fields in WCW. Only quantum jump remains the genuinely quantal aspect of quantum physics.

- During these years TGD led to a rather profound generalization of the space-time concept. Quite general properties of the theory led to the notion of many-sheeted space-time with sheets representing physical subsystems of various sizes. At the beginning of 90s I became dimly aware of the importance of p-adic number fields and soon ended up with the idea that p-adic thermodynamics for a conformally invariant system allows to understand elementary particle massivation with amazingly few input assumptions. The attempts to understand p-adicity from basic principles led gradually to the vision about physics as a generalized number theory as an approach complementary to the physics as an infinite-dimensional spinor geometry of WCW approach. One of its elements was a generalization of the number concept obtained by fusing real numbers and various p-adic numbers along common rationals. The number theoretical trinity involves besides p-adic number fields also quaternions and octonions and the notion of infinite prime.

- TGD inspired theory of consciousness entered the scheme after 1995 as I started to write a book about consciousness. Gradually it became difficult to say where physics ends and consciousness theory begins since consciousness theory could be seen as a generalization of quantum measurement theory by identifying quantum jump as a moment of consciousness and by replacing the observer with the notion of self identified as a system which is conscious as long as it can avoid entanglement with environment. The somewhat cryptic statement "Everything is conscious and consciousness can be only lost" summarizes the basic philosophy neatly.

The idea about p-adic physics as physics of cognition and intentionality emerged also rather naturally and implies perhaps the most dramatic generalization of the space-time concept in which most points of p-adic space-time sheets are infinite in real sense and the projection to the real imbedding space consists of discrete set of points. One of the most fascinating outcomes was the observation that the entropy based on p-adic norm can be negative. This observation led to the vision that life can be regarded as something in the intersection of real and p-adic worlds. Negentropic entanglement has interpretation as a correlate for various positively colored aspects of conscious experience and means also the possibility of strongly correlated states stable under state function reduction and different from the conventional bound states and perhaps playing key role in the energy metabolism of living matter.

If one requires consistency of Negentropy Mazimization Pronciple with standard measurement theory, negentropic entanglement defined in terms of number theoretic negentropy is necessarily associated with a density matrix proportional to unit matrix and is maximal and is characterized by the dimension n of the unit matrix. Negentropy is positive and maximal for a p-adic unique prime dividing n.

- One of the latest threads in the evolution of ideas is not more than nine years old. Learning about the paper of Laurent Nottale about the possibility to identify planetary orbits as Bohr orbits with a gigantic value of gravitational Planck constant made once again possible to see the obvious. Dynamical quantized Planck constant is strongly suggested by quantum classical correspondence and the fact that space-time sheets identifiable as quantum coherence regions can have arbitrarily large sizes. Second motivation for the hierarchy of Planck constants comes from bio-electromagnetism suggesting that in living systems Planck constant could have large values making macroscopic quantum coherence possible. The interpretation of dark matter as a hierarchy of phases of ordinary matter characterized by the value of Planck constant is very natural.

  During summer 2010 several new insights about the mathematical structure and interpretation of TGD emerged. One of these insights was the realization that the postulated hierarchy of Planck constants might follow from the basic structure of quantum TGD. The point is that due to the extreme non-linearity of the classical action principle the correspondence between canonical momentum densities and time derivatives of the imbedding space coordinates is one-to-many and the natural description of the situation is in terms of local singular covering spaces of the imbedding space. One could speak about effective value of Planck constant $h_{eff} = n \times h$ coming as a multiple of minimal value of Planck constant. Quite recently it became clear that the non-determinism of Kähler action is indeed the fundamental justification for the hierarchy: the integer n can be also interpreted as the integer characterizing the dimension of unit matrix characterizing negentropic entanglement made possible by the many-sheeted character of the space-time surface.

  Due to conformal invariance acting as gauge symmetry the n degenerate space-time sheets must be replaced with conformal equivalence classes of space-time sheets and conformal transformations correspond to quantum critical deformations leaving the ends of space-time surfaces invariant. Conformal invariance would be broken: only the sub-algebra for which conformal weights are divisible by n act as gauge symmetries. Thus deep connections between conformal invariance related to quantum criticality, hierarchy of Planck constants, negentropic entanglement, effective p-adic topology, and non-determinism of Kähler action perhaps reflecting p-adic non-determinism emerges.

  The implications of the hierarchy of Planck constants are extremely far reaching so that the significance of the reduction of this hierarchy to the basic mathematical structure distinguishing between TGD and competing theories cannot be under-estimated.

From the point of view of particle physics the ultimate goal is of course a practical construction recipe for the S-matrix of the theory. I have myself regarded this dream as quite too ambitious taking into account how far reaching re-structuring and generalization of the basic mathematical structure of quantum physics is required. It has indeed turned out that the dream about explicit formula is unrealistic before one has understood what happens in quantum jump. Symmetries and general physical principles have turned out to be the proper guide line here. To give some impressions about what is required some highlights are in order.

- With the emergence of ZEO the notion of S-matrix was replaced with M-matrix defined between positive and negative energy parts of zero energy states. M-matrix can be interpreted as a complex square root of density matrix representable as a diagonal and positive square root of density matrix and unitary S-matrix so that quantum theory in ZEO can be said to define a square root of thermodynamics at least formally. M-matrices in turn bombine to form the rows of unitary U-matrix defined between zero energy states.

- A decisive step was the strengthening of the General Coordinate Invariance to the requirement that the formulations of the theory in terms of light-like 3-surfaces identified as 3-surfaces at which the induced metric of space-time surfaces changes its signature and in terms of space-like 3-surfaces are equivalent. This means effective 2-dimensionality in the sense that partonic 2-surfaces defined as intersections of these two kinds of surfaces plus 4-D tangent space data at partonic 2-surfaces code for the physics. Quantum classical correspondence requires the coding of the quantum numbers characterizing quantum states assigned to the partonic 2-surfaces to the geometry of space-time surface. This is achieved by adding to the modified Dirac action a measurement interaction term assigned with light-like 3-surfaces.

- The replacement of strings with light-like 3-surfaces equivalent to space-like 3-surfaces means enormous generalization of the super conformal symmetries of string models. A further generalization of these symmetries to non-local Yangian symmetries generalizing the recently discovered Yangian symmetry of $\mathcal{N}=4$ supersymmetric Yang-Mills theories is highly suggestive. Here the replacement of point like particles with partonic 2-surfaces means the replacement of conformal symmetry of Minkowski space with infinite-dimensional super-conformal algebras. Yangian symmetry provides also a further refinement to the notion of conserved quantum numbers allowing to define them for bound states using non-local energy conserved currents.

- A further attractive idea is that quantum TGD reduces to almost topological quantum field theory. This is possible if the Kähler action for the preferred extremals defining WCW Kähler function reduces to a 3-D boundary term. This takes place if the conserved currents are so called Beltrami fields with the defining property that the coordinates associated with flow lines extend to single global coordinate variable. This ansatz together with the weak form of electric-magnetic duality reduces the Kähler action to Chern-Simons term with the condition that the 3-surfaces are extremals of Chern-Simons action subject to the constraint force defined by the weak form of electric magnetic duality. It is the latter constraint which prevents the trivialization of the theory to a topological quantum field theory. Also the identification of the Kähler function of WCW as Dirac determinant finds support as well as the description of the scattering amplitudes in terms of braids with interpretation in terms of finite measurement resolution coded to the basic structure of the solutions of field equations.

- In standard QFT Feynman diagrams provide the description of scattering amplitudes. The beauty of Feynman diagrams is that they realize unitarity automatically via the so called Cutkosky rules. In contrast to Feynman's original beliefs, Feynman diagrams and virtual particles are taken only as a convenient mathematical tool in quantum field theories. QFT approach is however plagued by UV and IR divergences and one must keep mind open for the possibility that a genuine progress might mean opening of the black box of the virtual particle.

  In TGD framework this generalization of Feynman diagrams indeed emerges unavoidably. Light-like 3-surfaces replace the lines of Feynman diagrams and vertices are replaced by 2-D partonic 2-surfaces. Zero energy ontology and the interpretation of parton orbits as light-like "wormhole throats" suggests that virtual particle do not differ from on mass shell particles only in that the four- and three- momenta of wormhole throats fail to be parallel. The two throats of the wormhole contact defining virtual particle would contact carry on mass shell quantum numbers but for virtual particles the four-momenta need not be parallel and can also have opposite signs of energy.

The localization of the nodes of induced spinor fields to 2-D string world sheets (and possibly also to partonic 2-surfaces) implies a stringy formulation of the theory analogous to stringy variant of twistor formalism with string world sheets having interpretation as 2-braids. In TGD framework fermionic variant of twistor Grassmann formalism leads to a stringy variant of twistor diagrammatics in which basic fermions can be said to be on mass-shell but carry non-physical helicities in the internal lines. This suggests the generalization of the Yangian symmetry to infinite-dimensional super-conformal algebras.

What I have said above is strongly biased view about the recent situation in quantum TGD. This vision is single man's view and doomed to contain unrealistic elements as I know from experience. My dream is that young critical readers could take this vision seriously enough to try to demonstrate that some of its basic premises are wrong or to develop an alternative based on these or better premises. I must be however honest and tell that 32 years of TGD is a really vast bundle of thoughts and quite a challenge for anyone who is not able to cheat himself by taking the attitude of a blind believer or a light-hearted debunker trusting on the power of easy rhetoric tricks.

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During last decade Tapio Tammi has helped me quite concretely by providing the necessary computer facilities and being one of the few persons in Finland with whom to discuss about my work. I have had also stimulating discussions with Samuli Penttinen who has also helped to get through the economical situations in which there seemed to be no hope. The continual updating of fifteen online books means quite a heavy bureaucracy at the level of bits and without a systemization one ends up with endless copying and pasting and internal consistency is soon lost. Pekka Rapinoja has offered his help in this respect and I am especially grateful for him for my Python skills. Also Matti Vallinkoski has helped me in computer related problems.

The collaboration with Lian Sidorov was extremely fruitful and she also helped me to survive economically through the hardest years. The participation to CASYS conferences in Liege has been an important window to the academic world and I am grateful for Daniel Dubois and Peter Marcer for making this participation possible. The discussions and collaboration with Eduardo de Luna and Istvan Dienes stimulated the hope that the communication of new vision might not be a mission impossible after all. Also blog discussions have been very useful. During these years I have received innumerable email contacts from people around the world. In particualr, I am grateful for Mark McWilliams and Ulla Matfolk for providing links to possibly interesting web sites and articles. These contacts have helped me to avoid the depressive feeling of being some kind of Don Quixote of Science and helped me to widen my views: I am grateful for all these people.

In the situation in which the conventional scientific communication channels are strictly closed it is important to have some loop hole through which the information about the work done can at least in principle leak to the publicity through the iron wall of the academic censorship. Without any exaggeration I can say that without the world wide web I would not have survived as a scientist nor as individual. Homepage and blog are however not enough since only the formally published result is a result in recent day science. Publishing is however impossible without a direct support from power holders- even in archives like arXiv.org.

Situation changed for five years ago as Andrew Adamatsky proposed the writing of a book about TGD when I had already got used to the thought that my work would not be published during my life time. The Prespacetime Journal and two other journals related to quantum biology and consciousness - all of them founded by Huping Hu - have provided this kind of loop holes. In particular, Dainis Zeps, Phil Gibbs, and Arkadiusz Jadczyk deserve my gratitude for their kind help in the preparation of an article series about TGD catalyzing a considerable progress in the understanding of quantum TGD. Also the viXra archive founded by Phil Gibbs and its predecessor Archive Freedom have been of great help: Victor Christianto deserves special thanks for doing the hard work needed to run Archive Freedom. Also the Neuroquantology Journal founded by Sultan Tarlaci deserves a special mention for its publication policy. And last but not least: there are people who experience as a fascinating intellectual challenge to spoil the practical working conditions of a person working with something which might be called unified theory: I am grateful for the people who have helped me to survive through the virus attacks, an activity which has taken roughly one month per year during the last half decade and given a strong hue of grey to my hair.

For a person approaching his sixty year birthday it is somewhat easier to overcome the hard

feelings due to the loss of academic human rights than for an inpatient youngster. Unfortunately the economic situation has become increasingly difficult during the twenty years after the economic depression in Finland which in practice meant that Finland ceased to be a constitutional state in the strong sense of the word. It became possible to depose people like me from the society without fear about public reactions and the classification as dropout became a convenient tool of ridicule to circumvent the ethical issues. During last few years when the right wing has held the political power this trend has been steadily strengthening. In this kind of situation the concrete help from individuals has been and will be of utmost importance. Against this background it becomes obvious that this kind of work is not possible without the support from outside and I apologize for not being able to mention all the people who have helped me during these years.

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Chapter 1

Introduction

1.1 Basic Ideas Of Topological Geometrodynamics (TGD)

Standard model describes rather successfully both electroweak and strong interactions but sees them as totally separate and contains a large number of parameters which it is not able to predict. For about four decades ago unified theories known as Grand Unified Theories (GUTs) trying to understand electroweak interactions and strong interactions as aspects of the same fundamental gauge interaction assignable to a larger symmetry group emerged. Later superstring models trying to unify even gravitation and strong and weak interactions emerged. The shortcomings of both GUTs and superstring models are now well-known. If TGD - whose basic idea emerged 37 years ago - would emerge now it would be seen as an attempt trying to solve the difficulties of these approaches to unification.

The basic physical picture behind TGD corresponds to a fusion of two rather disparate approaches: namely TGD as a Poincare invariant theory of gravitation and TGD as a generalization of the old-fashioned string model.

1.1.1 Basic Vision Very Briefly

T(opological) G(eometro)D(ynamics) is one of the many attempts to find a unified description of basic interactions. The development of the basic ideas of TGD to a relatively stable form took time of about half decade [K1].

The basic vision and its relationship to existing theories is now rather well understood.

1. Space-times are representable as 4-surfaces in the 8-dimensional imbedding space $H = M^4 \times CP_2$, where M^4 is 4-dimensional (4-D) Minkowski space and CP_2 is 4-D complex projective space (see Appendix).

2. Induction procedure (a standard procedure in fiber bundle theory, see Appendix) allows to geometrize various fields. Space-time metric characterizing gravitational fields corresponds to the induced metric obtained by projecting the metric tensor of H to the space-time surface. Electroweak gauge potentials are identified as projections of the components of CP_2 spinor connection to the space-time surface, and color gauge potentials as projections ofCP_2 Killing vector fields representing color symmetries. Also spinor structure can be induced: induced spinor gamma matrices are projections of gamma matrices of H and induced spinor fields just H spinor fields restricted to space-time surface. Spinor connection is also projected. The interpretation is that distances are measured in imbedding space metric and parallel translation using spinor connection of imbedding space.

   The induction procedure applies to octonionic structure and the conjecture is that for preferred extremals the induced octonionic structure is quaternionic: again one just projects the octonion units. I have proposed that one can lift space-time surfaces in H to the Cartesian product of the twistor spaces of M^4 and CP_2, which are the only 4-manifolds allowing twistor space with Kähler structure. Now the twistor structure would be induced in some sense, and should co-incide with that associated with the induced metric. Clearly, the 2-spheres defining the fibers of twistor spaces of M^4 and CP_2 must allow identification: this 2-sphere defines the S^2 fiber of the twistor space of space-time surface. This poses constraint on the imbedding of the twistor space of space-time surfaces as sub-manifold in the Cartesian product of twistor spaces.

3. Geometrization of quantum numbers is achieved. The isometry group of the geometry of CP_2 codes for the color gauge symmetries of strong interactions. Vierbein group codes for electroweak symmetries, and explains their breaking in terms of CP_2 geometry so that standard model gauge group results. There are also important deviations from standard model: color quantum numbers are not spin-like but analogous to orbital angular momentum: this difference is expected to be seen only in CP_2 scale. In contrast to GUTs, quark and lepton numbers are separately conserved and family replication has a topological explanation in terms of topology of the partonic 2-surface carrying fermionic quantum numbers.

   M^4 and CP_2 are unique choices for many other reasons. For instance, they are the unique 4-D space-times allowing twistor space with Kähler structure. M^4 light-cone boundary allows a huge extension of 2-D conformal symmetries. Imbedding space H has a number theoretic interpretation as 8-D space allowing octonionic tangent space structure. M^4 and CP_2 allow quaternionic structures. Therefore standard model symmetries have number theoretic meaning.

4. Induced gauge potentials are expressible in terms of imbedding space coordinates and their gradients and general coordinate invariance implies that there are only 4 field like variables locally. Situation is thus extremely simple mathematically. The objection is that one loses linear superposition of fields. The resolution of the problem comes from the generalization of the concepts of particle and space-time.

   Space-time surfaces can be also particle like having thus finite size. In particular, space-time regions with Euclidian signature of the induced metric (temporal and spatial dimensions in the same role) emerge and have interpretation as lines of generalized Feynman diagrams. Particle in space-time can be identified as a topological inhomogenuity in background space-time surface which looks like the space-time of general relativity in long length scales.

   One ends up with a generalization of space-time surface to many-sheeted space-time with space-time sheets having extremely small distance of about 10^4 Planck lengths (CP_2 size). As one adds a particle to this kind of structure, it touches various space-time sheets and thus interacts with the associated classical fields. Their effects superpose linearly in good approximation and linear superposition of fields is replaced with that for their effects.

   This resolves the basic objection. It also leads to the understanding of how the space-time of general relativity and quantum field theories emerges from TGD space-time as effective space-time when the sheets of many-sheeted space-time are lumped together to form a region of Minkowski space with metric replaced with a metric identified as the sum of empty Minkowski metric and deviations of the metrics of sheets from empty Minkowski metric. Gauge potentials are identified as sums of the induced gauge potentials. TGD is therefore a microscopic theory from which standard model and general relativity follow as a topological simplification however forcing to increase dramatically the number of fundamental field variables.

5. A further objection is that classical weak fields identified as induced gauge fields are long ranged and should cause large parity breaking effects due to weak interactions. These effects are indeed observed but only in living matter. A possible resolution of problem is implied by the condition that the modes of the induced spinor fields have well-defined electromagnetic charge. This forces their localization to 2-D string world sheets in the generic case having vanishing weak gauge fields so that parity breaking effects emerge just as they do in standard model. Also string model like picture emerges from TGD and one ends up with a rather concrete view about generalized Feynman diagrammatics. A possible objection is that the Kähler-Dirac gamma matrices do not define an integrable distribution of 2-planes defining string world sheet.

An even strong condition would be that the induced classical gauge fields at string world sheet vanish: this condition is allowed by the topological description of particles. The CP_2 projection of string world sheet would be 1-dimensional. Also the number theoretical condition that octonionic and ordinary spinor structures are equivalent guaranteeing that fermionic dynamics is associative leads to the vanishing of induced gauge fields.

The natural action would be given by string world sheet area, which is present only in the space-time regions with Minkowskian signature. Gravitational constant would be present as a fundamental constant in string action and the ratio $\hbar/G/R^2$ would be determined by quantum criticality condition. The hierarchy of Planck constants $h_{eff}/h = n$ assigned to dark matter in TGD framework would allow to circumvent the objection that only objects of length of order Planck length are possible since string tension given by $T = 1/\hbar_{eff}G$ apart from numerical factor could be arbitrary small. This would make possible gravitational bound states as partonic 2-surfaces as structures connected by strings and solve the basic problem of super string theories. This option allows the natural interpretation of M^4 type vacuum extremals with CP_2 projection, which is Lagrange manifold as good approximations for space-time sheets at macroscopic length scales. String area does not contribute to the Kähler function at all.

Whether also induced spinor fields associated with Kähler-Dirac action and de-localized inside entire space-time surface should be allowed remains an open question: super-conformal symmetry strongly suggests their presence. A possible interpretation for the corresponding spinor modes could be in terms of dark matter, sparticles, and hierarchy of Planck constants.

It is perhaps useful to make clear what TGD is not and also what new TGD can give to physics.

1. TGD is *not* just General Relativity made concrete by using imbeddings: the 4-surface property is absolutely essential for unifying standard model physics with gravitation and to circumvent the incurable conceptual problems of General Relativity. The many-sheeted space-time of TGD gives rise only at macroscopic limit to GRT space-time as a slightly curved Minkowski space. TGD is *not* a Kaluza-Klein theory although color gauge potentials are analogous to gauge potentials in these theories.

   TGD space-time is 4-D and its dimension is due to completely unique conformal properties of light-cone boundary and 3-D light-like surfaces implying enormous extension of the ordinary conformal symmetries. Light-like 3-surfaces represent orbits of partonic 2-surfaces and carry fundamental fermions at 1-D boundaries of string world sheets. TGD is *not* obtained by performing Poincare gauging of space-time to introduce gravitation and plagued by profound conceptual problems.

2. TGD is *not* a particular string model although string world sheets emerge in TGD very naturally as loci for spinor modes: their 2-dimensionality makes among other things possible quantum deformation of quantization known to be physically realized in condensed matter, and conjectured in TGD framework to be crucial for understanding the notion of finite measurement resolution. Hierarchy of objects of dimension up to 4 emerge from TGD: this obviously means analogy with branes of super-string models.

   TGD is *not* one more item in the collection of string models of quantum gravitation relying on Planck length mystics. Dark matter becomes an essential element of quantum gravitation and quantum coherence in astrophysical scales is predicted just from the assumption that strings connecting partonic 2-surfaces serve are responsible for gravitational bound states.

   TGD is *not* a particular string model although AdS/CFT duality of super-string models generalizes due to the huge extension of conformal symmetries and by the identification of WCW gamma matrices as Noether super-charges of super-symplectic algebra having a natural conformal structure.

3. TGD is *not* a gauge theory. In TGD framework the counterparts of also ordinary gauge symmetries are assigned to super-symplectic algebra (and its Yangian [A27] [B39, B30, B31]), which is a generalization of Kac-Moody algebras rather than gauge algebra and suffers a

fractal hierarchy of symmetry breakings defining hierarchy of criticalities. TGD is *not* one more quantum field theory like structure based on path integral formalism: path integral is replaced with functional integral over 3-surfaces, and the notion of classical space-time becomes exact part of the theory. Quantum theory becomes formally a purely classical theory of WCW spinor fields: only state function reduction is something genuinely quantal.

4. TGD view about spinor fields is *not* the standard one. Spinor fields appear at three levels. Spinor modes of the imbedding space are analogs of spinor modes charactering incoming and outgoing states in quantum field theories. Induced second quantized spinor fields at space-time level are analogs of stringy spinor fields. Their modes are localized by the well-definedness of electro-magnetic charge and by number theoretic arguments at string world sheets. Kähler-Dirac action is fixed by supersymmetry implying that ordinary gamma matrices are replaced by what I call Kähler-Dirac gamma matrices - this something new. WCW spinor fields, which are classical in the sense that they are not second quantized, serve as analogs of fields of string field theory and imply a geometrization of quantum theory.

5. TGD is in some sense an extremely conservative geometrization of entire quantum physics: *no* additional structures such as gauge fields as independent dynamical degrees of freedom are introduced: Kähler geometry and associated spinor structure are enough. "Topological" in TGD should not be understood as an attempt to reduce physics to torsion (see for instance [B29]) or something similar. Rather, TGD space-time is topologically non-trivial in all scales and even the visible structures of everyday world represent non-trivial topology of space-time in TGD Universe.

6. Twistor space - or rather, a generalization of twistor approach replacing masslessness in 4-D sense with masslessness in 8-D sense and thus allowing description of also massive particles - emerges as a technical tool, and its Kähler structure is possible only for $H = M^4 \times CP_2$. What is genuinely new is the infinite-dimensional character of the Kähler geometry making it highly unique, and its generalization to p-adic number fields to describe correlates of cognition. Also the hierarchies of Planck constants $h_{eff} = n \times h$ reducing to the quantum criticality of TGD Universe and p-adic length scales and Zero Energy Ontology represent something genuinely new.

The great challenge is to construct a mathematical theory around these physically very attractive ideas and I have devoted the last thirty seven years for the realization of this dream and this has resulted in eight online books about TGD and nine online books about TGD inspired theory of consciousness and of quantum biology.

1.1.2 Two Vision About TGD And Their Fusion

As already mentioned, TGD can be interpreted both as a modification of general relativity and generalization of string models.

TGD as a Poincare invariant theory of gravitation

The first approach was born as an attempt to construct a Poincare invariant theory of gravitation. Space-time, rather than being an abstract manifold endowed with a pseudo-Riemannian structure, is regarded as a surface in the 8-dimensional space $H = M^4_{\times} CP_2$, where M^4 denotes Minkowski space and $CP_2 = SU(3)/U(2)$ is the complex projective space of two complex dimensions [A53, A62, A43, A58].

The identification of the space-time as a sub-manifold [A54, A75] of $M^4 \times CP_2$ leads to an exact Poincare invariance and solves the conceptual difficulties related to the definition of the energy-momentum in General Relativity.

It soon however turned out that sub-manifold geometry, being considerably richer in structure than the abstract manifold geometry, leads to a geometrization of all basic interactions. First, the geometrization of the elementary particle quantum numbers is achieved. The geometry of CP_2 explains electro-weak and color quantum numbers. The different H-chiralities of H-spinors correspond to the conserved baryon and lepton numbers. Secondly, the geometrization of the field concept results. The projections of the CP_2 spinor connection, Killing vector fields of CP_2 and of H-metric to four-surface define classical electro-weak, color gauge fields and metric in X^4.

The choice of H is unique from the condition that TGD has standard model symmetries. Also number theoretical vision selects $H = M^4 \times CP_2$ uniquely. M^4 and CP_2 are also unique spaces allowing twistor space with Kähler structure.

TGD as a generalization of the hadronic string model

The second approach was based on the generalization of the mesonic string model describing mesons as strings with quarks attached to the ends of the string. In the 3-dimensional generalization 3-surfaces correspond to free particles and the boundaries of the 3- surface correspond to partons in the sense that the quantum numbers of the elementary particles reside on the boundaries. Various boundary topologies (number of handles) correspond to various fermion families so that one obtains an explanation for the known elementary particle quantum numbers. This approach leads also to a natural topological description of the particle reactions as topology changes: for instance, two-particle decay corresponds to a decay of a 3-surface to two disjoint 3-surfaces.

This decay vertex does not however correspond to a direct generalization of trouser vertex of string models. Indeed, the important difference between TGD and string models is that the analogs of string world sheet diagrams do not describe particle decays but the propagation of particles via different routes. Particle reactions are described by generalized Feynman diagrams for which 3-D light-like surface describing particle propagating join along their ends at vertices. As 4-manifolds the space-time surfaces are therefore singular like Feynman diagrams as 1-manifolds.

Quite recently, it has turned out that fermionic strings inside space-time surfaces define an exact part of quantum TGD and that this is essential for understanding gravitation in long length scales. Also the analog of AdS/CFT duality emerges in that the Kähler metric can be defined either in terms of Kähler function identifiable as Kähler action assignable to Euclidian space-time regions or Kähler action + string action assignable to Minkowskian regions.

The recent view about construction of scattering amplitudes is very "stringy". By strong form of holography string world sheets and partonic 2-surfaces provide the data needed to construct scattering amplitudes. Space-time surfaces are however needed to realize quantum-classical correspondence necessary to understand the classical correlates of quantum measurement. There is a huge generalization of the duality symmetry of hadronic string models. Scattering amplitudes can be regarded as sequences of computational operations for the Yangian of super-symplectic algebra. Product and co-product define the basic vertices and realized geometrically as partonic 2-surfaces and algebraically as multiplication for the elements of Yangian identified as super-symplectic Noether charges assignable to strings. Any computational sequences connecting given collections of algebraic objects at the opposite boundaries of causal diamond (CD) produce identical scattering amplitudes.

Fusion of the two approaches via a generalization of the space-time concept

The problem is that the two approaches to TGD seem to be mutually exclusive since the orbit of a particle like 3-surface defines 4-dimensional surface, which differs drastically from the topologically trivial macroscopic space-time of General Relativity. The unification of these approaches forces a considerable generalization of the conventional space-time concept. First, the topologically trivial 3-space of General Relativity is replaced with a "topological condensate" containing matter as particle like 3-surfaces "glued" to the topologically trivial background 3-space by connected sum operation. Secondly, the assumption about connectedness of the 3-space is given up. Besides the "topological condensate" there could be "vapor phase" that is a "gas" of particle like 3-surfaces and string like objects (counterpart of the "baby universes" of GRT) and the non-conservation of energy in GRT corresponds to the transfer of energy between different sheets of the space-time and possibly existence vapour phase.

What one obtains is what I have christened as many-sheeted space-time (see **Fig.** `http://tgdtheory.fi/appfigures/manysheeted.jpg` or **Fig. ??** in the appendix of this book). One particular aspect is topological field quantization meaning that various classical fields assignable to a physical system correspond to space-time sheets representing the classical fields to that particular system. One can speak of the field body of a particular physical system. Field body consists of topological light rays, and electric and magnetic flux quanta. In Maxwell's theory system does not possess this kind of field identity. The notion of magnetic body is one of the key players in TGD inspired theory of consciousness and quantum biology.

This picture became more detailed with the advent of zero energy ontology (ZEO). The basic notion of ZEO is causal diamond (CD) identified as the Cartesian product of CP_2 and of the intersection of future and past directed light-cones and having scale coming as an integer multiple of CP_2 size is fundamental. CDs form a fractal hierarchy and zero energy states decompose to products of positive and negative energy parts assignable to the opposite boundaries of CD defining the ends of the space-time surface. The counterpart of zero energy state in positive energy ontology is the pair of initial and final states of a physical event, say particle reaction.

At space-time level ZEO means that 3-surfaces are pairs of space-like 3-surfaces at the opposite light-like boundaries of CD. Since the extremals of Kähler action connect these, one can say that by holography the basic dynamical objects are the space-time surface connecting these 3-surfaces. This changes totally the vision about notions like self-organization: self-organization by quantum jumps does not take for a 3-D system but for the entire 4-D field pattern associated with it.

General Coordinate Invariance (GCI) allows to identify the basic dynamical objects as space-like 3-surfaces at the ends of space-time surface at boundaries of CD: this means that space-time surface is analogous to Bohr orbit. An alternative identification is as light-like 3-surfaces at which the signature of the induced metric changes from Minkowskian to Euclidian and interpreted as lines of generalized Feynman diagrams. Also the Euclidian 4-D regions would have similar interpretation. The requirement that the two interpretations are equivalent, leads to a strong form of General Coordinate Invariance. The outcome is effective 2-dimensionality stating that the partonic 2-surfaces identified as intersections of the space-like ends of space-time surface and light-like wormhole throats are the fundamental objects. That only effective 2-dimensionality is in question is due to the effects caused by the failure of strict determinism of Kähler action. In finite length scale resolution these effects can be neglected below UV cutoff and above IR cutoff. One can also speak about strong form of holography.

1.1.3 Basic Objections

Objections are the most powerful tool in theory building. The strongest objection against TGD is the observation that all classical gauge fields are expressible in terms of four imbedding space coordinates only- essentially CP_2 coordinates. The linear superposition of classical gauge fields taking place independently for all gauge fields is lost. This would be a catastrophe without many-sheeted space-time. Instead of gauge fields, only the effects such as gauge forces are superposed. Particle topologically condenses to several space-time sheets simultaneously and experiences the sum of gauge forces. This transforms the weakness to extreme economy: in a typical unified theory the number of primary field variables is countered in hundreds if not thousands, now it is just four.

Second objection is that TGD space-time is quite too simple as compared to GRT space-time due to the imbeddability to 8-D imbedding space. One can also argue that Poincare invariant theory of gravitation cannot be consistent with General Relativity. The above interpretation allows to understand the relationship to GRT space-time and how Equivalence Principle (EP) follows from Poincare invariance of TGD. The interpretation of GRT space-time is as effective space-time obtained by replacing many-sheeted space-time with Minkowski space with effective metric determined as a sum of Minkowski metric and sum over the deviations of the induced metrices of space-time sheets from Minkowski metric. Poincare invariance suggests strongly classical EP for the GRT limit in long length scales at least. One can consider also other kinds of limits such as the analog of GRT limit for Euclidian space-time regions assignable to elementary particles. In this case deformations of CP_2 metric define a natural starting point and CP_2 indeed defines a gravitational instanton with very large cosmological constant in Einstein-Maxwell theory. Also gauge potentials of standard model correspond classically to superpositions of induced gauge potentials over space-time sheets.

Topological field quantization

Topological field quantization distinguishes between TGD based and more standard - say Maxwellian - notion of field. In Maxwell's fields created by separate systems superpose and one cannot tell which part of field comes from which system except theoretically. In TGD these fields correspond to different space-time sheets and only their effects on test particle superpose. Hence physical systems have well-defined field identifies - field bodies - in particular magnetic bodies.

The notion of magnetic body carrying dark matter with non-standard large value of Planck constant has become central concept in TGD inspired theory of consciousness and living matter, and by starting from various anomalies of biology one ends up to a rather detailed view about the role of magnetic body as intentional agent receiving sensory input from the biological body and controlling it using EEG and its various scaled up variants as a communication tool. Among other thins this leads to models for cell membrane, nerve pulse, and EEG.

1.1.4 P-Adic Variants Of Space-Time Surfaces

There is a further generalization of the space-time concept inspired by p-adic physics forcing a generalization of the number concept through the fusion of real numbers and various p-adic number fields. One might say that TGD space-time is adelic. Also the hierarchy of Planck constants forces a generalization of the notion of space-time but this generalization can be understood in terms of the failure of strict determinism for Kähler action defining the fundamental variational principle behind the dynamics of space-time surfaces.

A very concise manner to express how TGD differs from Special and General Relativities could be following. Relativity Principle (Poincare Invariance), General Coordinate Invariance, and Equivalence Principle remain true. What is new is the notion of sub-manifold geometry: this allows to realize Poincare Invariance and geometrize gravitation simultaneously. This notion also allows a geometrization of known fundamental interactions and is an essential element of all applications of TGD ranging from Planck length to cosmological scales. Sub-manifold geometry is also crucial in the applications of TGD to biology and consciousness theory.

1.1.5 The Threads In The Development Of Quantum TGD

The development of TGD has involved several strongly interacting threads: physics as infinite-dimensional geometry; TGD as a generalized number theory, the hierarchy of Planck constants interpreted in terms of dark matter hierarchy, and TGD inspired theory of consciousness. In the following these threads are briefly described.

The theoretical framework involves several threads.

1. Quantum T(opological) G(eometro)D(ynamics) as a classical spinor geometry for infinite-dimensional WCW, p-adic numbers and quantum TGD, and TGD inspired theory of consciousness and of quantum biology have been for last decade of the second millenium the basic three strongly interacting threads in the tapestry of quantum TGD.

2. The discussions with Tony Smith initiated a fourth thread which deserves the name "TGD as a generalized number theory". The basic observation was that classical number fields might allow a deeper formulation of quantum TGD. The work with Riemann hypothesis made time ripe for realization that the notion of infinite primes could provide, not only a reformulation, but a deep generalization of quantum TGD. This led to a thorough and extremely fruitful revision of the basic views about what the final form and physical content of quantum TGD might be. Together with the vision about the fusion of p-adic and real physics to a larger coherent structure these sub-threads fused to the "physics as generalized number theory" thread.

3. A further thread emerged from the realization that by quantum classical correspondence TGD predicts an infinite hierarchy of macroscopic quantum systems with increasing sizes, that it is not at all clear whether standard quantum mechanics can accommodate this hierarchy, and that a dynamical quantized Planck constant might be necessary and strongly suggested by the failure of strict determinism for the fundamental variational principle. The identification

of hierarchy of Planck constants labelling phases of dark matter would be natural. This also led to a solution of a long standing puzzle: what is the proper interpretation of the predicted fractal hierarchy of long ranged classical electro-weak and color gauge fields. Quantum classical correspondences allows only single answer: there is infinite hierarchy of p-adically scaled up variants of standard model physics and for each of them also dark hierarchy. Thus TGD Universe would be fractal in very abstract and deep sense.

The chronology based identification of the threads is quite natural but not logical and it is much more logical to see p-adic physics, the ideas related to classical number fields, and infinite primes as sub-threads of a thread which might be called "physics as a generalized number theory". In the following I adopt this view. This reduces the number of threads to four.

TGD forces the generalization of physics to a quantum theory of consciousness, and represent TGD as a generalized number theory vision leads naturally to the emergence of p-adic physics as physics of cognitive representations. The eight online books [K83, K62, K50, K101, K71, K100, K99, K69] about TGD and nine online books about TGD inspired theory of consciousness and of quantum biology [K75, K9, K56, K8, K33, K38, K40, K68, K96] are warmly recommended to the interested reader.

Quantum TGD as spinor geometry of World of Classical Worlds

A turning point in the attempts to formulate a mathematical theory was reached after seven years from the birth of TGD. The great insight was "Do not quantize". The basic ingredients to the new approach have served as the basic philosophy for the attempt to construct Quantum TGD since then and have been the following ones:

1. Quantum theory for extended particles is free(!), classical(!) field theory for a generalized Schrödinger amplitude in the configuration space CH ("world of classical worlds", WCW) consisting of all possible 3-surfaces in H. "All possible" means that surfaces with arbitrary many disjoint components and with arbitrary internal topology and also singular surfaces topologically intermediate between two different manifold topologies are included. Particle reactions are identified as topology changes [A69, A81, A93]. For instance, the decay of a 3-surface to two 3-surfaces corresponds to the decay $A \rightarrow B+C$. Classically this corresponds to a path of WCW leading from 1-particle sector to 2-particle sector. At quantum level this corresponds to the dispersion of the generalized Schrödinger amplitude localized to 1-particle sector to two-particle sector. All coupling constants should result as predictions of the theory since no nonlinearities are introduced.

2. During years this naive and very rough vision has of course developed a lot and is not anymore quite equivalent with the original insight. In particular, the space-time correlates of Feynman graphs have emerged from theory as Euclidian space-time regions and the strong form of General Coordinate Invariance has led to a rather detailed and in many respects unexpected visions. This picture forces to give up the idea about smooth space-time surfaces and replace space-time surface with a generalization of Feynman diagram in which vertices represent the failure of manifold property. I have also introduced the word "world of classical worlds" (WCW) instead of rather formal "configuration space". I hope that "WCW" does not induce despair in the reader having tendency to think about the technicalities involved!

3. WCW is endowed with metric and spinor structure so that one can define various metric related differential operators, say Dirac operator, appearing in the field equations of the theory [1]

4. WCW Dirac operator appearing in Super-Virasoro conditions, imbedding space Dirac operator whose modes define the ground states of Super-Virasoro representations, Kähler-Dirac operator at space-time surfaces, and the algebraic variant of M^4 Dirac operator appearing in

[1] There are four kinds of Dirac operators in TGD. The geometrization of quantum theory requires Kähler metric definable either in terms of Kähler function identified as Kähler action for Euclidian space-time regions or as anti-commutators for WCW gamma matrices identified as conformal Noether super-charges associated with the second quantized modified Dirac action consisting of string world sheet term and possibly also Kähler Dirac action in Minkowskian space-time regions. These two possible definitions reflect a duality analogous to AdS/CFT duality.

propagators. The most ambitious dream is that zero energy states correspond to a complete solution basis for the Dirac operator of WCW so that this classical free field theory would dictate M-matrices defined between positive and negative energy parts of zero energy states which form orthonormal rows of what I call U-matrix as a matrix defined between zero energy states. Given M-matrix in turn would decompose to a product of a hermitian square root of density matrix and unitary S-matrix.

M-matrix would define time-like entanglement coefficients between positive and negative energy parts of zero energy states (all net quantum numbers vanish for them) and can be regarded as a hermitian square root of density matrix multiplied by a unitary S-matrix. Quantum theory would be in well-defined sense a square root of thermodynamics. The orthogonality and hermiticity of the M-matrices commuting with S-matrix means that they span infinite-dimensional Lie algebra acting as symmetries of the S-matrix. Therefore quantum TGD would reduce to group theory in well-defined sense.

In fact the Lie algebra of Hermitian M-matrices extends to Kac-Moody type algebra obtained by multiplying hermitian square roots of density matrices with powers of the S-matrix. Also the analog of Yangian algebra involving only non-negative powers of S-matrix is possible and would correspond to a hierarchy of CDs with the temporal distances between tips coming as integer multiples of the CP_2 time.

The M-matrices associated with CDs are obtained by a discrete scaling from the minimal CD and characterized by integer n are naturally proportional to a representation matrix of scaling: $S(n) = S^n$, where S is unitary S-matrix associated with the minimal CD [K91]. This conforms with the idea about unitary time evolution as exponent of Hamiltonian discretized to integer power of S and represented as scaling with respect to the logarithm of the proper time distance between the tips of CD.

U-matrix elements between M-matrices for various CDs are proportional to the inner products $Tr[S^{-n_1} \circ H^i H^j \circ S^{n_2} \lambda]$, where λ represents unitarily the discrete Lorentz boost relating the moduli of the active boundary of CD and H^i form an orthonormal basis of Hermitian square roots of density matrices. $\circ$ tells that S acts at the active boundary of CD only. It turns out possible to construct a general representation for the U-matrix reducing its construction to that of S-matrix. S-matrix has interpretation as exponential of the Virasoro generator L_{-1} of the Virasoro algebra associated with super-symplectic algebra.

5. By quantum classical correspondence the construction of WCW spinor structure reduces to the second quantization of the induced spinor fields at space-time surface. The basic action is so called modified Dirac action (or Kähler-Dirac action) in which gamma matrices are replaced with the modified (Kähler-Dirac) gamma matrices defined as contractions of the canonical momentum currents with the imbedding space gamma matrices. In this manner one achieves super-conformal symmetry and conservation of fermionic currents among other things and consistent Dirac equation. The Kähler-Dirac gamma matrices define as anti-commutators effective metric, which might provide geometrization for some basic observables of condensed matter physics. One might also talk about bosonic emergence in accordance with the prediction that the gauge bosons and graviton are expressible in terms of bound states of fermion and anti-fermion.

6. An important result relates to the notion of induced spinor connection. If one requires that spinor modes have well-defined em charge, one must assume that the modes in the generic situation are localized at 2-D surfaces - string world sheets or perhaps also partonic 2-surfaces - at which classical W boson fields vanish. Covariantly constant right handed neutrino generating super-symmetries forms an exception. The vanishing of also Z^0 field is possible for Kähler-Dirac action and should hold true at least above weak length scales. This implies that string model in 4-D space-time becomes part of TGD. Without these conditions classical weak fields can vanish above weak scale only for the GRT limit of TGD for which gauge potentials are sums over those for space-time sheets.

   The localization simplifies enormously the mathematics and one can solve exactly the Kähler-Dirac equation for the modes of the induced spinor field just like in super string models.

At the light-like 3-surfaces at which the signature of the induced metric changes from Euclidian to Minkowskian so that $\sqrt{g_4}$ vanishes one can pose the condition that the algebraic analog of massless Dirac equation is satisfied by the nodes so that Kähler-Dirac action gives massless Dirac propagator localizable at the boundaries of the string world sheets.

The evolution of these basic ideas has been rather slow but has gradually led to a rather beautiful vision. One of the key problems has been the definition of Kähler function. Kähler function is Kähler action for a preferred extremal assignable to a given 3-surface but what this preferred extremal is? The obvious first guess was as absolute minimum of Kähler action but could not be proven to be right or wrong. One big step in the progress was boosted by the idea that TGD should reduce to almost topological QFT in which braids would replace 3-surfaces in finite measurement resolution, which could be inherent property of the theory itself and imply discretization at partonic 2-surfaces with discrete points carrying fermion number.

It took long time to realize that there is no discretization in 4-D sense - this would lead to difficulties with basic symmetries. Rather, the discretization occurs for the parameters characterizing co-dimension 2 objects representing the information about space-time surface so that they belong to some algebraic extension of rationals. These 2-surfaces - string world sheets and partonic 2-surfaces - are genuine physical objects rather than a computational approximation. Physics itself approximates itself, one might say! This is of course nothing but strong form of holography.

1. TGD as almost topological QFT vision suggests that Kähler action for preferred extremals reduces to Chern-Simons term assigned with space-like 3-surfaces at the ends of space-time (recall the notion of causal diamond (CD)) and with the light-like 3-surfaces at which the signature of the induced metric changes from Minkowskian to Euclidian. Minkowskian and Euclidian regions would give at wormhole throats the same contribution apart from coefficients and in Minkowskian regions the $\sqrt{g_4}$ factorc coming from metric would be imaginary so that one would obtain sum of real term identifiable as Kähler function and imaginary term identifiable as the ordinary Minkowskian action giving rise to interference effects and stationary phase approximation central in both classical and quantum field theory.

   Imaginary contribution - the presence of which I realized only after 33 years of TGD - could also have topological interpretation as a Morse function. On physical side the emergence of Euclidian space-time regions is something completely new and leads to a dramatic modification of the ideas about black hole interior.

2. The manner to achieve the reduction to Chern-Simons terms is simple. The vanishing of Coulomb contribution to Kähler action is required and is true for all known extremals if one makes a general ansatz about the form of classical conserved currents. The so called weak form of electric-magnetic duality defines a boundary condition reducing the resulting 3-D terms to Chern-Simons terms. In this manner almost topological QFT results. But only "almost" since the Lagrange multiplier term forcing electric-magnetic duality implies that Chern-Simons action for preferred extremals depends on metric.

TGD as a generalized number theory

Quantum T(opological)D(ynamics) as a classical spinor geometry for infinite-dimensional configuration space ("world of classical worlds", WCW), p-adic numbers and quantum TGD, and TGD inspired theory of consciousness, have been for last ten years the basic three strongly interacting threads in the tapestry of quantum TGD. The fourth thread deserves the name "TGD as a generalized number theory". It involves three separate threads: the fusion of real and various p-adic physics to a single coherent whole by requiring number theoretic universality discussed already, the formulation of quantum TGD in terms of hyper-counterparts of classical number fields identified as sub-spaces of complexified classical number fields with Minkowskian signature of the metric defined by the complexified inner product, and the notion of infinite prime.

1. p-Adic TGD and fusion of real and p-adic physics to single coherent whole

The p-adic thread emerged for roughly ten years ago as a dim hunch that p-adic numbers might be important for TGD. Experimentation with p-adic numbers led to the notion of canonical identification mapping reals to p-adics and vice versa. The breakthrough came with the successful p-adic mass calculations using p-adic thermodynamics for Super-Virasoro representations with the super-Kac-Moody algebra associated with a Lie-group containing standard model gauge group. Although the details of the calculations have varied from year to year, it was clear that p-adic physics reduces not only the ratio of proton and Planck mass, the great mystery number of physics, but all elementary particle mass scales, to number theory if one assumes that primes near prime powers of two are in a physically favored position. Why this is the case, became one of the key puzzles and led to a number of arguments with a common gist: evolution is present already at the elementary particle level and the primes allowed by the p-adic length scale hypothesis are the fittest ones.

It became very soon clear that p-adic topology is not something emerging in Planck length scale as often believed, but that there is an infinite hierarchy of p-adic physics characterized by p-adic length scales varying to even cosmological length scales. The idea about the connection of p-adics with cognition motivated already the first attempts to understand the role of the p-adics and inspired "Universe as Computer" vision but time was not ripe to develop this idea to anything concrete (p-adic numbers are however in a central role in TGD inspired theory of consciousness). It became however obvious that the p-adic length scale hierarchy somehow corresponds to a hierarchy of intelligences and that p-adic prime serves as a kind of intelligence quotient. Ironically, the almost obvious idea about p-adic regions as cognitive regions of space-time providing cognitive representations for real regions had to wait for almost a decade for the access into my consciousness.

In string model context one tries to reduces the physics to Planck scale. The price is the inability to say anything about physics in long length scales. In TGD p-adic physics takes care of this shortcoming by predicting the physics also in long length scales.

There were many interpretational and technical questions crying for a definite answer.

1. What is the relationship of p-adic non-determinism to the classical non-determinism of the basic field equations of TGD? Are the p-adic space-time region genuinely p-adic or does p-adic topology only serve as an effective topology? If p-adic physics is direct image of real physics, how the mapping relating them is constructed so that it respects various symmetries? Is the basic physics p-adic or real (also real TGD seems to be free of divergences) or both? If it is both, how should one glue the physics in different number field together to get *the* Physics? Should one perform p-adicization also at the level of the WCW? Certainly the p-adicization at the level of super-conformal representation is necessary for the p-adic mass calculations.

2. Perhaps the most basic and most irritating technical problem was how to precisely define p-adic definite integral which is a crucial element of any variational principle based formulation of the field equations. Here the frustration was not due to the lack of solution but due to the too large number of solutions to the problem, a clear symptom for the sad fact that clever inventions rather than real discoveries might be in question. Quite recently I however learned that the problem of making sense about p-adic integration has been for decades central problem in the frontier of mathematics and a lot of profound work has been done along same intuitive lines as I have proceeded in TGD framework. The basic idea is certainly the notion of algebraic continuation from the world of rationals belonging to the intersection of real world and various p-adic worlds.

Despite various uncertainties, the number of the applications of the poorly defined p-adic physics has grown steadily and the applications turned out to be relatively stable so that it was clear that the solution to these problems must exist. It became only gradually clear that the solution of the problems might require going down to a deeper level than that represented by reals and p-adics.

The key challenge is to fuse various p-adic physics and real physics to single larger structures. This has inspired a proposal for a generalization of the notion of number field by fusing real numbers and various p-adic number fields and their extensions along rationals and possible common algebraic numbers. This leads to a generalization of the notions of imbedding space and space-time concept and one can speak about real and p-adic space-time sheets. One can talk about adelic space-time, imbedding space, and WCW.

The notion of p-adic manifold [K104] identified as p-adic space-time surface solving p-adic analogs of field equations and having real space-time sheet as chart map provided a possible solution of the basic challenge of relating real and p-adic classical physics. One can also speak of real space-time surfaces having p-adic space-time surfaces as chart maps (cognitive maps, "thought bubbles"). Discretization required having interpretation in terms of finite measurement resolution is unavoidable in this approach and this leads to problems with symmetries: canonical identification does not commute with symmetries.

It is now clear that much more elegant approach based on abstraction exists [K111]. The map of real preferred extremals to p-adic ones is not induced from a local correspondence between points but is global. Discretization occurs only for the parameters characterizing string world sheets and partonic 2-surfaces so that they belong to some algebraic extension of rationals. Restriction to these 2-surfaces is possible by strong form of holography. Adelization providing number theoretical universality reduces to algebraic continuation for the amplitudes from this intersection of reality and various p-adicities - analogous to a back of a book - to various number fields. There are no problems with symmetries but canonical identification is needed: various group invariant of the amplitude are mapped by canonical identification to various p-adic number fields. This is nothing but a generalization of the mapping of the p-adic mass squared to its real counterpart in p-adic mass calculations.

This leads to surprisingly detailed predictions and far reaching conjectures. For instance, the number theoretic generalization of entropy concept allows negentropic entanglement central for the applications to living matter (see **Fig.** `http://tgdtheory.fi/appfigures/cat.jpg` or **Fig. ??** in the appendix of this book). One can also understand how preferred p-adic primes could emerge as so called ramified primes of algebraic extension of rationals in question and characterizing string world sheets and partonic 2-surfaces. Preferred p-adic primes would be ramified primes for extensions for which the number of p-adic continuations of two-surfaces to space-time surfaces (imaginations) allowing also real continuation (realization of imagination) would be especially large. These ramifications would be winners in the fight for number theoretical survival. Also a generalization of p-adic length scale hypothesis emerges from NMP [K41].

The characteristic non-determinism of the p-adic differential equations suggests strongly that p-adic regions correspond to "mind stuff", the regions of space-time where cognitive representations reside. This interpretation implies that p-adic physics is physics of cognition. Since Nature is probably a brilliant simulator of Nature, the natural idea is to study the p-adic physics of the cognitive representations to derive information about the real physics. This view encouraged by TGD inspired theory of consciousness clarifies difficult interpretational issues and provides a clear interpretation for the predictions of p-adic physics.

2. The role of classical number fields

The vision about the physical role of the classical number fields relies on certain speculative questions inspired by the idea that space-time dynamics could be reduced to associativity or co-associativity condition. Associativity means here associativity of tangent spaces of space-time region and co-associativity associativity of normal spaces of space-time region.

1. Could space-time surfaces X^4 be regarded as associative or co-associative ("quaternionic" is equivalent with "associative") surfaces of H endowed with octonionic structure in the sense that tangent space of space-time surface would be associative (co-associative with normal space associative) sub-space of octonions at each point of X^4 [K74]. This is certainly possible and an interesting conjecture is that the preferred extremals of Kähler action include associative and co-associative space-time regions.

2. Could the notion of compactification generalize to that of number theoretic compactification in the sense that one can map associative (co-associative) surfaces of M^8 regarded as octonionic linear space to surfaces in $M^4 \times CP_2$ [K74] ? This conjecture - $M^8 - H$ duality - would give for $M^4 \times CP_2$ deep number theoretic meaning. CP_2 would parametrize associative planes of octonion space containing fixed complex plane $M^2 \subset M^8$ and CP_2 point would thus characterize the tangent space of $X^4 \subset M^8$. The point of M^4 would be obtained by projecting the point of $X^4 \subset M^8$ to a point of M^4 identified as tangent space of X^4. This would guarantee that the dimension of space-time surface in H would be four. The conjecture is that the preferred extremals of Kähler action include these surfaces.

3. $M^8 - H$ duality can be generalized to a duality $H \rightarrow H$ if the images of the associative surface in M^8 is associative surface in H. One can start from associative surface of H and assume

that it contains the preferred M^2 tangent plane in 8-D tangent space of H or integrable distribution $M^2(x)$ of them, and its points to H by mapping M^4 projection of H point to itself and associative tangent space to CP_2 point. This point need not be the original one! If the resulting surface is also associative, one can iterate the process indefinitely. WCW would be a category with one object.

4. G_2 defines the automorphism group of octonions, and one might hope that the maps of octonions to octonions such that the action of Jacobian in the tangent space of associative or co-associative surface reduces to that of G_2 could produce new associative/co-associative surfaces. The action of G_2 would be analogous to that of gauge group.

5. One can also ask whether the notions of commutativity and co-commutativity could have physical meaning. The well-definedness of em charge as quantum number for the modes of the induced spinor field requires their localization to 2-D surfaces (right-handed neutrino is an exception) - string world sheets and partonic 2-surfaces. This can be possible only for Kähler action and could have commutativity and co-commutativity as a number theoretic counterpart. The basic vision would be that the dynamics of Kähler action realizes number theoretical geometrical notions like associativity and commutativity and their co-notions.

The notion of number theoretic compactification stating that space-time surfaces can be regarded as surfaces of either M^8 or $M^4 \times CP_2$. As surfaces of M^8 identifiable as space of hyper-octonions they are hyper-quaternionic or co-hyper-quaternionic- and thus maximally associative or co-associative. This means that their tangent space is either hyper-quaternionic plane of M^8 or an orthogonal complement of such a plane. These surface can be mapped in natural manner to surfaces in $M^4 \times CP_2$ [K74] provided one can assign to each point of tangent space a hyper-complex plane $M^2(x) \subset M^4 \subset M^8$. One can also speak about $M^8 - H$ duality.

This vision has very strong predictive power. It predicts that the preferred extremals of Kähler action correspond to either hyper-quaternionic or co-hyper-quaternionic surfaces such that one can assign to tangent space at each point of space-time surface a hyper-complex plane $M^2(x) \subset M^4$. As a consequence, the M^4 projection of space-time surface at each point contains $M^2(x)$ and its orthogonal complement. These distributions are integrable implying that space-time surface allows dual slicings defined by string world sheets Y^2 and partonic 2-surfaces X^2. The existence of this kind of slicing was earlier deduced from the study of extremals of Kähler action and christened as Hamilton-Jacobi structure. The physical interpretation of $M^2(x)$ is as the space of non-physical polarizations and the plane of local 4-momentum.

Number theoretical compactification has inspired large number of conjectures. This includes dual formulations of TGD as Minkowskian and Euclidian string model type theories, the precise identification of preferred extremals of Kähler action as extremals for which second variation vanishes (at least for deformations representing dynamical symmetries) and thus providing space-time correlate for quantum criticality, the notion of number theoretic braid implied by the basic dynamics of Kähler action and crucial for precise construction of quantum TGD as almost-topological QFT, the construction of WCW metric and spinor structure in terms of second quantized induced spinor fields with modified Dirac action defined by Kähler action realizing the notion of finite measurement resolution and a connection with inclusions of hyper-finite factors of type II_1 about which Clifford algebra of WCW represents an example.

The two most important number theoretic conjectures relate to the preferred extremals of Kähler action. The general idea is that classical dynamics for the preferred extremals of Kähler action should reduce to number theory: space-time surfaces should be either associative or co-associative in some sense.

Associativity (co-associativity) would be that tangent (normal) spaces of space-time surfaces associative (co-associative) in some sense and thus quaternionic (co-quaternionic). This can be formulated in two manners.

1. One can introduce octonionic tangent space basis by assigning to the “free” gamma matrices octonion basis or in terms of octonionic representation of the imbedding space gamma matrices possible in dimension $D = 8$.

2. Associativity (quaternionicity) would state that the projections of octonionic basic vectors or induced gamma matrices basis to the space-time surface generates associative (quaternionic)

sub-algebra at each space-time point. Co-associativity is defined in analogous manner and can be expressed in terms of the components of second fundamental form.

3. For gamma matrix option induced rather than Kähler-Dirac gamma matrices must be in question since Kähler-Dirac gamma matrices can span lower than 4-dimensional space and are not parallel to the space-time surfaces as imbedding space vectors.

3. Infinite primes

The discovery of the hierarchy of infinite primes and their correspondence with a hierarchy defined by a repeatedly second quantized arithmetic quantum field theory gave a further boost for the speculations about TGD as a generalized number theory.

After the realization that infinite primes can be mapped to polynomials possibly representable as surfaces geometrically, it was clear how TGD might be formulated as a generalized number theory with infinite primes forming the bridge between classical and quantum such that real numbers, p-adic numbers, and various generalizations of p-adics emerge dynamically from algebraic physics as various completions of the algebraic extensions of rational (hyper-)quaternions and (hyper-)octonions. Complete algebraic, topological and dimensional democracy would characterize the theory.

The infinite primes at the first level of hierarchy, which represent analogs of bound states, can be mapped to irreducible polynomials, which in turn characterize the algebraic extensions of rationals defining a hierarchy of algebraic physics continuable to real and p-adic number fields. The products of infinite primes in turn define more general algebraic extensions of rationals. The interesting question concerns the physical interpretation of the higher levels in the hierarchy of infinite primes and integers mappable to polynomials of $n > 1$ variables.

1.1.6 Hierarchy Of Planck Constants And Dark Matter Hierarchy

By quantum classical correspondence space-time sheets can be identified as quantum coherence regions. Hence the fact that they have all possible size scales more or less unavoidably implies that Planck constant must be quantized and have arbitrarily large values. If one accepts this then also the idea about dark matter as a macroscopic quantum phase characterized by an arbitrarily large value of Planck constant emerges naturally as does also the interpretation for the long ranged classical electro-weak and color fields predicted by TGD. Rather seldom the evolution of ideas follows simple linear logic, and this was the case also now. In any case, this vision represents the fifth, relatively new thread in the evolution of TGD and the ideas involved are still evolving.

Dark matter as large $\hbar$ phases

D. Da Rocha and Laurent Nottale [E1] have proposed that Schrödinger equation with Planck constant $\hbar$ replaced with what might be called gravitational Planck constant $\hbar_{gr} = \frac{GmM}{v_0}$ ($\hbar = c = 1$). v_0 is a velocity parameter having the value $v_0 = 144.7 \pm .7$ km/s giving $v_0/c = 4.6 \times 10^{-4}$. This is rather near to the peak orbital velocity of stars in galactic halos. Also subharmonics and harmonics of v_0 seem to appear. The support for the hypothesis coming from empirical data is impressive.

Nottale and Da Rocha believe that their Schrödinger equation results from a fractal hydrodynamics. Many-sheeted space-time however suggests that astrophysical systems are at some levels of the hierarchy of space-time sheets macroscopic quantum systems. The space-time sheets in question would carry dark matter.

Nottale's hypothesis would predict a gigantic value of h_{gr}. Equivalence Principle and the independence of gravitational Compton length on mass m implies however that one can restrict the values of mass m to masses of microscopic objects so that h_{gr} would be much smaller. Large h_{gr} could provide a solution of the black hole collapse (IR catastrophe) problem encountered at the classical level. The resolution of the problem inspired by TGD inspired theory of living matter is that it is the dark matter at larger space-time sheets which is quantum coherent in the required time scale [K66].

It is natural to assign the values of Planck constants postulated by Nottale to the space-time sheets mediating gravitational interaction and identifiable as magnetic flux tubes (quanta) possibly carrying monopole flux and identifiable as remnants of cosmic string phase of primordial cosmology. The magnetic energy of these flux quanta would correspond to dark energy and magnetic tension would give rise to negative "pressure" forcing accelerate cosmological expansion. This leads to a rather detailed vision about the evolution of stars and galaxies identified as bubbles of ordinary and dark matter inside magnetic flux tubes identifiable as dark energy.

Certain experimental findings suggest the identification $h_{eff} = n\times = h_{gr}$. The large value of h_{gr} can be seen as a manner to reduce the string tension of fermionic strings so that gravitational (in fact all!) bound states can be described in terms of strings connecting the partonic 2-surfaces defining particles (analogous to AdS/CFT description). The values $h_{eff}/h = n$ can be interpreted in terms of a hierarchy of breakings of super-conformal symmetry in which the super-conformal generators act as gauge symmetries only for a sub-algebras with conformal weights coming as multiples of n. Macroscopic quantum coherence in astrophysical scales is implied. If also Kähler-Dirac action is present, part of the interior degrees of freedom associated with the Kähler-Dirac part of conformal algebra become physical. A possible is that tfermionic oscillator operators generate super-symmetries and sparticles correspond almost by definition to dark matter with $h_{eff}/h = n > 1$. One implication would be that at least part if not all gravitons would be dark and be observed only through their decays to ordinary high frequency graviton ($E = hf_{high} = h_{eff}f_{low}$) of bunch of n low energy gravitons.

Hierarchy of Planck constants from the anomalies of neuroscience and biology

The quantal ELF effects of ELF em fields on vertebrate brain have been known since seventies. ELF em fields at frequencies identifiable as cyclotron frequencies in magnetic field whose intensity is about 2/5 times that of Earth for biologically important ions have physiological effects and affect also behavior. What is intriguing that the effects are found only in vertebrates (to my best knowledge). The energies for the photons of ELF em fields are extremely low - about 10^{-10} times lower than thermal energy at physiological temperatures- so that quantal effects are impossible in the framework of standard quantum theory. The values of Planck constant would be in these situations large but not gigantic.

This inspired the hypothesis that these photons correspond to so large a value of Planck constant that the energy of photons is above the thermal energy. The proposed interpretation was as dark photons and the general hypothesis was that dark matter corresponds to ordinary matter with non-standard value of Planck constant. If only particles with the same value of Planck constant can appear in the same vertex of Feynman diagram, the phases with different value of Planck constant are dark relative to each other. The phase transitions changing Planck constant can however make possible interactions between phases with different Planck constant but these interactions do not manifest themselves in particle physics. Also the interactions mediated by classical fields should be possible. Dark matter would not be so dark as we have used to believe.

The hypothesis $h_{eff} = h_{gr}$ - at least for microscopic particles - implies that cyclotron energies of charged particles do not depend on the mass of the particle and their spectrum is thus universal although corresponding frequencies depend on mass. In bio-applications this spectrum would correspond to the energy spectrum of bio-photons assumed to result from dark photons by h_{eff} reducing phase transition and the energies of bio-photons would be in visible and UV range associated with the excitations of bio-molecules.

Also the anomalies of biology (see for instance [K57, K58, K93]) support the view that dark matter might be a key player in living matter.

Does the hierarchy of Planck constants reduce to the vacuum degeneracy of Kähler action?

This starting point led gradually to the recent picture in which the hierarchy of Planck constants is postulated to come as integer multiples of the standard value of Planck constant. Given integer multiple $\hbar = n\hbar_0$ of the ordinary Planck constant $\hbar_0$ is assigned with a multiple singular covering of the imbedding space [K22]. One ends up to an identification of dark matter as phases with non-standard value of Planck constant having geometric interpretation in terms of these coverings providing generalized imbedding space with a book like structure with pages labelled by Planck constants or integers characterizing Planck constant. The phase transitions changing the value of

Planck constant would correspond to leakage between different sectors of the extended imbedding space. The question is whether these coverings must be postulated separately or whether they are only a convenient auxiliary tool.

The simplest option is that the hierarchy of coverings of imbedding space is only effective. Many-sheeted coverings of the imbedding space indeed emerge naturally in TGD framework. The huge vacuum degeneracy of Kähler action implies that the relationship between gradients of the imbedding space coordinates and canonical momentum currents is many-to-one: this was the very fact forcing to give up all the standard quantization recipes and leading to the idea about physics as geometry of the "world of classical worlds". If one allows space-time surfaces for which all sheets corresponding to the same values of the canonical momentum currents are present, one obtains effectively many-sheeted covering of the imbedding space and the contributions from sheets to the Kähler action are identical. If all sheets are treated effectively as one and the same sheet, the value of Planck constant is an integer multiple of the ordinary one. A natural boundary condition would be that at the ends of space-time at future and past boundaries of causal diamond containing the space-time surface, various branches co-incide. This would raise the ends of space-time surface in special physical role.

A more precise formulation is in terms of presence of large number of space-time sheets connecting given space-like 3-surfaces at the opposite boundaries of causal diamond. Quantum criticality presence of vanishing second variations of Kähler action and identified in terms of conformal invariance broken down to to sub-algebras of super-conformal algebras with conformal weights divisible by integer n is highly suggestive notion and would imply that n sheets of the effective covering are actually conformal equivalence classes of space-time sheets with same Kähler action and same values of conserved classical charges (see **Fig.** `http://tgdtheory.fi/appfigures/planckhierarchy.jpg` or **Fig. ??** the appendix of this book). n would naturally correspond the value of h_{eff} and its factors negentropic entanglement with unit density matrix would be between the n sheets of two coverings of this kind. p-Adic prime would be largest prime power factor of n.

Dark matter as a source of long ranged weak and color fields

Long ranged classical electro-weak and color gauge fields are unavoidable in TGD framework. The smallness of the parity breaking effects in hadronic, nuclear, and atomic length scales does not however seem to allow long ranged electro-weak gauge fields. The problem disappears if long range classical electro-weak gauge fields are identified as space-time correlates for massless gauge fields created by dark matter. Also scaled up variants of ordinary electro-weak particle spectra are possible. The identification explains chiral selection in living matter and unbroken $U(2)_{ew}$ invariance and free color in bio length scales become characteristics of living matter and of bio-chemistry and bio-nuclear physics.

The recent view about the solutions of Kähler- Dirac action assumes that the modes have a well-defined em charge and this implies that localization of the modes to 2-D surfaces (right-handed neutrino is an exception). Classical W boson fields vanish at these surfaces and also classical Z^0 field can vanish. The latter would guarantee the absence of large parity breaking effects above intermediate boson scale scaling like h_{eff}.

1.1.7 Twistors And TGD

8-dimensional generalization of ordinary twistors is highly attractive approach to TGD [K76]. The reason is that M^4 and CP_2 are completely exceptional in the sense that they are the only 4-D manifolds allowing twistor space with Kähler structure [A63]. The twistor space of $M^4 \times CP_2$ is Cartesian product of those of M^4 and CP_2. The obvious idea is that space-time surfaces allowing twistor structure if they are orientable are representable as surfaces in H such that the properly induced twistor structure co-incides with the twistor structure defined by the induced metric. This condition would define the dynamics, and the conjecture is that this dynamics is equivalent with the identification of space-time surfaces as preferred extremals of Kähler action. The dynamics of space-time surfaces would be lifted to the dynamics of twistor spaces, which are sphere bundles over space-time surfaces. What is remarkable that the powerful machinery of complex analysis becomes available.

The condition that the basic formulas for the twistors in M^8 serving as tangent space of imbedding space generalize. This is the case if one introduces octonionic sigma matrices allowing twistor representation of 8-momentum serving as dual for four-momentum and color quantum numbers. The conditions that octonionic spinors are equivalent with ordinary requires that the induced gamma matrices generate quaternionic sub-algebra at given point of string world sheet. This is however not enough: the charge matrices defined by sigma matrices can also break associativity and induced gauge fields must vanish: the CP_2 projection of string world sheet would be one-dimensional at most. This condition is symplectically invariant. Note however that for the interior dynamics of induced spinor fields octonionic representations of Clifford algebra cannot be equivalent with the ordinary one.

One can assign 4-momentum both to the spinor harmonics of the imbedding space representing ground states of superconformal representations and to light-like boundaries of string world sheets at the orbits of partonic 2-surfaces. The two four-momenta should be identifical by quantum classical correspondence: this is nothing but a concretization of Equivalence Principle. Also a connection with string model emerges.

Twistor approach developed rapidly during years. Witten's twistor string theory generalizes: the most natural counterpart of Witten's twistor strings is partonic 2-surface. The notion of positive Grassmannian has emerged and TGD provides a possible generalization and number theoretic interpretation of this notion. TGD generalizes the observation that scattering amplitudes in twistor Grassmann approach correspond to representations for permutations. Since 2-vertex is the only fermionic vertex in TGD, OZI rules for fermions generalizes, and scattering amplitudes are representations for braidings. Braid interpretation gives further support for the conjecture that non-planar diagrams can be reduced to ordinary ones by a procedure analogous to the construction of braid (knot) invariants by gradual un-braiding (un-knotting).

1.2 Bird's Eye Of View About The Topics Of The Book

This book is devoted to a detailed representation of what quantum TGD in its recent form. Quantum TGD relies on two different views about physics: physics as an infinite-dimensional spinor geometry and physics as a generalized number theory. The most important guiding principle is quantum classical correspondence whose most profound implications follow almost trivially from the basic structure of the classical theory forming an exact part of quantum theory. A further mathematical guideline is the mathematics associated with hyper-finite factors of type II_1 about which the spinors of the world of classical worlds represent a canonical example.

1. Quantum classical correspondence

Quantum classical correspondence has turned out to be the most important guiding principle concerning the interpretation of the theory.

1. Quantum classical correspondence and the properties of the simplest extremals of Kähler action have served as the basic guideline in the attempts to understand the new physics predicted by TGD. The most dramatic predictions follow without even considering field equations in detail by using quantum classical correspondence and form the backbone of TGD and TGD inspired theory of living matter in particular.

   The notions of many-sheeted space-time, topological field quantization and the notion of field/magnetic body, follow from simple topological considerations. The observation that space-time sheets can have arbitrarily large sizes and their interpretation as quantum coherence regions forces to conclude that in TGD Universe macroscopic and macro-temporal quantum coherence are possible in arbitrarily long scales.

2. Also long ranged classical color and electro-weak fields are an unavoidable prediction It however took a considerable time to make the obvious conclusion: TGD Universe is fractal containing fractal copies of standard model physics at various space-time sheets and labeled by the collection of p-adic primes assignable to elementary particles and by the level of dark matter hierarchy characterized partially by the value of Planck constant labeling the pages of the book like structure formed by singular covering spaces of the imbedding space $M^4 \times CP_2$ glued together along a four-dimensional back. Particles at different pages are dark relative to

each other since purely local interactions defined in terms of the vertices of Feynman diagram involve only particles at the same page.

3. The new view about energy and time finding a justification in the framework of zero energy ontology means that the sign of the inertial energy depends on the time orientation of the space-time sheet and that negative energy space-time sheets serve as correlates for communications to the geometric future. This alone leads to profoundly new views about metabolism, long term memory, and realization of intentional action.

4. The general properties of Kähler action, in particular its vacuum degeneracy and the failure of the classical determinism in the conventional sense, have also strong implications. Space-time surface as a generalization of Bohr orbit provides not only a representation of quantum states but also of sequences of quantum jumps and thus contents of consciousness. Vacuum degeneracy implies spin glass degeneracy in 4-D sense reflecting quantum criticality which is the fundamental characteristic of TGD Universe.

5. The detailed study of the simplest extremals of Kähler action interpreted as correlates for asymptotic self organization patterns provides additional insights. CP_2 type extremals representing elementary particles, cosmic strings, vacuum extremals, topological light rays ("massless extremal", ME), flux quanta of magnetic and electric fields represent the basic extremals. Pairs of wormhole throats identifiable as parton pairs define a completely new kind of particle carrying only color quantum numbers in ideal case and I have proposed their interpretation as quantum correlates for Boolean cognition. MEs and flux quanta of magnetic and electric fields are of special importance in living matter.

   Topological light rays have interpretation as space-time correlates of "laser beams" of ordinary or dark photons or their electro-weak and gluonic counterparts. Neutral MEs carrying em and Z^0 fields are ideal for communication purposes and charged W MEs ideal for quantum control. Magnetic flux quanta containing dark matter are identified as intentional agents quantum controlling the behavior of the corresponding biological body parts utilizing negative energy W MEs. Bio-system in turn is populated by electrets identifiable as electric flux quanta.

2. Physics as infinite-dimensional geometry in the "world of classical worlds"

Physics as infinite-dimensional Kähler geometry of the "world of classical worlds" with classical spinor fields representing the quantum states of the universe and gamma matrix algebra geometrizing fermionic statistics is the first vision.

The mere existence of infinite-dimensional non-flat Kähler geometry has impressive implications. Configuration space must decompose to a union of infinite-dimensional symmetric spaces labelled by zero modes having interpretation as classical dynamical degrees of freedom assumed in quantum measurement theory. Infinite-dimensional symmetric space has maximal isometry group identifiable as a generalization of Kac Moody group obtained by replacing finite-dimensional group with the group of canonical transformations of $\delta M^4_+ \times CP_2$, where δM^4_+ is the boundary of 4-dimensional future light-cone. The infinite-dimensional Clifford algebra of configuration space gamma matrices in turn can be expressed as direct sum of von Neumann algebras known as hyperfinite factors of type II_1 having very close connections with conformal field theories, quantum and braid groups, and topological quantum field theories.

3. Physics as a generalized number theory

Second vision is physics as a generalized number theory. This vision forces to fuse real physics and various p-adic physics to a single coherent whole having rational physics as their intersection and poses extremely strong conditions on real physics.

A further aspect of this vision is the reduction of the classical dynamics of space-time sheets to number theory with space-time sheets identified as what I have christened hyper-quaternionic sub-manifolds of hyper-octonionic imbedding space. Field equations would state that space-time surfaces are Kähler calibrations with Kähler action density reducing to a closed 4-form at space-time surfaces. Hence TGD would define a generalized topological quantum field theory with conserved

Noether charges (in particular rest energy) serving as generalized topological invariants having extremum in the set of topologically equivalent 3-surfaces.

Infinite primes, integers, and rationals define the third aspect of this vision. The construction of infinite primes is structurally similar to a repeated second quantization of an arithmetic quantum field theory and involves also bound states. Infinite rationals can be also represented as space-time surfaces somewhat like finite numbers can be represented as space-time points.

4. The organization of the book

The first part of the book describes basic quantum TGD in its recent form.

1. The properties of the preferred extremals of Kähler action are crucial for the construction and the discussion of known extremals is therefore included.

2. General coordinate invariance and generalized super-conformal symmetries - the latter present only for 4-dimensional space-time surfaces and for 4-D Minkowski space - define the basic symmetries of quantum TGD.

3. In zero energy ontology S-matrix is replaced with M-matrix and identified as time-like entanglement coefficients between positive and negative energy parts of zero energy states assignable to the past and future boundaries of 4-surfaces inside causal diamond defined as intersection of future and past directed light-cones. M-matrix is a product of diagonal density matrix and unitary S-matrix and there are reasons to believe that S-matrix is universal. Generalized Feynman rules based on the generalization of Feynman diagrams obtained by replacing lines with light-like 3-surfaces and vertices with 2-D surfaces at which the lines meet.

4. A category theoretical formulation of quantum TGD is considered. Finite measurement resolution realized in terms of a fractal hierarchy of causal diamonds inside causal diamonds leads to a stringy formulation of quantum TGD involving effective replacement of the 3-D light-like surface with a collection of braid strands representing the ends of strings. A formulation in terms of category theoretic concepts is proposed and leads to a hierarchy of algebras forming what is known as operads.

5. Twistors emerge naturally in TGD framework and could allow the formulation of low energy limit of the theory in the approximation that particles are massless. The replacement of massless plane waves with states for which amplitudes are localized are light-rays is suggestive in twistor theoretic framework. Twistors could allow also a dual representation of space-time surfaces in terms of surfaces of $X \times CP_2$, where X is 8-D twistor space or its 6-D projective variant. These surfaces would have dimension higher than four in non-perturbative phases meaning an analogy with branes. In full theory a massive particles must be included but represent a problem in approach based on standard twistors. The interpretation of massive particles in 4-D sense as massless particles in 8-D sense would resolve the problem and requires a generalization of twistor concept involving in essential manner the triality of vector and spinor representations of $SO(7,1)$.

6. In TGD Universe bosons are in well-defined sense bound states of fermion and anti-fermion. This leads to the notion of bosonic emergence meaning that the fundamental action is just Dirac action coupled to gauge potentials and bosonic action emerges as part of effective action as one functionally integrates over the spinor fields. This kind of approach predicts the evolution of all coupling constants if one is able to fix the necessary UV cutoffs of mass and hyperbolic angle in loop integrations. The guess for the hyperbolic cutoff motivated by the geometric view about finite measurement resolution predicts coupling constant evolution which is consistent with that predicted by standard model. The condition that all N-vertices defined by fermiomic loops vanish for $N > 3$ when incoming particles are massless gives hopes of fixing completely the hyperbolic cutoff from fundamental principles.

1.3 Sources

The eight online books about TGD [K83, K62, K101, K71, K50, K100, K99, K69] and nine online books about TGD inspired theory of consciousness and quantum biology [K75, K9, K56, K8, K33, K38, K40, K68, K96] are warmly recommended for the reader willing to get overall view about what is involved.

My homepage (`http://tinyurl.com/ybv8dt4n`) contains a lot of material about TGD. In particular, a TGD glossary at `http://tinyurl.com/yd6jf3o7`).

I have published articles about TGD and its applications to consciousness and living matter in *Journal of Non-Locality* (`http://tinyurl.com/ycyrxj4o` founded by Lian Sidorov and in *Prespacetime Journal* (`http://tinyurl.com/ycvktjhn`), *Journal of Consciousness Research and Exploration* (`http://tinyurl.com/yba4f672`), and *DNA Decipher Journal* (`http://tinyurl.com/y9z52khg`), all of them founded by Huping Hu. One can find the list about the articles published at `http://tinyurl.com/ybv8dt4n`. I am grateful for these far-sighted people for providing a communication channel, whose importance one cannot overestimate.

1.4 The contents of the book

1.4.1 Part I: The recent view about field equations

Basic extremals of the Kähler action

The physical interpretation of the Kähler function and the TGD based space-time concept are the basic themes of this book. The aim is to develop what might be called classical TGD at fundamental level. The strategy is simple: try to guess the general physical consequences of the geometry of the "world of classical worlds" (WCW) and of the TGD based gauge field concept and study the simplest extremals of Kähler action and try to abstract general truths from their properties.

The fundamental underlying assumptions are the following:

1. The notion of preferred extremals emerged during the period when I believed that positive energy ontology applies in TGD. In this framework the 4-surface associated with given 3-surface defined by Kähler function K as a preferred extremal of the Kähler action is identifiable as a classical space-time. Number theoretically preferred extremals would decompose to associative and co-associative regions. The reduction of the classical theory to the level of the Kähler-Dirac action implies that the preferred extremals are critical in the sense of allowing infinite number of deformations for which the second variation of Kähler action vanishes [?] It is not clear whether criticality and associativeity are consistent with each other. A further natural conjecture is that these critical deformations should act as conformal symmetries of light-like wormhole contacts at which the signature of the induced metric changes and preserve their light-likeness.

   Due to the preferred extremal property classical space-time can be also regarded as a generalized Bohr orbit - at least in positive energy ontology - so that the quantization of the various parameters associated with a typical extremal of the Kähler action is expected to take place in general. In TGD quantum states corresponds to quantum superpositions of these classical space-times so that this classical space-time is certainly not some kind of effective quantum average space-time.

2. In ZEO one can also consider the possibility that there is no selection of preferred extremal at all! The two space-like 3-surfaces at the ends of CD define the space-time surface connecting them apart from conformal symmetries acting as critical deformations. If 3-surface is identified as union of both space-like 3-surfaces and the light-like surfaces defining parton orbits connecting then, the conformal equivalence class of the preferred extremal is unique without any additional conditions! This conforms with the view about hierarchy of Planck constants requiring that the conformal equivalence classes of light-like surfaces must be counted as physical degrees of freedom and also with the idea that these surface together define analog for the Wilson loop. Actually all the discussions of this chapter are about extremals in general so that the attribute "preferred" is not relevant for them.

3. The bosonic vacuum functional of the theory is the exponent of the Kähler function $\Omega_B = exp(K)$. This assumption is the only assumption about the dynamics of the theory and is necessitated by the requirement of divergence cancellation in perturbative approach.

4. Renormalization group invariance and spin glass analogy. The value of the Kähler coupling strength is such that the vacuum functional $exp(K)$ is analogous to the exponent $exp(H/T)$ defining the partition function of a statistical system at critical temperature. This allows Kähler coupling strength to depend on zero modes of the configuration space metric and as already found there is very attractive hypothesis determining completely the dependence of the Kähler coupling strength on the zero modes based on p-adic considerations motivated by the spin glass analogy. Coupling constant evolution would be replaced by effective discrete evolution with respect to p-adic length scale and angle variable defined by the phases appearing in the algebraic extension of p-adic numbers in question.

5. In spin degrees of freedom the massless Dirac equation for the induced spinor fields with Kähler-Dirac action defines classical theory: this is in complete accordance with the proposed definition of the WCW spinor structure.

The geometrization of the classical gauge fields in terms of the induced gauge field concept is also important concerning the physical interpretation. Electro-weak gauge potentials correspond to the space-time projections of the spinor connection of CP_2, gluonic gauge potentials to the projections of the Killing vector fields of CP_2 and gravitational field to the induced metric. The topics to be discussed in this part of the book are summarized briefly in the following.

What the selection of preferred extremals of Kähler action might mean has remained a long standing problem and real progress occurred only quite recently (I am writing this towards the end of year 2003).

1. The vanishing of Lorentz 4-force for the induced Kähler field means that the vacuum 4-currents are in a mechanical equilibrium. Lorentz 4-force vanishes for all known solutions of field equations which inspires the hypothesis that all preferred extremals of Kähler action satisfy the condition. The vanishing of the Lorentz 4-force in turn implies local conservation of the ordinary energy momentum tensor. The corresponding condition is implied by Einstein's equations in General Relativity. The hypothesis would mean that the solutions of field equations are what might be called generalized Beltrami fields. The condition implies that vacuum currents can be non-vanishing only provided the dimension D_{CP_2} of the CP_2 projection of the space-time surface is less than four so that in the regions with $D_{CP_2} = 4$, Maxwell's vacuum equations are satisfied.

2. The hypothesis that Kähler current is proportional to a product of an arbitrary function ψ of CP_2 coordinates and of the instanton current generalizes Beltrami condition and reduces to it when electric field vanishes. Instanton current has a vanishing divergence for $D_{CP_2} < 4$, and Lorentz 4-force indeed vanishes. Four 4-dimensional projection the scalar function multiplying the instanton current can make it divergenceless. The remaining task would be the explicit construction of the imbeddings of these fields and the demonstration that field equations can be satisfied.

3. By quantum classical correspondence the non-deterministic space-time dynamics should mimic the dissipative dynamics of the quantum jump sequence. Beltrami fields appear in physical applications as asymptotic self organization patterns for which Lorentz force and dissipation vanish. This suggests that preferred extemals of Kähler action correspond to space-time sheets which at least asymptotically satisfy the generalized Beltrami conditions so that one can indeed assign to the final 3-surface a unique 4-surface apart from effects related to non-determinism. Preferred extremal property abstracted to purely algebraic generalized Beltrami conditions makes sense also in the p-adic context.

This chapter is mainly devoted to the study of the basic extremals of the Kähler action besides the detailed arguments supporting the view that the preferred extrema satisfy generalized Beltrami conditions at least asymptotically.

The newest results discussed in the last section about the weak form of electric-magnetic duality suggest strongly that Beltrami property is general and together with the weak form of electric-magnetic duality allows a reduction of quantum TGD to almost topological field theory with Kähler function allowing expression as a Chern-Simons term.

The surprising implication of the duality is that Kähler form of CP_2 must be replaced with that for $S^2 \times CP_2$ in order to obtain a WCW metric which is non-trivial in M^4 degrees of freedom. This modification implies much richer vacuum structure than the original Kähler action which is a good news as far as the description of classical gravitational fields in terms of small deformations of vacuum extremals with the four-momentum density of the topologically condensed matter given by Einstein's equations is considered. The breaking of Lorentz invariace from $SO(3,1)$ to $SO(3)$ is implied already by the geometry of CD but is extremely small for a given causal diamond (CD). Since a wave function over the Lorentz boosts and translates of CD is allowed, there is no actual breaking of Poincare invariance at the level of the basic theory. Beltrami property leads to a rather explicit construction of the general solution of field equations based on the hydrodynamic picture implying that single particle quantum numbers are conserved along flow lines defined by the instanton current. The construction generalizes also to the fermionic sector.

About Identification of the Preferred extremals of Kähler Action

Preferred extremal of Kähler action have remained one of the basic poorly defined notions of TGD. There are pressing motivations for understanding what the attribute "preferred" really means. Symmetries give a clue to the problem. The conformal invariance of string models naturally generalizes to 4-D invariance defined by quantum Yangian of quantum affine algebra (Kac-Moody type algebra) characterized by two complex coordinates and therefore explaining naturally the effective 2-dimensionality [K89]. Preferred extremal property should rely on this symmetry.

In Zero Energy Ontology (ZEO) preferred extremals are space-time surfaces connecting two space-like 3-surfaces at the ends of space-time surfaces at boundaries of causal diamond (CD). A natural looking condition is that the symplectic Noether charges associated with a sub-algebra of symplectic algebra with conformal weights n-multiples of the weights of the entire algebra vanish for preferred extremals. These conditions would be classical counterparts the the condition that super-symplectic sub-algebra annihilates the physical states. This would give a hierarchy of super-symplectic symmetry breakings and quantum criticalities having interpretation in terms of hierarchy of Planck constants $h_{eff} = n \times h$ identified as a hierarchy of dark matter. n could be interpreted as the number of space-time conformal gauge equivalence classes for space-time sheets connecting the 3-surfaces at the ends of space-time surface.

There are also many other proposals for what preferred extremal property could mean or imply. The weak form of electric-magnetic duality combined with the assumption that the contraction of the Kähler current with Kähler gauge potential vanishes for preferred extremals implies that Kähler action in Minkowskian space-time regions reduces to Chern-Simons terms at the light-like orbits of wormhole throats at which the signature of the induced metric changes its signature from Minkowskian to Euclidian. In regions with 4-D CP_2 projection (wormhole contacts) also a 3-D contribution not assignable to the boundary of the region might be possible. These conditions pose strong physically feasible conditions on extremals and might be true for preferred extremals too.

Number theoretic vision leads to a proposal that either the tangent space or normal space of given point of space-time surface is associative and thus quaternionic. Also the formulation in terms of quaternion holomorphy and quaternion-Kähler property is an attractive possibility. So called $M^8 - H$ duality is a variant of this vision and would mean that one can map associative/co-associative space-time surfaces from M^8 to H and also iterate this mapping from H to H to generate entire category of preferred extremals. The signature of M^4 is a general technical problem. For instance, the holomorphy in 2 complex variables could correspond to what I have called Hamilton-Jacobi property. Associativity/co-associativity of the tangent space makes sense also in Minkowskian signature.

In this chapter various views about preferred extremal property are discussed.

1.4.2 WCW Spinor Structure

Quantum TGD should be reducible to the classical spinor geometry of the configuration space ("world of classical worlds" (WCW)). The possibility to express the components of WCW Kähler metric as anti-commutators of WCW gamma matrices becomes a practical tool if one assumes that WCW gamma matrices correspond to Noether super charges for super-symplectic algebra of WCW. The possibility to express the Kähler metric also in terms of Kähler function identified as Kähler for Euclidian space-time regions leads to a duality analogous to AdS/CFT duality.

Physical states should correspond to the modes of the WCW spinor fields and the identification of the fermionic oscillator operators as super-symplectic charges is highly attractive. WCW spinor fields cannot, as one might naively expect, be carriers of a definite spin and unit fermion number. Concerning the construction of the WCW spinor structure there are some important clues.

1. Geometrization of fermionic statistics in terms of WCW spinor structure

The great vision has been that the second quantization of the induced spinor fields can be understood geometrically in terms of the WCW spinor structure in the sense that the anti-commutation relations for WCW gamma matrices require anti-commutation relations for the oscillator operators for free second quantized induced spinor fields.

1. One must identify the counterparts of second quantized fermion fields as objects closely related to the WCW spinor structure. Ramond model has as its basic field the anti-commuting field $\Gamma^k(x)$, whose Fourier components are analogous to the gamma matrices of the WCW and which behaves like a spin 3/2 fermionic field rather than a vector field. This suggests that the complexified gamma matrices of the WCW are analogous to spin 3/2 fields and therefore expressible in terms of the fermionic oscillator operators so that their anti-commutativity naturally derives from the anti-commutativity of the fermionic oscillator operators.

   As a consequence, WCW spinor fields can have arbitrary fermion number and there would be hopes of describing the whole physics in terms of WCW spinor field. Clearly, fermionic oscillator operators would act in degrees of freedom analogous to the spin degrees of freedom of the ordinary spinor and bosonic oscillator operators would act in degrees of freedom analogous to the "orbital" degrees of freedom of the ordinary spinor field.

2. The classical theory for the bosonic fields is an essential part of the WCW geometry. It would be very nice if the classical theory for the spinor fields would be contained in the definition of the WCW spinor structure somehow. The properties of the modified massless Dirac operator associated with the induced spinor structure are indeed very physical. The modified massless Dirac equation for the induced spinors predicts a separate conservation of baryon and lepton numbers. The differences between quarks and leptons result from the different couplings to the CP_2 Kähler potential. In fact, these properties are shared by the solutions of massless Dirac equation of the imbedding space.

3. Since TGD should have a close relationship to the ordinary quantum field theories it would be highly desirable that the second quantized free induced spinor field would somehow appear in the definition of the WCW geometry. This is indeed true if the complexified WCW gamma matrices are linearly related to the oscillator operators associated with the second quantized induced spinor field on the space-time surface and/or its boundaries. There is actually no deep reason forbidding the gamma matrices of the WCW to be spin half odd-integer objects whereas in the finite-dimensional case this is not possible in general. In fact, in the finite-dimensional case the equivalence of the spinorial and vectorial vielbeins forces the spinor and vector representations of the vielbein group $SO(D)$ to have same dimension and this is possible for $D = 8$-dimensional Euclidian space only. This coincidence might explain the success of 10-dimensional super string models for which the physical degrees of freedom effectively correspond to an 8-dimensional Euclidian space.

4. It took a long time to realize that the ordinary definition of the gamma matrix algebra in terms of the anti-commutators $\{\gamma_A, \gamma_B\} = 2g_{AB}$ must in TGD context be replaced with $\{\gamma_A^\dagger, \gamma_B\} = iJ_{AB}$, where J_{AB} denotes the matrix elements of the Kähler form of the WCW.

The presence of the Hermitian conjugation is necessary because WCW gamma matrices carry fermion number. This definition is numerically equivalent with the standard one in the complex coordinates. The realization of this delicacy is necessary in order to understand how the square of the WCW Dirac operator comes out correctly.

2. Kähler-Dirac equation for induced spinor fields

Super-symmetry between fermionic and and WCW degrees of freedom dictates that Kähler-Dirac action is the unique choice for the Dirac action

There are several approaches for solving the Kähler-Dirac (or Kähler-Dirac) equation.

1. The most promising approach assumes that the solutions are restricted on 2-D stringy world sheets and/or partonic 2-surfaces. This strange looking view is a rather natural consequence of both strong form of holography and of number theoretic vision, and also follows from the notion of finite measurement resolution having discretization at partonic 2-surfaces as a geometric correlate. Furthermore, the conditions stating that electric charge is well-defined for preferred extremals forces the localization of the modes to 2-D surfaces in the generic case. This also resolves the interpretational problems related to possibility of strong parity breaking effects since induce W fields and possibly also Z^0 field above weak scale, vahish at these surfaces.

   The condition that also spinor dynamics is associative suggests strongly that the localization to 2-D surface occurs always (for right-handed neutrino the above conditions does not apply this). The induced gauge potentials are the possible source of trouble but the holomorphy of spinor modes completely analogous to that encountered in string models saves the situation. Whether holomorphy could be replaced with its quaternionic counterpart in Euclidian regions is not clear (this if W fields vanish at the entire space-time surface so that 4-D modes are possible). Neither it is clear whether the localization to 2-D surfaces occurs also in Euclidian regions with 4-D CP_2 projection.

2. One expects that stringy approach based on 4-D generalization of conformal invariance or its 2-D variant at 2-D preferred surfaces should also allow to understand the Kähler-Dirac equation. Conformal invariance indeed allows to write the solutions explicitly using formulas similar to encountered in string models. In accordance with the earlier conjecture, all modes of the Kähler-Dirac operator generate badly broken super-symmetries.

3. Well-definedness of em charge is not enough to localize spinor modes at string world sheets. Covariantly constant right-handed neutrino certainly defines solutions de-localized inside entire space-time sheet. This need not be the case if right-handed neutrino is not covarianty constant since the non-vanishing CP_2 part for the induced gamma matrices mixes it with left-handed neutrino. For massless extremals (at least) the CP_2 part however vanishes and right-handed neutrino allows also massless holomorphic modes de-localized at entire space-time surface and the de-localization inside Euclidian region defining the line of generalized Feynman diagram is a good candidate for the right-handed neutrino generating the least broken super-symmetry. This super-symmetry seems however to differ from the ordinary one in that ν_R is expected to behave like a passive spectator in the scattering. Also for the left-handed neutrino solutions localized inside string world sheet the condition that coupling to right-handed neutrino vanishes is guaranteed if gamma matrices are either purely Minkowskian or CP_2 like inside the world sheet.

awcwspin

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   The condition that also spinor dynamics is associative suggests strongly that the localization to 2-D surface occurs always (for right-handed neutrino the above conditions does not apply this). The induced gauge potentials are the possible source of trouble but the holomorphy of spinor modes completely analogous to that encountered in string models saves the situation. Whether holomorphy could be replaced with its quaternionic counterpart in Euclidian regions is not clear (this if W fields vanish at the entire space-time surface so that 4-D modes are possible). Neither it is clear whether the localization to 2-D surfaces occurs also in Euclidian regions with 4-D CP_2 projection.

2. One expects that stringy approach based on 4-D generalization of conformal invariance or its 2-D variant at 2-D preferred surfaces should also allow to understand the Kähler-Dirac equation. Conformal invariance indeed allows to write the solutions explicitly using formulas similar to encountered in string models. In accordance with the earlier conjecture, all modes of the Kähler-Dirac operator generate badly broken super-symmetries.

3. Well-definedness of em charge is not enough to localize spinor modes at string world sheets. Covariantly constant right-handed neutrino certainly defines solutions de-localized inside entire space-time sheet. This need not be the case if right-handed neutrino is not covarianty constant since the non-vanishing CP_2 part for the induced gamma matrices mixes it with left-handed neutrino. For massless extremals (at least) the CP_2 part however vanishes and right-handed neutrino allows also massless holomorphic modes de-localized at entire space-time surface and the de-localization inside Euclidian region defining the line of generalized Feynman diagram is a good candidate for the right-handed neutrino generating the least broken super-symmetry. This super-symmetry seems however to differ from the ordinary one in that ν_R is expected to behave like a passive spectator in the scattering. Also for the left-handed neutrino solutions localized inside string world sheet the condition that coupling to right-handed neutrino vanishes is guaranteed if gamma matrices are either purely Minkowskian or CP_2 like inside the world sheet.

1.4.3 Recent View about Kähler Geometry and Spin Structure of "World of Classical Worlds"

The construction of Kähler geometry of WCW ("world of classical worlds") is fundamental to TGD program. I ended up with the idea about physics as WCW geometry around 1985 and made a breakthrough around 1990, when I realized that Kähler function for WCW could correspond to Kähler action for its preferred extremals defining the analogs of Bohr orbits so that classical theory with Bohr rules would become an exact part of quantum theory and path integral would be replaced with genuine integral over WCW. The motivating construction was that for loop spaces leading to a unique Kähler geometry. The geometry for the space of 3-D objects is even more complex than that for loops and the vision still is that the geometry of WCW is unique from the mere existence of Riemann connection.

This chapter represents the updated version of the construction providing a solution to the problems of the previous construction. The basic formulas remain as such but the expressions for WCW super-Hamiltonians defining WCW Hamiltonians (and matrix elements of WCW metric) as their anticommutator are replaced with those following from the dynamics of the Kähler-Dirac action.

Can one apply Occam's razor as a general purpose debunking argument to TGD?

Occarm's razor have been used to debunk TGD. The following arguments provide the information needed by the reader to decide himself. Considerations are at three levels.

The level of "world of classical worlds" (WCW) defined by the space of 3-surfaces endowed with Kähler structure and spinor structure and with the identification of WCW space spinor fields as quantum states of the Universe: this is nothing but Einstein's geometrization program applied to quantum theory. Second level is space-time level.

Space-time surfaces correspond to preferred extremals of Käction in $M^4 \times CP_2$. The number of field like variables is 4 corresponding to 4 dynamically independent imbedding space coordinates. Classical gauge fields and gravitational field emerge from the dynamics of 4-surfaces. Strong form of holography reduces this dynamics to the data given at string world sheets and partonic 2-surfaces and preferred extremals are minimal surface extremals of Kähler action so that the classical dynamics in space-time interior does not depend on coupling constants at all which are visible via boundary conditions only. Continuous coupling constant evolution is replaced with a sequence of phase transitions between phases labelled by critical values of coupling constants: loop corrections vanish in given phase. Induced spinor fields are localized at string world sheets to guarantee well-definedness of em charge.

At imbedding space level the modes of imbedding space spinor fields define ground states of super-symplectic representations and appear in QFT-GRT limit. GRT involves post-Newtonian approximation involving the notion of gravitational force. In TGD framework the Newtonian force correspond to a genuine force at imbedding space level.

I was also asked for a summary about what TGD is and what it predicts. I decided to add this summary to this chapter although it is goes slightly outside of its title.

1.4.4 Part II: General Theory

Construction of Quantum Theory: Symmetries

This chapter provides a summary about the role of symmetries in the construction of quantum TGD. In fact, the general definition of geometry is as a structure characterized by given symmetries. The discussions are based on the general vision that quantum states of the Universe correspond to the modes of classical spinor fields in the "world of the classical worlds" (WCW) identified as the infinite-dimensional WCW of light-like 3-surfaces of $H = M^4 \times CP_2$ (more or less-equivalently, the corresponding 4-surfaces defining generalized Bohr orbits). The following topics are discussed on basis of this vision.

1. Physics as infinite-dimensional Kähler geometry

1. The basic idea is that it is possible to reduce quantum theory to WCW geometry and spinor structure. The geometrization of loop spaces inspires the idea that the mere existence of Riemann connection fixes WCW Kähler geometry uniquely. Accordingly, WCW can be regarded as a union of infinite-dimensional symmetric spaces labeled by zero modes labeling classical non-quantum fluctuating degrees of freedom.

   The huge symmetries of the WCW geometry deriving from the light-likeness of 3-surfaces and from the special conformal properties of the boundary of 4-D light-cone would guarantee the maximal isometry group necessary for the symmetric space property. Quantum criticality is the fundamental hypothesis allowing to fix the Kähler function and thus dynamics of TGD uniquely. Quantum criticality leads to surprisingly strong predictions about the evolution of coupling constants.

2. WCW spinors correspond to Fock states and anti-commutation relations for fermionic oscillator operators correspond to anti-commutation relations for the gamma matrices of the WCW. WCW gamma matrices contracted with Killing vector fields give rise to a super-symplectic algebra which together with Hamiltonians of the WCW forms what I have used to call super-symplectic algebra.

   Super-symplectic degrees of freedom represent completely new degrees of freedom and have no electroweak couplings. In the case of hadrons super-symplectic quanta correspond to what has been identified as non-perturbative sector of QCD: they define TGD correlate for the degrees of freedom assignable to hadronic strings. They are responsible for the most of the mass of hadron and resolve spin puzzle of proton.

3. Besides super-symplectic symmetries there are Super-Kac Moody symmetries assignable to light-like 3-surfaces and together these algebras extend the conformal symmetries of string models to dynamical conformal symmetries instead of mere gauge symmetries. The construction of the representations of these symmetries is one of the main challenges of quantum TGD. Modular invariance is one aspect of conformal symmetries and plays a key role in the understanding of elementary particle vacuum functionals and the description of family replication phenomenon in terms of the topology of partonic 2-surfaces.

4. Kähler-Dirac equation (or Kähler-Dirac equation) gives also rise to a hierarchy super-conformal algebras assignable to zero modes. These algebras follow from the existence of conserved fermionic currents. The corresponding deformations of the space-time surface correspond to vanishing second variations of Kähler action and provide a realization of quantum criticality. This led to a breakthrough in the understanding of the Kähler-Dirac action via the addition of a measurement interaction term to the action allowing to obtain among other things stringy propagator and the coding of quantum numbers of super-conformal representations to the geometry of space-time surfaces required by quantum classical correspondence.

   A crucial feature of the Kähler-Dirac equation is the localization of the modes to 2-D surfaces with vanishing induced W fields (this in generic situation and for all modes but covariantly constant right-handed neutrino): this is needed in order to have modes with well-defined em charge. Also Z^0 fields can be vanish and is expected to do so - at least above weak scale. This implies that all elementary particles are string like objects in very concrete sense.

2. p-adic physics and p-adic variants of basic symmetries

p-Adic mass calculations relying on p-adic length scale hypothesis led to an understanding of elementary particle masses using only super-conformal symmetries and p-adic thermodynamics. The need to fuse real physics and various p-adic physics to single coherent whole led to a generalization of the notion of number obtained by gluing together reals and p-adics together along common rationals and algebraics. The interpretation of p-adic space-time sheets is as correlates for cognition and intentionality. p-Adic and real space-time sheets intersect along common rationals and algebraics and the subset of these points defines what I call number theoretic braid in terms of which both WCW geometry and S-matrix elements should be expressible. Thus one would obtain number theoretical discretization which involves no adhoc elements and is inherent to the physics of TGD.

3. Hierarchy of Planck constants and dark matter hierarchy

The realization for the hierarchy of Planck constants proposed as a solution to the dark matter puzzle leads to a profound generalization of quantum TGD through a generalization of the notion of imbedding space to characterize quantum criticality. The resulting space has a book like structure with various almost-copies of the imbedding space representing the pages of the book meeting at quantum critical sub-manifolds. A particular page of the book can be seen as an n-fold singular covering or factor space of CP_2 or of a causal diamond (CD) of M^4 defined as an intersection of the future and past directed light-cones. Therefore the cyclic groups Z_n appear as discrete symmetry groups. The extension of imbedding space can be seen as a formal tool allowing an elegant description of the multi-sheetednes due to the non-determinism of Kähler action. At the space-like ends the sheets fuse together so that a singular covering is in question.

The original intuition was the the space-time would be n-sheeted for $h_{eff} = n$. Quantum criticality expected on basis of the vacuum degeneracy of Kähler action suggests that conformal symmetries act as critical deformations respecting the light-likeness of partonic orbits at which the signature of the induced metric changes from Minkowskian to Euclidian. Therefore one would have n conformal equivalence classes of physically equivalent space-time sheets. A hierarchy of breakings of conformal symmetry is expected on basis of ordinary catastrophe theory. These breakings would correspond to the hierarchy defined by the sub-algebras of conformal algebra or associated algebra for which conformal weights are divisible by n. This defines infinite number of inclusion hierarchies

.. $\subset C(n_1) \subset C(n_3)$... such that n_{i+1} divides n_i. These hierarchies could correspond to inclusion hierarchies of hyper-finite factors and conformal algebra acting as gauge transformations would naturally define the notion of finite measurement resolution.

4. Number theoretical symmetries

TGD as a generalized number theory vision leads to the idea that also number theoretical symmetries are important for physics.

1. There are good reasons to believe that the strands of number theoretical braids - ends of string world sheets - can be assigned with the roots of a polynomial with suggests the interpretation corresponding Galois groups as purely number theoretical symmetries of quantum TGD. Galois groups are subgroups of the permutation group S_∞ of infinitely manner objects acting as the Galois group of algebraic numbers. The group algebra of S_∞ is HFF which can be mapped to the HFF defined by configuration space spinors. This picture suggest a number theoretical gauge invariance stating that S_∞ acts as a gauge group of the theory and that global gauge transformations in its completion correspond to the elements of finite Galois groups represented as diagonal groups of $G \times G \times ....$ of the completion of S_∞.

2. HFFs inspire also an idea about how entire TGD emerges from classical number fields, actually their complexifications. In particular, SU(3) acts as subgroup of octonion automorphisms leaving invariant preferred imaginary unit. If space-time surfaces are hyper-quaternionic (meaning that the octonionic counterparts of the Kähler-Dirac gamma matrices span complex quaternionic sub-algebra of octonions) and contain at each point a preferred plane M^2 of M^4, one ends up with $M^8 - H$ duality stating that space-time surfaces can be equivalently regarded as surfaces in M^8 or $M^4 \times CP_2$. One can actually generalize M^2 to a two-dimensional Minkowskian sub-manifold of M^4. One ends up with quantum TGD by considering associative sub-algebras of the local octonionic Clifford algebra of M^8 or H. so that TGD could be seen as a generalized number theory.

Zero Energy Ontology and Matrices

During years the basic mathematical and conceptual building bricks of quantum TGD have become rather obvious.

One important building brick is Zero Energy Ontology (ZEO). ZEO forces to generalize the notion of S-matrix by introducing M-matrices and U-matrix and allows a new view about observer based on TGD inspired theory of consciousness.

Second building brick consists of various hierarchies and connections between them. There is the hierarchy of quantum criticalities for super-symplectic algebra and its Yangian extension acting as a spectrum generating algebra. This hierarchy is closely related to the hierarchy of Planck constants $h_{eff} = n \times h$. The hierarchies of criticalities correspond also to fractal hierarchies of breakings of super-symplectic gauge conformal symmetry: only the sub-algebra isomorphic to the original gauge algebra acts as gauge algebra after the breaking. At each step one criticality is reduced and the number of physical degrees of freedom increases. There is also a natural connection between these hierarchies with hierarchies of hyperfinite factors of type II_1 (HFFs) and their inclusions providing a description for the notion of measurement resolution. Also the construction of zero energy states using super-symplectic Yangian provides a concrete realization for the notion of finite measurement resolution in the structure of zero energy states and manifesting in the structure of space-time surfaces serving as classical correlates of quantum states.

There are also other important building bricks but in this chapter only ZEO and hyper-finite factors are discussed.

What Scattering Amplitudes Should Look Like?

During years I have spent a lot of time and effort in attempts to imagine various options for the construction of S-matrix - in Zero Energy Ontology (ZEO) M- and U-matrices - and it seems that there are quite many strong constraints, which might lead to a more or less unique final result if some young analytically blessed brain decided to transform these assumptions to concrete calculational recipes.

The realization that WCW spinors correspond to von Neumann algebras known as hyper-finite factors of type II_1 meant a turning point also in the attempts to construct S-matrix. A sequence of trials and errors led rapidly to the generalization of the quantum measurement theory and re-interpretation of S-matrix elements as entanglement coefficients of zero energy states in accordance with the ZEO applied already earlier in TGD inspired cosmology. ZEO motivated the replacement of the term "S-matrix" with "M-matrix".

The general mathematical concepts are not enough to get to the level of concrete scattering amplitudes. The notion of preferred extremal inspiring the notion of generalized Feynman diagram is central in bringing in this concretia. The very notion of preferred extremals means that ordinary Feynman diagrams providing a visualization of path integral are not in question. Generalized Feynman diagrams have 4-D Euclidian space-time regions (wormhole contacts) as lines, and light-like partonic orbits of 2-surfaces as 3-D lines. String world sheets carrying fermions are also present and have 1-D boundaries at the light-like orbits of partonic 2-surfaces carrying fermion number and light-like 8-momenta suggesting strongly 8-D generalization of twistor approach.

The resulting objects could be indeed seen as generalizations of twistor diagrams rather than Feynman diagrams. The preferred extremal property strongly encourages the old and forgotten TGD inspried idea as sequences of algebraic operations with product and co-product representing 3-vertices. The sequences connect given states at the opposite boundaries of CD and have minimal length. The algebraic structure in question would be the Yangian of the super-symplectic algebra with generators identified as super-symplectic charges assignable to strings connecting partonic 2-surfaces.

The purpose of this chapter is to collect to single chapter various general ideas about the construction of M-matrix and give a brief summary about intuitive picture behind various matrices. Also a general vision about generalized Feynman diagrams is formulated. A more detailed construction requires the introduction of generalization of twistor approach to 8-D context.

Does Riemann Zeta Code for Generic Coupling Constant Evolution?

A general model for the coupling constant evolution is proposed. The analogy of Riemann zeta and fermionic zeta $\zeta_F(s)/\zeta_F(2s)$ with complex square root of a partition function natural in Zero Energy Ontology suggests that the the poles of $\zeta_F(ks)$, $k = 1/2$, correspond to complexified critical temperatures identifiable as inverse of Kähler coupling strength itself having interpretation as inverse of critical temperature. One can actually replace the argument s of ζ_F with Möbius transformed argument $w = (as+b)/(cs+d)$ with a, b, c, d real numbers, rationals, or even integers. For α_K $w = (s+b)/2$ is proper choices and gives zeros of $\zeta(s)$ and $s = 2-b$ as poles. The identification $\alpha_K = \alpha_{U(1)}$ leads to a prediction for α_{em}, which deviates by .7 per cent from the experimental value at low energies (atomic scale) if the experimental value of the Weinberg angle is used. The conjecture generalizes also to weak, color and gravitational interactions when general Möbius transformation leaving upper half-plane invariant is allowed. One ends up with a general model predicting successfully the entire electroweak coupling constant evolution successfully from the values of fine structure constant at atomic or electron scale and in weak scale.

Could $\mathcal{N} = 2$ Super-conformal Theories Be Relevant For TGD?

TGD has as is symmetries super-conformal symmetry (SCS), which is a huge extension of the ordinary SCS. For instance, the infinite-dimensional symplectic group plays the role of finite-dimension Lie-group as Kac-Moody group and the conformal weights for the generators of algebra corresponds to the zeros of fermionic zeta and their number of generators is therefore infinite.

The relationship of TGD SCS to super-conformal field theories (SCFTs) known as minimal models has remained without definite answer. The most general super-conformal algebra (SCA) assignable to string world sheets by strong form of holography has $\mathcal{N}$ equal to the number of spin states of leptonic and quark type fundamental spinors but the space-time SUSY is badly broken for it. Covariant constancy of the generating spinor modes is replaced with holomorphy - kind of "half covariant constancy". Right-handed neutrino and antineutrino are excellent candidates for generating $\mathcal{N} = 2$ SCS with a minimal breaking of the corresponding space-time SUSY.

$\mathcal{N} = 2$ SCS has also some inherent problems. The critical space-time dimension is $D = 4$ but the existence of complex structure seems to require the space-time has metric signature different from Minkowskian: here TGD suggests a solution. $\mathcal{N} = 2$ SCFTs are claimed also to reduce to topological QFTs under some conditions: this need not be a problem since TGD can be characterized as almost topological QFT. What looks like a further problem is that p-adic mass calculations require half-integer valued negative conformal weight for the ground state (and vanishing weight for massless states). One can however shift the scaling generator L_0 to get rid of problem: the shift has physical interpretation in TGD framework and must be half integer valued which poses the constraint $h = K/2$, $K = 0, 1, 2..$ on the representations of SCA.

$\mathcal{N} = 2$ SCA allows a spectral flow taking Ramond representations to Neveu-Scwartz variant of algebra. The physical interpretation is as super-symmetry mapping fermionic states to bosonic states. The representations of $\mathcal{N} = 2$ SCA allowing degenerate states with positive central charge c and non-vanishing ground state conformal weight h give rise to minimal models allowing ADE classification, construction of partition functions, and even of n-point functions. This could make S-matrix of TGD exactly solvable in the fermionic sector. The ADE hierarchy suggests a direct interpretation in terms of orbifold hierarchy assignable to the hierarchy of Planck constants associated with the super-symplectic algebra: primary fields would correspond to orbifolds identified as coset spaces of ADE groups. Also an interpretation in terms of inclusions of hyper-finite factors is highly suggestive.

1.4.5 Part III: Twistors and TGD

TGD Variant of Twistor Story

Twistor Grassmannian formalism has made a breakthrough in $\mathcal{N} = 4$ supersymmetric gauge theories and the Yangian symmetry suggests that much more than mere technical breakthrough is in question. Twistors seem to be tailor made for TGD but it seems that the generalization of twistor structure to that for 8-D imbedding space $H = M^4 \times CP_2$ is necessary. M^4 (and S^4 as its Euclidian counterpart) and CP_2 are indeed unique in the sense that they are the only 4-D spaces allowing twistor space with Kähler structure.

The Cartesian product of twistor spaces $P_3 = SU(2, 2)/SU(2, 1) \times U(1)$ and F_3 defines twistor space for the imbedding space H and one can ask whether this generalized twistor structure could allow to understand both quantum TGD and classical TGD defined by the extremals of Kähler action. In the following I summarize the background and develop a proposal for how to construct extremals of Kähler action in terms of the generalized twistor structure. One ends up with a scenario in which space-time surfaces are lifted to twistor spaces by adding CP_1 fiber so that the twistor spaces give an alternative representation for generalized Feynman diagrams.

There is also a very closely analogy with superstring models. Twistor spaces replace Calabi-Yau manifolds and the modification recipe for Calabi-Yau manifolds by removal of singularities can be applied to remove self-intersections of twistor spaces and mirror symmetry emerges naturally. The overall important implication is that the methods of algebraic geometry used in super-string theories should apply in TGD framework.

The physical interpretation is totally different in TGD. The landscape is replaced with twistor spaces of space-time surfaces having interpretation as generalized Feynman diagrams and twistor spaces as sub-manifolds of $P_3 \times F_3$ replace Witten's twistor strings.

The classical view about twistorialization of TGD makes possible a more detailed formulation of the previous ideas about the relationship between TGD and Witten's theory and twistor Grassmann approach. Furthermore, one ends up to a formulation of the scattering amplitudes in terms of Yangian of the super-symplectic algebra relying on the idea that scattering amplitudes are sequences consisting of algebraic operations (product and co-product) having interpretation as vertices in the Yangian extension of super-symplectic algebra. These sequences connect given initial and final states and having minimal length. One can say that Universe performs calculations.

From Principles to Diagrams

The recent somewhat updated view about the road from general principles to diagrams is discussed. A more explicit realization of twistorialization as lifting of the preferred extremal X^4 of Kähler action to corresponding 6-D twistor space X^6 identified as surface in the 12-D product of twistor spaces of M^4 and CP_2 allowing Kähler structure suggests itself. Contrary to the original expectations, the twistorial approach is not mere reformulation but leads to a first principle identification of cosmological constant and perhaps also of gravitational constant and to a modification of the dynamics of Kähler action however preserving the known extremals and basic properties of Kähler action and allowing to interpret induced Kähler form in terms of preferred imaginary unit defining twistor structure.

Second new element is the fusion of twistorial approach with the vision that diagrams are representations for computations. This as also quantum criticality demands that the diagrams should allow huge symmetries allowing to transform them to braided generalizations of tree-diagrams. Several guiding principles are involved and what is new is the observation that they indeed seem to form a coherent whole.

About Twistor Lift of TGD

The twistor lift of classical TGD is attractive physically but it is still unclear whether it satisfies all constraints. The basic implication of twistor lift would be the understanding of gravitational and cosmological constants. Cosmological constant removes the infinite vacuum degeneracy of Kähler action but because of the extreme smallness of cosmological constant Λ playing the role of inverse of gauge coupling strength, the situation for nearly vacuum extremals of Kähler action in the recent cosmology is non-perturbative. Cosmological constant and thus twistor lift make sense only in zero energy ontology (ZEO) involving causal diamonds (CDs) in an essential manner.

One motivation for introducing the hierarchy of Planck constants was that the phase transition increasing Planck constant makes possible perturbation theory in strongly interacting system. Nature itself would take care about the converge of the perturbation theory by scaling Kähler coupling strength α_K to α_K/n, $n = h_{eff}/h$. This hierarchy might allow to construct gravitational perturbation theory as has been proposed already earlier. This would for gravitation to be quantum coherent in astrophysical and even cosmological scales.

In this chapter twistor lift is studied in detail.

1. The first working hypothesis is that the values of $\alpha_K(M^4)$ and $\alpha_K(CP_2)$ are widely different with $\alpha_K(M^4)$ being extremely large so that M^4 part of the 6-D Kähler action gives in dimensional reduction extremely small cosmological term. The first interesting finding is that allowing Kähler coupling strength $\alpha_K(CP_2)$ to correspond to zeros of zeta implies that for complex zeros the preferred extremals for $\alpha_K(M^4)$ having different phase are mimimal surface extremals of Kähler action so that the values of coupling constants do not matter and extremals depend on couplings only through the boundary conditions stating the vanishing of certain super-symplectic conserved charges.

2. The other working hypothesis is $\alpha_K(M^4) = \alpha_K(CP_2)$. The small effective value of cosmological constant is obtained if the Kähler action and volume term tend to cancel each other. In this case minimal surface extremals of Kähler action correspond naturally to asymptotic dynamics near the boundaries of CDs. This option looks more natural.

Both options lead to a generalization of Chladni mechanism to a “dynamics of avoidance” meaning that at least asymptotically the two dynamics decouple. This leads to an interpretation with profound implications for the views about what happens in particle physics experiment and in quantum measurement, for consciousness theory and for quantum biology.

A related observation is that a fundamental length scale of biology - size scale of neuron and axon - would correspond to the p-adic length scale assignable to vacuum energy density assignable to cosmological constant and be therefore a fundamental physics length scale.

Some Questions Related to the Twistor Lift of TGD

In this chapter I consider questions related to both classical and quantum aspects of twistorialization.

1. The first group of questions relates to the twistor lift of classical TGD. What does the induction of the twistor structure really mean? Can the analog of Kähler form assignable to M^4 suggested by the symmetry between M^4 and CP_2 and by number theoretical vision appear in the theory. What would be the physical implications? How does gravitational

coupling emerge at fundamental level? Could one regard the localization of spinor modes to string world sheets as a localization to Lagrangian sub-manifolds of space-time surface with vanishing induced Kähler form. Lagrangian sub-manifolds would be commutative in the sense of Poisson bracket. How this relates to the idea that string world sheets correspond complex (commutative) surfaces of quaternionic space-time surface in octonionic imbedding space?

During the re-processing of the details related to twistor lift, it became clear that the earlier variant for the twistor lift can be criticized and allows an alternative. This option led to a much simpler view about twistor lift, to the conclusion that minimal surface extremals of Kähler action represent only asymptotic situation (external particles in scattering), and also to a re-interpretation for the p-adic evolution of the cosmological constant: cosmological term would correspond to the *entire* 4-D action and the cancellation of Kähler action and cosmological term would lead to the small value of the effective cosmological constant.

2. Second group of questions relates to the construction of scattering amplitudes. The idea is to generalize the usual construction for massless states. In TGD all single particle states are massless in 8-D sense and this gives excellent hopes about the applicability of 8-D twistor approach. M^8-H duality turns out to be the key to the construction. Also the holomorphy of twistor amplitudes in helicity spinors λ_i and independence on $\tilde{\lambda}_i$ is crucial. The basic vertex corresponds to 4-fermion vertex for which the simplest expression can be written immediately. $n>4$-fermion scattering amplitudes can be also written immediately.

   If scattering diagrams correspond to computations as number theoretic vision suggests, the diagrams should be reducible to tree diagrams by moves generalizing the old-fashioned hadronic duality. This condition reduces to the vanishing of loops which in terms of BCFW recursion formula states that the twistor diagrams correspond to closed objects in what might be called WCFW homology.

The Recent View about Twistorialization in TGD Framework

The recent view about the twistorialization in TGD framework is discussed.

1. A proposal made already earlier is that scattering diagrams as analogs of twistor diagrams are constructible as tree diagrams for CDs connected by free particle lines. Loop contributions are not even well-defined in zero energy ontology (ZEO) and are in conflict with number theoretic vision. The coupling constant evolution would be discrete and associated with the scale of CDs (p-adic coupling constant evolution) and with the hierarchy of extensions of rationals defining the hierarchy of adelic physics.

2. Logarithms appear in the coupling constant evolution in QFTs. The identification of their number theoretic versions as rational number valued functions required by number-theoretical universality for both the integer characterizing the size scale of CD and for the hierarchy of Galois groups leads to an answer to a long-standing question what makes small primes and primes near powers of them physically special. The primes $p\in\{2,3,5\}$ indeed turn out to be special from the point of view of number theoretic logarithm.

3. The reduction of the scattering amplitudes to tree diagrams is in conflict with unitarity in 4-D situation. The imaginary part of the scattering amplitude would have discontinuity proportional to the scattering rate only for many-particle states with light-like total momenta. Scattering rates would vanish identically for the physical momenta for many-particle states.

   In TGD framework the states would be however massless in 8-D sense. Massless pole corresponds now to a continuum for M^4 mass squared and one would obtain the unitary cuts from a pole at $P^2=0$! Scattering rates would be non-vanishing only for many-particle states having light-like 8-momentum, which would pose a powerful condition on the construction of many-particle states.

   This idea does not make sense for incoming/outgoing particles, which light-like momenta unless they are parallel: their total momentum cannot be light-like in the general case. Rather, $P^2=0$ applies to the states formed inside CDs from groups of incoming and outgoing particles. BCFW deformation $p_i \to p_i + zr_i$ describes what happens for the single-particle momenta: they cease to be light-like but the total momenta for subgroups of particles in factorization channels are complex and light-like. This strong form of conformal symmetry has highly non-trivial implications concerning color confinement.

4. The key idea is number theoretical discretization in terms of "cognitive representations" as space-time time points with M^8-coordinates in an extension of rationals and therefore shared by both real and various p-adic sectors of the adele. Discretization realizes measurement resolution, which becomes an inherent aspect of physics rather than something forced by observed as outsider. This fixes the space-time surface completely as a zero locus of real or imaginary part of octonionic polynomial.

   This must imply the reduction of "world of classical worlds" (WCW) corresponding to a fixed number of points in the extension of rationals to a finite-dimensional discretized space with maximal symmetries and Kähler structure.

   The simplest identification for the reduced WCW would be as complex Grassmannian - a more general identification would be as a flag manifold. More complex options can of course be considered. The Yangian symmetries of the twistor Grassmann approach known to act as diffeomorphisms respecting the positivity of Grassmannian and emerging also in its TGD variant would have an interpretation as general coordinate invariance for the reduced WCW. This would give a completely unexpected connection with supersymmetric gauge theories and TGD.

5. M^8 picture implies the analog of SUSY realized in terms of polynomials of super-octonions whereas H picture suggests that supersymmetry is broken in the sense that many-fermion states as analogs of components of super-field at partonic 2-surfaces are not local. This requires breaking of SUSY. At M^8 level the breaking could be due to the reduction of Galois group to its subgroup G/H, where H is normal subgroup leaving the point of cognitive representation defining space-time surface invariant. As a consequence, local many-fermion composite in M^8 would be mapped to a non-local one in H by $M^8 - H$ correspondence.

1.4.6 Part IV: Category theory and TGD

Category Theory and Quantum TGD

Possible applications of category theory to quantum TGD are discussed. The so called 2-plectic structure generalizing the ordinary symplectic structure by replacing symplectic 2-form with 3-form and Hamiltonians with Hamiltonian 1-forms has a natural place in TGD since the dynamics of the light-like 3-surfaces is characterized by Chern-Simons type action. The notion of planar operad was developed for the classification of hyper-finite factors of type II_1 and its mild generalization allows to understand the combinatorics of the generalized Feynman diagrams obtained by gluing 3-D light-like surfaces representing the lines of Feynman diagrams along their 2-D ends representing the vertices.

The fusion rules for the symplectic variant of conformal field theory, whose existence is strongly suggested by quantum TGD, allow rather precise description using the basic notions of category theory and one can identify a series of finite-dimensional nilpotent algebras as discretized versions of field algebras defined by the fusion rules. These primitive fusion algebras can be used to construct more complex algebras by replacing any algebra element by a primitive fusion algebra. Trees with arbitrary numbers of branches in any node characterize the resulting collection of fusion algebras forming an operad. One can say that an exact solution of symplectic scalar field theory is obtained.

Conformal fields and symplectic scalar field can be combined to form symplecto-formal fields. The combination of symplectic operad and Feynman graph operad leads to a construction of Feynman diagrams in terms of n-point functions of conformal field theory. M-matrix elements with a finite measurement resolution are expressed in terms of a hierarchy of symplecto-conformal n-point functions such that the improvement of measurement resolution corresponds to an algebra homomorphism mapping conformal fields in given resolution to composite conformal fields in improved resolution. This expresses the idea that composites behave as independent conformal fields. Also other applications are briefly discussed.

Years after writing this chapter a very interesting new TGD related candidate for a category emerged. The preferred extremals of Kähler action would form a category if the proposed duality mapping associative (co-associative) 4-surfaces of imbedding space respects associativity (co-associativity). The duality would allow to construct new preferred extremals of Kähler action.

Could categories, tensor networks, and Yangians provide the tools for handling the complexity of TGD?

TGD Universe is extremely simple locally but the presence of various hierarchies make it to look extremely complex globally. Category theory and quantum groups, in particular Yangian or its TGD generalization are most promising tools to handle this complexity. The arguments developed in the sequel suggest the following overall view.

1. Positive and negative energy parts of zero energy states can be regarded as tensor networks identifiable as categories. The new element is that one does not have only particles (objects) replaced with partonic 2-surfaces but also strings connecting them (morphisms). Morphisms and functors provide a completely new element not present in standard model. For instance, S-matrix would be a functor between categories. Various hierarchies of of TGD would in turn translate to hierarchies of categories.

2. TGD view about generalized Feynman diagrams relies on two general ideas. First, the twistor lift of TGD replaces space-time surfaces with their twistor-spaces getting their twistor structure as induced twistor structure from the product of twistor spaces of M^4 and CP_2. Secondly, topological scattering diagrams are analogous to computations and can be reduced to tree diagrams with braiding. This picture fits very nicely with the picture suggested by fusion categories. At fermionic level the basic interaction is 2+2 scattering of fermions occurring at the vertices identifiable as partonic 2-surface and re-distributes the fermion lines between partonic 2-surfaces. This interaction is highly analogous to what happens in braiding interaction but vertices expressed in terms of twistors depend on momenta of fermions.

3. Braiding transformations take place inside the light-like orbits of partonic 2-surfaces defining boundaries of space-time regions with Minkowskian and Euclidian signature of induced metric respectively permuting two braid strands. R-matrix satisfying Yang-Baxter equation characterizes this operation algebraically.

4. Reconnections of fermionic strings connecting partonic 2-surfaces are possible and suggest interpretation in terms of 2-braiding generalizing ordinary braiding: string world sheets get knotted in 4-D space-time forming 2-knots and strings form 1-knots in 3-D space. Reconnection induces an exchange of braid strands defined by the boundaries of the string world sheet and therefore exchange of fermion lines defining boundaries of string world sheets. A generalization of quantum algebras to include also algebraic representation for reconnection is needed. Also reconnection might reduce to a braiding type operation.

Yangians look especially natural quantum algebras from TGD point of view. They are bi-algebras with co-product Δ. This makes the algebra multi-local raising hopes about the understanding of bound states. Δ-iterates of single particle system would give many-particle systems with non-trivial interactions reducing to kinematics.

One should assign Yangian to various Kac-Moody algebras (SKMAs) involved and even with super-conformal algebra (SSA), which however reduces effectively to SKMA for finite-dimensional Lie group if the proposed gauge conditions meaning vanishing of Noether charges for some subalgebra H of SSA isomorphic to it and for its commutator $[SSA, H]$ with the entire SSA. Strong form of holography (SH) implying almost 2-dimensionality motivates these gauge conditions. Each SKMA would define a direct summand with its own parameter defining coupling constant for the interaction in question.

Are higher structures needed in the categorification of TGD?

The notion of higher structures promoted by John Baez looks very promising notion in the attempts to understand various structures like quantum algebras and Yangians in TGD framework. The stimulus for this article came from the nice explanations of the notion of higher structure by Urs Screiber. The basic idea is simple: replace "=" as a blackbox with an operational definition with a proof for $A = B$. This proof is called homotopy generalizing homotopy in topological sense. n-structure emerges when one realizes that also the homotopy is defined only up to homotopy in turn defined only up...

In TGD framework the notion of measurement resolution defines in a natural manner various kinds of "="s and this gives rise to resolution hierarchies. Hierarchical structures are characteristic for TGD: hierarchy of space-time sheet, hierarchy of p-adic length scales, hierarchy of Planck constants and dark matters, hierarchy of inclusions of hyperfinite factors, hierarchy of extensions of rationals defining adeles in adelic TGD and corresponding hierarchy of Galois groups represented geometrically, hierarchy of infinite primes, self hierarchy, etc...

In this article the idea of n-structure is studied in more detail. A rather radical idea is a formulation of quantum TGD using only cognitive representations consisting of points of space-time surface with imbedding space coordinates in extension of rationals defining the level of adelic hierarchy. One would use only these discrete points sets and Galois groups. Everything would reduce to number theoretic discretization at space-time level perhaps reducing to that at partonic 2-surfaces with points of cognitive representation carrying fermion quantum numbers.

Even the "world of classical worlds " (WCW) would discretize: cognitive representation would define the coordinates of WCW point. One would obtain cognitive representations of scattering amplitudes using a fusion category assignable to the representations of Galois groups: something diametrically opposite to the immense complexity of the WCW but perhaps consistent with it. Also a generalization of McKay's correspondence suggests itself: only those irreps of the Lie group associated with Kac-Moody algebra that remain irreps when reduced to a subgroup defined by a Galois group of Lie type are allowed as ground states. Also the relation to number theoretic Langlands correspondence is very interesting.

Is Non-associative Physics and Language Possible Only in Many-Sheeted Space-Time?

Language is an essentially non-associative structure as the necessity to parse linguistic expressions essential also for computation using the hierarchy of brackets makes obvious. Hilbert space operators are associative so that non-associative quantum physics does not seem plausible without an extension of what one means with physics. Associativity of the classical physics at the level of *single* space-time sheet in the sense that tangent or normal spaces of space-time sheets are associative as sub-spaces of the octonionic tangent space of 8-D imbedding space $M^4 \times CP_2$ is one of the key conjectures of TGD. But what about many-sheeted space-time? The sheets of the many-sheeted space-time form hierarchies labelled by p-adic primes and values of Planck constants $h_{eff} = n \times h$. Could these hierarchies provide space-time correlates for the parsing hierarchies of language and music, which in TGD framework can be seen as kind of dual for the spoken language? For instance, could the braided flux tubes inside larger braided flux tubes inside... realize the parsing hierarchies of language, in particular topological quantum computer programs? And could the great differences between organisms at very different levels of evolution but having very similar genomes be understood in terms of widely different numbers of levels in the parsing hierarchy of braided flux tubes- that is in terms of magnetic bodies as indeed proposed. If the intronic portions of DNA connected by magnetic flux tubes to the lipids of lipid layers of nuclear and cellular membranes make them topological quantum computers, the parsing hierarchy could be realized at the level of braided magnetic bodies of DNA. The mathematics needed to describe the breaking of associativity at fundamental level seems to exist. The hierarchy of braid group algebras forming an operad combined with the notions of quasi-bialgebra and quasi-Hopf algebra discovered by Drinfeld are highly suggestive concerning the realization of weak breaking of associativity.

1.4.7 Part V: Miscellaneous topics

Does the QFT Limit of TGD Have Space-Time Super-Symmetry?

Contrary to the original expectations, TGD seems to allow a generalization of the space-time super-symmetry. This became clear with the increased understanding of the Kähler-Dirac action. The introduction of a measurement interaction term to the action allows to understand how stringy propagator results and provides profound insights about physics predicted by TGD.

The appearance of the momentum (and possibly also color quantum numbers) in the measurement interaction couples space-time degrees of freedom to quantum numbers and allows also to define SUSY algebra at fundamental level as anti-commutation relations of fermionic oscillator operators. Depending on the situation a finite-dimensional SUSY algebra or the fermionic part of super-conformal algebra with an infinite number of oscillator operators results. The addition of a fermion in particular mode would define particular super-symmetry. Zero energy ontology implies that fermions as wormhole throats correspond to chiral super-fields assignable to positive or negative energy SUSY algebra whereas bosons as wormhole contacts with two throats correspond to the direct sum of positive and negative energy algebra and fields which are chiral or antichiral with respect to both positive and negative energy theta parameters. This super-symmetry is badly broken due to the dynamics of the Kähler-Dirac operator which also mixes M^4 chiralities inducing massivation. Since righthanded neutrino has no electro-weak couplings the breaking of the corresponding super-symmetry should be weakest.

The question is whether this SUSY has a realization as a SUSY algebra at space-time level and whether the QFT limit of TGD could be formulated as a generalization of SUSY QFT. There are several problems involved.

1. In TGD framework super-symmetry means addition of fermion to the state and since the number of spinor modes is larger states with large spin and fermion numbers are obtained. This picture does not fit to the standard view about super-symmetry. In particular, the identification of theta parameters as Majorana spinors and super-charges as Hermitian operators is not possible.

2. The belief that Majorana spinors are somehow an intrinsic aspect of super-symmetry is however only a belief. Weyl spinors meaning complex theta parameters are also possible. Theta parameters can also carry fermion number meaning only the supercharges carry fermion number and are non-hermitian. The general classification of super-symmetric theories indeed demonstrates that for $D = 8$ Weyl spinors and complex and non-hermitian super-charges are possible. The original motivation for Majorana spinors might come from MSSM assuming that right handed neutrino does not exist. This belief might have also led to string theories in D=10 and D=11 as the only possible candidates for TOE after it turned out that chiral anomalies cancel.

3. The massivation of particles is basic problem of both SUSYs and twistor approach. The fact that particles which are massive in M^4 sense can be interpreted as massless particles in $M^4 \times CP_2$ suggests a manner to understand super-symmetry breaking and massivation in TGD framework. The octonionic realization of twistors is one possibility in this framework and quaternionicity condition guaranteeing associativity leads to twistors which are almost equivalent with ordinary 4-D twistors.

4. The first approach is based on an approximation assuming only the super-multiplets generated by right-handed neutrino or both right-handed neutrino and its antineutrino. The assumption that right-handed neutrino has fermion number opposite to that of the fermion associated with the wormhole throat implies that bosons correspond to $\mathcal{N} = (1,1)$ SUSY and fermions to $\mathcal{N} = 1$ SUSY identifiable also as a short representation of $\mathcal{N} = (1,1)$ SUSY algebra trivial with respect to positive or negative energy algebra. This means a deviation from the standard view but the standard SUSY gauge theory formalism seems to apply in this case.

5. A more ambitious approach would put the modes of induced spinor fields up to some cutoff into super-multiplets. At the level next to the one described above the lowest modes of the induced spinor fields would be included. The very large value of $\mathcal{N}$ means that $\mathcal{N} \leq \ni\in$ SUSY cannot define the QFT limit of TGD for higher cutoffs. One must generalize SUSYs gauge theories to arbitrary value of $\mathcal{N}$ but there are reasons to expect that the formalism becomes rather complex. More ambitious approach working at TGD however suggest a more general manner to avoid this problem.

   (a) One of the key predictions of TGD is that gauge bosons and Higgs can be regarded as bound states of fermion and antifermion located at opposite throats of a wormhole contact. This implies bosonic emergence meaning that it QFT limit can be defined in terms of Dirac action. The resulting theory was discussed in detail in [K54] and it was shown that bosonic propagators and vertices can be constructed as fermionic loops so that all coupling constant follow as predictions. One must however pose cutoffs in mass squared and hyperbolic angle assignable to the momenta of fermions appearing in the loops in order to obtain finite theory and to avoid massivation of bosons. The resulting coupling constant evolution is consistent with low energy phenomenology if the cutoffs in hyperbolic angle as a function of p-adic length scale is chosen suitably.

(b) The generalization of bosonic emergence that the TGD counterpart of SUSY is obtained by the replacement of Dirac action with action for chiral super-field coupled to vector field as the action defining the theory so that the propagators of bosons and all their super-counterparts would emerge as fermionic loops.

(c) The huge super-symmetries give excellent hopes about the cancelation of infinities so that this approach would work even without the cutoffs in mass squared and hyperbolic angle assignable to the momenta of fermions appearing in the loops. Cutoffs have a physical motivation in zero energy ontology but it could be an excellent approximation to take them to infinity. Alternatively, super-symmetric dynamics provides cutoffs dynamically.

6. The condition that $\mathcal{N} = \infty$ variants for chiral and vector superfields exist fixes completely the identification of these fields in zero energy ontology.

(a) In this framework chiral fields are generalizations of induced spinor fields and vector fields those of gauge potentials obtained by replacing them with their super-space counterparts. Chiral condition reduces to analyticity in theta parameters thanks to the different definition of hermitian conjugation in zero energy ontology (θ is mapped to a derivative with respect to theta rather than to $\overline{\theta}$) and conjugated super-field acts on the product of all theta parameters.

(b) Chiral action is a straightforward generalization of the Dirac action coupled to gauge potentials. The counterpart of YM action can emerge only radiatively as an effective action so that the notion emergence is now unavoidable and indeed basic prediction of TGD.

(c) The propagators associated with the monomials of n theta parameters behave as $1/p^n$ so that only $J = 0, 1/2, 1$ states propagate in normal manner and correspond to normal particles. The presence of monomials with number of thetas higher than 2 is necessary for the propagation of bosons since by the standard argument fermion and scalar loops cancel each other by super-symmetry. This picture conforms with the identification of graviton as a bound state of wormhole throats at opposite ends of string like object.

(d) This formulation allows also to use Kähler-Dirac gamma matrices in the measurement interaction defining the counterpart of super variant of Dirac operator. Poincare invariance is not lost since momenta and color charges act on the tip of CD rather than the coordinates of the space-time sheet. Hence what is usually regarded as a quantum theory in the background defined by classical fields follows as exact theory. This feeds all data about space-time sheet associated with the maximum of Kähler function. In this approach WCW as a Kähler manifold is replaced by a cartesian power of CP_2, which is indeed quaternionic Kähler manifold. The replacement of light-like 3-surfaces with number theoretic braids when finite measurement resolution is introduced, leads to a similar replacement.

(e) Quantum TGD as a "complex square root" of thermodynamics approach suggests that one should take a superposition of the amplitudes defined by the points of a coherence region (identified in terms of the slicing associated with a given wormhole throat) by weighting the points with the Kähler action density. The situation would be highly analogous to a spin glass system since the Kähler-Dirac gamma matrices defining the propagators would be analogous to the parameters of spin glass Hamiltonian allowed to have a spatial dependence. This would predict the proportionality of the coupling strengths to Kähler coupling strength and bring in the dependence on the size of CD coming as a power of 2 and give rise to p-adic coupling constant evolution. Since TGD Universe is analogous to 4-D spin glass, also a sum over different preferred extremals assignable to a given coherence regions and weighted by $exp(K)$ is probably needed.

(f) In TGD Universe graviton is necessarily a bi-local object and the emission and absorption of graviton are bi-local processes involving two wormhole contacts: a pair of particles rather than single particle emits graviton. This is definitely something new and defies a description in terms of QFT limit using point like particles. Graviton like states would be entangled states of vector bosons at both ends of stringy curve so that gravitation could be regarded as a square of YM interactions in rather concrete sense. The notion of emergence would suggest that graviton propagator is defined by a bosonic loop. Since bosonic loop is dimensionless, IR cutoff defined by the largest CD present must be actively involved. At QFT limit one can hope a description as a bi-local process using a bi-local generalization of the QFT limit. It turns out that surprisingly simple candidate for the bi-local action exists.

This statement has become somewhat misleading. It has turned out that all elementary particle in TGD framework are bi-local objects: one can assign to them both closed magnetic flux tubes behaving like strings and closed strings carrying fermion number. For other elementary particles than graviton second wormhole contact carries only neutrino pair neutralizing electroweak-isospin so that above weak scale they correspond to single em charged wormhole contact.

Coupling Constant Evolution in Quantum TGD

How to calculate or at least "understand" the correlation functions and coupling constant evolution has remained a basic unresolved challenge. Basically the unability to calculate is of course due to the lack of understanding.

Zero energy ontology, the construction of M-matrix as time like entanglement coefficients defining Connes tensor product characterizing finite measurement resolution in terms of inclusion of hyper-finite factors of type II_1, the realization that symplectic invariance of N-point functions providing a detailed mechanism eliminating UV divergences, and the understanding of the relationship between super-symplectic and super Kac-Moody symmetries. p-Adic length scale hypothesis suggests that continuous coupling constant evolution is replaced by discrete p-adic coupling constant evolution and that number theoretical constraints are of crucial importance. These are the pieces of the puzzle whose combination makes possible a rather concrete vision about coupling constant evolution in TGD Universe and one can even speak about rudimentary form of generalized Feynman rules. This was the picture behind previous updating.

Several steps of progress have however occurred since then.

1. A crucial step in progress has been the understanding of how GRT space-time emerges from the many-sheeted space-time of TGD. At classical level Equivalence Principle (EP) follows from the interpretation of GRT space-time as effective space-time obtained by replacing many-sheeted space-time with Minkowski space with effective metric determined as a sum of Minkowski metric and sum over the deviations of the induced metrices of space-time sheets from Minkowski metric. Poincare invariance suggests strongly classical EP for the GRT limit in long length scales at least. One can consider also other kinds of limits such as the analog of GRT limit for Euclidian space-time regions assignable to elementary particles. In this case deformations of CP_2 metric define a natural starting point and CP_2 indeed defines a gravitational instanton with very large cosmological constant in Einstein-Maxwell theory. Also gauge potentials of standard model correspond classically to superpositions of induced gauge potentials over space-time sheets.

2. Second powerful idea is quantum classical correspondence in statistical sense stating that the statistical properties of a preferred extremal in quantum superposition of them are same as those of the zero energy state in question. This principle would be quantum generalization of ergodic theorem stating that the time evolution of a single member of ensemble represents the ensemble statistically. This principle would allow to deduce correlation functions and

S-matrix from the statistical properties of single preferred extremal alone using classical intuition. Also coupling constant evolution would be coded by the statistical properties of the representative preferred extremal.

This idea can be formulated more convincingly in terms of a generalization of the AdS/CFT duality to TGD framework motivated by the generalization of conformal symmetry. In full generality this principle would state that all predictions of the theory can be expressed either in terms of classical fields in the interior of the space-time surface or in terms of scattering amplitudes formulated in terms of fundamental fermions defining the building bricks of elementary particles. The implication would that correlation functions can be also identified as those for classical induce gauge and gravitational fields.

3. Third powerful vision inspired by the notion of preferred extremal - I gave up the vision for years as too crazy - is that scattering amplitudes correspond to sequences of computations and that all computations connecting collectings of algebraic objects produce same scattering amplitudes [K76]. All scattering amplitudes could be reduced to minimal tree diagrams by moving the ends of the lines and snipping away the loops. The 8-D generalization of twistor approach to TGD allows to identify the arithmetics as that of super-symplectic Yangian and basic vertices in the construction correspond to product and co-product in Yangian.

4. The fourth new ingredient is the dramatic increase in the understanding of the hierarchy of Planck constants $h_{eff} = n \times h$. The hierarchy corresponds to hierarchy of quantum criticalities at which the sub-algebra of super-symplectic algebra with natural conformal structure changes. Sub-algebras are labelled by integer n: the conformal weights of the sub-algebra come as multiples of n. One has infinite number of hierarchies $n_{i+1} = \prod_{k<i+1} m_k$ which relate naturally to the hierarchies of inclusions of hyper-finite factors. The sub-algebra acts as gauge symmetries whereas the other generators of the full algebra fail to do so. Therefore the increase of n means that gauge degrees of freedom become physical ones. One can assign coupling constant evolution also with these hierarchies and the natural conjecture is that coupling constants for given value of n are renormalization group invariances.

   Especially intereting are the implications for the understanding of gravitational binding assuming that strings connecting partonic 2-surfaces are responsible for the formation of bound states. This leads together with the generalization of AdS/CFT corresponds and localization of fermions to string world sheets to a prediction that Kähler action is expressible as string area in the effective metric defined by the anti-commutators of Kähler-Dirac gamma matrices. This predicts that the size scale of bound states scales as h_{eff} and it is possible to obtain bound states of macroscopic size unlike for ordinary string area action for which their sizes would be given by Planck length.

Part I

THE RECENT VIEW ABOUT FIELD EQUATIONS

Chapter 2

Basic Extremals of the Kähler Action

2.1 Introduction

In this chapter the classical field equations associated with the Kähler action are studied. The study of the extremals of the Kähler action has turned out to be extremely useful for the development of TGD. Towards the end of year 2003 quite dramatic progress occurred in the understanding of field equations and it seems that field equations might be in well-defined sense exactly solvable. The progress made during next five years led to a detailed understanding of quantum TGD at the fundamental parton level and this provides considerable additional insights concerning the interpretation of field equations.

2.1.1 About The Notion Of Preferred Extremal

The notion of preferred extremal has been central in classical TGD although the known solutions could be preferred or not: the main challenge has been to understand what "preferred" could mean.

In zero energy ontology (ZEO) one can also consider the releaving possibility that all extremals are preferred ones! The two space-like 3-surfaces at the ends of CD define the space-time surface connecting them apart from conformal symmetries acting as critical deformations. If 3-surface is identified as union of both space-like 3-surfaces and the light-like surfaces defining parton orbits connecting then, the conformal equivalence class of the preferred extremal is unique without any additional conditions! This conforms with the view about hierarchy of Planck constants requiring that the conformal equivalence classes of light-like surfaces must be counted as physical degrees of freedom and also with the idea that these surface together define analog for the Wilson loop. The non-determinism of Kähler action suggests that "preferred" could be obsolete in given length scale resolution.

Actually all the discussions of this chapter are about known extremals in general so that the attribute "preferred" is not relevant for them.

2.1.2 Beltrami Fields And Extremals

The vanishing of Lorentz 4-force for the induced Kähler field means that the vacuum 4-currents are in a mechanical equilibrium. Lorentz 4-force vanishes for all known solutions of field equations which inspires the hypothesis that preferred extremals satisfy the condition. The vanishing of the Lorentz 4-force in turn implies a local conservation of the ordinary energy momentum tensor. The corresponding condition is implied by Einstein's equations in General Relativity. The hypothesis would mean that the solutions of field equations are what might be called generalized Beltrami fields. If Kähler action is defined by CP_2 Kähler form alone, the condition implies that vacuum currents can be non-vanishing only provided the dimension D_{CP_2} of the CP_2 projection of the space-time surface is less than four so that in the regions with $D_{CP_2} = 4$, Maxwell's vacuum equations are satisfied.

The hypothesis that Kähler current is proportional to a product of an arbitrary function ψ of CP_2 coordinates and of the instanton current generalizes Beltrami condition and reduces to it when electric field vanishes. Instanton current has vanishing divergence for $D_{CP_2} < 4$, and Lorentz 4-force indeed vanishes. The remaining task would be the explicit construction of the imbeddings of these fields and the demonstration that field equations can be satisfied.

Under additional conditions magnetic field reduces to what is known as Beltrami field. Beltrami fields are known to be extremely complex but highly organized structures. The natural conjecture is that topologically quantized many-sheeted magnetic and Z^0 magnetic Beltrami fields and their generalizations serve as templates for the helical molecules populating living matter, and explain both chirality selection, the complex linking and knotting of DNA and protein molecules, and even the extremely complex and self-organized dynamics of biological systems at the molecular level.

Field equations can be reduced to algebraic conditions stating that energy momentum tensor and second fundamental form have no common components (this occurs also for minimal surfaces in string models) and only the conditions stating that Kähler current vanishes, is light-like, or proportional to instanton current, remain and define the remaining field equations. The conditions guaranteeing topologization to instanton current can be solved explicitly. Solutions can be found also in the more general case when Kähler current is not proportional to instanton current. On basis of these findings there are strong reasons to believe that classical TGD is exactly solvable.

An important outcome is the notion of Hamilton-Jacobi structure meaning dual slicings of M^4 projection of preferred extremals to string world sheets and partonic 2-surfaces. The necessity of this slicing was discovered years later from number theoretic compactification and is now a key element of quantum TGD allowing to deduce Equivalence Principle in its stringy form from quantum TGD and formulate and understand quantum TGD in terms of Kähler-Dirac action assignable to Kähler action. The conservation of Noether charges associated with Kähler-Dirac action requires the vanishing of the second second variation of Kähler action for preferred extremals. Preferred extremals would thus define space-time representation for quantum criticality. Infinite-dimensional variant for the hierarchy of criticalities analogous to the hierarchy assigned to the extrema of potential function with levels labeled by the rank of the matrix defined by the second derivatives of the potential function in catastrophe theory would suggest itself.

A natural interpretation for deformations would be as conformal gauge symmetries due to the non-determinism of Kähler action. They would transform to each other preferred extremals having fixed 3-surfaces as ends at the boundaries of the causal diamond. They would preserve the value of Kähler action and those of conserved charges. The assumption is that there are n gauge equivalence classes of these surfaces and that n defines the value of the effective Planck constant $h_{eff} = n \times h$ in the effective GRT type description replacing many-sheeted space-time with single sheeted one.

2.1.3 In What Sense Field Equations Could Mimic Dissipative Dynamics?

By quantum classical correspondence the non-deterministic space-time dynamics should mimic the dissipative dynamics of the quantum jump sequence. The nontrivial question is what this means in TGD framework.

1. Beltrami fields appear in physical applications as asymptotic self organization patterns for which Lorentz force and dissipation vanish. This suggests that preferred extremals of Kähler action correspond to space-time sheets which at least asymptotically satisfy generalized Beltrami conditions so that one can indeed assign to the final (rather than initial!) 3-surface a unique 4-surface apart from effects related to non-determinism. Preferred extremal property of Kähler action abstracted to purely algebraic generalized Beltrami conditions would make sense also in the p-adic context. The general solution ansatz discussed in the last section of the chapter assumes that all conserved isometry currents are proportional to instanton current so that various charges are conserved separately for all flow lines: this means essentially the integrability of the theory. This ansatz is forced by the hypothesis that TGD reduces to almost topological QFT and this idea. The basic consequence is that dissipation is impossible classically.

2. A more radical view inspired by zero energy ontology is that the light-like 3-surfaces and corresponding space-time regions with Euclidian signature defining generalized Feynman diagrams provide a space-time representation of dissipative dynamics just as they provide this representation in quantum field theory. Minkowskian regions would represent empty space so that the vanishing of Lorentz 4-force and absence of dissipation would be natural. This would mean very precise particle field duality and the topological pattern associated with the generalized Feynman diagram would represent dissipation. One could also interprete dissipation as transfer of energy between sheets of the many-sheeted space time and thus as an essentially topological phenomenon. This option seems to be the only viable one.

2.1.4 The Dimension Of CP_2 Projection As Classifier For The Fundamental Phases Of Matter

The dimension D_{CP_2} of CP_2 projection of the space-time sheet encountered already in p-adic mass calculations classifies the fundamental phases of matter. For $D_{CP_2} = 4$ empty space Maxwell equations hold true. The natural guess would be that this phase is chaotic and analogous to de-magnetized phase. $D_{CP_2} = 2$ phase is analogous to ferromagnetic phase: highly ordered and relatively simple. It seems however that preferred extremals can correspond only to small perturbations of these extremals resulting by topological condensation of CP_2 type vacuum extremals and through topological condensation to larger space-time sheets. $D_{CP_2} = 3$ is the analog of spin glass and liquid crystal phases, extremely complex but highly organized by the properties of the generalized Beltrami fields. This phase could be seen as the boundary between chaos and order and corresponds to life emerging in the interaction of magnetic bodies with bio-matter. It is possible only in a finite temperature interval (note however the p-adic hierarchy of critical temperatures) and characterized by chirality just like life.

The original proposal was that $D(CP_2) = 4$ phase is completely chaotic. This is not true if the reduction to almost topological QFT takes place. This phase must correspond to Maxwellian phase with a vanishing Kähler current as concluded already earlier. Various isometry currents are however proportional to the instanton current and conserved along the flow lines of the instanton current whose flow parameter extends to a global coordinate. Hence a completely chaotic phase is not in question even in this case.

2.1.5 Specific Extremals Of Kähler Action

The study of extremals of Kähler action represents more than decade old layer in the development of TGD.

1. The huge vacuum degeneracy is the most characteristic feature of Kähler action (any 4-surface having CP_2 projection which is Legendre sub-manifold is vacuum extremal, Legendre sub-manifolds of CP_2 are in general 2-dimensional). This vacuum degeneracy is behind the spin glass analogy and leads to the p-adic TGD. As found in the second part of the book, various particle like vacuum extremals also play an important role in the understanding of the quantum TGD.

2. The so called CP_2 type vacuum extremals have finite, negative action and are therefore an excellent candidate for real particles whereas vacuum extremals with vanishing Kähler action are candidates for the virtual particles. These extremals have one dimensional M^4 projection, which is light like curve but not necessarily geodesic and locally the metric of the extremal is that of CP_2: the quantization of this motion leads to Virasoro algebra. Space-times with topology $CP_2\#CP_2\#...CP_2$ are identified as the generalized Feynman diagrams with lines thickened to 4-manifolds of "thickness" of the order of CP_2 radius. The quantization of the random motion with light velocity associated with the CP_2 type extremals in fact led to the discovery of Super Virasoro invariance, which through the construction of the configuration space geometry, becomes a basic symmetry of quantum TGD.

3. There are also various non-vacuum extremals.

(a) String like objects, with string tension of same order of magnitude as possessed by the cosmic strings of GUTs, have a crucial role in TGD inspired model for the galaxy formation and in the TGD based cosmology.

(b) The so called massless extremals describe non-linear plane waves propagating with the velocity of light such that the polarization is fixed in given point of the space-time surface. The purely TGD:eish feature is the light like Kähler current: in the ordinary Maxwell theory vacuum gauge currents are not possible. This current serves as a source of coherent photons, which might play an important role in the quantum model of bio-system as a macroscopic quantum system.

(c) In the so called Maxwell phase, ordinary Maxwell equations for the induced Kähler field would be satisfied in an excellent approximation. It is however far from clear whether this kind of extremals exist. Their non-existence would actually simplify the theory enormously since all extremals would have quantal character. The recent view indeed is that Maxwell phase makes sense only as as genuinely many-sheeted structure and solutions of Maxwell's equation appear only at the level of effective space-time obtained by replacing many-sheeted space-time with Minkowski space with effective metric determined as a sum of Minkowski metric and sum over the deviations of the induced metrices of space-time sheets from Minkowski metric. Gauge potentials in effective space-time are determined in the same manner. Since the gauge potentials sum up, it is possible to understand how field configurations of Maxwell's theory emerge at this limit.

2.1.6 The Weak Form Of Electric-Magnetic Duality And Modification Of Kähler Action

The newest results discussed in the last section about the weak form of electric-magnetic duality suggest strongly that Beltrami property is general and together with the weak form of electric-magnetic duality allows a reduction of quantum TGD to almost topological field theory with Kähler function allowing expression as a Chern-Simons term.

Generalized Beltrami property leads to a rather explicit construction of the general solution of field equations based on the hydrodynamic picture implying that single particle quantum numbers are conserved along flow lines defined by the instanton current. The construction generalizes also to the fermionic sector and there are reasons to hope that TGD is completely integrable theory.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [L12].

2.2 General Considerations

The solution families of field equations studied in this chapter were found already during eighties. The physical interpretation turned out to be the really tough problem. What is the principle selecting preferred extremals of Kähler action as analogs of Bohr orbits assigning to 3-surface X^3 a unique space-time surface $X^4(X^3)$? Does Equivalence Principle hold true and if so, in what sense? These have been the key questions. The realization that light-like 3-surfaces X^3_l associated with the light-like wormhole throats at which the signature of the induced metric changes from Minkowskian to Euclidian led to the formulation of quantum TGD in terms of second quantized induced spinor fields at these surfaces. Together with the notion of number theoretical compactification this approach allowed to identify the conditions characterizing the preferred extremals. What is remarkable that these conditions are consistent with what is known about extremals.

Also a connection with string models emerges and partial understanding of the space-time realization of Equivalence Principle suggests itself. However, much more general argument allows to understand how GRT space-time appears from the many-sheeted space-time of TGD (see **Fig.** `http://tgdtheory.fi/appfigures/manysheeted.jpg` or **Fig.** 9 in the appendix of this book) as effective concept [K79]: this more general view is not in conflict with the much earlier proposal discussed below.

In this section the theoretical background behind field equations is briefly summarized. I will not repeat the discussion of previous two chapters [K29, K30] summarizing the general vision about many-sheeted space-time, and consideration will be restricted to those aspects of vision leading to direct predictions about the properties of preferred extremals of Kähler action.

2.2.1 Number Theoretical Compactification And $M^8 - H$ Duality

The notion of hyper-quaternionic and octonionic manifold makes sense but it not plausible that $H = M^4 \times CP_2$ could be endowed with a hyper-octonionic manifold structure. Situation changes if H is replaced with hyper-octonionic M^8. Suppose that $X^4 \subset M^8$ consists of hyper-quaternionic and co-hyper-quaternionic regions. The basic observation is that the hyper-quaternionic sub-spaces of M^8 with a fixed hyper-complex structure (containing in their tangent space a fixed hyper-complex subspace M^2 or at least one of the light-like lines of M^2) are labeled by points of CP_2. Hence each hyper-quaternionic and co-hyper-quaternionic four-surface of M^8 defines a 4-surface of $M^4 \times CP_2$. One can loosely say that the number-theoretic analog of spontaneous compactification occurs: this of course has nothing to do with dynamics.

This picture was still too naive and it became clear that not all known extremals of Kähler action contain fixed $M^2 \subset M^4$ or light-like line of M^2 in their tangent space.

1. The first option represents the minimal form of number theoretical compactification. M^8 is interpreted as the tangent space of H. Only the 4-D tangent spaces of light-like 3-surfaces X^3_l (wormhole throats or boundaries) are assumed to be hyper-quaternionic or co-hyper-quaternionic and contain fixed M^2 or its light-like line in their tangent space. Hyper-quaternionic regions would naturally correspond to space-time regions with Minkowskian signature of the induced metric and their co-counterparts to the regions for which the signature is Euclidian. What is of special importance is that this assumption solves the problem of identifying the boundary conditions fixing the preferred extremals of Kähler action since in the generic case the intersection of M^2 with the 3-D tangent space of X^3_l is 1-dimensional. The surfaces $X^4(X^3_l) \subset M^8$ would be hyper-quaternionic or co-hyper-quaternionic but would not allow a local mapping between the 4-surfaces of M^8 and H.

2. One can also consider a more local map of $X^4(X^3_l) \subset H$ to $X^4(X^3_l) \subset M^8$. The idea is to allow $M^2 \subset M^4 \subset M^8$ to vary from point to point so that $S^2 = SO(3)/SO(2)$ characterizes the local choice of M^2 in the interior of X^4. This leads to a quite nice view about strong geometric form of $M^8 - H$ duality in which M^8 is interpreted as tangent space of H and $X^4(X^3_l) \subset M^8$ has interpretation as tangent for a curve defined by light-like 3-surfaces at X^3_l and represented by $X^4(X^3_l) \subset H$. Space-time surfaces $X^4(X^3_l) \subset M^8$ consisting of hyper-quaternionic and co-hyper-quaternionic regions would naturally represent a preferred extremal of E^4 Kähler action. The value of the action would be same as CP_2 Kähler action. $M^8 - H$ duality would apply also at the induced spinor field and at the level of WCW . The possibility to assign $M^2(x) \subset M^4$ to each point of M^4 projection $P_{M^4}(X^4(X^3_l))$ is consistent with what is known about extremals of Kähler action with only one exception: CP_2 type vacuum extremals. In this case M^2 can be assigned to the normal space.

3. Strong form of $M^8 - H$ duality satisfies all the needed constraints if it represents Kähler isometry between $X^4(X^3_l) \subset M^8$ and $X^4(X^3_l) \subset H$. This implies that light-like 3-surface is mapped to light-like 3-surface and induced metrics and Kähler forms are identical so that also Kähler action and field equations are identical. The only differences appear at the level of induced spinor fields at the light-like boundaries since due to the fact that gauge potentials are not identical.

4. The map of $X^3_l \subset H \rightarrow X^3_l \subset M^8$ would be crucial for the realization of the number theoretical universality. $M^8 = M^4 \times E^4$ allows linear coordinates as those preferred coordinates in which the points of imbedding space are rational/algebraic. Thus the point of $X^4 \subset H$ is algebraic if it is mapped to algebraic point of M^8 in number theoretic compactification. This of course restricts the symmetry groups to their rational/algebraic variants but this does not have practical meaning. Number theoretical compactification could thus be motivated by the number theoretical universality.

5. The possibility to use either M^8 or H picture might be extremely useful for calculational purposes. In particular, M^8 picture based on $SO(4)$ gluons rather than $SU(3)$ gluons could perturbative description of low energy hadron physics. The strong $SO(4)$ symmetry of low energy hadron physics can be indeed seen direct experimental support for the M^8-H duality.

Number theoretical compactification has quite deep implications for quantum TGD and is actually responsible for most of the progress in the understanding of the mathematical structure of quantum TGD. A very powerful prediction is that preferred extremals should allow slicings to either stringy world sheets or dual partonic 2-surfaces as well as slicing by light-like 3-surfaces. Both predictions are consistent with what is known about extremals.

1. If the distribution of planes $M^2(x)$ is integrable, it is possible to slice $X^4(X^3)$ to a union of 2-dimensional surfaces having interpretation as string world sheets and dual 2-dimensional copies of partonic surfaces X^2. This decomposition defining 2+2 Kaluza-Klein type structure could realize quantum gravitational holography and might allow to understand Equivalence Principle at space-time level in the sense that dimensional reduction defined by the integral of Kähler action over the 2-dimensional space labeling stringy world sheets gives rise to the analog of stringy action and one obtains string model like description of quantum TGD as dual for a description based on light-like partonic 3-surfaces. String tension is not however equal to the inverse of gravitational constant as one might naively expect but the connection is more delicate. As already mentioned, TGD-GRT connection and EP can be understood at general level only from very general arguments [K79].

2. Second implication is the slicing of $X^4(X^3_l)$ to light-like 3-surfaces Y^3_l "parallel" to X^3_l. Also this slicing realizes quantum gravitational holography if one requires General Coordinate Invariance in the sense that the Dirac determinant differs for two 3-surfaces Y^3_l in the slicing only by an exponent of a real part of a holomorphic function of WCW complex coordinates giving no contribution to the Kähler metric.

3. The square of the Dirac determinant would be equal to the modulus squared for the exponent of vacuum functional and would be formally defined as the product of conformal weights assignable to the modes of the Dirac operator at string world sheets at the ends of strings at partonic 2-surfaces defining the ends of Y^3_l. The detailed definition requires to specify what one means with the conformal weights assignable with the modes of the Kähler-Dirac operator.

4. The localization of the modes of Kähler-Dirac operator to 2-D surfaces (string world sheets and possibly partonic 2-surfaces) [K88] following from the condition that electromagnetic charges of the modes is well-defined is very strong restriction and reduces Dirac determinant to a product of Dirac determinants assignable with these 2-surfaces.

2.2.2 Preferred Extremal Property As Classical Correlate For Quantum Criticality, Holography, And Quantum Classical Correspondence

The Noether currents assignable to the Kähler-Dirac equation are conserved only if the first variation of the Kähler-Dirac operator D_K defined by Kähler action vanishes. This is equivalent with the vanishing of the second variation of Kähler action -at least for the variations corresponding to dynamical symmetries having interpretation as dynamical degrees of freedom which are below measurement resolution and therefore effectively gauge symmetries. The natural identification would be as conformal symmetries. The weaker condition would mean that the inner product defined by the integral of $D_\alpha \partial L_K/\partial h^k_\alpha \delta h^k$ over the space-time surface vanishes for the deformations defining dynamical symmetries but the field equations are not satisfied completely generally. The weaker condition would mean that the inner product defined by the integral of $D_\alpha \partial L_K/\partial h^k_\alpha \delta h^k$ over the space-time surface vanishes for the deformations defining dynamical symmetries but the field equations are not satisfied completely generally.

The vanishing of the second variation in interior of $X^4(X^3_l)$ is what corresponds exactly to quantum criticality so that the basic vision about quantum dynamics of quantum TGD would lead directly to a precise identification of the preferred extremals. Something which I should have noticed for more than decade ago!

For instance, the natural expectation is that the number of critical deformations is infinite and corresponds to conformal symmetries naturally assignable to criticality. The number n of conformal equivalence classes of the deformations can be finite and n would naturally relate to the hierarchy of Planck constants $h_{eff} = n \times h$ (see **Fig.** `http://tgdtheory.fi/appfigures/planckhierarchy.jpg` or **Fig. ??**in the appendix of this book).

The vanishing of second variations of preferred extremals -at least for deformations representing dynamical symmetries, suggests a generalization of catastrophe theory of Thom, where the rank of the matrix defined by the second derivatives of potential function defines a hierarchy of criticalities with the tip of bifurcation set of the catastrophe representing the complete vanishing of this matrix. In the recent case this theory would be generalized to infinite-dimensional context. There are three kind of variables now but quantum classical correspondence (holography) allows to reduce the types of variables to two.

1. The variations of $X^4(X_l^3)$ vanishing at the intersections of $X^4(X_l^3)$ with the light-like boundaries of causal diamonds CD would represent behavior variables. At least the vacuum extremals of Kähler action would represent extremals for which the second variation vanishes identically (the "tip" of the multi-furcation set).

2. The zero modes of Kähler function would define the control variables interpreted as classical degrees of freedom necessary in quantum measurement theory. By effective 2-dimensionality (or holography or quantum classical correspondence) meaning that the configuration space metric is determined by the data coming from partonic 2-surfaces X^2 at intersections of X_l^3 with boundaries of CD, the interiors of 3-surfaces X^3 at the boundaries of CDs in rough sense correspond to zero modes so that there is indeed huge number of them. Also the variables characterizing 2-surface, which cannot be complexified and thus cannot contribute to the Kähler metric of WCW represent zero modes. Fixing the interior of the 3-surface would mean fixing of control variables. Extremum property would fix the 4-surface and behavior variables if boundary conditions are fixed to sufficient degree.

3. The complex variables characterizing X^2 would represent third kind of variables identified as quantum fluctuating degrees of freedom contributing to the WCW metric. Quantum classical correspondence requires 1-1 correspondence between zero modes and these variables. This would be essentially holography stating that the 2-D "causal boundary" X^2 of $X^3(X^2)$ codes for the interior. Preferred extremal property identified as criticality condition would realize the holography by fixing the values of zero modes once X^2 is known and give rise to the holographic correspondence $X^2 \rightarrow X^3(X^2)$. The values of behavior variables determined by extremization would fix then the space-time surface $X^4(X_l^3)$ as a preferred extremal.

4. Clearly, the presence of zero modes would be absolutely essential element of the picture. Quantum criticality, quantum classical correspondence, holography, and preferred extremal property would all represent more or less the same thing. One must of course be very cautious since the boundary conditions at X_l^3 involve normal derivative and might bring in delicacies forcing to modify the simplest heuristic picture.

The basic question is whether number theoretic view about preferred extremals imply absolute minimization or something analogous to it.

1. The number theoretic conditions defining preferred extremals are purely algebraic and make sense also p-adically and this is enough since p-adic variants of field equations make sense although the notion of Kähler action does not make sense as integral. Despite this the identification of the vacuum functional as exponent of Kähler function as Dirac determinant allows to define the exponent of Kähler function as a p-adic number [K88].

2. The general objection against all extremization principles is that they do not make sense p-adically since p-adic numbers are not well-ordered.

3. These observations do not encourage the idea about equivalence of the two approaches. On the other hand, real and p-adic sectors are related by algebraic continuation and it could be quite enough if the equivalence were true in real context alone.

The finite-dimensional analogy allows to compare absolute minimization and criticality with each other.

1. Absolute minimization would select the branch of Thom's catastrophe surface with the smallest value of potential function for given values of control variables. In general this value would not correspond to criticality since absolute minimization says nothing about the values of control variables (zero modes).

2. Criticality forces the space-time surface to belong to the bifurcation set and thus fixes the values of control variables, that is the interior of 3-surface assignable to the partonic 2-surface, and realized holography. If the catastrophe has more than $N = 3$ sheets, several preferred extremals are possible for given values of control variables fixing $X^3(X^2)$ unless one assumes that absolute minimization or some other criterion is applied in the bifurcation set. In this sense absolute minimization might make sense in the real context and if the selection is between finite number of alternatives is in question, it should be possible carry out the selection in number theoretically universal manner.

It must be emphasized that there are several proposals for what preferred extremal property could mean. For instance, one can consider the identification of space-time surface as quaternionic sub-manifold meaning that tangent space of space-time surface can be regarded as quaternionic sub-manifold of complexified octonions defining tangent space of imbedding space. One manner to define "quaternionic sub-manifold" is by introducing octonionic representation of imbedding space gamma matrices identified as tangent space vectors. It must be also assumed that the tangent space contains a preferred complex (commutative) sub-space at each point and defining an integrable distribution having identification as string world sheet (also slicing of space-time sheet by string world sheets can be considered). Associativity and commutativity would define the basic dynamical principle. A closely related approach is based on so called Hamilton-Jacobi structure [K7] defining also this kind of slicing and the approaches could be equivalent. A further approach is based on the identification of preferred extremal property as quantum criticality [K7].

2.2.3 Can One Determine Experimentally The Shape Of The Space-Time Surface?

The question "Can one determine experimentally the shape of the space-time surface?" does not relate directly to the topic of this chapter in technical sense, and the only excuse for its inclusion is the title of this section plus the fact that the general conceptual framework behind quantum TGD assumes an affirmative answer to this question. If physics were purely classical physics, operationalism in the strong sense of the word would require that one can experimentally determine the shape of the space-time as a surface of the imbedding space with arbitrary accuracy by measuring suitable classical observables. In quantum physics situation is considerably more complex and quantum effects are both a blessing and a curse.

Measuring classically the shape of the space-time surface

Consider first the purely classical situation to see what is involved.

1. All classical gauge fields are expressible in terms of CP_2 coordinates and their space-time gradients so that the measurement of four field quantities with some finite resolution in some space-time volume could in principle give enough information to deduce the remaining field quantities. The requirement that space-time surface corresponds to an extremal of Kähler action gives a further strong consistency constraint and one can in principle test whether this constraint is satisfied. A highly over-determined system is in question.

2. The freedom to choose the space-time coordinates freely causes complications and it seems that one must be able to determine also the distances between the points at which the field quantities are determined. At purely classical Riemannian level this boils down to the measurement of the induced metric defining classical gravitational field. In macroscopic length

scales one could base the approach to iterative procedure in which one starts from the assumption that the coordinates used are Minkowski coordinates and gravitational corrections are very weak.

3. The measurement of induced Kähler form in some space-time volume determines space-time surface only modulo canonical transformations of CP_2 and isometries of the imbedding space. If one measures classical electromagnetic field, which is not canonical invariant in general case, with some precision, one can determine to what kind of surface space-time region corresponds apart from the action of the isometries of H.

Quantum measurement of the shape of the space-time surface

In practice the measurement of the shape of the space-time surface is necessarily a bootstrap procedure based on the model for space-time region and on the requirement of internal consistency. Many-sheeted space-time and quantum phenomena produce considerable complications but also provide universal measurement standards.

Consider first how quantum effects could help to measure classical fields and distances.

1. The measurement of distances by measuring first induced metric at each point of space-time sheet is rather unpractical procedure. Many-sheeted space-time however comes in rescue here. p-Adic length scale hypothesis provides a hierarchy of natural length scales and one can use p-adic length and time scales as natural units of length and time: space-time sheets serve as meter sticks. For instance, length measurement reduces in principle to a finite number of operations using various space-time sheets with standardized lengths given by p-adic length scales. Also various transition frequencies and corresponding wavelengths provide universal time and length units. Atomic clock provides a standard example of this kind of time unit. A highly nontrivial implication is the possibility to deduce the composition of distant star from its spectral lines. Without p-adic length scale hypothesis the scales for the mass spectra of the elementary particles would be variable and atomic spectra would vary from point to point in TGD universe.

   Do the p-adic length scales correspond to the length units of the induced metric or of M^4_+ metric? If the topological condensation a meter stick space-time sheet at a larger space-time sheet does not stretch the meter stick but only bends it, the length topologically condensed meter stick in the induced metric equals to its original length measured using M^4_+ metric.

2. If superconducting order parameters are expressible in terms of the CP_2 coordinates (there is evidence for this, see the chapter "Macroscopic quantum phenomena and CP_2 geometry"), one might determine directly the CP_2 coordinates as functions of Minkowski coordinates and this would allow to estimate all classical fields directly and thus to deduce strong consistency constraints.

3. At quantum level only the fluxes of the classical fields through surface areas with some minimum size determined by the length scale resolution can be measured. In case of magnetic fields the quantization of the magnetic flux simplifies the situation dramatically. Topological field quantization quite generally modifies the measurement of continuous field variables to the measurement of fluxes. Interestingly, the construction of WCW geometry uses as WCW coordinates various electric and magnetic fluxes over 2-dimensional cross sections of 3-surface.

Quantum effects introduce also difficulties and restrictions.

1. Canonical transformations localized with respect to the boundary of the light cone or more general light like surfaces act as isometries of WCW and one can determine the space-time surface only modulo these isometries. Even more, only the values of the non-quantum fluctuating zero modes characterizing the shape and size of the space-time surface are measurable with arbitrary precision in quantum theory. At the level of conscious experience quantum fluctuating degrees of freedom correspond to sensory qualia like color having no classical geometric content.

2. Space-time surface is replaced by a new one in each quantum jump (or rather the superposition of perceptively equivalent space-time surfaces). Only in the approximation that the change of the space-time region in single quantum jump is negligible, the measurement of the shape of space-time surface makes sense. The physical criterion for this is that dissipation is negligible. The change of the space-time region in single quantum jump can indeed be negligible if the measurement is performed with a finite resolution.

3. Conscious experience of self is an average over quantum jumps defining moments of consciousness. In particular, only the average increment of the zero modes is experienced and this means that one cannot fix the space-time surface apart from canonical transformation affecting the zero modes. Again the notion of measurement resolution comes in rescue.

4. The possibility of coherent states of photons and gravitons brings in a further quantum complication since the effective classical em and gravitational fields are superpositions of classical field and the order parameter describing the coherent state. In principle the extremely strong constraints between the classical field quantities allow to measure both the order parameters of the coherent phases and classical fields.

Quantum holography and the shape of the space-time surface

If the Dirac determinant asssognable to the mass squared eigenvalue spectrum of the Kähler-Dirac operator $D_K(X^2)$ equals to the exponent of Kähler action of a preferred extremal, it is fair to say that a lot of information about the shape of the space-time surface is coded to physical observables, which eigenvalues indeed represent. Quantum gravitational holography due to the Bohr orbit like character of space-time surface reduces the amount of information needed. Only a finite number of eigenvalues is involved and the eigen modes are associated with the 3-D light-like wormhole throats rather than with the space-time surface itself. If the eigenvalues were known or could be measured with infinite accuracy, one could in principle fix the boundary conditions at X_l^3 and solve field equations determining the preferred extremal of Kähler action.

What is of course needed is the complete knowledge of the light-like 3-surfaces X_l^3. Needless to say, in practice a complete knowledge of X_l^3 is impossible since measurement resolution is finite. The notion number theoretic braid provides a precise realization for the finite measurement accuracy at space-time level. At the level of WCW spinors fields (world of classical worlds) just the fact that the number of eigenvalues is finite is correlate for the finite measurement accuracy. Furthermore, quantum states are actually quantum superpositions of 3-surfaces, which means that one can only speak about quantum average space-time surface for which the phase factors coding for the quantum numbers of elementary particles assigned to the strands of number theoretic braids are stationary so that correlation of classical gauge charges with quantum gauge charges is obtained.

2.3 The Vanishing Of Super-Conformal Charges As A Gauge Conditions Selecting Preferred Extremals Of Kähler Action

Classical TGD [K7] involves several key questions waiting for clearcut answers.

1. The notion of preferred extremal emerges naturally in positive energy ontology, where Kähler metric assigns a unique (apart from gauge symmetries) preferred extremal to given 3-surface at M^4 time= constant section of imbedding space $H = M^4 \times CP_2$. This would quantize the initial values of the time derivatives of imbedding coordinates and this could correspond to the Bohr orbitology in quantum mechanics.

2. In zero energy ontology (ZEO) initial conditions are replaced by boundary conditions. One fixes only the 3-surfaces at the opposite boundaries of CD and in an ideal situation there would exist a unique space-time surface connecting them. One must however notice that the existence of light-like wormhole throat orbits at which the signature of the induced metric changes ($det(g_4) = 0$) its signature might change the situation. Does the attribute

"preferred" become obsolete and does one lose the beautiful Bohr orbitology, which looks intuitively compelling and would realize quantum classical correspondence?

3. Intuitively it has become clear that the generalization of super-conformal symmetries by replacing 2-D manifold with metrically 2-D but topologically 3-D light-like boundary of causal diamond makes sense. Generalized super-conformal symmetries should apply also to the wormhole throat orbits which are also metrically 2-D and for which conformal symmetries respect $detg(g_4) = 0$ condition. Quantum classical correspondence demands that the generalized super-confornal invariance has a classical counterpart. How could this classical counterpart be realized?

4. Holography is one key aspect of TGD and mean that 3-surfaces dictate everything. In positive energy ontology the content of this statement would be rather obvious and reduce to Bohr orbitology but in ZEO situation is different. On the other hand, TGD strongly suggests strong form of holography based stating that partonic 2-surfaces (the ends of wormhole throat orbits at boundaries of CD) and tangent space data at them code for quantum physics of TGD. General coordinate invariance would be realized in strong sense: one could formulate the theory either in terms of space-like 3-surfaces at the ends of CD or in terms of light-like wormhole throat orbits. This would realize Bohr orbitology also in ZEO by reducing the boundary conditions to those at partonic 2-surfaces. How to realize this explicitly at the level of field equations? This has been the challenge.

Answering questions is extremely useful activity. During last years Hamed has posed continually questions related to the basic TGD. At this time Hamed asked about the derivation of field equations of TGD. In "simple" field theories involving some polynomial non-linearities the deduction of field equations is of course totally trivial process but in the extremely non-linear geometric framework of TGD situation is quite different.

While answering the questions I made what I immediately dare to call a breakthrough discovery in the mathematical understanding of TGD. To put it concisely: one can assume that the variations at the light-like boundaries of CD vanish for all conformal variations which are not isometries. For isometries the contributions from the ends of CD cancel each other so that the corresponding variations need not vanish separately at boundaries of CD! This is extremely simple and profound fact. This would be nothing but the realisation of the analogs of conformal symmetries classically and give precise content for the notion of preferred external, Bohr orbitology, and strong form of holography. And the condition makes sense only in ZEO!

I attach below the answers to the questions of Hamed almost as such apart from slight editing and little additions, re-organization, and correction of typos.

2.3.1 Field Equations For Kähler Action

Hamed made some questions relating to the derivation of field equations for the extremals of Kähler action which led to the recent progress. I comment first these questions since they lead naturally to the basic new idea.

The physical interpretation of the canonical momentum current

Hamed asked about the physical meaning of $T^n_k \equiv \partial L/\partial(\partial_n h^k)$ - normal components of canonical momentum labelled by the label k of imbedding space coordinates - it is good to start from the physical meaning of a more general vector field

$$T^{\alpha}_k \equiv \frac{\partial L}{\partial(\partial_{\alpha} h^k)}$$

with both imbedding space indices k and space-time indices α - canonical momentum currents. L refers to Kähler action.

1. One can start from the analogy with Newton's equations derived from action principle (Lagrangian). Now the analogs are the partial derivatives $\partial L/\partial(dx^k/dt)$. For a particle in potential one obtains just the momentum. Therefore the term canonical momentum current/density: one has kind of momentum current for each imbedding space coordinate.

2. By contracting with generators of imbedding space isometries (Poincare and color) one indeed obtains conserved currents associated with isometries by Noether's theorem:

$$j^{A\alpha} = T_k^{\alpha} j^{Ak} \ .$$

By field equations the divergences of these currents vanish and one obtains conserved charged-classical four-momentum and color charges:

$$D_\alpha T^{A\alpha} = 0 \ .$$

3. The normal component of conserved current must vanish at boundaries with one time-like direction if one has such:

$$T^{An} = 0.$$

Now one has wormhole throat orbits which are not genuine boundaries albeit analogous to them and one must be very careful. The quantity T_k^n determines the values of normal components of currents and must vanish at possible space-like boundaries.

Note that in TGD field equations reduce to the conservation of isometry currents as in hydrodynamics where basic equations are just conservation laws.

The basic steps in the derivation of field equations

First a general recipe for deriving field equations from Kähler action - or any action as a matter of fact.

1. At the first step one writes an expression of the variation of the Kähler action as sum of variations with respect to the induced metric g and induced Kähler form J. The partial derivatives in question are energy momentum tensor and contravariant Kähler form.

2. After this the variations of g and J are expressed in terms of variations of imbedding space coordinates, which are the primary dynamical variables.

3. The integral defining the variation can be decomposed to a total divergence plus a term vanishing for extremals for all variations: this gives the field equations. Total divergence term gives a boundary term and it vanishes by boundary conditions if the boundaries in question have time-like direction.

   If the boundary is space-like, the situation is more delicate in TGD framework: this will be considered in the sequel. In TGD situation is also delicate also because the light-like 3-surfaces which are common boundaries of regions with Minkowskian or Euclidian signature of the induced metric are not ordinary topological boundaries. Therefore a careful treatment of both cases is required in order to not to miss important physics.

Expressing this summary more explicitly, the variation of the Kahler action with respect to the gradients of the imbedding space coordinates reduces to the integral of

$$T_k^{\alpha}\partial_\alpha \delta h^k + \frac{\partial K}{\partial h^k}\delta h^k \ .$$

The latter term comes only from the dependence of the imbedding space metric and Kähler form on imbedding space coordinates. One can use a simple trick. Assume that they do not depend at all on imbedding space coordinates, derive field equations, and replaced partial derivatives by covariant derivatives at the end. Covariant derivative means covariance with respect to both space-time and imbedding space vector indices for the tensorial quantities involved. The trick works because imbedding space metric and Kähler form are covariantly constant quantities.

The integral of the first term $T_k^{\alpha}\partial_\alpha \delta h^k$ decomposes to two parts.

1. The first term, whose vanishing gives rise to field equations, is integral of

$$D_\alpha T^\alpha_k \delta h^k \ .$$

2. The second term is integral of

$$\partial_\alpha (T^\alpha_k \delta h^k) \ .$$

   This term reduces as a total divergence to a 3-D surface integral over the boundary of the region of fixed signature of the induced metric consisting of the ends of CD and wormhole throat orbits (boundary of region with fixed signature of induced metric). This term vanishes if the normal components T^n_k of canonical momentum currents vanishes at the boundary like region.

In the sequel the boundary terms are discussed explicitly and it will be found that their treatment indeed involves highly non-trivial physics.

Complex isometry charges and twistorialization

TGD space-time contains regions of both Minkowskian and Euclidian signature of metric. This has some highly non-trivial consequences.

1. Should one assume that $\sqrt{det(g_4)}$ is imaginary in Minkowskian and real in Euclidian region? For Kähler action this is sensible and Euclidian region would give a real negative contribution giving rise to exponent of Kähler function of WCW ("world of classical worlds") making the functional integral convergent. Minkowskian regions would give imaginary contribution to the exponent causing interference effects absolutely essential in quantum field theory. This contribution would correspond to Morse function for WCW .

   The implication would be that the classical four-momenta in Euclidian/Minkowskian regions are imaginary/real. What could the interpretation be? Should one accept as a fact that four-momenta are complex.

2. Twistor approach to TGD is now in quite good shape [K76]. $M^4 \times CP_2$ is the unique choice is one requires that the Cartesian factors allow twistor space with Kähler structure [A63] and classical TGD allows twistor formulation.

   In the recent formulation the fundamental fermions are assumed to propagate with light-like momenta along wormhole throats. At gauge theory limit particles must have massless or massive four-momenta. One can however also consider the possibility of complex massless momenta and in the standard twistor approach on mass shell massless particles appearing in graphs indeed have complex momenta. These complex momenta should by quantum classical correspondence correspond directly to classical complex momenta.

3. A funny question popping in mind is whether the massivation of particles could be such that the momenta remain massless in complex sense! The complex variant of light-likeness condition would be

$$p_{re}^2 = p_{Im}^2 \ \ , \ \ p_{re} \cdot p_{Im} = 0 \ .$$

   Could one interpret p_{Im}^2 as the mass squared of the particle? Or could p_{Im}^2 code for the decay width of an unstable particle? This option does not look feasible.

4. The complex momenta could provide an elegant 4-D space-time level representation for the isometry quantum numbers at the level of imbedding space. The ground states of the super-conformal representations have as building bricks the spinor harmonics of the imbedding space which correspond to the analogs of massless particles in 8-D sense [K39]. Indeed, the condition giving mass squared eigenvalues for the spinor harmonics is just massless condition in $M^4 \times CP_2$.

At the space-time level these conditions must be replaced by 4-D conditions and complex masslessness would be the elegant manner to realizes this. Also the massivation of massless states by p-adic thermodynamics could have similar description.

This interpretation would also conform with $M^8 - M^4 \times CP_2$ duality [K111] at the level of momentum space.

2.3.2 Boundary Conditions At Boundaries Of CD

In positive energy ontology one would formulate boundary conditions as initial conditions by fixing both the 3-surface and associated canonical momentum densities at either end of CD (positions and momenta of particles in mechanics). This would bring asymmetry between boundaries of CD. In ZEO the basic boundary condition is that space-time surfaces have as their ends the members of pairs of surfaces at the ends of CD. Besides this one can have additional boundary conditions and the notion of preferred extremal suggests this.

Do boundary conditions realize quantum classical correspondence?

In TGD framework one must carefully consider the boundary conditions at the boundaries of CDs. What is clear that the time-like boundary contributions from the boundaries of CD to the variation must vanish.

1. This is true if the variations are assumed to vanish at the ends of CD. This might be however too strong a condition.

2. One cannot demand the vanishing of T^t_k (t refers to time coordinate as normal coordinate) since this would give only vacuum extremals. One could however require quantum classical correspondence for any Cartan sub-algebra of isometries whose elements define maximal set of isometry generators. The eigenvalues of quantal variants of isometry charge assignable to second quantized induced spinors at the ends of space-time surface are equal to the classical charges. Is this actually a formulation of Equivalence Principle, is not quite clear to me.

Do boundary conditions realize preferred extremal property as a choice of conformal gauge?

While writing this a completely new idea popped to my mind. What if one poses the vanishing of the boundary terms at boundaries of CDs as additional boundary conditions for *all* variations *except isometries* ? Of perhaps for all conformal variations (conformal in TGD sense)? This would *not* imply vanishing of isometry charges since the variations coming from the opposite ends of CD cancel each other! It soon became clear that this would allow to meet all the challenges listed in the beginning!

1. These conditions would realize Bohr orbitology also to ZEO approach and define what "preferred extremal" means.

2. The conditions would be very much like super-Virasoro conditions stating that the superconformal generators with non-vanishing conformal weight annihilate states or create zero norm states but no conditions are posed on generators with vanishing conformal weight (now isometries). One could indeed assume only deformations, which are local isometries assignable to the generalised conformal algebra of the $\delta M^4_+/-\times CP_2$. For arbitrary variations one would not require the vanishing. This could be the long sought for precise formulation of superconformal invariance at the level of classical field equations!

   It is enough co consider the weaker conditions that the conformal charges defined as integrals of corresponding Noether currents vanish. These conditions would be direct equivalents of quantal conditions.

3. The natural interpretation would be as a fixing of conformal gauge. This fixing would be motivated by the fact that WCW Kähler metric must possess isometries associated with the conformal algebra and can depend only on the tangent data at partonic 2-surfaces as became

clear already for more than two decades ago. An alternative, non-practical option would be to allow all 3-surfaces at the ends of CD: this would lead to the problem of eliminating the analog of the volume of gauge group from the functional integral.

4. The conditions would also define precisely the notion of holography and its reduction to strong form of holography in which partonic 2-surfaces and their tangent space data code for the dynamics.

Needless to say, the modification of this approach could make sense also at partonic orbits.

2.3.3 Boundary Conditions At Parton Orbits

The contributions from the orbits of wormhole throats are singular since the contravariant form of the induced metric develops components which are infinite ($det(g_4) = 0$). The contributions are real at Euclidian side of throat orbit and imaginary at the Minkowskian side so that they must be treated as independently.

Conformal gauge choice, preferred extremal property, hierarchy of Planck constants, and TGD as almost topological QFT

The generalization of the boundary conditions as a classical realization conformal gauge invariance is natural.

1. One can consider the possibility that under rather general conditions the normal components $T^n_k\sqrt{det(g_4)}$ approach to zero at partonic orbits since $det(g_4)$ is vanishing. Note however the appearance of contravariant appearing twice as index raising operator in Kähler action. If so, the vanishing of $T^n_k\sqrt{det(g_4)}$ need not fix completely the "boundary" conditions. In fact, I assign to the wormhole throat orbits conformal gauge symmetries so that just this is expected on physical grounds.

2. Generalized conformal invariance would suggest that the variations defined as integrals of $T^n_k\sqrt{det(g_4)}\delta h^k$ vanish in a non-trivial manner for the conformal algebra associated with the light-like wormhole throats with deformations respecting $det(g_4) = 0$ condition. Also the variations defined by infinitesimal isometries (zero conformal weight sector) should vanish since otherwise one would lose the conservation laws for isometry charges. The conditions for isometries might reduce to $T^n_k\sqrt{det(g_4)} \rightarrow 0$ at partonic orbits. Also now the interpretation would be in terms of fixing of conformal gauge.

3. Even $T^n_k\sqrt{g} = 0$ condition need not fix the partonic orbit completely. The Gribov ambiguity meaning that gauge conditions do not fix uniquely the gauge potential could have counterpart in TGD framework. It could be that there are several conformally non-equivalent space-time surfaces connecting 3-surfaces at the opposite ends of CD.

   If so, the boundary values at wormhole throats orbits could matter to some degree: very natural in boundary value problem thinking but new in initial value thinking. This would conform with the non-determinism of Kähler action implying criticality and the possibility that the 3-surfaces at the ends of CD are connected by several space-time surfaces which are physically non-equivalent.

4. The hierarchy of Planck [K22] constants assigned to dark matter, quantum criticality and even criticality indeed relies on the assumption that $h_{eff} = n \times h$ corresponds to n-fold coverings having n space-time sheets which coincide at the ends of CD and that conformal symmetries act on the sheets as gauge symmetries. One would have as Gribov copies n conformal equivalence classes of wormhole throat orbits and corresponding space-time surfaces. Depending on whether one fixes the conformal gauge one has n equivalence classes of space-time surfaces or just one representative from each conformal equivalent class.

5. There is also the question about the correspondence with the weak form of electric magnetic duality [K7]. This duality plus the condition that $j^\alpha A_\alpha = 0$ in the interior of space-time surface imply the reduction of Kähler action to Chern-Simons terms. This would suggest

that the boundary variation of the Kähler action reduces to that for Chern-Simons action which is indeed well-defined for light-like 3-surfaces.

If so, the gauge fixing would reduce to variational equations for Chern-Simons action! A weaker condition is that classical conformal charges vanish. This would give a nice connection to the vision about TGD as almost topological QFT. In TGD framework these conditions do not imply the vanishing of Kähler form at boundaries. The conditions are satisfied if the CP_2 projection of the partonic orbit is 2-D: the reason is that Chern-Simons term vanishes identically in this case.

Fractal hierarchy of conformal symmetry breakings

A further intuitively natural hypothesis is that there is a fractal hierarchy of breakings of conformal symmetry.

1. Only the generators of conformal sub-algebra with conformal weight multiple of n act as gauge symmetries. This would give infinite hierarchies of breakings of conformal symmetry interpreted in terms of criticality: in the hierarchy n_i divides n_{i+1}.

   Similar degeneracy would be associated with both the parton orbits and the space-like ends at CD boundaries and I have considered the possibility that the integer n appearing in h_{eff} has decomposition $n = n_1 n_2$ corresponding to the degeneracies associated with the two kinds of boundaries. Alternatively, one could have just $n = n_1 = n_2$ from the condition that the two conformal symmetries are 3-dimensional manifestations of single 4-D analog of conformal symmetry.

2. In the symmetry breaking $n_i \rightarrow n_{i+1}$ the conformal charges, which vanished earlier, would become non-vanishing. Could one require that they are conserved that is the contributions of the boundary terms at the ends of CD cancel each other? If so, one would have dynamical conformal symmetry.

What could the proper interpretation of the conformal hierarchies $n_i \rightarrow n_{i+1}$?

1. Could one interpret the hierarchy in terms of increasing measurement resolution? Conformal degrees of freedom below measurement resolution would be gauge degrees of freedom and the conformal hierarchies would correspond to an inclusion hierarchies for hyper-finite factors of type II_1 [K87]. If $h_{eff} = n \times h$ defines the conformal gauge sub-algebra, the improvement of the resolution would scale up the Compton scales and would quite concretely correspond to a zoom analogous to that done for Mandelbrot fractal to get new details visible. From the point of view of cognition the improving resolution would fit nicely with the recent view about h_{eff}/h as a kind of intelligence quotient.

   This interpretation might make sense for the symplectic algebra of $\delta M^4_\pm \times CP_2$ for which the light-like radial coordinate r_M of light-cone boundary takes the role of complex coordinate. The reason is that symplectic algebra acts as isometries.

2. Suppose that the Kähler action has vanishing variation under deformations defined by the broken conformal symmetries so that the corresponding conformal charges As a consequence, Kähler function would be critical with respect to the corresponding variations. The components of WCW Kähler metric expressible in terms of second derivatives of Kähler function can be however non-vanishing and have also components, which correspond to WCW coordinates associated with different partonic 2-surfaces. This conforms with the idea that conformal algebras extend to Yangian algebras generalizing the Yangian symmetry of $\mathcal{N} = 4$ symmetric gauge theories.

   In this kind of situation one could consider the interpretation in terms of criticality: the lower the criticality, the larger then value of h_{eff} and h and the higher the resolution.

3. n gives also the number of space-time sheets in the singular covering. Could the interpretation be in terms measurement resolution for counting the number of space-time sheets. Our recent quantum physics would only see single space-time sheet representing visible manner and dark matter would become visible only for $n > 1$.

As should have become clear, the derivation of field equations in TGD framework is not just an application of a formal recipe as in field theories and a lot of non-trivial physics is involved!

2.4 General View About Field Equations

In this section field equations are deduced and discussed in general level. The fact that the divergence of the energy momentum tensor, Lorentz 4-force, does not vanish in general, in principle makes possible the mimicry of even dissipation and of the second law. For asymptotic self organization patterns for which dissipation is absent the Lorentz 4-force must vanish. This condition is guaranteed if Kähler current is proportional to the instanton current in the case that CP_2 projection of the space-time sheet is smaller than four and vanishes otherwise. An attractive identification for the vanishing of Lorentz 4-force is as a condition equivalent with the selection of preferred extremal of Kähler action. This condition implies that covariant divergence of energy momentum tensor vanishes and in General Relativity context this leads to Einstein's equations. If preferred extremals correspond to absolute minima this principle would be essentially equivalent with the second law of thermodynamics. There are however could reasons to keep the identification of preferred extremely property open.

2.4.1 Field Equations

The requirement that Kähler action is stationary leads to the following field equations in the interior of the four-surface

$$\begin{aligned} D_\beta(T^{\alpha\beta}h^k_\alpha) &- j^\alpha J^k{}_l\partial_\alpha h^l = 0 \ , \\ T^{\alpha\beta} &= J^{\nu\alpha}J_\nu{}^\beta - \frac{1}{4}g^{\alpha\beta}J^{\mu\nu}J_{\mu\nu} \ . \end{aligned} \tag{2.4.1}$$

Here $T^{\alpha\beta}$ denotes the traceless canonical energy momentum tensor associated with the Kähler action. An equivalent form for the first equation is

$$\begin{aligned} T^{\alpha\beta}H^k_{\alpha\beta} &- j^\alpha(J_\alpha{}^\beta h^k{}_\beta + J^k{}_l\partial_\alpha h^l) = 0 \ . \\ H^k_{\alpha\beta} &= D_\beta\partial_\alpha h^k \ . \end{aligned} \tag{2.4.2}$$

$H^k_{\alpha\beta}$ denotes the components of the

second fundamental form and $j^\alpha = D_\beta J^{\alpha\beta}$ is the gauge current associated with the Kähler field.

On the boundaries of X^4 and at wormhole throats the field equations are given by the expression

$$\frac{\partial L_K}{\partial_n h^k} = T^{n\beta}\partial_\beta h^k - J^{n\alpha}(J_\alpha{}^\beta\partial_\beta h^k + J^k{}_l)\partial_\alpha h^k) = 0 \ . \tag{2.4.3}$$

At wormhole throats problems are caused by the vanishing of metric determinant implying that contravariant metric is singular.

For M^4 coordinates boundary conditions are satisfied if one assumes

$$T^{n\beta} = 0 \tag{2.4.4}$$

stating that there is no flow of four-momentum through the boundary component or wormhole throat. This means that there is no energy exchange between Euclidian and Minkowskian regions so that Euclidian regions provide representations for particles as autonomous units. This is in accordance with the general picture [K30]. Note that momentum transfer with external world necessarily involves generalized Feynman diagrams also at classical level.

For CP_2 coordinates the boundary conditions are more delicate. The construction of WCW spinor structure [K88] led to the conditions

$$g_{ni} = 0 \ , \ J_{ni} = 0 \ . \tag{2.4.5}$$

$J^{ni} = 0$ does not and should not follow from this condition since contravariant metric is singular. It seems that limiting procedure is necessary in order to see what comes out.

The condition that Kähler electric charge defined as a gauge flux is non-vanishing would require that the quantity $J^{nr}\sqrt{g}$ is finite (here r refers to the light-like coordinate of X^3_l). Also $g^{nr}\sqrt{g_4}$ which is analogous to gravitational flux if n is interpreted as time coordinate could be non-vanishing. These conditions are consistent with the above condition if one has

$$\begin{array}{llll} J_{ni} = 0 \ , & g_{ni} = 0 \ , & J_{ir} = 0 \ , & g_{ir} = 0 \ , \\ J^{nk} = 0 \ \ k \neq r \ , & g^{nk} = 0 \ \ k \neq r \ , & J^{nr}\sqrt{g_4} \neq 0 \ , & g^{nr}\sqrt{g_4} \neq 0 \ . \end{array} \tag{2.4.6}$$

The interpretation of this conditions is rather transparent.

1. The first two conditions state that covariant form of the induced Kähler electric field is in direction normal to X^3_l and metric separate into direct sum of normal and tangential contributions. Fifth and sixth condition state the same in contravariant form for $k \neq n$.

2. Third and fourth condition state that the induced Kähler field at X^3_l is purely magnetic and that the metric of x^3_l reduces to a block diagonal form. The reduction to purely magnetic field is of obvious importance as far as the understanding of the generalized eigen modes of the Kähler-Dirac operator is considered [K88].

3. The last two conditions must be understood as a limit and $\neq$ means only the possibility of non-vanishing Kähler gauge flux or analog of gravitational flux through X^3_l.

4. The vision inspired by number theoretical compactification allows to identify r and n in terms of the light-like coordinates assignable to an integrable distribution of planes $M^2(x)$ assumed to be assignable to M^4 projection of $X^4(X^3_l)$. Later it will be found that Hamilton-Jacobi structure assignable to the extremals indeed means the existence of this kind of distribution meaning slicing of $X^4(X^3_l)$ both by string world sheets and dual partonic 2-surfaces as well as by light-like 3-surfaces Y^3_l.

5. The physical analogy for the situation is the surface of an ideal conductor. It would not be surprising that these conditions are satisfied by all induced gauge fields.

2.4.2 Topologization And Light-Likeness Of The Kähler Current As Alternative Manners To Guarantee Vanishing Of Lorentz 4-Force

The general solution of 4-dimensional Einstein-Yang Mills equations in Euclidian 4-metric relies on self-duality of the gauge field, which topologizes gauge charge. This topologization can be achieved by a weaker condition, which can be regarded as a dynamical generalization of the Beltrami condition. An alternative manner to achieve vanishing of the Lorentz 4-force is light-likeness of the Kähler 4-current. This does not require topologization.

Topologization of the Kähler current for $D_{CP_2} = 3$: covariant formulation

The condition states that Kähler 4-current is proportional to the instanton current whose divergence is instanton density and vanishes when the dimension of CP_2 projection is smaller than four: $D_{CP_2} < 4$. For $D_{CP_2} = 2$ the instanton 4-current vanishes identically and topologization is equivalent with the vanishing of the Kähler current.

If the simplest vision about light-like 3-surfaces as basic dynamical objects is accepted $D_{CP_2} = 2$, corresponds to a non-physical situation and only the deformations of these surfaces - most naturally resulting by gluing of CP_2 type vacuum extremals on them - can represent preferred extremals of Kähler action. One can however speak about $D_{CP_2}=2$ phase if 4-surfaces are obtained are obtained in this manner.

$$j^\alpha \equiv D_\beta J^{\alpha\beta} = \psi\times j_I^\alpha = \psi\times\epsilon^{\alpha\beta\gamma\delta}J_{\beta\gamma}A_\delta \ . \tag{2.4.7}$$

Here the function ψ is an arbitrary function $\psi(s^k)$ of CP_2 coordinates s^k regarded as functions of space-time coordinates. It is essential that ψ depends on the space-time coordinates through the CP_2 coordinates only. Hence the representation as an imbedded gauge field is crucial element of the solution ansatz.

The field equations state the vanishing of the divergence of the 4-current. This is trivially true for instanton current for $D_{CP_2}<4$. Also the contraction of $\nabla\psi$ (depending on space-time coordinates through CP_2 coordinates only) with the instanton current is proportional to the winding number density and therefore vanishes for $D_{CP_2}<4$.

The topologization of the Kähler current guarantees the vanishing of the Lorentz 4-force. Indeed, using the self-duality condition for the current, the expression for the Lorentz 4-force reduces to a term proportional to the instanton density:

$$\begin{aligned} j^\alpha J_{\alpha\beta} &= \psi\times j_I^\alpha J_{\alpha\beta} \\ &= \psi\times\epsilon^{\alpha\mu\nu\delta}J_{\mu\nu}A_\delta J_{\alpha\beta} \ . \end{aligned} \tag{2.4.8}$$

Since all vector quantities appearing in the contraction with the four-dimensional permutation tensor are proportional to the gradients of CP_2 coordinates, the expression is proportional to the instanton density, and thus winding number density, and vanishes for $D_{CP_2}<4$.

Remarkably, the topologization of the Kähler current guarantees also the vanishing of the term $j^\alpha J^{kl}\partial_\alpha s^k$ in the field equations for CP_2 coordinates. This means that field equations reduce in both M^4_+ and CP_2 degrees of freedom to

$$T^{\alpha\beta}H^k_{\alpha\beta} = 0 \ . \tag{2.4.9}$$

These equations differ from the equations of minimal surface only by the replacement of the metric tensor with energy momentum tensor. The earlier proposal that quaternion conformal invariance in a suitable sense might provide a general solution of the field equations could be seen as a generalization of the ordinary conformal invariance of string models. If the topologization of the Kähler current implying effective dimensional reduction in CP_2 degrees of freedom is consistent with quaternion conformal invariance, the quaternion conformal structures must differ for the different dimensions of CP_2 projection.

Topologization of the Kähler current for $D_{CP_2}=3$: non-covariant formulation

In order to gain a concrete understanding about what is involved it is useful to repeat these arguments using the 3-dimensional notation. The components of the instanton 4-current read in three-dimensional notation as

$$\overline{j}_I = \overline{E}\times\overline{A}+\phi\overline{B} \ , \quad \rho_I = \overline{B}\cdot\overline{A} \ . \tag{2.4.10}$$

The self duality conditions for the current can be written explicitly using 3-dimensional notation and read

$$\begin{aligned} \nabla\times\overline{B}-\partial_t\overline{E} &= \overline{j} = \psi\overline{j}_I = \psi\left(\phi\overline{B}+\overline{E}\times\overline{A}\right) \ , \\ \nabla\cdot E &= \rho = \psi\rho_I \ . \end{aligned} \tag{2.4.11}$$

For a vanishing electric field the self-duality condition for Kähler current reduces to the Beltrami condition

$$\nabla \times \overline{B} = \alpha \overline{B} \quad , \quad \alpha = \psi \phi \quad . \tag{2.4.12}$$

The vanishing of the divergence of the magnetic field implies that α is constant along the field lines of the flow. When ϕ is constant and $\overline{A}$ is time independent, the condition reduces to the Beltrami condition with $\alpha = \phi = constant$, which allows an explicit solution [B16].

One can check also the vanishing of the Lorentz 4-force by using 3-dimensional notation. Lorentz 3-force can be written as

$$\rho_I \overline{E} + \overline{j} \times \overline{B} = \psi \overline{B} \cdot \overline{AE} + \psi \left(\overline{E} \times \overline{A} + \phi \overline{B} \right) \times \overline{B} = 0 \quad . \tag{2.4.13}$$

The fourth component of the Lorentz force reads as

$$\overline{j} \cdot \overline{E} = \psi \overline{B} \cdot \overline{E} + \psi \left(\overline{E} \times \overline{A} + \phi \overline{B} \right) \cdot \overline{E} = 0 \quad . \tag{2.4.14}$$

The remaining conditions come from the induction law of Faraday and could be guaranteed by expressing $\overline{E}$ and $\overline{B}$ in terms of scalar and vector potentials.

The density of the Kähler electric charge of the vacuum is proportional to the the helicity density of the so called helicity charge $\rho = \psi \rho_I = \psi B \cdot A$. This charge is topological charge in the sense that it does not depend on the induced metric at all. Note the presence of arbitrary function ψ of CP_2 coordinates.

Further conditions on the functions appearing in the solution ansatz come from the 3 independent field equations for CP_2 coordinates. What is remarkable that the generalized self-duality condition for the Kähler current allows to understand the general features of the solution ansatz to very high degree without any detailed knowledge about the detailed solution. The question whether field equations allow solutions consistent with the self duality conditions of the current will be dealt later. The optimistic guess is that the field equations and topologization of the Kähler current relate to each other very intimately.

Vanishing or light likeness of the Kähler current guarantees vanishing of the Lorentz 4-force for $D_{CP_2} = 2$

For $D_{CP_2} = 2$ one can always take two CP_2 coordinates as space-time coordinates and from this it is clear that instanton current vanishes so that topologization gives a vanishing Kähler current. In particular, the Beltrami condition $\nabla \times \overline{B} = \alpha \overline{B}$ is not consistent with the topologization of the instanton current for $D_{CP_2} = 2$.

$D_{CP_2} = 2$ case can be treated in a coordinate invariant manner by using the two coordinates of CP_2 projection as space-time coordinates so that only a magnetic or electric field is present depending on whether the gauge current is time-like or space-like. Light-likeness of the gauge current provides a second manner to achieve the vanishing of the Lorentz force and is realized in case of massless extremals having $D_{CP_2} = 2$: this current is in the direction of propagation whereas magnetic and electric fields are orthogonal to it so that Beltrami conditions is certainly not satisfied.

Under what conditions topologization of Kähler current yields Beltrami conditions?

Topologization of the Kähler 4-current gives rise to magnetic Beltrami fields if either of the following conditions is satisfied.

1. The $\overline{E} \times \overline{A}$ term contributing besides $\phi \overline{B}$ term to the topological current vanishes. This requires that $\overline{E}$ and $\overline{A}$ are parallel to each other

$$\overline{E} = \nabla \Phi - \partial_t \overline{A} = \beta \overline{A} \tag{2.4.15}$$

This condition is analogous to the Beltrami condition. Now only the 3-space has as its coordinates time coordinate and two spatial coordinatesand$\overline{B}$ is replaced with $\overline{A}$. Since E and B are orthogonal, this condition implies $\overline{B} \cdot \overline{A} = 0$ so that Kähler charge density is vanishing.

2. The vector $\overline{E} \times \overline{A}$ is parallel to $\overline{B}$.

$$\overline{E} \times \overline{A} = \beta \overline{B} \tag{2.4.16}$$

The condition is consistent with the orthogonality of $\overline{E}$ and $\overline{B}$ but implies the orthogonality of $\overline{A}$ and $\overline{B}$ so that electric charge density vanishes

In both cases vector potential fails to define a contact structure since $B \cdot A$ vanishes (contact structures are discussed briefly below), and there exists a global coordinate along the field lines of $\overline{A}$ and the full contact structure is lost again. Note however that the Beltrami condition for magnetic field means that magnetic field defines a contact structure irrespective of whether $\overline{B} \cdot \overline{A}$ vanishes or not. The transition from the general case to Beltrami field would thus involve the replacement

$$(\overline{A}, \overline{B}) \rightarrow_{\nabla\times} (\overline{B}, \overline{j})$$

induced by the rotor.

One must of course take these considerations somewhat cautiously since the inner product depends on the induced 4-metric and it might be that induced metric could allow small vacuum charge density and make possible genuine contact structure.

Hydrodynamic analogy

The field equations of TGD are basically hydrodynamic equations stating the local conservation of the currents associated with the isometries of the imbedding space. Therefore it is intriguing that Beltrami fields appear also as solutions of ideal magnetohydrodynamics equations and as steady solutions of non-viscous incompressible flow described by Euler equations [B66].

In hydrodynamics the role of the magnetic field is taken by the velocity field. This raises the idea that the incompressible flow could occur along the field lines of some natural vector field. The considerations of the last section show that the instanton current defines a universal candidate as far as the general solution of the field equations is considered. All conserved currents defined by the isometry charges would be parallel to the instanton current: one can say each flow line of instanton current is a carrier of conserved quantum numbers. Perhaps even the flow lines of an incompressible hydrodynamic flow could in reasonable approximation correspond to those of instanton current.

The conservation laws are satisfied for each flow line separately and therefore it seems that one cannot have the analog of viscous hydrodynamic flow in this framework. One the other hand, quantum classical correspondence requires that also dissipative effects have space-time correlates. Does something go badly wrong?

The following argument suggests a way out of the problem. Dissipation is certaily due to the quantum jumps at scales below that associated with causal diamond (CD) associated with the observer and is thus assignable to sub-CDs. The quantum jumps for sub-CDs would eventually lead to a thermal ensemble of sub-CDs.

The usual description of dissipation in terms of viscocity and similar parameters emerges at the GRT-QFT limit of TGD replacing in long length scales the many-sheeted space-time (see **Fig.** `http://tgdtheory.fi/appfigures/manysheeted.jpg` or **Fig.** 9 in the appendix of this book) with a piece of Minkowski space with effective metric defined by the sum of Minkowski metric and deviations of the induced metrics of space-time sheets from Minkowski metric. This lumping of space-time sheets means that induced gauge fields and gravitational fields from various space-time sheet sum up and become random (by central limit theorems). Thus locally the dynamics is dissipation free for individual space-time sheets and dissipation emerges at the level of GRT space-time carrying effective metric and effective gauge fields.

The stability of generalized Beltrami fields

The stability of generalized Beltrami fields is of high interest since unstable points of space-time sheets are those around which macroscopic changes induced by quantum jumps are expected to be localized.

1. Contact forms and contact structures

The stability of Beltrami flows has been studied using the theory of contact forms in three-dimensional Riemann manifolds contact. Contact form is a one-form A (that is covariant vector field A_α) with the property $A \wedge dA \neq 0$. In the recent case the induced Kähler gauge potential A_α and corresponding induced Kähler form $J_{\alpha\beta}$ for any 3-sub-manifold of space-time surface define a contact form so that the vector field $A^\alpha = g^{\alpha\beta}A_\beta$ is not orthogonal with the magnetic field $B^\alpha = \epsilon^{\alpha\beta\delta}J_{\beta\gamma}$. This requires that magnetic field has a helical structure. Induced metric in turn defines the Riemann structure.

If the vector potential defines a contact form, the charge density associated with the topologized Kähler current must be non-vanishing. This can be seen as follows.

1. The requirement that the flow lines of a one-form X_μ defined by the vector field X^μ as its dual allows to define a global coordinate x varying along the flow lines implies that there is an integrating factor ϕ such that $\phi X = dx$ and therefore $d(\phi X) = 0$. This implies $dlog(\phi) \wedge X = -dX$. From this the necessary condition for the existence of the coordinate x is $X \wedge dX = 0$. In the three-dimensional case this gives $\overline{X} \cdot (\nabla \times \overline{X}) = 0$.

2. This condition is by definition not satisfied by the vector potential defining a contact form so that one cannot identify a global coordinate varying along the flow lines of the vector potential. The condition $\overline{B} \cdot \overline{A} \neq 0$ states that the charge density for the topologized Kähler current is non-vanishing. The condition that the field lines of the magnetic field allow a global coordinate requires $\overline{B} \cdot \nabla \times \overline{B} = 0$. The condition is not satisfied by Beltrami fields with $\alpha \neq 0$. Note that in this case magnetic field defines a contact structure.

Contact structure requires the existence of a vector ξ satisfying the condition $A(\xi) = 0$. The vector field ξ defines a plane field, which is orthogonal to the vector field A^α. Reeb field in turn is a vector field for which $A(X) = 1$ and $dA(X;) = 0$ hold true. The latter condition states the vanishing of the cross product $X \times B$ so that X is parallel to the Kähler magnetic field B^α and has unit projection in the direction of the vector field A^α. Any Beltrami field defines a Reeb field irrespective of the Riemannian structure.

2. Stability of the Beltrami flow and contact structures

Contact structures are used in the study of the topology and stability of the hydrodynamical flows [B44], and one might expect that the notion of contact structure and its proper generalization to the four-dimensional context could be useful in TGD framework also. An example giving some idea about the complexity of the flows defined by Beltrami fields is the Beltrami field in R^3 possessing closed orbits with all possible knot and link types simultaneously [B44] !

Beltrami flows associated with Euler equations are known to be unstable [B44]. Since the flow is volume preserving, the stationary points of the Beltrami flow are saddle points at which also vorticity vanishes and linear instabilities of Navier-Stokes equations can develop. From the point of view of biology it is interesting that the flow is stabilized by vorticity which implies also helical structures. The stationary points of the Beltrami flow correspond in TGD framework to points at which the induced Kähler magnetic field vanishes. They can be unstable by the vacuum degeneracy of Kähler action implying classical non-determinism. For generalized Beltrami fields velocity and vorticity (both divergence free) are replaced by Kähler current and instanton current.

More generally, the points at which the Kähler 4-current vanishes are expected to represent potential instabilities. The instanton current is linear in Kähler field and can vanish in a gauge invariant manner only if the induced Kähler field vanishes so that the instability would be due to the vacuum degeneracy also now. Note that the vanishing of the Kähler current allows also the generation of region with $D_{CP_2} = 4$. The instability of the points at which induce Kähler field vanish is manifested in quantum jumps replacing the generalized Beltrami field with a new one such that something new is generated around unstable points. Thus the regions in which induced

Kähler field becomes weak are the most interesting ones. For example, unwinding of DNA could be initiated by an instability of this kind.

2.4.3 How To Satisfy Field Equations?

The topologization of the Kähler current guarantees also the vanishing of the term $j^\alpha J^{k_l}\partial_\alpha s^k$ in the field equations for CP_2 coordinates. This means that field equations reduce in both M^4_+ and CP_2 degrees of freedom to

$$T^{\alpha\beta}H^k_{\alpha\beta} = 0 . \tag{2.4.17}$$

These equations differ from the equations of minimal surface only by the replacement of the metric tensor with energy momentum tensor. The following approach utilizes the properties of Hamilton Jacobi structures of M^4_+ introduced in the study of massless extremals and contact structures of CP_2 emerging naturally in the case of generalized Beltrami fields.

String model as a starting point

String model serves as a starting point.

1. In the case of Minkowskian minimal surfaces representing string orbit the field equations reduce to purely algebraic conditions in light cone coordinates (u,v) since the induced metric has only the component g_{uv}, whereas the second fundamental form has only diagonal components H^k_{uu} and H^k_{vv}.
2. For Euclidian minimal surfaces (u,v) is replaced by complex coordinates $(w,\overline{w})$ and field equations are satisfied because the metric has only the component $g^{w\overline{w}}$ and second fundamental form has only components of type H^k_{ww} and $H^{\overline{k}}_{\overline{w}\overline{w}}$. The mechanism should generalize to the recent case.

The general form of energy momentum tensor as a guideline for the choice of coordinates

Any 3-dimensional Riemann manifold allows always a orthogonal coordinate system for which the metric is diagonal. Any 4-dimensional Riemann manifold in turn allows a coordinate system for which 3-metric is diagonal and the only non-diagonal components of the metric are of form g^{ti}. This kind of coordinates might be natural also now. When $\overline{E}$ and $\overline{B}$ are orthogonal, energy momentum tensor has the form

$$T = \begin{pmatrix} \frac{E^2+B^2}{2} & 0 & 0 & EB \\ 0 & \frac{E^2+B^2}{2} & 0 & 0 \\ 0 & 0 & \frac{-E^2+B^2}{2} & 0 \\ EB & 0 & 0 & \frac{E^2-B^2}{2} \end{pmatrix} \tag{2.4.18}$$

in the tangent space basis defined by time direction and longitudinal direction $\overline{E}\times\overline{B}$, and transversal directions $\overline{E}$ and $\overline{B}$. Note that T is traceless.

The optimistic guess would be that the directions defined by these vectors integrate to three orthogonal coordinates of X^4 and together with time coordinate define a coordinate system containing only g^{ti} as non-diagonal components of the metric. This however requires that the fields in question allow an integrating factor and, as already found, this requires $\nabla\times X\cdot X=0$ and this is not the case in general.

Physical intuition suggests however that X^4 coordinates allow a decomposition into longitudinal and transversal degrees freedom. This would mean the existence of a time coordinate t and longitudinal coordinate z the plane defined by time coordinate and vector $\overline{E}\times\overline{B}$ such that the coordinates $u=t-z$ and $v=t+z$ are light like coordinates so that the induced metric would have only the component g^{uv} whereas g^{vv} and g^{uu} would vanish in these coordinates. In the transversal space-time directions complex space-time coordinate coordinate w could be introduced. Metric could have also non-diagonal components besides the components $g^{w\overline{w}}$ and g^{uv}.

Hamilton Jacobi structures in M^4_+

Hamilton Jacobi structure in M^4_+ can understood as a generalized complex structure combing transversal complex structure and longitudinal hyper-complex structure so that notion of holomorphy and Kähler structure generalize.

1. Denote by m^i the linear Minkowski coordinates of M^4. Let (S^+, S^-, E^1, E^2) denote local coordinates of M^4_+ defining a *local* decomposition of the tangent space M^4 of M^4_+ into a direct, not necessarily orthogonal, sum $M^4 = M^2 \oplus E^2$ of spaces M^2 and E^2. This decomposition has an interpretation in terms of the longitudinal and transversal degrees of freedom defined by local light-like four-velocities $v_\pm = \nabla S_\pm$ and polarization vectors $\epsilon_i = \nabla E^i$ assignable to light ray. Assume that E^2 allows complex coordinates $w = E^1 + iE^2$ and $\overline{w} = E^1 - iE^2$. The simplest decomposition of this kind corresponds to the decomposition $(S^+ \equiv u = t+z, S^- \equiv v = t-z, w = x+iy, \overline{w} = x-iy)$.

2. In accordance with this physical picture, S^+ and S^- define light-like curves which are normals to light-like surfaces and thus satisfy the equation:

$$(\nabla S_\pm)^2 = 0 \quad .$$

   The gradients of $S_\pm$ are obviously analogous to local light like velocity vectors $v = (1, \overline{v})$ and $\tilde{v} = (1, -\overline{v})$. These equations are also obtained in geometric optics from Hamilton Jacobi equation by replacing photon's four-velocity with the gradient ∇S: this is consistent with the interpretation of massless extremals as Bohr orbits of em field. $S_\pm = constant$ surfaces can be interpreted as expanding light fronts. The interpretation of $S_\pm$ as Hamilton Jacobi functions justifies the term Hamilton Jacobi structure.

   The simplest surfaces of this kind correspond to $t = z$ and $t = -z$ light fronts which are planes. They are dual to each other by hyper complex conjugation $u = t - z \rightarrow v = t + z$. One should somehow generalize this conjugation operation. The simplest candidate for the conjugation $S^+ \rightarrow S^-$ is as a conjugation induced by the conjugation for the arguments: $S^+(t-z, t+z, x, y) \rightarrow S^-(t-z, t+z, x, y) = S^+(t+z, t-z, x, -y)$ so that a dual pair is mapped to a dual pair. In transversal degrees of freedom complex conjugation would be involved.

3. The coordinates $(S_\pm, w, \overline{w})$ define local light cone coordinates with the line element having the form

$$\begin{aligned} ds^2 &= g_{+-}dS^+dS^- + g_{w\overline{w}}dwd\overline{w} \\ &+ g_{+w}dS^+dw + g_{+\overline{w}}dS^+d\overline{w} \\ &+ g_{-w}dS^-dw + g_{-\overline{w}}dS^-d\overline{w} \ . \end{aligned} \tag{2.4.19}$$

   Conformal transformations of M^4_+ leave the general form of this decomposition invariant. Also the transformations which reduces to analytic transformations $w \rightarrow f(w)$ in transversal degrees of freedom and hyper-analytic transformations $S^+ \rightarrow f(S^+), S^- \rightarrow f(S^-)$ in longitudinal degrees of freedom preserve this structure.

4. The basic idea is that of generalized Kähler structure meaning that the notion of Kähler function generalizes so that the non-vanishing components of metric are expressible as

$$\begin{aligned} g_{w\overline{w}} &= \partial_w\partial_{\overline{w}}K \ , \quad g_{+-} = \partial_{S^+}\partial_{S^-}K \ , \\ g_{w\pm} &= \partial_w\partial_{S^\pm}K \ , \quad g_{\overline{w}\pm} = \partial_{\overline{w}}\partial_{S^\pm}K \ . \end{aligned} \tag{2.4.20}$$

   for the components of the metric. The expression in terms of Kähler function is coordinate invariant for the same reason as in case of ordinary Kähler metric. In the standard light-cone coordinates the Kähler function is given by

$$K = w_0\overline{w}_0 + uv \ , \quad w_0 = x + iy \ , \quad u = t - z \ , \quad v = t + z \ . \tag{2.4.21}$$

The Christoffel symbols satisfy the conditions

$$\{^{\ k}_{w\ \overline{w}}\} = 0 \ , \quad \{^{\ k}_{+-}\} = 0 \ . \tag{2.4.22}$$

If energy momentum tensor has only the components $T^{w\overline{w}}$ and T^{+-}, field equations are satisfied in M^4_+ degrees of freedom.

5. The Hamilton Jacobi structures related by these transformations can be regarded as being equivalent. Since light-like 3- surface is, as the dynamical evolution defined by the light front, fixed by the 2-surface serving as the light source, these structures should be in one-one correspondence with 2-dimensional surfaces with two surfaces regarded as equivalent if they correspond to different time=constant snapshots of the same light front, or are related by a conformal transformation of M^4_+. Obviously there should be quite large number of them. Note that the generating two-dimensional surfaces relate also naturally to quaternion conformal invariance and corresponding Kac Moody invariance for which deformations defined by the M^4 coordinates as functions of the light-cone coordinates of the light front evolution define Kac Moody algebra, which thus seems to appear naturally also at the level of solutions of field equations.

The task is to find all possible local light cone coordinates defining one-parameter families 2-surfaces defined by the condition $S_i = constant$, $i = +\ or\ = -$, dual to each other and expanding with light velocity. The basic open questions are whether the generalized Kähler function indeed makes sense and whether the physical intuition about 2-surfaces as light sources parameterizing the set of all possible Hamilton Jacobi structures makes sense.

Hamilton Jacobi structure means the existence of foliations of the M^4 projection of X^4 by 2-D surfaces analogous to string word sheets labeled by w and the dual of this foliation defined by partonic 2-surfaces labeled by the values of S_i. Also the foliation by light-like 3-surfaces Y^3_l labeled by $S_\pm$ with $S_\mp$ serving as light-like coordinate for Y^3_l is implied. This is what number theoretic compactification and $M^8 - H$ duality predict when space-time surface corresponds to hyper-quaternionic surface of M^8 [K30, K74].

Contact structure and generalized Kähler structure of CP_2 projection

In the case of 3-dimensional CP_2 projection it is assumed that one can introduce complex coordinates $(\xi, \overline{\xi})$ and the third coordinate s. These coordinates would correspond to a contact structure in 3-dimensional CP_2 projection defining transversal symplectic and Kähler structures. In these coordinates the transversal parts of the induced CP_2 Kähler form and metric would contain only components of type $g_{w\overline{w}}$ and $J_{w\overline{w}}$. The transversal Kähler field $J_{w\overline{w}}$ would induce the Kähler magnetic field and the components J_{sw} and $J_{s\overline{w}}$ the Kähler electric field.

It must be emphasized that the non-integrability of the contact structure implies that J cannot be parallel to the tangent planes of $s = constant$ surfaces, s cannot be parallel to neither A nor the dual of J, and ξ cannot vary in the tangent plane defined by J. A further important conclusion is that for the solutions with 3-dimensional CP_2 projection topologized Kähler charge density is necessarily non-vanishing by $A \wedge J \neq 0$ whereas for the solutions with $D_{CP_2} = 2$ topologized Kähler current vanishes.

Also the CP_2 projection is assumed to possess a generalized Kähler structure in the sense that all components of the metric except s_{ss} are derivable from a Kähler function by formulas similar to M^4_+ case.

$$s_{w\overline{w}} = \partial_w\partial_{\overline{w}}K \ , \quad s_{ws} = \partial_w\partial_s K \ , \quad s_{\overline{w}s} = \partial_{\overline{w}}\partial_s K \ . \tag{2.4.23}$$

Generalized Kähler property guarantees that the vanishing of the Christoffel symbols of CP_2 (rather than those of 3-dimensional projection), which are of type $\{^{\ k}_{\xi\ \overline{\xi}}\}$.

$$\{^{\ k}_{\xi\ \overline{\xi}}\} \ = \ 0 \ . \tag{2.4.24}$$

Here the coordinates of CP_2 have been chosen in such a manner that three of them correspond to the coordinates of the projection and fourth coordinate is constant at the projection. The upper index k refers also to the CP_2 coordinate, which is constant for the CP_2 projection. If energy momentum tensor has only components of type T^{+-} and $T^{w\overline{w}}$, field equations are satisfied even when if non-diagonal Christoffel symbols of CP_2 are present. The challenge is to discover solution ansatz, which guarantees this property of the energy momentum tensor.

A stronger variant of Kähler property would be that also s_{ss} vanishes so that the coordinate lines defined by s would define light like curves in CP_2. The topologization of the Kähler current however implies that CP_2 projection is a projection of a 3-surface with strong Kähler property. Using $(s, \xi, \overline{\xi}, S^-)$ as coordinates for the space-time surface defined by the ansatz $(w = w(\xi, s), S^+ = S^+(s))$ one finds that g_{ss} must be vanishing so that stronger variant of the Kähler property holds true for $S^- = constant$ 3-surfaces.

The topologization condition for the Kähler current can be solved completely generally in terms of the induced metric using $(\xi, \overline{\xi}, s)$ and some coordinate of M^4_+, call it x^4, as space-time coordinates. Topologization boils down to the conditions

$$\begin{aligned} \partial_\beta(J^{\alpha\beta}\sqrt{g}) \ &= \ 0 \text{ for } \ \alpha \in \{\xi, \overline{\xi}, s\} \ , \\ g^{4i} \neq 0 \ . \end{aligned} \tag{2.4.25}$$

Thus 3-dimensional empty space Maxwell equations and the non-orthogonality of X^4 coordinate lines and the 3-surfaces defined by the lift of the CP_2 projection.

A solution ansatz yielding light-like current in $D_{CP_2} = 3$ case

The basic idea is that of generalized Kähler structure and solutions of field equations as maps or deformations of canonically imbedded M^4_+ respecting this structure and guaranteeing that the only non-vanishing components of the energy momentum tensor are $T^{\xi\xi}$ and T^{s-} in the coordinates $(\xi, \overline{\xi}, s, S^-)$.

1. The coordinates (w, S^+) are assumed to holomorphic functions of the CP_2 coordinates (s, ξ)

   $$S^+ = S^+(s) \ \ , \quad w = w(\xi, s) \ \ . \tag{2.4.26}$$

   Obviously S^+ could be replaced with S^-. The ansatz is completely symmetric with respect to the exchange of the roles of (s, w) and (S^+, ξ) since it maps longitudinal degrees of freedom to longitudinal ones and transverse degrees of freedom to transverse ones.

2. Field equations are satisfied if the only non-vanishing components of the energy momentum tensor are of type $T^{\xi\overline{\xi}}$ and T^{s-}. The reason is that the CP_2 Christoffel symbols for projection and projections of M^4_+ Christoffel symbols are vanishing for these lower index pairs.

3. By a straightforward calculation one can verify that the only manner to achieve the required structure of energy momentum tensor is to assume that the induced metric in the coordinates $(\xi, \overline{\xi}, s, S^-)$ has as non-vanishing components only $g_{\xi\overline{\xi}}$ and g_{s-}

   $$g_{ss} = 0 \ \ , \quad g_{\xi s} = 0 \ \ , \quad g_{\overline{\xi} s} = 0 \ \ . \tag{2.4.27}$$

   Obviously the space-time surface must factorize into an orthogonal product of longitudinal and transversal spaces.

4. The condition guaranteeing the product structure of the metric is

$$\begin{aligned} s_{ss} &= m_{+w}\partial_s w(\xi,s)\partial_s S^+(s) + m_{+\overline{w}}\partial_s \overline{w}(\xi,s)\partial_s S^+(s) \ , \\ s_{s\xi} &= m_{+w}\partial_\xi w(\xi)\partial_s S^+(s) \ , \\ s_{s\overline{\xi}} &= m_{+w}\partial_{\overline{\xi}} w(\overline{\xi})\partial_s S^+(s) \ . \end{aligned} \tag{2.4.28}$$

Thus the function of dynamics is to diagonalize the metric and provide it with strong Kähler property. Obviously the CP_2 projection corresponds to a light-like surface for all values of S^- so that space-time surface is foliated by light-like surfaces and the notion of generalized conformal invariance makes sense for the entire space-time surface rather than only for its boundary or elementary particle horizons.

5. The requirement that the Kähler current is proportional to the instanton current means that only the j^- component of the current is non-vanishing. This gives the following conditions

$$\begin{aligned} & j^\xi\sqrt{g} = \partial_\beta(J^{\xi\beta}\sqrt{g}) = 0 \ , \quad j^{\overline{\xi}}\sqrt{g} = \partial_\beta(J^{\overline{\xi}\beta}\sqrt{g}) = 0 \ , \\ & j^+\sqrt{g} = \partial_\beta(J^{+\beta}\sqrt{g}) = 0 \ . \end{aligned} \tag{2.4.29}$$

Since $J^{+\beta}$ vanishes, the condition

$$\sqrt{g}j^+ = \partial_\beta(J^{+\beta}\sqrt{g}) \quad = \quad 0 \tag{2.4.30}$$

is identically satisfied. Therefore the number of field equations reduces to three.

The physical interpretation of the solution ansatz deserves some comments.

1. The light-like character of the Kähler current brings in mind CP_2 extremals for which CP_2 projection is light like. This suggests that the topological condensation of CP_2 type extremal occurs on $D_{CP_2} = 3$ helical space-time sheet representing zitterbewegung. In the case of many-body system light-likeness of the current does not require that particles are massless if particles of opposite charges can be present. Field tensor has the form $(J^{\xi\overline{\xi}}, J^{\xi -}, J^{\overline{\xi}-})$. Both helical magnetic field and electric field present as is clear when one replaces the coordinates (S^+, S^-) with time-like and space-like coordinate. Magnetic field dominates but the presence of electric field means that genuine Beltrami field is not in question.

2. Since the induced metric is product metric, 3-surface is metrically product of 2-dimensional surface X^2 and line or circle and obeys product topology. If preferred extremals correspond to asymptotic self-organization patterns, the appearance of the product topology and even metric is not so surprising. Thus the solutions can be classified by the genus of X^2. An interesting question is how closely the explanation of family replication phenomenon in terms of the topology of the boundary component of elementary particle like 3-surface relates to this. The heaviness and instability of particles which correspond to genera $g > 2$ (sphere with more than two handles) might have simple explanation as absence of (stable) $D_{CP_2} = 3$ solutions of field equations with genus $g > 2$.

3. The solution ansatz need not be the most general. Kähler current is light-like and already this is enough to reduce the field equations to the form involving only energy momentum tensor. One might hope of finding also solution ansätze for which Kähler current is time-like or space-like. Space-likeness of the Kähler current might be achieved if the complex coordinates $(\xi, \overline{\xi})$ and hyper-complex coordinates (S^+, S^-) change the role. For this solution ansatz electric field would dominate. Note that the possibility that Kähler current is always light-like cannot be excluded.

4. Suppose that CP_2 projection quite generally defines a foliation of the space-time surface by light-like 3-surfaces, as is suggested by the conformal invariance. If the induced metric has Minkowskian signature, the fourth coordinate x^4 and thus also Kähler current must be time-like or light-like so that magnetic field dominates. Already the requirement that the metric is non-degenerate implies $g_{s4} \neq 0$ so that the metric for the $\xi = constant$ 2-surfaces has a Minkowskian signature. Thus space-like Kähler current does not allow the lift of the CP_2 projection to be light-like.

Are solutions with time-like or space-like Kähler current possible in $D_{CP_2} = 3$ case?

As noticed in the section about number theoretical compactification, the flow of gauge currents along slices Y_l^3 of $X^4(X_l^3)$ "parallel" to X_l^3 requires only that gauge currents are parallel to Y_l^3 and can thus space-like. The following ansatz gives good hopes for obtaining solutions with space-like and perhaps also time-like Kähler currents.

1. Assign to light-like coordinates coordinates (T, Z) by the formula $T = S^+ + S^-$ and $Z = S^+ - S^-$. Space-time coordinates are taken to be $(\xi, \overline{\xi}, s)$ and coordinate Z. The solution ansatz with time-like Kähler current results when the roles of T and Z are changed. It will however found that same solution ansatz can give rise to both space-like and time-like Kähler current.

2. The solution ansatz giving rise to a space-like Kähler current is defined by the equations

$$T = T(Z, s) \quad , \quad w = w(\xi, s) \quad . \tag{2.4.31}$$

If T depends strongly on Z, the g_{ZZ} component of the induced metric becomes positive and Kähler current time-like.

3. The components of the induced metric are

$$\begin{array}{ll} g_{ZZ} = m_{ZZ} + m_{TT}\partial_Z T \partial_s T \ , & g_{Zs} = m_{TT}\partial_Z T \partial_s T \ , \\ g_{ss} = s_{ss} + m_{TT}\partial_s T \partial_s T \ , & g_{w\overline{w}} = s_{w\overline{w}} + m_{w\overline{w}}\partial_\xi w \partial_{\overline{\xi}} \overline{w} \ , \\ g_{s\xi} = s_{s\xi} \ , & g_{s\overline{\xi}} = s_{s\overline{\xi}} \ . \end{array} \tag{2.4.32}$$

Topologized Kähler current has only Z-component and 3-dimensional empty space Maxwell's equations guarantee the topologization.

In CP_2 degrees of freedom the contractions of the energy momentum tensor with Christoffel symbols vanish if T^{ss}, $T^{\xi s}$ and $T^{\xi\xi}$ vanish as required by internal consistency. This is guaranteed if the condition

$$J^{\xi s} \ = \ 0 \tag{2.4.33}$$

holds true. Note however that $J^{\xi Z}$ is non-vanishing. Therefore only the components $T^{\xi\overline{\xi}}$ and $T^{Z\xi}$, $T^{Z\overline{\xi}}$ of energy momentum tensor are non-vanishing, and field equations reduce to the conditions

$$\begin{aligned} \partial_{\overline{\xi}}(J^{\xi\overline{\xi}}\sqrt{g}) + \partial_Z(J^{\xi Z}\sqrt{g}) \ &= \ 0 \ , \\ \partial_\xi(J^{\overline{\xi}\xi}\sqrt{g}) + \partial_Z(J^{\overline{\xi}Z}\sqrt{g}) \ &= \ 0 \ . \end{aligned} \tag{2.4.34}$$

In the special case that the induced metric does not depend on z-coordinate equations reduce to holomorphicity conditions. This is achieve if T depends linearly on Z: $T = aZ$.

The contractions with M^4_+ Christoffel symbols come from the non-vanishing of $T^{Z\xi}$ and vanish if the Hamilton Jacobi structure satisfies the conditions

$$\begin{aligned} &\{{}^{\ k}_{T\ w}\} = 0 \ , \quad \{{}^{\ k}_{T\ \overline{w}}\} = 0 \ , \\ &\{{}^{\ k}_{Z\ w}\} = 0 \ , \quad \{{}^{\ k}_{Z\ \overline{w}}\} = 0 \end{aligned} \tag{2.4.35}$$

hold true. The conditions are equivalent with the conditions

$$\{{}^{\ k}_{\pm\ w}\} = 0 \ , \quad \{{}^{\ k}_{\pm\ \overline{w}}\} = 0 \ . \tag{2.4.36}$$

These conditions possess solutions (standard light cone coordinates are the simplest example). Also the second derivatives of $T(s,Z)$ contribute to the second fundamental form but they do not give rise to non-vanishing contractions with the energy momentum tensor. The cautious conclusion is that also solutions with time-like or space-like Kähler current are possible.

$D_{CP_2} = 4$ **case**

The preceding discussion was for $D_{CP_2} = 3$ and one should generalize the discussion to $D_{CP_2} = 4$ case.

1. Hamilton Jacobi structure for M^4_+ is expected to be crucial also now.
2. One might hope that for $D_{CP_2} = 4$ the Kähler structure of CP_2 defines a foliation of CP_2 by 3-dimensional contact structures. This requires that there is a coordinate varying along the field lines of the normal vector field X defined as the dual of the three-form $A\wedge dA = A\wedge J$. By the previous considerations the condition for this reads as $dX = d(log\phi)\wedge X$ and implies $X\wedge dX = 0$. Using the self duality of the Kähler form one can express X as $X^k = J^{kl}A_l$. By a brief calculation one finds that $X\wedge dX \propto X$ holds true so that (somewhat disappointingly) a foliation of CP_2 by contact structures does not exist.

For $D_{CP_2} = 4$ case Kähler current vanishes and this case corresponds to what I have called earlier Maxwellian phase since empty space Maxwell's equations would be indeed satisfied, provided this phase exists at all. It however seems that Maxwell phase is probably realized differently.

1. Solution ansatz with a 3-dimensional M^4_+ projection

The basic idea is that the complex structure of CP_2 is preserved so that one can use complex coordinates (ξ^1,ξ^2) for CP_2 in which CP_2 Christoffel symbols and energy momentum tensor have automatically the desired properties. This is achieved the second light like coordinate, say v, is non-dynamical so that the induced metric does not receive any contribution from the longitudinal degrees of freedom. In this case one has

$$S^+ = S^+(\xi^1,\xi^2) \ , \quad w = w(\xi^1,\xi^2) \ , \quad S^- = constant \ . \tag{2.4.37}$$

The induced metric does possesses only components of type $g_{i\bar{j}}$ if the conditions

$$g_{+w} = 0 \ , \quad g_{+\overline{w}} = 0 \ . \tag{2.4.38}$$

This guarantees that energy momentum tensor has only components of type $T^{i\bar{j}}$ in coordinates (ξ^1,ξ^2) and their contractions with the Christoffel symbols of CP_2 vanish identically. In M^4_+ degrees of freedom one must pose the conditions

$$\{{}^{\ k}_{w+}\} = 0 \ , \quad \{{}^{\ k}_{\overline{w}+}\} = 0 \ , \quad \{{}^{\ k}_{++}\} = 0 \ . \tag{2.4.39}$$

on Christoffel symbols. These conditions are satisfied if the the M^4_+ metric does not depend on S^+:

$$\partial_+ m_{kl} \quad = \quad 0 \ . \tag{2.4.40}$$

This means that m_{-w} and $m_{-\overline{w}}$ can be non-vanishing but like m_{+-} they cannot depend on S^+.

The second derivatives of S^+ appearing in the second fundamental form are also a source of trouble unless they vanish. Hence S^+ must be a linear function of the coordinates ξ^k:

$$S^+ \quad = \quad a_k\xi^k + \overline{a}_k\overline{\xi}^k \ . \tag{2.4.41}$$

Field equations are the counterparts of empty space Maxwell equations $j^\alpha = 0$ but with M^4_+ coordinates (u, w) appearing as dynamical variables and entering only through the induced metric. By holomorphy the field equations can be written as

$$\partial_j(J^{j\bar{i}}\sqrt{g}) = 0 \ \ , \quad \partial_{\bar{j}}(J^{\bar{j}i}\sqrt{g}) = 0 \ \ , \tag{2.4.42}$$

and can be interpreted as conditions stating the holomorphy of the contravariant Kähler form.

What is remarkable is that the M^4_+ projection of the solution is 3-dimensional light like surface and that the induced metric has Euclidian signature. Light front would become a concrete geometric object with one compactified dimension rather than being a mere conceptualization. One could see this as topological quantization for the notion of light front or of electromagnetic shock wave, or perhaps even as the realization of the particle aspect of gauge fields at classical level.

If the latter interpretation is correct, quantum classical correspondence would be realized very concretely. Wave and particle aspects would both be present. One could understand the interactions of charged particles with electromagnetic fields both in terms of absorption and emission of topological field quanta and in terms of the interaction with a classical field as particle topologically condenses at the photonic light front.

For CP_2 type extremals for which M^4_+ projection is a light like curve correspond to a special case of this solution ansatz: transversal M^4_+ coordinates are constant and S^+ is now arbitrary function of CP_2 coordinates. This is possible since M^4_+ projection is 1-dimensional.

2. Are solutions with a 4-dimensional M^4_+ projection possible?

The most natural solution ansatz is the one for which CP_2 complex structure is preserved so that energy momentum tensor has desired properties. For four-dimensional M^4_+ projection this ansatz does not seem to make promising since the contribution of the longitudinal degrees of freedom implies that the induced metric is not anymore of desired form since the components $g_{ij} = m_{+-}(\partial_{\xi^i}S^+\partial_{\xi^j}S^- + m_{+-}\partial_{\xi^i}S^-\partial_{\xi^j}S^+)$ are non-vanishing.

1. The natural dynamical variables are still Minkowski coordinates $(w, \overline{w}, S^+, S^-)$ for some Hamilton Jacobi structure. Since the complex structure of CP_2 must be given up, CP_2 coordinates can be written as (ξ, s, r) to stress the fact that only "one half" of the Kähler structure of CP_2 is respected by the solution ansatz.

2. The solution ansatz has the same general form as in $D_{CP_2} = 3$ case and must be symmetric with respect to the exchange of M^4_+ and CP_2 coordinates. Transverse coordinates are mapped to transverse ones and longitudinal coordinates to longitudinal ones:

$$(S^+, S^-) = (S^+(s, r), S^-(s, r)) \ \ , \quad w = w(\xi) \ . \tag{2.4.43}$$

This ansatz would describe ordinary Maxwell field in M^4_+ since the roles of M^4_+ coordinates and CP_2 coordinates are interchangeable.

It is however far from obvious whether there are any solutions with a 4-dimensional M^4_+ projection. That empty space Maxwell's equations would allow only the topologically quantized light fronts as its solutions would realize quantum classical correspondence very concretely.

The recent view conforms with this intuition. The Maxwell phase is certainly physical notion but would correspond effective fields experience by particle in many-sheeted space-time. Test particle topological condenses to all the space-time sheets with projection to a given region of Minkowski space and experiences essentially the sum of the effects caused by the induced gauge fields at different sheets. This applies also to gravitational fields interpreted as deviations from Minkowski metric.

The transition to GRT and QFT picture means the replacement of many-sheeted space-time with piece of Minkowski space with effective metric defined as the sum of Minkowski metric and deviations of the induced metrics of space-time sheets from Minkowski metric. Effective gauge potentials are sums of the induced gauge potentials. Hence the rather simple topologically quantized induced gauge fields associated with space-time sheets become the classical fields in the sense of Maxwell's theory and gauge theories.

$D_{CP_2} = 2$ case

Hamilton Jacobi structure for M^4_+ is assumed also for $D_{CP_2} = 2$, whereas the contact structure for CP_2 is in $D_{CP_2} = 2$ case replaced by the induced Kähler structure. Topologization yields vanishing Kähler current. Light-likeness provides a second manner to achieve vanishing Lorentz force but one cannot exclude the possibility of time- and space-like Kähler current.

1. Solutions with vanishing Kähler current

1. String like objects, which are products $X^2 \times Y^2 \subset M^4_+ \times CP_2$ of minimal surfaces Y^2 of M^4_+ with geodesic spheres S^2 of CP_2 and carry vanishing gauge current. String like objects allow considerable generalization from simple Cartesian products of $X^2 \times Y^2 \subset M^4 \times S^2$. Let $(w, \overline{w}, S^+, S^-)$ define the Hamilton Jacobi structure for M^4_+. $w = constant$ surfaces define minimal surfaces X^2 of M^4_+. Let ξ denote complex coordinate for a sub-manifold of CP_2 such that the imbedding to CP_2 is holomorphic: $(\xi^1, \xi^2) = (f^1(\xi), f^2(\xi))$. The resulting surface $Y^2 \subset CP_2$ is a minimal surface and field equations reduce to the requirement that the Kähler current vanishes: $\partial_{\overline{\xi}}(J^{\xi\overline{\xi}}\sqrt{g_2}) = 0$. One-dimensional strings are deformed to 3-dimensional cylinders representing magnetic flux tubes. The oscillations of string correspond to waves moving along string with light velocity, and for more general solutions they become TGD counterparts of Alfven waves associated with magnetic flux tubes regarded as oscillations of magnetic flux lines behaving effectively like strings. It must be emphasized that Alfven waves are a phenomenological notion not really justified by the properties of Maxwell's equations.

2. Also electret type solutions with the role of the magnetic field taken by the electric field are possible. $(\xi, \overline{\xi}, u, v)$ would provide the natural coordinates and the solution ansatz would be of the form

$$(s, r) = (s(u, v), r(u, v)) \quad , \quad \xi = constant \quad , \tag{2.4.44}$$

   and corresponds to a vanishing Kähler current.

3. Both magnetic and electric fields are necessarily present only for the solutions carrying non-vanishing electric charge density (proportional to $\overline{B} \cdot \overline{A}$). Thus one can ask whether more general solutions carrying both magnetic and electric field are possible. As a matter fact, one must first answer the question what one really means with the magnetic field. By choosing the coordinates of 2-dimensional CP_2 projection as space-time coordinates one can define what one means with magnetic and electric field in a coordinate invariant manner. Since the CP_2 Kähler form for the CP_2 projection with $D_{CP_2} = 2$ can be regarded as a pure Kähler magnetic field, the induced Kähler field is either magnetic field or electric field.

The form of the ansatz would be

$$(s,r) = (s,r)\,(u,v,w,\overline{w}) \ \ , \quad \xi = constant \ \ . \tag{2.4.45}$$

As a matter fact, CP_2 coordinates depend on two properly chosen M^4 coordinates only.

1. Solutions with light-like Kähler current

There are large classes of solutions of field equations with a light-like Kähler current and 2-dimensional CP_2 projection.

1. Massless extremals for which CP_2 coordinates are arbitrary functions of one transversal coordinate $e = f(w,\overline{w})$ defining local polarization direction and light like coordinate u of M^4_+ and carrying in the general case a light like current. In this case the holomorphy does not play any role.

2. The string like solutions thickened to magnetic flux tubes carrying TGD counterparts of Alfven waves generalize to solutions allowing also light-like Kähler current. Also now Kähler metric is allowed to develop a component between longitudinal and transversal degrees of freedom so that Kähler current develops a light-like component. The ansatz is of the form

$$\xi^i = f^i(\xi) \ \ , \ \ w = w(\xi) \ \ , \ \ S^- = s^- \ \ , \ \ S^+ = s^+ + f(\xi,\overline{\xi}) \ \ .$$

Only the components $g_{+\xi}$ and $g_{+\overline{\xi}}$ of the induced metric receive contributions from the modification of the solution ansatz. The contravariant metric receives contributions to $g^{-\xi}$ and $g^{-\overline{\xi}}$ whereas $g^{+\xi}$ and $g^{+\overline{\xi}}$ remain zero. Since the partial derivatives $\partial_\xi\partial_+h^k$ and $\partial_{\overline{\xi}}\partial_+h^k$ and corresponding projections of Christoffel symbols vanish, field equations are satisfied. Kähler current develops a non-vanishing component j^-. Apart from the presence of the electric field, these solutions are highly analogous to Beltrami fields.

Could $D_{CP_2} = 2 \rightarrow 3$ transition occur in rotating magnetic systems?

I have studied the imbeddings of simple cylindrical and helical magnetic fields in various applications of TGD to condensed matter systems, in particular in attempts to understand the strange findings about rotating magnetic systems [K77].

Let S^2 be the homologically non-trivial geodesic sphere of CP_2 with standard spherical coordinates $(U \equiv cos(\theta),\Phi)$ and let (t,ρ,ϕ,z) denote cylindrical coordinates for a cylindrical space-time sheet. The simplest possible space-time surfaces $X^4 \subset M^4_+ \times S^2$ carrying helical Kähler magnetic field depending on the radial cylindrical coordinate ρ, are given by:

$$\begin{array}{ll} U = U(\rho) \ \ , & \Phi = n\phi + kz \ \ , \\ J_{\rho\phi} = n\partial_\rho U \ \ , & J_{\rho z} = k\partial_\rho U \ \ . \end{array} \tag{2.4.46}$$

This helical field is not Beltrami field as one can easily find. A more general ansatz corresponding defined by

$$\Phi = \omega t + kz + n\phi$$

would in cylindrical coordinates give rise to both helical magnetic field and radial electric field depending on ρ only. This field can be obtained by simply replacing the vector potential with its rotated version and provides the natural first approximation for the fields associated with rotating magnetic systems.

A non-vanishing vacuum charge density is however generated when a constant magnetic field is put into rotation and is implied by the condition $\overline{E} = \overline{v} \times \overline{B}$ stating vanishing of the Lorentz force. This condition does not follow from the induction law of Faraday although Faraday observed this effect first. This is also clear from the fact that the sign of the charge density depends on the direction of rotation.

The non-vanishing charge density is not consistent with the vanishing of the Kähler 4-current and requires a 3-dimensional CP_2 projection and topologization of the Kähler current. Beltrami condition cannot hold true exactly for the rotating system. The conclusion is that rotation induces a phase transition $D_{CP_2} = 2 \rightarrow 3$. This could help to understand various strange effects related to the rotating magnetic systems [K77]. For instance, the increase of the dimension of CP_2 projection could generate join along boundaries contacts/flux tubes and wormhole contacts leading to the transfer of charge between different space-time sheets. The possibly resulting flow of gravitational flux to larger space-time sheets might help to explain the claimed antigravity effects.

2.4.4 $D_{CP_2} = 3$ Phase Allows Infinite Number Of Topological Charges Characterizing The Linking Of Magnetic Field Lines

When space-time sheet possesses a $D = 3$-dimensional CP_2 projection, one can assign to it a non-vanishing and conserved topological charge characterizing the linking of the magnetic field lines defined by Chern-Simons action density $A \wedge dA/4\pi$ for induced Kähler form. This charge can be seen as classical topological invariant of the linked structure formed by magnetic field lines.

The topological charge can also vanish for $D_{CP_2} = 3$ space-time sheets. In Darboux coordinates for which Kähler gauge potential reads as $A = P_k dQ^k$, the surfaces of this kind result if one has $Q^2 = f(Q^1)$ implying $A = fdQ^1$, $f = P_1 + P_2\partial_{Q_1}Q^2$, which implies the condition $A \wedge dA = 0$. For these space-time sheets one can introduce Q^1 as a global coordinate along field lines of A and define the phase factor $exp(i\int A_\mu dx^\mu)$ as a wave function defined for the entire space-time sheet. This function could be interpreted as a phase of an order order parameter of super-conductor like state and there is a high temptation to assume that quantum coherence in this sense is lost for more general $D_{CP_2} = 3$ solutions.

Chern-Simons action is known as helicity in electrodynamics [B47]. Helicity indeed describes the linking of magnetic flux lines as is easy to see by interpreting magnetic field as incompressible fluid flow having A as vector potential: $B = \nabla \times A$. One can write A using the inverse of $\nabla\times$ as $A = (1/\nabla\times)B$. The inverse is non-local operator expressible as

$$\frac{1}{\nabla\times}B(r) = \int dV'\frac{(r-r')}{|r-r'|^3} \times B(r') \ ,$$

as a little calculation shows. This allows to write $\int A \cdot B$ as

$$\int dV A \cdot B = \int dV dV' B(r) \cdot \left(\frac{(r-r')}{|r-r'|^3} \times B(r')\right) \ ,$$

which is completely analogous to the Gauss formula for linking number when linked curves are replaced by a distribution of linked curves and an average is taken.

For $D_{CP_2} = 3$ field equations imply that Kähler current is proportional to the helicity current by a factor which depends on CP_2 coordinates, which implies that the current is automatically divergence free and defines a conserved charge for $D = 3$-dimensional CP_2 projection for which the instanton density vanishes identically. Kähler charge is not equal to the helicity defined by the inner product of magnetic field and vector potential but to a more general topological charge.

The number of conserved topological charges is infinite since the product of any function of CP_2 coordinates with the helicity current has vanishing divergence and defines a topological charge. A very natural function basis is provided by the scalar spherical harmonics of $SU(3)$ defining Hamiltonians of CP_2 canonical transformations and possessing well defined color quantum numbers. These functions define and infinite number of conserved charges which are also classical knot invariants in the sense that they are not affected at all when the 3-surface interpreted as a map from CP_2 projection to M^4_+ is deformed in M^4_+ degrees of freedom. Also canonical transformations induced by Hamiltonians in irreducible representations of color group affect these invariants via Poisson bracket action when the $U(1)$ gauge transformation induced by the canonical transformation corresponds to a single valued scalar function. These link invariants are additive in union whereas the quantum invariants defined by topological quantum field theories are multiplicative.

Also non-Abelian topological charges are well-defined. One can generalize the topological current associated with the Kähler form to a corresponding current associated with the induced electro-weak gauge fields whereas for classical color gauge fields the Chern-Simons form vanishes identically. Also in this case one can multiply the current by CP_2 color harmonics to obtain an infinite number of invariants in $D_{CP_2} = 3$ case. The only difference is that $A \wedge dA$ is replaced by $Tr(A \wedge (dA + 2A \wedge A/3))$.

There is a strong temptation to assume that these conserved charges characterize colored quantum states of the conformally invariant quantum theory as a functional of the light-like 3-surface defining boundary of space-time sheet or elementary particle horizon surrounding wormhole contacts. They would be TGD analogs of the states of the topological quantum field theory defined by Chern-Simons action as highest weight states associated with corresponding Wess-Zumino-Witten theory. These charges could be interpreted as topological counterparts of the isometry charges of WCW defined by the algebra of canonical transformations of CP_2.

The interpretation of these charges as contributions of light-like boundaries to WCW Hamiltonians would be natural. The dynamics of the induced second quantized spinor fields relates to that of Kähler action by a super-symmetry, so that it should define super-symmetric counterparts of these knot invariants. The anti-commutators of these super charges cannot however contribute to WCW Kähler metric so that topological zero modes are in question. These Hamiltonians and their super-charge counterparts would be responsible for the topological sector of quantum TGD.

2.4.5 Preferred Extremal Property And The Topologization/Light-Likeness Of Kähler Current?

The basic question is under what conditions the Kähler current is either topologized or light-like so that the Lorentz force vanishes. Does this hold for all preferred extremals of Kähler action? Or only asymptotically as suggested by the fact that generalized Beltrami fields can be interpreted as asymptotic self-organization patterns, when dissipation has become insignificant. Or does topologization take place in regions of space-time surface having Minkowskian signature of the induced metric? And what asymptotia actually means? Do absolute minima of Kähler action correspond to preferred extremals?

One can challenge the interpretation in terms of asymptotic self organization patterns assigned to the Minkowskian regions of space-time surface.

1. Zero energy ontology challenges the notion of approach to asymptotia in Minkowskian sense since the dynamics of light-like 3-surfaces is restricted inside finite volume $CD \subset M^4$ since the partonic 2-surfaces representing their ends are at the light-like boundaries of causal diamond in a given p-adic time scale.

2. One can argue that generic non-asymptotic field configurations have $D_{CP_2} = 4$, and would thus carry a vanishing Kähler four-current if Beltrami conditions were satisfied universally rather than only asymptotically. $j^\alpha = 0$ would obviously hold true also for the asymptotic configurations, in particular those with $D_{CP_2} < 4$ so that empty space Maxwell's field equations would be universally satisfied for asymptotic field configurations with $D_{CP_2} < 4$. The weak point of this argument is that it is 3-D light-like 3-surfaces rather than space-time surfaces which are the basic dynamical objects so that the generic and only possible case corresponds to $D_{CP_2} = 3$ for X_l^3. It is quite possible that preferred extremal property implies that $D_{CP_2} = 3$ holds true in the Minkowskian regions since these regions indeed represent empty space. Geometrically this would mean that the CP_2 projection does not change as the light-like coordinate labeling Y_l^3 varies. This conforms nicely with the notion of quantum gravitational holography.

3. The failure of the generalized Beltrami conditions would mean that Kähler field is completely analogous to a dissipative Maxwell field for which also Lorentz force vanishes since $\overline{j} \cdot \overline{E}$ is non-vanishing (note that isometry currents are conserved although energy momentum tensor is not). Quantum classical correspondence states that classical space-time dynamics is by its classical non-determinism able to mimic the non-deterministic sequence of quantum jumps at space-time level, in particular dissipation in various length scales defined by the hierarchy of space-time sheets. Classical fields would represent "symbolically" the average dynamics, in particular dissipation, in shorter length scales. For instance, vacuum 4-current would be a symbolic representation for the average of the currents consisting of elementary particles. This would seem to support the view that $D_{CP_2} = 4$ Minkowskian regions are present. The

weak point of this argument is that there is fractal hierarchy of length scales represented by the hierarchy of causal diamonds (CDs) and that the resulting hierarchy of generalized Feynman graphs might be enough to represent dissipation classically.

4. One objection to the idea is that second law realized as an asymptotic vanishing of Lorentz-Kähler force implies that all space-like 3-surfaces approaching same asymptotic state have the same value of Kähler function assuming that the Kähler function assignable to space-like 3-surface is same for all space-like sections of $X^4(X_l^3)$ (assuming that one can realize general coordinate invariance also in this sense). This need not be the case. In any case, this need not be a problem since it would mean an additional symmetry extending general coordinate invariance. The exponent of Kähler function would be highly analogous to a partition function defined as an exponent of Hamiltonian with Kähler coupling strength playing the role of temperature.

It seems that asymptotic self-organization pattern need not be correct interpretation for non-dissipating regions, and the identification of light-like 3-surfaces as generalized Feynman diagrams encourages an alternative interpretation.

1. M^8-H duality states that also the H counterparts of co-hyper-hyperquaternionic surfaces of M^8 are preferred extremals of Kähler action. CP_2 type vacuum extremals represent the basic example of these and a plausible conjecture is that the regions of space-time with Euclidian signature of the induced metric represent this kind of regions. If this conjecture is correct, dissipation could be assigned with regions having Euclidian signature of the induced metric. This makes sense since dissipation has quantum description in terms of Feynman graphs and regions of Euclidian signature indeed correspond to generalized Feynman graphs. This argument would suggest that generalized Beltrami conditions or light-likeness hold true inside Minkowskian regions rather than only asymptotically.

2. One could of course play language games and argue that asymptotia is with respect to the Euclidian time coordinate inside generalized Feynman graps and is achieved exactly when the signature of the induced metric becomes Minkowskian. This is somewhat artificial attempt to save the notion of asymptotic self-organization pattern since the regions outside Feynman diagrams represent empty space providing a holographic representations for the matter at X_l^3 so that the vanishing of $j^\alpha F_{\alpha\beta}$ is very natural.

3. What is then the correct identification of asymptotic self-organization pattern. Could correspond to the negative energy part of the zero energy state at the upper light-like boundary δM^4_- of CD? Or in the case of phase conjugate state to the positive energy part of the state at δM^4_+? An identification consistent with the fractal structure of zero energy ontology and TGD inspired theory of consciousness is that the entire zero energy state reached by a sequence of quantum jumps represents asymptotic self-organization pattern represented by the asymptotic generalized Feynman diagram or their superposition. Biological systems represent basic examples about self-organization, and one cannot avoid the questions relating to the relationship between experience and geometric time. A detailed discussion of these points can be found in [K4].

Absolute minimization of Kähler action was the first guess for the criterion selecting preferred extremals. Absolute minimization in a strict sense of the word does not make sense in the p-adic context since p-adic numbers are not well-ordered, and one cannot even define the action integral as a p-adic number. The generalized Beltrami conditions and the boundary conditions defining the preferred extremals are however local and purely algebraic and make sense also p-adically. If absolute minimization reduces to these algebraic conditions, it would make sense.

2.4.6 Generalized Beltrami Fields And Biological Systems

The following arguments support the view that generalized Beltrami fields play a key role in living systems, and that $D_{CP_2}=2$ corresponds to ordered phase, $D_{CP_2}=3$ to spin glass phase and $D_{CP_2}=4$ to chaos, with $D_{CP_2}=3$ defining life as a phenomenon at the boundary between order and chaos. If the criteria suggested by the number theoretic compactification are accepted, it is not clear whether D_{CP_2} extremals can define preferred extremals of Kähler action. For instance, cosmic strings are not preferred extremals and the Y_l^3 associated with MEs allow only covariantly constant right handed neutrino eigenmode of $D_K(X^2)$. The topological condensation of CP_2 type vacuum extremals around $D_{CP_2} = 2$ type extremals is however expected to give preferred extremals and if the density of the condensate is low enough one can still speak about $D_{CP_2} = 2$ phase. A natural guess is also that the deformation of $D_{CP_2} = 2$ extremals transforms light-like gauge currents to space-like topological currents allowed by $D_{CP_2} = 3$ phase.

Why generalized Beltrami fields are important for living systems?

Chirality, complexity, and high level of organization make $D_{CP_2} = 3$ generalized Beltrami fields excellent candidates for the magnetic bodies of living systems.

1. Chirality selection is one of the basic signatures of living systems. Beltrami field is characterized by a chirality defined by the relative sign of the current and magnetic field, which means parity breaking. Chirality reduces to the sign of the function ψ appearing in the topologization condition and makes sense also for the generalized Beltrami fields.

2. Although Beltrami fields can be extremely complex, they are also extremely organized. The reason is that the function α is constant along flux lines so that flux lines must in the case of compact Riemann 3-manifold belong to 2-dimensional $\alpha = constant$ closed surfaces, in fact two-dimensional invariant tori [B66] .

For generalized Beltrami fields the function ψ is constant along the flow lines of the Kähler current. Space-time sheets with 3-dimensional CP_2 projection serve as an illustrative example. One can use the coordinates for the CP_2 projection as space-time coordinates so that one space-time coordinate disappears totally from consideration. Hence the situation reduces to a flow in a 3-dimensional sub-manifold of CP_2. One can distinguish between three types of flow lines corresponding to space-like, light-like and time-like topological current. The 2-dimensional $\psi = constant$ invariant manifolds are sub-manifolds of CP_2. Ordinary Beltrami fields are a special case of space-like flow with flow lines belonging to the 2-dimensional invariant tori of CP_2. Time-like and light-like situations are more complex since the flow lines need not be closed so that the 2-dimensional $\psi = constant$ surfaces can have boundaries.

For periodic self-organization patterns flow lines are closed and $\psi = constant$ surfaces of CP_2 must be invariant tori. The dynamics of the periodic flow is obtained from that of a steady flow by replacing one spatial coordinate with effectively periodic time coordinate. Therefore topological notions like helix structure, linking, and knotting have a dynamical meaning at the level of CP_2 projection. The periodic generalized Beltrami fields are highly organized also in the temporal domain despite the potentiality for extreme topological complexity.

For these reasons topologically quantized generalized Beltrami fields provide an excellent candidate for a generic model for the dynamics of biological self-organization patterns. A natural guess is that many-sheeted magnetic and Z^0 magnetic fields and their generalizations serve as templates for the helical molecules populating living matter, and explain both chiral selection, the complex linking and knotting of DNA and protein molecules, and even the extremely complex and self-organized dynamics of biological systems at the molecular level.

The intricate topological structures of DNA, RNA, and protein molecules are known to have a deep significance besides their chemical structure, and they could even define something analogous to the genetic code. Usually the topology and geometry of bio-molecules is believed to reduce to chemistry. TGD suggests that space-like generalized Beltrami fields serve as templates for the formation of bio-molecules and bio-structures in general. The dynamics of bio-systems would in turn utilize the time-like Beltrami fields as templates. There could even exist a mapping from the topology of magnetic flux tube structures serving as templates for bio-molecules to the templates of self-organized dynamics. The helical structures, knotting, and linking of bio-molecules would thus define a symbolic representation, and even coding for the dynamics of the bio-system analogous to written language.

$D_{CP_2} = 3$ systems as boundary between $D_{CP_2} = 2$ order and $D_{CP_2} = 4$ chaos

The dimension of CP_2 projection is basic classifier for the asymptotic self-organization patterns.

1. $D_{CP_2} = 4$ phase, dead matter, and chaos

$D_{CP_2} = 4$ corresponds to the ordinary Maxwellian phase in which Kähler current and charge density vanish and there is no topologization of Kähler current. By its maximal dimension this phase would naturally correspond to disordered phase, ordinary "dead matter". If one assumes that Kähler charge corresponds to either em charge or Z^0 charge then the signature of this state of matter would be em neutrality or Z^0 neutrality.

2. $D_{CP_2} = 2$ phase as ordered phase

By the low dimension of CP_2 projection $D_{CP_2} = 2$ phase is the least stable phase possible only at cold space-time sheets. Kähler current is either vanishing or light-like, and Beltrami fields are not possible. This phase is highly ordered and much like a topological quantized version of ferro-magnet. In particular, it is possible to have a global coordinate varying along the field lines of the vector potential also now. The magnetic and Z^0 magnetic body of any system is a candidate for this kind of system. Z^0 field is indeed always present for vacuum extremals having $D_{CP_2} = 2$ and the vanishing of em field requires that that $sin^2(\theta_W)$ (θ_W is Weinberg angle) vanishes.

3. $D_{CP_2} = 3$ corresponds to living matter

$D_{CP_2} = 3$ corresponds to highly organized phase characterized in the case of space-like Kähler current by complex helical structures necessarily accompanied by topologized Kähler charge density $\propto \overline{A} \cdot \overline{B} \neq 0$ and Kähler current $\overline{E} \times \overline{A} + \phi \overline{B}$. For time like Kähler currents the helical structures are replaced by periodic oscillation patterns for the state of the system. By the non-maximal dimension of CP_2 projection this phase must be unstable against too strong external perturbations and cannot survive at too high temperatures. Living matter is thus excellent candidate for this phase and it might be that the interaction of the magnetic body with living matter makes possible the transition from $D_{CP_2} = 2$ phase to the self-organizing $D_{CP_2} = 3$ phase.

Living matter which is indeed populated by helical structures providing examples of space-like Kähler current. Strongly charged lipid layers of cell membrane might provide example of time-like Kähler current. Cell membrane, micro-tubuli, DNA, and proteins are known to be electrically charged and Z^0 charge plays key role in TGD based model of catalysis discussed in [K25]. For instance, denaturing of DNA destroying its helical structure could be interpreted as a transition leading from $D_{CP_2} = 3$ phase to $D_{CP_2} = 4$ phase. The prediction is that the denatured phase should be electromagnetically (or Z^0) neutral.

Beltrami fields result when Kähler charge density vanishes. For these configurations magnetic field and current density take the role of the vector potential and magnetic field as far as the contact structure is considered. For Beltrami fields there exist a global coordinate along the field lines of the vector potential but not along those of the magnetic field. As a consequence, the covariant consistency condition $(\partial_s - qeA_s)\Psi = 0$ frequently appearing in the physics of super conducting systems would make sense along the flow lines of the vector potential for the order parameter of Bose-Einstein condensate. If Beltrami phase is super-conducting, then the state of the system must change in the transition to a more general phase. It is impossible to assign slicing of 4-surface by 3-D surfaces labeled by a coordinate t varying along the flow lines. This means that one cannot speak about a continuous evolution of Schrödinger amplitude with t playing the role of time coordinate. One could perhaps say that the entire space-time sheet represents single quantum event which cannot be decomposed to evolution. This would conform with the assignment of macroscopic and macro-temporal quantum coherence with living matter.

The existence of these three phases brings in mind systems allowing chaotic de-magnetized phase above critical temperature T_c, spin glass phase at the critical point, and ferromagnetic phase below T_c. Similar analogy is provided by liquid phase, liquid crystal phase possible in the vicinity of the critical point for liquid to solid transition, and solid phase. Perhaps one could regard $D_{CP_2} = 3$ phase and life as a boundary region between $D_{CP_2} = 2$ order and $D_{CP_2} = 4$ chaos. This would naturally explain why life as it is known is possible in relatively narrow temperature interval.

Can one assign a continuous Schrödinger time evolution to light-like 3-surfaces?

Alain Connes wrote [A30] about factors of various types using as an example Schrödinger equation for various kinds of foliations of space-time to time=constant slices. If this kind of foliation does not exist, one cannot speak about time evolution of Schrödinger equation at all. Depending on the character of the foliation one can have factor of type I, II, or III. For instance, torus with slicing $dx = ady$ in flat coordinates, gives a factor of type I for rational values of a and factor of type II for irrational values of a.

1. 3-D foliations and type III factors

Connes mentioned 3-D foliations V which give rise to type III factors. Foliation property requires a slicing of V by a one-form v to which slices are orthogonal (this requires metric).

1. The foliation property requires that v multiplied by suitable scalar is gradient. This gives the integrability conditions $dv = w \wedge v$, $w = -d\psi/\psi = -dlog(\psi)$. Something proportional to $log(\psi)$ can be taken as a third coordinate varying along flow lines of v: the flow defines a continuous sequence of maps of 2-dimensional slice to itself.

2. If the so called Godbillon-Vey invariant defined as the integral of $dw \wedge w$ over V is non-vanishing, factor of type III is obtained using Schrödinger amplitudes for which the flow lines of foliation define the time evolution. The operators of the algebra in question are transversal operators acting on Schrödinger amplitudes at each slice. Essentially Schrödinger equation in 3-D space-time would be in question with factor of type III resulting from the exotic choice of the time coordinate defining the slicing.

2. What happens in case of light-like 3-surfaces?

In TGD light-like 3-surfaces are natural candidates for V and it is interesting to look what happens in this case. Light-likeness is of course a disturbing complication since orthogonality condition and thus contravariant metric is involved with the definition of the slicing. Light-likeness is not however involved with the basic conditions.

1. The one-form v defined by the induced Kähler gauge potential A defining also a braiding is a unique identification for v. If foliation exists, the braiding flow defines a continuous sequence of maps of partonic 2-surface to itself.

2. Physically this means the possibility of a super-conducting phase with order parameter satisfying covariant constancy equation $D\psi = (d/dt - ieA)\psi = 0$. This would describe a supra current flowing along flow lines of A.

3. If the integrability fails to be true, one *cannot* assign Schrödinger time evolution with the flow lines of v. One might perhaps say that 3-surface behaves like single quantum event not allowing slicing by a continuous Schrödinger time evolution.

4. The condition that the modes of the induced spinor field have well-defined em charge implies that CP_2 projection for the region of space-time in which induced spinor field is non-vanishing is 2-dimensional. In the generic case a localization to 2-surfaces - string world sheets and possibly partonic 2-surface. At light-like 3-surfaces this implies that modes are localized at 1-D curves so that the hydrodynamic picture is realized [K88].

3. Extremals of Kähler action

Some comments relating to the interpretation of the classification of the extremals of Kähler action by the dimension of their CP_2 projection are in order. It has been already found that the extremals can be classified according to the dimension D of the CP_2 projection of space-time sheet in the case that $A_a = 0$ holds true.

1. For $D_{CP_2} = 2$ integrability conditions for the vector potential can be satisfied for $A_a = 0$ so that one has generalized Beltrami flow and one can speak about Schrödinger time evolution associated with the flow lines of vector potential defined by covariant constancy condition $D\psi = 0$ makes sense. Kähler current is vanishing or light-like. This phase is analogous to a super-conductor or a ferromagnetic phase. For non-vanishing A_a the Beltrami flow property is lost but the analogy with ferromagnetism makes sense still.

2. For $D_{CP_2} = 3$ foliations are lost. The phase is dominated by helical structures. This phase is analogous to spin glass phase around phase transition point from ferromagnetic to non-magnetized phase and expected to be important in living matter systems.

3. $D_{CP_2} = 4$ is analogous to a chaotic phase with vanishing Kähler current and to a phase without magnetization. The interpretation in terms of non-quantum coherent "dead" matter is suggestive.

An interesting question is whether the ordinary 8-D imbedding space which defines one sector of the generalized imbedding space could correspond to $A_a = 0$ phase. If so, then all states for this sector would be vacua with respect to M^4 quantum numbers. M^4-trivial zero energy states in this sector could be transformed to non-trivial zero energy states by a leakage to other sectors.

2.4.7 About Small Perturbations Of Field Equations

The study of small perturbations of the known solutions of field equations is a standard manner to get information about the properties of the solutions, their stability in particular. Fourier expansion is the standard manner to do the perturbation theory. In the recent case an appropriate modification of this ansatz might make sense if the solution in question is representable as a map $M^4_+ \rightarrow CP_2$, and the perturbations are rapidly varying when compared to the components of the induced metric and Kähler form so that one can make adiabatic approximation and approximate them as being effectively constant. Presumably also restrictions on directions of wave 4-vectors $k_\mu = (\omega, \overline{k}))$ are necessary so that the direction of wave vector adapts to the slowly varying background as in ray optics. Also Hamilton Jacobi structure is expected to modify the most straightforward approach. The four CP_2 coordinates are the dynamical variables so that the situation is relatively simple.

A completely different approach is inspired by the physical picture. In this approach one glues CP_2 type vacuum extremal to a known extremal and tries to deduce the behavior of the deformed extremal in the vicinity of wormhole throat by posing the general conditions on the slicing by light-like 3-surfaces Y^3_l. This approach is not followed now.

Generalized plane waves

Individual plane waves are geometrically very special since they represent a deformation of the space-time surface depending on single coordinate only. Despite this one might hope that plane waves or their appropriate modifications allowing to algebraize the treatment of small perturbations could give useful information also now.

1. Lorentz invariance plus the translational invariance due to the assumption that the induced metric and Kähler form are approximately constant encourage to think that the coordinates reduce Minkowski coordinates locally with the orientation of the local Minkowski frame depending slowly on space-time position. Hamilton Jacobi $(S^+, S^-, w, \overline{w})$ are a good candidate for this kind of coordinates. The properties of the Hamilton Jacobi structure and of the solution ansatz suggest that excitations are generalized plane waves in longitudinal degrees of freedom only so that four-momentum would be replaced by the longitudinal momentum. In transverse degrees of freedom one might expect that holomorphic plane-waves $exp(ik_T w)$, where k_T is transverse momentum, make algebraization possible.

   For time-like longitudinal momenta one can choose the local M^4 coordinates in such a manner that longitudinal momentum reduces to $(\omega_0, 0)$, where ω_0 plays the role of rest mass and is analogous to the plasma frequency serving as an infrared cutoff for plasma waves. In these coordinates the simplest candidates for excitations with time-like momentum would be of form $\Delta s^k = \epsilon a^k exp(i\omega_0 u)$, where s^k are some real coordinates for CP_2, a^k are Fourier coefficients, and time-like coordinate is defined as $u = S^+ + S^-$. The excitations moving with light velocity correspond to $\omega_0 = 0$, and one must treat this case separately using plane wave $exp(i\omega S^\pm)$, where ω has continuum of values.

2. It is possible that only some preferred CP_2 coordinates are excited in longitudinal degrees of freedom. For $D_{CP_2} = 3$ ansatz the simplest option is that the complex CP_2 coordinate

ξ depends analytically on w and the longitudinal CP_2 coordinate s obeys the plane wave ansatz. $\xi(w) = a \times exp(ik_T w)$, where k_T is transverse momentum allows the algebraization of the solution ansatz also in the transversal degrees of freedom so that a dispersion relation results. For imaginary values of k_T and ω the equations are real.

2. General form for the second variation of the field equations

For time-like four-momentum the second variation of field equations contains three kinds of terms. There are terms quadratic in ω_0 and coming from the second derivatives of the deformation, terms proportional to $i\omega_0$ coming from the variation with respect to the derivatives of CP_2 coordinates, and terms which do not depend on ω_0 and come from the variations of metric and Kähler form with respect to the CP_2 coordinates.

In standard perturbation theory the terms proportional to $i\omega_0$ would have interpretation as analogs of dissipative terms. This forces to assume that ω_0 is complex: note that in purely imaginary ω_0 the equations are real. The basic assumption is that Kähler action is able to mimic dissipation despite the fact that energy and momentum are conserved quantities. The vanishing of the Lorentz force has an interpretation as the vanishing of the dissipative effects. This would suggest that the terms proportional to $i\omega_0$ vanish for the perturbations of the solution preserving the non-dissipative character of the asymptotic solutions. This might quite well result from the vanishing of the contractions with the deformation of the energy momentum tensor with the second fundamental form and of energy momentum tensor with the deformation of the second fundamental form coming from first derivatives.

Physical intuition would suggest that dissipation-less propagation is possible only along special directions. Thus the vanishing of the linear terms should occur only for special directions of the longitudinal momentum vector, say for light-like four-momenta in the direction of coordinate lines of S^+ or S^-. Quite generally, the sub-space of allowed four-momenta is expected to depend on position since the components of metric and Kähler form are slowly varying. This dependence is completely analogous with that appearing in the Hamilton Jacobi (ray-optics) approach to the approximate treatment of wave equations and makes sense if the phase of the plane wave varies rapidly as compared to the variation of CP_2 coordinates for the unperturbed solution.

Complex values of ω_0 are also possible, and would allow to deduce important information about the rate at which small deviations from asymptotia vanish as well as about instabilities of the asymptotic solutions. In particular, for imaginary values of ω_0 one obtains completely well-defined solution ansatz representing exponentially decaying or increasing perturbation.

High energy limit

One can gain valuable information by studying the perturbations at the limit of very large four-momentum. At this limit the terms which are quadratic in the components of momentum dominate and come from the second derivatives of the CP_2 coordinates appearing in the second fundamental form. The resulting equations reduce for all CP_2 coordinates to the same condition

$$T^{\alpha\beta}k_\alpha k_\beta = 0 \ .$$

This condition is generalization of masslessness condition with metric replaced by the energy momentum tensor, which means that light velocity is replaced by an effective light velocity. In fact, energy momentum tensor effectively replaces metric also in the modified Dirac equation whose form is dictated by super symmetry. Light-like four momentum is a rather general solution to the condition and corresponds to $\omega_0 = 0$ case.

Reduction of the dispersion relation to the graph of swallowtail catastrophe

Also the general structure of the equations for small perturbations allows to deduce highly non-trivial conclusions about the character of perturbations.

1. The equations for four CP_2 coordinates are simultaneously satisfied if the determinant associated with the equations vanishes. This condition defines a 3-dimensional surface in the 4-dimensional space defined by ω_0 and coordinates of 3-space playing the role of slowly varying control parameters. 4×4 determinant results and corresponds to a polynomial which is

of order $d = 8$ in ω_0. If the determinant is real, the polynomial can depend on ω_0^2 only so that a fourth order polynomial in $w = \omega_0^2$ results.

2. Only complex roots are possible in the case that the terms linear in $i\omega_0$ are non-vanishing. One might hope that the linear term vanishes for certain choices of the direction of slowly varying four-momentum vector $k^\mu(x)$ at least. For purely imaginary values of ω_0 the equations determinant are real always. Hence catastrophe theoretic description applies in this case at least, and the so called swallow tail [A52] with three control parameters applies to the situation.

3. The general form of the vanishing determinant is

$$D(w, a, b, c) = w^4 - ew^3 - cw^2 - bw - a \ .$$

The transition from the oscillatory to purely dissipative case changes only the sign of w. By the shift $w = \hat{w} + e/4$ the determinant reduces to the canonical form

$$D(\hat{w}, a, b, c) = \hat{w}^4 - c\hat{w}^2 - b\hat{w} - a$$

of the swallowtail catastrophe. This catastrophe has three control variables, which basically correspond to the spatial 3-coordinates on which the induced metric and Kähler form depend. The variation of these coefficients at the space-time sheet of course covers only a finite region of the parameter space of the swallowtail catastrophe. The number of real roots for $w = \omega_0^2$ is four, two, or none since complex roots appear in complex conjugate pairs for a real polynomial. The general shape of the region of 3-space is that for a portion of swallow tail catastrophe (see **Fig. 2.1**).

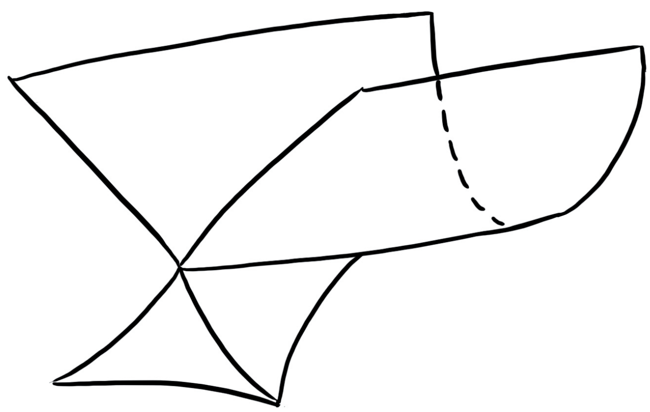

Figure 2.1: The projection of the bifurcation set of the swallowtail catastrophe to the 3-dimensional space of control variables. The potential function has four extrema in the interior of the swallowtail bounded by the triangles, no extrema in the valley above the swallowtail, and 2 extrema elsewhere.

4. The dispersion relation for the "rest mass" ω_0 (decay rate for the imaginary value of ω_0) has at most four real branches, which conforms with the fact that there are four dynamical variables. In real case ω_0 is analogous to plasma frequency acting as an infrared cutoff for the frequencies of plasma excitations. To get some grasp on the situation notice that for $a = 0$ the swallowtail reduces to $\hat{w} = 0$ and

$$\hat{w}^3 - c\hat{w} - b = 0 \ ,$$

which represents the cusp catastrophe easy to illustrate in 3-dimensional space. Cusp (see **Fig. 2.2**) in turn reduces for $b = 0$ to $\hat{w} = 0$ and fold catastrophe $\hat{w} = \pm\sqrt{c}$. Thus the catastrophe surface becomes 4-sheeted for $c \geq 0$ for sufficiently small values of the parameters a and b. The possibility of negative values of $\hat{w}$ in principle allows $\omega^2 = \hat{w} + e/4 < 0$ solutions identifiable as exponentially decaying or amplified perturbations. At the high frequency limit the 4 branches degenerate to a single branch $T^{\alpha\beta}k_\alpha k_\beta = 0$, which as a special case gives light-like four-momenta corresponding to $\omega_0 = 0$ and the origin of the swallowtail catastrophe.

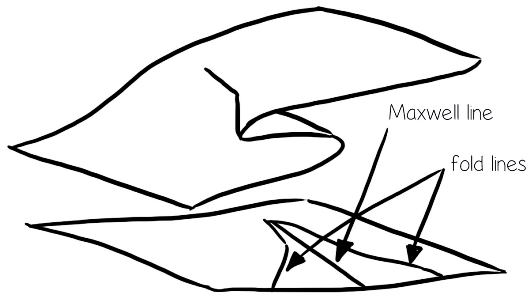

Figure 2.2: Cusp catastrophe. Vertical direction corresponds to the behavior variable and orthogonal directions to control variables.

5. It is quite possible that the imaginary terms proportional to $i\omega_0$ cannot be neglected in the time-like case. The interpretation would be as dissipative effects. If these effects are not too large, an approximate description in terms of butterfly catastrophe makes still sense. Note however that the second variation contains besides gravitational terms potentially large dissipative terms coming from the variation of the induced Kähler form and from the variation of CP_2 Christoffel symbols.

6. Additional complications are encountered at the points, where the induced Kähler field vanishes since the second variation vanishes identically at these points. By the arguments represented earlier, these points quite generally represent instabilities.

2.5 Vacuum Extremals

Vacuum extremals come as two basic types: CP_2 type vacuum extremals for which the induced Kähler field and Kähler action are non-vanishing and the extremals for which the induced Kähler field vanishes. The deformations of both extremals are expected to be of fundamental importance in TGD universe. Vacuum extremals are not gravitational vacua and they are indeed fundamental in TGD inspired cosmology.

2.5.1 CP_2 Type Extremals

CP_2 type vacuum extremals

These extremals correspond to various isometric imbeddings of CP_2 to $M^4_+ \times CP_2$. One can also drill holes to CP_2. Using the coordinates of CP_2 as coordinates for X^4 the imbedding is given by the formula

$$\begin{aligned} m^k &= m^k(u) \ , \\ m_{kl}\dot{m}^k\dot{m}^l &= 0 \ , \end{aligned} \tag{2.5.1}$$

where $u(s^k)$ is an arbitrary function of CP_2 coordinates. The latter condition tells that the curve representing the projection of X^4 to M^4 is light like curve. One can choose the functions $m^i, i = 1,2,3$ freely and solve m^0 from the condition expressing light likeness so that the number of this kind of extremals is very large.

The induced metric and Kähler field are just those of CP_2 and energy momentum tensor $T^{\alpha\beta}$ vanishes identically by the self duality of the Kähler form of CP_2. Also the canonical current $j^\alpha = D_\beta J^{\alpha\beta}$ associated with the Kähler form vanishes identically. Therefore the field equations in the interior of X^4 are satisfied. The field equations are also satisfied on the boundary components of CP_2 type extremal because the non-vanishing boundary term is, besides the normal component of Kähler electric field, also proportional to the projection operator to the normal space and vanishes identically since the induced metric and Kähler form are identical with the metric and Kähler form of CP_2.

As a special case one obtains solutions for which M^4 projection is light like geodesic. The projection of $m^0 = constant$ surfaces to CP_2 is $u = constant$ 3-sub-manifold of CP_2. Geometrically these solutions correspond to a propagation of a massless particle. In a more general case the interpretation as an orbit of a massless particle is not the only possibility. For example, one can imagine a situation, where the center of mass of the particle is at rest and motion occurs along a circle at say (m^1, m^2) plane. The interpretation as a massive particle is natural. Amusingly, there is nice analogy with the classical theory of Dirac electron: massive Dirac fermion moves also with the velocity of light (zitterbewegung). The quantization of this random motion with light velocity leads to Virasoro conditions and this led to a breakthrough in the understanding of the p-adic QFT limit of TGD. Furthermore, it has turned out that Super Virasoro invariance is a general symmetry of WCW geometry and quantum TGD and appears both at the level of imbedding space and space-time surfaces.

The action for all extremals is same and given by the Kähler action for the imbedding of CP_2. The value of the action is given by

$$S = -\frac{\pi}{8\alpha_K} \ . \tag{2.5.2}$$

To derive this expression we have used the result that the value of Lagrangian is constant: $L = 4/R^4$, the volume of CP_2 is $V(CP_2) = \pi^2 R^4/2$ and the definition of the Kähler coupling strength $k_1 = 1/16\pi\alpha_K$ (by definition, πR is the length of CP_2 geodesics). Four-momentum vanishes for these extremals so that they can be regarded as vacuum extremals. The value of the action is negative so that these vacuum extremals are indeed favored by the minimization of the Kähler action.

The absolute minimization of Kähler action was the original suggestion for what preferred extremal property could mean, and suggested that ordinary vacuums with vanishing Kähler action density are unstable against the generation of CP_2 type extremals. The same conclusion however follows also from the mere vacuum degeneracy of Käbler action. There are even reasons to expect that CP_2 type extremals are for TGD what black holes are for GRT. This identification seems reasonable: the 4-D lines of generalized Feynman graphs [K32] would be regions with Euclidian signature of induced metric and identifiable as deformations of CP_2 type vacuum extremals, and even TGD counterparts of blackholes would be analogous to lines of Feynman diagrams. Their M^4 projection would be of course arbitrarily of macroscopic size. The nice generalization of the area law for the entropy of black hole [K28] supports this view.

In accordance with the basic ideas of TGD topologically condensed vacuum extremals should somehow correspond to massive particles. The properties of the CP_2 type vacuum extremals are in accordance with this interpretation. Although these objects move with a velocity of light, the motion can be transformed to a mere zitterbewegung so that the center of mass motion is trivial. Even the generation of the rest mass could might be understood classically as a consequence of the minimization of action. Long range Kähler fields generate negative action for the topologically condensed vacuum extremal (momentum zero massless particle) and Kähler field energy in turn is identifiable as the rest mass of the topologically condensed particle.

An interesting feature of these objects is that they can be regarded as gravitational instantons [A58]. A further interesting feature of CP_2 type extremals is that they carry nontrivial classical color charges. The possible relationship of this feature to color confinement raises interesting questions. Could one model classically the formation of the color singlets to take place through the emission of “colorons”: states with zero momentum but non-vanishing color? Could these peculiar states reflect the infrared properties of the color interactions?

Are CP_2 type non-vacuum extremals possible?

The isometric imbeddings of CP_2 are all vacuum extremals so that these extremals as such cannot correspond to physical particles. One obtains however non-vacuum extremals as deformations of these solutions. There are several types of deformations leading to non-vacuum solutions. In order to describe some of them, recall the expressions of metric and Kähler form of CP_2 in the coordinates (r,Θ,Ψ,Φ) [A53] are given by

$$\begin{aligned}
\frac{ds^2}{R^2} &= \frac{dr^2}{(1+r^2)^2}+\frac{r}{2(1+r^2))^2}(d\Psi+cos(\Theta)d\Phi)^2\\
&+ \frac{r^2}{(4(1+r^2)}(d\Theta^2+sin^2\Theta d\Phi^2)\ ,\\
J &= \frac{r}{(1+r^2)}dr\wedge(d\Psi+cos(\Theta)d\Phi)\\
&- \frac{r^2}{(2(1+r^2)}sin(\Theta)d\Theta\wedge d\Phi\ .
\end{aligned} \tag{2.5.3}$$

The scaling of the line element is defined so that πR is the length of the CP_2 geodesic line. Note that Φ and Ψ appear as “cyclic” coordinates in metric and Kähler form: this feature plays important role in the solution ansatze to be described.

Let $M^4 = M^2\times E^2$ denote the decomposition of M^4 to a product of 2-dimensional Minkowski space and 2-dimensional Euclidian plane. This decomposition corresponds physically to the decomposition of momentum degrees of freedom for massless particle: E^2 corresponds to polarization degrees of freedom.

There are several types of non-vacuum extremals.

“Virtual particle” extremals: the mass spectrum is continuous (also Euclidian momenta are allowed) but these extremals reduce to vacuum extremals in the massless limit.

2. Massless extremals.

Consider first an example of virtual particle extremal. The simplest extremal of this type is obtained in the following form

$$m^k = a^k\Psi+b^k\Phi\ . \tag{2.5.4}$$

Here a^k and b^k are some constant quantities. Field equations are equivalent to the conditions expressing four-momentum conservation and are identically satisfied the reason being that induced metric and Kähler form do not depend on the coordinates Ψ and Φ.

Extremal describes 3-surface, which moves with constant velocity in M^4. Four-momentum of the solution can be both space and time like. In the massless limit solution however reduces to a vacuum extremal. Therefore the interpretation as an off mass shell massless particle seems appropriate.

Massless extremals are obtained from the following solution ansatz.

$$\begin{aligned}
m^0 &= m^3 = a\Psi+b\Phi\ ,\\
(m^1,m^2) &= (m^1(r,\Theta),m^2(r,\Theta))\ .
\end{aligned} \tag{2.5.5}$$

Only E^2 degrees of freedom contribute to the induced metric and the line element is obtained from

$$ds^2 = ds^2_{CP_2} - (dm^1)^2 - (dm^2)^2 \ . \tag{2.5.6}$$

Field equations reduce to conservation condition for the componenents of four-momentum in E^2 plane. By their cyclicity the coordinates Ψ and Φ disappear from field equations and one obtains essentially current conservation condition for two-dimensional field theory defined in space spanned by the coordinates r and Θ.

$$\begin{aligned} (J^i_a)_{,i} &= 0 \ , \\ J^i_a &= T^{ij} f^a_{,j} \sqrt{g} \ . \end{aligned} \tag{2.5.7}$$

Here the index i and a refer to r and Θ and to E^2 coordinates m^1 and m^2 respectively. T^{ij} denotes the canonical energy momentum tensor associated with Kähler action. One can express the components of T^{ij} in terms of induced metric and CP_2 metric in the following form

$$T^{ij} = (-g^{ik} g^{jl} + g^{ij} g^{kl}/2) s_{kl} \ . \tag{2.5.8}$$

This expression holds true for all components of the energy momentum tensor.

Since field equations are essentially two-dimensional conservation conditions they imply that components of momentum currents can be regarded as vector fields of some canonical transformations

$$J^i_a = \varepsilon^{ij} H^a_{,j} \ , \tag{2.5.9}$$

where ε^{ij} denotes two-dimensional constant symplectic form. An open problem is whether one could solve field equations exactly and whether there exists some nonlinear superposition principle for the solutions of these equations. Solutions are massless since transversal momentum densities vanish identically.

Consider as a special case the solution obtained by assuming that one E^2 coordinate is constant and second coordinate is function $f(r)$ of the variable r only. Field equations reduce to the following form

$$f_{,r} = \pm \frac{k}{(1+r^2)^{1/3}} \sqrt{r^2 - k^2(1+r^2)^{4/3}} \ . \tag{2.5.10}$$

The solution is well defined only for sufficiently small values of the parameter k appearing as integration constant and becomes ill defined at two singular values of the variable r. Boundary conditions are identically satisfied at the singular values of r since the radial component of induced metric diverges at these values of r. The result leads to suspect that the generation of boundary components dynamically is a general phenomenon so that all non-vacuum solutions have boundary components in accordance with basic ideas of TGD.

$CP_2\#CP_2\#...\#CP_2$: s as generalized Feynman graphs

There are reasons to believe that point like particles might be identified as CP_2 type extremals in TGD approach. Also the geometric counterparts of the massless on mass shell particles and virtual particles have been identified. It is natural to extend this idea to the level of particle interactions: the lines of Feynman diagrams of quantum field theory are thickened to four-manifolds, which are in a good approximation CP_2 type vacuum extremals. This would mean that generalized Feynman graphs are essentially connected sums of CP_2: s (see **Fig. 2.3**): $X^4 = CP_2\#CP_2....\#CP_2$).

Unfortunately, this picture seems to be oversimplified. First, it is questionable whether the cross sections for the scattering of CP_2 type extremals have anything to do with the cross sections associated with the standard gauge interactions. A naive geometric argument suggests that the

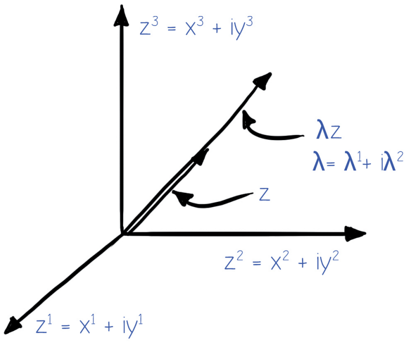

Figure 2.3: Topological sum of CP_2: s as Feynman graph with lines thickened to four-manifolds

cross section should reflect the geometric size of the scattered objects and therefore be of the order of CP_2 radius for topologically non-condensed CP_2 type extremals. The observed cross sections would result at the first level of condensation, where particles are effectively replaced by surfaces with size of order Compton length. Secondly, the $h_{vac} = -D$ rule, considered in the previous chapter, suggests that only real particles correspond to the CP_2 type extremals whereas virtual particles in general correspond to the vacuum extremals with a vanishing Kähler action. The reason is that the negative exponent of the Kähler action reduces the contribution of the CP_2 type extremals to the functional integral very effectively. Therefore the exchanges of CP_2 type extremals are suppressed by the negative exponent of the Kähler action very effectively so that geometric scattering cross section is obtained.

2.5.2 Vacuum Extremals With Vanishing Kähler Field

Vacuum extremals correspond to 4-surfaces with vanishing Kähler field and therefore to gauge field zero configurations of gauge field theory. These surfaces have CP_2 projection, which is Legendre manifold. The condition expressing Legendre manifold property is obtained in the following manner. Kähler potential of CP_2 can be expressed in terms of the canonical coordinates (P_i, Q_i) for CP_2 as

$$A = \sum_k P_k dQ^k \ . \tag{2.5.11}$$

The conditions

$$P_k = \partial_{Q^k} f(Q^i) \ , \tag{2.5.12}$$

where $f(Q^i)$ is arbitrary function of its arguments, guarantee that Kähler potential is pure gauge. It is clear that canonical transformations, which act as local $U(1)$ gauge transformations, transform different vacuum configurations to each other so that vacuum degeneracy is enormous. Also M^4_+ diffeomorphisms act as the dynamical symmetries of the vacuum extremals. Some sub-group of these symmetries extends to the isometry group of the WCW in the proposed construction of the configuration space metric. The vacuum degeneracy is still enhanced by the fact that the topology of the four-surface is practically free.

Vacuum extremals are certainly not absolute minima of the action. For the induced metric having Minkowski signature the generation of Kähler electric fields lowers the action. For Euclidian signature both electric and magnetic fields tend to reduce the action. Therefore the generation of Euclidian regions of space-time is expected to occur. CP_2 type extremals, identifiable as real (as contrast to virtual) elementary particles, can be indeed regarded as these Euclidian regions.

Particle like vacuum extremals can be classified roughly by the number of the compactified dimensions D having size given by CP_2 length. Thus one has $D_{CP_2} = 3$ for CP_2 type extremals, $D_{CP_2} = 2$ for string like objects, $D_{CP_2} = 1$ for membranes and $D_{CP_2} = 0$ for pieces of M^4. As already mentioned, the rule $h_{vac} = -D$ relating the vacuum weight of the Super Virasoro representation to the number of compactified dimensions of the vacuum extremal is very suggestive. $D < 3$ vacuum extremals would correspond in this picture to virtual particles, whose contribution to the generalized Feynman diagram is not suppressed by the exponential of Kähler action unlike that associated with the virtual CP_2 type lines.

M^4 type vacuum extremals (representable as maps $M^4_+ \rightarrow CP_2$ by definition) are also expected to be natural idealizations of the space-time at long length scales obtained by smoothing out small scale topological inhomogenuities (particles) and therefore they should correspond to space-time of GRT in a reasonable approximation.

In both cases the vanishing of Kähler action per volume in long length scales makes vacuum extremals excellent idealizations for the smoothed out space-time surface. Robertson-Walker cosmologies provide a good example in this respect. As a matter fact the smoothed out space-time is not a mere fictive concept since larger space-time sheets realize it as a essential part of the Universe.

Several absolute minima could be possible and the non-determinism of the vacuum extremals is not expected to be reduced completely. The remaining degeneracy could be even infinite. A good example is provided by the vacuum extremals representable as maps $M^4_+ \rightarrow D^1$, where D^1 is one-dimensional curve of CP_2. This degeneracy could be interpreted as a space-time correlate for the non-determinism of quantum jumps with maximal deterministic regions representing quantum states in a sequence of quantum jumps.

2.6 Non-Vacuum Extremals

2.6.1 Cosmic Strings

Cosmic strings are extremals of type $X^2 \times S^2$, where X^2 is minimal surface in M^4_+ (analogous to the orbit of a bosonic string) and S^2 is the homologically non-trivial geodesic sphere of CP_2. The action of these extremals is positive and thus absolute minima are certainly not in question. One can however consider the possibility that these extremals are building blocks of the absolute minimum space-time surfaces since the absolute minimization of the Kähler action is global rather than a local principle. A more general approach gives up absolute minimization as definition of preferred extremal property and there are indeed several proposals for what preferred extremal property could mean. Cosmic strings can contain also Kähler charged matter in the form of small holes containing elementary particle quantum numbers on their boundaries and the negative Kähler electric action for a topologically condensed cosmic string could cancel the Kähler magnetic action.

The string tension of the cosmic strings is given by

$$T = \frac{1}{8\alpha_K R^2} \simeq .2210^{-6}\frac{1}{G} \ , \tag{2.6.1}$$

where $\alpha_K \simeq \alpha_{em}$ has been used to get the numerical estimate. The string tension is of the same order of magnitude as the string tension of the cosmic strings of GUTs and this leads to the model of the galaxy formation providing a solution to the dark matter puzzle as well as to a model for large voids as caused by the presence of a strongly Kähler charged cosmic string. Cosmic strings play also fundamental role in the TGD inspired very early cosmology.

2.6.2 Massless Extremals

Massless extremals (or topological light rays) are characterized by massless wave vector p and polarization vector ε orthogonal to this wave vector. Using the coordinates of M^4 as coordinates for X^4 the solution is given as

$$
\begin{array}{ll}
s^k = f^k(u,v) \ , & \\
u = p\cdot m \ , & v = \varepsilon\cdot m \ , \\
p\cdot\varepsilon = 0 \ , & p^2 = 0 \ .
\end{array}
$$

CP_2 coordinates are arbitrary functions of $p \cdot m$ and $\varepsilon \cdot m$. Clearly these solutions correspond to plane wave solutions of gauge field theories. It is important to notice however that linear super position doesn't hold as it holds in Maxwell phase. Gauge current is proportional to wave vector and its divergence vanishes as a consequence. Also cylindrically symmetric solutions for which the transverse coordinate is replaced with the radial coordinate $\rho = \sqrt{m_1^2 + m_2^2}$ are possible. In fact, v can be *any* function of the coordinates m^1, m^2 transversal to the light like vector p.

Boundary conditions on the boundaries of the massless extremal are satisfied provided the normal component of the energy momentum tensor vanishes. Since energy momentum tensor is of the form $T^{\alpha\beta} \propto p^\alpha p^\beta$ the conditions $T^{n\beta} = 0$ are satisfied if the M^4 projection of the boundary is given by the equations of form

$$
\begin{array}{l}
H(p\cdot m, \varepsilon\cdot m, \varepsilon_1\cdot m) = 0 \ , \\
\qquad \varepsilon\cdot p = 0 \ , \qquad \varepsilon_1 \cdot p = 0 \ , \quad \varepsilon\cdot\varepsilon_1 = 0 \ .
\end{array} \tag{2.6.2}
$$

where H is arbitrary function of its arguments. Recall that for M^4 type extremals the boundary conditions are also satisfied if Kähler field vanishes identically on the boundary.

The following argument suggests that there are not very many manners to satisfy boundary conditions in case of M^4 type extremals. The boundary conditions, when applied to M^4 coordinates imply the vanishing of the normal component of energy momentum tensor. Using coordinates, where energy momentum tensor is diagonal, the requirement boils down to the condition that at least one of the eigen values of $T^{\alpha\beta}$ vanishes so that the determinant $det(T^{\alpha\beta})$ must vanish on the boundary: this condition defines 3-dimensional surface in X^4. In addition, the normal of this surface must have same direction as the eigen vector associated with the vanishing eigen value: this means that three additional conditions must be satisfied and this is in general true in single point only. The boundary conditions in CP_2 coordinates are satisfied provided that the conditions

$$J^{n\beta} J^k_{\ l} \partial_\beta s^l = 0$$

are satisfied. The identical vanishing of the normal components of Kähler electric and magnetic fields on the boundary of massless extremal property provides a manner to satisfy all boundary conditions but it is not clear whether there are any other manners to satisfy them.

The characteristic feature of the massless extremals is that in general the Kähler gauge current is non-vanishing. In ordinary Maxwell electrodynamcis this is not possible. This means that these extremals are accompanied by vacuum current, which contains in general case both weak and electromagnetic terms as well as color part.

A possible interpretation of the solution is as the exterior space-time to a topologically condensed particle with vanishing mass described by massless CP_2 type extremal, say photon or neutrino. In general the surfaces in question have boundaries since the coordinates s^k are bounded this is in accordance with the general ideas about topological condensation. The fact that massless plane wave is associated with CP_2 type extremal combines neatly the wave and particle aspects at geometrical level.

The fractal hierarchy of space-time sheets implies that massless extremals should interesting also in long length scales. The presence of a light like electromagnetic vacuum current implies the generation of coherent photons and also coherent gravitons are generated since the Einstein tensor is also non-vanishing and light like (proportional to $k^\alpha k^\beta$). Massless extremals play an important role in the TGD based model of bio-system as a macroscopic quantum system. The possibility of vacuum currents is what makes possible the generation of the highly desired coherent photon states.

2.6.3 Does GRT really allow gravitational radiation: could cosmological constant save the situation?

In Facebook discussion Nils Grebäck mentioned Weyl tensor and I learned something that I should have noticed long time ago. Wikipedia article (see `http://tinyurl.com/y7fsnzk8`) lists the basic properties of Weyl tensor as the traceless part of curvature tensor, call it R. Weyl tensor C is vanishing for conformally flat space-times. In dimensions D=2,3 Weyl tensor vanishes identically so that they are always conformally flat: this obviously makes the dimension $D = 3$ for space very special. Interestingly, one can have non-flat space-times with nonvanishing Weyl tensor but the vanishing Schouten/Ricci/Einstein tensor and thus also with vanishing energy momentum tensor.

The rest of curvature tensor R can be expressed in terms of so called Kulkarni-Nomizu product $P \cdot g$ of Schouten tensor P and metric tensor g: $R = C + P \cdot g$, which can be also transformed to a definition of Weyl tensor using the definition of curvature tensor in terms of Christoffel symbols as the fundamental definition. Kulkarni-Nomizu product $\cdot$ is defined as tensor product of two 2-tensors with symmetrization with respect to first and second index pairs plus antisymmetrization with respect to second and fourth indices.

Schouten tensor P is expressible as a combination of Ricci tensor Ric defined by the trace of R with respect to the first two indices and metric tensor g multiplied by curvature scalar s (rather than R in order to use index free notation without confusion with the curvature tensor). The expression reads as

$$P = \frac{1}{D-2}\left[Ric - \frac{s}{2(D-1)}g\right] \quad .$$

Note that the coefficients of Ric and g differ from those for Einstein tensor. Ricci tensor and Einstein tensor are proportional to energy momentum tensor by Einstein equations relate to the part.

Weyl tensor is assigned with gravitational radiation in GRT. What I see as a serious interpretational problem is that by Einstein's equations gravitational radiation would carry no energy and momentum in absence of matter. One could argue that there are no free gravitons in GRT if this interpretation is adopted! This could be seen as a further argument against GRT besides the problems with the notions of energy and momentum: I had not realized this earlier.

Interestingly, in TGD framework so called massless extremals (MEs) [K7, K112, K52] are four-surfaces, which are extremals of Kähler action, have Weyl tensor equal to curvature tensor and therefore would have interpretation in terms of gravitons. Now these extremals are however non-vacuum extremals.

1. Massless extremals correspond to graphs of possibly multi-valued maps from M^4 to CP_2. CP_2 coordinates are arbitrary functions of variables $u = k \cot m$ and $w = \epsilon \cdot m$. k is light-like wave vector and ϵ space-like polarization vector orthogonal to k so that the interpretation in terms of massless particle with polarization is possible. ME describes in the most general case a wave packet preserving its shape and propagating with maximal signal velocity along a kind of tube analogous to wave guide so that they are ideal for precisely targeted communications and central in TGD inspired quantum biology. MEs do not have Maxwellian counterparts. For instance, MEs can carry light-like gauge currents parallel to them: this is not possible in Maxwell's theory.

2. I have discussed a generalization of this solution ansatz so that the directions defined by light-like vector k and polarization vector ϵ orthogonal to it are not constant anymore but define a slicing of M^4 by orthogonal curved surfaces (analogs of string world sheets and space-like surfaces orthogonal to them). MEs in their simplest form at least are minimal surfaces and actually extremals of practically any general coordinate invariance action principle. For instance, this is the case if the volume term suggested by the twistor lift of Kähler action [L22] and identifiable in terms of cosmological constant is added to Kähler action.

3. MEs carry non-trivial induced gauge fields and gravitational fields identified in terms of the induced metric. I have identified them as correlates for particles, which correspond to pairs of wormhole contacts between two space-times such that at least one of them is ME. MEs

would accompany to both gravitational radiation and other forms or radiation classically and serve as their correlates. For massless extremals the metric tensor is of form

$$g = m + a\epsilon \otimes \epsilon + bk \otimes k + c(\epsilon \otimes kv + k \otimes \epsilon) \ ,$$

where m is the metric of empty Minkowski space. The curvature tensor is necessarily quadrilinear in polarization vector ϵ and light-like wave vector k (light-like ifor both M^4 and ME metric) and from the general expression of Weyl tensor C in terms of R and g it is equal to curvature tensor: $C = R$.

Hence the interpretation as graviton solution conforms with the GRT interpretation. Now however the energy momentum tensor for the induced Kähler form is non-vanishing and bilinear in velocity vector k and the interpretational problem is avoided.

What is interesting that also at GRT limit cosmological constant saves gravitons from reducing to vacuum solutions. The deviation of the energy density given by cosmological term from that for Minkowski metric is identifiable as gravitonic energy density. The mysterious cosmological constant would be necessary for making gravitons non-vacuum solutions. The value of graviton amplitude would be determined by the continuity conditions for Einstein's equations with cosmological term. The p-adic evolution of cosmological term predicted by TGD is however difficult to understand in GRT framework.

2.6.4 Generalization Of The Solution Ansatz Defining Massless Extremals (MEs)

The solution ansatz for MEs has developed gradually to an increasingly general form and the following formulation is the most general one achieved hitherto. Rather remarkably, it rather closely resembles the solution ansatz for the CP_2 type extremals and has direct interpretation in terms of geometric optics. Equally remarkable is that the latest generalization based on the introduction of the local light cone coordinates was inspired by quantum holography principle.

The solution ansatz for MEs has developed gradually to an increasingly general form and the following formulation is the most general one achieved hitherto. Rather remarkably, it rather closely resembles the solution ansatz for the CP_2 type extremals and has direct interpretation in terms of geometric optics. Equally remarkable is that the latest generalization based on the introduction of the local light cone coordinates was inspired by quantum holography principle.

Local light cone coordinates

The solution involves a decomposition of M^4_+ tangent space localizing the decomposition of Minkowski space to an orthogonal direct sum $M^2 \oplus E^2$ defined by light-like wave vector and polarization vector orthogonal to it. This decomposition defines what might be called local light cone coordinates.

1. Denote by m^i the linear Minkowski coordinates of M^4. Let (S^+, S^-, E^1, E^2) denote local coordinates of M^4_+ defining a *local* decomposition of the tangent space M^4 of M^4_+ into a direct *orthogonal* sum $M^4 = M^2 \oplus E^2$ of spaces M^2 and E^2. This decomposition has interpretation in terms of the longitudinal and transversal degrees of freedom defined by local light-like four-velocities $v_\pm = \nabla S_\pm$ and polarization vectors $\epsilon_i = \nabla E^i$ assignable to light ray.

2. With these assumptions the coordinates $(S_\pm, E^i)$ define local light cone coordinates with the metric element having the form

$$ds^2 = 2g_{+-}dS^+dS^- + g_{11}(dE^1)^2 + g_{22}(dE^2)^2 \ . \tag{2.6.3}$$

   If complex coordinates are used in transversal degrees of freedom one has $g_{11} = g_{22}$.

3. This family of light cone coordinates is not the most general family since longitudinal and transversal spaces are orthogonal. One can also consider light-cone coordinates for which one non-diagonal component, say m_{1+}, is non-vanishing if the solution ansatz is such that longitudinal and transversal spaces are orthogonal for the induced metric.

A conformally invariant family of local light cone coordinates

The simplest solutions to the equations defining local light cone coordinates are of form $S_\pm = k\cdot m$ giving as a special case $S_\pm = m^0 \pm m^3$. For more general solutions of from

$$S_\pm = m^0 \pm f(m^1, m^2, m^3) \ , \ \ (\nabla_3 f)^2 = 1 \ ,$$

where f is an otherwise arbitrary function, this relationship reads as

$$S^+ + S^- = 2m^0 \ .$$

This condition defines a natural rest frame. One can integrate f from its initial data at some two-dimensional $f = constant$ surface and solution describes curvilinear light rays emanating from this surface and orthogonal to it. The flow velocity field $\overline{v} = \nabla f$ is irrotational so that closed flow lines are not possible in a connected region of space and the condition $\overline{v}^2 = 1$ excludes also closed flow line configuration with singularity at origin such as $v = 1/\rho$ rotational flow around axis.

One can identify E^2 as a local tangent space spanned by polarization vectors and orthogonal to the flow lines of the velocity field $\overline{v} = \nabla f(m^1, m^2, m^3)$. Since the metric tensor of any 3-dimensional space allows always diagonalization in suitable coordinates, one can always find coordinates (E^1, E^2) such that (f, E^1, E^2) form orthogonal coordinates for $m^0 = constant$ hyperplane. Obviously one can select the coordinates E^1 and E^2 in infinitely many manners.

Closer inspection of the conditions defining local light cone coordinates

Whether the conformal transforms of the local light cone coordinates $\{S_\pm = m^0 \pm f(m^1, m^2, m^3), E^i\}$ define the only possible compositions $M^2 \oplus E^2$ with the required properties, remains an open question. The best that one might hope is that any function S^+ defining a family of light-like curves defines a local decomposition $M^4 = M^2 \oplus E^2$ with required properties.

1. Suppose that S^+ and S^- define light-like vector fields which are not orthogonal (proportional to each other). Suppose that the polarization vector fields $\epsilon_i = \nabla E^i$ tangential to local E^2 satisfy the conditions $\epsilon_i \cdot \nabla S^+ = 0$. One can formally integrate the functions E^i from these condition since the initial values of E^i are given at $m^0 = constant$ slice.

2. The solution to the condition $\nabla S_+ \cdot \epsilon_i = 0$ is determined only modulo the replacement

   $$\epsilon_i \rightarrow \hat{\epsilon}_i = \epsilon_i + k\nabla S_+ \ ,$$

   where k is any function. With the choice

   $$k = -\frac{\nabla E^i \cdot \nabla S^-}{\nabla S^+ \cdot \nabla S^-}$$

   one can satisfy also the condition $\hat{\epsilon}_i \cdot \nabla S^- = 0$.

3. The requirement that also $\hat{\epsilon}_i$ is gradient is satisfied if the integrability condition

   $$k = k(S^+)$$

   is satisfiedin this case $\hat{\epsilon}_i$ is obtained by a gauge transformation from ϵ_i. The integrability condition can be regarded as an additional, and obviously very strong, condition for S^- once S^+ and E^i are known.

4. The problem boils down to that of finding local momentum and polarization directions defined by the functions S^+, S^- and E^1 and E^2 satisfying the orthogonality and integrability conditions

   $$(\nabla S^+)^2 = (\nabla S^-)^2 = 0 \ , \quad \nabla S^+ \cdot \nabla S^- \neq 0 \ ,$$

   $$\nabla S^+ \cdot \nabla E^i = 0 \ , \qquad \frac{\nabla E^i \cdot \nabla S^-}{\nabla S^+ \cdot \nabla S^-} = k_i(S^+) \ .$$

The number of integrability conditions is 3+3 (all derivatives of k_i except the one with respect to S^+ vanish): thus it seems that there are not much hopes of finding a solution unless some discrete symmetry relating S^+ and S^- eliminates the integrability conditions altogether.

A generalization of the spatial reflection $f \rightarrow -f$ working for the separable Hamilton Jacobi function $S_\pm = m^0 \pm f$ ansatz could relate S^+ and S^- to each other and trivialize the integrability conditions. The symmetry transformation of M^4_+ must perform the permutation $S^+ \leftrightarrow S^-$, preserve the light-likeness property, map E^2 to E^2, and multiply the inner products between M^2 and E^2 vectors by a mere conformal factor. This encourages the conjecture that all solutions are obtained by conformal transformations from the solutions $S_\pm = m^0 \pm f$.

General solution ansatz for MEs for given choice of local light cone coordinates

Consider now the general solution ansatz assuming that a local wave-vector-polarization decomposition of M^4_+ tangent space has been found.

1. Let $E(S^+, E^1, E^2)$ be an arbitrary function of its arguments: the gradient ∇E defines at each point of E^2 an S^+-dependent (and thus time dependent) polarization direction orthogonal to the direction of local wave vector defined by ∇S^+. Polarization vector depends on E^2 position only.

2. Quite a general family of MEs corresponds to the solution family of the field equations having the general form

$$s^k = f^k(S^+, E) \ ,$$

   where s^k denotes CP_2 coordinates and f^k is an arbitrary function of S^+ and E. The solution represents a wave propagating with light velocity and having definite S^+ dependent polarization in the direction of ∇E. By replacing S^+ with S^- one obtains a dual solution. Field equations are satisfied because energy momentum tensor and Kähler current are light-like so that all tensor contractions involved with the field equations vanish: the orthogonality of M^2 and E^2 is essential for the light-likeness of energy momentum tensor and Kähler current.

3. The simplest solutions of the form $S_\pm = m^0 \pm m^3$, $(E^1, E^2) = (m^1, m^2)$ and correspond to a cylindrical MEs representing waves propagating in the direction of the cylinder axis with light velocity and having polarization which depends on point (E^1, E^2) and S^+ (and thus time). For these solutions four-momentum is light-like: for more general solutions this cannot be the case. Polarization is in general case time dependent so that both linearly and circularly polarized waves are possible. If m^3 varies in a finite range of length L, then "free" solution represents geometrically a cylinder of length L moving with a light velocity. Of course, ends could be also anchored to the emitting or absorbing space-time surfaces.

4. For the general solution the cylinder is replaced by a three-dimensional family of light like curves and in this case the rectilinear motion of the ends of the cylinder is replaced with a curvilinear motion with light velocity unless the ends are anchored to emitting/absorbing space-time surfaces. The non-rotational character of the velocity flow suggests that the freely moving particle like 3-surface defined by ME cannot remain in a infinite spatial volume. The most general ansatz for MEs should be useful in the intermediate and nearby regions of a radiating object whereas in the far away region radiation solution is excepted to decompose to cylindrical ray like MEs for which the function $f(m^1, m^2, m^2)$ is a linear function of m^i.

5. One can try to generalize the solution ansatz further by allowing the metric of M^4_+ to have components of type g_{i+} or g_{i-} in the light cone coordinates used. The vanishing of T^{11}, T^{+1}, and T^{--} is achieved if $g_{i\pm} = 0$ holds true for the induced metric. For $s^k = s^k(S^+, E^1)$ ansatz neither $g_{2\pm}$ nor g_{1-} is affected by the imbedding so that these components of the metric must vanish for the Hamilton Jacobi structure:

$$ds^2 = 2g_{+-}dS^+dS^- + 2g_{1+}dE^1dS^+ + g_{11}(dE^1)^2 + g_{22}(dE^2)^2 \ . \tag{2.6.4}$$

$g_{1+} = 0$ can be achieved by an additional condition

$$m_{1+} = s_{kl}\partial_1 s^k \partial_+ s^k \ . \tag{2.6.5}$$

The diagonalization of the metric seems to be a general aspect of preferred extremals. The absence of metric correlations between space-time degrees of freedom for asymptotic self-organization patterns is somewhat analogous to the minimization of non-bound entanglement in the final state of the quantum jump.

Are the boundaries of space-time sheets quite generally light like surfaces with Hamilton Jacobi structure?

Quantum holography principle naturally generalizes to an approximate principle expected to hold true also in non-cosmological length and time scales.

1. The most general ansatz for topological light rays or massless extremals (MEs) inspired by the quantum holographic thinking relies on the introduction of the notion of local light cone coordinates S_+, S_-, E_1, E_2. The gradients ∇S_+ and ∇S_- define two light like directions just like Hamilton Jacobi functions define the direction of propagation of wave in geometric optics. The two polarization vector fields ∇E_1 and ∇E_2 are orthogonal to the direction of propagation defined by either S_+ or S_-. Since also E_1 and E_2 can be chosen to be orthogonal, the metric of M^4_+ can be written locally as $ds^2 = g_{+-}dS_+dS_- + g_{11}dE_1^2 + g_{22}dE_2^2$. In the earlier ansatz S_+ and S_- where restricted to the variables $k \cdot m$ and $\tilde{k} \cdot m$, where k and $\tilde{k}$ correspond to light like momentum and its mirror image and m denotes linear M^4 coordinates: these MEs describe cylindrical structures with constant direction of wave propagation expected to be most important in regions faraway from the source of radiation.

2. Boundary conditions are satisfied if the 3-dimensional boundaries of MEs have one light like direction (S_+ or S_- is constant). This means that the boundary of ME has metric dimension $d = 2$ and is characterized by an infinite-dimensional super-symplectic and super-conformal symmetries just like the boundary of the imbedding space $M^4_+ \times CP_2$: The boundaries are like moments for mini big bangs (in TGD based fractal cosmology big bang is replaced with a silent whisper amplified to not necessarily so big bang).

3. These observations inspire the conjecture that boundary conditions for M^4 like space-time sheets fixed by the absolute minimization of Kähler action quite generally require that space-time boundaries correspond to light like 3-surfaces with metric dimension equal to $d = 2$. This does not yet imply that light like surfaces of imbedding space would take the role of the light cone boundary: these light like surface could be seen only as a special case of causal determinants analogous to event horizons.

2.6.5 Maxwell Phase

"Maxwell phase" corresponds to small deformations of the M^4 type vacuum extremals. Since energy momentum tensor is quadratic in Kähler field the term proportional to the contraction of the energy momentum tensor with second fundamental form drops from field equations and one obtains in lowest order the following field equations

$$j^\alpha J^k_{\ l} s^l_{,\alpha} = 0 \ . \tag{2.6.6}$$

These equations are satisfied if Maxwell's equations

$$j^\alpha = 0 \tag{2.6.7}$$

hold true. Massless extremals and Maxwell phase clearly exclude each other and it seems that they must corresponds to different space-time sheets.

The explicit construction of these extremals reduces to the task of finding an imbedding for an arbitrary free Maxwell field to H. One can also allow source terms corresponding to the presence of the point like charges: these should correspond to the regions of the space-time, where the flat space-time approximation of the space-time fails. The regions where the approximation defining the Maxwell phase fails might correspond to a topologically condensed CP_2 type extremals, for example. As a consequence, Kähler field is superposition of radiation type Kähler field and of Coulomb term. A second possibility is the generation of "hole" with similar Coulombic Kähler field.

An important property of the Maxwell phase (also of massless extremals) is its approximate canonical invariance. Canonical transformations do not spoil the extremal property of the four-surface in the approximation used, since it corresponds to a mere $U(1)$ gauge transformation. This implies the counterpart of the vacuum degeneracy, that is, the existence of an enormous number of four-surfaces with very nearly the same action. Also there is an approximate $Diff(M^4_+)$ invariance.

The canonical degeneracy has some very interesting consequences concerning the understanding of the electro-weak symmetry breaking and color confinement. Kähler field is canonical invariant and satisfies Maxwells equations. This is in accordance with the identification of Kähler field as $U(1)$ part of the electro-weak gauge field. Electromagnetic gauge field is a superposition of Kähler field and Z^0 field$\gamma = 3J - sin^2(\theta_W)Z^0/2$ so that also electromagnetic gauge field is long ranged assuming that Z^0 and W^+ fields are short ranged. These fields are not canonical invariants and their behavior seems to be essentially random, which implies short range correlations and the consequent massivation.

There is an objection against this argument. For the known $D < 4$ solutions of field equations weak fields are not random at all. These situations could represent asymptotic configurations assignable to space-time sheets. This conforms with the interpretation that weak gauge fields are essentially massless within the asymptotic space-time sheets representing weak bosons. Gauge fields are however transferred between space-time sheets through # contacts modellable as pieces of CP_2 type extremals having $D_{CP_2} = 4$. In contrast to Kähler and color gauge fluxes, weak gauge fluxes are not conserved in the Euclidian time evolution between the 3-D causal horizons separating the Euclidian # contact from space-time sheets with Minkowskian signature. This non-conservation implying the loss of coherence in the transfer of fields between space-time sheets is a plausible mechanism for the loss of correlations and massivation of the weak gauge fields.

Classical gluon fields are proportional to Kähler field and to the Hamiltonians associated with the color isometry generators.

$$g^A_{\alpha\beta} = kH^A J_{\alpha\beta} \ . \tag{2.6.8}$$

This implies that the direction of gluon fields in color algebra is random. One can always perform a canonical transformation, which reduces to a global color rotation in some arbitrary small region of space-time and reduces to identity outside this region. The proportionality of a gluon field to Kähler form implies that there is a classical long range correlation in X^4 degrees of freedom: in this sense classical gluon fields differ from massive electro-weak fields in Maxwell phase.

2.6.6 Stationary, Spherically Symmetric Extremals

The stationary, spherically symmetric extremals of the Kähler action imbeddable in $M^4 \times S^2$, where S^2 is geodesic sphere, are the simplest extremals, which one can study as models for the space-time surrounding a topologically condensed particle, say CP_2 type vacuum extremal. In the region near the particle the spherical symmetry is an unrealistic assumption since it excludes the presence of magnetic fields needed to cancel the total Kähler action. The stationarity is also unrealistic assumption since zitterbewegung seems to provide a necessary mechanism for generating Kähler magnetic field and for satisfying boundary conditions. Also the imbeddability to $M^4 \times S^2$ implies unrealistic relationship between Z^0 and photon charges.

According to the general wisdom, the generation of a Kähler electric field must take place in order to minimize the action and it indeed turns out that the extremal is characterized by essentially $1/r^2$ Kähler electric field. The necessary presence of a hole or of a topologically condensed object is also demonstrated: it is impossible to find extremals well defined in the region surrounding the origin. It is impossible to satisfy boundary conditions at a hole: this is in accordance with the idea that Euclidian region corresponding to a CP_2 type extremal performing zitterbewegung is generated. In case of CP_2 extremal radius is of the order of the Compton length of the particle and in case of a "hole" of the order of Planck length. The value of the vacuum frequency ω is of order of particle mass whereas for macroscopic vacuum extremals it must be of the order of $1/R$. This does not lead to a contradiction if the concept of a many-sheeted space-time is accepted.

The Poincare energy of the exterior region is considerably smaller than the gravitational mass; this conforms with the interpretation that gravitational mass is sum of absolute values of positive and negative inertial masses associated with matter and negative energy antimatter. It is quite possible that classical considerations cannot provide much understanding concerning the inertial masses of topologically condensed particles. Electro-weak gauge forces are considerably weaker than the gravitational force at large distances, when the value of the frequency parameter ω is of order $1/R$. Both these desirable properties fail to be true if CP_2 radius is of order Planck length as believed earlier.

In light of the general ideas about topological condensation it is clear that in planetary length scales these kind of extremals cannot provide a realistic description of space-time. Indeed, spherically symmetric extremals predict a wrong rate for the precession of the perihelion of Mercury. Scwhartschild and Reissner-Nordström metric do this and indeed allow imbedding as vacuum extremals for which the inertial masses of positive energy matter and negative energy antimatter sum up to zero.

This does not yet resolve the interpretational challenge due to the unavoidable long range color and weak gauge fields. A dark matter hierarchy giving rise to a hierarchy of color and electro-weak physics characterized by increasing values of weak and confinement scales explains these fields. # contacts involve a pair of causal horizons at which the Euclidian metric signature of # contact transforms to Minkowskian one. These causal horizons have interpretation as partons so that # contact can be regarded as a bound state of partons bound together by a gravitational instanton (CP_2 type extremal). # contacts provide basic example of dark matter creating long ranged weak fields.

An important result is the correlation between the sign of the vacuum frequency ω and that of the Kähler charge, which is of opposite sign for fermions and anti-fermions. This suggests an explanation for matter-antimatter asymmetry. Matter and antimatter condense stably on disjoint regions of the space-time surface at different space-time sheets. Stable antimatter could correspond to negative time orientation and negative energy. This leads to a model for the primordial generation of matter as spontaneous generation of zero energy # contacts between space-time sheets of opposite time orientations. If CP conjugation is not exact symmetry, # contacts and their CP conjugates are created with slightly different rates and this gives rise to CP asymmetry at each of the two space-time sheets involved. After the splitting of # contacts and subsequent annihilation of particles and antiparticles at each space-time sheet, the two space-time sheets contain only positive energy matter and negative energy antimatter.

General solution ansatz

The general form of the solution ansatz is obtained by assuming that the space-time surface in question is a sub-manifold of $M^4 \times S^2$, where S^2 is the homologically non-trivial geodesic sphere of CP_2. S^2 is most conveniently realized as $r = \infty$ surface of CP_2, for which all values of the coordinate Ψ correspond to same point of CP_2 so that one can use Θ and Φ as the coordinates of S^2.

The solution ansatz is given by the expression

$$\begin{aligned} cos(\Theta) &= u(r) \ , \\ \Phi &= \omega t \ , \\ m^0 &= \lambda t \ , \\ r_M &= r \ , \quad \theta_M = \theta \ , \quad \phi_M = \phi \ . \end{aligned} \tag{2.6.9}$$

The induced metric is given by the expression

$$ds^2 = \left[\lambda^2 - \frac{R^2}{4}\omega^2(1-u^2)\right] dt^2 - (1+\frac{R^2}{4}\theta_{,r}^2)dr^2 - r^2 d\Omega^2 \ . \tag{2.6.10}$$

The value of the parameter λ is fixed by the condition $g_{tt}(\infty) = 1$:

$$\lambda^2 - \frac{R^2}{4}\omega^2(1-u(\infty)^2) = 1 \ . \tag{2.6.11}$$

From the condition $e^0 \wedge e^3 = 0$ the non-vanishing components of the induced Kähler field are given by the expression

$$J_{tr} = \frac{\omega}{4}u_{,r} \ . \tag{2.6.12}$$

Geodesic sphere property implies that Z^0 and photon fields are proportional to Kähler field:

$$\begin{aligned} \gamma &= (3-p/2)J \ , \\ Z^0 &= J \ . \end{aligned} \tag{2.6.13}$$

From this formula one obtains the expressions

$$\begin{aligned} Q_{em} &= \frac{(3-p/2)}{4\pi\alpha_{em}}Q_K \ , \quad Q_Z = \frac{1}{4\pi\alpha_Z}Q \ , \\ Q &\equiv \frac{J_{tr}4\pi r^2}{\sqrt{-g_{rr}g_{tt}}} \ . \end{aligned} \tag{2.6.14}$$

for the electromagnetic and Z^0 charges of the solution using e and g_Z as unit.

Field equations can be written as conditions for energy momentum conservation (two equations is in principle all what is needed in the case of geodesic sphere). Energy conservation holds identically true and conservation of momentum, say, in z-direction gives the equation

$$(T^{rr}z_{,r})_{,r} + (T^{\theta\theta}z_{,\theta})_{,\theta} = 0 \ . \tag{2.6.15}$$

Using the explicit expressions for the components of the energy momentum tensor

$$\begin{aligned} T^{rr} &= g^{rr}L/2 \ , \\ T^{\theta\theta} &= -g^{\theta\theta}L/2 \ , \\ L &= g^{tt}g^{rr}(J_{tr})^2\sqrt{g}/2 \ , \end{aligned} \tag{2.6.16}$$

and the following notations

$$\begin{aligned} A &= g^{tt}g^{rr}r^2\sqrt{-g_{tt}g_{rr}} \ , \\ X &\equiv (J_{tr})^2 \ , \end{aligned} \tag{2.6.17}$$

the field equations reduce to the following form

$$(g^{rr}AX)_{,r} - \frac{2AX}{r} = 0 \ . \tag{2.6.18}$$

In the approximation $g^{rr} = 1$ this equation can be readily integrated to give $AX = C/r^2$. Integrating Eq. (3.2.7), one obtains integral equation for X

$$J_{tr} = \frac{q}{r_c}(|g_{rr}|^3 g_{tt})^{1/4} exp(\int_{r_c}^{r} dr \frac{g_{rr}}{r})\frac{1}{r} \ , \tag{2.6.19}$$

where q is integration constant, which is related to the charge parameter of the long range Kähler electric field associated with the solution. r_c denotes the critical radius at which the solution ceases to be well defined.

The inspection of this formula shows that J_{tr} behaves essentially as $1/r^2$ Coulomb field. This behavior doesn't depend on the detailed properties of the solution ansatz (for example the imbeddability to $M^4 \times S^2$): stationarity and spherical symmetry is what matters only. The compactness of CP_2 means that stationary, spherically symmetric solution is not possible in the region containing origin. This is in concordance with the idea that either a hole surrounds the origin or there is a topologically condensed CP_2 extremal performing zitterbewegung near the origin and making the solution non-stationary and breaking spherical symmetry.

Second integration gives the following integral equation for CP_2 coordinate $u = cos(\Theta)$

$$u(r) = u_0 + \frac{4q}{\omega}\int_{r_c}^{r}(-g_{rr}^3 g_{tt})^{1/4}\frac{1}{r} exp(\int_{r_c}^{r} dr \frac{g_{rr}}{r}) \ . \tag{2.6.20}$$

Here u_0 denotes the value of the coordinate u at $r = r_0$.

The form of the field equation suggests a natural iterative procedure for the numerical construction of the solution for large values of r.

$$u_n(r) = T_{n-1} \ , \tag{2.6.21}$$

where T_{n-1} is evaluated using the induced metric associated with u_{n-1}. The physical content of the approximation procedure is clear: estimate the gravitational effects using lower order solution since these are expected to be small.

A more convenient manner to solve u is based on Taylor expansion around the point $V \equiv 1/r = 0$. The coefficients appearing in the power series expansion $u = \sum_n u_n A^n V^n \ : \ A = q/\omega$ can be solved by calculating successive derivatives of the integral equation for u.

The lowest order solution is simply

$$u_0 = u_\infty \ , \tag{2.6.22}$$

and the corresponding metric is flat metric. In the first order one obtains for $u(r)$ the expression

$$u = u_\infty - \frac{4q}{\omega r} \ , \tag{2.6.23}$$

which expresses the fact that Kähler field behaves essentially as $1/r^2$ Coulomb field. The behavior of u as a function of r is identical with that obtained for the imbedding of the Reissner-Nordström solution.

To study the properties of the solution we fix the signs of the parameters in the following manner:

$$u_\infty < 0 \ , \quad q < 0 \ , \quad \omega > 0 \tag{2.6.24}$$

(reasons become clear later).

Concerning the behavior of the solution one can consider two different cases.

1. The condition $g_{tt}>0$ hold true for all values of Θ. In this case u decreases and the rate of decrease gets faster for small values of r. This means that in the lowest order the solution becomes certainly ill defined at a critical radius $r=r_c$ given by the the condition $u=1$: the reason is that u cannot get values large than one. The expression of the critical radius is given by

$$\begin{aligned} r_c &\geq \frac{4q}{(|u_\infty|+1)\omega} \\ &= \frac{4\alpha Q_{em}}{(3-p/2)}\frac{1}{(|u_\infty|+1)\omega} \ . \end{aligned} \tag{2.6.25}$$

   The presence of the critical radius for the actual solution is also a necessity as the inspection of the expression for J_{tr} shows: $\partial_r\theta$ grows near the origin without bound and $u=1$ is reached at some finite value of r. Boundary conditions require that the quantity $X=T^{rr}\sqrt{g}$ vanishes at critical radius (no momentum flows through the boundary). Substituting the expression of J_{tr} from the field equation to T^{rr} the expression for X reduces to a form, from which it is clear that X cannot vanish. The cautious conclusion is that boundary conditions cannot be satisfied and the underlying reason is probably the stationarity and spherical symmetry of the solution. Physical intuition suggests that that CP_2 type extremal performing zitterbewegung is needed to satisfy the boundary conditions.

2. g_{tt} vanishes for some value of Θ. In this case the radial derivative of u together with g_{tt} can become zero for some value of $r=r_c$. Boundary conditions can be satisfied only provided $r_c=0$. Thus it seems that for the values of ω satisfying the condition $\omega^2=\frac{4\lambda^2}{R^2 sin^2(\Theta_0)}$ it might be possible to find a globally defined solution. The study of differential equation for u however shows that the ansatz doesn't work. The conclusion is that although the boundary is generated it is not possible to satisfy boundary conditions.

A direct calculation of the coefficients u_n from power series expansion gives the following third order polynomial approximation for u ($V=1/r$)

$$\begin{aligned} u &= \sum_n u_n A^n V^n \ , \\ u_0 &= u_\infty(<0) \ , \quad u_1=1 \ , \\ u_2 &= K|u_\infty| \ , \quad u_3=K(1+4K|u_\infty|) \ , \\ A &\equiv \frac{4q}{\omega} \ , \quad K\equiv \omega^2\frac{R^2}{4} \ . \end{aligned} \tag{2.6.26}$$

The coefficients u_2 and u_3 are indeed positive which means that the value of the critical radius gets larger at least in these orders.

Solution contains three parameters: Kähler electric flux $Q=4\pi q$, parameter ωR and parameter u_∞. The latter parameters can be regarded as parameters describing the properties of a flat vacuum extremal (lowest order solution) to which particle like solution is glued and are analogous to the parameters describing symmetry broken vacuum in gauge theories.

Solution is not a realistic model for topological condensation

The solution does not provide realistic model for topological condensation although it gives indirect support for some essential assumptions of TGD based description of Higgs mechanism.

1. When the value of ω is of the order of CP_2 mass the solution could be interpreted as the "exterior metric" of a "hole".
   i) The radius of the hole is of the order of CP_2 length and its mass is of the order of CP_2 mass.

ii) Kähler electric field is generated and charge renormalization takes place classically at CP_2 length scales as is clear from the expression of $Q(r)$: $Q(r) \propto (\frac{-g_{rr}}{g_{tt}})^{1/4}$ and charge increases at short distances.
iii) The existence of the critical radius is unavoidable but boundary conditions cannot be satisfied. The failure to satisfy boundary conditions might be related to stationarity or to the absence of magnetic field. The motion of the boundary component with velocity of light might be the only manner to satisfy boundary conditions. Second possibility is the breaking of spherical symmetry by the generation of a static magnetic field.
iv) The absence of the Kähler magnetic field implies that the Kähler action has an infinite magnitude and the probability of the configuration is zero. A more realistic solution ansatz would break spherical symmetry containing dipole type magnetic field in the nearby region of the hole. The motion of the boundary with a velocity of light could serves as an alternative mechanism for the generation of magnetic field. The third possibility, supported by physical intuition, is that one must give up "hole" type extremal totally.

2. For sufficiently large values of r and for small values of ω (of the order of elementary particle mass scale), the solution might provide an approximate description for the region surrounding elementary particle. Although it is not possible to satisfy boundary conditions the order of magnitude estimate for the size of critical radius ($r_c \simeq \alpha/\omega$) should hold true for more realistic solutions, too. The order of magnitude for the critical radius is smaller than Compton length or larger if the vacuum parameter ω is larger than the mass of the particle. In macroscopic length scales the value of ω is of order $1/R$. This does not lead to a contradiction if the many-sheeted space-time concept is accepted so that $\omega < m$ corresponds to elementary particle space-time sheet. An unrealistic feature of the solution is that the relationship between Z^0 and em charges is not correct: Z^0 charge should be very small in these length scales.

Exterior solution cannot be identified as a counterpart of Schwartshild solution

The first thing, which comes into mind is to ask whether one might identify exterior solution as the TGD counterpart of the Schwartshild solution. The identification of gravitational mass as absolute value of inertial mass which is negative for antimatter implies that vacuum extremals are vacua only with respect to the inertial four-momentum and have a non-vanishing gravitational four-momentum. Hence, in the approximation that the net density of inertial mass vanishes, vacuum extremals provide the proper manner to model matter, and the identification of the ansatz for a spherically symmetric extremal as the counterpart of Scwhartschild metric is certainly not possible. It is however useful to show explicitly that the identification is indeed unrealistic. The solution is consistent with Equivalence Principle but the electro-weak gauge forces are considerably weaker than gravitational forces. A wrong perihelion shift is also predicted so that the identification as an exterior metric of macroscopic objects is out of question.

1. Is Equivalence Principle respected?

The following calculation demonstrates that Equivalence Principle might not be satisfied for the solution ansatz (which need not actually define a preferred extremal!).

The gravitational mass of the solution is determined from the asymptotic behavior of g_{tt} and is given by

$$M_{gr} = \frac{R^2}{G}\omega q u_\infty \ , \tag{2.6.27}$$

and is proportional to the Kähler charge q of the solution.

One can estimate the gravitational mass density also by applying Newtonian approximation to the time component of the metric $g_{tt} = 1 - 2\Phi_{gr}$. One obtains Φ_{gr} corresponds in the lowest order approximation to a solution of Einstein's equations with the source consisting of a mass point at origin and the energy density of the Kähler electric field. The effective value of gravitational constant is however $G_{eg} = 8R^2\alpha_K$. Thus the only sensible interpretation is that the density of Kähler (inertial) energy is only a fraction $G/G_{eq} \equiv \epsilon \simeq .22 \times 10^{-6}$ of the density of gravitational mass. Hence the densities of positive energy matter and negative energy antimatter cancel each other in a good approximation.

The work with cosmic strings lead to a possible interpretation of the solution as a space-time sheet containing topologically condensed magnetic flux tube idealizable as a point. The negative Kähler electric action must cancel the positive Kähler magnetic action. The resulting structure in turn can condense to a vacuum extremal and Schwartshild metric is a good approximation for the metric.

One can estimate the contribution of the exterior region ($r > r_c$) to the inertial mass of the system and Equivalence principle requires this to be a fraction of order ϵ about the gravitational mass unless the region $r < r_c$ contains negative inertial mass density, which is of course quite possible. Approximating the metric with a flat metric and using first order approximation for $u(r)$ the energy reduces just to the standard Coulomb energy of charged sphere with radius r_c

$$\begin{aligned} M_I(ext) &= \frac{1}{32\pi\alpha_K}\int_{r>r_c} E^2\sqrt{g}d^3x \\ &\simeq \frac{\lambda q^2}{2\alpha_K r_c} \ , \\ \lambda &= \sqrt{1+\frac{R^2}{4}\omega^2(1-u_\infty^2)}\ (>1) \ . \end{aligned} \tag{2.6.28}$$

Approximating the metric with flat metric the contribution of the region $r > r_c$ to the energy of the solution is given by

$$M_I(ext) = \frac{1}{8\alpha_K}\lambda q\omega(1+|u_\infty|) \ . \tag{2.6.29}$$

The contribution is proportional to Kähler charge as expected. The ratio of external inertial and gravitational masses is given by the expression

$$\begin{aligned} \frac{M_I(ext)}{M_{gr}} &= \frac{G}{4R^2\alpha_K}x \ , \\ x &= \frac{(1+|u_\infty|)}{|u_\infty|} > 1 \ . \end{aligned} \tag{2.6.30}$$

In the approximation used the ratio of external inertial and gravitational masses is of order 10^{-6} for $R \sim 10^4\sqrt{G}$ implied by the p-adic length scale hypothesis and for $x \sim 1$. The result conforms with the above discussed interpretation.

The result forces to challenge the underlying implicit assumptions behind the calculation.

1. Many-sheeted space-time means that single space-time sheet need not be a good approximation for astrophysical systems. The GRT limit of TGD can be interpreted as obtained by lumping many-sheeted space-time time to Minkowski space with effective metric defined as sum M^4 metric and sum of deviations from M^4 metric for various space-time sheets involved [K79]. This effective metric should correspond to that of General Relativity and Einstein's equations reflect the underlying Poincare invariance. Gravitational and cosmological constants follow as predictions and EP is satisfied.

2. The systems considered need not be preferred extremals of Kähler action so that one cannot take the results obtained too seriously. For vacuum extremals one does not encounter this problem at all and it could be that vacuum extremals with induced metric identified as GRT metric are a good approximation in astrophysical systems. This requires that single-sheetedness is a good approximation. TGD based single-sheeted models for astrophysical and cosmological systems rely on this assumption.

2. Z^0 and electromagnetic forces are much weaker than gravitational force

The extremal in question carries Kähler charge and therefore also Z^0 and electromagnetic charge. This implies long range gauge interactions, which ought to be weaker than gravitational interaction in the astrophysical scales. This is indeed the case as the following argument shows.

Expressing the Kähler charge using Planck mass as unit and using the relationships between gauge fields one obtains a direct measure for the strength of the Z^0 force as compared with the strength of gravitational force.

$$Q_Z \equiv \varepsilon_Z M_{gr}\sqrt{G} \ . \tag{2.6.31}$$

The value of the parameter ε_Z should be smaller than one. A transparent form for this condition is obtained, when one writes $\Phi = \omega t = \Omega m^0 : \Omega = \lambda\omega$:

$$\varepsilon_Z = \frac{\alpha_K}{\alpha_Z}\frac{1}{\pi(1+|u_\infty|)\Omega R}\sqrt{\frac{G}{R}} \ . \tag{2.6.32}$$

The order of magnitude is determined by the values of the parameters $\sqrt{\frac{G}{R^2}} \sim 10^{-4}$ and ΩR. Global Minkowskian signature of the induced metric implies the condition $\Omega R < 2$ for the allowed values of the parameter ΩR. In macroscopic length scales one has $\Omega R \sim 1$ so that Z^0 force is by a factor of order 10^{-4} weaker than gravitational force. In elementary particle length scales with $\omega \sim m$ situation is completely different as expected.

3. The shift of the perihelion is predicted incorrectly

The g_{rr} component of Reissner-Nordström and TGD metrics are given by the expressions

$$g_{rr} = -\frac{1}{(1-\frac{2GM}{r})} \ , \tag{2.6.33}$$

and

$$g_{rr} \simeq 1 - \frac{\frac{Rq}{\omega^2}}{\left[1-(u_\infty - \frac{4q}{\omega r})^2\right] r^4} \ , \tag{2.6.34}$$

respectively. For reasonable values of q, ω and u_∞ the this terms is extremely small as compared with $1/r$ term so that these expressions differ by $1/r$ term.

The absence of the $1/r$ term from g_{rr}-component of the metric predicts that the shift of the perihelion for elliptic plane orbits is about $2/3$ times that predicted by GRT so that the identification as a metric associated with objects of a planetary scale leads to an experimental contradiction. Reissner-Nordström solutions are obtained as vacuum extremals so that standard predictions of GRT are obtained for the planetary motion.

One might hope that the generalization of the form of the spherically symmetric ansatz by introducing the same modification as needed for the imbedding of Reissner-Nordstrm̈ metric might help. The modification would read as

$$\begin{aligned}
cos(\Theta) &= u(r) \ , \\
\Phi &= \omega t + f(r) \ , \\
m^0 &= \lambda t + h(r) \ , \\
r_M &= r \ , \quad \theta_M = \theta \ , \quad \phi_M = \phi \ .
\end{aligned} \tag{2.6.35}$$

The vanishing of the g_{tr} component of the metric gives the condition

$$\lambda\partial_r h - \frac{R^2}{4}sin^2(\Theta)\omega\partial_r f = 0 \ . \tag{2.6.36}$$

The expression for the radial component of the metric transforms to

$$g_{rr} \simeq \partial_r h^2 - 1 - \frac{R^2}{4}(\partial_r \Theta)^2 - \frac{R^2}{4} sin^2(\Theta)\partial_r f^2 \ , \qquad (2.6.37)$$

Essentially the same perihelion shift as for Schwartschild metric is obtained if g_{rr} approaches asymptotically to its expression for Schwartschild metric. This is guaranteed if the following conditions hold true:

$$f(r)_{r\rightarrow\infty} \rightarrow \omega r \ , \ \Lambda^2 - 1 = \frac{R^2\omega^2}{4} sin^2(\Theta_\infty) \ll \frac{2GM}{\langle r\rangle} \ . \qquad (2.6.38)$$

In the second equation $\langle r\rangle$ corresponds to the average radius of the planetary orbit.

The field equations for this ansatz can be written as conditions for energy momentum and color charge conservation. Two equations are enough to determine the functions $\Theta(r)$ and $f(r)$. The equation for momentum conservation is same as before. Second field equation corresponds to the conserved isometry current associated with the color isometry $\Phi \rightarrow \Phi + \epsilon$ and gives equation for f.

$$[T^{rr} f_{,r} s_{\Phi\Phi}\sqrt{g}]_{,r} = 0 \ . \qquad (2.6.39)$$

The conservation laws associated with other infinitesimal $SU(2)$ rotations of S^2_I should be satisfied identically. This equation can be readily integrated to give

$$T^{rr} f_{,r} s_{\Phi\Phi}\sqrt{g_{tt}g_{rr}} = \frac{C}{r^2} \ . \qquad (2.6.40)$$

Unfortunately, the result is inconsistent with the $1/r^4$ behavior of T^{rr} and $f \rightarrow \omega r$ implies by correct red shift.

It seems that the only possible way out of the difficulty is to replace spherical symmetry with a symmetry with respect to the rotations around z-axis. The simplest modification of the solution ansatz is as follows:

$$m^0 = \lambda t + h(\rho) \ , \quad \Phi = \omega t + k\rho \ .$$

Thanks to the linear dependence of Φ on ρ, the conservation laws for momentum and color isospin reduce to the same condition. The ansatz induces a small breaking of spherical symmetry by adding to $g_{\rho\rho}$ the term

$$(\partial_\rho h)^2 - \frac{R^2}{4} sin^2(\Theta)k^2 \ .$$

One might hope that in the plane $\theta = \pi/2$, where $r = \rho$ holds true, the ansatz could behave like Schwartschild metric if the conditions discussed above are posed (including the condition $k = \omega$). The breaking of the spherical symmetry in the planetary system would be coded already to the gravitational field of Sun.

Also the study of the imbeddings of Reissner-Nordström metric as vacuum extremals and the investigation of spherically symmetric (inertial) vacuum extremals for which gravitational four-momentum is conserved [K79] leads to the conclusion that the loss of spherical symmetry due to rotation is inevitable characteristic of realistic solutions.

2.6.7 Maxwell Hydrodynamics As A Toy Model For TGD

The field equations of TGD are extremely non-linear and all known solutions have been discovered by symmetry arguments. Chern-Simons term plays essential role also in the construction of solutions of field equations and at partonic level defines braiding for light-like partonic 3-surfaces expected to play key role in the construction of S-matrix. The inspiration for this section came from Terence Tao's blog posting *2006 ICM: Etienne Ghys, "Knots and dynamics"* [A87] giving an elegant summary about amazing mathematical results related to knots, links, braids and hydrodynamical flows in dimension $D = 3$. Posting tells about really amazing mathematical results related to knots.

Chern-Simons term as helicity invariant

Tao mentions helicity as an invariant of fluid flow. Chern-Simons action defined by the induced Kähler gauge potential for light-like 3-surfaces has interpretation as helicity when Kähler gauge potential is identified as fluid velocity. This flow can be continued to the interior of space-time sheet. Also the dual of the induced Kähler form defines a flow at the light-like partonic surfaces but not in the interior of space-time sheet. The lines of this flow can be interpreted as magnetic field lines. This flow is incompressible and represents a conserved charge (Kähler magnetic flux).

The question is which of these flows should define number theoretical braids. Perhaps both of them can appear in the definition of S-matrix and correspond to different kinds of partonic matter (electric/magnetic charges, quarks/leptons?, ...). Second kind of matter could not flow in the interior of space-time sheet. Or could interpretation in terms of electric magnetic duality make sense?

Helicity is not gauge invariant and this is as it must be in TGD framework since CP_2 symplectic transformations induce $U(1)$ gauge transformation, which deforms space-time surface an modifies induced metric as well as classical electroweak fields defined by induced spinor connection. Gauge degeneracy is transformed to spin glass degeneracy.

Maxwell hydrodynamics

In TGD Maxwell's equations are replaced with field equations which express conservation laws and are thus hydrodynamical in character. With this background the idea that the analogy between gauge theory and hydrodynamics might be applied also in the reverse direction is natural. Hence one might ask what kind of relativistic hydrodynamics results if assumes that the action principle is Maxwell action for the four-velocity u^α with the constraint term saying that light velocity is maximal signal velocity.

1. For massive particles the length of four-velocity equals to 1: $u^\alpha u_\alpha = 1$. In massless case one has $u^\alpha u_\alpha = 0$. Geometrically this means that one has sigma model with target space which is 3-D Lobatschevski space or at light-cone boundary. This condition means the addition of constraint term

$$\lambda(u^\alpha u_\alpha - \epsilon) \tag{2.6.41}$$

   to the Maxwell action. $\epsilon = 1/0$ holds for massive/massless flow. In the following the notation of electrodynamics is used to make easier the comparison with electrodynamics.

2. The constraint term destroys gauge invariance by allowing to express A^0 in terms of A^i but in general the constraint is not equivalent to a choice of gauge in electrodynamics since the solutions to the field equations with constraint term are not solutions of field equations without it. One obtains field equations for an effectively massive em field with Lagrange multiplier λ having interpretation as photon mass depending on space-time point:

$$\begin{aligned} j^\alpha &= \partial_\beta F^{\alpha\beta} = \lambda A^\alpha \ , \\ A^\alpha &\equiv u^\alpha \ , \ \ F^{\alpha\beta} = \partial^\beta A^\alpha - \partial^\alpha A^\beta \ . \end{aligned} \tag{2.6.42}$$

3. In electrodynamic context the natural interpretation would be in terms of spontaneous massivation of photon and seems to occur for both values of ϵ. The analog of em current given by λA^α is in general non-vanishing and conserved. This conservation law is quite strong additional constraint on the hydrodynamics. What is interesting is that breaking of gauge invariance does not lead to a loss of charge conservation.

4. One can solve λ by contracting the equations with A_α to obtain

$$\lambda = j^\alpha A_\alpha$$

for $\epsilon = 1$. For $\epsilon = 0$ one obtains

$$j^\alpha A_\alpha = 0$$

stating that the field does not dissipate energy: λ can be however non-vanishing unless field equations imply $j^\alpha = 0$. One can say that for $\epsilon = 0$ spontaneous massivation can occur. For $\epsilon = 1$ massivation is present from the beginning and dissipation rate determines photon mass: a natural interpretation for $\epsilon = 1$ would be in terms of thermal massivation of photon. Non-tachyonicity fixes the sign of the dissipation term so that the thermodynamical arrow of time is fixed by causality.

5. For $\epsilon = 0$ massless plane wave solutions are possible and one has

$$\partial_\alpha \partial_\beta A^\beta = \lambda A_\alpha \ .$$

$\lambda = 0$ is obtained in Lorentz gauge which is consistent with the condition $\epsilon = 0$. Also superpositions of plane waves with same polarization and direction of propagation are solutions of field equations: these solutions represent dispersionless precisely targeted pulses. For superpositions of plane waves λ with 4-momenta, which are not all parallel λ is non-vanishing so that non-linear self interactions due to the constraint can be said to induce massivation. In asymptotic states for which gauge symmetry is not broken one expects a decomposition of solutions to regions of space-time carrying this kind of pulses, which brings in mind final states of particle reactions containing free photons with fixed polarizations.

6. Gradient flows satisfying the conditions

$$A_\alpha = \partial_\alpha \Phi \ , \ A^\alpha A_\alpha = \epsilon \tag{2.6.43}$$

give rise to identically vanishing hydrodynamical gauge fields and $\lambda = 0$ holds true. These solutions are vacua since energy momentum tensor vanishes identically. There is huge number of this kind of solutions and spin glass degeneracy suggests itself. Small deformations of these vacuum flows are expected to give rise to non-vacuum flows.

7. The counterparts of charged solutions are of special interest. For $\epsilon = 0$ the solution $(u^0, u^r) = (Q/r)(1,1)$ is a solution of field equations outside origin and corresponds to electric field of a point charge Q. In fact, for $\epsilon = 0$ any ansatz $(u^0, u^r) = f(r)(1,1)$ satisfies field equations for a suitable choice of $\lambda(r)$ since the ratio of equations associate with j^0 and j^r gives an equation which is trivially satisfied. For $\epsilon = 1$ the ansatz $(u^0, u^r) = (cosh(u), sinh(u))$ expressing solution in terms of hyperbolic angle linearizes the field equation obtained by dividing the equations for j^0 and j^r to eliminate λ. The resulting equation is

$$\partial_r^2 u + \frac{2\partial_r u}{r} = 0$$

for ordinary Coulomb potential and one obtains $(u^0, u^r) = (cosh(u_0 + k/r), sinh(u_0 + k/r))$. The charge of the solution at the limit $r \to \infty$ approaches to the value $Q = sinh(u_0)k$ and diverges at the limit $r \to 0$. The charge increases exponentially as a function of $1/r$ near origin rather than logarithmically as in QED and the interpretation in terms of thermal screening suggests itself. Hyperbolic ansatz might simplify considerably the field equations also in the general case.

Similarities with TGD

There are strong similarities with TGD which suggests that the proposed model might provide a toy model for the dynamics defined by Kähler action.

1. Also in TGD field equations are essentially hydrodynamical equations stating the conservation of various isometry charges. Gauge invariance is broken for the induced Kähler field although Kähler charge is conserved. There is huge vacuum degeneracy corresponding to vanishing of induced Kähler field and the interpretation is in terms of spin glass degeneracy.

2. Also in TGD dissipation rate vanishes for the known solutions of field equations and a possible interpretation is as space-time correlates for asymptotic non-dissipating self organization patterns.

3. In TGD framework massless extremals represent the analogs for superpositions of plane waves with fixed polarization and propagation direction and representing targeted and dispersionless propagation of signal. Gauge currents are light-like and non-vanishing for these solutions. The decomposition of space-time surface to space-time sheets representing particles is much more general counterpart for the asymptotic solutions of Maxwell hydrodynamics with vanishing λ.

4. In TGD framework one can consider the possibility that the four-velocity assignable to a macroscopic quantum phase is proportional to the induced Kähler gauge potential. In this kind of situation one could speak of a quantal variant of Maxwell hydrodynamics, at least for light-like partonic 3-surfaces. For instance, the condition

$$D^\alpha D_\alpha \Psi = 0 \ , \ D_\alpha \Psi = (\partial_\alpha - iq_K A_\alpha)\Psi$$

   for the order parameter of the quantum phase corresponds at classical level to the condition $p^\alpha = q_K Q^\alpha + l^\alpha$, where q_K is Kähler charge of fermion and l^α is a light-like vector field naturally assignable to the partonic boundary component. This gives $u^\alpha = (q_K Q^\alpha + l^\alpha)/m$, $m^2 = p^\alpha p_\alpha$, which is somewhat more general condition. The expressibility of u^α in terms of the vector fields provided by the induced geometry is very natural.

   The value ϵ depends on space-time region and it would seem that also $\epsilon = -1$ is possible meaning tachyonicity and breaking of causality. Kähler gauge potential could however have a time-like pure gauge component in M^4 possibly saving the situation. The construction of quantum TGD at parton level indeed forces to assume that Kähler gauge potential has Lorentz invariant M^4 component $A_a = constant$ in the direction of the light-cone proper time coordinate axis a. Note that the decomposition of WCW to sectors consisting of space-time sheets inside future or past light-cone of M^4 is an essential element of the construction of WCW geometry and does not imply breaking of Poincare invariance. Without this component $u_\alpha u^\alpha$ could certainly be negative. The contribution of M^4 component could prevent this for preferred extremals.

If TGD is taken seriously, these similarities force to ask whether Maxwell hydrodynamics might be interpreted as a nonlinear variant of electrodynamics. Probably not: in TGD em field is proportional to the induced Kähler form only in special cases and is in general non-vanishing also for vacuum extremals.

Chapter 3

About Identification of the Preferred extremals of Kähler Action

3.1 Introduction

Preferred extremal of Kähler action have remained one of the basic poorly defined notions of TGD. There are pressing motivations for understanding what the attribute "preferred" really means. Symmetries give a clue to the problem. The conformal invariance of string models naturally generalizes to 4-D invariance defined by quantum Yangian of quantum affine algebra (Kac-Moody type algebra) characterized by two complex coordinates and therefore explaining naturally the effective 2-dimensionality [K76]. Preferred extremal property should rely on this symmetry.

In Zero Energy Ontology (ZEO) preferred extremals are space-time surfaces connecting two space-like 3-surfaces at the ends of space-time surfaces at boundaries of causal diamond (CD). A natural looking condition is that the symplectic Noether charges associated with a sub-algebra of symplectic algebra with conformal weights n-multiples of the weights of the entire algebra vanish for preferred extremals. These conditions would be classical counterparts the the condition that super-symplectic sub-algebra annihilates the physical states. This would give a hierarchy of super-symplectic symmetry breakings and quantum criticalities having interpretation in terms of hierarchy of Planck constants $h_{eff} = n \times h$ identified as a hierarchy of dark matter. n could be interpreted as the number of space-time conformal gauge equivalence classes for space-time sheets connecting the 3-surfaces at the ends of space-time surface.

There are also many other proposals for what preferred extremal property could mean or imply. The weak form of electric-magnetic duality combined with the assumption that the contraction of the Kähler current with Kähler gauge potential vanishes for preferred extremals implies that Kähler action in Minkowskian space-time regions reduces to Chern-Simons terms at the light-like orbits of wormhole throats at which the signature of the induced metric changes its signature from Minkowskian to Euclidian. In regions with 4-D CP_2 projection (wormhole contacts) also a 3-D contribution not assignable to the boundary of the region might be possible. These conditions pose strong physically feasible conditions on extremals and might be true for preferred extremals too.

Number theoretic vision leads to a proposal that either the tangent space or normal space of given point of space-time surface is associative and thus quaternionic. Also the formulation in terms of quaternion holomorphy and quaternion-Kähler property is an attractive possibility. So called $M^8 - H$ duality is a variant of this vision and would mean that one can map associative/co-associative space-time surfaces from M^8 to H and also iterate this mapping from H to H to generate entire category of preferred extremals. The signature of M^4 is a general technical problem. For instance, the holomorphy in 2 complex variables could correspond to what I have called Hamilton-Jacobi property. Associativity/co-associativity of the tangent space makes sense also in Minkowskian signature.

In this chapter various views about preferred extremal property are discussed.

3.1.1 Preferred Extremals As Critical Extremals

The study of the Kähler-Dirac equation leads to a detailed view about criticality. Quantum criticality [D5] fixes the values of Kähler coupling strength as the analog of critical temperature. Quantum criticality implies that second variation of Kähler action vanishes for critical deformations and the existence of conserved current except in the case of Cartan algebra of isometries. Quantum criticality allows to fix the values of couplings appearing in the measurement interaction by using the condition $K \rightarrow K + f + \overline{f}$. p-Adic coupling constant evolution can be understood also and corresponds to scale hierarchy for the sizes of causal diamonds (CDs).

The discovery that the hierarchy of Planck constants realized in terms of singular covering spaces of $CD \times CP_2$ can be understood in terms of the extremely non-linear dynamics of Kähler action implying 1-to-many correspondence between canonical momentum densities and time derivatives of the imbedding space coordinates led to a further very concrete understanding of the criticality at space-time level and its relationship to zero energy ontology [K34].

Criticality is accompanied by conformal invariance and this leads to the proposal that critical deformations correspond to Kac-Moody type conformal algebra respecting the light-likeness of the partonic orbits and acting trivially at partonic 2-surfaces. Sub-algebras of conformal algebras with conformal weights divisible by integer n would act as gauge symmetries and these algebras would form an inclusion hierarchy defining hierarchy of symmetry breakings. n would also characterize the value of Planck constant $h_{eff} = n \times h$ assignable to various phases of dark matter.

3.1.2 Construction Of Preferred Extremals

There has been considerable progress in the understanding of both preferred extremals and Kähler-Dirac equation.

1. For preferred extremals the generalization of conformal invariance to 4-D situation is very attractive idea and leads to concrete conditions formally similar to those encountered in string model [K7]. In particular, Einstein's equations with cosmological constant would solve consistency conditions and field equations would reduce to a purely algebraic statements analogous to those appearing in equations for minimal surfaces if one assumes that space-time surface has Hermitian structure or its Minkowskian variant Hamilton-Jacobi structure (Appendix). The older approach based on basic heuristics for massless equations, on effective 3-dimensionality, weak form of electric magnetic duality, and Beltrami flows is also promising. An alternative approach is inspired by number theoretical considerations and identifies space-time surfaces as associative or co-associative sub-manifolds of octonionic imbedding space [K74].

   The basic step of progress was the realization that the known extremals of Kähler action - certainly limiting cases of more general extremals - can be deformed to more general extremals having interpretation as preferred extremals.

   (a) The generalization boils down to the condition that field equations reduce to the condition that the traces $Tr(TH^k)$ for the product of energy momentum tensor and second fundamental form vanish. In string models energy momentum tensor corresponds to metric and one obtains minimal surface equations. The equations reduce to purely algebraic conditions stating that T and H^k have no common components. Complex structure of string world sheet makes this possible.

   Stringy conditions for metric stating $g_{zz} = g_{\overline{z}\overline{z}} = 0$ generalize. The condition that field equations reduce to $Tr(TH^k) = 0$ requires that the terms involving Kähler gauge current in field equations vanish. This is achieved if Einstein's equations hold true (one can consider also more general manners to satisfy the conditions). The conditions guaranteeing the vanishing of the trace in turn boil down to the existence of Hermitian structure in the case of Euclidian signature and to the existence of its analog - Hamilton-Jacobi structure - for Minkowskian signature (Appendix). These conditions state that certain components of the induced metric vanish in complex coordinates or Hamilton-Jacobi coordinates.

In string model the replacement of the imbedding space coordinate variables with quantized ones allows to interpret the conditions on metric as Virasoro conditions. In the recent case a generalization of classical Virasoro conditions to four-dimensional ones would be in question. An interesting question is whether quantization of these conditions could make sense also in TGD framework at least as a useful trick to deduce information about quantum states in WCW degrees of freedom.

The interpretation of the extended algebra as Yangian [A27] [B30] suggested previously [K76] to act as a generalization of conformal algebra in TGD Universe is attractive. There is also the conjecture that preferred extremals could be interpreted as quaternionic of co-quaternionic 4-surface of the octonionic imbedding space with octonionic representation of the gamma matrices defining the notion of tangent space quanternionicity.

3.2 Weak Form Electric-Magnetic Duality And Its Implications

The notion of electric-magnetic duality [B5] was proposed first by Olive and Montonen and is central in $\mathcal{N} = 4$ supersymmetric gauge theories. It states that magnetic monopoles and ordinary particles are two different phases of theory and that the description in terms of monopoles can be applied at the limit when the running gauge coupling constant becomes very large and perturbation theory fails to converge. The notion of electric-magnetic self-duality is more natural since for CP_2 geometry Kähler form is self-dual and Kähler magnetic monopoles are also Kähler electric monopoles and Kähler coupling strength is by quantum criticality renormalization group invariant rather than running coupling constant. The notion of electric-magnetic (self-)duality emerged already two decades ago in the attempts to formulate the Kähler geometric of world of classical worlds. Quite recently a considerable step of progress took place in the understanding of this notion [K15] . What seems to be essential is that one adopts a weaker form of the self-duality applying at partonic 2-surfaces. What this means will be discussed in the sequel.

Every new idea must be of course taken with a grain of salt but the good sign is that this concept leads to precise predictions. The point is that elementary particles do not generate monopole fields in macroscopic length scales: at least when one considers visible matter. The first question is whether elementary particles could have vanishing magnetic charges: this turns out to be impossible. The next question is how the screening of the magnetic charges could take place and leads to an identification of the physical particles as string like objects identified as pairs magnetic charged wormhole throats connected by magnetic flux tubes.

(a) The first implication is a new view about electro-weak massivation reducing it to weak confinement in TGD framework. The second end of the string contains particle having electroweak isospin neutralizing that of elementary fermion and the size scale of the string is electro-weak scale would be in question. Hence the screening of electro-weak force takes place via weak confinement realized in terms of magnetic confinement.

(b) This picture generalizes to the case of color confinement. Also quarks correspond to pairs of magnetic monopoles but the charges need not vanish now. Rather, valence quarks would be connected by flux tubes of length of order hadron size such that magnetic charges sum up to zero. For instance, for baryonic valence quarks these charges could be $(2, -1, -1)$ and could be proportional to color hyper charge.

(c) The highly non-trivial prediction making more precise the earlier stringy vision is that elementary particles are string like objects: this could become manifest at LHC energies.

(d) The weak form electric-magnetic duality together with Beltrami flow property of Kähler leads to the reduction of Kähler action to Chern-Simons action so that TGD reduces to almost topological QFT and that Kähler function is explicitly calculable. This has enormous impact concerning practical calculability of the theory.

(e) One ends up also to a general solution ansatz for field equations from the condition that the theory reduces to almost topological QFT. The solution ansatz is inspired by the idea that all isometry currents are proportional to Kähler current which is integrable in the sense that the flow parameter associated with its flow lines defines a global coordinate. The proposed solution ansatz would describe a hydrodynamical flow with the property that isometry charges are conserved along the flow lines (Beltrami flow). A general ansatz satisfying the integrability conditions is found.

The strongest form of the solution ansatz states that various classical and quantum currents flow along flow lines of the Beltrami flow defined by Kähler current. Intuitively this picture is attractive. A more general ansatz would allow several Beltrami flows meaning multi-hydrodynamics. The integrability conditions boil down to two scalar functions: the first one satisfies massless d'Alembert equation in the induced metric and the gradients of the scalar functions are orthogonal. The interpretation in terms of momentum and polarization directions is natural.

3.2.1 Could A Weak Form Of Electric-Magnetic Duality Hold True?

Holography means that the initial data at the partonic 2-surfaces should fix the WCW metric. A weak form of this condition allows only the partonic 2-surfaces defined by the wormhole throats at which the signature of the induced metric changes. A stronger condition allows all partonic 2-surfaces in the slicing of space-time sheet to partonic 2-surfaces and string world sheets. Number theoretical vision suggests that hyper-quaternionicity *resp.* co-hyperquaternionicity constraint could be enough to fix the initial values of time derivatives of the imbedding space coordinates in the space-time regions with Minkowskian *resp.* Euclidian signature of the induced metric. This is a condition on modified gamma matrices and hyper-quaternionicity states that they span a hyper-quaternionic sub-space.

Definition of the weak form of electric-magnetic duality

One can also consider alternative conditions possibly equivalent with this condition. The argument goes as follows.

(a) The expression of the matrix elements of the metric and Kähler form of WCW in terms of the Kähler fluxes weighted by Hamiltonians of $\delta M^4_\pm$ at the partonic 2-surface X^2 looks very attractive. These expressions however carry no information about the 4-D tangent space of the partonic 2-surfaces so that the theory would reduce to a genuinely 2-dimensional theory, which cannot hold true. One would like to code to the WCW metric also information about the electric part of the induced Kähler form assignable to the complement of the tangent space of $X^2 \subset X^4$.

(b) Electric-magnetic duality of the theory looks a highly attractive symmetry. The trivial manner to get electric magnetic duality at the level of the full theory would be via the identification of the flux Hamiltonians as sums of of the magnetic and electric fluxes. The presence of the induced metric is however troublesome since the presence of the induced metric means that the simple transformation properties of flux Hamiltonians under symplectic transformations -in particular color rotations- are lost.

(c) A less trivial formulation of electric-magnetic duality would be as an initial condition which eliminates the induced metric from the electric flux. In the Euclidian version of 4-D YM theory this duality allows to solve field equations exactly in terms of instantons. This approach involves also quaternions. These arguments suggest that the duality in some form might work. The full electric magnetic duality is certainly too strong and implies that space-time surface at the partonic 2-surface corresponds to piece of CP_2 type vacuum extremal and can hold only in the deep interior of the region with Euclidian signature. In the region surrounding wormhole throat at both sides the condition must be replaced with a weaker condition.

(d) To formulate a weaker form of the condition let us introduce coordinates (x^0, x^3, x^1, x^2) such (x^1, x^2) define coordinates for the partonic 2-surface and (x^0, x^3) define coordinates labeling partonic 2-surfaces in the slicing of the space-time surface by partonic 2-surfaces and string world sheets making sense in the regions of space-time sheet with Minkowskian signature. The assumption about the slicing allows to preserve general coordinate invariance. The weakest condition is that the generalized Kähler electric fluxes are apart from constant proportional to Kähler magnetic fluxes. This requires the condition

$$J^{03}\sqrt{g_4} = KJ_{12} . \tag{3.2.1}$$

A more general form of this duality is suggested by the considerations of [K34] reducing the hierarchy of Planck constants to basic quantum TGD and also reducing Kähler function for preferred extremals to Chern-Simons terms [B1] at the boundaries of CD and at light-like wormhole throats. This form is following

$$J^{n\beta}\sqrt{g_4} = K\epsilon \times \epsilon^{n\beta\gamma\delta}J_{\gamma\delta}\sqrt{g_4} . \tag{3.2.2}$$

Here the index n refers to a normal coordinate for the space-like 3-surface at either boundary of CD or for light-like wormhole throat. ϵ is a sign factor which is opposite for the two ends of CD. It could be also opposite of opposite at the opposite sides of the wormhole throat. Note that the dependence on induced metric disappears at the right hand side and this condition eliminates the potentials singularity due to the reduction of the rank of the induced metric at wormhole throat.

(e) Information about the tangent space of the space-time surface can be coded to the WCW metric with loosing the nice transformation properties of the magnetic flux Hamiltonians if Kähler electric fluxes or sum of magnetic flux and electric flux satisfying this condition are used and K is symplectic invariant. Using the sum

$$J_e + J_m = (1+K)J_{12} , \tag{3.2.3}$$

where J denotes the Kähler magnetic flux, , makes it possible to have a non-trivial WCW metric even for $K = 0$, which could correspond to the ends of a cosmic string like solution carrying only Kähler magnetic fields. This condition suggests that it can depend only on Kähler magnetic flux and other symplectic invariants. Whether local symplectic coordinate invariants are possible at all is far from obvious, If the slicing itself is symplectic invariant then K could be a non-constant function of X^2 depending on string world sheet coordinates. The light-like radial coordinate of the light-cone boundary indeed defines a symplectically invariant slicing and this slicing could be shifted along the time axis defined by the tips of CD.

Electric-magnetic duality physically

What could the weak duality condition mean physically? For instance, what constraints are obtained if one assumes that the quantization of electro-weak charges reduces to this condition at classical level?

(a) The first thing to notice is that the flux of J over the partonic 2-surface is analogous to magnetic flux

$$Q_m = \frac{e}{\hbar}\oint BdS = n .$$

n is non-vanishing only if the surface is homologically non-trivial and gives the homology charge of the partonic 2-surface.

(b) The expressions of classical electromagnetic and Z^0 fields in terms of Kähler form [L2] , [L2] read as

$$\begin{aligned} \gamma &= \frac{eF_{em}}{\hbar} = 3J - sin^2(\theta_W)R_{03} \ , \\ Z^0 &= \frac{g_Z F_Z}{\hbar} = 2R_{03} \ . \end{aligned} \tag{3.2.4}$$

Here R_{03} is one of the components of the curvature tensor in vielbein representation and F_{em} and F_Z correspond to the standard field tensors. From this expression one can deduce

$$J = \frac{e}{3\hbar}F_{em} + sin^2(\theta_W)\frac{g_Z}{6\hbar}F_Z \ . \tag{3.2.5}$$

(c) The weak duality condition when integrated over X^2 implies

$$\begin{aligned} \frac{e^2}{3\hbar}Q_{em} + \frac{g_Z^2 p}{6}Q_{Z,V} &= K\oint J = Kn \ , \\ Q_{Z,V} &= \frac{I_V^3}{2} - Q_{em} \ , \ p = sin^2(\theta_W) \ . \end{aligned} \tag{3.2.6}$$

Here the vectorial part of the Z^0 charge rather than as full Z^0 charge $Q_Z = I_L^3 + sin^2(\theta_W)Q_{em}$ appears. The reason is that only the vectorial isospin is same for left and right handed components of fermion which are in general mixed for the massive states. The coefficients are dimensionless and expressible in terms of the gauge coupling strengths and using $\hbar = r\hbar_0$ one can write

$$\begin{aligned} \alpha_{em}Q_{em} &+ p\frac{\alpha_Z}{2}Q_{Z,V} = \frac{3}{4\pi}\times rnK \ , \\ \alpha_{em} &= \frac{e^2}{4\pi\hbar_0} \ , \ \alpha_Z = \frac{g_Z^2}{4\pi\hbar_0} = \frac{\alpha_{em}}{p(1-p)} \ . \end{aligned} \tag{3.2.7}$$

(d) There is a great temptation to assume that the values of Q_{em} and Q_Z correspond to their quantized values and therefore depend on the quantum state assigned to the partonic 2-surface. The linear coupling of the Kähler-Dirac operator to conserved charges implies correlation between the geometry of space-time sheet and quantum numbers assigned to the partonic 2-surface. The assumption of standard quantized values for Q_{em} and Q_Z would be also seen as the identification of the fine structure constants α_{em} and α_Z. This however requires weak isospin invariance.

The value of K from classical quantization of Kähler electric charge

The value of K can be deduced by requiring classical quantization of Kähler electric charge.

(a) The condition that the flux of $F^{03} = (\hbar/g_K)J^{03}$ defining the counterpart of Kähler electric field equals to the Kähler charge g_K would give the condition $K = g_K^2/\hbar$, where g_K is Kähler coupling constant which should invariant under coupling constant evolution by quantum criticality. Within experimental uncertainties one has $\alpha_K = g_K^2/4\pi\hbar_0 = \alpha_{em} \simeq 1/137$, where α_{em} is finite structure constant in electron length scale and $\hbar_0$ is the standard value of Planck constant.

(b) The quantization of Planck constants makes the condition highly non-trivial. The most general quantization of r is as rationals but there are good arguments favoring the quantization as integers corresponding to the allowance of only singular coverings of CD andn CP_2. The point is that in this case a given value of Planck constant corresponds to a finite number pages of the "Big Book". The quantization of the Planck constant implies a further quantization of K and would suggest that K scales as $1/r$ unless the spectrum of values of Q_{em} and Q_Z allowed by the quantization condition scales as r. This is quite possible and the interpretation would be that each of the r sheets of the covering carries (possibly same) elementary charge. Kind of discrete variant of a full Fermi sphere would be in question. The interpretation in terms of anyonic phases [K55] supports this interpretation.

(c) The identification of J as a counterpart of $eB/\hbar$ means that Kähler action and thus also Kähler function is proportional to $1/\alpha_K$ and therefore to $\hbar$. This implies that for large values of $\hbar$ Kähler coupling strength $g_K^2/4\pi$ becomes very small and large fluctuations are suppressed in the functional integral. The basic motivation for introducing the hierarchy of Planck constants was indeed that the scaling $\alpha \rightarrow \alpha/r$ allows to achieve the convergence of perturbation theory: Nature itself would solve the problems of the theoretician. This of course does not mean that the physical states would remain as such and the replacement of single particles with anyonic states in order to satisfy the condition for K would realize this concretely.

(d) The condition $K = g_K^2/\hbar$ implies that the Kähler magnetic charge is always accompanied by Kähler electric charge. A more general condition would read as

$$K = n \times \frac{g_K^2}{\hbar}, n \in Z \ . \tag{3.2.8}$$

This would apply in the case of cosmic strings and would allow vanishing Kähler charge possible when the partonic 2-surface has opposite fermion and anti-fermion numbers (for both leptons and quarks) so that Kähler electric charge should vanish. For instance, for neutrinos the vanishing of electric charge strongly suggests $n = 0$ besides the condition that abelian Z^0 flux contributing to em charge vanishes.

It took a year to realize that this value of K is natural at the Minkowskian side of the wormhole throat. At the Euclidian side much more natural condition is

$$K = \frac{1}{hbar} \ . \tag{3.2.9}$$

In fact, the self-duality of CP_2 Kähler form favours this boundary condition at the Euclidian side of the wormhole throat. Also the fact that one cannot distinguish between electric and magnetic charges in Euclidian region since all charges are magnetic can be used to argue in favor of this form. The same constraint arises from the condition that the action for CP_2 type vacuum extremal has the value required by the argument leading to a prediction for gravitational constant in terms of the square of CP_2 radius and α_K the effective replacement $g_K^2 \rightarrow 1$ would spoil the argument.

The boundary condition $J_E = J_B$ for the electric and magnetic parts of Kählwer form at the Euclidian side of the wormhole throat inspires the question whether all Euclidian regions could be self-dual so that the density of Kähler action would be just the instanton density. Self-duality follows if the deformation of the metric induced by the deformation of the canonically imbedded CP_2 is such that in CP_2 coordinates for the Euclidian region the tensor $(g^{\alpha\beta}g^{\mu\nu} - g^{\alpha\nu}g^{\mu\beta})/\sqrt{g}$ remains invariant. This is certainly the case for CP_2 type vacuum extremals since by the light-likeness of M^4 projection the metric remains invariant. Also conformal scalings of the induced metric would satisfy this condition. Conformal scaling is not consistent with the degeneracy of the 4-metric at the wormhole.

Reduction of the quantization of Kähler electric charge to that of electromagnetic charge

The best manner to learn more is to challenge the form of the weak electric-magnetic duality based on the induced Kähler form.

(a) Physically it would seem more sensible to pose the duality on electromagnetic charge rather than Kähler charge. This would replace induced Kähler form with electromagnetic field, which is a linear combination of induced Kahler field and classical Z^0 field

$$\begin{aligned} \gamma &= 3J - sin^2\theta_W R_{03} \ , \\ Z^0 &= 2R_{03} \ . \end{aligned} \tag{3.2.10}$$

Here $Z_0 = 2R_{03}$ is the appropriate component of CP_2 curvature form [L2]. For a vanishing Weinberg angle the condition reduces to that for Kähler form.

(b) For the Euclidian space-time regions having interpretation as lines of generalized Feynman diagrams Weinberg angle should be non-vanishing. In Minkowskian regions Weinberg angle could however vanish. If so, the condition guaranteeing that electromagnetic charge of the partonic 2-surfaces equals to the above condition stating that the em charge assignable to the fermion content of the partonic 2-surfaces reduces to the classical Kähler electric flux at the Minkowskian side of the wormhole throat. One can argue that Weinberg angle must increase smoothly from a vanishing value at both sides of wormhole throat to its value in the deep interior of the Euclidian region.

(c) The vanishing of the Weinberg angle in Minkowskian regions conforms with the physical intuition. Above elementary particle length scales one sees only the classical electric field reducing to the induced Kähler form and classical Z^0 fields and color gauge fields are effectively absent. Only in phases with a large value of Planck constant classical Z^0 field and other classical weak fields and color gauge field could make themselves visible. Cell membrane could be one such system [K60]. This conforms with the general picture about color confinement and weak massivation.

The GRT limit of TGD suggests a further reason for why Weinberg angle should vanish in Minkowskian regions.

(a) The value of the Kähler coupling strength mut be very near to the value of the fine structure constant in electron length scale and these constants can be assumed to be equal.

(b) GRT limit of TGD with space-time surfaces replaced with abstract 4-geometries would naturally correspond to Einstein-Maxwell theory with cosmological constant which is non-vanishing only in Euclidian regions of space-time so that both Reissner-Nordström metric and CP_2 are allowed as simplest possible solutions of field equations [K79]. The extremely small value of the observed cosmological constant needed in GRT type cosmology could be equal to the large cosmological constant associated with CP_2 metric multiplied with the 3-volume fraction of Euclidian regions.

(c) Also at GRT limit quantum theory would reduce to almost topological QFT since Einstein-Maxwell action reduces to 3-D term by field equations implying the vanishing of the Maxwell current and of the curvature scalar in Minkowskian regions and curvature scalar + cosmological constant term in Euclidian regions. The weak form of electric-magnetic duality would guarantee also now the preferred extremal property and prevent the reduction to a mere topological QFT.

(d) GRT limit would make sense only for a vanishing Weinberg angle in Minkowskian regions. A non-vanishing Weinberg angle would make sense in the deep interior of the Euclidian regions where the approximation as a small deformation of CP_2 makes sense.

The weak form of electric-magnetic duality has surprisingly strong implications for the basic view about quantum TGD as following considerations show.

3.2.2 Magnetic Confinement, The Short Range Of Weak Forces, And Color Confinement

The weak form of electric-magnetic duality has surprisingly strong implications if one combines it with some very general empirical facts such as the non-existence of magnetic monopole fields in macroscopic length scales.

How can one avoid macroscopic magnetic monopole fields?

Monopole fields are experimentally absent in length scales above order weak boson length scale and one should have a mechanism neutralizing the monopole charge. How electroweak interactions become short ranged in TGD framework is still a poorly understood problem. What suggests itself is the neutralization of the weak isospin above the intermediate gauge boson Compton length by neutral Higgs bosons. Could the two neutralization mechanisms be combined to single one?

(a) In the case of fermions and their super partners the opposite magnetic monopole would be a wormhole throat. If the magnetically charged wormhole contact is electromagnetically neutral but has vectorial weak isospin neutralizing the weak vectorial isospin of the fermion only the electromagnetic charge of the fermion is visible on longer length scales. The distance of this wormhole throat from the fermionic one should be of the order weak boson Compton length. An interpretation as a bound state of fermion and a wormhole throat state with the quantum numbers of a neutral Higgs boson would therefore make sense. The neutralizing throat would have quantum numbers of $X_{-1/2} = \nu_L\overline{\nu}_R$ or $X_{1/2} = \overline{\nu}_L\nu_R$. $\nu_L\overline{\nu}_R$ would not be neutral Higgs boson (which should correspond to a wormhole contact) but a super-partner of left-handed neutrino obtained by adding a right handed neutrino. This mechanism would apply separately to the fermionic and anti-fermionic throats of the gauge bosons and corresponding space-time sheets and leave only electromagnetic interaction as a long ranged interaction.

(b) One can of course wonder what is the situation situation for the bosonic wormhole throats feeding gauge fluxes between space-time sheets. It would seem that these wormhole throats must always appear as pairs such that for the second member of the pair monopole charges and I_V^3 cancel each other at both space-time sheets involved so that one obtains at both space-time sheets magnetic dipoles of size of weak boson Compton length. The proposed magnetic character of fundamental particles should become visible at TeV energies so that LHC might have surprises in store!

Well-definedness of electromagnetic charge implies stringiness

Well-definedness of electromagnetic charged at string world sheets carrying spinor modes is very natural constraint and not trivially satisfied because classical W boson fields are present. As a matter fact, all weak fields should be effectively absent above weak scale. How this is possible classical weak fields identified as induced gauge fields are certainly present.

The condition that em charge is well defined for spinor modes implies that the space-time region in which spinor mode is non-vanishing has 2-D CP_2 projection such that the induced W boson fields are vanishing. The vanishing of classical Z^0 field can be poses as additional condition - at least in scales above weak scale. In the generic case this requires that the spinor mode is restricted to 2-D surface: string world sheet or possibly also partonic 2-surface. This implies that TGD reduces to string model in fermionic sector. Even for preferred extremals with 2-D projecting the modes are expected to allow restriction to 2-surfaces. This localization is possible only for Kähler-Dirac action.

A word of warning is however in order. The GRT limit or rather limit of TGD as Einstein Yang-Mills theory replaces the sheets of many-sheeted space-time with Minkowski space with effective metric obtained by summing to Minkowski metric the deviations of the induced metrics of space-time sheets from Minkowski metric. For gauge potentials a similar identification applies. YM-Einstein equations coupled with matter and with non-vanishing cosmological constant are expected on basis of Poincare invariance. One cannot exclude the possibility that the sums of weak gauge potentials from different space-time sheet tend to vanish above weak scale and that well-definedness of em charge at classical level follows from the effective absence of classical weak gauge fields.

Magnetic confinement and color confinement

Magnetic confinement generalizes also to the case of color interactions. One can consider also the situation in which the magnetic charges of quarks (more generally, of color excited leptons and quarks) do not vanish and they form color and magnetic singles in the hadronic length scale. This would mean that magnetic charges of the state $q_{\pm 1/2} - X_{\mp 1/2}$ representing the physical quark would not vanish and magnetic confinement would accompany also color confinement. This would explain why free quarks are not observed. To how degree then quark confinement corresponds to magnetic confinement is an interesting question.

For quark and antiquark of meson the magnetic charges of quark and antiquark would be opposite and meson would correspond to a Kähler magnetic flux so that a stringy view about meson emerges. For valence quarks of baryon the vanishing of the net magnetic charge takes place provided that the magnetic net charges are $(\pm 2, \mp 1, \mp 1)$. This brings in mind the spectrum of color hyper charges coming as $(\pm 2, \mp 1, \mp 1)/3$ and one can indeed ask whether color hyper-charge correlates with the Kähler magnetic charge. The geometric picture would be three strings connected to single vertex. Amusingly, the idea that color hypercharge could be proportional to color hyper charge popped up during the first year of TGD when I had not yet discovered CP_2 and believed on $M^4 \times S^2$.

p-Adic length scale hypothesis and hierarchy of Planck constants defining a hierarchy of dark variants of particles suggest the existence of scaled up copies of QCD type physics and weak physics. For p-adically scaled up variants the mass scales would be scaled by a power of $\sqrt{2}$ in the most general case. The dark variants of the particle would have the same mass as the original one. In particular, Mersenne primes $M_k = 2^k - 1$ and Gaussian Mersennes $M_{G,k} = (1+i)^k - 1$ has been proposed to define zoomed copies of these physics. At the level of magnetic confinement this would mean hierarchy of length scales for the magnetic confinement.

One particular proposal is that the Mersenne prime M_{89} should define a scaled up variant of the ordinary hadron physics with mass scaled up roughly by a factor $2^{(107-89)/2} = 512$. The size scale of color confinement for this physics would be same as the weal length scale. It would look more natural that the weak confinement for the quarks of M_{89} physics takes place in some shorter scale and M_{61} is the first Mersenne prime to be considered. The mass scale of M_{61} weak bosons would be by a factor $2^{(89-61)/2} = 2^{14}$ higher and about 1.6×10^4 TeV. M_{89} quarks would have virtually no weak interactions but would possess color interactions with weak confinement length scale reflecting themselves as new kind of jets at collisions above TeV energies.

In the biologically especially important length scale range 10 nm -2500 nm there are as many as four scaled up electron Compton lengths $L_e(k) = \sqrt{5}L(k)$: they are associated with Gaussian Mersennes $M_{G,k}$, $k = 151, 157, 163, 167$. This would suggest that the existence of scaled up scales of magnetic-, weak- and color confinement. An especially interesting possibly testable prediction is the existence of magnetic monopole pairs with the size scale in this range. There are recent claims about experimental evidence for magnetic monopole pairs [D3] .

Magnetic confinement and stringy picture in TGD sense

The connection between magnetic confinement and weak confinement is rather natural if one recalls that electric-magnetic duality in super-symmetric quantum field theories means that the descriptions in terms of particles and monopoles are in some sense dual descriptions. Fermions would be replaced by string like objects defined by the magnetic flux tubes and bosons as pairs of wormhole contacts would correspond to pairs of the flux tubes. Therefore the sharp distinction between gravitons and physical particles would disappear.

The reason why gravitons are necessarily stringy objects formed by a pair of wormhole contacts is that one cannot construct spin two objects using only single fermion states at wormhole throats. Of course, also super partners of these states with higher spin obtained by adding fermions and anti-fermions at the wormhole throat but these do not give rise to graviton like states [K24] . The upper and lower wormhole throat pairs would be quantum superpositions of fermion anti-fermion pairs with sum over all fermions. The reason is that otherwise one cannot realize graviton emission in terms of joining of the ends of light-like 3-surfaces together. Also now magnetic monopole charges are necessary but now there is no need to assign the entities $X_\pm$ with gravitons.

Graviton string is characterized by some p-adic length scale and one can argue that below this length scale the charges of the fermions become visible. Mersenne hypothesis suggests that some Mersenne prime is in question. One proposal is that gravitonic size scale is given by electronic Mersenne prime M_{127}. It is however difficult to test whether graviton has a structure visible below this length scale.

What happens to the generalized Feynman diagrams is an interesting question. It is not at all clear how closely they relate to ordinary Feynman diagrams. All depends on what one is ready to assume about what happens in the vertices. One could of course hope that zero energy ontology could allow some very simple description allowing perhaps to get rid of the problematic aspects of Feynman diagrams.

(a) Consider first the recent view about generalized Feynman diagrams which relies ZEO. A highly attractive assumption is that the particles appearing at wormhole throats are on mass shell particles. For incoming and outgoing elementary bosons and their super partners they would be positive it resp. negative energy states with parallel on mass shell momenta. For virtual bosons they the wormhole throats would have opposite sign of energy and the sum of on mass shell states would give virtual net momenta. This would make possible twistor description of virtual particles allowing only massless particles (in 4-D sense usually and in 8-D sense in TGD framework). The notion of virtual fermion makes sense only if one assumes in the interaction region a topological condensation creating another wormhole throat having no fermionic quantum numbers.

(b) The addition of the particles $X^\pm$ replaces generalized Feynman diagrams with the analogs of stringy diagrams with lines replaced by pairs of lines corresponding to fermion and $X_{\pm 1/2}$. The members of these pairs would correspond to 3-D light-like surfaces glued together at the vertices of generalized Feynman diagrams. The analog of 3-vertex would not be splitting of the string to form shorter strings but the replication of the entire string to form two strings with same length or fusion of two strings to single string along all their points rather than along ends to form a longer string. It is not clear whether the duality symmetry of stringy diagrams can hold true for the TGD variants of stringy diagrams.

(c) How should one describe the bound state formed by the fermion and $X^\pm$? Should one describe the state as superposition of non-parallel on mass shell states so that the composite state would be automatically massive? The description as superposition of on mass shell states does not conform with the idea that bound state formation requires binding energy. In TGD framework the notion of negentropic entanglement has been suggested to make possible the analogs of bound states consisting of on mass shell states so that the binding energy is zero [K41] . If this kind of states are in question the description of virtual states in terms of on mass shell states is not lost. Of course, one cannot exclude the possibility that there is infinite number of this kind of states serving as analogs for the excitations of string like object.

(d) What happens to the states formed by fermions and $X_{\pm 1/2}$ in the internal lines of the Feynman diagram? Twistor philosophy suggests that only the higher on mass shell excitations are possible. If this picture is correct, the situation would not change in an essential manner from the earlier one.

The highly non-trivial prediction of the magnetic confinement is that elementary particles should have stringy character in electro-weak length scales and could behaving to become manifest at LHC energies. This adds one further item to the list of non-trivial predictions of TGD about physics at LHC energies [K42] .

3.2.3 Could Quantum TGD Reduce To Almost Topological QFT?

There seems to be a profound connection with the earlier unrealistic proposal that TGD reduces to almost topological quantum theory in the sense that the counterpart of Chern-Simons action assigned with the wormhole throats somehow dictates the dynamics. This proposal can be formulated also for the Kähler-Dirac action action. I gave up this proposal but the following argument shows that Kähler action with weak form of electric-magnetic duality effectively reduces to Chern-Simons action plus Coulomb term.

(a) Kähler action density can be written as a 4-dimensional integral of the Coulomb term $j_K^\alpha A_\alpha$ plus and integral of the boundary term $J^{n\beta}A_\beta\sqrt{g_4}$ over the wormhole throats and of the quantity $J^{0\beta}A_\beta\sqrt{g_4}$ over the ends of the 3-surface.

(b) If the self-duality conditions generalize to $J^{n\beta} = 4\pi\alpha_K\epsilon^{n\beta\gamma\delta}J_{\gamma\delta}$ at throats and to $J^{0\beta} = 4\pi\alpha_K\epsilon^{0\beta\gamma\delta}J_{\gamma\delta}$ at the ends, the Kähler function reduces to the counterpart of Chern-Simons action evaluated at the ends and throats. It would have same value for each branch and the replacement $h \rightarrow n \times h$ would effectively describe this. Boundary conditions would however give $1/n$ factor so that $\hbar$ would disappear from the Kähler function! It is somewhat surprising that Kähler action gives Chern-Simons action in the vacuum sector defined as sector for which Kähler current is light-like or vanishes.

Holography encourages to ask whether also the Coulomb interaction terms could vanish. This kind of dimensional reduction would mean an enormous simplification since TGD would reduce to an almost topological QFT. The attribute "almost" would come from the fact that one has non-vanishing classical Noether charges defined by Kähler action and non-trivial quantum dynamics in M^4 degrees of freedom. One could also assign to space-time surfaces conserved four-momenta which is not possible in topological QFTs. For this reason the conditions guaranteeing the vanishing of Coulomb interaction term deserve a detailed analysis.

(a) For the known extremals j_K^α either vanishes or is light-like ("massless extremals" for which weak self-duality condition does not make sense [K7]) so that the Coulomb term vanishes identically in the gauge used. The addition of a gradient to A induces terms located at the ends and wormhole throats of the space-time surface but this term must be cancelled by the other boundary terms by gauge invariance of Kähler action. This implies that the M^4 part of WCW metric vanishes in this case. Therefore massless extremals as such are not physically realistic: wormhole throats representing particles are needed.

(b) The original naive conclusion was that since Chern-Simons action depends on CP_2 coordinates only, its variation with respect to Minkowski coordinates must vanish so that the WCW metric would be trivial in M^4 degrees of freedom. This conclusion is in conflict with quantum classical correspondence and was indeed too hasty. The point is that the allowed variations of Kähler function must respect the weak electro-magnetic duality which relates Kähler electric field depending on the induced 4-metric at 3-surface to the Kähler magnetic field. Therefore the dependence on M^4 coordinates creeps via a Lagrange multiplier term

$$\int \Lambda_\alpha(J^{n\alpha} - K\epsilon^{n\alpha\beta\gamma}J_{\beta\ gamma})\sqrt{g_4}d^3x \quad . \tag{3.2.11}$$

The (1,1) part of second variation contributing to M^4 metric comes from this term.

(c) This erratic conclusion about the vanishing of M^4 part WCW metric raised the question about how to achieve a non-trivial metric in M^4 degrees of freedom. The proposal was a modification of the weak form of electric-magnetic duality. Besides CP_2 Kähler form there would be the Kähler form assignable to the light-cone boundary reducing to that for $r_M = constant$ sphere - call it J^1. The generalization of the weak form of self-duality would be $J^{n\beta} = \epsilon^{n\beta\gamma\delta}K(J_{\gamma\delta} + \epsilon J^1_{\gamma\delta})$. This form implies that the boundary term gives a non-trivial contribution to the M^4 part of the WCW metric even without the constraint from electric-magnetic duality. Kähler charge is not affected unless the partonic 2-surface contains the tip of CD in its interior. In this case the value of Kähler charge is shifted by a topological contribution. Whether this term can survive depends on whether the resulting vacuum extremals are consistent with the basic facts about classical gravitation.

(d) The Coulombic interaction term is not invariant under gauge transformations. The good news is that this might allow to find a gauge in which the Coulomb term vanishes. The vanishing condition fixing the gauge transformation ϕ is

$$j_K^\alpha \partial_\alpha \phi = -j^\alpha A_\alpha \ . \tag{3.2.12}$$

This differential equation can be reduced to an ordinary differential equation along the flow lines j_K by using $dx^\alpha/dt = j_K^\alpha$. Global solution is obtained only if one can combine the flow parameter t with three other coordinates- say those at the either end of CD to form space-time coordinates. The condition is that the parameter defining the coordinate differential is proportional to the covariant form of Kähler current: $dt = \phi j_K$. This condition in turn implies $d^2t = d(\phi j_K) = d(\phi j_K) = d\phi \wedge j_K + \phi dj_K = 0$ implying $j_K \wedge dj_K = 0$ or more concretely,

$$\epsilon^{\alpha\beta\gamma\delta} j_\beta^K \partial_\gamma j_{delta}^K \ = \ 0 \ . \tag{3.2.13}$$

j_K is a four-dimensional counterpart of Beltrami field [B16] and could be called generalized Beltrami field.

The integrability conditions follow also from the construction of the extremals of Kähler action [K7] . The conjecture was that for the extremals the 4-dimensional Lorentz force vanishes (no dissipation): this requires $j_K \wedge J = 0$. One manner to guarantee this is the topologization of the Kähler current meaning that it is proportional to the instanton current: $j_K = \phi j_I$, where $j_I = {}^*(J \wedge A)$ is the instanton current, which is not conserved for 4-D CP_2 projection. The conservation of j_K implies the condition $j_I^\alpha \partial_\alpha \phi = \partial_\alpha j^\alpha \phi$ and from this ϕ can be integrated if the integrability condition $j_I \wedge dj_I = 0$ holds true implying the same condition for j_K. By introducing at least 3 or CP_2 coordinates as space-time coordinates, one finds that the contravariant form of j_I is purely topological so that the integrability condition fixes the dependence on M^4 coordinates and this selection is coded into the scalar function ϕ. These functions define families of conserved currents $j_K^\alpha \phi$ and $j_I^\alpha \phi$ and could be also interpreted as conserved currents associated with the critical deformations of the space-time surface.

(e) There are gauge transformations respecting the vanishing of the Coulomb term. The vanishing condition for the Coulomb term is gauge invariant only under the gauge transformations $A \rightarrow A + \nabla\phi$ for which the scalar function the integral $\int j_K^\alpha \partial_\alpha \phi$ reduces to a total divergence a giving an integral over various 3-surfaces at the ends of CD and at throats vanishes. This is satisfied if the allowed gauge transformations define conserved currents

$$D_\alpha(j^\alpha \phi) \ = \ 0 \ . \tag{3.2.14}$$

As a consequence Coulomb term reduces to a difference of the conserved charges $Q_\phi^e = \int j^0 \phi \sqrt{g_4} d^3x$ at the ends of the CD vanishing identically. The change of the Chern-Simons type term is trivial if the total weighted Kähler magnetic flux $Q_\phi^m = \sum \int J \phi dA$ over wormhole throats is conserved. The existence of an infinite number of conserved weighted magnetic fluxes is in accordance with the electric-magnetic duality. How these fluxes relate to the flux Hamiltonians central for WCW geometry is not quite clear.

(f) The gauge transformations respecting the reduction to almost topological QFT should have some special physical meaning. The measurement interaction term in the Kähler-Dirac interaction corresponds to a critical deformation of the space-time sheet and is realized as an addition of a gauge part to the Kähler gauge potential of CP_2. It would be natural to identify this gauge transformation giving rise to a conserved charge so that the conserved charges would provide a representation for the charges associated with the infinitesimal critical deformations not affecting Kähler action. The gauge transformed Kähler gauge potential couples to the Kähler-Dirac equation and its effect could be visible in the value of Kähler function and therefore also in the properties of the preferred extremal. The effect on WCW metric would however vanish since K would transform only by an addition of a real part of a holomorphic function.

(g) A first guess for the explicit realization of the quantum classical correspondence between quantum numbers and space-time geometry is that the deformation of the preferred extremal due to the addition of the measurement interaction term is induced by a $U(1)$ gauge transformation induced by a transformation of $\delta CD \times CP_2$ generating the gauge transformation represented by ϕ. This interpretation makes sense if the fluxes defined by Q^m_ϕ and corresponding Hamiltonians affect only zero modes rather than quantum fluctuating degrees of freedom.

(h) Later a simpler proposal assuming Kähler action with Chern-Simons term at partonic orbits and Kähler-Dirac action with Chern-Simons Dirac term at partonic orbits emerged. Measurement interaction terms would correspond to Lagrange multiplier terms at the ends of space-time surface fixing the values of classical conserved charges to their quantum values. Super-symmetry requires the assignment of this kind of term also to Kähler-Dirac action as boundary term.

    Kähler-Dirac equation gives rise to a boundary condition at space-like ends of the space-time surface stating that the action of the Kähler-Dirac gamma matrix in normal direction annihilates the spinor modes. The normal vector would be light-like and the value of the incoming on mass shell four-momentum would be coded to the geometry of the space-time surface and string world sheet.

    One can assign to partonic orbits Chern-Simons Dirac action and now the condition would be that the action of C-S-D operator equals to that of massless M^4 Dirac operator. C-S-D Dirac action would give rise to massless Dirac propagator. Twistor Grassmann approach suggests that also the virtual fermions reduce effectively to massless on-shell states but have non-physical helicity.

To sum up, one could understand the basic properties of WCW metric in this framework. Effective 2-dimensionality would result from the existence of an infinite number of conserved charges in two different time directions (genuine conservation laws plus gauge fixing). The infinite-dimensional symmetric space for given values of zero modes corresponds to the Cartesian product of the WCWs associated with the partonic 2-surfaces at both ends of CD and the generalized Chern-Simons term decomposes into a sum of terms from the ends giving single particle Kähler functions and to the terms from light-like wormhole throats giving interaction term between positive and negative energy parts of the state. Hence Kähler function could be calculated without any knowledge about the interior of the space-time sheets and TGD would reduce to almost topological QFT as speculated earlier. Needless to say this would have immense boost to the program of constructing WCW Kähler geometry.

3.3 An attempt to understand preferred extremals of Kähler action

Preferred extremal of Kähler action is one of the basic poorly defined notions of TGD. There are pressing motivations for understanding what "preferred" really means. For instance, the conformal invariance of string models naturally generalizes to 4-D invariance defined by quantum Yangian of quantum affine algebra (Kac-Moody type algebra) characterized by two complex coordinates and therefore explaining naturally the effective 2-dimensionality [K89]. The problem is however how to assign a complex coordinate with the string world sheet having Minkowskian signature of metric. One can hope that the understanding of preferred extremals could allow to identify two preferred complex coordinates whose existence is also suggested by number theoretical vision giving preferred role for the rational points of partonic 2-surfaces in preferred coordinates. The best one could hope is a general solution of field equations in accordance with the hints that TGD is integrable quantum theory.

3.3.1 What "preferred" could mean?

The first question is what preferred extremal could mean.

(a) In positive energy ontology preferred extremal would be a space-time surface assignable to given 3-surface and unique in the ideal situation: since one cannot pose conditions to the normal derivatives of imbedding space coordinates at 3-surface, there is infinity of extremals. Some additional conditions are required and space-time surface would be analogous to Bohr orbit: hence the attribute "preferred". The problem would be to understand what "preferred" could mean. The non-determinism of Kähler action however destroyed this dream in its original form and led to zero energy ontology (ZEO).

(b) In ZEO one considers extremals as space-time surfaces connecting two space-like 3-surfaces at the boundaries. One might hope that these 4-surfaces are unique. The non-determinism of Kähler action suggests that this is not the case. At least there is conformal invariance respecting the light-likeness of the 3-D parton orbits at which the signature of the induced metric changes: the conformal transformations would leave the space-like 3-D ends or at least partonic 2-surfaces invariant. This non-determinism would correspond to quantum criticality.

(c) Effective 2-dimensionality follows from strong form of general coordinate invariance (GCI) stating that light-like partonic orbits and space-like 3-surfaces at the ends of space-time surface are equivalent physically: partonic 2-surfaces and their 4-D tangent space data would determine everything. One can however worry about how effective 2-dimensionality relates to the fact that the modes of the induced spinor field are localized at string world sheets and partonic 2-surface. Are the tangent space data equivalent with the data characterizing string world sheets as surfaces carrying vanishing electroweak fields?

    There is however a problem: the hierarchy of Planck constants (dark matter) requires that the conformal equivalence classes of light-like surfaces must be counted as physical degrees of freedom so that either space-like or light-like surfaces do not seem to be quite enough.

    Should one then include also the light-like partonic orbits to the what one calls 3-surface? The resulting connected 3-surfaces would define analogs of Wilson loops. Could the conformal equivalence class of the preferred extremal be unique without any additional conditions? If so, one could get rid of the attribute "preferred". The fractal character of the many-sheeted space-time however suggests that one can have this kind of uniqueness only in given length scale resolution and that "radiative corrections" due to the non-determinism are always present.

These considerations show that the notion of preferred extremal is still far from being precisely defined and it is not even clear whether the attribute "preferred" is needed. If not then the question is what are the extremals of Kähler action.

3.3.2 What is known about extremals?

A lot is is known about properties of extremals and just by trying to integrate all this understanding, one might gain new visions. The problem is that all these arguments are heuristic and rely heavily on physical intuition. The following considerations relate to the space-time regions having Minkowskian signature of the induced metric. The attempt to generalize the construction also to Euclidian regions could be very rewarding. Only a humble attempt to combine various ideas to a more coherent picture is in question.

The core observations and visions are following.

(a) Hamilton-Jacobi coordinates for M^4 (discussed in this chapter) define natural preferred coordinates for Minkowskian space-time sheet and might allow to identify string world sheets for X^4 as those for M^4. Hamilton-Jacobi coordinates consist of light-like coordinate m and its dual defining local 2-plane $M^2 \subset M^4$ and complex transversal complex coordinates $(w, \overline{w})$ for a plane E^2_x orthogonal to M^2_x at each point of M^4. Clearly, hyper-complex analyticity and complex analyticity are in question.

(b) Space-time sheets allow a slicing by string world sheets (partonic 2-surfaces) labelled by partonic 2-surfaces (string world sheets).

(c) The quaternionic planes of octonion space containing preferred hyper-complex plane are labelled by CP_2, which might be called CP_2^{mod} [K74]. The identification $CP_2 = CP_2^{mod}$ motivates the notion of $M^8 - -M^4 \times CP_2$ duality [K14]. It also inspires a concrete solution ansatz assuming the equivalence of two different identifications of the quaternionic tangent space of the space-time sheet and implying that string world sheets can be regarded as strings in the 6-D coset space $G_2/SU(3)$. The group G_2 of octonion automorphisms has already earlier appeared in TGD framework.

(d) The duality between partonic 2-surfaces and string world sheets in turn suggests that the $CP_2 = CP_2^{mod}$ conditions reduce to string model for partonic 2-surfaces in $CP_2 = SU(3)/U(2)$. String model in both cases could mean just hypercomplex/complex analyticity for the coordinates of the coset space as functions of hyper-complex/complex coordinate of string world sheet/partonic 2-surface.

The considerations of this section lead to a revival of an old very ambitious and very romantic number theoretic idea.

(a) To begin with express octonions in the form $o = q_1 + Iq_2$, where q_i is quaternion and I is an octonionic imaginary unit in the complement of fixed a quaternionic sub-space of octonions. Map preferred coordinates of $H = M^4 \times CP_2$ to octonionic coordinate, form an arbitrary octonion analytic function having expansion with real Taylor or Laurent coefficients to avoid problems due to non-commutativity and non-associativity. Map the outcome to a point of H to get a map $H \to H$. This procedure is nothing but a generalization of Wick rotation to get an 8-D generalization of analytic map.

(b) Identify the preferred extremals of Kähler action as surfaces obtained by requiring the vanishing of the imaginary part of an octonion analytic function. Partonic 2-surfaces and string world sheets would correspond to commutative sub-manifolds of the space-time surface and of imbedding space and would emerge naturally. The ends of braid strands at partonic 2-surface would naturally correspond to the poles of the octonion analytic functions. This would mean a huge generalization of conformal invariance of string models to octonionic conformal invariance and an exact solution of the field equations of TGD and presumably of quantum TGD itself.

3.3.3 Basic ideas about preferred extremals

The slicing of the space-time sheet by partonic 2-surfaces and string world sheets

The basic vision is that space-time sheets are sliced by partonic 2-surfaces and string world sheets. The challenge is to formulate this more precisely at the level of the preferred extremals of Kähler action.

(a) Almost topological QFT property means that the Kähler action reduces to Chern-Simons terms assignable to 3-surfaces. This is guaranteed by the vanishing of the Coulomb term in the action density implied automatically if conserved Kähler current is proportional to the instanton current with proportionality coefficient some scalar function.

(b) The field equations reduce to the conservation of isometry currents. An attractive ansatz is that the flow lines of these currents define global coordinates. This means that these currents are Beltrami flows [B16] so that corresponding 1-forms J satisfy the condition $J \wedge dJ = 0$. These conditions are satisfied if

$$J = \Phi \nabla \Psi$$

hold true for conserved currents. From this one obtains that Ψ defines global coordinate varying along flow lines of J.

(c) A possible interpretation is in terms of local polarization and momentum directions defined by the scalar functions involved and natural additional conditions are that the gradients of Ψ and Φ are orthogonal:

$$\nabla \Phi \cdot \nabla \Psi = 0 \ ,$$

and that the Ψ satisfies massless d'Alembert equation

$$\nabla^2 \Psi = 0$$

as a consequence of current conservation. If Ψ defines a light-like vector field - in other words

$$\nabla \Psi \cdot \nabla \Psi = 0 \ ,$$

the light-like dual of Φ -call it Φ_c- defines a light-like like coordinate and Φ and Φ_c defines a light-like plane at each point of space-time sheet.

If also Φ satisfies d'Alembert equation

$$\nabla^2 \Phi = 0 \ ,$$

also the current

$$K = \Psi \nabla \Phi$$

is conserved and its flow lines define a global coordinate in the polarization plane orthogonal to time-lik plane defined by local light-like momentum direction.

If Φ allows a continuation to an analytic function of the transversal complex coordinate, one obtains a coordinatization of space-time surface by Ψ and its dual (defining hyper-complex coordinate) and $w, \overline{w}$. Complex analyticity and its hyper-complex variant would allow to provide space-time surface with four coordinates very much analogous with Hamilton-Jacobi coordinates of M^4.

This would mean a decomposition of the tangent space of space-time surface to orthogonal planes defined by light-like momentum and plane orthogonal to it. If the flow lines of J defined Beltrami flow it seems that the distribution of momentum planes is integrable.

(d) General arguments suggest that the space-time sheets allow a slicing by string world sheets parametrized by partonic 2-surfaces or vice versa. This would mean a intimate connection with the mathematics of string models. The two complex coordinates assignable to the Yangian of affine algebra would naturally relate to string world sheets and partonic 2-surfaces and the highly non-trivial challenge is to identify them appropriately.

Hamilton-Jacobi coordinates for M^4

The earlier attempts to construct preferred extremals [K7] led to the realization that so called Hamilton-Jacobi coordinates (m, w) for M^4 define its slicing by string world sheets parametrized by partonic 2-surfaces. m would be pair of light-like conjugate coordinates associated with an integrable distribution of planes M^2 and w would define a complex coordinate for the integrable distribution of 2-planes E^2 orthogonal to M^2. There is a great temptation to assume that these coordinates define preferred coordinates for M^4.

(a) The slicing is very much analogous to that for space-time sheets and the natural question is how these slicings relate. What is of special interest is that the momentum plane M^2 can be defined by massless momentum. The scaling of this vector does not matter so that these planes are labelled by points z of sphere S^2 telling the direction of the line $M^2 \cap E^3$, when one assigns rest frame and therefore S^2 with the preferred time coordinate defined by the line connecting the tips of CD. This direction vector can be mapped to a twistor consisting of a spinor and its conjugate. The complex scalings of the twistor $(u, \overline{u}) \to \lambda u, \overline{u}/\lambda)$ define the same plane. Projective twistor like entities defining CP_1 having only one complex component instead of three are in question. This complex number defines with certain prerequisites a local coordinate for space-time sheet and together with the complex coordinate of E^2 could serve as a pair of complex coordinates (z, w) for space-time sheet. This brings strongly in mind the two complex coordinates appearing in the expansion of the generators of quantum Yangian of quantum affine algebra [K89].

(b) The coordinate Ψ appearing in Beltrami flow defines the light-like vector field defining M^2 distribution. Its hyper-complex conjugate would define Ψ_c and conjugate light-like direction. An attractive possibility is that Φ allows analytic continuation to a holomorphic function of w. In this manner one would have four coordinates for M^4 also for space-time sheet.

(c) The general vision is that at each point of space-time surface one can decompose the tangent space to $M^2(x) \subset M^4 = M^2_x \times E^2_x$ representing momentum plane and polarization plane $E^2 \subset E^2_x \times T(CP_2)$. The moduli space of planes $E^2 \subset E^6$ is 8-dimensional and parametrized by $SO(6)/SO(2) \times SO(4)$ for a given E^2_x. How can one achieve this selection and what conditions it must satisfy? Certainly the choice must be integrable but this is not the only condition.

Space-time surfaces as associative/co-associative surfaces

The idea that number theory determines classical dynamics in terms of associativity condition means that space-time surfaces are in some sense quaternionic surfaces of an octonionic space-time. It took several trials before the recent form of this hypothesis was achieved.

(a) Octonionic structure is defined in terms of the octonionic representaton of gamma matrices of the imbedding space existing only in dimension $D = 8$ since octonion units are in one-one correspondence with tangent vectors of the tangent space. Octonionic real unit corresponds to a preferred time axes (and rest frame) identified naturally as that connecting the tips of CD. What modified gamma matrices mean depends on variational principle for space-time surface. For volume action one would obtain induced gamma matrices. For Kähler action one obtains something different. In particular, the modified gamma matrices do not define vector basis identical with tangent vector basis of space-time surface.

(b) Quaternionicity means that the modified gamma matrices defined as contractions of gamma matrices of H with canonical momentum densities for Kähler action span quaternionic sub-space of the octonionic tangent space [K88, ?]. A further condition is that each quaternionic space defined in this manner contains a preferred hyper-complex subspace of octonions.

(c) The sub-space defined by the modified gamma matrices does not co-incide with the tangent space of space-time surface in general so that the interpretation of this condition is far from obvious. The canonical momentum densities need not define four independent vectors at given point. For instance, for massless extremals these densities are proportional to light-like vector so that the situation is degenerate and the space in question reduces to 2-D hyper-complex sub-space since light-like vector defines plane M^2.

The obvious questions are following.

(a) Does the analog of tangent space defined by the octonionic modified gammas contain the local tangent space $M^2 \subset M^4$ for preferred extremals? For massless extremals [K7] this condition would be true. The orthogonal decomposition $T(X^4) = M^2 \oplus_\perp E^2$ can be defined at each point if this is true. For massless extremals also the functions Ψ and Φ can be identified.

(b) One should answer also the following delicate question. Can M^2 really depend on point x of space-time? CP_2 as a moduli space of quaternionic planes emerges naturally if M^2 is *same* everywhere. It however seems that one should allow an integrable distribution of M^2_x such that M^2_x is same for all points of a given partonic 2-surface.
How could one speak about fixed CP_2 (the imbedding space) at the entire space-time sheet even when M^2_x varies?

   i. Note first that G_2 (see `http://tinyurl.com/y9rrs7un`) defines the Lie group of octonionic automorphisms and G_2 action is needed to change the preferred hyper-octonionic sub-space. Various $SU(3)$ subgroups of G_2 are related by G_2 automorphism. Clearly, one must assign to each point of a string world sheet in the slicing parameterizing the partonic 2-surfaces an element of G_2. One would have Minkowskian string model with G_2 as a target space. As a matter fact, this string model is defined in the target space $G_2/SU(3)$ having dimension $D = 6$ since $SU(3)$ automorphisms leave given $SU(3)$ invariant.

   ii. This would allow to identify at each point of the string world sheet standard quaternionic basis - say in terms of complexified basis vectors consisting of two hyper-complex units and octonionic unit q_1 with "color isospin" $I_3 = 1/2$ and "color hypercharge" $Y = -1/3$ and its conjugate $\overline{q}_1$ with opposite color isospin and hypercharge.

   iii. The CP_2 point assigned with the quaternionic basis would correspond to the $SU(3)$ rotation needed to rotate the standard basis to this basis and would actually correspond to the first row of $SU(3)$ rotation matrix. Hyper-complex analyticity is the basic property of the solutions of the field equations representing Minkowskian string world sheets. Also now the same assumption is highly natural. In the case of string models in Minkowski space, the reduction of the induced metric to standard form implies Virasoro conditions and similar conditions are expected also now. There is no need to introduce action principle -just the hyper-complex analycitity is enough-since Kähler action already defines it.

(c) The WZW model (see `http://tinyurl.com/ydxcvfhv`) inspired approach to the situation would be following. The parameterization corresponds to a map $g : X^2 \rightarrow G_2$ for which g defines a flat G_2 connection at string world sheet. WZW type action would give rise to this kind of situation. The transition $G_2 \rightarrow G_2/SU(3)$ would require that one gauges $SU(3)$ degrees of freedom by bringing in $SU(3)$ connection. Similar procedure for $CP_2 = SU(3)/U(2)$ would bring in $SU(3)$ valued chiral field and $U(2)$ gauge field. Instead of introducing these connections one can simply introduce $G_2/SU(3)$ and $SU(3)/U(2)$ valued chiral fields. What this observation suggests that this ansatz indeed predicts gluons and electroweak gauge bosons assignable to string like objects so that the mathematical picture would be consistent with physical intuition.

The two interpretations of CP_2

An old observation very relevant for what I have called $M^8 - H$ duality [K14] is that the moduli space of quaternionic sub-spaces of octonionic space (identifiable as M^8) containing preferred hyper-complex plane is CP_2. Or equivalently, the space of two planes whose addition extends hyper-complex plane to some quaternionic subspace can be parametrized by CP_2. This CP_2 can be called it CP_2^{mod} to avoid confusion. In the recent case this would mean that the space $E^2(x) \subset E_x^2 \times T(CP_2)$ is represented by a point of CP_2^{mod}. On the other hand, the imbedding of space-time surface to H defines a point of "real" CP_2. This gives two different CP_2s.

(a) The highly suggestive idea is that the identification $CP_2^{mod} = CP_2$ (apart from isometry) is crucial for the construction of preferred extremals. Indeed, the projection of the space-time point to CP_2 would fix the local polarization plane completely. This condition for $E^2(x)$ would be purely local and depend on the values of CP_2 coordinates only. Second condition for $E^2(x)$ would involve the gradients of imbedding space coordinates including those of CP_2 coordinates.

(b) The conditions that the planes M_x^2 form an integrable distribution at space-like level and that M_x^2 is determined by the modified gamma matrices. The integrability of this distribution for M^4 could imply the integrability for X^2. X^4 would differ from M^4 only by a deformation in degrees of freedom transversal to the string world sheets defined by the distribution of M^2s.

   Does this mean that one can begin from vacuum extremal with constant values of CP_2 coordinates and makes them non-constant but allows to depend only on transversal degrees of freedom? This condition is too strong even for simplest massless extremals for which CP_2 coordinates depend on transversal coordinates defined by $\epsilon \cdot m$ and $\epsilon \cdot k$. One could however allow dependence of CP_2 coordinates on light-like M^4 coordinate since the modification of the induced metric is light-like so that light-like coordinate remains light-like coordinate in this modification of the metric.

   Therefore, if one generalizes directly what is known about massless extremals, the most general dependence of CP_2 points on the light-like coordinates assignable to the distribution of M_x^2 would be dependence on either of the light-like coordinates of Hamilton-Jacobi coordinates but not both.

3.3.4 What could be the construction recipe for the preferred extremals assuming $CP_2 = CP_2^{mod}$ identification?

The crucial condition is that the planes $E^2(x)$ determined by the point of $CP_2 = CP_2^{mod}$ identification and by the tangent space of $E_x^2 \times CP_2$ are same. The challenge is to transform this condition to an explicit form. $CP_2 = CP_2^{mod}$ identification should be general coordinate invariant. This requires that also the representation of E^2 as (e^2, e^3) plane is general coordinate invariant suggesting that the use of preferred CP_2 coordinates - presumably complex Eguchi-Hanson coordinates - could make life easy. Preferred coordinates are also suggested by number theoretical vision. A careful consideration of the situation would be required.

The modified gamma matrices define a quaternionic sub-space analogous to tangent space of X^4 but not in general identical with the tangent space: this would be the case only if the action were 4-volume. I will use the notation $T_x^m(X^4)$ about the modified tangent space and call the vectors of $T_x^m(X^4)$ modified tangent vectors. I hope that this would not cause confusion.

$CP_2 = CP_2^{mod}$ condition

Quaternionic property of the counterpart of $T_x^m(X^4)$ allows an explicit formulation using the tangent vectors of $T_x^m(X^4)$.

(a) The unit vector pair (e_2, e_3) should correspond to a unique tangent vector of H defined by the coordinate differentials dh^k in some natural coordinates used. Complex Eguchi-Hanson coordinates [L2] are a natural candidate for CP_2 and require complexified octonionic imaginary units. If octonionic units correspond to the tangent vector basis of H uniquely, this is possible.

(b) The pair (e_2, e_3) as also its complexification $(q_1 = e_2 + ie_3, \overline{q}_1 = e_2 - ie_3)$ is expressible as a linear combination of octonionic units $I_2, ...I_7$ should be mapped to a point of $CP_2^{mod} = CP_2$ in canonical manner. This mapping is what should be expressed explicitly. One should express given (e_2, e_3) in terms of $SU(3)$ rotation applied to a standard vector. After that one should define the corresponding CP_2 point by the bundle projection $SU(3) \rightarrow CP_2$.

(c) The tangent vector pair

$$(\partial_w h^k, \partial_{\overline{w}} h^k)$$

defines second representation of the tangent space of $E^2(x)$. This pair should be equivalent with the pair $(q_1, \overline{q}_1)$. Here one must be however very cautious with the choice of coordinates. If the choice of w is unique apart from constant the gradients should be unique. One can use also real coordinates (x, y) instead of $(w = x + iy, \overline{w} = x - iy)$ and the pair (e_2, e_3). One can project the tangent vector pair to the standard vielbein basis which must correspond to the octonionic basis

$$(\partial_x h^k, \partial_y h^k) \rightarrow (\partial_x h^k e_k^A e_A, \partial_y h^k e_k^A) e_A) \leftrightarrow (e_2, e_3) \ ,$$

where the e_A denote the octonion units in 1-1 correspondence with vielbein vectors. This expression can be compared to the expression of (e_2, e_3) derived from the knowledge of CP_2 projection.

Formulation of quaternionicity condition in terms of octonionic structure constants

One can consider also a formulation of the quaternionic tangent planes in terms of (e_2, e_3) expressed in terms of octonionic units deducible from the condition that unit vectors obey quaternionic algebra. The expressions for octonionic (see `http://tinyurl.com/5m5lqr`) *resp.* quaternionic (see `http://tinyurl.com/3rr79p9`) structure constants can be found at [A14] *resp.* [A18].

(a) The ansatz is

$$\begin{aligned} \{E_k\} &= \{1, I_1, E_2, E_3\} \ , \\ E_2 &= E_{2k} e^k \equiv \sum_{k=2}^{7} E_{2k} e^k \ , \ E_3 = E_{3k} e^k \equiv \sum_{k=2}^{7} E_{3k} e^k \ , \\ |E_2| &= 1 \ , \ |E_3| = 1 \ . \end{aligned} \tag{3.3.1}$$

(b) The multiplication table for octonionic units expressible in terms of octonionic triangle (see `http://tinyurl.com/5m5lqr`) [A14] gives

$$f^{1kl} E_{2k} = E_{3l} \ , \ f^{1kl} E_{3k} = -E_{2l} \ , \ f^{klr} E_{2k} E_{3l} = \delta_1^r \ . \tag{3.3.2}$$

Here the indices are raised by unit metric so that there is no difference between lower and upper indices. Summation convention is assumed. Also the contribution of the real unit is present in the structure constants of third equation but this contribution must vanish.

(c) The conditions are linear and quadratic in the coefficients E_{2k} and E_{3k} and are expected to allow an explicit solution. The first two conditions define homogenous equations which must allow solution. The coefficient matrix acting on (E_2, E_3) is of the form

$$\left(\begin{array}{cc} f_1 & 1 \\ -1 & f_1 \end{array} \right) \ ,$$

where 1 denotes unit matrix. The vanishing of the determinant of this matrix should be due to the highly symmetric properties of the structure constants. In fact the equations can be written as eigen conditions

$$f_1 \circ (E_2 \pm iE_3) = \mp i(E_2 \pm iE_3) \ ,$$

and one can say that the structure constants are eigenstates of the hermitian operator defined by I_1 analogous to color hyper charge. Both values of color hyper charged are obtained.

Explicit expression for the $CP_2 = CP_2^{mod}$ conditions

The symmetry under $SU(3)$ allows to construct the solutions of the above equations directly.

(a) One can introduce complexified basis of octonion units transforming like $(1, 1, 3, \overline{3})$ under SU(3). Note the analogy of triplet with color triplet of quarks. One can write complexified basis as $(1, e_1, (q_1, q_2, q_3), (\overline{q}_1\overline{q}_2, \overline{q}_3))$. The expressions for complexified basis elements are

$$(q_1, q_2, q_3) = \frac{1}{\sqrt{2}}(e_2 + ie_3, e_4 + ie_5, e_6 + ie_7) \ .$$

These options can be seen to be possible by studying octonionic triangle in which all lines containing 3 units defined associative triple: any pair of octonion units at this kind of line can be used to form pair of complexified unit and its conjugate. In the tangent space of $M^4 \times CP_2$ the basis vectors q_1, and q_2 are mixtures of E_x^2 and CP_2 tangent vectors. q_3 involves only CP_2 tangent vectors and there is a temptation to interpret it as the analog of the quark having no color isospin.

(b) The quaternionic basis is real and must transform like $(1, 1, q_1, \overline{q}_1)$, where q_1 is any quark in the triplet and $\overline{q}_1$ its conjugate in antitriplet. Having fixed some basis one can perform SU(3) rotations to get a new basis. The action of the rotation is by 3×3 special unitary matrix. The over all phases of its rows do not matter since they induce only a rotation in (e_2, e_3) plane not affecting the plane itself. The action of $SU(3)$ on q_1 is simply the action of its first row on (q_1, q_2, q_3) triplet:

$$\begin{aligned} q_1 \ &\rightarrow \ (Uq)_1 = U_{11}q_1 + U_{12}q_2 + U_{13}q_3 \equiv z_1q_1 + z_2q_2 + z_3q_3 \\ &= \ z_1(e_2 + ie_3) + z_2(e_4 + ie_5) + z_3(e_6 + ie_7) \ . \end{aligned} \tag{3.3.3}$$

The triplets (z_1, z_2, z_3) defining a complex unit vector and point of S^5. Since overall phase does not matter a point of CP_2 is in question. The new real octonion units are given by the formulas

$$\begin{aligned} e_2 \ &\rightarrow \ Re(z_1)e_2 + Re(z_2)e_4 + Re(z_3)e_6 - Im(z_1)e_3 - Im(z_2)e_5 - Im(z_3)e_7 \ , \\ e_3 \ &\rightarrow \ Im(z_1)e_2 + Im(z_2)e_4 + Im(z_3)e_6 + Re(z_1)e_3 + Re(z_2)e_5 + Re(z_3)e_7 \ . \end{aligned} \tag{3.3.4}$$

For instance the CP_2 coordinates corresponding to the coordinate patch (z_1, z_2, z_3) with $z_3 \neq 0$ are obtained as $(\xi_1, \xi_2) = (z_1/z_3, z_2/z_3)$.

Using these expressions the equations expressing the conjecture $CP_2 = CP_2^{mod}$ equivalence can be expressed explicitly as first order differential equations. The conditions state the equivalence

$$(e_2, e_3) \ \leftrightarrow \ (\partial_x h^k e_k^A e_A, \partial_y h^k e_k^A e_A) \ , \tag{3.3.5}$$

where e_A denote octonion units. The comparison of two pairs of vectors requires normalization of the tangent vectors on the right hand side to unit vectors so that one takes unit vector in the direction of the tangent vector. After this the vectors can be equated. This allows to expresses the contractions of the partial derivatives with vielbein vectors with the 6 components of e_2 and e_3. Each condition gives 6+6 first order partial differential equations which are non-linear by the presence of the overal normalization factor for the right hand side. The equations are invariant under scalings of (x, y). The very special form of these equations suggests that some symmetry is involved.

It must be emphasized that these equations make sense only in preferred coordinates: ordinary Minkowski coordinates and Hamilton-Jacobi coordinates for M^4 and Eguchi-Hanson complex coordinates in which $SU(2)\times U(1)$ is represented linearly for CP_2. These coordinates are preferred because they carry deep physical meaning.

Does TGD boil down to two string models?

It is good to look what have we obtained. Besides Hamilton-Jacobi conditions, and $CP_2 = CP_2^{mod}$ conditions one has what one might call string model with 6-dimensional $G_2/SU(3)$ as targent space. The orbit of string in $G_2/SU(3)$ allows to deduce the G_2 rotation identifiable as a point of $G_2/SU(3)$ defining what one means with standard quaternionic plane at given point of string world sheet. The hypothesis is that hyper-complex analyticity solves these equations.

The conjectured electric-magnetic duality implies duality between string world sheet and partonic 2-surfaces central for the proposed mathematical applications of TGD [K35, K36, K72, K90]. This duality suggests that the solutions to the $CP_2 = CP_2^{mod}$ conditions could reduce to holomorphy with respect to the coordinate w for partonic 2-surface plus the analogs of Virasoro conditions. The dependence on light-like coordinate would appear as a parametric dependence.

If this were the case, TGD would reduce at least partially to what might be regarded as dual string models in $G_2/SU(3)$ and $SU(3)/U(2)$ and also to string model in M^4 and X^4! In the previous arguments one ends up to string models in moduli spaces of string world sheets and partonic 2-surfaces. TGD seems to yield an inflation of string models! This not actually surprising since the slicing of space-time sheets by string world sheets and partonic 2-surfaces implies automatically various kinds of maps having interpretation in terms of string orbits.

3.4 In What Sense TGD Could Be An Integrable Theory?

During years evidence supporting the idea that TGD could be an integrable theory in some sense has accumulated. The challenge is to show that various ideas about what integrability means form pieces of a bigger coherent picture. Of course, some of the ideas are doomed to be only partially correct or simply wrong. Since it is not possible to know beforehand what ideas are wrong and what are right the situation is very much like in experimental physics and it is easy to claim (and has been and will be claimed) that all this argumentation is useless speculation. This is the price that must be paid for real thinking.

Integrable theories allow to solve nonlinear classical dynamics in terms of scattering data for a linear system. In TGD framework this translates to quantum classical correspondence. The solutions of Kähler-Dirac equation define the scattering data. This data should define a real analytic function whose octonionic extension defines the space-time surface as a surface for which its imaginary part in the representation as bi-quaternion vanishes. There are excellent hopes about this thanks to the reduction of the Kähler-Dirac equation to geometric optics.

In the following I will first discuss briefly what integrability means in (quantum) field theories, list some bits of evidence for integrability in TGD framework, discuss once again the question whether the different pieces of evidence are consistent with other and what one really means with various notions. An an outcome I represent what I regard as a more coherent view about integrability of TGD. The notion of octonion analyticity developed in the previous section is essential for the for what follows.

3.4.1 What Integrable Theories Are?

The following is an attempt to get some bird's eye of view about the landscape of integrable theories.

Examples of integrable theories

Integrable theories are typically non-linear 1+1-dimensional (quantum) field theories. Solitons and various other particle like structures are the characteristic phenomenon in these theories. Scattering matrix is trivial in the sense that the particles go through each other in the scattering and suffer only a phase change. In particular, momenta are conserved. Korteveg-de Vries equation (see `http://tinyurl.com/3cyt8hk`) [B3] was motivated by the attempt to explain the experimentally discovered shallow water wave preserving its shape and moving with a constant velocity. Sine-Gordon equation (see `http://tinyurl.com/yafl243x`) [B8] describes geometrically constant curvature surfaces and defines a Lorentz invariant non-linear field theory in 1+1-dimensional space-time, which can be applied to Josephson junctions (in TGD inspired quantum biology it is encountered in the model of nerve pulse [K60]). Non-linear Schrödinger equation (see `http://tinyurl.com/y88efbo7`) [B6] having applications to optics and water waves represents a further example. All these equations have various variants.

From TGD point of view conformal field theories represent an especially interesting example of integrable theories. (Super-)conformal invariance is the basic underlying symmetry and by its infinite-dimensional character implies infinite number of conserved quantities. The construction of the theory reduces to the construction of the representations of (super-)conformal algebra. One can solve 2-point functions exactly and characterize them in terms of (possibly anomalous) scaling dimensions of conformal fields involved and the coefficients appearing in 3-point functions can be solved in terms of fusion rules leading to an associative algebra for conformal fields. The basic applications are to 2-dimensional critical thermodynamical systems whose scaling invariance generalizes to conformal invariance. String models represent second application in which a collection of super-conformal field theories associated with various genera of 2-surface is needed to describe loop corrections to the scattering amplitudes. Also moduli spaces of conformal equivalence classes become important.

Topological quantum field theories (see `http://tinyurl.com/lsvx7g3`) are also examples of integrable theories. Because of its independence on the metric Chern-Simons action (see `http://tinyurl.com/ydgsqm2c`) is in 3-D case the unique action defining a topological quantum field theory. The calculations of knot invariants (for TGD approach see [K35]), topological invariants of 3-manifolds and 4-manifolds, and topological quantum computation (see `http://tinyurl.com/dkpo4y`) (for a model of DNA as topological quantum computer see [K20]) represent applications of this approach. TGD as almost topological QFT means that the Kähler action for preferred extremals reduces to a surface term by the vanishing of Coulomb term in action and by the weak form of electric-magnetic duality reduces to Chern-Simons action. Both Euclidian and Minkowskian regions give this kind of contribution.

$\mathcal{N} = 4$ SYM is the a four-dimensional and very nearly realistic candidate for an integral quantum field theory. The observation that twistor amplitudes allow also a dual of the 4-D conformal symmetry motivates the extension of this symmetry to its infinite-dimensional Yangian variant [A27]. Also the enormous progress in the construction of scattering amplitudes suggests integrability. In TGD framework Yangian symmetry would emerge naturally by extending the symplectic variant of Kac-Moody algebra from light-cone boundary to the interior of causal diamond and the Kac-Moody algebra from light-like 3-surface representing wormhole throats at which the signature of the induced metric changes to the space-time interior [K76].

About mathematical methods

The mathematical methods used in integrable theories are rather refined and have contributed to the development of the modern mathematical physics. Mention only quantum groups, conformal algebras, and Yangian algebras.

The basic element of integrability is the possibility to transform the non-linear classical problem for which the interaction is characterized by a potential function or its analog to a linear scattering problem depending on time. For instance, for the ordinary Schrödinger function one can solve potential once single solution of the equation is known. This does not work in practice. One can however gather information about the asymptotic states in scattering to deduce the potential. One cannot do without information about bound state energies too.

In TGD framework asymptotic states correspond to partonic 2-surfaces at the two light-like boundaries of CD (more precisely: the largest CD involved and defining the IR resolution for momenta). From the scattering data coding information about scattering for various values of energy of the incoming particle one deduced the potential function or its analog.

(a) The basic tool is inverse scattering transform known as Gelfand-Marchenko-Levitan (GML) transform (see `http://tinyurl.com/y9f7ybln`) described in simple terms in [B13].

   i. In 1+1 dimensional case the S-matrix characterizing scattering is very simple since the only thing that can take place in scattering is reflection or transmission. Therefore the S-matrix elements describe either of these processes and by unitarity the sum of corresponding probabilities equals to 1. The particle can arrive to the potential either from left or right and is characterized by a momentum. The transmission coefficient can have a pole meaning complex (imaginary in the simplest case) wave vector serving as a signal for the formation of a bound state or resonance. The scattering data are represented by the reflection and transmission coefficients as function of time.

   ii. One can deduce an integral equation for a propagator like function $K(t,x)$ describing how delta pulse moving with light velocity is scattered from the potential and is expressible in terms of time integral over scattering data with contributions from both scattering states and bound states. The derivation of GML transform [B13] uses time reversal and time translational invariance and causality defined in terms of light velocity. After some tricks one obtains the integral equation as well as an expression for the time independent potential as $V(x) = K(x,x)$. The argument can be generalized to more complex problems to deduce the GML transform.

(b) The so called Lax pair (see `http://tinyurl.com/yc93nw53`) is one manner to describe integrable systems [B4]. Lax pair consists of two operators L and M. One studies what might be identified as "energy" eigenstates satisfying $L(x,t)\Psi = \lambda\Psi$. λ does not depend on time and one can say that the dynamics is associated with x coordinate whereas as t is time coordinate parametrizing different variants of eigenvalue problem with the same spectrum for L. The operator $M(t)$ does not depend on x at all and the independence of λ on time implies the condition

$$\partial_t L = [L,M] \ .$$

This equation is analogous to a quantum mechanical evolution equation for an operator induced by time dependent "Hamiltonian" M and gives the non-linear classical evolution equation when the commutator on the right hand side is a multiplicative operator (so that it does not involve differential operators acting on the coordinate x). Non-linear classical dynamics for the time dependent potential emerges as an integrability condition.

One could say that $M(t)$ introduces the time evolution of $L(t,x)$ as an automorphism which depends on time and therefore does not affect the spectrum. One has $L(t,x) =$

$U(t)L(0,x)U^{-1}(t)$ with $dU(t)/dt = M(t)U(t)$. The time evolution of the analog of the quantum state is given by a similar equation.

(c) A more refined view about Lax pair is based on the observation that the above equation can be generalized so that M depends also on x. The generalization of the basic equation for $M(x,t)$ reads as

$$\partial_t L - \partial_x M - [L, M] = 0 \ .$$

The condition has interpretation as a vanishing of the curvature of a gauge potential having components $A_x = L, A_t = M$. This generalization allows a beautiful geometric formulation of the integrability conditions and extends the applicability of the inverse scattering transform. The monodromy of the flat connection becomes important in this approach. Flat connections in moduli spaces are indeed important in topological quantum field theories and in conformal field theories.

(d) There is also a connection with the so called Riemann-Hilbert problem (see `http://tinyurl.com/ybay4qjg`) [A20]. The monodromies of the flat connection define monodromy group and Riemann-Hilbert problem concerns the existence of linear differential equations having a given monodromy group. Monodromy group emerges in the analytic continuation of an analytic function and the action of the element of the monodromy group tells what happens for the resulting many-valued analytic function as one turns around a singularity once ("mono-"). The linear equations obviously relate to the linear scattering problem. The flat connection (M, L) in turn defines the monodromy group. What is needed is that the functions involved are analytic functions of (t, x) replaced with a complex or hyper-complex variable. Again Wick rotation is involved. Similar approach generalizes also to higher dimensional moduli spaces with complex structures.

In TGD framework the effective 2-dimensionality raises the hope that this kind of mathematical apparatus could be used. An interesting possibility is that finite measurement resolution could be realized in terms of a gauge group or Kac-Moody type group represented by trivial gauge potential defining a monodromy group for n-point functions. Monodromy invariance would hold for the full n-point functions constructed in terms of analytic n-point functions and their conjugates. The ends of braid strands are natural candidates for the singularities around which monodromies are defined.

3.4.2 Why TGD Could Be Integrable Theory In Some Sense?

There are many indications that TGD could be an integrable theory in some sense. The challenge is to see which ideas are consistent with each other and to build a coherent picture where everything finds its own place.

(a) 2-dimensionality or at least effective 2-dimensionality seems to be a prerequisite for integrability. Effective 2-dimensionality is suggested by the strong form of General Coordinate Invariance implying also holography and generalized conformal invariance predicting infinite number of conservation laws. The dual roles of partonic 2-surfaces and string world sheets supports a four-dimensional generalization of conformal invariance. Twistor considerations [K86] indeed suggest that Yangian invariance and Kac-Moody invariances combine to a 4-D analog of conformal invariance induced by 2-dimensional one by algebraic continuation.

(b) Octonionic representation of imbedding space Clifford algebra and the identification of the space-time surfaces as quaternionic space-time surfaces would define a number theoretically natural generalization of conformal invariance. The reason for using gamma matrix representation is that vector field representation for octonionic units does not exist. The problem concerns the precise meaning of the octonionic representation of gamma matrices.

Space-time surfaces could be quaternionic also in the sense that conformal invariance is analytically continued from string curve to 8-D space by octonion real-analyticity. The question is whether the Clifford algebra based notion of tangent space quaternionicity is equivalent with octonionic real-analyticity based notion of quaternionicity.

The notions of co-associativity and co-quaternionicity make also sense and one must consider seriously the possibility that associativity-co-associativity dichotomy corresponds to Minkowskian-Euclidian dichotomy.

(c) Field equations define hydrodynamic Beltrami flows satisfying integrability conditions of form $J \wedge dJ = 0$.

   i. One can assign local momentum and polarization directions to the preferred extremals and this gives a decomposition of Minkowskian space-time regions to massless quanta analogous to the 1+1-dimensional decomposition to solitons. The linear superposition of modes with 4-momenta with different directions possible for free Maxwell action does not look plausible for the preferred extremals of Kähler action. This rather quantal and solitonic character is in accordance with the quantum classical correspondence giving very concrete connection between quantal and classical particle pictures. For 4-D volume action one does not obtain this kind of decomposition. In 2-D case volume action gives superposition of solutions with different polarization directions so that the situation is nearer to that for free Maxwell action and is not like soliton decomposition.

   ii. Beltrami property in strong sense allows to identify 4 preferred coordinates for the space-time surface in terms of corresponding Beltrami flows. This is possible also in Euclidian regions using two complex coordinates instead of hyper-complex coordinate and complex coordinate. The assumption that isometry currents are parallel to the same light-like Beltrami flow implies hydrodynamic character of the field equations in the sense that one can say that each flow line is analogous to particle carrying some quantum numbers. This property is not true for all extremals (say cosmic strings).

   iii. The tangent bundle theoretic view about integrability is that one can find a Lie algebra of vector fields in some manifold spanning the tangent space of a lower-dimensional manifolds and is expressed in terms of Frobenius theorem (see `http://tinyurl.com/of6vfz5`) [A5]. The gradients of scalar functions defining Beltrami flows appearing in the ansatz for preferred exremals would define these vector fields and the slicing. Partonic 2-surfaces would correspond to two complex conjugate vector fields (local polarization direction) and string world sheets to light-like vector field and its dual (light-like momentum directions). This slicing generalizes to the Euclidian regions.

(d) Infinite number of conservation laws is the signature of integrability. Classical field equations follow from the condition that the vector field defined by Kähler-Dirac gamma matrices has vanishing divergence and can be identified an integrability condition for the Kähler-Dirac equation guaranteeing also the conservation of super currents so that one obtains an infinite number of conserved charges.

(e) Quantum criticality is a further signal of integrability. 2-D conformal field theories describe critical systems so that the natural guess is that quantum criticality in TGD framework relates to the generalization of conformal invariance and to integrability. Quantum criticality implies that Kähler coupling strength is analogous to critical temperature. This condition does affects classical field equations only via boundary conditions expressed as weak form of electric magnetic duality at the wormhole throats at which the signature of the metric changes.

   For finite-dimensional systems the vanishing of the determinant of the matrix defined by the second derivatives of potential is similar signature and applies in catastrophe theory. Therefore the existence of vanishing second variations of Kähler action should characterize criticality and define a property of preferred extremals. The vanishing of second variations indeed leads to an infinite number of conserved currents [K23, K7] following the conditions that the deformation of Kähler-Dirac gamma matrix is also divergenceless and that the Kähler-Dirac equation associated with it is satisfied.

3.4.3 Could TGD Be An Integrable Theory?

Consider first the abstraction of integrability in TGD framework. Quantum classical correspondence could be seen as a correspondence between linear quantum dynamics and non-linear classical dynamics. Integrability would realize this correspondence. In integrable models such as Sine-Gordon equation particle interactions are described by potential in 1+1 dimensions. This too primitive for the purposes of TGD. The vertices of generalized Feynman diagrams take care of this. At lines one has free particle dynamics so that the situation could be much simpler than in integrable models if one restricts the considerations to the lines or Minkowskian space-time regions surrounding them.

The non-linear dynamics for the space-time sheets representing incoming lines of generalized Feynman diagram should be obtainable from the linear dynamics for the induced spinor fields defined by Kähler-Dirac operator. There are two options.

(a) Strong form of the quantum classical correspondence states that each solution for the linear dynamics of spinor fields corresponds to space-time sheet. This is analogous to solving the potential function in terms of a single solution of Schrödinger equation. Coupling of space-time geometry to quantum numbers via measurement interaction term is a proposal for realizing this option. It is however the quantum numbers of positive/negative energy parts of zero energy state which would be visible in the classical dynamics rather than those of induced spinor field modes.

(b) Only overall dynamics characterized by scattering data- the counterpart of S-matrix for the Kähler-Dirac operator- is mapped to the geometry of the space-time sheet. This is much more abstract realization of quantum classical correspondence.

(c) Can these two approaches be equivalent? This might be the case since quantum numbers of the state are not those of the modes of induced spinor fields.

What the scattering data could be for the induced spinor field satisfying Kähler-Dirac equation?

(a) If the solution of field equation has hydrodynamic character, the solutions of the Kähler-Dirac equation can be localized to light-like Beltrami flow lines of hydrodynamic flow. These correspond to basic solutions and the general solution is a superposition of these. There is no dispersion and the dynamics is that of geometric optics at the basic level. This means geometric optics like character of the spinor dynamics.

    Solutions of the Kähler-Dirac equation are completely analogous to the pulse solutions defining the fundamental solution for the wave equation in the argument leading from wave equation with external time independent potential to Marchenko-Gelfand-Levitan equation allowing to identify potential in terms of scattering data. There is however no potential present now since the interactions are described by the vertices of Feynman diagram where the particle lines meet. Note that particle like regions are Euclidian and that this picture applies only to the Minkowskian exteriors of particles.

(b) Partonic 2-surfaces at the ends of the line of generalized Feynman diagram are connected by flow lines. Partonic 2-surfaces at which the signature of the induced metric changes are in a special position. Only the imaginary part of the bi-quaternionic value of the octonion valued map is non-vanishing at these surfaces which can be said to be co-complex 2-surfaces. By geometric optics behavior the scattering data correspond to a diffeomorphism mapping initial partonic 2-surface to the final one in some preferred complex coordinates common to both ends of the line.

(c) What could be these preferred coordinates? Complex coordinates for S^2 at light-cone boundary define natural complex coordinates for the partonic 2-surface. With these coordinates the diffeomorphism defining scattering data is diffeomorphism of S^2. Suppose that this map is real analytic so that maps "real axis" of S^2 to itself. This map would be same as the map defining the octonionic real analyticity as algebraic extension of the complex real analytic map. By octonionic analyticity one can make large number of alternative choices for the coordinates of partonic 2-surface.

(d) There can be non-uniqueness due to the possibility of $G_2/SU(3)$ valued map characterizing the local octonionic units. The proposal is that the choice of octonionic imaginary units can depend on the point of string like orbit: this would give string model in $G_2/SU(3)$. Conformal invariance for this string model would imply analyticity and helps considerably but would not probably fix the situation completely since the element of the coset space would constant at the partonic 2-surfaces at the ends of CD. One can of course ask whether the $G_2/SU(3)$ element could be constant for each propagator line and would change only at the 2-D vertices?

This would be the inverse scattering problem formulated in the spirit of TGD. There could be also dependence of space-time surface on quantum numbers of quantum states but not on individual solution for the induced spinor field since the scattering data of this solution would be purely geometric.

3.5 Do Geometric Invariants Of Preferred Extremals Define Topological Invariants Of Space-time Surface And Code For Quantumphysics?

The recent progress in the understanding of preferred extremals [K7] led to a reduction of the field equations to conditions stating for Euclidian signature the existence of Kähler metric. The resulting conditions are a direct generalization of corresponding conditions emerging for the string world sheet and stating that the 2-metric has only non-diagonal components in complex/hypercomplex coordinates. Also energy momentum of Kähler action and has this characteristic (1, 1) tensor structure. In Minkowskian signature one obtains the analog of 4-D complex structure combining hyper-complex structure and 2-D complex structure.

The construction lead also to the understanding of how Einstein's equations with cosmological term follow as a consistency condition guaranteeing that the covariant divergence of the Maxwell's energy momentum tensor assignable to Kähler action vanishes. This gives $T = kG + \Lambda g$. By taking trace a further condition follows from the vanishing trace of T:

$$R = \frac{4\Lambda}{k} . \tag{3.5.1}$$

That any preferred extremal should have a constant Ricci scalar proportional to cosmological constant is very strong prediction. Note that the accelerating expansion of the Universe would support positive value of Λ. Note however that both Λ and $k \propto 1/G$ are both parameters characterizing one particular preferred extremal. One could of course argue that the dynamics allowing only constant curvature space-times is too simple. The point is however that particle can topologically condense on several space-time sheets meaning effective superposition of various classical fields defined by induced metric and spinor connection.

The following considerations demonstrate that preferred extremals can be seen as canonical representatives for the constant curvature manifolds playing central role in Thurston's geometrization theorem (see `http://tinyurl.com/y8bbzlnr`) [A24] known also as hyperbolization theorem implying that geometric invariants of space-time surfaces transform to topological invariants. The generalization of the notion of Ricci flow to Maxwell flow in the space of metrics and further to Kähler flow for preferred extremals in turn gives a rather detailed vision about how preferred extremals organize to one-parameter orbits. It is quite possible that Kähler flow is actually discrete. The natural interpretation is in terms of dissipation and self organization.

Quantum classical correspondence suggests that this line of thought could be continued even further: could the geometric invariants of the preferred extremals could code not only for space-time topology but also for quantum physics? How to calculate the correlation functions and coupling constant evolution has remained a basic unresolved challenge of quantum

TGD. Could the correlation functions be reduced to statistical geometric invariants of preferred extemals? The latest (means the end of 2012) and perhaps the most powerful idea hitherto about coupling constant evolution is quantum classical correspondence in statistical sense stating that the statistical properties of a preferred extremal in quantum superposition of them are same as those of the zero energy state in question. This principle would be quantum generalization of ergodic theorem stating that the time evolution of a single member of ensemble represents the ensemble statistically. This principle would allow to deduce correlation functions and S-matrix from the statistical properties of single preferred extremal alone using classical intuition. Also coupling constant evolution would be coded by the statistical properties of the representative preferred extremal.

3.5.1 Preferred Extremals Of Kähler Action As Manifolds With Constant Ricci Scalar Whose Geometric Invariants Are Topological-Invariants

An old conjecture inspired by the preferred extremal property is that the geometric invariants of space-time surface serve as topological invariants. The reduction of Kähler action to 3-D Chern-Simons terms (see `http://tinyurl.com/ybp86sho`) [K7] gives support for this conjecture as a classical counterpart for the view about TGD as almost topological QFT. The following arguments give a more precise content to this conjecture in terms of existing mathematics.

(a) It is not possible to represent the scaling of the induced metric as a deformation of the space-time surface preserving the preferred extremal property since the scale of CP_2 breaks scale invariance. Therefore the curvature scalar cannot be chosen to be equal to one numerically. Therefore also the parameter $R = 4\Lambda/k$ and also Λ and k separately characterize the equivalence class of preferred extremals as is also physically clear.

Also the volume of the space-time sheet closed inside causal diamond CD remains constant along the orbits of the flow and thus characterizes the space-time surface. Λ and even $k \propto 1/G$ can indeed depend on space-time sheet and p-adic length scale hypothesis suggests a discrete spectrum for Λ/k expressible in terms of p-adic length scales: $\Lambda/k \propto 1/L_p^2$ with $p \simeq 2^k$ favored by p-adic length scale hypothesis. During cosmic evolution the p-adic length scale would increase gradually. This would resolve the problem posed by cosmological constant in GRT based theories.

(b) One could also see the preferred extremals as 4-D counterparts of constant curvature 3-manifolds in the topology of 3-manifolds. An interesting possibility raised by the observed negative value of Λ is that most 4-surfaces are constant negative curvature 4-manifolds. By a general theorem coset spaces (see `http://tinyurl.com/y8d3udpr`) H^4/Γ, where $H^4 = SO(1,4)/SO(4)$ is hyperboloid of M^5 and Γ a torsion free discrete subgroup of $SO(1,4)$ [A8]. It is not clear to me, whether the constant value of Ricci scalar implies constant sectional curvatures and therefore hyperbolic space property. It could happen that the space of spaces with constant Ricci curvature contain a hyperbolic manifold as an especially symmetric representative. In any case, the geometric invariants of hyperbolic metric are topological invariants.

By Mostow rigidity theorem (see `http://tinyurl.com/yacbu8sk`) [A12] finite-volume hyperbolic manifold is unique for $D > 2$ and determined by the fundamental group of the manifold. Since the orbits under the Kähler flow preserve the curvature scalar the manifolds at the orbit must represent different imbeddings of one and hyperbolic 4-manifold. In 2-D case the moduli space for hyperbolic metric for a given genus $g > 0$ is defined by Teichmueller parameters and has dimension $6(g-1)$. Obviously the exceptional character of $D = 2$ case relates to conformal invariance. Note that the moduli space in question (see `http://tinyurl.com/ybowqm5v`) plays a key role in p-adic mass calculations [K12].

In the recent case Mostow rigidity theorem could hold true for the Euclidian regions and maybe generalize also to Minkowskian regions. If so then both "topological" and "geometro" in "Topological GeometroDynamics" would be fully justified. The fact that geometric invariants become topological invariants also conforms with "TGD as almost topological QFT" and allows the notion of scale to find its place in topology. Also the dream about exact solvability of the theory would be realized in rather convincing manner.

These conjectures are the main result independent of whether the generalization of the Ricci flow discussed in the sequel exists as a continuous flow or possibly discrete sequence of iterates in the space of preferred extremals of Kähler action. My sincere hope is that the reader could grasp how far reaching these result really are.

3.5.2 Is There A Connection Between Preferred Extremals And AdS$_4$/CFT Correspondence?

The preferred extremals satisfy Einstein Maxwell equations with a cosmological constant and have negative scalar curvature for negative value of Λ. 4-D space-times with hyperbolic metric provide canonical representation for a large class of four-manifolds and an interesting question is whether these spaces are obtained as preferred extremals and/or vacuum extremals.

4-D hyperbolic space with Minkowski signature is locally isometric with AdS$_4$. This suggests at connection with AdS$_4$/CFT correspondence of M-theory. The boundary of AdS would be now replaced with 3-D light-like orbit of partonic 2-surface at which the signature of the induced metric changes. The metric 2-dimensionality of the light-like surface makes possible generalization of 2-D conformal invariance with the light-like coordinate taking the role of complex coordinate at light-like boundary. AdS could represent a special case of a more general family of space-time surfaces with constant Ricci scalar satistying Einstein-Maxwell equations and generalizing the AdS$_4$/CFT correspondence. There is however a strong objection from cosmology: the accelerated expansion of the Universe requires positive value of Λ and favors De Sitter Space dS_4 instead of AdS_4.

These observations provide motivations for finding whether AdS$_4$ and/or dS_4 allows an imbedding as a vacuum extremal to $M^4 \times S^2 \subset M^4 \times CP_2$, where S^2 is a homologically trivial geodesic sphere of CP_2. It is easy to guess the general form of the imbedding by writing the line elements of, M^4, S^2, and AdS$_4$.

(a) The line element of M^4 in spherical Minkowski coordinates (m, r_M, θ, ϕ) reads as

$$ds^2 = dm^2 - dr_M^2 - r_M^2 d\Omega^2 \ . \tag{3.5.2}$$

(b) Also the line element of S^2 is familiar:

$$ds^2 = -R^2(d\Theta^2 + sin^2(\theta)d\Phi^2) \ . \tag{3.5.3}$$

(c) By visiting in Wikipedia (see `http://tinyurl.com/y9hw95ql`) one learns that in spherical coordinate the line element of AdS$_4$/dS_4 is given by

$$\begin{aligned} ds^2 &= A(r)dt^2 - \frac{1}{A(r)}dr^2 - r^2 d\Omega^2 \ , \\ A(r) &= 1 + \epsilon y^2 \ , \ y = \frac{r}{r_0} \ , \\ \epsilon &= 1 \ \text{for } AdS_4 \ , \ \epsilon = -1 \ \text{for } dS_4 \ . \end{aligned} \tag{3.5.4}$$

(d) From these formulas it is easy to see that the ansatz is of the same general form as for the imbedding of Schwartschild-Nordstöm metric:

$$\begin{aligned} &m = \Lambda t + h(y) \ , \quad r_M = r \ , \\ &\Theta = s(y) \ , \quad \Phi = \omega(t + f(y)) \ . \end{aligned} \tag{3.5.5}$$

The non-trivial conditions on the components of the induced metric are given by

$$\begin{aligned} g_{tt} &= \Lambda^2 - x^2 sin^2(\Theta) = A(r) \ , \\ g_{tr} &= \frac{1}{r_0}\left[\Lambda \frac{dh}{dy} - x^2 sin^2(\theta)\frac{df}{dr}\right] = 0 \ , \\ g_{rr} &= \frac{1}{r_0^2}\left[(\frac{dh}{dy})^2 - 1 - x^2 sin^2(\theta)(\frac{df}{dy})^2 - R^2(\frac{d\Theta}{dy})^2\right] = -\frac{1}{A(r)} \ , \\ x &= R\omega \ . \end{aligned} \tag{3.5.6}$$

By some simple algebraic manipulations one can derive expressions for $sin(\Theta)$, df/dr and dh/dr.

(a) For $\Theta(r)$ the equation for g_{tt} gives the expression

$$\begin{aligned} sin(\Theta) &= \pm\frac{P^{1/2}}{x} \ , \\ P &= \Lambda^2 - A = \Lambda^2 - 1 - \epsilon y^2 \ . \end{aligned} \tag{3.5.7}$$

The condition $0 \leq sin^2(\Theta) \leq 1$ gives the conditions

$$\begin{array}{rcll} (\Lambda^2 - x^2 - 1)^{1/2} \leq & y & \leq (\Lambda^2-1)^{1/2} & \text{for } \epsilon = 1 \ (AdS_4) \ , \\ (-\Lambda^2+1)^{1/2} \leq & y & \leq (x^2+1-\Lambda^2)^{1/2} & \text{for } \epsilon = -1 \ (dS_4) \ . \end{array} \tag{3.5.8}$$

Only a spherical shell is possible in both cases. The model for the final state of star considered in [K79] predicted similar layer layer like structure and inspired the proposal that stars quite generally have an onion-like structure with radii of various shells characterize by p-adic length scale hypothesis and thus coming in some powers of $\sqrt{2}$. This brings in mind also Titius-Bode law.

(b) From the vanishing of g_{tr} one obtains

$$\frac{dh}{dy} = \frac{P}{\Lambda}\frac{df}{dy} \ . \tag{3.5.9}$$

(c) The condition for g_{rr} gives

$$(\frac{df}{dy})^2 = \frac{r_0^2}{AP}[A^{-1} - R^2(\frac{d\Theta}{dy})^2] \ . \tag{3.5.10}$$

Clearly, the right-hand side is positive if $P \geq 0$ holds true and $Rd\Theta/dy$ is small. One can express $d\Theta/dy$ using chain rule as

$$(\frac{d\Theta}{dy})^2 = \frac{x^2y^2}{P(P-x^2)} \ . \tag{3.5.11}$$

One obtains

$$(\frac{df}{dy})^2 = \Lambda r_0^2 \frac{y^2}{AP}\left[\frac{1}{1+y^2} - x^2(\frac{R}{r_0})^2\frac{1}{P(P-x^2)}\right] \ . \tag{3.5.12}$$

The right hand side of this equation is non-negative for certain range of parameters and variable y. Note that for $r_0 \gg R$ the second term on the right hand side can be neglected. In this case it is easy to integrate $f(y)$.

The conclusion is that both AdS_4 and dS^4 allow a local imbedding as a vacuum extremal. Whether also an imbedding as a non-vacuum preferred extremal to $M^4 \times S^2$, S^2 a homologically non-trivial geodesic sphere is possible, is an interesting question.

3.5.3 Generalizing Ricci Flow To Maxwell Flow For 4-Geometries And Kähler Flow For Space-Time Surfaces

The notion of Ricci flow has played a key part in the geometrization of topological invariants of Riemann manifolds. I certainly did not have this in mind when I choose to call my unification attempt "Topological Geometrodynamics" but this title strongly suggests that a suitable generalization of Ricci flow could play a key role in the understanding of also TGD.

Ricci flow and Maxwell flow for 4-geometries

The observation about constancy of 4-D curvature scalar for preferred extremals inspires a generalization of the well-known volume preserving Ricci flow (see `http://tinyurl.com/2cwlzh9l`) [A19] introduced by Richard Hamilton. Ricci flow is defined in the space of Riemann metrics as

$$\frac{dg_{\alpha\beta}}{dt} = -2R_{\alpha\beta} + 2\frac{R_{avg}}{D}g_{\alpha\beta} \quad . \tag{3.5.13}$$

Here R_{avg} denotes the average of the scalar curvature, and D is the dimension of the Riemann manifold. The flow is volume preserving in average sense as one easily checks ($\langle g^{\alpha\beta}dg_{\alpha\beta}/dt\rangle = 0$). The volume preserving property of this flow allows to intuitively understand that the volume of a 3-manifold in the asymptotic metric defined by the Ricci flow is topological invariant. The fixed points of the flow serve as canonical representatives for the topological equivalence classes of 3-manifolds. These 3-manifolds (for instance hyperbolic 3-manifolds with constant sectional curvatures) are highly symmetric. This is easy to understand since the flow is dissipative and destroys all details from the metric.

What happens in the recent case? The first thing to do is to consider what might be called Maxwell flow in the space of all 4-D Riemann manifolds allowing Maxwell field.

(a) First of all, the vanishing of the trace of Maxwell's energy momentum tensor codes for the volume preserving character of the flow defined as

$$\frac{dg_{\alpha\beta}}{dt} = T_{\alpha\beta} \quad . \tag{3.5.14}$$

Taking covariant divergence on both sides and assuming that d/dt and D_α commute, one obtains that $T^{\alpha\beta}$ is divergenceless.

This is true if one assumes Einstein's equations with cosmological term. This gives

$$\frac{dg_{\alpha\beta}}{dt} = kG_{\alpha\beta} + \Lambda g_{\alpha\beta} = kR_{\alpha\beta} + (-\frac{kR}{2} + \Lambda)g_{\alpha\beta} \quad . \tag{3.5.15}$$

The trace of this equation gives that the curvature scalar is constant. Note that the value of the Kähler coupling strength plays a highly non-trivial role in these equations and it is quite possible that solutions exist only for some critical values of α_K. Quantum criticality should fix the allow value triplets (G, Λ, α_K) apart from overall scaling

$$(G, \Lambda, \alpha_K) \to (xG, \Lambda/x, x\alpha_K \quad .$$

Fixing the value of G fixes the values remaining parameters at critical points. The rescaling of the parameter t induces a scaling by x.

(b) By taking trace one obtains the already mentioned condition fixing the curvature to be constant, and one can write

$$\frac{dg_{\alpha\beta}}{dt} = kR_{\alpha\beta} - \Lambda g_{\alpha\beta} \ . \tag{3.5.16}$$

Note that in the recent case $R_{avg} = R$ holds true since curvature scalar is constant. The fixed points of the flow would be Einstein manifolds (see http://tinyurl.com/ybrnakuu) [A4, A35] satisfying

$$R_{\alpha\beta} = \frac{\Lambda}{k} g_{\alpha\beta} \tag{3.5.17}$$

.

(c) It is by no means obvious that continuous flow is possible. The condition that Einstein-Maxwell equations are satisfied might pick up from a completely general Maxwell flow a discrete subset as solutions of Einstein-Maxwell equations with a cosmological term. If so, one could assign to this subset a sequence of values t_n of the flow parameter t.

(d) I do not know whether 3-dimensionality is somehow absolutely essential for getting the topological classification of closed 3-manifolds using Ricci flow. This ignorance allows me to pose some innocent questions. Could one have a canonical representation of 4-geometries as spaces with constant Ricci scalar? Could one select one particular Einstein space in the class four-metrics and could the ratio Λ/k represent topological invariant if one normalizes metric or curvature scalar suitably. In the 3-dimensional case curvature scalar is normalized to unity. In the recent case this normalization would give $k = 4\Lambda$ in turn giving $R_{\alpha\beta} = g_{\alpha\beta}/4$. Does this mean that there is only single fixed point in local sense, analogous to black hole toward which all geometries are driven by the Maxwell flow? Does this imply that only the 4-volume of the original space would serve as a topological invariant?

Maxwell flow for space-time surfaces

One can consider Maxwell flow for space-time surfaces too. In this case Kähler flow would be the appropriate term and provides families of preferred extremals. Since space-time surfaces inside CD are the basic physical objects are in TGD framework, a possible interpretation of these families would be as flows describing physical dissipation as a four-dimensional phenomenon polishing details from the space-time surface interpreted as an analog of Bohr orbit.

(a) The flow is now induced by a vector field $j^k(x,t)$ of the space-time surface having values in the tangent bundle of imbedding space $M^4 \times CP_2$. In the most general case one has Kähler flow without the Einstein equations. This flow would be defined in the space of all space-time surfaces or possibly in the space of all extremals. The flow equations reduce to

$$h_{kl} D_\alpha j^k(x,t) D_\beta h^l = \frac{1}{2} T_{\alpha\beta} \ . \tag{3.5.18}$$

The left hand side is the projection of the covariant gradient $D_\alpha j^k(x,t)$ of the flow vector field $j^k(x,t)$ to the tangent space of the space-time surface. D_{alpha} is covariant derivative taking into account that j^k is imbedding space vector field. For a fixed point space-time surface this projection must vanish assuming that this space-time surface reachable. A good guess for the asymptotia is that the divergence of Maxwell energy momentum tensor vanishes and that Einstein's equations with cosmological constant are well-defined.

Asymptotes corresponds to vacuum extremals. In Euclidian regions CP_2 type vacuum extremals and in Minkowskian regions to any space-time surface in any 6-D sub-manifold $M^4 \times Y^2$, where Y^2 is Lagrangian sub-manifold of CP_2 having therefore vanishing induced Kähler form. Symplectic transformations of CP_2 combined with diffeomorphisms of M^4 give new Lagrangian manifolds. One would expect that vacuum extremals are approached but never reached at second extreme for the flow.

If one assumes Einstein's equations with a cosmological term, allowed vacuum extremals must be Einstein manifolds. For CP_2 type vacuum extremals this is the case. It is quite possible that these fixed points do not actually exist in Minkowskian sector, and could be replaced with more complex asymptotic behavior such as limit, chaos, or strange attractor.

(b) The flow could be also restricted to the space of preferred extremals. Assuming that Einstein Maxwell equations indeed hold true, the flow equations reduce to

$$h_{kl}D_\alpha j^k(x,t)\partial_\beta h^l = \frac{1}{2}(kR_{\alpha\beta} - \Lambda g_{\alpha\beta}) . \quad (3.5.19)$$

Preferred extremals would correspond to a fixed sub-manifold of the general flow in the space of all 4-surfaces.

(c) One can also consider a situation in which $j^k(x,t)$ is replaced with $j^k(h,t)$ defining a flow in the entire imbedding space. This assumption is probably too restrictive. In this case the equations reduce to

$$(D_r j_l(x,t) + D_l j_r)\partial_\alpha h^r \partial_\beta h^l = kR_{\alpha\beta} - \Lambda g_{\alpha\beta} . \quad (3.5.20)$$

Here D_r denotes covariant derivative. Asymptotia is achieved if the tensor $D_k j_l + D_k j_l$ becomes orthogonal to the space-time surface. Note for that Killing vector fields of H the left hand side vanishes identically. Killing vector fields are indeed symmetries of also asymptotic states.

It must be made clear that the existence of a continuous flow in the space of preferred extremals might be too strong a condition. Already the restriction of the general Maxwell flow in the space of metrics to solutions of Einstein-Maxwell equations with cosmological term might lead to discretization, and the assumption about reprentability as 4-surface in $M^4 \times CP_2$ would give a further condition reducing the number of solutions. On the other hand, one might consiser a possibility of a continuous flow in the space of constant Ricci scalar metrics with a fixed 4-volume and having hyperbolic spaces as the most symmetric representative.

Dissipation, self organization, transition to chaos, and coupling constant evolution

A beautiful connection with concepts like dissipation, self-organization, transition to chaos, and coupling constant evolution suggests itself.

(a) It is not at all clear whether the vacuum extremal limits of the preferred extremals can correspond to Einstein spaces except in special cases such as CP_2 type vacuum extremals isometric with CP_2. The imbeddability condition however defines a constraint force which might well force asymptotically more complex situations such as limit cycles and strange attractors. In ordinary dissipative dynamics an external energy feed is essential prerequisite for this kind of non-trivial self-organization patterns.

In the recent case the external energy feed could be replaced by the constraint forces due to the imbeddability condition. It is not too difficult to imagine that the flow (if it exists!) could define something analogous to a transition to chaos taking place in a stepwise manner for critical values of the parameter t. Alternatively, these discrete values could correspond to those values of t for which the preferred extremal property holds true for a general Maxwell flow in the space of 4-metrics. Therefore the preferred extremals of Kähler action could emerge as one-parameter (possibly discrete) families describing dissipation and self-organization at the level of space-time dynamics.

(b) For instance, one can consider the possibility that in some situations Einstein's equations split into two mutually consistent equations of which only the first one is independent

$$\begin{aligned} xJ^{\alpha}_{\ \nu}J^{\nu\beta} &= R^{\alpha\beta} \ , \\ L_K &= xJ^{\alpha}_{\ \nu}J^{\nu\beta} = 4\Lambda \ , \\ x &= \frac{1}{16\pi\alpha_K} \ . \end{aligned} \tag{3.5.21}$$

Note that the first equation indeed gives the second one by tracing. This happens for CP_2 type vacuum extremals.

Kähler action density would reduce to cosmological constant which should have a continuous spectrum if this happens always. A more plausible alternative is that this holds true only asymptotically. In this case the flow equation could not lead arbitrary near to vacuum extremal, and one can think of situation in which $L_K = 4\Lambda$ defines an analog of limiting cycle or perhaps even strange attractor. In any case, the assumption would allow to deduce the asymptotic value of the action density which is of utmost importance from calculational point of view: action would be simply $S_K = 4\Lambda V_4$ and one could also say that one has minimal surface with Λ taking the role of string tension.

(c) One of the key ideas of TGD is quantum criticality implying that Kähler coupling strength is analogous to critical temperature. Second key idea is that p-adic coupling constant evolution represents discretized version of continuous coupling constant evolution so that each p-adic prime would correspond a fixed point of ordinary coupling constant evolution in the sense that the 4-volume characterized by the p-adic length scale remains constant. The invariance of the geometric and thus geometric parameters of hyperbolic 4-manifold under the Kähler flow would conform with the interpretation as a flow preserving scale assignable to a given p-adic prime. The continuous evolution in question (if possible at all!) might correspond to a fixed p-adic prime. Also the hierarchy of Planck constants relates to this picture naturally. Planck constant $\hbar_{eff} = n\hbar$ corresponds to a multi-furcation generating n-sheeted structure and certainly affecting the fundamental group.

(d) One can of course question the assumption that a continuous flow exists. The property of being a solution of Einstein-Maxwell equations, imbeddability property, and preferred extremal property might allow allow only discrete sequences of space-time surfaces perhaps interpretable as orbit of an iterated map leading gradually to a fractal limit. This kind of discrete sequence might be also be selected as preferred extremals from the orbit of Maxwell flow without assuming Einstein-Maxwell equations. Perhaps the discrete p-adic coupling constant evolution could be seen in this manner and be regarded as an iteration so that the connection with fractality would become obvious too.

Does a 4-D counterpart of thermodynamics make sense?

The interpretation of the Kähler flow in terms of dissipation, the constancy of R, and almost constancy of L_K suggest an interpretation in terms of 4-D variant of thermodynamics natural in zero energy ontology (ZEO), where physical states are analogs for pairs of initial and final states of quantum event are quantum superpositions of classical time evolutions. Quantum theory becomes a "square root" of thermodynamics so that 4-D analog of thermodynamics might even replace ordinary thermodynamics as a fundamental description. If so this 4-D thermodynamics should be qualitatively consistent with the ordinary 3-D thermodynamics.

(a) The first naive guess would be the interpretation of the action density L_K as an analog of energy density $e = E/V_3$ and that of R as the analog to entropy density $s = S/V_3$.

The asymptotic states would be analogs of thermodynamical equilibria having constant values of L_K and R.

(b) Apart from an overall sign factor ϵ to be discussed, the analog of the first law $de = Tds - pdV/V$ would be

$$dL_K = kdR + \Lambda\frac{dV_4}{V_4} \ .$$

One would have the correspondences $S \to \epsilon RV_4$, $e \to \epsilon L_K$ and $k \to T$, $p \to -\Lambda$. $k \propto 1/G$ indeed appears formally in the role of temperature in Einstein's action defining a formal partition function via its exponent. The analog of second law would state the increase of the magnitude of ϵRV_4 during the Kähler flow.

(c) One must be very careful with the signs and discuss Euclidian and Minkowskian regions separately. Concerning purely thermodynamic aspects at the level of vacuum functional Euclidian regions are those which matter.

  i. For CP_2 type vacuum extremals $L_K \propto E^2 + B^2$, $R = \Lambda/k$, and Λ are positive. In thermodynamical analogy for $\epsilon = 1$ this would mean that pressure is negative.

  ii. In Minkowskian regions the value of $R = \Lambda/k$ is negative for $\Lambda < 0$ suggested by the large abundance of 4-manifolds allowing hyperbolic metric and also by cosmological considerations. The asymptotic formula $L_K = 4\Lambda$ considered above suggests that also Kähler action is negative in Minkowskian regions for magnetic flux tubes dominating in TGD inspired cosmology: the reason is that the magnetic contribution to the action density $L_K \propto E^2 - B^2$ dominates.

Consider now in more detail the 4-D thermodynamics interpretation in Euclidian and Minkowskian regions assuming that the evolution by quantum jumps has Kähler flow as a space-time correlate.

(a) In Euclidian regions the choice $\epsilon = 1$ seems to be more reasonable one. In Euclidian regions $-\Lambda$ as the analog of pressure would be negative, and asymptotically (that is for CP_2 type vacuum extremals) its value would be proportional to $\Lambda \propto 1/GR^2$, where R denotes CP_2 radius defined by the length of its geodesic circle.

A possible interpretation for negative pressure is in terms of string tension effectively inducing negative pressure (note that the solutions of the Kähler-Dirac equation indeed assign a string to the wormhole contact). The analog of the second law would require the increase of RV_4 in quantum jumps. The magnitudes of L_K, R, V_4 and Λ would be reduced and approach their asymptotic values. In particular, V_4 would approach asymptotically the volume of CP_2.

(b) In Minkowskian regions Kähler action contributes to the vacuum functional a phase factor analogous to an imaginary exponent of action serving in the role of Morse function so that thermodynamics interpretation can be questioned. Despite this one can check whether thermodynamic interpretation can be considered. The choice $\epsilon = -1$ seems to be the correct choice now. $-\Lambda$ would be analogous to a negative pressure whose gradually decreases. In 3-D thermodynamics it is natural to assign negative pressure to the magnetic flux tube like structures as their effective string tension defined by the density of magnetic energy per unit length. $-R \geq 0$ would entropy and $-L_K \geq 0$ would be the analog of energy density.

$R = \Lambda/k$ and the reduction of Λ during cosmic evolution by quantum jumps suggests that the larger the volume of CD and thus of (at least) Minkowskian space-time sheet the smaller the negative value of Λ.

Assume the recent view about state function reduction explaining how the arrow of geometric time is induced by the quantum jump sequence defining experienced time [K4]. According to this view zero energy states are quantum superpositions over CDs of various size scales but with common tip, which can correspond to either the upper or lower light-like boundary of CD. The sequence of quantum jumps the gradual increase of the average size of CD in the quantum superposition and therefore that of average value of V_4. On the other hand, a gradual decrease of both $-L_K$ and $-R$ looks physically very natural. If Kähler flow describes the effect of dissipation by quantum jumps in ZEO then the space-time surfaces would gradually approach nearly vacuum extremals with constant value of entropy density $-R$ but gradually increasing 4-volume so that the analog of second law stating the increase of $-RV_4$ would hold true.

(c) The interpretation of $-R > 0$ as negentropy density assignable to entanglement is also possible and is consistent with the interpretation in terms of second law. This interpretation would only change the sign factor ϵ in the proposed formula. Otherwise the above arguments would remain as such.

3.5.4 Could Correlation Functions, S-Matrix, And Coupling Constant Evolution Be Coded The Statistical Properties Of Preferred Extremals?

How to calculate the correlation functions and coupling constant evolution has remained a basic unresolved challenge. Generalized Feynman diagrams provide a powerful vision which however does not help in practical calculations. Some big idea has been lacking.

Quantum classical correspondence states that all aspects of quantum states should have correlates in the geometry of preferred extremals. In particular, various elementary particle propagators should have a representation as properties of preferred extremals. This would allow to realize the old dream about being able to say something interesting about coupling constant evolution although it is not yet possible to calculate the M-matrices and U-matrix. The general structure of U-matrix is however understood [K91]. Hitherto everything that has been said about coupling constant evolution has been rather speculative arguments except for the general vision that it reduces to a discrete evolution defined by p-adic length scales. General first principle definitions are however much more valuable than ad hoc guesses even if the latter give rise to explicit formulas.

In quantum TGD and also at its QFT limit various correlation functions in given quantum state should code for its properties. By quantum classical correspondence these correlation functions should have counterparts in the geometry of preferred extremals. Even more: these classical counterparts for a given preferred extremal ought to be identical with the quantum correlation functions for the superposition of preferred extremals. This correspondence could be called quantum ergodicity by its analogy with ordinary ergodicity stating that the member of ensemble becomes representative of ensemble.

This principle would be a quantum generalization of ergodic theorem stating that the time evolution of a single member of ensemble represents the ensemble statistically. This symmetry principle analogous to holography might allow to fix S-matrix uniquely even in the case that the hermitian square root of the density matrix appearing in the M-matrix would lead to a breaking of quantum ergodicity as also 4-D spin glass degeneracy suggests.

This principle would allow to deduce correlation functions from the statistical properties of single preferred extremal alone using just classical intuition. Also coupling constant evolution would be coded by the statistical properties of preferred extremals. Quantum ergodicity would mean an enormous simplification since one could avoid the horrible conceptual complexities involved with the functional integrals over WCW .

This might of course be too optimistic guess. If a sub-algebra of symplectic algebra acts as gauge symmmetries of the preferred extremals in the sense that corresponding Noether charges vanish, it can quite well be that correlations functions correspond to averages for extremals belonging to single conformal equivalence class.

(a) The marvellous implication of quantum ergodicity would be that one could calculate everything solely classically using the classical intuition - the only intuition that we have. Quantum ergodicity would also solve the paradox raised by the quantum classical correspondence for momentum eigenstates. Any preferred extremal in their superposition defining momentum eigenstate should code for the momentum characterizing the superposition itself. This is indeed possible if every extremal in the superposition codes the momentum to the properties of classical correlation functions which are identical for all of them.

(b) The only manner to possibly achieve quantum ergodicity is in terms of the statistical properties of the preferred extremals. It should be possible to generalize the ergodic theorem stating that the properties of statistical ensemble are represented by single space-time evolution in the ensemble of time evolutions. Quantum superposition of classical worlds would effectively reduce to single classical world as far as classical correlation functions are considered. The notion of finite measurement resolution suggests that one must state this more precisely by adding that classical correlation functions are calculated in a given UV and IR resolutions meaning UV cutoff defined by the smallest CD and IR cutoff defined by the largest CD present.

(c) The skeptic inside me immediately argues that TGD Universe is 4-D spin glass so that this quantum ergodic theorem must be broken. In the case of the ordinary spin classes one has not only statistical average for a fixed Hamiltonian but a statistical average over Hamiltonians. There is a probability distribution over the coupling parameters appearing in the Hamiltonian. Maybe the quantum counterpart of this is needed to predict the physically measurable correlation functions.

    Could this average be an ordinary classical statistical average over quantum states with different classical correlation functions? This kind of average is indeed taken in density matrix formalism. Or could it be that the square root of thermodynamics defined by ZEO actually gives automatically rise to this average? The eigenvalues of the "hermitian square root" of the density matrix would code for components of the state characterized by different classical correlation functions. One could assign these contributions to different "phases".

(d) Quantum classical correspondence in statistical sense would be very much like holography (now individual classical state represents the entire quantum state). Quantum ergodicity would pose a rather strong constraint on quantum states. This symmetry principle could actually fix the spectrum of zero energy states to a high degree and fix therefore the M-matrices given by the product of hermitian square root of density matrix and unitary S-matrix and unitary U-matrix constructible as inner products of M-matrices associated with CDs with various size scales [K91].

(e) In TGD inspired theory of consciousness the counterpart of quantum ergodicity is the postulate that the space-time geometry provides a symbolic representation for the quantum states and also for the contents of consciousness assignable to quantum jumps between quantum states. Quantum ergodicity would realize this strongly self-referential looking condition. The positive and negative energy parts of zero energy state would be analogous to the initial and final states of quantum jump and the classical correlation functions would code for the contents of consciousness like written formulas code for the thoughts of mathematician and provide a sensory feedback.

How classical correlation functions should be defined?

(a) General Coordinate Invariance and Lorentz invariance are the basic constraints on the definition. These are achieved for the space-time regions with Minkowskian signature and 4-D M^4 projection if linear Minkowski coordinates are used. This is equivalent with the contraction of the indices of tensor fields with the space-time projections of M^4 Killing vector fields representing translations. Accepting ths generalization, there is no need to restrict oneself to 4-D M^4 projection and one can also consider also Euclidian regions identifiable as lines of generalized Feynman diagrams.

    Quantum ergodicity very probably however forces to restrict the consideration to Minkowskian and Euclidian space-time regions and various phases associated with them. Also CP_2 Killing vector fields can be projected to space-time surface and give a representation for classical gluon fields. These in turn can be contracted with M^4 Killing vectors giving rise to gluon fields as analogs of graviton fields but with second polarization index replaced with color index.

(b) The standard definition for the correlation functions associated with classical time evolution is the appropriate starting point. The correlation function $G_{XY}(\tau)$ for two dynamical variables $X(t)$ and $Y(t)$ is defined as the average $G_{XY}(\tau) = \int_T X(t)Y(t+\tau)dt/T$ over an interval of length T, and one can also consider the limit $T \rightarrow \infty$. In the recent case one would replace τ with the difference $m_1 - m_2 = m$ of M^4 coordinates of two points at the preferred extremal and integrate over the points of the extremal to get the average. The finite time interval T is replaced with the volume of causal diamond in a given length scale. Zero energy state with given quantum numbers for positive and negative energy parts of the state defines the initial and final states between which the fields appearing in the correlation functions are defined.

(c) What correlation functions should be considered? Certainly one could calculate correlation functions for the induced spinor connection given electro-weak propagators and correlation functions for CP_2 Killing vector fields giving correlation functions for gluon fields using the description in terms of Killing vector fields. If one can uniquely separate from the Fourier transform uniquely a term of form $Z/(p^2 - m^2)$ by its momentum dependence, the coefficient Z can be identified as coupling constant squared for the corresponding gauge potential component and one can in principle deduce coupling constant evolution purely classically. One can imagine of calculating spinorial propagators for string world sheets in the same manner. Note that also the dependence on color quantum numbers would be present so that in principle all that is needed could be calculated for a single preferred extremal without the need to construct QFT limit and to introduce color quantum numbers of fermions as spin like quantum numbers (color quantum numbers corresponds to CP_2 partial wave for the tip of the CD assigned with the particle).

Many detailed speculations about coupling constant evolution to be discussed in the sections below must be taken as innovative guesses doomed to have the eventual fate of guesses. The notion of quantum ergodicity could however be one of the really deep ideas about coupling constant evolution comparable to the notion of p-adic coupling constant evolution. Quantum Ergodicity (briefly QE) would also state something extremely non-trivial also about the construction of correlation functions and S-matrix. Because this principle is so new, the rest of the chapter does not yet contain any applications of QE. This should not lead the reader to under-estimate the potential power of QE.

3.6 About Deformations Of Known Extremals Of Kähler Action

I have done a considerable amount of speculative guesswork to identify what I have used to call preferred extremals of Kähler action. The difficulty is that the mathematical problem at hand is extremely non-linear and that I do not know about existing mathematical literature relevant to the situation. One must proceed by trying to guess the general constraints on the preferred extremals which look physically and mathematically plausible. The hope is that this net of constraints could eventually chrystallize to Eureka! Certainly the recent speculative picture involves also wrong guesses. The need to find explicit ansatz for the deformations of known extremals based on some common principles has become pressing. The following considerations represent an attempt to combine the existing information to achieve this.

3.6.1 What Might Be The Common Features Of The Deformations Of Known Extremals

The dream is to discover the deformations of all known extremals by guessing what is common to all of them. One might hope that the following list summarizes at least some common features.

Effective three-dimensionality at the level of action

(a) Holography realized as effective 3-dimensionality also at the level of action requires that it reduces to 3-dimensional effective boundary terms. This is achieved if the contraction $j^\alpha A_\alpha$ vanishes. This is true if j^α vanishes or is light-like, or if it is proportional to instanton current in which case current conservation requires that CP_2 projection of the space-time surface is 3-dimensional. The first two options for j have a realization for known extremals. The status of the third option - proportionality to instanton current - has remained unclear.

(b) As I started to work again with the problem, I realized that instanton current could be replaced with a more general current $j = *B \wedge J$ or concretely: $j^\alpha = \epsilon^{\alpha\beta\gamma\delta} B_\beta J_{\gamma\delta}$, where B is vector field and CP_2 projection is 3-dimensional, which it must be in any case. The contractions of j appearing in field equations vanish automatically with this ansatz.

(c) Almost topological QFT property in turn requires the reduction of effective boundary terms to Chern-Simons terms: this is achieved by boundary conditions expressing weak form of electric magnetic duality. If one generalizes the weak form of electric-magnetic duality to $J = \Phi * J$ one has $B = d\Phi$ and j has a vanishing divergence for 3-D CP_2 projection. This is clearly a more general solution ansatz than the one based on proportionality of j with instanton current and would reduce the field equations in concise notation to $Tr(TH^k) = 0$.

(d) Any of the alternative properties of the Kähler current implies that the field equations reduce to $Tr(TH^k) = 0$, where T and H^k are shorthands for Maxwellian energy momentum tensor and second fundamental form and the product of tensors is obvious generalization of matrix product involving index contraction.

Could Einstein's equations emerge dynamically?

For j^α satisfying one of the three conditions, the field equations have the same form as the equations for minimal surfaces except that the metric g is replaced with Maxwell energy momentum tensor T.

(a) This raises the question about dynamical generation of small cosmological constant Λ: $T = \Lambda g$ would reduce equations to those for minimal surfaces. For $T = \Lambda g$ Kähler-Dirac gamma matrices would reduce to induced gamma matrices and the Kähler-Dirac operator would be proportional to ordinary Dirac operator defined by the induced gamma matrices. One can also consider weak form for $T = \Lambda g$ obtained by restricting the consideration to a sub-space of tangent space so that space-time surface is only "partially" minimal surface but this option is not so elegant although necessary for other than CP_2 type vacuum extremals.

(b) What is remarkable is that $T = \Lambda g$ implies that the divergence of T which in the general case equals to $j^\beta J^\alpha_\beta$ vanishes. This is guaranteed by one of the conditions for the Kähler current. Since also Einstein tensor has a vanishing divergence, one can ask whether the condition to $T = \kappa G + \Lambda g$ could the general condition. This would give Einstein's equations with cosmological term besides the generalization of the minimal surface equations. GRT would emerge dynamically from the non-linear Maxwell's theory although in slightly different sense as conjectured [K79] ! Note that the expression for G involves also second derivatives of the imbedding space coordinates so that actually a partial differential equation is in question. If field equations reduce to purely algebraic ones, as the basic conjecture states, it is possible to have $Tr(GH^k) = 0$ and $Tr(gH^k) = 0$ separately so that also minimal surface equations would hold true.

What is amusing that the first guess for the action of TGD was curvature scalar. It gave analogs of Einstein's equations as a definition of conserved four-momentum currents. The recent proposal would give the analog of ordinary Einstein equations as a dynamical constraint relating Maxwellian energy momentum tensor to Einstein tensor and metric.

(c) Minimal surface property is physically extremely nice since field equations can be interpreted as a non-linear generalization of massless wave equation: something very natural for non-linear variant of Maxwell action. The theory would be also very "stringy" although the fundamental action would not be space-time volume. This can however hold true only for Euclidian signature. Note that for CP_2 type vacuum extremals Einstein tensor is proportional to metric so that for them the two options are equivalent. For their small deformations situation changes and it might happen that the presence of G is necessary. The GRT limit of TGD discussed in [K79] [L10] indeed suggests that CP_2 type solutions satisfy Einstein's equations with large cosmological constant and that the small observed value of the cosmological constant is due to averaging and small volume fraction of regions of Euclidian signature (lines of generalized Feynman diagrams).

(d) For massless extremals and their deformations $T = \Lambda g$ cannot hold true. The reason is that for massless extremals energy momentum tensor has component T^{vv} which actually quite essential for field equations since one has $H^k_{vv} = 0$. Hence for massless extremals and their deformations $T = \Lambda g$ cannot hold true if the induced metric has Hamilton-Jacobi structure meaning that g^{uu} and g^{vv} vanish. A more general relationship of form $T = \kappa G + \Lambda G$ can however be consistent with non-vanishing T^{vv} but require that deformation has at most 3-D CP_2 projection (CP_2 coordinates do not depend on v).

(e) The non-determinism of vacuum extremals suggest for their non-vacuum deformations a conflict with the conservation laws. In, also massless extremals are characterized by a non-determinism with respect to the light-like coordinate but like-likeness saves the situation. This suggests that the transformation of a properly chosen time coordinate of vacuum extremal to a light-like coordinate in the induced metric combined with Einstein's equations in the induced metric of the deformation could allow to handle the non-determinism.

Are complex structure of CP_2 and Hamilton-Jacobi structure of M^4 respected by the deformations?

The complex structure of CP_2 and Hamilton-Jacobi structure of M^4 could be central for the understanding of the preferred extremal property algebraically.

(a) There are reasons to believe that the Hermitian structure of the induced metric ((1, 1) structure in complex coordinates) for the deformations of CP_2 type vacuum extremals could be crucial property of the preferred extremals. Also the presence of light-like direction is also an essential elements and 3-dimensionality of M^4 projection could be essential. Hence a good guess is that allowed deformations of CP_2 type vacuum extremals are such that (2, 0) and (0, 2) components the induced metric and/or of the energy momentum tensor vanish. This gives rise to the conditions implying Virasoro conditions in string models in quantization:

$$g_{\xi^i\xi^j} = 0 \ , \quad g_{\overline{\xi}^i\overline{\xi}^j} = 0 \ , \quad i,j = 1,2 \ . \tag{3.6.1}$$

Holomorphisms of CP_2 preserve the complex structure and Virasoro conditions are expected to generalize to 4-dimensional conditions involving two complex coordinates. This means that the generators have two integer valued indices but otherwise obey an algebra very similar to the Virasoro algebra. Also the super-conformal variant of this algebra is expected to make sense.

These Virasoro conditions apply in the coordinate space for CP_2 type vacuum extremals. One expects similar conditions hold true also in field space, that is for M^4 coordinates.

(b) The integrable decomposition $M^4(m) = M^2(m)+E^2(m)$ of M^4 tangent space to longitudinal and transversal parts (non-physical and physical polarizations) - Hamilton-Jacobi structure- could be a very general property of preferred extremals and very natural since non-linear Maxwellian electrodynamics is in question. This decomposition led rather early to the introduction of the analog of complex structure in terms of what I called

Hamilton-Jacobi coordinates $(u, v, w, \overline{w})$ for M^4. (u, v) defines a pair of light-like coordinates for the local longitudinal space $M^2(m)$ and $(w, \overline{w})$ complex coordinates for $E^2(m)$. The metric would not contain any cross terms between $M^2(m)$ and $E^2(m)$: $g_{uw} = g_{vw} = g_{u\overline{w}} = g_{v\overline{w}} = 0$.

A good guess is that the deformations of massless extremals respect this structure. This condition gives rise to the analog of the constraints leading to Virasoro conditions stating the vanishing of the non-allowed components of the induced metric. $g_{uu} = g_{vv} = g_{ww} = g_{\overline{w}\overline{w}} = g_{uw} = g_{vw} = g_{u\overline{w}} = g_{v\overline{w}} = 0$. Again the generators of the algebra would involve two integers and the structure is that of Virasoro algebra and also generalization to super algebra is expected to make sense. The moduli space of Hamilton-Jacobi structures would be part of the moduli space of the preferred extremals and analogous to the space of all possible choices of complex coordinates. The analogs of infinitesimal holomorphic transformations would preserve the modular parameters and give rise to a 4-dimensional Minkowskian analog of Virasoro algebra. The conformal algebra acting on CP_2 coordinates acts in field degrees of freedom for Minkowskian signature.

Field equations as purely algebraic conditions

If the proposed picture is correct, field equations would reduce basically to purely algebraically conditions stating that the Maxwellian energy momentum tensor has no common index pairs with the second fundamental form. For the deformations of CP_2 type vacuum extremals T is a complex tensor of type (1, 1) and second fundamental form H^k a tensor of type (2, 0) and (0, 2) so that $Tr(TH^k) =$ is true. This requires that second light-like coordinate of M^4 is constant so that the M^4 projection is 3-dimensional. For Minkowskian signature of the induced metric Hamilton-Jacobi structure replaces conformal structure. Here the dependence of CP_2 coordinates on second light-like coordinate of $M^2(m)$ only plays a fundamental role. Note that now T^{vv} is non-vanishing (and light-like). This picture generalizes to the deformations of cosmic strings and even to the case of vacuum extremals.

3.6.2 What Small Deformations Of CP_2 Type Vacuum Extremals Could Be?

I was led to these arguments when I tried find preferred extremals of Kähler action, which would have 4-D CP_2 and M^4 projections - the Maxwell phase analogous to the solutions of Maxwell's equations that I conjectured long time ago. It however turned out that the dimensions of the projections can be $(D_{M^4} \leq 3, D_{CP_2} = 4)$ or $(D_{M^4} = 4, D_{CP_2} \leq 3)$. What happens is essentially breakdown of linear superposition so that locally one can have superposition of modes which have 4-D wave vectors in the same direction. This is actually very much like quantization of radiation field to photons now represented as separate space-time sheets and one can say that Maxwellian superposition corresponds to union of separate photonic space-time sheets in TGD.

Approximate linear superposition of fields is fundamental in standard physics framework and is replaced in TGD with a linear superposition of effects of classical fields on a test particle topologically condensed simultaneously to several space-time sheets. One can say that linear superposition is replaced with a disjoint union of space-time sheets. In the following I shall restrict the consideration to the deformations of CP_2 type vacuum extremals.

Solution ansatz

I proceed by the following arguments to the ansatz.

(a) Effective 3-dimensionality for action (holography) requires that action decomposes to vanishing $j^\alpha A_\alpha$ term + total divergence giving 3-D "boundary" terms. The first term certainly vanishes (giving effective 3-dimensionality) for

$$D_\beta J^{\alpha\beta} = j^\alpha = 0 \ .$$

Empty space Maxwell equations, something extremely natural. Also for the proposed GRT limit these equations are true.

(b) How to obtain empty space Maxwell equations $j^\alpha = 0$? The answer is simple: assume self duality or its slight modification:

$$J = *J$$

holding for CP_2 type vacuum extremals or a more general condition

$$J = k * J \ ,$$

In the simplest situation k is some constant not far from unity. * is Hodge dual involving 4-D permutation symbol. $k = constant$ requires that the determinant of the induced metric is apart from constant equal to that of CP_2 metric. It does not require that the induced metric is proportional to the CP_2 metric, which is not possible since M^4 contribution to metric has Minkowskian signature and cannot be therefore proportional to CP_2 metric.

One can consider also a more general situation in which k is scalar function as a generalization of the weak electric-magnetic duality. In this case the Kähler current is non-vanishing but divergenceless. This also guarantees the reduction to $Tr(TH^k) = 0$. In this case however the proportionality of the metric determinant to that for CP_2 metric is not needed. This solution ansatz becomes therefore more general.

(c) Field equations reduce with these assumptions to equations differing from minimal surfaces equations only in that metric g is replaced by Maxwellian energy momentum tensor T. Schematically:

$$Tr(TH^k) = 0 \ ,$$

where T is the Maxwellian energy momentum tensor and H^k is the second fundamental form - asymmetric 2-tensor defined by covariant derivative of gradients of imbedding space coordinates.

How to satisfy the condition $Tr(TH^k) = 0$?

It would be nice to have minimal surface equations since they are the non-linear generalization of massless wave equations. It would be also nice to have the vanishing of the terms involving Kähler current in field equations as a consequence of this condition. Indeed, $T = \kappa G + \Lambda g$ implies this. In the case of CP_2 vacuum extremals one cannot distinguish between these options since CP_2 itself is constant curvature space with $G \propto g$. Furthermore, if G and g have similar tensor structure the algebraic field equations for G and g are satisfied separately so that one obtains minimal surface property also now. In the following minimal surface option is considered.

(a) The first opton is achieved if one has

$$T = \Lambda g \ .$$

Maxwell energy momentum tensor would be proportional to the metric! One would have dynamically generated cosmological constant! This begins to look really interesting since it appeared also at the proposed GRT limit of TGD [L10] (see `http://tinyurl.com/hzkldnb`). Note that here also non-constant value of Λ can be considered and would correspond to a situation in which k is scalar function: in this case the the determinant condition can be dropped and one obtains just the minimal surface equations.

(b) Very schematically and forgetting indices and being sloppy with signs, the expression for T reads as

$$T = JJ - g/4Tr(JJ) \ .$$

Note that the product of tensors is obtained by generalizing matrix product. This should be proportional to metric.

Self duality implies that $Tr(JJ)$ is just the instanton density and does not depend on metric and is constant.

For CP_2 type vacuum extremals one obtains

$$T = -g + g = 0 \ .$$

Cosmological constant would vanish in this case.

(c) Could it happen that for deformations a small value of cosmological constant is generated?

The condition would reduce to

$$JJ = (\Lambda - 1)g \ .$$

Λ must relate to the value of parameter k appearing in the generalized self-duality condition. For the most general ansatz Λ would not be constant anymore.

This would generalize the defining condition for Kähler form

$$JJ = -g \ \ (i^2 = -1 \ \ geometrically)$$

stating that the square of Kähler form is the negative of metric. The only modification would be that index raising is carried out by using the induced metric containing also M^4 contribution rather than CP_2 metric.

(d) Explicitly:

$$J_{\alpha\mu}J^{\mu}_{\ \beta} = (\Lambda - 1)g_{\alpha\beta} \ .$$

Cosmological constant would measure the breaking of Kähler structure. By writing $g = s + m$ and defining index raising of tensors using CP_2 metric and their product accordingly, this condition can be also written as

$$Jm = (\Lambda - 1)mJ \ .$$

If the parameter k is constant, the determinant of the induced metric must be proportional to the CP_2 metric. If k is scalar function, this condition can be dropped. Cosmological constant would not be constant anymore but the dependence on k would drop out from the field equations and one would hope of obtaining minimal surface equations also now. It however seems that the dimension of M^4 projection cannot be four. For 4-D M^4 projection the contribution of the M^2 part of the M^4 metric gives a non-holomorphic contribution to CP_2 metric and this spoils the field equations.

For $T = \kappa G + \Lambda g$ option the value of the cosmological constant is large - just as it is for the proposed GRT limit of TGD [K79] [L10]. The interpretation in this case is that the average value of cosmological constant is small since the portion of space-time volume containing generalized Feynman diagrams is very small.

More detailed ansatz for the deformations of CP_2 type vacuum extremals

One can develop the ansatz to a more detailed form. The most obvious guess is that the induced metric is apart from constant conformal factor the metric of CP_2. This would guarantee self-duality apart from constant factor and $j^\alpha = 0$. Metric would be in complex CP_2 coordinates tensor of type (1, 1) whereas CP_2 Riemann connection would have only purely holomorphic or anti-holomorphic indices. Therefore CP_2 contributions in $Tr(TH^k)$ would vanish identically. M^4 degrees of freedom however bring in difficulty. The M^4 contribution to the induced metric should be proportional to CP_2 metric and this is impossible due to the different signatures. The M^4 contribution to the induced metric breaks its Kähler property but would preserve Hermitian structure.

A more realistic guess based on the attempt to construct deformations of CP_2 type vacuum extremals is following.

(a) Physical intuition suggests that M^4 coordinates can be chosen so that one has integrable decomposition to longitudinal degrees of freedom parametrized by two light-like coordinates u and v and to transversal polarization degrees of freedom parametrized by complex coordinate w and its conjugate. M^4 metric would reduce in these coordinates to a direct sum of longitudinal and transverse parts. I have called these coordinates Hamilton-Jacobi coordinates.

(b) w would be holomorphic function of CP_2 coordinates and therefore satisfy the analog of massless wave equation. This would give hopes about rather general solution ansatz. u and v cannot be holomorphic functions of CP_2 coordinates. Unless wither u or v is constant, the induced metric would receive contributions of type (2, 0) and (0, 2) coming from u and v which would break Kähler structure and complex structure. These contributions would give no-vanishing contribution to all minimal surface equations. Therefore either u or v is constant: the coordinate line for non-constant coordinate -say u- would be analogous to the M^4 projection of CP_2 type vacuum extremal.

(c) With these assumptions the induced metric would remain $(1,1)$ tensor and one might hope that $Tr(TH^k)$ contractions vanishes for all variables except u because the there are no common index pairs (this if non-vanishing Christoffel symbols for H involve only holomorphic or anti-holomorphic indices in CP_2 coordinates). For u one would obtain massless wave equation expressing the minimal surface property.

(d) If the value of k is constant the determinant of the induced metric must be proportional to the determinant of CP_2 metric. The induced metric would contain only the contribution from the transversal degrees of freedom besides CP_2 contribution. Minkowski contribution has however rank 2 as CP_2 tensor and cannot be proportional to CP_2 metric. It is however enough that its determinant is proportional to the determinant of CP_2 metric with constant proportionality coefficient. This condition gives an additional non-linear condition to the solution. One would have wave equation for u (also w and its conjugate satisfy massless wave equation) and determinant condition as an additional condition.

    The determinant condition reduces by the linearity of determinant with respect to its rows to sum of conditions involved 0, 1, 2 rows replaced by the transversal M^4 contribution to metric given if M^4 metric decomposes to direct sum of longitudinal and transversal parts. Derivatives with respect to derivative with respect to particular CP_2 complex coordinate appear linearly in this expression they can depend on u via the dependence of transversal metric components on u. The challenge is to show that this equation has (or does not have) non-trivial solutions.

(e) If the value of k is scalar function the situation changes and one has only the minimal surface equations and Virasoro conditions.

What makes the ansatz attractive is that special solutions of Maxwell empty space equations are in question, equations reduces to non-linear generalizations of Euclidian massless wave equations, and possibly space-time dependent cosmological constant pops up dynamically. These properties are true also for the GRT limit of TGD [L10] (see `http://tinyurl.com/hzkldnb`).

3.6.3 Hamilton-Jacobi Conditions In Minkowskian Signature

The maximally optimistic guess is that the basic properties of the deformations of CP_2 type vacuum extremals generalize to the deformations of other known extremals such as massless extremals, vacuum extremals with 2-D CP_2 projection which is Lagrangian manifold, and cosmic strings characterized by Minkowskian signature of the induced metric. These properties would be following.

(a) The recomposition of M^4 tangent space to longitudinal and transversal parts giving Hamilton-Jacobi structure. The longitudinal part has hypercomplex structure but the second light-like coordinate is constant: this plays a crucial role in guaranteeing the vanishing of contractions in $Tr(TH^k)$. It is the algebraic properties of g and T which are crucial. T can however have light-like component T^{vv}. For the deformations of CP_2 type vacuum extremals $(1,1)$ structure is enough and is guaranteed if second light-like coordinate of M^4 is constant whereas w is holomorphic function of CP_2 coordinates.

(b) What could happen in the case of massless extremals? Now one has 2-D CP_2 projection in the initial situation and CP_2 coordinates depend on light-like coordinate u and single real transversal coordinate. The generalization would be obvious: dependence on single light-like coordinate u and holomorphic dependence on w for complex CP_2 coordinates. The constraint is $T = \Lambda g$ cannot hold true since T^{vv} is non-vanishing (and light-like). This property restricted to transversal degrees of freedom could reduce the field equations to minimal surface equations in transversal degrees of freedom. The transversal part of energy momentum tensor would be proportional to metric and hence covariantly constant. Gauge current would remain light-like but would not be given by $j = *d\phi \wedge J$. $T = \kappa G + \Lambda g$ seems to define the attractive option.

It therefore seems that the essential ingredient could be the condition

$$T = \kappa G + \lambda g \ ,$$

which has structure (1, 1) in both $M^2(m)$ and $E^2(m)$ degrees of freedom apart from the presence of T^{vv} component with deformations having no dependence on v. If the second fundamental form has (2, 0)+(0, 2) structure, the minimal surface equations are satisfied provided Kähler current satisfies on of the proposed three conditions and if G and g have similar tensor structure.

One can actually pose the conditions of metric as complete analogs of stringy constraints leading to Virasoro conditions in quantization to give

$$g_{uu} = 0 \ , \quad g_{vv} = 0 \ , \quad g_{ww} = 0 \ , \quad g_{\overline{ww}} = 0 \ . \tag{3.6.2}$$

This brings in mind the generalization of Virasoro algebra to four-dimensional algebra for which an identification in terms of non-local Yangian symmetry [A27] [B39, B30, B31] has been proposed [K76]. The number of conditions is four and the same as the number of independent field equations. One can consider similar conditions also for the energy momentum tensor T but allowing non-vanishing component T^{vv} if deformations has no v-dependence. This would solve the field equations if the gauge current vanishes or is light-like. On this case the number of equations is 8. First order differential equations are in question and they can be also interpreted as conditions fixing the coordinates used since there is infinite number of manners to choose the Hamilton-Jacobi coordinates.

One can can try to apply the physical intuition about general solutions of field equations in the linear case by writing the solution as a superposition of left and right propagating solutions:

$$\xi^k = f_+^k(u,w) + f_+^k(v,w) \ . \tag{3.6.3}$$

This could guarantee that second fundamental form is of form (2, 0)+(0, 2) in both M^2 and E^2 part of the tangent space and these terms if $Tr(TH^k)$ vanish identically. The remaining terms involve contractions of T^{uw}, $T^{u\overline{w}}$ and T^{vw}, $T^{v\overline{w}}$ with second fundamental form. Also these terms should sum up to zero or vanish separately. Second fundamental form has components coming from f_+^k and f_-^k

Second fundamental form H^k has as basic building bricks terms $\hat{H}^k$ given by

$$\hat{H}^k_{\alpha\beta} \quad = \quad \partial_\alpha\partial_\beta h^k + \left({k \atop l\ m}\right)\partial_\alpha h^l \partial_\beta h^m \quad . \tag{3.6.4}$$

For the proposed ansatz the first terms give vanishing contribution to H^k_{uv}. The terms containing Christoffel symbols however give a non-vanishing contribution and one can allow only f_+^k or f_-^k as in the case of massless extremals. This reduces the dimension of CP_2 projection to $D = 3$.

What about the condition for Kähler current? Kähler form has components of type $J_{w\overline{w}}$ whose contravariant counterpart gives rise to space-like current component. J_{uw} and $J_{u\overline{w}}$ give rise to light-like currents components. The condition would state that the $J^{w\overline{w}}$ is covariantly constant. Solutions would be characterized by a constant Kähler magnetic field. Also electric field is represent. The interpretation both radiation and magnetic flux tube makes sense.

3.6.4 Deformations Of Cosmic Strings

In the physical applications it has been assumed that the thickening of cosmic strings to Kähler magnetic flux tubes takes place. One indeed expects that the proposed construction generalizes also to the case of cosmic strings having the decomposition $X^4 = X^2 \times Y^2 \subset M^4 \times CP_2$, where X^2 is minimal surface and Y^2 a complex homologically non-trivial submanifold of CP_2. Now the starting point structure is Hamilton-Jacobi structure for $M^2_m \times Y^2$ defining the coordinate space.

(a) The deformation should increase the dimension of either CP_2 or M^4 projection or both. How this thickening could take place? What comes in mind that the string orbits X^2 can be interpreted as a distribution of longitudinal spaces $M^2(x)$ so that for the deformation w coordinate becomes a holomorphic function of the natural Y^2 complex coordinate so that M^4 projection becomes 4-D but CP_2 projection remains 2-D. The new contribution to the X^2 part of the induced metric is vanishing and the contribution to the Y^2 part is of type $(1, 1)$ and the ansatz $T = \kappa G + \Lambda g$ might be needed as a generalization of the minimal surface equations The ratio of κ and G would be determined from the form of the Maxwellian energy momentum tensor and be fixed at the limit of undeformed cosmic strong to $T = (ag(Y^2) - bg(Y^2)$. The value of cosmological constant is now large, and overall consistency suggests that $T = \kappa G + \Lambda g$ is the correct option also for the CP_2 type vacuum extremals.

(b) One could also imagine that remaining CP_2 coordinates could depend on the complex coordinate of Y^2 so that also CP_2 projection would become 4-dimensional. The induced metric would receive holomorphic contributions in Y^2 part. As a matter fact, this option is already implied by the assumption that Y^2 is a complex surface of CP_2.

3.6.5 Deformations Of Vacuum Extremals?

What about the deformations of vacuum extremals representable as maps from M^4 to CP_2?

(a) The basic challenge is the non-determinism of the vacuum extremals. One should perform the deformation so that conservation laws are satisfied. For massless extremals there is also non-determinism but it is associated with the light-like coordinate so that there are no problems with the conservation laws. This would suggest that a properly chosen time coordinate consistent with Hamilton-Jacobi decomposition becomes light-like coordinate in the induced metric. This poses a conditions on the induced metric.

(b) Physical intuition suggests that one cannot require $T = \Lambda g$ since this would mean that the rank of T is maximal whereas the original situation corresponds to the vanishing of T. For small deformations rank two for T looks more natural and one could think that T is proportional to a projection of metric to a 2-D subspace. The vision about the long length scale limit of TGD is that Einstein's equations are satisfied and this would suggest $T = kG$ or $T = \kappa G + \Lambda g$. The rank of T could be smaller than four for this ansatz and this conditions binds together the values of κ and G.

(c) These extremals have CP_2 projection which in the generic case is 2-D Lagrangian sub-manifold Y^2. Again one could assume Hamilton-Jacobi coordinates for X^4. For CP_2 one could assume Darboux coordinates (P_i, Q_i), $i = 1, 2$, in which one has $A = P_i dQ^i$, and that $Y^2 \subset CP_2$ corresponds to $Q_i = constant$. In principle P_i would depend on arbitrary manner on M^4 coordinates. It might be more convenient to use as coordinates (u, v) for M^2 and (P_1, P_2) for Y^2. This covers also the situation when M^4 projection is not 4-D. By its 2-dimensionality Y^2 allows always a complex structure defined by its induced metric: this complex structure is not consistent with the complex structure of CP_2 (Y^2 is not complex sub-manifold).

Using Hamilton-Jacobi coordinates the pre-image of a given point of Y^2 is a 2-dimensional sub-manifold X^2 of X^4 and defines also 2-D sub-manifold of M^4. The following picture suggests itself. The projection of X^2 to M^4 can be seen for a suitable choice of Hamilton-Jacobi coordinates as an analog of Lagrangian sub-manifold in M^4 that is as surface for which v and $Im(w)$ vary and u and $Re(w)$ are constant. X^2 would be obtained by allowing u and $Re(w)$ to vary: as a matter fact, (P_1, P_2) and $(u, Re(w))$ would be related to each other. The induced metric should be consistent with this picture. This would requires $g_{uRe(w)} = 0$.

For the deformations Q_1 and Q_2 would become non-constant and they should depend on the second light-like coordinate v only so that only g_{uu} and g_{uw} and $gu\overline{w}$ $g_{w,w}$ and $g_{\overline{w},\overline{w}}$ receive contributions which vanish. This would give rise to the analogs of Virasoro conditions guaranteeing that T is a tensor of form $(1,1)$ in both M^2 and E^2 indices and that there are no cross components in the induced metric. A more general formulation states that energy momentum tensor satisfies these conditions. The conditions on T might be equivalent with the conditions for g and G separately.

(d) Einstein's equations provide an attractive manner to achieve the vanishing of effective 3-dimensionality of the action. Einstein equations would be second order differential equations and the idea that a deformation of vacuum extremal is in question suggests that the dynamics associated with them is in directions transversal to Y^2 so that only the deformation is dictated partially by Einstein's equations.

(e) Lagrangian manifolds do not involve complex structure in any obvious manner. One could however ask whether the deformations could involve complex structure in a natural manner in CP_2 degrees of freedom so that the vanishing of g_{ww} would be guaranteed by holomorphy of CP_2 complex coordinate as function of w.

One should get the complex structure in some natural manner: in other words, the complex structure should relate to the geometry of CP_2 somehow. The complex coordinate defined by say $z = P_1 + iQ^1$ for the deformation suggests itself. This would suggest that at the limit when one puts $Q_1 = 0$ one obtains $P_1 = P_1(Re(w))$ for the vacuum extremals and the deformation could be seen as an analytic continuation of real function to region of complex plane. This is in spirit with the algebraic approach. The vanishing of Kähler current requires that the Kähler magnetic field is covariantly constant: $D_z J^{z\overline{z}} = 0$ and $D_{\overline{z}} J^{z\overline{z}} = 0$.

(f) One could consider the possibility that the resulting 3-D sub-manifold of CP_2 can be regarded as contact manifold with induced Kähler form non-vanishing in 2-D section with natural complex coordinates. The third coordinate variable- call it s- of the contact manifold and second coordinate of its transversal section would depend on time space-time coordinates for vacuum extremals. The coordinate associated with the transversal section would be continued to a complex coordinate which is holomorphic function of w and u.

(g) The resulting thickened magnetic flux tubes could be seen as another representation of Kähler magnetic flux tubes: at this time as deformations of vacuum flux tubes rather than cosmic strings. For this ansatz it is however difficult to imagine deformations carrying Kähler electric field.

3.6.6 About The Interpretation Of The Generalized Conformal Algebras

The long-standing challenge has been finding of the direct connection between the super-conformal symmetries assumed in the construction of the geometry of the "world of classical worlds" (WCW) and possible conformal symmetries of field equations. 4-dimensionality and Minkowskian signature have been the basic problems. The recent construction provides new insights to this problem.

(a) In the case of string models the quantization of the Fourier coefficients of coordinate variables of the target space gives rise to Kac-Moody type algebra and Virasoro algebra generators are quadratic in these. Also now Kac-Moody type algebra is expected. If one were to perform a quantization of the coefficients in Laurents series for complex CP_2 coordinates, one would obtain interpretation in terms of $su(3) = u(2)+t$ decomposition, where t corresponds to CP_3: the oscillator operators would correspond to generators in t and their commutator would give generators in $u(2)$. SU(3)/SU(2) coset representation for Kac-Moody algebra would be in question. Kac-Moody algebra would be associated with the generators in both M^4 and CP_2 degrees of freedom. This kind of Kac-Moody algebra appears in quantum TGD.

(b) The constraints on induced metric imply a very close resemblance with string models and a generalization of Virasoro algebra emerges. An interesting question is how the two algebras acting on coordinate and field degrees of freedom relate to the super-conformal algebras defined by the symplectic group of $\delta M^4_+ \times CP_2$ acting on space-like 3-surfaces at boundaries of CD and to the Kac-Moody algebras acting on light-like 3-surfaces. It has been conjectured that these algebras allow a continuation to the interior of space-time surface made possible by its slicing by 2-surfaces parametrized by 2-surfaces. The proposed construction indeed provides this kind of slicings in both M^4 and CP_2 factor.

(c) In the recent case, the algebras defined by the Fourier coefficients of field variables would be Kac-Moody algebras. Virasoro algebra acting on preferred coordinates would be expressed in terms of the Kac-Moody algebra in the standard Sugawara construction applied in string models. The algebra acting on field space would be analogous to the conformal algebra assignable to the symplectic algebra so that also symplectic algebra is present. Stringy pragmatist could imagine quantization of symplectic algebra by replacing CP_2 coordinates in the expressions of Hamiltonians with oscillator operators. This description would be counterpart for the construction of spinor harmonics in WCW and might provide some useful insights.

(d) For given type of space-time surface either CP_2 or M^4 corresponds to Kac-Moody algebra but not both. From the point of view of quantum TGD it looks as that something were missing. An analogous problem was encountered at GRT limit of TGD [L10]. When Euclidian space-time regions are allowed Einstein-Maxwell action is able to mimic standard model with a surprising accuracy but there is a problem: one obtains either color charges or M^4 charges but not both. Perhaps it is not enough to consider either CP_2 type vacuum extremal or its exterior but both to describe particle: this would give the direct product of the Minkowskian and Euclidian algebras acting on tensor product. This does not however seem to be consistent with the idea that the two descriptions are duality related (the analog of T-duality).

3.7 Appendix: Hamilton-Jacobi Structure

In the following the definition of Hamilton-Jacobi structure is discussed in detail.

3.7.1 Hermitian And Hyper-Hermitian Structures

The starting point is the observation that besides the complex numbers forming a number field there are hyper-complex numbers. Imaginary unit i is replaced with e satisfying $e^2 = 1$. One obtains an algebra but not a number field since the norm is Minkowskian norm $x^2 - y^2$, which vanishes at light-cone $x = y$ so that light-like hypercomplex numbers $x \pm e$) do not have inverse. One has "almost" number field.

Hyper-complex numbers appear naturally in 2-D Minkowski space since the solutions of a massless field equation can be written as $f = g(u = t-ex)+h(v = t+ex)$ whith $e^2 = 1$ realized by putting $e = 1$. Therefore Wick rotation relates sums of holomorphic and antiholomorphic functions to sums of hyper-holomorphic and anti-hyper-holomorphic functions. Note that u and v are hyper-complex conjugates of each other.

Complex n-dimensional spaces allow Hermitian structure. This means that the metric has in complex coordinates $(z_1, ..., z_n)$ the form in which the matrix elements of metric are non-vanishing only between z_i and complex conjugate of z_j. In 2-D case one obtains just $ds^2 = g_{z\overline{z}}dzd\overline{z}$. Note that in this case metric is conformally flat since line element is proportional to the line element $ds^2 = dzd\overline{z}$ of plane. This form is always possible locally. For complex n-D case one obtains $ds^2 = g_{i\overline{j}}dz^id\overline{z}^j$. $g_{i\overline{j}} = \overline{g_{j\overline{i}}}$ guaranteeing the reality of ds^2. In 2-D case this condition gives $g_{z\overline{z}} = \overline{g_{z\overline{z}}}$.

How could one generalize this line element to hyper-complex n-dimensional case. In 2-D case Minkowski space M^2 one has $ds^2 = g_{uv}dudv$, $g_{uv} = 1$. The obvious generalization would be the replacement $ds^2 = g_{u_iv_j}du^idv^j$. Also now the analogs of reality conditions must hold with respect to $u_i \leftrightarrow v_i$.

3.7.2 Hamilton-Jacobi Structure

Consider next the path leading to Hamilton-Jacobi structure.

4-D Minkowski space $M^4 = M^2 \times E^2$ is Cartesian product of hyper-complex M^2 with complex plane E^2, and one has $ds^2 = dudv + dzd\overline{z}$ in standard Minkowski coordinates. One can also consider more general integrable decompositions of M^4 for which the tangent space $TM^4 = M^4$ at each point is decomposed to $M^2(x) \times E^2(x)$. The physical analogy would be a position dependent decomposition of the degrees of freedom of massless particle to longitudinal ones ($M^2(x)$: light-like momentum is in this plane) and transversal ones ($E^2(x)$: polarization vector is in this plane). Cylindrical and spherical variants of Minkowski coordinates define two examples of this kind of coordinates (it is perhaps a good exercise to think what kind of decomposition of tangent space is in question in these examples). An interesting mathematical problem highly relevant for TGD is to identify all possible decompositions of this kind for empty Minkowski space.

The integrability of the decomposition means that the planes $M^2(x)$ are tangent planes for 2-D surfaces of M^4 analogous to Euclidian string world sheet. This gives slicing of M^4 to Minkowskian string world sheets parametrized by euclidian string world sheets. The question is whether the sheets are stringy in a strong sense: that is minimal surfaces. This is not the case: for spherical coordinates the Euclidian string world sheets would be spheres which are not minimal surfaces. For cylindrical and spherical coordinates howeverr $M^2(x)$ integrate to plane M^2, which is minimal surface.

Integrability means in the case of $M^2(x)$ the existence of light-like vector field J whose flow lines define a global coordinate. Its existence implies also the existence of its conjugate and together these vector fields give rise to $M^2(x)$ at each point. This means that one has $J = \Psi\nabla\Phi$: Φ indeed defines the global coordinate along flow lines. In the case of M^2 either the coordinate u or v would be the coordinate in question. This kind of flows are called Beltrami flows. Obviously the same holds for the transversal planes E^2.

One can generalize this metric to the case of general 4-D space with Minkowski signature of metric. At least the elements g_{uv} and $g_{z\overline{z}}$ are non-vanishing and can depend on both u, v

and $z, \overline{z}$. They must satisfy the reality conditions $g_{z\overline{z}} = \overline{g_{z\overline{z}}}$ and $g_{uv} = \overline{g_{vu}}$ where complex conjugation in the argument involves also $u \leftrightarrow v$ besides $z \leftrightarrow \overline{z}$.

The question is whether the components g_{uz}, g_{vz}, and their complex conjugates are non-vanishing if they satisfy some conditions. They can. The direct generalization from complex 2-D space would be that one treats u and v as complex conjugates and therefore requires a direct generalization of the hermiticity condition

$$g_{uz} = \overline{g_{v\overline{z}}} \ \ , \ \ g_{vz} = \overline{g_{u\overline{z}}} \ \ .$$

This would give complete symmetry with the complex 2-D (4-D in real sense) spaces. This would allow the algebraic continuation of hermitian structures to Hamilton-Jacobi structures by just replacing i with e for some complex coordinates.

Chapter 4

WCW Spinor Structure

4.1 Introduction

Quantum TGD should be reducible to the classical spinor geometry of the configuration space ("world of classical worlds" (WCW)). The possibility to express the components of WCW Kähler metric as anti-commutators of WCW gamma matrices becomes a practical tool if one assumes that WCW gamma matrices correspond to Noether super charges for super-symplectic algebra of WCW. The possibility to express the Kähler metric also in terms of Kähler function identified as Kähler for Euclidian space-time regions leads to a duality analogous to AdS/CFT duality.

4.1.1 Basic Principles

Physical states should correspond to the modes of the WCW spinor fields and the identification of the fermionic oscillator operators as super-symplectic charges is highly attractive. WCW spinor fields cannot, as one might naively expect, be carriers of a definite spin and unit fermion number. Concerning the construction of the WCW spinor structure there are some important clues.

Geometrization of fermionic statistics in terms of WCW spinor structure

The great vision has been that the second quantization of the induced spinor fields can be understood geometrically in terms of the WCW spinor structure in the sense that the anti-commutation relations for WCW gamma matrices require anti-commutation relations for the oscillator operators for free second quantized induced spinor fields.

(a) One must identify the counterparts of second quantized fermion fields as objects closely related to the configuration space spinor structure. [B56] has as its basic field the anti-commuting field $\Gamma^k(x)$, whose Fourier components are analogous to the gamma matrices of the configuration space and which behaves like a spin 3/2 fermionic field rather than a vector field. This suggests that the are analogous to spin 3/2 fields and therefore expressible in terms of the fermionic oscillator operators so that their naturally derives from the anti-commutativity of the fermionic oscillator operators.

As a consequence, WCW spinor fields can have arbitrary fermion number and there would be hopes of describing the whole physics in terms of WCW spinor field. Clearly, fermionic oscillator operators would act in degrees of freedom analogous to the spin degrees of freedom of the ordinary spinor and bosonic oscillator operators would act in degrees of freedom analogous to the "orbital" degrees of freedom of the ordinary spinor field.

(b) The classical theory for the bosonic fields is an essential part of the WCW geometry. It would be very nice if the classical theory for the spinor fields would be contained in the definition of the WCW spinor structure somehow. The properties of the associated with the induced spinor structure are indeed very physical. The modified massless Dirac equation for the induced spinors predicts a separate conservation of baryon and lepton numbers. Contrary to the long held belief it seems that covariantly constant right handed neutrino does not generate . The differences between quarks and leptons result from the different couplings to the CP_2 Kähler potential. In fact, these properties are shared by the solutions of massless Dirac equation of the imbedding space.

(c) Since TGD should have a close relationship to the ordinary quantum field theories it would be highly desirable that the second quantized free induced spinor field would somehow appear in the definition of the WCW geometry. This is indeed true if the complexified WCW gamma matrices are linearly related to the oscillator operators associated with the second quantized induced spinor field on the space-time surface and its boundaries. There is actually no deep reason forbidding the gamma matrices of the WCW to be spin half odd-integer objects whereas in the finite-dimensional case this is not possible in general. In fact, in the finite-dimensional case the equivalence of the spinorial and vectorial vielbeins forces the spinor and vector representations of the vielbein group $SO(D)$ to have same dimension and this is possible for $D = 8$-dimensional Euclidian space only. This coincidence might explain the success of 10-dimensional super string models for which the physical degrees of freedom effectively correspond to an 8-dimensional Euclidian space.

(d) It took a long time to realize that the ordinary definition of the gamma matrix algebra in terms of the anti-commutators $\{\gamma_A, \gamma_B\} = 2g_{AB}$ must in TGD context be replaced with

$$\{\gamma_A^\dagger, \gamma_B\} = iJ_{AB} \quad .$$

where J_{AB} denotes the matrix elements of the Kähler form of the WCW. The presence of the Hermitian conjugation is necessary because WCW gamma matrices carry fermion number. This definition is numerically equivalent with the standard one in the complex coordinates. The realization of this delicacy is necessary in order to understand how the square of the WCW Dirac operator comes out correctly.

(e) TGD as a generalized number theory vision leads to the understanding of how the second quantization of the induced spinor fields should be carried out and space-time conformal symmetries allow to explicitly solve the Dirac equation associated with the Kähler-Dirac action in the interior and at the 3-D light like causal determinants. An essentially new element is the notion of number theoretic braid forced by the fact that the Kähler-Dirac operator allows only finite number of generalized eigen modes so that the number of fermionic oscillator operators is finite. As a consequence, anti-commutation relations can be satisfied only for a finite set of points defined by the number theoretic braid, which is uniquely identifiable. The interpretation is in terms of finite measurement resolution. The finite Clifford algebra spanned by the fermionic oscillator operators is interpreted as the factor space $\mathcal{M}/\mathcal{N}$ of infinite hyper-finite factors of type II_1 defined by WCW Clifford algebra $\mathcal{N}$ and included Clifford algebra $\mathcal{M} \subset \mathcal{N}$ interpreted as the characterizer of the finite measurement resolution. Note that the finite number of eigenvalues guarantees that Dirac determinant identified as the exponent of Kähler function is finite. Finite number of eigenvalues is also essential for number theoretic universality.

Identification of WCW gamma matrices as super Hamiltonians and expression of WCW Kähler metric

The basic super-algebra corresponds to the fermionic oscillator operators and can be regarded as a generalization $\mathcal{N}$ super algebras by replacing $\mathcal{N}$ with the number of solutions of the

Kähler-Dirac equation which can be infinite. This leads to QFT SUSY limit of TGD different in many respects crucially from standard SUSYs.

WCW gamma matrices are identified as super generators of super-symplectic and are expressible in terms of these oscillator operators. In the original proposal super-symplectic and super charges were assumed to be expressible as integrals over 2-dimensional partonic surfaces X^2 and interior degrees of freedom of X^4 can be regarded as zero modes representing classical variables in one-one correspondence with quantal degrees of freedom at X_l^3 as indeed required by quantum measurement theory.

It took quite long time to realize that it is possible to second quantize induced spinor fields by using just the standard canonical quantization. The only new element is the replacement of the ordinary gamma matrices with K-D gamma matrices identified as canonical momentum currents contracted with the imbedding space gamma matrices. This allows to deduce super-generators of super-symplectic algebra as Noether supercharges assignable to the fermionic strings connecting partonic 2-surfaces. Their anti-commutators giving the matrix elements of WCW Kähler metric can be deduced explicitly. This is a decisive calculational advantage since the formal expression of the matrix elements in terms of second derivatives of Kähler function is not possible to calculate with the recent understanding. WCW gamma matrices provide also a natural identification for the counterparts of fermionic oscillator operators creating physical states.

One can also deduce the fermionic Hamiltonians as conserved Noether charges. The expressions for Hamiltonians generalized the earlier expressions as Hamiltonian fluxes in the sense that the imbedding space Hamiltonian is replaced with the corresponding fermionic Noether charge. This replacement is analogous to a transition from field theory to string models requiring the replacement of points of partonic 2-surfaces with stringy curves connecting the points of two partonic 2-surfaces. One can consider also several strings emanating from a given partonic 2-surface. This leads to an extension of the super-symplectic algebra to a Yangian, whose generators are multi-local (multi-stringy) operators. This picture does not mean loss of effective 2-dimensionality implied by strong form of general coordinate invariance but allows genuine generalization of super-conformal invariance in 4-D context.

4.1.2 Kähler-Dirac Action

Supersymmetry fixes the interior part of Kähler-Dirac uniquely. The K-D gamma matrices are contractions of the canonical momentum currents of Kähler action with the imbedding space gamma matrices and this gives field equations consistent with hermitian conjugation. The modes of K-D equation must be restricted to 2-D string world sheets with vanishing induced W boson fields in order that they have a well-defined em charge. It is not yet clear whether this restriction is part of variational principle or whether it is a property of spinor modes. For the latter option modes one can have 4-D modes if the space-time surface has CP_2 projection carrying vanishing W gauge potentials. Also covariantly constant right-handed neutrino defines this kind of mode.

The boundary terms of Kähler action and Kähler-Dirac action

A long standing question has been whether Kähler action could contain Chern-Simons term cancelling the Chern-Simons contribution of Kähler action at space-time interior at partonic orbit reducing to Chern-Simons terms so that only the contribution at space-like ends of space-time surface at the boundaries of causal diamond (CD) remains. This is however not necessary and super-symmetry would require Chern-Simons-Dirac term as boundary term in Dirac action. This however has unphysical implications since C-S-D Dirac operator acts on CP_2 coordinates only.

The intuitive expectation is that fermionic propagators assignable to string boundaries at light-like partonic orbits are needed in the construction of the scattering amplitudes. These boundaries can be locally space-like or light-like. One could add 1-D massles Dirac action with gamma matrices defined in the induced metric, which is by supersymmetry accompanied by the action defined by geodesic length, which however vanishes for light-like curves. Massless Dirac equation at the boundary of string world sheet fixes the boundary conditions for the spinor modes at the string world sheet. This option seems to be the most plausible at this moment.

Kähler-Dirac equation for induced spinor fields

It has become clear that Kähler-Dirac action with induced spinor fields localized at string world sheets carrying vanishing classical W fields, and the light-like boundaries of the string world sheets at light-like orbits of partonic 2-surfaces carrying massless Dirac operator for induced gamma matrices is the most natural looking option.

The light-like momentum associated with the boundary is a light-like curve of imbedding space and defines light-like 8-momentum, whose M^4 projection is in general time-like. This leads to an 8-D generalization of twistor formalism. The squares of the M^4 and CP_2 parts of the 8-momentum could be identified as mass squared for the imbedding space spinor mode assignable to the ground state of super-symplectic representation. This would realize quantum classical correspondence for fermions. The four-momentum assignable to fermion line would have identification as gravitational four-momentum and that associated with the mode of imbedding space spinor field as inertial four-momentum.

There are several approaches for solving the Kähler-Dirac (or Kähler-Dirac) equation.

(a) The most promising approach assumes that the solutions are restricted on 2-D stringy world sheets and/or partonic 2-surfaces. This strange looking view is a rather natural consequence of both strong form of holography and of number theoretic vision, and also follows from the notion of finite measurement resolution having discretization at partonic 2-surfaces as a geometric correlate. Furthermore, the conditions stating that electric charge is well-defined for preferred extremals forces the localization of the modes to 2-D surfaces in the generic case. This also resolves the interpretational problems related to possibility of strong parity breaking effects since induce W fields and possibly also Z^0 field above weak scale, vanish at these surfaces.

(b) One expects that stringy approach based on 4-D generalization of conformal invariance or its 2-D variant at 2-D preferred surfaces should also allow to understand the Kähler-Dirac equation. Conformal invariance indeed allows to write the solutions explicitly using formulas similar to encountered in string models. In accordance with the earlier conjecture, all modes of the Kähler-Dirac operator generate badly broken super-symmetries.

(c) Well-definedness of em charge is not enough to localize spinor modes at string world sheets. Covariantly constant right-handed neutrino certainly defines solutions de-localized inside entire space-time sheet. This need not be the case if right-handed neutrino is not covarianty constant since the non-vanishing CP_2 part for the induced gamma matrices mixes it with left-handed neutrino. For massless extremals (at least) the CP_2 part however vanishes and right-handed neutrino allows also massless holomorphic modes de-localized at entire space-time surface and the de-localization inside Euclidian region defining the line of generalized Feynman diagram is a good candidate for the right-handed neutrino generating the least broken super-symmetry.This super-symmetry seems however to differ from the ordinary one in that ν_R is expected to behave like a passive spectator in the scattering. Also for the left-handed neutrino solutions localized inside string world sheet the condition that coupling to right-handed neutrino vanishes is guaranteed if gamma matrices are either purely Minkowskian or CP_2 like inside the world sheet.

Quantum criticality and K-D action

A detailed view about the physical role of quantum criticality results. Quantum criticality fixes the values of Kähler coupling strength as the analog of critical temperature. The recent formulation of quantum criticality states the existence of hierarchy of sub-algebras of super-symplectic algebras isomorphic with the original algebra. The conformal weights of given sub-algebra are n-multiples of those of the full algebra. n would also characterize the value of Planck constant $h_{eff} = n \times h$ assignable to various phases of dark matter. These sub-algebras correspond to a hierarchy of breakings of super-symplectic gauge symmetry to a sub-algebra. Accordingly the super-symplectic Noether charges of the sub-algebra annihilate physical states and the corresponding classical Noether charges vanish for Kähler action at the ends of space-time surfaces. This defines the notion of preferred extremal. These sub-algebras form an inclusion hierarchy defining a hierarchy of symmetry breakings. n would also characterize the value of Planck constant $h_{eff} = n \times h$ assignable to various phases of dark matter.

Quantum criticality implies that second variation of Kähler action vanishes for critical deformations defined by the sub-algebra and vanishing of the corresponding Noether charges and super-charges for physical stats. It is not quite clear whether the charges corresponding to broken super-symplectic symmetries are conserved. If this is the case, Kähler action is invariant under brokent symplectic transformations although the second variation is non-vanishing so these deformations contribute to Kähler metric and are thus quantum fluctuating dynamical degrees of freedom.

Quantum classical correspondence

Quantum classical correspondence (QCC) requires a coupling between quantum and classical and this coupling should also give rise to a generalization of quantum measurement theory. The big question mark is how to realize this coupling.

(a) As already described, the massless Dirac equation for induced gamma matrices at the boundary of string world sheets gives as solutions for which local 8-momentum is light-like. The M^4 part of this momentum is in general time-like and can be identified as the 8-momentum of incoming fermion assignable to an imbedding space spinor mode. The interpretation is as equivalence of gravitational and inertial masses.

(b) QCC can be realized at the level of WCW Dirac operator and Kähler-Dirac operator contains only interior term. The vanishing of the normal component of fermion current replaces Chern-Simons Dirac operator at various boundary like surfaces. I have proposed that WCW spinor fields with given quantum charges in Cartan algebra are superpositions of space-time surfaces with same classical charges. A stronger form of QCC at the level of WCW would be that classical correlation functions for various geometric observables are identical with quantal correlation functions.

    QCC could be realized at the level of WCW by putting it in by hand. One can of course consider also the possibility that the equality of quantal and classical Cartan charges is realized by adding constraint terms realized using Lagrange multipliers at the space-like ends of space-time surface at the boundaries of CD. This procedure would be very much like the thermodynamical procedure used to fix the average energy or particle number of the the system with Lagrange multipliers identified as temperature or chemical potential. Since quantum TGD in zero energy ontology (ZEO) can be regarded as square root of thermodynamics, the procedure looks logically sound.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [L12].

4.2 WCW Spinor Structure: General Definition

The basic problem in constructing WCW spinor structure is clearly the construction of the explicit representation for the gamma matrices of WCW . One should be able to identify the space, where these gamma matrices act as well as the counterparts of the "free" gamma matrices, in terms of which the gamma matrices would be representable using generalized vielbein coefficients.

4.2.1 Defining Relations For Gamma Matrices

The ordinary definition of the gamma matrix algebra is in terms of the anti-commutators

$$\{\gamma_A, \gamma_B\} = 2g_{AB} \quad .$$

This definition served implicitly also as a basic definition of the gamma matrix algebra in TGD context until the difficulties related to the understanding of WCW d'Alembertian defined in terms of the square of the Dirac operator forced to reconsider the definition. If WCW allows Kähler structure, the most general definition allows to replace the metric any covariantly constant Hermitian form. In particular, g_{AB} can be replaced with

$$\{\Gamma_A^\dagger, \Gamma_B\} = iJ_{AB} \quad , \tag{4.2.1}$$

where J_{AB} denotes the matrix element of the Kähler form of WCW . The reason is that gamma matrices carry fermion number and are non-hermitian in all coordinate systems. This definition is numerically equivalent with the standard one in the complex coordinates but in arbitrary coordinates situation is different since in general coordinates iJ_{kl} is a nontrivial positive square root of g_{kl}. The realization of this delicacy is necessary in order to understand how the square of WCW Dirac operator comes out correctly. Obviously, what one must do is the equivalent of replacing $D^2 = (\Gamma^k D_k)^2$ with $D\hat{D}$ with $\hat{D}$ defined as

$$\hat{D} = iJ^{kl}\Gamma_l^\dagger D_k \quad .$$

4.2.2 General Vielbein Representations

There are two ideas, which make the solution of the problem obvious.

(a) Since the classical time development in bosonic degrees of freedom (induced gauge fields) is coded into the geometry of WCW it seems natural to expect that same applies in the case of the spinor structure. The time development of the induced spinor fields dictated by TGD counterpart of the massless Dirac action should be coded into the definition of the WCW spinor structure. This leads to the challenge of defining what classical spinor field means.

(b) Since classical scalar field in WCW corresponds to second quantized boson fields of the imbedding space same correspondence should apply in the case of the fermions, too. The spinor fields of WCW should correspond to second quantized fermion field of the imbedding space and the space of the configuration space spinors should be more or less identical with the Fock space of the second quantized fermion field of imbedding space or $X^4(X^3)$. Since classical spinor fields at space-time surface are obtained by restricting the spinor structure to the space-time surface, one might consider the possibility that life is really simple: the second quantized spinor field corresponds to the free spinor field of the imbedding space satisfying the counterpart of the massless Dirac equation and more or less standard anti-commutation relations. Unfortunately life is not *so* simple as the construction of WCW spinor structure demonstrates: second quantization must be performed for induced spinor fields.

It is relatively simple to fill in the details once these basic ideas are accepted.

(a) The only natural candidate for the second quantized spinor field is just the on X^4. Since this field is free field, one can indeed perform second quantization and construct fermionic oscillator operator algebra with unique anti-commutation relations. The space of WCW spinors can be identified as the associated with these oscillator operators. This space depends on 3-surface and strictly speaking one should speak of the Fock bundle having WCW as its base space.

(b) The gamma matrices of WCW (or rather fermionic Kac Moody generators) are representable as super positions of the fermionic oscillator algebra generators:

$$\begin{aligned} \Gamma_A^+ &= E_A^n a_n^\dagger \\ \Gamma_A^- &= \bar{E}_A^n a_n \\ iJ_{A\bar{B}} &= \sum_n E_A^n \bar{E}_B^n \end{aligned} \tag{4.2.2}$$

where E_A^n are the vielbein coefficients. Induced spinor fields can possess zero modes and there is no oscillator operators associated with these modes. Since oscillator operators are spin 1/2 objects, WCW gamma matrices are analogous to spin 3/2 spinor fields (in a very general sense). Therefore the generalized vielbein and WCW metric is analogous to the pair of spin 3/2 and spin 2 fields encountered in super gravitation! Notice that the contractions $j^{Ak}\Gamma_k$ of the complexified gamma matrices with the isometry generators are genuine spin 1/2 objects labeled by the quantum numbers labeling isometry generators. In particular, in CP_2 degrees of freedom these fermions are color octets.

(c) A further great idea inspired by the symplectic and Kähler structures of WCW is that configuration gamma matrices are actually generators of super-symplectic symmetries. This simplifies enormously the construction allows to deduce explicit formulas for the gamma matrices.

4.2.3 Inner Product For WCW Spinor Fields

The conjugation operation for WCW spinor s corresponds to the standard $ket \rightarrow bra$ operation for the states of the Fock space:

$$\begin{aligned} \Psi &\leftrightarrow |\Psi\rangle \\ \bar{\Psi} &\leftrightarrow \langle\Psi| \end{aligned} \tag{4.2.3}$$

The inner product for WCW spinor s at a given point of WCW is just the standard Fock space inner product, which is unitary.

$$\bar{\Psi}_1(X^3)\Psi_2(X^3) = \langle\Psi_1|\Psi_2\rangle_{|X^3} \tag{4.2.4}$$

WCW inner product for two WCW spinor fields is obtained as the integral of the Fock space inner product over the whole WCW using the vacuum functional $exp(K)$ as a weight factor

$$\langle\Psi_1|\Psi_2\rangle = \int \langle\Psi_1|\Psi_2\rangle_{|X^3} exp(K)\sqrt{G}dX^3 \tag{4.2.5}$$

This inner product is obviously unitary. A modified form of the inner product is obtained by including the factor $exp(K/2)$ in the definition of the spinor field. In fact, the construction of the central extension for the isometry algebra leads automatically to the appearance of this factor in vacuum spinor field.

The inner product differs from the standard inner product for, say, Minkowski space spinors in that integration is over the entire WCW rather than over a time= constant slice of the WCW . Also the presence of the vacuum functional makes it different from the finite dimensional inner product. These are not un-physical features. The point is that (apart from classical non-determinism forcing to generalized the concept of 3-surface) Diff^4 invariance dictates the behavior of WCW spinor field completely: it is determined form its values at the moment of the big bang. Therefore there is no need to postulate any Dirac equation to determine the behavior and therefore no need to use the inner product derived from dynamics.

4.2.4 Holonomy Group Of The Vielbein Connection

Generalized vielbein allows huge gauge symmetry. An important constraint on physical observables is that they do not depend at all on the gauge chosen to represent the gamma matrices. This is indeed achieved using vielbein connection, which is now quadratic in fermionic oscillator operators. The holonomy group of the vielbein connection is the WCW counterpart of the electro-weak gauge group and its algebra is expected to have same general structure as the algebra of the WCW isometries. In particular, the generators of this algebra should be labeled by conformal weights like the elements of Kac Moody algebras. In present case however conformal weights are complex as the construction of WCW geometry demonstrates.

4.2.5 Realization Of WCW Gamma Matrices In Terms Of Super Symmetry Generators

In string models super symmetry generators behave effectively as gamma matrices and it is very tempting to assume that WCW gamma matrices can be regarded as generators of the symplectic algebra extended to super-symplectic Kac Moody type algebra. The experience with string models suggests also that radial Virasoro algebra extends to Super Virasoro algebra. There are good reasons to expect that WCW Dirac operator and its square give automatically a realization of this algebra. It this is indeed the case, then WCW spinor structure as well as Dirac equation reduces to mere group theory.

One can actually guess the general form of the super-symplectic algebra. The form is a direct generalization of the ordinary super Kac Moody algebra. The complexified super generators S_A are identifiable as WCW gamma matrices:

$$\Gamma_A = S_A \ . \tag{4.2.6}$$

The anti-commutators $\{\Gamma_A^\dagger, \Gamma_B\}_+ = i2J_{A,B}$ define a Hermitian matrix, which is proportional to the Kähler form of the configuration space rather than metric as usually. Only in complex coordinates the anti-commutators equal to the metric numerically. This is, apart from the multiplicative constant n, is expressible as the Poisson bracket of the WCW Hamiltonians H_A and H_B. Therefore one should be able to identify super generators $S_A(r_M)$ for each values of r_M as the counterparts of fluxes. The anti-commutators between the super generators S_A and their Hermitian conjugates should read as

$$\{S_A, S_B^\dagger\}_+ = iQ_m(H_{[A,B]}) \ . \tag{4.2.7}$$

and should be induced directly from the anti-commutation relations of free second quantized spinor fields of the imbedding space restricted to the light cone boundary.

The commutation relations between s and super generators follow solely from the transformation properties of the super generators under symplectic transformations, which are same as for the Hamiltonians themselves

$$\{H_{Am}, S_{Bn}\}_- = S_{[Am,Bn]} , \qquad (4.2.8)$$

and are of the same form as in the case of Super-Kac-Moody algebra.

The task is to derive an explicit representation for the super generators S_A in both cases. For obvious reason the spinor fields restricted to the 3-surfaces on the light cone boundary $\delta M^4_+ \times CP_2$ can be used. Leptonic/quark like oscillator operators are used to construct Ramond/NS type algebra.

What is then the strategy that one should follow?

(a) WCW Hamiltonians correspond to either magnetic or electric flux Hamiltonians and the conjecture is that these representations are equivalent. It turns out that this electric-magnetic duality generalizes to the level of super charges. It also turns out that quark representation is the only possible option whereas leptonic super charges super-symmetrize the ordinary function algebra of the light cone boundary.

(b) The simplest option would be that second quantized imbedding space spinors could be used in the definition of super charges. This turns out to not work and one must second quantize the induced spinor fields.

(c) The task is to identify a super-symmetric variational principle for the induced spinors: ordinary Dirac action does not work. It turns out that in the most plausible scenario the Kähler-Dirac action varied with respect to *both* imbedding space coordinates and spinor fields is the fundamental action principle. The c-number parts of the conserved symplectic charges associated with this action give rise to bosonic conserved charges defining WCW Hamiltonians. The second quantization of the spinor fields reduces to the requirement that super charges and Hamiltonians generate super-symplectic algebra determining the anti-commutation relations for the induced spinor fields.

4.2.6 Central Extension As Symplectic Extension At WCW Level

The earlier attempts to understand the emergence of central extension of super-symplectic algebra were based on the notion of symplectic extension. This general view is not given up although it seems that this abstract approach is not very practical. Symplectic extension emerged originally in the attempts to construct formal expression for the WCW Dirac equation. The rather obvious idea was that the Dirac equation reduces to super Virasoro conditions with Super Virasoro generators involving the Dirac operator of the imbedding space. The basic difficulty was the necessity to assign to the gamma matrices of the imbedding space fermion number. In the recent formulation the Dirac operator of H does not appear in in the Super Virasoro conditions so that this problem disappears.

The proposal that Super Virasoro conditions should replaced with conditions stating that the commutator of super-symplectic and super Kac-Moody algebras annihilates physical states, looks rather feasible. One could call these conditions as WCW Dirac equation but at this moment I feel that this would be just play with words and mask the group theoretical content of these conditions. In any case, the formulas for the symplectic extension and action of isometry generators on WCW spinor deserve to be summarized.

Symplectic extension

The Abelian extension of the super-symplectic algebra is obtained by an extremely simple trick. Replace the ordinary derivatives appearing in the definition of, say spinorial isometry generator, by the covariant derivatives defined by a coupling to a multiple of the Kähler potential.

$$\begin{aligned} j^{Ak}\partial_k &\rightarrow j^{Ak}D_k \ , \\ D_k &= \partial_k + ikA_k/2 \ . \end{aligned} \tag{4.2.9}$$

where A_k denotes Kähler potential. The reality of the parameter k is dictated by the Hermiticity requirement and also by the requirement that Abelian extension reduces to the standard form in Cartan algebra. k is expected to be integer also by the requirement that covariant derivative corresponds to connection (quantization of magnetic charge).

The commutation relations for the centrally extended generators J^A read:

$$[J^A, J^B] = J^{[A,B]} + ikj^{Ak}J_{kl}j^{Bl} \equiv J^{[A,B]} + ikJ_{AB} \ . \tag{4.2.10}$$

Since Kähler form defines symplectic structure in WCW one can express Abelian extension term as a Poisson bracket of two Hamiltonians

$$J_{AB} \equiv j^{Ak}J_{kl}j^{Bl} = \{H^A, H^B\} \ . \tag{4.2.11}$$

Notice that Poisson bracket is well defined also when Kähler form is degenerate.

The extension indeed has acceptable properties:

(a) Jacobi-identities reduce to the form

$$\sum_{cyclic} H^{[A,[B,C]]} = 0 \ , \tag{4.2.12}$$

and therefore to the Jacobi identities of the original Lie- algebra in Hamiltonian representation.

(b) In the Cartan algebra Abelian extension reduces to a constant term since the Poisson bracket for two commuting generators must be a multiple of a unit matrix. This feature is clearly crucial for the non-triviality of the Abelian extension and is encountered already at the level of ordinary (q,p) Poisson algebra: although the differential operators ∂_p and ∂_q commute the Poisson bracket of the corresponding Hamiltonians p and q is nontrivial: $\{p,q\} = 1$. Therefore the extension term commutes with the generators of the Cartan subalgebra. Extension is also local $U(1)$ extension since Poisson algebra differs from the Lie-algebra of the vector fields in that it contains constant Hamiltonian ("1" in the commutator), which commutes with all other Hamiltonians and corresponds to a vanishing vector field.

(c) For the generators not belonging to Cartan sub-algebra of CH isometries Abelian extension term is not annihilated by the generators of the original algebra and in this respect the extension differs from the standard central extension for the loop algebras. It must be however emphasized that for the super-symplectic algebra generators correspond to products of δM^4_+ and CP_2 Hamiltonians and this means that generators of say δM^4_+-local $SU(3)$ Cartan algebra are non-commuting and the commutator is completely analogous to central extension term since it is symmetric with respect to $SU(3)$ generators.

(d) The proposed method yields a trivial extension in the case of Diff^4. The reason is the (four-dimensional!) Diff degeneracy of the Kähler form. Abelian extension term is given by the contraction of the Diff^4 generators with the Kähler potential

$$j^{Ak}J_{kl}j^{Bl} = 0 \ , \tag{4.2.13}$$

which vanishes identically by the Diff degeneracy of the Kähler form. Therefore neither 3- or 4-dimensional Diff invariance is not expected to cause any difficulties. Recall that 4-dimensional Diff degeneracy is what is needed to eliminate time like vibrational excitations from the spectrum of the theory. By the way, the fact that the loop space metric is not Diff degenerate makes understandable the emergence of Diff anomalies in string models [B56, B49] .

(e) The extension is trivial also for the other zero norm generators of the tangent space algebra, in particular for the $k_2 = Im(k) = 0$ symplectic generators possible present so that these generators indeed act as genuine $U(1)$ transformations.

(f) Concerning the solution of WCW Dirac equation the maximum of Kähler function is expected to be special, much like origin of Minkowski space and symmetric space property suggests that the construction of solutions reduces to this point. At this point the generators and Hamiltonians of the algebra h in the defining Cartan decomposition $g = h + t$ should vanish. h corresponds to integer values of $k_1 = Re(k)$ for Cartan algebra of super-symplectic algebra and integer valued conformal weights n for Super Kac-Moody algebra. The algebra reduces at the maximum to an exceptionally simple form since only central extension contributes to the metric and Kähler form. In the ideal case the elements of the metric and Kähler form could be even diagonal. The degeneracy of the metric might of course pose additional complications.

Super symplectic action on WCW spinor s

The generators of symplectic transformations are obtained in the spinor representation of the isometry group of WCW by the following formal construction. Take isometry generator in the spinor representation and add to the covariant derivative D_k defined by vielbein connection the coupling to the multiple of the Kähler potential: $D_k \to D_k + ikAk/2$.

$$\begin{aligned} J^A &= j^{Ak}D_k + D_l j_k \Sigma^{kl}/2 \ , \\ &\to \hat{J}^A = j^{Ak}(D_k + ikA_k/2) + D_l j_k^A \Sigma^{kl}/2 \ , \end{aligned} \tag{4.2.14}$$

This induces the required central term to the commutation relations. Introduce complex coordinates and define bosonic creation and annihilation operators as $(1,0)$ and $(0,1)$ parts of the modified isometry generators

$$\begin{aligned} B_A^\dagger &= J_+^A = j^{Ak}(D_k + ... \ , \\ B_A &= J_-^A = j^{A\bar{k}}(D_{\bar{k}} + ... \ . \end{aligned} \tag{4.2.15}$$

where "k" refers now to complex coordinates and "$\bar{k}$" to their conjugates.

Fermionic generators are obtained as the contractions of the complexified gamma matrices with the isometry generators

$$\begin{aligned} \Gamma_A^\dagger &= j^{Ak}\Gamma_k \ , \\ \Gamma_A &= j^{A\bar{k}}\Gamma_{\bar{k}} \ . \end{aligned} \tag{4.2.16}$$

Notice that the bosonic Cartan algebra generators obey standard oscillator algebra commutation relations and annihilate fermionic Cartan algebra generators. Hermiticity condition holds in the sense that creation type generators are hermitian conjugates of the annihilation operator type generators. There are two kinds of representations depending on whether one uses leptonic or quark like oscillator operators to construct the gammas. These will be assumed to correspond to Ramond and NS type generators with the radial plane waves being labeled by integer and half odd integer indices respectively.

The non-vanishing commutators between the Cartan algebra bosonic generators are given by the matrix elements of the Kähler form in the basis of formed by the isometry generators

$$[B_A^\dagger, B_B] \quad = \quad J(j^{A\dagger}, j^B) \equiv J_{\bar{A}B} \quad . \tag{4.2.17}$$

and are isometry invariant quantities. The commutators between local $SU(3)$ generators not belonging to Cartan algebra are just those of the local gauge algebra with Abelian extension term added.

The anti-commutators between the fermionic generators are given by the elements of the metric (as opposed to Kähler form in the case of bosonic generators) in the basis formed by the isometry generators

$$\{\Gamma_A\dagger, \Gamma_B\} \quad = \quad 2g(j^{A\dagger}, j^B) \equiv 2g_{\bar{A}B} \quad . \tag{4.2.18}$$

and are invariant under isometries. Numerically the commutators and anti-commutators differ only the presence of the imaginary unit and the scale factor R relating the metric and Kähler form to each other (the factor R is same for CP_2 metric and Kähler form).

The commutators between bosonic and fermionic generators are given by

$$[B_A, \Gamma_B] \quad = \quad \Gamma_{[A,B]} \quad . \tag{4.2.19}$$

The presence of vielbein and rotation terms in the representation of the isometry generators is essential for obtaining these nice commutations relations. The commutators vanish identically for Cartan algebra generators. From the commutation relations it is clear that Super Kac Moody algebra structure is directly related to the Kähler structure of WCW : the anti-commutator of fermionic generators is proportional to the metric and the commutator of the bosonic generators is proportional to the Kähler form. It is this algebra, which should generate the solutions of the field equations of the theory.

The vielbein and rotational parts of the bosonic isometry generators are quadratic in the fermionic oscillator operators and this suggests the interpretation as the fermionic contribution to the isometry currents. This means that the action of the bosonic generators is essentially non-perturbative since it creates fermion anti-fermion pairs besides exciting bosonic degrees of freedom.

4.2.7 WCW Clifford Algebra As AHyper-Finite Factor Of Type II_1

The naive expectation is that the trace of the unit matrix associated with the Clifford algebra spanned by WCW sigma matrices is infinite and thus defines an excellent candidate for a source of divergences in perturbation theory. This potential source of infinities remained unnoticed until it became clear that there is a connection with von Neumann algebras [A67]. In fact, for a separable Hilbert space defines a standard representation for so called [A55]. This guarantees that the trace of the unit matrix equals to unity and there is no danger about divergences.

Philosophical ideas behind von Neumann algebras

The goal of von Neumann was to generalize the algebra of quantum mechanical observables. The basic ideas behind the von Neumann algebra are dictated by physics. The algebra elements allow Hermitian conjugation * and observables correspond to Hermitian operators. Any measurable function $f(A)$ of operator A belongs to the algebra and one can say that non-commutative measure theory is in question.

The predictions of quantum theory are expressible in terms of traces of observables. Density matrix defining expectations of observables in ensemble is the basic example. The highly non-trivial requirement of von Neumann was that identical a priori probabilities for a detection of states of infinite state system must make sense. Since quantum mechanical expectation values are expressible in terms of operator traces, this requires that unit operator has unit trace: $tr(Id) = 1$.

In the finite-dimensional case it is easy to build observables out of minimal projections to 1-dimensional eigen spaces of observables. For infinite-dimensional case the probably of projection to 1-dimensional sub-space vanishes if each state is equally probable. The notion of observable must thus be modified by excluding 1-dimensional minimal projections, and allow only projections for which the trace would be infinite using the straightforward generalization of the matrix algebra trace as the dimension of the projection.

The non-trivial implication of the fact that traces of projections are never larger than one is that the eigen spaces of the density matrix must be infinite-dimensional for non-vanishing projection probabilities. Quantum measurements can lead with a finite probability only to mixed states with a density matrix which is projection operator to infinite-dimensional subspace. The simple von Neumann algebras for which unit operator has unit trace are known as factors of type II_1 [A55].

The definitions of adopted by von Neumann allow however more general algebras. Type I_n algebras correspond to finite-dimensional matrix algebras with finite traces whereas I_∞ associated with a separable infinite-dimensional Hilbert space does not allow bounded traces. For algebras of type III non-trivial traces are always infinite and the notion of trace becomes useless.

von Neumann, Dirac, and Feynman

The association of algebras of type I with the standard quantum mechanics allowed to unify matrix mechanism with wave mechanics. Note however that the assumption about continuous momentum state basis is in conflict with separability but the particle-in-box idealization allows to circumvent this problem (the notion of space-time sheet brings the box in physics as something completely real).

Because of the finiteness of traces von Neumann regarded the factors of type II_1 as fundamental and factors of type III as pathological. The highly pragmatic and successful approach of Dirac based on the notion of delta function, plus the emergence of Feynman graphs, the possibility to formulate the notion of delta function rigorously in terms of distributions, and the emergence of path integral approach meant that von Neumann approach was forgotten by particle physicists.

Algebras of type II_1 have emerged only much later in conformal and topological quantum field theories [A86, A50] allowing to deduce invariants of knots, links and 3-manifolds. Also algebraic structures known as bi-algebras, Hopf algebras, and ribbon algebras [A41, A59] relate closely to type II_1 factors. In topological quantum computation [B45] based on braid groups [A95] modular S-matrices they play an especially important role.

Clifford algebra of WCW as von Neumann algebra

The Clifford algebra of WCW provides a school example of a hyper-finite factor of type II_1, which means that fermionic sector does not produce divergence problems. Super-symmetry means that also "orbital" degrees of freedom corresponding to the deformations of 3-surface define similar factor. The general theory of hyper-finite factors of type II_1 is very rich and leads to rather detailed understanding of the general structure of S-matrix in TGD framework. For instance, there is a unitary evolution operator intrinsic to the von Neumann algebra defining in a natural manner single particle time evolution. Also a connection with 3-dimensional topological quantum field theories and knot theory, conformal field theories, braid groups, quantum groups, and quantum counterparts of quaternionic and octonionic division algebras emerges naturally. These aspects are discussed in detail in [K87].

4.3 Under What Conditions Electric Charge Is Conserved For The Kähler-Dirac Equation?

One might think that talking about the conservation of electric charge at 21st century is a waste of time. In TGD framework this is certainly not the case.

(a) In quantum field theories there are two manners to define em charge: as electric flux over 2-D surface sufficiently far from the source region or in the case of spinor field quantum mechanically as combination of fermion number and vectorial isospin. The latter definition is quantum mechanically more appropriate.

(b) There is however a problem. In standard approach to gauge theory Dirac equation in presence of charged classical gauge fields does not conserve electric charge as quantum number: electron is transformed to neutrino and vice versa. Quantization solves the problem since the non-conservation can be interpreted in terms of emission of gauge bosons. In TGD framework this does not work since one does not have path integral quantization anymore. Preferred extremals carry classical gauge fields and the question whether em charge is conserved arises. Heuristic picture suggests that em charge must be conserved.

It seems that one should pose the well-definedness of spinorial em charge as an additional condition. Well-definedness of em charge is not the only problem. How to avoid large parity breaking effects due to classical Z^0 fields? How to avoid the problems due to the fact that color rotations induced vielbein rotation of weak fields? Does this require that classical weak fields vanish in the regions where the modes of induced spinor fields are non-vanishing?

This condition might be one of the conditions defining what it is to be a preferred extremal/solution of Kähler Dirac equation. It is not however trivial whether this kind of additional condition can be posed unless it follows automatically from the recent formulation for Kähler action and Kähler Dirac action. The common answer to these questions is restriction of the modes of induced spinor field to 2-D string world sheets (and possibly also partonic 2-surfaces) such that the induced weak fields vanish. This makes string/parton picture part of TGD. The vanishing of classical weak fields has also number theoretic interpretation: space-time surfaces would have quaternionic (hyper-complex) tangent space and the 2-surfaces carrying spinor fields complex (hyper-complex) tangent space.

4.3.1 Conservation Of EM Charge For Kähler Dirac Equation

What does the conservation of em charge imply in the case of the Kähler-Dirac equation? The obvious guess that the em charged part of the Kähler-Dirac operator must annihilate the solutions, turns out to be correct as the following argument demonstrates.

(a) Em charge as coupling matrix can be defined as a linear combination $Q = aI + bI_3$, $I_3 = J_{kl}\Sigma^{kl}$, where I is unit matrix and I_3 vectorial isospin matrix, J_{kl} is the Kähler form of CP_2, Σ^{kl} denotes sigma matrices, and a and b are numerical constants different for quarks and leptons. Q is covariantly constant in $M^4 \times CP_2$ and its covariant derivatives at space-time surface are also well-defined and vanish.

(b) The modes of the Kähler-Dirac equation should be eigen modes of Q. This is the case if the Kähler-Dirac operator D commutes with Q. The covariant constancy of Q can be used to derive the condition

$$\begin{aligned} [D,Q]\,\Psi &= D_1\Psi = 0 \ , \\ D &= \hat{\Gamma}^{\mu}D_{\mu} \ , \ D_1 = [D,Q] = \hat{\Gamma}^{\mu}_1 D_{\mu} \ , \ \hat{\Gamma}^{\mu}_1 = \left[\hat{\Gamma}^{\mu},Q\right] \ . \end{aligned} \tag{4.3.1}$$

Covariant constancy of J is absolutely essential: without it the resulting conditions would not be so simple.

It is easy to find that also $[D_1,Q]\Psi = 0$ and its higher iterates $[D_n,Q]\Psi = 0$, $D_n = [D_{n-1},Q]$ must be true. The solutions of the Kähler-Dirac equation would have an additional symmetry.

(c) The commutator $D_1 = [D,Q]$ reduces to a sum of terms involving the commutators of the vectorial isospin $I_3 = J_{kl}\Sigma^{kl}$ with the CP_2 part of the gamma matrices:

$$D_1 = [Q,D] = [I_3,\Gamma_r]\partial_{\mu}s^rT^{\alpha\mu}D_{\alpha} \ . \tag{4.3.2}$$

In standard complex coordinates in which $U(2)$ acts linearly the complexified gamma matrices can be chosen to be eigenstates of vectorial isospin. Only the charged flat space complexified gamma matrices Γ^A denoted by Γ^+ and Γ^- possessing charges +1 and -1 contribute to the right hand side. Therefore the additional Dirac equation $D_1\Psi = 0$ states

$$\begin{aligned} D_1\Psi &= [Q,D]\Psi = I_3(A)e_{Ar}\Gamma^A\partial_{\mu}s^rT^{\alpha\mu}D_{\alpha}\Psi \\ &= (e_{+r}\Gamma^+ - e_{-r}\Gamma^-)\partial_{\mu}s^rT^{\alpha\mu}D_{\alpha}\Psi = 0 \ . \end{aligned} \tag{4.3.3}$$

The next condition is

$$D_2\Psi = [Q,D]\Psi = (e_{+r}\Gamma^+ + e_{-r}\Gamma^-)\partial_{\mu}s^rT^{\alpha\mu}D_{\alpha}\Psi = 0 \ . \tag{4.3.4}$$

Only the relative sign of the two terms has changed. The remaining conditions give nothing new.

(d) These equations imply two separate equations for the two charged gamma matrices

$$\begin{aligned} D_+\Psi &= T^{\alpha}_+\Gamma^+D_{\alpha}\Psi = 0 \ , \\ D_-\Psi &= T^{\alpha}_-\Gamma^-D_{\alpha}\Psi = 0 \ , \\ T^{\alpha}_{\pm} &= e_{\pm r}\partial_{\mu}s^rT^{\alpha\mu} \ . \end{aligned} \tag{4.3.5}$$

These conditions state what one might have expected: the charged part of the Kähler-Dirac operator annihilates separately the solutions. The reason is that the classical W fields are proportional to $e_{r\pm}$.

The above equations can be generalized to define a decomposition of the energy momentum tensor to charged and neutral components in terms of vierbein projections. The equations state that the analogs of the Kähler-Dirac equation defined by charged components of the energy momentum tensor are satisfied separately.

(e) In complex coordinates one expects that the two equations are complex conjugates of each other for Euclidian signature. For the Minkowskian signature an analogous condition should hold true. The dynamics enters the game in an essential manner: whether the equations can be satisfied depends on the coefficients a and b in the expression $T = aG + bg$ implied by Einstein's equations in turn guaranteeing that the solution ansatz generalizing minimal surface solutions holds true [K7].

(f) As a result one obtains three separate Dirac equations corresponding to the neutral part $D_0\Psi = 0$ and charged parts $D_\pm\Psi = 0$ of the Kähler-Dirac equation. By acting on the equations with these Dirac operators one obtains also that the commutators $[D_+, D_-]$, $[D_0, D_\pm]$ and also higher commutators obtained from these annihilate the induced spinor field model. Therefore entire -possibly- infinite-dimensional algebra would annihilate the induced spinor fields. In string model the counterpart of Dirac equation when quantized gives rise to Super-Virasoro conditions. This analogy would suggest that Kähler-Dirac equation gives rise to the analog of Super-Virasoro conditions in 4-D case. But what the higher conditions mean? Could they relate to the proposed generalization to Yangian algebra [A27] [B39, B30, B31]? Obviously these conditions resemble structurally Virasoro conditions $L_n|phys\rangle = 0$ and their supersymmetric generalizations, and might indeed correspond to a generalization of these conditions just as the field equations for preferred extremals could correspond to the Virasoro conditions if one takes seriously the analogy with the quantized string.

What could this additional symmetry mean from the point of view of the solutions of the Kähler-Dirac equation? The field equations for the preferred extremals of Kähler action reduce to purely algebraic conditions in the same manner as the field equations for the minimal surfaces in string model. Could this happen also for the Kähler-Dirac equation and could the condition on charged part of the Dirac operator help to achieve this?

This argument was very general and one can ask for simple manners to realize these conditions. Obviously the vanishing of classical W fields in the region where the spinor mode is non-vanishing defines this kind of condition.

4.3.2 About The Solutions Of Kähler Dirac Equation For Known Extremals

To gain perpective consider first Dirac equation in in H. Quite generally, one can construct the solutions of the ordinary Dirac equation in H from covariantly constant right-handed neutrino spinor playing the role of fermionic vacuum annihilated by the second half of complexified gamma matrices. Dirac equation reduces to Laplace equation for a scalar function and solution can be constructed from this “vacuum” by multiplying with the spherical harmonics of CP_2 and applying Dirac operator [K39]. Similar construction works quite generally thanks to the existence of covariantly constant right handed neutrino spinor. Spinor harmonics of CP_2 are only replaced with those of space-time surface possessing either hermitian structure or Hamilton-Jacobi structure (corresponding to Euclidian and Minkowskian signatures of the induced metric [K7, K88]). What is remarkable is that these solutions possess well-defined em charge although classical W boson fields are present.

This in sense that H d'Alembertian commutes with em charge matrix defined as a linear combination of unit matrix and the covariantly constant matrix $J^{kl}\Sigma_{kl}$ since the commutators of the covariant derivatives give constant Ricci scalar and $J^{kl}\Sigma_{kl}$ term to the d'Alembertian besides scalar d'Alembertian commuting with em charge. Dirac operator itself does not commute with em charge matrix since gamma matrices not commute with em charge matrix.

Consider now Kähler Dirac operator. The square of Kähler Dirac operator contains commutator of covariant derivatives which contains contraction $[\Gamma^\mu, \Gamma^\nu]\, F^{weak}_{\mu\nu}$ which is quadratic in sigma matrices of $M^4 \times CP_2$ and does not reduce to a constant term commuting which em charge matrix. Therefore additional condition is required even if one is satisfies with the commutativity of d'Alembertian with em charge. Stronger condition would be commutativity with the Kähler Dirac operator and this will be considered in the following.

To see what happens one must consider space-time regions with Minkowskian and Euclidian signature. What will be assumed is the existence of Hamilton-Jacobi structure [K7] meaning complex structure in Euclidian signature and hyper-complex plus complex structure in Minkowskian signature. The goal is to get insights about what the condition that spinor modes have a well-defined em charge eigenvalue requires. Or more concretely: is the localization at string world sheets guaranteeing well-defined value of em charge predicted by Kähler Dirac operator or must one introduce this condition separately? One can also ask whether this condition reduces to commutativity/co-commutativity in number theoretic vision.

(a) CP_2 type vacuum extremals serve as a convenient test case for the Euclidian signature. In this case the Kähler-Dirac equation reduces to the massless ordinary Dirac equation in CP_2 allowing only covariantly constant right-handed neutrino as solution. Only part of CP_2 so that one give up the constraint that the solution is defined in the entire CP_2. In this case holomorphic solution ansatz obtained by assuming that solutions depend on the coordinates ξ^i, $i = 1, 2$ but not on their conjugates and that the gamma matrices $\Gamma^{\bar{i}}$, $i = 1, 2$, annihilate the solutions, works. The solutions ansatz and its conjugate are of exactly the same form as in case string models where one considers string world sheets instead of CP_2 region.

The solutions are not restricted to 2-D string world sheets and it is not clear whether one can assign to them a well-defined em charge in any sense. Note that for massless Dirac equation in H one obtains all CP_2 harmonics as solutions, and it is possible to talk about em charge of the solution although solution itself is not restricted to 2-D surface of CP_2.

(b) For massless extremals and a very wide class of solutions produced by Hamilton-Jacobi structure - perhaps all solutions representable locally as graphs for map $M^4 \to CP_2$ - canonical momentum densities are light-like and solutions are hyper-holomorphic in the coordinates associated with with string world sheet and annihilated by the conjugate gamma and arbitrary functions in transversal coordinates. This allows localization to string world sheets. The localization is now strictly dynamical and implied by the properties of Kähler Dirac operator.

(c) For string like objects one obtains massless Dirac equation in $X^2 \times Y^2 \subset M^4 \times Y^2$, Y^2 a complex 2-surface in CP_2. Homologically trivial geodesic sphere corresponds to the simplest choice for Y^2. Modified Dirac operator reduces to a sum of massless Dirac operators associated with X^2 and Y^2. The most general solutions would have Y^2 mass. Holomorphic solutions reduces to product of hyper-holomorphic and holomorphic solutions and massless 2-D Dirac equation is satisfied in both factors.

For instance, for S^2 a geodesic sphere and $X^2 = M^2$ one obtains M^2 massivation with mass squared spectrum given by Laplace operator for S^2. Conformal and hyper-conformal symmetries are lost, and one might argue that this is quite not what one wants. One must be however resist the temptation to make too hasty conclusions since the massivation of string like objects is expected to take place. The question is whether it takes place already at the level of fundamental spinor fields or only at the level of elementary particles constructed as many-fermion states of them as twistor Grassmann approach assuming massless M^4 propagators for the fundamental fermions strongly suggests [K76].

(d) For vacuum extremals the Kähler Dirac operator vanishes identically so that it does not make sense to speak about solutions.

What can one conclude from these observations?

(a) The localization of solutions to 2-D string world sheets follows from Kähler Dirac equation only for the Minkowskian regions representable as graphs of map $M^4 \to CP_2$ locally. For string like objects and deformations of CP_2 type vacuum extremals this is not expected to take place.

(b) It is not clear whether one can speak about well-defined em charge for the holomorphic spinors annihilated by the conjugate gamma matrices or their conjugates. As noticed, for imbedding space spinor harmonics this is however possible.

(c) Strong form of conformal symmetry and the condition that em charge is well-defined for the nodes suggests that the localization at 2-D surfaces at which the charged parts of induced electroweak gauge fields vanish must be assumed as an additional condition.

Number theoretic vision would suggest that these surfaces correspond to 2-D commutative or co-commutative surfaces. The string world sheets inside space-time surfaces would not emerge from theory but would be defined as basic geometric objects.

This kind of condition would also allow duals of string worlds sheets as partonic 2-surfaces identified number theoretically as co-commutative surfaces. Commutativity and co-commutativity would become essential elemenents of the number theoretical vision.

(d) The localization of solutions of the Kähler-Dirac action at string world sheets and partonic 2-surfaces as a constraint would mean induction procedure for Kähler-Dirac matrices from SX^4 to X^2 - that is projection. The resulting em neutral gamma matrices would correspond to tangent vectors of the string world sheet. The vanishing of the projections of charged parts of energy momentum currents would define these surfaces. The conditions would apply both in Minkowskian and Euclidian regions. An alternative interpretation would be number theoretical: these surface would be commutative or co-commutative.

4.3.3 Concrete Realization Of The Conditions Guaranteeing Well-Defined Em Charge

Well-definedness of the em charge is the fundamental condiiton on spinor modes. Physical intuition suggests that also classical Z^0 field should vanish - at least in scales longer than weak scale. Above the condition guaranteeing vanishing of em charge has been discussed at very general level. It has however turned out that one can understand situation by simply posing the simplest condition that one can imagine: the vanishing of classical W and possibly also Z^0 fields inducing mixing of different charge states.

(a) Induced W fields mean that the modes of Kähler-Dirac equation do not in general have well-defined em charge. The problem disappears if the induced W gauge fields vanish. This does not yet guarantee that couplings to classical gauge fields are physical in long scales. Also classical Z^0 field should vanish so that the couplings would be purely vectorial. Vectoriality might be true in long enough scales only. If W and Z^0 fields vanish in all scales then electroweak forces are due to the exchanges of corresponding gauge bosons described as string like objects in TGD and represent non-trivial space-time geometry and topology at microscopic scale.

(b) The conditions solve also another long-standing interpretational problem. Color rotations induce rotations in electroweak-holonomy group so that the vanishing of all induced weak fields also guarantees that color rotations do not spoil the property of spinor modes to be eigenstates of em charge.

One can study the conditions quite concretely by using the formulas for the components of spinor curvature [L2] (`http://tinyurl.com/z86o5qk`).

(a) The representation of the covariantly constant curvature tensor is given by

$$\begin{array}{lllllll} R_{01} & = & e^0\wedge e^1 - e^2\wedge e^3 \ , & R_{23} & = & e^0\wedge e^1 - e^2\wedge e^3 \ , \\ R_{02} & = & e^0\wedge e^2 - e^3\wedge e^1 \ , & R_{31} & = & -e^0\wedge e^2 + e^3\wedge e^1 \ , \\ R_{03} & = & 4e^0\wedge e^3 + 2e^1\wedge e^2 \ , & R_{12} & = & 2e^0\wedge e^3 + 4e^1\wedge e^2 \ . \end{array} \tag{4.3.6}$$

$R_{01} = R_{23}$ and $R_{03} = -R_{31}$ combine to form purely left handed classical W boson fields and Z^0 field corresponds to $Z^0 = 2R_{03}$.

Kähler form is given by

$$J = 2(e^0\wedge e^3 + e^1\wedge e^2) \ . \tag{4.3.7}$$

(b) The vanishing of classical weak fields is guaranteed by the conditions

$$\begin{aligned} e^0 \wedge e^1 - e^2 \wedge e^3 = 0 \ \ , \\ e^0 \wedge e^2 - e^3 \wedge e^1 \ \ , \\ 4e^0 \wedge e^3 + 2e^1 \wedge e^2 \ \ . \end{aligned} \tag{4.3.8}$$

(c) There are many manners to satisfy these conditions. For instance, the condition $e^1 = a \times e^0$ and $e^2 = -a \times e^3$ with arbitrary a which can depend on position guarantees the vanishing of classical W fields. The CP_2 projection of the tangent space of the region carrying the spinor mode must be 2-D.

Also classical Z^0 vanishes if $a^2 = 2$ holds true. This guarantees that the couplings of induced gauge potential are purely vectorial. One can consider other alternaties. For instance, one could require that only classical Z^0 field or induced Kähler form is non-vanishing and deduce similar condition.

(d) The vanishing of the weak part of induced gauge field implies that the CP_2 projection of the region carrying spinor mode is 2-D. Therefore the condition that the modes of induced spinor field are restricted to 2-surfaces carrying no weak fields sheets guarantees well-definedness of em charge and vanishing of classical weak couplings. This condition does not imply string world sheets in the general case since the CP_2 projection of the space-time sheet can be 2-D.

How string world sheets could emerge?

(a) Additional consistency condition to neutrality of string world sheets is that Kähler-Dirac gamma matrices have no components orthogonal to the 2-surface in question. Hence various fermionic would flow along string world sheet.

(b) If the Kähler-Dirac gamma matrices at string world sheet are expressible in terms of two non-vanishing gamma matrices parallel to string world sheet and sheet and thus define an integrable distribution of tangent vectors, this is achieved. What is important that modified gamma matrices can indeed span lower than 4-D space and often do so as already described. Induced gamma matrices defined always 4-D space so that the restriction of the modes to string world sheets is not possible.

(c) String models suggest that string world sheets are minimal surfaces of space-time surface or of imbedding space but it might not be necessary to pose this condition separately.

In the proposed scenario string world sheets emerge rather than being postulated from beginning.

(a) The vanishing conditions for induced weak fields allow also 4-D spinor modes if they are true for entire spatime surface. This is true if the space-time surface has 2-D projection. One can expect that the space-time surface has foliation by string world sheets and the general solution of K-D equation is continuous superposition of the 2-D modes in this case and discrete one in the generic case.

(b) If the CP_2 projection of space-time surface is homologically non-trivial geodesic sphere S^2, the field equations reduce to those in $M^4 \times S^2$ since the second fundamental form for S^2 is vanishing. It is possible to have geodesic sphere for which induced gauge field has only em component?

(c) If the CP_2 projection is complex manifold as it is for string like objects, the vanishing of weak fields might be also achieved.

(d) Does the phase of cosmic strings assumed to dominate primordial cosmology correspond to this phase with no classical weak fields? During radiation dominated phase 4-D string like objects would transform to string world sheets.Kind of dimensional transmutation would occur.

Right-handed neutrino has exceptional role in K-D action.

(a) Electroweak gauge potentials do not couple to ν_R at all. Therefore the vanishing of W fields is un-necessary if the induced gamma matrices do not mix right handed neutrino with left-handed one. This is guaranteed if M^4 and CP_2 parts of Kähler-Dirac operator annihilate separately right-handed neutrino spinor mode. Also ν_R modes can be interpreted as continuous superpositions of 2-D modes and this allows to define overlap integrals for them and induced spinor fields needed to define WCW gamma matrices and super-generators.

(b) For covariantly constant right-handed neutrino mode defining a generator of super-symmetries is certainly a solution of K-D. Whether more general solutions of K-D exist remains to be checked out.

4.3.4 Connection With Number Theoretic Vision?

The interesting potential connection of the Hamilton-Jacobi vision to the number theoretic vision about field equations has been already mentioned.

(a) The vision that associativity/co-associativity defines the dynamics of space-time surfaces boils down to $M^8 - H$ duality stating that space-time surfaces can be regarded as associative/co-associative surfaces either in M^8 or H [K74, K111]. Associativity reduces to hyper-quaternionicity implying that that the tangent/normal space of space-time surface at each point contains preferred sub-space $M^2(x) \subset M^8$ and these sub-spaces forma an integrable distribution. An analogous condition is involved with the definition of Hamilton-Jacobi structure.

(b) The octonionic representation of the tangent space of M^8 and H effectively replaces $SO(7,1)$ as tangent space group with its octonionic analog obtained by the replacement of sigma matrices with their octonionic counterparts defined by anti-commutators of gamma matrices. By non-associativity the resulting algebra is not ordinary Lie-algebra and exponentiates to a non-associative analog of Lie group. The original wrong belief was that the reduction takes place to the group G_2 of octonionic automorphisms acting as a subgroup of $SO(7)$. One can ask whether the conditions on the charged part of energy momentum tensor could relate to the reduction of $SO(7,1)$

(c) What puts bells ringing is that the Kähler-Dirac equation for the octonionic representation of gamma matrices allows the conservation of electromagnetic charge in the proposed sense. The reason is that the left handed sigma matrices (W charges are left-handed) in the octonionic representation of gamma matrices vanish identically! What remains are vectorial=right-handed em and Z^0 charge which becomes proportional to em charge since its left-handed part vanishes. All spinor modes have a well-defined em charge in the octonionic sense defined by replacing imbedding space spinor locally by its octonionic variant? Maybe this could explain why H spinor modes can have well-defined em charge contrary to the naive expectations.

(d) The non-associativity of the octonionic spinors is however a problem. Even non-commutativity poses problems - also at space-time level if one assumes quaternion-real analyticity for the spinor modes. Could one assume commutativity or co-commutativity for the induced spinor modes? This would mean restriction to associative or co-associative 2-surfaces and (hyper-)holomorphic depends on its (hyper-)complex coordinate. The outcome would be a localization to a hyper-commutative of commutative 2-surface, string world sheet or partonic 2-surface.

(e) These conditions could also be interpreted by saying that for the Kähler Dirac operator the octonionic induced spinors assumed to be commutative/co-commutative are equivalent with ordinary induced spinors. The well-definedness of em charge for ordinary spinors would correspond to commutativity/co-commutativity for octonionic spinors. Even the Dirac equations based on induced and Kähler-Dirac gamma matrices could be equivalent since it is essentially holomorphy which matters.

To sum up, these considerations inspire to ask whether the associativity/co-associativity of the space-time surface is equivalent with the reduction of the field equations to stringy field equations stating that certain components of the induced metric in complex/Hamilton-Jacobi coordinates vanish in turn guaranteeing that field equations reduce to algebraic identifies following from the fact that energy momentum tensor and second fundamental form have no common components? Commutativity/co-commutativity would characterize fermionic dynamics and would have physical representation as possibility to have em charge eigenspinors. This should be the case if one requires that the two solution ansätze are equivalent.

4.4 Representation Of WCW Metric As Anti-Commutators Of Gamma Matrices Identified As Symplectic Super-Charges

WCW gamma matrices identified as symplectic super Noether charges suggest an elegant representation of WCW metric and Kähler form, which seems to be more practical than the representations in terms of Kähler function or representations guessed by symmetry arguments.

This representation is equivalent with the somewhat dubious representation obtained using symmetry arguments - that is by assuming that that the half Poisson brackets of imbedding space Hamiltonians defining Kähler form and metric can be lifted to the level of WCW, if the conformal gauge conditions hold true for the spinorial conformal algebra, which is the TGD counterpart of the standard Kac-Moody type algebra of the ordinary strings models. For symplectic algebra the hierarchy of breakings of super-conformal gauge symmetry is possible but not for the standard conformal algebras associated with spinor modes at string world sheets.

4.4.1 Expression For WCW Kähler Metric As Anticommutators As Symplectic Super Charges

During years I have considered several variants for the representation of symplectic Hamiltonians and WCW gamma matrices and each of these proposals have had some weakness. The key question has been whether the Noether currents assignable to WCW Hamiltonians should play any role in the construction or whether one can use only the generalization of flux Hamiltonians.

The original approach based on flux Hamiltonians did not use Noether currents.

(a) Magnetic flux Hamiltonians do not refer to the space-time dynamics and imply genuine rather than only effective 2-dimensionality, which is more than one wants. If the sum of the magnetic and electric flux Hamiltonians and the weak form of self duality is assumed, effective 2-dimensionality might be achieved.

   The challenge is to identify the super-partners of the flux Hamiltonians and postulate correct anti-commutation relations for the induced spinor fields to achieve anti-commutation to flux Hamiltonians. It seems that this challenge leads to ad hoc constructions.

(b) For the purposes of generalization it is useful to give the expression of flux Hamiltonian. Apart from normalization factors one would have

$$Q(H_A) = \int_{X^2} H_A J_{\mu\nu} dx^\mu \wedge dx^\nu \quad .$$

   Here A is a label for the Hamiltonian of $\delta M^4_\pm \times CP_2$ decomposing to product of $\delta M^4_\pm$ and CP_2 Hamiltonians with the first one decomposing to a product of function of the radial light-like coordinate r_M and Hamiltonian depending on S^2 coordinates. It is natural to assume that Hamiltonians have well- defined $SO(3)$ and $SU(3)$ quantum numbers. This expressions serves as a natural starting point also in the new approach based on Noether charges.

The approach identifying the Hamiltonians as symplectic Noether charges is extremely natural from physics point of view but the fact that it leads to 3-D expressions involving the induced metric led to the conclusion that it cannot work. In hindsight this conclusion seems wrong: I had not yet realized how profound that basic formulas of physics really are. If the generalization of AdS/CFT duality works, Kähler action can be expressed as a sum of string area actions for string world sheets with string area in the effective metric given as the anti-commutator of the Kähler-Dirac gamma matrices for the string world sheet so that also now a reduction of dimension takes place. This is easy to understand if the classical Noether charges vanish for a sub-algebra of symplectic algebra for preferred extremals.

(a) If all end points for strings are possible, the recipe for constructing super-conformal generators would be simple. The imbedding space Hamiltonian H_A appearing in the expression of the flux Hamiltonian given above would be replaced by the corresponding symplectic quantum Noether charge $Q(H_A)$ associated with the string defined as 1-D integral along the string. By replacing Ψ or its conjugate with a mode of the induced spinor field labeled by electroweak quantum numbers and conformal weight nm one would obtain corresponding super-charged identifiable as WCW gamma matrices. The anti-commutators of the super-charges would give rise to the elements of WCW metric labelled by conformal weights n_1, n_2 not present in the naive guess for the metric. If one assumes that the fermionic super-conformal symmetries act as gauge symmetries only $n_i = 0$ gives a non-vanishing matrix element.

   Clearly, one would have weaker form of effective 2-dimensionality in the sense that Hamiltonian would be functional of the string emanating from the partonic 2-surface. The quantum Hamiltonian would also carry information about the presence of other wormhole contacts- at least one- when wormhole throats carry Kähler magnetic monopole flux. If only discrete set for the end points for strings is possible one has discrete sum making possible easy p-adicization. It might happen that integrability conditions for the tangent spaces of string world sheets having vanishing W boson fields do not allow all possible strings.

(b) The super charges obtained in this manner are not however entirely satisfactory. The problem is that they involve only single string emanating from the partonic 2-surface. The intuitive expectation is that there can be an arbitrarily large number of strings: as the number of strings is increased the resolution improves. Somehow the super-conformal algebra defined by Hamiltonians and super-Hamiltonians should generalize to allow tensor products of the strings providing more physical information about the 3-surface.

(c) Here the idea of Yangian symmetry [K76] suggests itself strongly. The notion of Yangian emerges from twistor Grassmann approach and should have a natural place in TGD. In Yangian algebra one has besides product also co-product, which is in some sense "time-reversal" of the product. What is essential is that Yangian algebra is also multi-local.

   The Yangian extension of the super-conformal algebra would be multi-local with respect to the points of partonic surface (or multi-stringy) defining the end points of string. The basic formulas would be schematically

$$O_1^A = f^A_{BC} T^B \otimes T^B \quad ,$$

   where a summation of B, C occurs and f^A_{BC} are the structure constants of the algebra. The operation can be iterated and gives a hierarchy of n-local operators. In the recent case the operators are n-local symplectic super-charges with unit fermion number and symplectic Noether charges with a vanishing fermion number. It would be natural to assume that also the n-local gamma matrix like entities contribute via their anti-commutators to WCW metric and give multi-local information about the partonic 2-surface and 3-surface.

   The operation generating the algebra well-defined if one an assumes that the second quantization of induced spinor fields is carried out using the standard canonical quantization. One could even assume that the points involved belong to different partonic 2-surfaces belonging even at opposite boundaries of CD. The operation is also well-defined if one assumes that induced spinor fields at different space-time points at boundaries of CD always anti-commute. This could make sense at boundary of CD but lead to problems with imbedding space-causality if assumed for the spinor modes at opposite boundaries of CD.

4.4.2 Handful Of Problems With A Common Resolution

Theory building could be compared to pattern recognition or to a solving a crossword puzzle. It is essential to make trials, even if one is aware that they are probably wrong. When stares long enough to the letters which do not quite fit, one suddenly realizes what one particular crossword must actually be and it is soon clear what those other crosswords are. In the following I describe an example in which this analogy is rather concrete.

I will first summarize the problems of ordinary Dirac action based on induced gamma matrices and propose Kähler-Dirac action as their solution.

Problems associated with the ordinary Dirac action

In the following the problems of the ordinary Dirac action are discussed and the notion of Kähler-Dirac action is introduced.

Minimal 2-surface represents a situation in which the representation of surface reduces to a complex-analytic map. This implies that induced metric is hermitian so that it has no diagonal components in complex coordinates $(z, \overline{z})$ and the second fundamental form has only diagonal components of type H^k_{zz}. This implies that minimal surface is in question since the trace of the second fundamental form vanishes. At first it seems that the same must happen also in the more general case with the consequence that the space-time surface is a minimal surface. Although many basic extremals of Kähler action are minimal surfaces, it seems difficult to believe that minimal surface property plus extremization of Kähler action could really boil down to the absolute minimization of Kähler action or some other general principle selecting preferred extremals as Bohr orbits [K15, K74].

This brings in mind a similar long-standing problem associated with the Dirac equation for the induced spinors. The problem is that right-handed neutrino generates super-symmetry only provided that space-time surface and its boundary are minimal surfaces. Although one could interpret this as a geometric symmetry breaking, there is a strong feeling that something goes wrong. Induced Dirac equation and super-symmetry fix the variational principle but this variational principle is not consistent with Kähler action.

One can also question the implicit assumption that Dirac equation for the induced spinors is consistent with the super-symmetry of the WCW geometry. Super-symmetry would obviously require that for vacuum extremals of Kähler action also induced spinor fields represent vacua. This is however not the case. This super-symmetry is however assumed in the construction of WCW geometry so that there is internal inconsistency.

Super-symmetry forces Kähler-Dirac equation

The above described three problems have a common solution. Nothing prevents from starting directly from the hypothesis of a super-symmetry generated by covariantly constant right-handed neutrino and finding a Dirac action which is consistent with this super-symmetry. Field equations can be written as

$$\begin{aligned} D_\alpha T^\alpha_k &= 0 \ , \\ T^\alpha_k &= \frac{\partial}{\partial h^k{}_\alpha} L_K \ . \end{aligned} \tag{4.4.1}$$

Here T^{α}_k is canonical momentum current of Kähler action. If super-symmetry is present one can assign to this current its super-symmetric counterpart

$$\begin{aligned} J^{\alpha k} &= \overline{\nu_R}\Gamma^k T^{\alpha}_l \Gamma^l \Psi \ , \\ D_{\alpha} J^{\alpha k} &= 0 \ . \end{aligned} \tag{4.4.2}$$

having a vanishing divergence. The isometry currents currents and super-currents are obtained by contracting T^{α_k} and J^{α_k} with the Killing vector fields of super-symmetries. Note also that the super current

$$J^{\alpha} = \overline{\nu_R} T^{\alpha}_l \Gamma^l \Psi \tag{4.4.3}$$

has a vanishing divergence.

By using the covariant constancy of the right-handed neutrino spinor, one finds that the divergence of the super current reduces to

$$D_{\alpha} J^{\alpha k} = \overline{\nu_R}\Gamma^k T^{\alpha}_l \Gamma^l D_{\alpha}\Psi \ . \tag{4.4.4}$$

The requirement that this current vanishes is guaranteed if one assumes that Kähler-Dirac equation

$$\begin{aligned} \hat{\Gamma}^{\alpha} D_{\alpha}\Psi &= 0 \ , \\ \hat{\Gamma}^{\alpha} &= T^{\alpha}_l \Gamma^l \ . \end{aligned} \tag{4.4.5}$$

This equation must be derivable from a Kähler-Dirac action. It indeed is. The action is given by

$$L = \overline{\Psi}\hat{\Gamma}^{\alpha} D_{\alpha}\Psi \ . \tag{4.4.6}$$

Thus the variational principle exists. For this variational principle induced gamma matrices are replaced with Kähler-Dirac gamma matrices and the requirement

$$D_{\mu}\hat{\Gamma}^{\mu} = 0 \tag{4.4.7}$$

guaranteeing that super-symmetry is identically satisfied if the bosonic field equations are satisfied. For the ordinary Dirac action this condition would lead to the minimal surface property. What sounds strange that the essentially hydrodynamical equations defined by Kähler action have fermionic counterpart: this is very far from intuitive expectations raised by ordinary Dirac equation and something which one might not guess without taking super-symmetry very seriously.

As a matter fact, any mode of Kähler-Dirac equation contracted with second quantized induced spinor field or its conjugate defines a conserved super charge. Also super-symplectic Noether charges and their super counterparts can be assigned to symplectic generators as Noether charges but they need not be conserved.

Second quantization of the K-D action

Second quantization of Kähler-Dirac action is crucial for the construction of the Kähler metric of world of classical worlds as anti-commutators of gamma matrices identified as super-symplectic Noether charges. To get a unique result, the anti-commutation relations must be fixed uniquely. This has turned out to be far from trivial.

1. Canonical quantization works after all

The canonical manner to second quantize fermions identifies spinorial canonical momentum densities and their conjugates as $\overline{\Pi} = \partial L_{K_D}/\partial_\Psi = \overline{\Psi}\Gamma^t$ and their conjugates. The vanishing of Γ^t at points, where the induced Kähler form J vanishes can cause problems since anti-commutation relations are not internally consistent anymore. This led me to give up the canonical quantization and to consider various alternatives consistent with the possibility that J vanishes. They were admittedly somewhat ad hoc. Correct (anti-)commutation relations for various fermionic Noether currents seem however to fix the anti-commutation relations to the standard ones. It seems that it is better to be conservative: the canonical method is heavily tested and turned out to work quite nicely.

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Consider first the 4-D situation without the localization to 2-D string world sheets. The canonical anti-commutation relations would state $\{\overline{\Pi}, \Psi\} = \delta^3(x, y)$ at the space-like boundaries of the string world sheet at either boundary of CD. At points where J and thus T^t vanishes, canonical momentum density vanishes identically and the equation seems to be inconsistent.

If fermions are localized at string world sheets assumed to always carry a non-vanishing J at their boundaries at the ends of space-time surfaces, the situation changes since Γ^t is non-vanishing. The localization to string world sheets, which are not vacua saves the situation. The problem is that the limit when string approaches vacuum could be very singular and discontinuous. In the case of elementary particle strings are associated with flux tubes carrying monopole fluxes so that the problem disappears.

It is better to formulate the anti-commutation relations for the modes of the induced spinor field. By starting from

$$\{\overline{\Pi}(x), \Psi(y)\} = \delta^1(x, y) \tag{4.4.8}$$

and contracting with $\Psi(x)$ and $\Pi(y)$ and integrating, one obtains using orthonormality of the modes of Ψ the result

$$\{b_m^\dagger, b_n\} = \gamma^0 \delta_{m,n} \tag{4.4.9}$$

holding for the nodes with non-vanishing norm. At the limit $J \rightarrow 0$ there are no modes with non-vanishing norm so that one avoids the conflict between the two sides of the equation.

The proposed anti-commutator would realize the idea that the fermions are massive. The following alternative starts from the assumption of 8-D light-likeness.

2. Does one obtain the analogy of SUSY algebra? In super Poincare algebra anti-commutators of super-generators give translation generator: anti-commutators are proportional to $p^k\sigma_k$. Could it be possible to have an anti-commutator proportional to the contraction of Dirac operator $p^k\sigma_k$ of 4-momentum with quaternionic sigma matrices having or 8-momentum with octonionic 8-matrices?

This would give good hopes that the GRT limit of TGD with many-sheeted space-time replaced with a slightly curved region of M^4 in long length scales has large $\mathcal{N}$ SUSY as an approximate symmetry: $\mathcal{N}$ would correspond to the maximal number of oscillator operators assignable to the partonic 2-surface. If conformal invariance is exact, it is just the number of fermion states for single generation in standard model.

(a) The first promising sign is that the action principle indeed assigns a conserved light-like 8-momentum to each fermion line at partonic 2-surface. Therefore octonionic representation of sigma matrices makes sense and the generalization of standard twistorialization of four-momentum also. 8-momentum can be characterized by a pair of octonionic 2-spinors $(\lambda, \overline{\lambda})$ such that one has $\lambda\overline{\lambda}) = p^k\sigma_k$.

(b) Since fermion line as string boundary is 1-D curve, the corresponding octonionic subspaces is just 1-D complex ray in octonion space and imaginary axes is defined by the associated imaginary octonion unit. Non-associativity and non-commutativity play no role and it is as if one had light like momentum in say z-direction.

(c) One can select the ininitial values of spinor modes at the ends of fermion lines in such a manner that they have well-defined spin and electroweak spin and one can also form linear superpositions of the spin states. One can also assume that the 8-D algebraic variant of Dirac equation correlating M^4 and CP_2 spins is satisfied.

One can introduce oscillator operators $b^{\dagger}_{m,\alpha}$ and $b_{n,\alpha}$ with α denoting the spin. The motivation for why electroweak spin is not included as an index is due to the correlation between spin and electroweak spin. Dirac equation at fermion line implies a complete correlation between directions of spin and electroweak spin: if the directions are same for leptons (convention only), they are opposite for antileptons and for quarks since the product of them defines imbedding space chirality which distinguishes between quarks and leptons. Instead of introducing electroweak isospin as an additional correlated index one can introduce 4 kinds of oscillator operators: leptonic and quark-like and fermionic and antifermionic.

(d) For definiteness one can consider only fermions in leptonic sector. In hope of getting the analog of SUSY algebra one could modify the fermionic anti-commutation relations such that one has

$$\{b^{\dagger}_{m,\alpha}, b_{n,\beta}\} = \pm i\epsilon_{\alpha\beta}\delta_{m,n} \quad . \tag{4.4.10}$$

Here α is spin label and ϵ is the standard antisymmetric tensor assigned to twistors. The anti-commutator is clearly symmetric also now. The anti-commmutation relations with different signs $\pm$ at the right-hand side distinguish between quarks and leptons and also between fermions and anti-fermions. $\pm = 1$ could be the convention for fermions in lepton sector.

(e) One wants combinations of oscillator operators for which one obtains anti-commutators having interpretation in terms of translation generators representing in terms of 8-momentum. The guess would be that the oscillator operators are given by

$$B^{\dagger}_n = b^{\dagger}_{m,\alpha}\lambda^{\alpha} \quad , \quad B_n = \overline{\lambda}^{\alpha} b_{m,\alpha} \quad . \tag{4.4.11}$$

The anti-commutator would in this case be given by

$$\begin{aligned} &\{B_m^\dagger, B_n\} = i\overline{\lambda}^{\alpha}\epsilon_{\alpha\beta}\lambda^\beta\delta_{m,n} \\ &= Tr(p^k\sigma_k)\delta_{m,n} = 2p^0\delta_{m,n} \quad . \end{aligned} \tag{4.4.12}$$

The inner product is positive for positive value of energy p^0. This form of anti-commutator obviously breaks Lorentz invariance and this us due the number theoretic selection of preferred time direction as that for real octonion unit. Lorentz invariance is saved by the fact that there is a moduli space for the choices of the quaternion units parameterized by Lorentz boosts for CD.

The anti-commutator vanishes for covariantly constant antineutrino so that it does not generate sparticle states. Only fermions with non-vanishing four-momentum do so and the resulting algebra is very much like that associated with a unitary representation of super Poincare algebra.

(f) The recipe gives one helicity state for lepton in given mode m (conformal weight). One has also antilepton with opposite helicity with $\pm = -1$ in the formula defining the anti-commutator. In the similar manner one obtains quarks and antiquarks.

(g) Contrary to the hopes, one did not obtain the anti-commutator $p^k\sigma_k$ but $Tr(p^0\sigma_0)$. $2p^0$ is however analogous to the action of Dirac operator $p^k\sigma_k$ to a massless spinor mode with "wrong" helicity giving $2p^0\sigma^0$. Massless modes with wrong helicity are expected to appear in the fermionic propagator lines in TGD variant of twistor approach. Hence one might hope that the resulting algebra is consistent with SUSY limit.

The presence of 8-momentum at each fermion line would allow also to consider the introduction of anti-commutators of form $p^k(8)\sigma_k$ directly making $\mathcal{N} = 8$ SUSY at parton level manifest. This expression restricts for time-like M^4 momenta always to quaternion and one obtains just the standard picture.

(h) Only the fermionic states with vanishing conformal weight seem to be realized if the conformal symmetries associated with the spinor modes are realized as gauge symmetries. Super-generators would correspond to the fermions of single generation standard model: 4+4 =8 states altogether. Interestingly, $\mathcal{N} = 8$ correspond to the maximal SUSY for super-gravity. Right-handed neutrino would obviously generate the least broken SUSY. Also now mixing of M^4 helicities induces massivation and symmetry breaking so that even this SUSY is broken. One must however distinguish this SUSY from the super-symplectic conformal symmetry. The space in which SUSY would be realized would be partonic 2-surfaces and this distinguishes it from the usual SUSY. Also the conservation of fermion number and absence of Majorana spinors is an important distinction.

3. What about quantum deformations of the fermionic oscillator algebra?

Quantum deformation introducing braid statistics is of considerable interest. Quantum deformations are essentially 2-D phenomenon, and the experimental fact that it indeed occurs gives a further strong support for the localization of spinors at string world sheets. If the existence of anyonic phases is taken completely seriously, it supports the existence of the hierarchy of Planck constants and TGD view about dark matter. Note that the localization also at partonic 2-surfaces cannot be excluded yet.

I have wondered whether quantum deformation could relate to the hierarchy of Planck constants in the sense that $n = h_{eff}/h$ corresponds to the value of deformation parameter $q = exp(i2\pi/n)$.

A q-deformation of Clifford algebra of WCW gamma matrices is required. Clifford algebra is characterized in terms of anti-commutators replaced now by q-anticommutators. The natural identification of gamma matrices is as complexified gamma matrices. For q-deformation q-anti-commutators would define WCW Kähler metric. The commutators of the supergenerators should still give anti-symmetric sigma matrices. The q-anticommutation relations should be same in the entire sector of WCW considered and be induced from the q-anticommutation relations for the oscillator operators of induced spinor fields at string world sheets, and reflect the fact that permutation group has braid group as covering group in 2-D case so that braid statistics becomes possible.

In [A56] (`http://tinyurl.com/y9e6pg4d`) the q-deformations of Clifford algebras are discussed, and this discussion seems to apply in TGD framework.

(a) It is assumed that a Lie-algebra g has action in the Clifford algebra. The q-deformations of Clifford algebra is required to be consistent with the q-deformation of the universal enveloping algebra Ug.

(b) The simplest situation corresponds to group $su(2)$ so that Clifford algebra elements are labelled by spin $\pm 1/2$. In this case the q-anticommutor for creation operators for spin up states reduces to an anti-commutator giving q-deformation I_q of unit matrix but for the spin down states one has genuine q-anti-commutator containing besides I_q also number operator for spin up states at the right hand side.

(c) The undeformed anti-commutation relations can be witten as

$$P^{+kl}_{ij}a_ka_l = 0 \ , \quad P^{+kl}_{ij}a^{\dagger}_ka^{\dagger}_l = 0 \ , \quad a^ia^{\dagger}_j + P^{ih}_{jk}a^{\dagger}_ha^k = \delta^i_j 1 \ . \tag{4.4.13}$$

Here $P^{kl}_{ij} = \delta^i_l\delta^j_k$ is the permutator and $P^{+kl}_{ij} = (1+P)/2$ is projector. The q-deformation reduces to a replacement of the permutator and projector with q-permutator P_q and q-projector and P^+_q, which are both fixed by the quantum group.

(d) Also the condition that deformed algebra has same Poincare series as the original one is posed. This says that the representation content is not changed that is the dimensions of summands in a representation as direct sum of graded sub-spaces are same for algebra and its q-deformation. If one has quantum group in a strict sense of the word (quasi-triangularity (genuine braid group) rather that triangularity requiring that the square of the deformed permutator P_q is unit matrix, one can have two situations.

   i. $g = sl(N)$ (special linear group such as $SL(2,F)$, $F = R, C$) or $g = Sp(N = 2n)$ (symplectic group such as $Sp(2) = SL(2,R)$), which is subgroup of $sl(N)$. Creation (annihilation-) operators must form the N-dimensional defining representation of g.

   ii. $g = sl(N)$ and one has direct sum of M N-dimensional defining representations of g. The M copies of representation are ordered so that they can be identified as strands of braid so that the deformation makes sense at the space-like ends of string world sheet naturally. q-projector is proportional to so called universal R-matrix.

(e) It is also shown that q-deformed oscillator operators can be expressed as polynomials of the ordinary ones.

The following argument suggest that the g must correspond to the minimal choices $sl(2,R)$ (or $su(2)$) in TGD framework.

(a) The q-Clifford algebra structure of WCW should be induced from that for the fermionic oscillator algebra. g cannot correspond to $su(2)_{spin}\times su(2)_{ew}$ since spin and weak isospin label fermionic oscillator operators beside conformal weights but must relate closely to this group. The physical reason is that the separate conservation of quark and lepton numbers and light-likeness in 8-D sense imply correlations between the components of the spinors and reduce g.

(b) For a given H-chirality (quark/ lepton) 8-D light-likeness forced by massless Dirac equation at the light-like boundary of the string world sheet at parton orbit implies correlation between M^4 and CP_2 chiralities. Hence there are 4+4 spinor components corresponding to fermions and antifermions with physical (creation oeprators) and unphysical (annihilation operators) polarizations. This allows two creation operators with given

H-chirality (quark or lepton) and fermion number. Same holds true for antifermions. By fermion number conservation one obtains a reduction to $SU(2)$ doublets and the quantum group would be $sl(2) = sp(2)$ for which "special linear" implies "symplectic".

4.5 Quantum Criticality And Kähler-Dirac Action

The precise mathematical formulation of quantum criticality has remained one of the basic challenges of quantum TGD. The belief has been that the existence of conserved current for Kähler-Dirac equation are possible if Kähler action is critical for the 3-surface in question in the sense that the deformation in question corresponds to vanishing of second variation of Kähler action. The vanishing of the second variation states that the deformation of the Kähler-Dirac gamma matrix is divergence free just like the Kähler-Dirac gamma matrix itself and is therefore very natural.

2-D conformal invariance accompanies 2-D criticality and allows to satisfy these conditions for spinor modes localized at 2-D surfaces - string world sheets and possibly also partonic 2-surfaces. This localization is in the generic case forced by the conditions that em charge is well-defined for the spinor modes: this requires that classical W fields vanish and also the vanishing of classical Z^0 field is natural -at least above weak scale. Only 2 Kähler-Dirac gamma matrices can be non-vanishing and this is possible only for Kähler-Dirac action.

4.5.1 What Quantum Criticality Could Mean?

Quantum criticality is one of the basic guiding principles of Quantum TGD. What it means mathematically is however far from clear and one can imagine several meanings for it.

(a) What is obvious is that quantum criticality implies quantization of Kähler coupling strength as a mathematical analog of critical temperature so that the theory becomes mathematically unique if only single critical temperature is possible. Physically this means the presence of long range fluctuations characteristic for criticality and perhaps assignable to the effective hierarchy of Planck constants having explanation in terms of effective covering spaces of the imbedding space. This hierarchy follows from the vacuum degeneracy of Kähler action, which in turn implies 4-D spin-glass degeneracy. It is easy to interpret the degeneracy in terms of criticality.

(b) At more technical level one would expect criticality to correspond to deformations of a given preferred extremal defining a vanishing second variation of Kähler Khler function or Kähler action.

   i. For Kähler function this criticality is analogous to thermodynamical criticality. The Hessian matrix defined by the second derivatives of free energy or potential function becomes degenerate at criticality as function of control variables which now would be naturally zero modes not contribution to Kähler metric of WCW but appearing as parameters in it. The bevavior variables correspond to quantum fluctuating degrees of freedom and according to catastrophe theory a big change can in quantum fluctuating degrees of freedom at criticality for zero modes. This would be control of quantum state by varying classical variables. Cusp catastrophe is standard example of this. One can imagined also a situation in which the roles of zero modes and behavior variables change and big jump in the values of zero modes is induced by small variation in behavior variables. This would mean quantum control of classical variables.

   ii. Zero modes controlling quantum fluctuating variables in Kähler function would correspond to vanishing of also second derivatives of potential function at extremum in certain directions so that the matrix defined by second derivatives does not have maximum rank. Entire hierarchy of criticalities is expected and a good finite-dimensional model is provided by the catastrophe theory of Thom [A52]. Cusp catastrophe (see `http://tinyurl.com/yddpfdgo`) [A3] is the simplest catastrophe one can think of, and here the folds of cusp where discontinuous jump occurs correspond to criticality with respect to one control variable and the tip to criticality with respect to both control variables.

(c) Quantum criticality makes sense also for Kähler action.

i. Now one considers space-time surface connecting which 3-surfaces at the boundaries of CD. The non-determinism of Kähler action allows the possibility of having several space-time sheets connecting the ends of space-time surface but the conditions that classical charges are same for them reduces this number so that it could be finite. Quantum criticality in this sense implies non-determinism analogous to that of critical systems since preferred extremals can co-incide and suffer this kind of bifurcation in the interior of CD. This quantum criticality can be assigned to the hierarchy of Planck constants and the integer n in $h_{eff} = n \times h$ [K22] corresponds to the number of degenerate space-time sheets with same Kähler action and conserved classical charges.

ii. Also now one expects a hierarchy of criticalitiesandsince criticality and conformal invariance are closely related, a natural conjecture is that the fractal hierarchy of sub-algebras of conformal algebra isomorphic to conformal algebra itself and having conformal weights coming as multiples of n corresponds to the hierarchy of Planck constants. This hierarchy would define a hierarchy of symmetry breakings in the sense that only the sub-algebra would act as gauge symmetries.

iii. The assignment of this hierarchy with super-symplectic algebra having conformal structure with respect to the light-like radial coordinate of light-cone boundary looks very attractive. An interesting question is what is the role of the super-conformal algebra associated with the isometries of light-cone boundary $R_+ \times S^2$ which are conformal transformations of sphere S^2 with a scaling of radial coordinate compensating the scaling induced by the conformal transformation. Does it act as dynamical or gauge symmetries?

(d) I have discussed what criticality could mean for Kähler-Dirac action [K88].

i. I have conjectured that it leads to the existence of additional conserved currents defined by the variations which do not affect the value of Kähler action. These arguments are far from being mathematically rigorous and the recent view about the solutions of the Kähler-Dirac equation predicting that the spinor modes are restricted to 2-D string world sheets requires a modification of these arguments.

ii. The basic challenge is to understand the mechanism making this kind of currents conserved: the same challenge is met already in the case of isometries since imbedding space coordinates appear as parameters in Kähler-Dirac action. Kähler-Dirac equation is satisfied if the first variation of the canonical momentum densities contracted with the imbedding space gamma matrices annihilates the spinor mode. Situation is analogous to massless Dirac equation: it does not imply the vanishing of four-momentum, only the vanishing of mass. One obtains conserved fermion current associated with deformations only if the deformation of the Kähler-Dirac gamma matrix is divergenceless just like the Kähler-Dirac gamma matrix itself. This conditions requires the vanishing of the second variation of Kähler action.

iii. It is far from obvious that these conditions can be satisfied. The localization of the spinor modes to string world sheets or partonic 2-surfaces guaranteeing in the generic case that em charge is well-defined for spinor modes implies holomorphy allowing to formulate current conservation for the deformations of the space-time surface for second quantized induced spinor field. The crux is that the deformation respects the holomorphy properties of the Kähler-Dirac gamma matrices at string world sheet and thus does not mix Γ^z with $\Gamma^{\overline{z}}$. The deformation of Γ^z has only z-component and also annihilates the holomorphic spinor.

This mechanism is possible only for Kähler-Dirac action since the Kähler-Dirac gamma matrices in directions orthogonal to the 2-surface must vanish and this is not possible for other actions. This also means that energy momentum tensor has rank 2 as a matrix. Cosmic string solutions are an exception since in this case

CP_2 projection of space-time surface is 2-D and conditions guaranteing vanishing of classical W fields can be satisfied without the restriction to 2-surface.

The vacuum degeneracy of Kähler action strongly suggests that the number of critical deformations is always infinite and that these deformations define an infinite inclusion hierarchy of super-conformal algebras. This inclusion hierarchy would correspond to a fractal hierarchy of breakings of super-conformal symmetry generalizing the symmetry breaking hierarchies of gauge theories. These super-conformal inclusion hierarchies would realize the inclusion hierarchies for hyper-finite factors of type II_1.

4.5.2 Quantum Criticality And Fermionic Representation Of Conserved Charges Associated With Second Variations Of Kähler Action

It is rather obvious that TGD allows a huge generalizations of conformal symmetries. The development of the understanding of conservation laws has been however slow. Kähler-Dirac action provides excellent candidates for quantum counterparts of Noether charges. The problem is that the imbedding space coordinates are in the role of classical external fields and induces spinor fields are second quantized so that it is not at all clear whether one obtains conserved charges.

What does the conservation of the fermionic Noether current require?

The obvious anser to the question of the title is that the conservation of the fermionic current requires the vanishing of the first variation of Kähler-Dirac action with respect to imbedding space coordinates. This is certainly true but need not mean vanishing of the second variation of Kähler action as thought first. Hence fermionic conserved currents might be obtained for much more general variations than critical ones.

(a) The Kähler-Dirac action assigns to a deformation of the space-time surface a conserved charge expressible as bilinears of fermionic oscillator operators only if the first variation of the Kähler-Dirac action under this deformation vanishes.

The vanishing of the first variation for the Kähler-Dirac action is equivalent with the vanishing of the second variation for the Kähler action. This can be seen by the explicit calculation of the second variation of the Kähler-Dirac action and by performing partial integration for the terms containing derivatives of Ψ and $\overline{\Psi}$ to give a total divergence representing the difference of the charge at upper and lower boundaries of the causal diamond plus a four-dimensional integral of the divergence term defined as the integral of the quantity

$$\begin{aligned} \Delta S_D &= \overline{\Psi}\Gamma^k D_\alpha J^\alpha_k \Psi \ , \\ J^\alpha_k &= \frac{\partial^2 L_K}{\partial h^k_\alpha \partial h^l_\beta}\delta h^k_\beta + \frac{\partial^2 L_K}{\partial h^k_\alpha \partial h^l}\delta h^l \ . \end{aligned} \tag{4.5.1}$$

Here h^k_β denote partial derivative of the imbedding space coordinates with respect to space-time coordinates. ΔS_D vanishes if this term vanishes:

$$D_\alpha J^\alpha_k = 0 \ .$$

The condition states the vanishing of the second variation of Kähler action. This can of course occur only for preferred deformations of X^4. One could consider the possibility that these deformations vanish at light-like 3-surfaces or at the boundaries of CD. Note that covariant divergence is in question so that J^α_k does not define conserved classical charge in the general case.

(b) This condition is however un-necessarily strong. It is enough that that the deformation of Dirac operator anihilates the spinor mode, which can also change in the deformation. It must be possible to compensate the change of the covariant derivative in the deformation by a gauge transformation which requires that deformations act as gauge transformations on induce gauge potentials. This gives additional constraint and strongly suggests Kac-Moody type algebra for the deformations. Conformal transformations would satisfy this constraint and are suggested by quantum criticality.

(c) It is essential that the Kähler-Dirac equation holds true so that the Kähler-Dirac action vanishes: this is needed to cancel the contribution to the second variation coming from the determinant of the induced metric. The condition that the Kähler-Dirac equation is satisfied for the deformed space-time surface requires that also Ψ suffers a transformation determined by the deformation. This gives

$$\delta\Psi = -\frac{1}{D} \times \Gamma^k J_k^\alpha \Psi \ . \tag{4.5.2}$$

Here $1/D$ is the inverse of the Kähler-Dirac operator defining the counterpart of the fermionic propagator.

(d) The fermionic conserved currents associated with the deformations are obtained from the standard conserved fermion current

$$J^\alpha = \overline{\Psi}\Gamma^\alpha\Psi \ . \tag{4.5.3}$$

Note that this current is conserved only if the space-time surface is extremal of Kähler action: this is also needed to guarantee Hermiticity and same form for the Kähler-Dirac equation for Ψ and its conjugate as well as absence of mass term essential for super-conformal invariance. Note also that ordinary divergence rather only covariant divergence of the current vanishes.

The conserved currents are expressible as sums of three terms. The first term is obtained by replacing Kähler-Dirac gamma matrices with their increments in the deformation keeping Ψ and its conjugate constant. Second term is obtained by replacing Ψ with its increment $\delta\Psi$. The third term is obtained by performing same operation for $\delta\overline{\Psi}$.

$$J^\alpha = \overline{\Psi}\Gamma^k J_k^\alpha\Psi + \overline{\Psi}\hat{\Gamma}^\alpha\delta\Psi + \delta\overline{\Psi}\hat{\Gamma}^\alpha\Psi \ . \tag{4.5.4}$$

These currents provide a representation for the algebra defined by the conserved charges analogous to a fermionic representation of Kac-Moody algebra.

(e) Also conserved super charges corresponding to super-conformal invariance are obtained. The first class of super currents are obtained by replacing Ψ or $\overline{\Psi}$ right handed neutrino spinor or its conjugate in the expression for the conserved fermion current and performing the above procedure giving two terms since nothing happens to the covariantly constant right handed-neutrino spinor. Second class of conserved currents is defined by the solutions of the Kähler-Dirac equation interpreted as c-number fields replacing Ψ or $\overline{\Psi}$ and the same procedure gives three terms appearing in the super current.

(f) The existence of vanishing of second variations is analogous to criticality in systems defined by a potential function for which the rank of the matrix defined by second derivatives of the potential function vanishes at criticality. Quantum criticality becomes the prerequisite for the existence of quantum theory since fermionic anti-commutation relations in principle can be fixed from the condition that the algebra in question is equivalent with the algebra formed by the vector fields defining the deformations of the space-time surface defining second variations. Quantum criticality in this sense would also select preferred extremals of Kähler action as analogs of Bohr orbits and the spectrum of preferred extremals would be more or less equivalent with the expected existence of infinite-dimensional symmetry algebras.

It is far from obvious that the criticality conditions or even the weaker conditions guaranteeing the existence of (say) isometry charges can be satisfied. It seems that the restriction of spinor modes to 2-D surfaces - string world sheets and possibly also partonic 2-surfaces - implied by the condition that em charge is well-define for them, is the manner to achieve this. The reason is that conformal invariance allows complexification of the Kähler-Dirac gamma matrices and allows to construct spinor modes as holomorphic modes and their conjugates. Holomorphy reduces K-D equation to algebraic condition that Γ^z annihilates the spinor mode. If this is true also the deformation of Γ^z then the existince of conserved current follows. It is essential that only two Kähler-Dirac gamma matrices are non-vanishing and this is possible only for Kähler-Dirac action.

About the general structure of the algebra of conserved charges

Some general comments about the structure of the algebra of conserved charges are in order.

(a) Any Cartan algebra of the isometry group $P \times SU(3)$ (there are two types of them for P corresponding to linear and cylindrical Minkowski coordinates) defines critical deformations (one could require that the isometries respect the geometry of CD). The corresponding second order charges for Kähler action are conserved but vanish since the corresponding conjugate coordinates are cyclic for the Kähler metric and Kähler form so that the conserved current is proportional to the gradient of a Killing vector field which is constant in these coordinates.

(b) Contrary to the original conclusion, the corresponding fermionic charges expressible as fermionic bilinears are first order in deformation and do not vanish! Four-momentum and color quantum numbers are defined for Kähler action as classical conserved quantities and for Kähler-Dirac action as quantal charges.

Critical manifold is infinite-dimensional for Kähler action

Some examples might help to understand what is involved.

(a) The action defined by four-volume gives a first glimpse about what one can expect. In this case Kähler-Dirac gamma matrices reduce to the induced gamma matrices. Second variations satisfy d'Alembert type equation in the induced metric so that the analogs of massless fields are in question. Mass term is present only if some dimensions are compact. The vanishing of excitations at light-like boundaries is a natural boundary condition and might well imply that the solution spectrum could be empty. Hence it is quite possible that four-volume action leads to a trivial theory.

(b) For the vacuum extremals of Kähler action the situation is different. There exists an infinite number of second variations and the classical non-determinism suggests that deformations vanishing at the light-like boundaries exist. For the canonical imbedding of M^4 the equation for second variations is trivially satisfied. If the CP_2 projection of the vacuum extremal is one-dimensional, the second variation contains a non-vanishing term and an equation analogous to massless d'Alembert equation for the increments of CP_2 coordinates is obtained. Also for the vacuum extremals of Kähler action with 2-D CP_2 projection all terms involving induced Kähler form vanish and the field equations reduce to d'Alembert type equations for CP_2 coordinates. A possible interpretation is as the classical analog of Higgs field. For the deformations of non-vacuum extremals this would suggest the presence of terms analogous to mass terms: these kind of terms indeed appear and are proportional to δs_k. M^4 degrees of freedom decouple completely and one obtains QFT type situation.

(c) The physical expectation is that at least for the vacuum extremals the critical manifold is infinite-dimensional. The notion of finite measurement resolution suggests infinite hierarchies of inclusions of hyper-finite factors of type II_1 possibly having interpretation in terms of inclusions of the super conformal algebras defined by the critical deformations.

(d) The properties of Kähler action give support for this expectation. The critical manifold is infinite-dimensional in the case of vacuum extremals. Canonical imbedding of M^4 would correspond to maximal criticality analogous to that encountered at the tip of the cusp catastrophe. The natural guess would be that as one deforms the vacuum extremal the previously critical degrees of freedom are transformed to non-critical ones. The dimension of the critical manifold could remain infinite for all preferred extremals of the Kähler action. For instance, for cosmic string like objects any complex manifold of CP_2 defines cosmic string like objects so that there is a huge degeneracy is expected also now. For CP_2 type vacuum extremals M^4 projection is arbitrary light-like curve so that also now infinite degeneracy is expected for the deformations.

This leads to the conjecture that the critical deformations correspond to sub-algebras of super-conformal algebras with conformal weights coming as integer multiples of fixed integer m. One would have infinite hierarchy of breakings of conformal symmetry labelled by m. The super-conformal algebras would be effectively m-dimensional. Since all commutators with the critical sub-algebra would create zero energy states. In ordinary conformal field theory one have maximal criticality corresponding to $m = 1$.

Critical super-algebra and zero modes

The relationship of the critical super-algebra to WCW geometry is interesting.

(a) The vanishing of the second variation plus the identification of Kähler function as a Kähler action for preferred extremals means that the critical variations are orthogonal to all deformations of the space-time surface with respect to the WCW metric.

The original expectation was that critical deformations correspond to zero modes but this interpretation need not be correct since critical deformations can leave 3-surface invariant but affect corresponding preferred extremal: this would conform with the non-deterministic character of the dynamics which is indeed the basic signature of criticality. Rather, critical deformations are limiting cases of ordinary deformations acting in quantum fluctuating degrees of freedom.

This conforms with the fact that WCW metric vanishes identically for canonically imbedded M^4 and that Kähler action has fourth order terms as first non-vanishing terms in perturbative expansion (for Kähler-Dirac the expansion is quadratic in deformation).

Therefore the super-conformal algebra associated with the critical deformations has genuine physical content.

(b) Since the action of X^4 local Hamiltonians of $\delta M^4_\times CP_2$ corresponds to the action in quantum fluctuating degrees of freedom, critical deformations cannot correspond to this kind of Hamiltonians.

(c) The notion of finite measurement resolution suggests that the degrees of freedom which are below measurement resolution correspond to vanishing gauge charges. The sub-algebras of critical super-conformal algebra for which charges annihilate physical states could correspond to this kind of gauge algebras.

(d) The conserved super charges associated with the vanishing second variations cannot give WCW metric as their anti-commutator. This would also lead to a conflict with the effective 2-dimensionality stating that WCW line-element is expressible as sum of contribution coming from partonic 2-surfaces as also with fermionic anti-commutation relations.

Connection with quantum criticality

The notion of quantum criticality of TGD Universe was originally inspired by the question how to make TGD unique if Kähler function for WCW is defined by the Kähler action for a preferred extremal assignable to a given 3-surface. Vacuum functional defined by the exponent of Kähler function is analogous to thermodynamical weight and the obviou idea with Kähler coupling strength taking the role of temperature. The obvious idea was that the value of Kähler coupling strength is analogous to critical temperature so that TGD would be more or less uniquely defined.

To understand the delicacies it is convenient to consider various variations of Kähler action first.

(a) The variation can leave 3-surface invariant but modify space-time surface such that Kähler action remains invariant. In this case infinitesimal deformation reduces to a diffeomorphism at space-like 3-surface and perhaps also at light-like 3-surfaces. In this case the correspondence between X^3 and $X^4(X^3)$ would not be unique and one would have non-deterministic dynamics characteristic for critical systems. This criticality would correspond to criticality of Kähler action at X^3. Note that the original working hypothesis was that $X^4(X^3)$ is unique. The failure of the strict classical determinism implying spin glass type vacuum degeneracy indeed suggets that this is the case.

(b) The variation could act on zero modes which do not affect Kähler metric which corresponds to (1, 1) part of Hessian in complex coordinates for WCW . Only the zero modes characterizing 3-surface appearing as parameters in the metric WCW would be affected and the result would be a generalization of conformal transformation. Kähler function would change but only due to the change in zero modes. These transformations do not seem to correspond to critical transformations since Kähler function changes.

(c) The variation could act on 3-surface both in zero modes and dynamical degrees of freedom represented by complex coordinates. It would of course affect also the space-time surface. Criticality for Kähler function would mean that Kähler metric has zero modes at X^3 meaning that (1, 1) part of Hessian is degenerate. This could mean that in the vicinity of X^3 the Kähler form has non-definite signature: physically this is unacceptable since inner product in Hilbert space would not be positive definite.

Critical transformations might relate closely to the coset space decomposition of WCW to a union of coset spaces G/H labelled by zero modes.

(a) The critical deformations leave 3-surface X^3 invariant as do also the transformations of H associated with X^3. If H affects $X^4(X^3)$ and corresponds to critical transformations then critical transformation would extend WCW to a bundle for which 3-surfaces would be base points and preferred extremals $X^4(X^3)$ would define the fiber. Gauge invariance with respect to H would generalize the assumption that $X^4(X^3)$ is unique.

(b) Critical deformations could correspond to H or sub-group of H (which dependes on X^3). For other 3-surfaces than X^3 the action of H is non-trivial as the case of $CP_2 = SU(3)/U(2)$ makes easy to understand.

(c) A possible identification of Lie-algebra of H is as a sub-algebra of Virasoro algebra associated with the symplectic transformations of $\delta M^4 \times CP_2$ and acting as diffeomorphisms for the light-like radial coordinate of δM^4_+. The sub-algebras of Virasoro algebra have conformal weights coming as integer multiplies of a given conformal weight m and form inclusion hierarchies suggesting a direct connection with finite measurement resolution realized in terms of inclusions of hyperfinite factors of type II_1. For $m > 1$ one would have breaking of maximal conformal symmetry. The action of these Virasoro algebra on symplectic algebra would make the corresponding sub-algebras gauge degrees of freedom so that the number of symplectic generators generating non-gauge transformations would be finite. This result is not surprising since also for 2-D critical systems criticality corresponds to conformal invariance acting as local scalings.

The vanishing of the second variation for some deformations means that the system is critical, in the recent case quantum critical. Basic example of criticality is bifurcation diagram for cusp catastrophe. Quantum criticality realized as the vanishing of the second variation gives hopes about a more or less unique identification of preferred extremals and considered alternative identifications such as absolute minimization of Kähler action which is just the opposite of criticality.

One must be very cautious here: there are two criticalities: one for the extremals of Kähler action with respect to deformations of four-surface and second for the Kähler function itself with respect to deformations of 3-surface: these criticalities are not equivalent since in the latter case variation respects preferred extremal property unlike in the first case.

(a) The criticality for preferred extremals would make 4-D criticality a property of all physical systems.

(b) The criticality for Kähler function would be 3-D and might hold only for very special systems. In fact, the criticality means that some eigenvalues for the Hessian of Kähler function vanish and for nearby 3-surfaces some eigenvalues are negative. On the other hand the Kähler metric defined by (1, 1) part of Hessian in complex coordinates must be positive definite. Thus criticality might imply problems.

    This allows and suggests non-criticality of Kähler function coming from Kähler action for Euclidian space-time regions: this is mathematically the simplest situation since in this case there are no zero modes causing troubles in Gaussian approximation to functional integral. The Morse function coming from Kähler action in Minkowskian as imaginary contribution analogous to that appearing in path integral could be critical and allow non-definite signature in principle. In fact this is expected by the defining properties of Morse function.

(c) The almost 2-dimensionality implied by strong form of holography suggests that the interior degrees of freedom of 3-surface can be regarded almost gauge degrees of freedom and that this relates directly to generalised conformal symmetries associated with symplectic isometries of WCW . These degrees of freedom are not critical in the sense inspired by G/H decomposition. The only plausible interaction seems to be that these degrees of freedom correspond to deformations in zero modes.

Both the super-symmetry of D_K and conservation Dirac Noether currents for Kähler-Dirac action have thus a connection with quantum criticality.

(a) Finite-dimensional critical systems defined by a potential function $V(x^1, x^2, ..)$ are characterized by the matrix defined by the second derivatives of the potential function and the rank of system classifies the levels in the hierarchy of criticalities. Maximal criticality corresponds to the complete vanishing of this matrix. Thom's catastrophe theory classifies these hierarchies, when the numbers of behavior and control variables are small (smaller than 5). In the recent case the situation is infinite-dimensional and the criticality conditions give additional field equations as existence of vanishing second variations of Kähler action.

(b) The vacuum degeneracy of Kähler action allows to expect that this kind infinite hierarchy of criticalities is realized. For a general vacuum extremal with at most 2-D CP_2 projection the matrix defined by the second variation vanishes because $J_{\alpha\beta} = 0$ vanishes and also the matrix $(J^{\alpha}_{\ k} + J_k^{\ \alpha})(J^{\beta}_{\ l} + J_l^{\ \beta})$ vanishes by the antisymmetry $J^{\alpha}_{\ k} = -J_k^{\ \alpha}$. The formulation of quantal version of Equivalence Principle (EP) in string picture demonstrates that the conservation of of fermionic Noether currents defining gravitational four-momentum and other Poincare quantum numbers requires that the deformation of the Kähler-Dirac equation obtained by replacing Kähler-Dirac gamma matrices with their deformations is also satisfied. Holomorphy can guarantee this. The original wrong conclusion was that this condition is equivalent with much stronger condition stating the vanishing of the second variation of Kähler action, which it is not. There is analogy for this: massless Dirac equation does not imply the vanishing of four-momentum.

(c) Conserved bosonic and fermionic Noether charges would characterize quantum criticality. In particular, the isometries of the imbedding space define conserved currents represented in terms of the fermionic oscillator operators if the second variations defined by the infinitesimal isometries vanish for the Kähler-Dirac action. For vacuum extremals the dimension of the critical manifold is infinite: maybe there is hierarchy of quantum criticalities for which this dimension decreases step by step but remains always infinite. This hierarchy could closely relate to the hierarchy of inclusions of hyper-finite factors of type II_1. Also the conserved charges associated with super-symplectic and Super Kac-Moody algebras would require infinite-dimensional critical manifold defined by the spectrum of second variations.

(d) Phase transitions are characterized by the symmetries of the phases involved with the transitions, and it is natural to expect that dynamical symmetries characterize the hierarchy of quantum criticalities. The notion of finite quantum measurement resolution based on the hierarchy of Jones inclusions indeed suggests the existence of a hierarchy of dynamical gauge symmetries characterized by gauge groups in ADE hierarchy [K22] with degrees of freedom below the measurement resolution identified as gauge degrees of freedom.

(e) Does this criticality have anything to do with the criticality against the phase transitions changing the value of Planck constant? If the geodesic sphere S^2_I for which induced Kähler form vanishes corresponds to the back of the CP_2 book (as one expects), this could be the case. The homologically non-trivial geodesic sphere S^12_{II} is as far as possible from vacuum extremals. If it corresponds to the back of CP_2 book, cosmic strings would be quantum critical with respect to phase transition changing Planck constant. They cannot however correspond to preferred extremals.

4.5.3 Preferred Extremal Property As Classical Correlate For Quantum Criticality, Holography, And Quantum Classical Correspondence

The Noether currents assignable to the Kähler-Dirac equation are conserved only if the first variation of the Kähler-Dirac operator D_K defined by Kähler action vanishes. This is equivalent with the vanishing of the second variation of Kähler action -at least for the variations corresponding to dynamical symmetries having interpretation as dynamical degrees of freedom which are below measurement resolution and therefore effectively gauge symmetries.

The vanishing of the second variation in interior of $X^4(X^3_l)$ is what corresponds exactly to quantum criticality so that the basic vision about quantum dynamics of quantum TGD would lead directly to a precise identification of the preferred extremals. Something which I should have noticed for more than decade ago! The question whether these extremals correspond to absolute minima remains however open.

The vanishing of second variations of preferred extremals -at least for deformations representing dynamical symmetries, suggests a generalization of catastrophe theory of Thom, where the rank of the matrix defined by the second derivatives of potential function defines a hierarchy of criticalities with the tip of bifurcation set of the catastrophe representing the complete vanishing of this matrix. In the recent case this theory would be generalized to infinite-dimensional context. There are three kind of variables now but quantum classical correspondence (holography) allows to reduce the types of variables to two.

(a) The variations of $X^4(X^3_l)$ vanishing at the intersections of $X^4(X^3_l)$ with the light-like boundaries of causal diamonds CD would represent behavior variables. At least the vacuum extremals of Kähler action would represent extremals for which the second variation vanishes identically (the "tip" of the multi-furcation set).

(b) The zero modes of Kähler function would define the control variables interpreted as classical degrees of freedom necessary in quantum measurement theory. By effective 2-dimensionality (or holography or quantum classical correspondence) meaning that the configuration space metric is determined by the data coming from partonic 2-surfaces X^2 at intersections of X^3_l with boundaries of CD, the interiors of 3-surfaces X^3 at the boundaries of CDs in rough sense correspond to zero modes so that there is indeed huge number of them. Also the variables characterizing 2-surface, which cannot be complexified and thus cannot contribute to the Kähler metric of WCW represent zero modes. Fixing the interior of the 3-surface would mean fixing of control variables. Extremum property would fix the 4-surface and behavior variables if boundary conditions are fixed to sufficient degree.

(c) The complex variables characterizing X^2 would represent third kind of variables identified as quantum fluctuating degrees of freedom contributing to the WCW metric. Quantum classical correspondence requires 1-1 correspondence between zero modes and these variables. This would be essentially holography stating that the 2-D "causal boundary" X^2 of $X^3(X^2)$ codes for the interior. Preferred extremal property identified as criticality condition would realize the holography by fixing the values of zero modes once X^2 is known and give rise to the holographic correspondence $X^2 \rightarrow X^3(X^2)$. The values of behavior variables determined by extremization would fix then the space-time surface $X^4(X^3_l)$ as a preferred extremal.

(d) Clearly, the presence of zero modes would be absolutely essential element of the picture. Quantum criticality, quantum classical correspondence, holography, and preferred extremal property would all represent more or less the same thing. One must of course be very cautious since the boundary conditions at X^3_l involve normal derivative and might bring in delicacies forcing to modify the simplest heuristic picture.

(e) There is a possible connection with the notion of self-organized criticality [B7] introduced to explain the behavior of systems like sand piles. Self-organization in these systems tends to lead "to the edge". The challenge is to understand how system ends up to a critical state, which by definition is unstable. Mechanisms for this have been discovered and based on phase transitions occurring in a wide range of parameters so that critical point extends to a critical manifold. In TGD Universe quantum criticality suggests a universal mechanism of this kind. The criticality for the preferred extremals of Kähler action would mean that classically all systems are critical in well-defined sense and the question is only about the degree of criticality. Evolution could be seen as a process leading gradually to increasingly critical systems. One must however distinguish between the criticality associated with the preferred extremals of Kähler action and the criticality caused by the spin glass like energy landscape like structure for the space of the maxima of Kähler function.

4.5.4 Quantum Criticality And Electroweak Symmetries

In the following quantum criticali and electroweak symmetries are discussed for Kähler-Dirac action.

What does one mean with quantum criticality?

Quantum criticality is one of the basic guiding principles of Quantum TGD. What it means mathematically is however far from clear and one can imagine several meanings for it.

(a) What is obvious is that quantum criticality implies quantization of Kähler coupling strength as a mathematical analog of critical temperature so that the theory becomes mathematically unique if only single critical temperature is possible. Physically this means the presence of long range fluctuations characteristic for criticality and perhaps assignable to the effective hierarchy of Planck constants having explanation in terms of effective covering spaces of the imbedding space. This hierarchy follows from the vacuum degeneracy of Kähler action, which in turn implies 4-D spin-glass degeneracy. It is easy to interpret the degeneracy in terms of criticality.

(b) At more technical level one would expect criticality to corresponds to deformations of a given preferred extremal defining a vanishing second variation of Kähler Khler function or Kähler action.

   i. For Kähler function this criticality is analogous to thermodynamical criticality. The Hessian matrix defined by the second derivatives of free energy or potential function becomes degenerate at criticality as function of control variables which now would be naturally zero modes not contribution to Kähler metric of WCW but appearing as parameters in it. The bevavior variables correspond to quantum fluctuating degrees of freedom and according to catastrophe theory a big change can in quantum fluctuating degrees of freedom at criticality for zero modes. This would be control of quantum state by varying classical variables. Cusp catastrophe is standard example of this. One can imagined also a situation in which the roles of zero modes and behavior variables change and big jump in the values of zero modes is induced by small variation in behavior variables. This would mean quantum control of classical variables.

ii. Zero modes controlling quantum fluctuating variables in Kähler function would correspond to vanishing of also second derivatives of potential function at extremum in certain directions so that the matrix defined by second derivatives does not have maximum rank. Entire hierarchy of criticalities is expected and a good finite-dimensional model is provided by the catastrophe theory of Thom [A52]. Cusp catastrophe (see `http://tinyurl.com/yddpfdgo`) [A3] is the simplest catastrophe one can think of, and here the folds of cusp where discontinuous jump occurs correspond to criticality with respect to one control variable and the tip to criticality with respect to both control variables.

(c) Quantum criticality makes sense also for Kähler action.

i. Now one considers space-time surface connecting which 3-surfaces at the boundaries of CD. The non-determinism of Kähler action allows the possibility of having several space-time sheets connecting the ends of space-time surface but the conditions that classical charges are same for them reduces this number so that it could be finite. Quantum criticality in this sense implies non-determinism analogous to that of critical systems since preferred extremals can co-incide and suffer this kind of bifurcation in the interior of CD. This quantum criticality can be assigned to the hierarchy of Planck constants and the integer n in $h_{eff} = n \times h$ [K22] corresponds to the number of degenerate space-time sheets with same Kähler action and conserved classical charges.

ii. Also now one expects a hierarchy of criticalitiesandsince criticality and conformal invariance are closely related, a natural conjecture is that the fractal hierarchy of sub-algebras of conformal algebra isomorphic to conformal algebra itself and having conformal weights coming as multiples of n corresponds to the hierarchy of Planck constants. This hierarchy would define a hierarchy of symmetry breakings in the sense that only the sub-algebra would act as gauge symmetries.

iii. The assignment of this hierarchy with super-symplectic algebra having conformal structure with respect to the light-like radial coordinate of light-cone boundary looks very attractive. An interesting question is what is the role of the super-conformal algebra associated with the isometries of light-cone boundary $R_+ \times S^2$ which are conformal transformations of sphere S^2 with a scaling of radial coordinate compensating the scaling induced by the conformal transformation. Does it act as dynamical or gauge symmetries?

(d) I have discussed what criticality could mean for Kähler-Dirac action [K88].

i. I have conjectured that it leads to the existence of additional conserved currents defined by the variations which do not affect the value of Kähler action. These arguments are far from being mathematically rigorous and the recent view about the solutions of the Kähler-Dirac equation predicting that the spinor modes are restricted to 2-D string world sheets requires a modification of these arguments.

ii. The basic challenge is to understand the mechanism making this kind of currents conserved: the same challenge is met already in the case of isometries since imbedding space coordinates appear as parameters in Kähler-Dirac action. The existence of conserved currents does not actually require the vanishing of the second variation of Kähler action as claimed earlier. It is enough that the first variation of the canonical momentum densities contracted with the imbedding space gamma matrices annihilates the spinor mode. Situation is analogous to massless Dirac equation:

it does not imply the vanishing of four-momentum, only the vanishing of mass. Hence conserved currents are obtained also outside the quantum criticality.

iii. It is far from obvious that these conditions can be satisfied. The localization of the spinor modes to string world sheets or partonic 2-surfaces guaranteeing in the generaic case that em charge is well-defined for spinor modes implies holomorphy allowing to formulate current conservation for currents associated with the deformations of the space-time surface for second quantized induced spinor field. The crux is that the deformation respects the holomorphy properties of the modified gamma matrices at string world sheet and thus does not mix Γ^z with $\Gamma^{\overline{z}}$. The deformation of Γ^z has only z-component and also annihilates the holomorphic spinor. This mechanism is possible only for Kähler-Dirac action since the Kähler-Dirac gamma matrices in directions orthogonal to the 2-surface must vanish and this is not possible for other actions. This also means that energy momentum tensor has rank 2 as matrix. Cosmic string solutions are an exception since in this case CP_2 projection of space-time surface is 2-D and conditions guaranteing vanishing of classical W fields can be satisfied.

In the following these arguments are formulated more precisely. The unexpected result is that critical deformations induce conformal scalings of the modified metric and electro-weak gauge transformations of the induced spinor connection at X^2. Therefore holomorphy brings in the Kac-Moody symmetries associated with isometries of H (gravitation and color gauge group) and quantum criticality those associated with the holonomies of H (electro-weak-gauge group) as additional symmetries.

The variation of modes of the induced spinor field in a variation of space-time surface respecting the preferred extremal property

Consider first the variation of the induced spinor field in a variation of space-time surface respecting the preferred extremal property. The deformation must be such that the deformed Kähler-Dirac operator D annihilates the modified mode. By writing explicitly the variation of the Kähler-Dirac action (the action vanishes by Kähler-Dirac equation) one obtains deformations and requiring its vanishing one obtains

$$\delta\Psi = D^{-1}(\delta D)\Psi \ . \tag{4.5.5}$$

D^{-1} is the inverse of the Kähler-Dirac operator defining the analog of Dirac propagator and δD defines vertex completely analogous to $\gamma^k\delta A_k$ in gauge theory context. The functional integral over preferred extremals can be carried out perturbatively by expressing δD in terms of δh^k and one obtains stringy perturbation theory around X^2 associated with the preferred extremal defining maximum of Kähler function in Euclidian region and extremum of Kähler action in Minkowskian region (stationary phase approximation).

What one obtains is stringy perturbation theory for calculating n-points functions for fermions at the ends of braid strands located at partonic 2-surfaces and representing intersections of string world sheets and partonic 2-surfaces at the light-like boundaries of CDs. δD- or more precisely, its partial derivatives with respect to functional integration variables - appear atthe vertices located anywhere in the interior of X^2 with outgoing fermions at braid ends. Bosonic propagators are replaced with correlation functions for δh^k. Fermionic propagator is defined by D^{-1}.

After 35 years or hard work this provides for the first time a reasonably explicit formula for the N-point functions of fermions. This is enough since by bosonic emergence these N-point functions define the basic building blocks of the scattering amplitudes. Note that bosonic emergence states that bosons corresponds to wormhole contacts with fermion and anti-fermion at the opposite wormhole throats.

What critical modes could mean for the induced spinor fields?

What critical modes could mean for the induced spinor fields at string world sheets and partonic 2-surfaces. The problematic part seems to be the variation of the Kähler-Dirac operator since it involves gradient. One cannot require that covariant derivative remains invariant since this would require that the components of the induced spinor connection remain invariant and this is quite too restrictive condition. Right handed neutrino solutions de-localized into entire X^2 are however an exception since they have no electro-weak gauge couplings and in this case the condition is obvious: Kähler-Dirac gamma matrices suffer a local scaling for critical deformations:

$$\delta\Gamma^{\mu} = \Lambda(x)\Gamma^{\mu} . \tag{4.5.6}$$

This guarantees that the Kähler-Dirac operator D is mapped to ΛD and still annihilates the modes of ν_R labelled by conformal weight, which thus remain unchanged.

What is the situation for the 2-D modes located at string world sheets? The condition is obvious. Ψ suffers an electro-weak gauge transformation as does also the induced spinor connection so that D_μ is not affected at all. Criticality condition states that the deformation of the space-time surfaces induces a conformal scaling of Γ^μ at X^2. It might be possible to continue this conformal scaling of the entire space-time sheet but this might be not necessary and this would mean that all critical deformations induced conformal transformations of the effective metric of the space-time surface defined by $\{\Gamma^\mu, \Gamma^\nu\} = 2G^{\mu\nu}$. Thus it seems that effective metric is indeed central concept (recall that if the conjectured quaternionic structure is associated with the effective metric, it might be possible to avoid problem related to the Minkowskian signature in an elegant manner).

In fact, one can consider even more general action of critical deformation: the modes of the induced spinor field would be mixed together in the infinitesimal deformation besides infinitesimal electroweak gauge transformation, which is same for all modes. This would extend electroweak gauge symmetry. Kähler-Dirac equation holds true also for these deformations. One might wonder whether the conjectured dynamically generated gauge symmetries assignable to finite measurement resolution could be generated in this manner.

The infinitesimal generator of a critical deformation J_M can be expressed as tensor product of matrix A_M acting in the space of zero modes and of a generator of infinitesimal electro-weak gauge transformation $T_M(x)$ acting in the same manner on all modes: $J_M = A_M \otimes T_M(x)$. A_M is a spatially constant matrix and $T_M(x)$ decomposes to a direct sum of left- and right-handed $SU(2) \times U(1)$ Lie-algebra generators. Left-handed Lie-algebra generator can be regarded as a quaternion and right handed as a complex number. One can speak of a direct sum of left-handed local quaternion $q_{M,L}$ and right-handed local complex number $c_{M,R}$. The commutator $[J_M, J_N]$ is given by $[J_M, J_N] = [A_M, A_N] \otimes \{T_M(x), T_N(x)\} + \{A_M, A_N\} \otimes [T_M(x), T_N(x)]$. One has $\{T_M(x), T_N(x)\} = \{q_{M,L}(x), q_{N,L}(x)\} \oplus \{c_{M,R}(x), c_{N,R}(x)\}$ and $[T_M(x), T_N(x)] = [q_{M,L}(x), q_{N,L}(x)]$. The commutators make sense also for more general gauge group but quaternion/complex number property might have some deeper role.

Thus the critical deformations would induce conformal scalings of the effective metric and dynamical electro-weak gauge transformations. Electro-weak gauge symmetry would be a dynamical symmetry restricted to string world sheets and partonic 2-surfaces rather than acting at the entire space-time surface. For 4-D de-localized right-handed neutrino modes the conformal scalings of the effective metric are analogous to the conformal transformations of M^4 for $\mathcal{N} = 4$ SYMs. Also ordinary conformal symmetries of M^4 could be present for string world sheets and could act as symmetries of generalized Feynman graphs since even virtual wormhole throats are massless. An interesting question is whether the conformal invariance associated with the effective metric is the analog of dual conformal invariance in $\mathcal{N} = 4$ theories.

Critical deformations of space-time surface are accompanied by conserved fermionic currents. By using standard Noetherian formulas one can write

$$J_i^\mu = \overline{\Psi}\Gamma^\mu \delta_i \Psi + \delta_i \overline{\Psi}\Gamma^\mu \Psi \ . \qquad (4.5.7)$$

Here $\delta\Psi_i$ denotes derivative of the variation with respect to a group parameter labeled by i. Since $\delta\Psi_i$ reduces to an infinitesimal gauge transformation of Ψ induced by deformation, these currents are the analogs of gauge currents. The integrals of these currents along the braid strands at the ends of string world sheets define the analogs of gauge charges. The interpretation as Kac-Moody charges is also very attractive and I have proposed that the 2-D Hodge duals of gauge potentials could be identified as Kac-Moody currents. If so, the 2-D Hodge duals of J would define the quantum analogs of dynamical electro-weak gauge fields and Kac-Moody charge could be also seen as non-integral phase factor associated with the braid strand in Abelian approximation (the interpretation in terms of finite measurement resolution is discussed earlier).

One can also define super currents by replacing $\overline{\Psi}$ or Ψ by a particular mode of the induced spinor field as well as c-number valued currents by performing the replacement for both $\overline{\Psi}$ or Ψ. As expected, one obtains a super-conformal algebra with all modes of induced spinor fields acting as generators of super-symmetries restricted to 2-D surfaces. The number of the charges which do not annihilate physical states as also the effective number of fermionic modes could be finite and this would suggest that the integer $\mathcal{N}$ for the supersymmetry in question is finite. This would conform with the earlier proposal inspired by the notion of finite measurement resolution implying the replacement of the partonic 2-surfaces with collections of braid ends.

Note that Kac-Moody charges might be associated with "long" braid strands connecting different wormhole throats as well as short braid strands connecting opposite throats of wormhole contacts. Both kinds of charges would appear in the theory.

What is the interpretation of the critical deformations?

Critical deformations bring in an additional gauge symmetry. Certainly not all possible gauge transformations are induced by the deformations of preferred extremals and a good guess is that they correspond to holomorphic gauge group elements as in theories with Kac-Moody symmetry. What is the physical character of this dynamical gauge symmetry?

(a) Do the gauge charges vanish? Do they annihilate the physical states? Do only their positive energy parts annihilate the states so that one has a situation characteristic for the representation of Kac-Moody algebras. Or could some of these charges be analogous to the gauge charges associated with the constant gauge transformations in gauge theories and be therefore non-vanishing in the absence of confinement. Now one has electro-weak gauge charges and these should be non-vanishing. Can one assign them to deformations with a vanishing conformal weight and the remaining deformations to those with non-vanishing conformal weight and acting like Kac-Moody generators on the physical states?

(b) The simplest option is that the critical Kac-Moody charges/gauge charges with non-vanishing positive conformal weight annihilate the physical states. Critical degrees of freedom would not disappear but make their presence known via the states labelled by different gauge charges assignable to critical deformations with vanishing conformal weight. Note that constant gauge transformations can be said to break the gauge symmetry also in the ordinary gauge theories unless one has confinement.

(c) The hierarchy of quantum criticalities suggests however entire hierarchy of electro-weak Kac-Moody algebras. Does this mean a hierarchy of electro-weak symmetries breakings in which the number of Kac-Moody generators not annihilating the physical states gradually increases as also modes with a higher value of positive conformal weight fail to annihilate the physical state?

The only manner to have a hierarchy of algebras is by assuming that only the generators satisfying $n\ mod\ N = 0$ define the sub-Kac-Moody algebra annihilating the physical states so that the generators with $n\ mod\ N \neq 0$ would define the analogs of gauge charges. I have suggested for long time ago the relevance of kind of fractal hierarchy of Kac-Moody and Super-Virasoro algebras for TGD but failed to imagine any concrete realization.

A stronger condition would be that the algebra reduces to a finite dimensional algebra in the sense that the actions of generators Q_n and Q_{n+kN} are identical. This would correspond to periodic boundary conditions in the space of conformal weights. The notion of finite measurement resolution suggests that the number of independent fermionic oscillator operators is proportional to the number of braid ends so that an effective reduction to a finite algebra is expected.

Whatever the correct interpretation is, this would obviously refine the usual view about electro-weak symmetry breaking.

These arguments suggests the following overall view. The holomorphy of spinor modes gives rise to Kac-Moody algebra defined by isometries and includes besides Minkowskian generators associated with gravitation also SU(3) generators associated with color symmetries. Vanishing second variations in turn define electro-weak Kac-Moody type algebra.

Note that criticality suggests that one must perform functional integral over WCW by decomposing it to an integral over zero modes for which deformations of X^4 induce only an electro-weak gauge transformation of the induced spinor field and to an integral over moduli corresponding to the remaining degrees of freedom.

4.5.5 The Emergence Of Yangian Symmetry And Gauge Potentials As Duals Of Kac-Moody Currents

Yangian symmetry plays a key role in $\mathcal{N} = 4$ super-symmetric gauge theories. What is special in Yangian symmetry is that the algebra contains also multi-local generators. In TGD framework multi-locality would naturally correspond to that with respect to partonic 2-surfaces and string world sheets and the proposal has been that the Super-Kac-Moody algebras assignable to string worlds sheets could generalize to Yangian.

Witten has written a beautiful exposition of Yangian algebras [B30]. Yangian is generated by two kinds of generators J^A and Q^A by a repeated formation of commutators. The number of commutations tells the integer characterizing the multi-locality and provides the Yangian algebra with grading by natural numbers. Witten describes a 2-dimensional QFT like situation in which one has 2-D situation and Kac-Moody currents assignable to real axis define the Kac-Moody charges as integrals in the usual manner. It is also assumed that the gauge potentials defined by the 1-form associated with the Kac-Moody current define a flat connection:

$$\partial_\mu j^A_\nu - \partial_\nu j^A_\nu + [j^A_\mu, j^A_\nu] = 0 \ . \tag{4.5.8}$$

This condition guarantees that the generators of Yangian are conserved charges. One can however consider alternative manners to obtain the conservation.

(a) The generators of first kind - call them J^A - are just the conserved Kac-Moody charges. The formula is given by

$$J_A = \int_{-\infty}^{\infty} dx j^{A0}(x,t) \ . \tag{4.5.9}$$

(b) The generators of second kind contain bi-local part. They are convolutions of generators of first kind associated with different points of string described as real axis. In the basic formula one has integration over the point of real axis.

$$Q^A = f^A_{BC} \int_{-\infty}^{\infty} dx \int_x^{\infty} dy j^{B0}(x,t) j^{C0}(y,t) - 2\int_{-\infty}^{\infty} j^A_x dx \ . \qquad (4.5.10)$$

These charges are indeed conserved if the curvature form is vanishing as a little calculation shows.

How to generalize this to the recent context?

(a) The Kac-Moody charges would be associated with the braid strands connecting two partonic 2-surfaces - Strands would be located either at the space-like 3-surfaces at the ends of the space-time surface or at light-like 3-surfaces connecting the ends. Kähler-Dirac equation would define Super-Kac-Moody charges as standard Noether charges. Super charges would be obtained by replacing the second quantized spinor field or its conjugate in the fermionic bilinear by particular mode of the spinor field. By replacing both spinor field and its conjugate by its mode one would obtain a conserved c-number charge corresponding to an anti-commutator of two fermionic super-charges. The convolution involving double integral is however not number theoretically attractive whereas single 1-D integrals might make sense.

(b) An encouraging observation is that the Hodge dual of the Kac-Moody current defines the analog of gauge potential and exponents of the conserved Kac-Moody charges could be identified as analogs for the non-integrable phase factors for the components of this gauge potential. This identification is precise only in the approximation that generators commute since only in this case the ordered integral $P(exp(i\int Adx))$ reduces to $P(exp(i\int Adx))$.Partonic 2-surfaces connected by braid strand would be analogous to nearby points of space-time in its discretization implying that Abelian approximation works. This conforms with the vision about finite measurement resolution as discretization in terms partonic 2-surfaces and braids.

This would make possible a direct identification of Kac-Moody symmetries in terms of gauge symmetries. For isometries one would obtain color gauge potentials and the analogs of gauge potentials for graviton field (in TGD framework the contraction with M^4 vierbein would transform tensor field to 4 vector fields). For Kac-Moody generators corresponding to holonomies one would obtain electroweak gauge potentials. Note that super-charges would give rise to a collection of spartners of gauge potentials automatically. One would obtain a badly broken SUSY with very large value of $\mathcal{N}$ defined by the number of spinor modes as indeed speculated earlier [K24].

(c) The condition that the gauge field defined by 1-forms associated with the Kac-Moody currents are trivial looks unphysical since it would give rise to the analog of topological QFT with gauge potentials defined by the Kac-Moody charges. For the duals of Kac-Moody currents defining gauge potentials only covariant divergence vanishes implying that curvature form is

$$F_{\alpha\beta} = \epsilon_{\alpha\beta}[j_\mu, j^\mu] \ , \qquad (4.5.11)$$

so that the situation does not reduce to topological QFT unless the induced metric is diagonal. This is not the case in general for string world sheets.

(d) It seems however that there is no need to assume that j_μ defines a flat connection. Witten mentions that although the discretization in the definition of J^A does not seem to be possible, it makes sense for Q^A in the case of $G = SU(N)$ for any representation of G. For general G and its general representation there exists no satisfactory definition of Q. For certain representations, such as the fundamental representation of $SU(N)$, the definition of Q^A is especially simple. One just takes the bi-local part of the previous formula:

$$Q^A = f^A_{BC} \sum_{i<j} J^B_i J^C_j \quad . \tag{4.5.12}$$

What is remarkable that in this formula the summation need not refer to a discretized point of braid but to braid strands ordered by the label i by requiring that they form a connected polygon. Therefore the definition of J^A could be just as above.

(e) This brings strongly in mind the interpretation in terms of twistor diagrams. Yangian would be identified as the algebra generated by the logarithms of non-integrable phase factors in Abelian approximation assigned with pairs of partonic 2-surfaces defined in terms of Kac-Moody currents assigned with the Kähler-Dirac action. Partonic 2-surfaces connected by braid strand would be analogous to nearby points of space-time in its discretization. This would fit nicely with the vision about finite measurement resolution as discretization in terms partonic 2-surfaces and braids.

The resulting algebra satisfies the basic commutation relations

$$\left[J^A, J^B\right] = f^{AB}_C J^C \quad , \quad \left[J^A, Q^B\right] = f^{AB}_C Q^C \quad . \tag{4.5.13}$$

plus the rather complex Serre relations described in [B30].

4.6 Kähler-Dirac Equation And Super-Symmetries

The previous considerations concerning super-conformal symmetries and space-time SUSY have been based on general arguments. The new vision about preferred extremals and Kähler-Dirac equation however leads to a rather detailed understanding of super-conformal symmetries at the level of field equations and is bound to modify the existing vision about super-conformal symmetries.

Whether TGD predicts some variant of space-time SUSY or not has been a long-standing issue: the reason is that TGD does not allow Majorana spinors since fermion number conservation is exact. The more precise formulation of field equations made possible by the realization that spinor modes are localized at string world sheets allows to conclude that the analog of broken $\mathcal{N} = 8$ SUSY is predicted at parton level and that right-handed neutrino generates the minimally broken $\mathcal{N} = 2$ sub-SUSY.

One important outcome of criticality is the identification of gauge potentials as duals of Kac-Moody currents at the boundaries of string world sheets: quantum gauge potentials are defined only where they are needed that is string curves defining the non-integrable phase factors. This gives also rise to the realization of the conjectured Yangian in terms of the Kac-Moody charges and commutators in accordance with the earlier conjecture.

4.6.1 Super-Conformal Symmetries

It is good to summarize first the basic ideas about Super-Virasoro representations. TGD allows two kinds of super-conformal symmetries.

(a) The first super-conformal symmetry is associated with $\delta M^4_\pm \times CP_2$ and corresponds to symplectic symmetries of $\delta M^4_\pm \times CP_2$. The reason for extension of conformal symmetries is metric 2-dimensionality of the light-like boundary $\delta M^4_\pm$ defining upper/lower boundary of causal diamond (CD). This super-conformal symmetry is something new and corresponds to replacing finite-dimensional Lie-group G for Kac-Moody symmetry with infinite-dimensional symplectic group. The light-like radial coordinate of $\delta M^4_\pm$ takes the role of the real part of complex coordinate z for ordinary conformal symmetry.

Together with complex coordinate of S^2 it defines 3-D restriction of Hamilton-Jacobi variant of 4-D super-conformal symmetries. One can continue the conformal symmetries from light-cone boundary to CD by forming a slicing by parallel copies of $\delta M^4_\pm$. There are two possible slicings corresponding to the choices δM^4_+ and δM^4_- assignable to the upper and lower boundaries of CD. These two choices correspond to two arrows of geometric time for the basis of zero energy states in ZEO.

(b) Super-symplectic degrees of freedom determine the electroweak and color quantum numbers of elementary particles. Bosonic emergence implies that ground states assignable to partonic 2-surfaces correspond to partial waves in $\delta M^4_\pm$ and one obtains color partial waves in particular. These partial waves correspond to the solutions for the Dirac equation in imbedding space and the correlation between color and electroweak quantum numbers is not quite correct. Super-Kac-Moody generators give the compensating color for massless states obtained from tachyonic ground states guaranteeing that standard correlation is obtained. Super-symplectic degrees are therefore directly visible in particle spectrum. One can say that at the point-like limit the WCW spinors reduce to tensor products of imbedding space spinors assignable to the center of mass degrees of freedom for the partonic 2-surfaces defining wormhole throats.

I have proposed a physical interpretation of super-symplectic vibrational degrees of freedom in terms of degrees of freedom assignable to non-perturbative QCD. These degrees of freedom would be responsible for most of the baryon masses but their theoretical understanding is lacking in QCD framework.

(c) The second super-conformal symmetry is assigned light-like 3-surfaces and to the isometries and holonomies of the imbedding space and is analogous to the super-Kac-Moody symmetry of string models. Kac-Moody symmetries could be assigned to the light-like deformations of light-like 3-surfaces. Isometries give tensor factor $E^2 \times SU(3)$ and holonomies factor $SU(2)_L \times U(1)$. Altogether one has 5 tensor factors to super-conformal algebra. That the number is just five is essential for the success p-adic mass calculations [K100, K39].

The construction of solutions of the Kähler-Dirac equation suggests strongly that the fermionic representation of the Super-Kac-Moody algebra can be assigned as conserved charges associated with the space-like braid strands at both the 3-D space-like ends of space-time surfaces and with the light-like (or space-like with a small deformation) associated with the light-like 3-surfaces. The extension to Yangian algebra involving higher multi-linears of super-Kac Moody generators is also highly suggestive. These charges would be non-local and assignable to several wormhole contacts simultaneously. The ends of braids would correspond points of partonic 2-surfaces defining a discretization of the partonic 2-surface having interpretation in terms of finite measurement resolution.

These symmetries would correspond to electroweak and strong gauge fields and to gravitation. The duals of the currents giving rise to Kac-Moody charges would define the counterparts of gauge potentials and the conserved Kac-Moody charges would define the counterparts of non-integrable phase factors in gauge theories. The higher Yangian charges would define generalization of non-integrable phase factors. This would suggest a rather direct connection with the twistorial program for calculating the scattering amplitudes implies also by zero energy ontology.

Quantization recipes have worked in the case of super-string models and one can ask whether the application of quantization to the coefficients of powers of complex coordinates or Hamilton-Jacobi coordinates could lead to the understanding of the 4-D variants of the conformal symmetries and give detailed information about the representations of the Kac-Moody algebra too.

4.6.2 WCW Geometry And Super-Conformal Symmetries

The vision about the geometry of WCW has been roughly the following and the recent steps of progress induce to it only small modifications if any.

(a) Kähler geometry is forced by the condition that hermitian conjugation allows geometrization. Kähler function is given by the Kähler action coming from space-time regions with Euclidian signature of the induced metric identifiable as lines of generalized Feynman diagrams. Minkowskian regions give imaginary contribution identifiable as the analog of Morse function and implying interference effects and stationary phase approximation. The vision about quantum TGD as almost topological QFT inspires the proposal that Kähler action reduces to 3-D terms reducing to Chern-Simons terms by the weak form of electric-magnetic duality. The recent proposal for preferred extremals is consistent with this property realizing also holography implied by general coordinate invariance. Strong form of general coordinate invariance implying effective 2-dimensionality in turn suggests that Kähler action is expressible string world sheets and possibly also areas of partonic 2-surfaces.

(b) The complexified gamma matrices of WCW come as hermitian conjugate pairs and anti-commute to the Kähler metric of WCW . Also bosonic generators of symplectic transformations of $\delta M^4_\pm \times CP_2$ a assumed to act as isometries of WCW geometry can be complexified and appear as similar pairs. The action of isometry generators coincides with that of symplectic generators at partonic 2-surfaces and string world sheets but elsewhere inside the space-time surface it is expected to be deformed from the symplectic action. The super-conformal transformations of $\delta M^4_\pm \times CP_2$ acting on the light-like radial coordinate of $\delta M^4_\pm$ act as gauge symmetries of the geometry meaning that the corresponding WCW vector fields have zero norm.

(c) WCW geometry has also zero modes which by definition do not contribute to WCW metric expect possibly by the dependence of the elements of WCW metric on zero modes through a conformal factor. In particular, induced CP_2 Kähler form and its analog for sphere $r_M = constant$ of light cone boundary are symplectic invariants, and one can define an infinite number of zero modes as invariants defined by Kähler fluxes over partonic 2-surfaces and string world sheets. This requires however the slicing of CD parallel copies of δM^4_+ or δM^4_-. The physical interpretation of these non-quantum fluctuating degrees of freedom is as classical variables necessary for the interpretation of quantum measurement theory. Classical variable would metaphorically correspond the position of the pointer of the measurement instrument.

(d) The construction receives a strong philosophical inspiration from the geometry of loop spaces. Loop spaces allow a unique Kähler geometry with maximal isometry group identifiable as Kac-Moody group. The reason is that otherwise Riemann connection does not exist. The only problem is that curvature scalar diverges since the Riemann tensor is by constant curvature property proportional to the metric. In 3-D case one would have union of constant curvature spaces labelled by zero modes and the situation is expected to be even more restrictive. The conjecture indeed is that WCW geometry exists only for $H = M^4 \times CP_2$: infinite-D Kähler geometric existence and therefore physics would be unique. One can also hope that Ricci scalar is finite and therefore zero by the constant curvature property so that Einstein's equations are satisfied.

(e) The matrix elements of WCW Kähler metric are given in terms of the anti-commutators of the fermionic Noether super-charges associated with symplectic isometry currents. A given mode of induced spinor field characterized by imbedding space chirality (quark or lepton), by spin and weak spin plus conformal weight n. If the super-conformal transformations for string modes act gauge transformations only the spinor modes with vanishing conformal weight correspond to non-zero modes of the WCW metric and the situation reduces essentially to the analog of $\mathcal{N} = 8$ SUSY.

The WCW Hamiltonians generating symplectic isometries correspond to the Hamiltonians spanning the symplectic group of $\delta M^4_\pm \times CP_2$. One can say that the space of quantum fluctuating degrees of freedom is this symplectic group of $\delta M^4_\pm \times CP_2$ or its subgroup or coset space: this must have very deep implications for the structure of the quantum TGD.

An interesting possibility is that the radial conformal weights of the symplectic algebra are linear combinations of the zeros of Riemann Zeta with integer coefficients. Also this option allows to realize the hierarchy of super-symplectic conformal symmetry breakings in terms of sub-algebras isomorphic to the entire super-symplectic algebra. WCW would have fractal structure corresponding to a hierarchy of quantum criticalities.

(f) The localization of the induced spinors to string world sheets means that the super-symplectic Noether charges are associated with strings connecting partonic 2-surfaces. The physically obvious fact that given partonic surface can be accompanied by an arbitrary number of strings, forces a generalization of the super-symplectic algebra to a Yangian containing infinite number of n-local variants of various super-symplectic Noether charges. For instance, four -momentum is accompanied by multi-stringy variants involving four-momentum P_0^A and angular momentum generators. At the first level of the hierarchy one has $P_1^A = f^A_{BC} P_0^B \otimes J^C$. This hierarchy might play crucial role in understanding of the four-momenta of bound states.

(g) Zero energy ontology brings in additional delicacies. Basic objects are now unions of partonic 2-surfaces at the ends of CD. One can generalize the expressions for the isometry generators in a straightforward manner by requiring that given isometry restricts to a symplectic transformation at partonic 2-surfaces and string world sheets.

(h) One could criticize the effective metric 2-dimensionality forced by the general consistency arguments as something non-physical. The WCW Hamiltonians are expressed using only the data at partonic 2-surfaces and string string world sheets: this includes also 4-D tangent space data via the weak form of electric-magnetic duality so that one has only effective 2-dimensionality. Obviously WCW geometry must huge large gauge symmetries besides zero modes. The hierarchy of super-symplectic symmetries indeed represent gauge symmetries of this kind.

Effective 2-dimensionality realizing strong form of holography in turn is induced by the strong form of general coordinate invariance. Light-like 3-surfaces at which the signature of the induced metric changes must be equivalent with the 3-D space-like ends of space-time surfaces at the light-boundaries of space-time surfaces as far as WCW geometry is considered. This requires that the data from their 2-D intersections defining partonic 2-surfaces should dictate the WCW geometry. Note however that Super-Kac-Moody charges giving information about the interiors of 3-surfaces appear in the construction of the physical states.

4.6.3 The Relationship Between Inertial Gravitational Masses

The relationship between inertial and gravitational masses and Equivalence Principle have been on of the longstanding problems in TGD. Not surprisingly, the realization how GRT space-time relates to the many-sheeted space-time of TGD finally allowed to solve the problem.

ZEO and non-conservation of Poincare charges in Poincare invariant theory of gravitation

In positive energy ontology the Poincare invarance of TGD is in sharpt contrast with the fact that GRT based cosmology predicts non-conservation of Poincare charges (as a matter fact, the definition of Poincare charges is very questionable for general solutions of field equations).

In zero energy ontology (ZEO) all conserved (that is Noether-) charges of the Universe vanish identically and their densities should vanish in scales below the scale defining the scale for observations and assignable to causal diamond (CD). This observation allows to imagine a ways out of what seems to be a conflict of Poincare invariance with cosmological facts.

ZEO would explain the local non-conservation of average energies and other conserved quantum numbers in terms of the contributions of sub-CDs analogous to quantum fluctuations. Classical gravitation should have a thermodynamical description if this interpretation is correct. The average values of the quantum numbers assignable to a space-time sheet would depend on the size of CD and possibly also its location in M^4. If the temporal distance between the tips of CD is interpreted as a quantized variant of cosmic time, the non-conservation of energy-momentum defined in this manner follows. One can say that conservation laws hold only true in given scale defined by the largest CD involved.

Equivalence Principle at quantum level

The interpretation of EP at quantum level has developed slowly and the recent view is that it reduces to quantum classical correspondence meaning that the classical charges of Kähler action can be identified with eigen values of quantal charges associated with Kähler-Dirac action.

(a) At quantum level I have proposed coset representations for the pair of super-symplectic algebras assignable to the light-like boundaries of CD and the Super Kac-Moody algebra assignable to the light-like 3-surfaces defining the orbits of partonic 2-surfaces as realization of EP. For coset representation the differences of super-conformal generators would annihilate the physical states so that one can argue that the corresponding four-momenta are identical. One could even say that one obtains coset representation for the "vibrational" parts of the super-conformal algebras in question. It is now clear that this idea does not work. Note however that coset representations occur naturally for the subalgebras of symplectic algebra and Super Kac-Moody algebra and are naturally induced by finite measurement resolution.

(b) The most recent view (2014) about understanding how EP emerges in TGD is described in [K79] and relies heavily on superconformal invariance and a detailed realisation of ZEO at quantum level. In this approach EP corresponds to quantum classical correspondence (QCC): four-momentum identified as classical conserved Noether charge for space-time sheets associated with Käbler action is identical with quantal four-momentum assignable to the representations of super-symplectic and super Kac-Moody algebras as in string models and having a realisation in ZEO in terms of wave functions in the space of causal diamonds (CDs).

(c) The latest realization is that the eigenvalues of quantal four-momentum can be identified as eigenvalues of the four-momentum operator assignable to the Kähler-Dirac equation. This realisation seems to be consistent with the p-adic mass calculations requiring that the super-conformal algebra acts in the tensor product of 5 tensor factors.

Equivalence Principle at classical level

How Einstein's equations and General Relativity in long length scales emerges from TGD has been a long-standing interpretational problem of TGD.

The first proposal making sense even when one does not assume ZEO is that vacuum extremals are only approximate representations of the physical situation and that small fluctuations around them give rise to an inertial four-momentum identifiable as gravitational four-momentum identifiable in terms of Einstein tensor. EP would hold true in the sense that the average gravitational four-momentum would be determined by the Einstein tensor assignable to the vacuum extremal. This interpretation does not however take into account the many-sheeted character of TGD spacetime and is therefore questionable.

The resolution of the problem came from the realization that GRT is only an effective theory obtained by endowing M^4 with effective metric.

(a) The replacement of superposition of fields with superposition of their effects means replacing superposition of fields with the set-theoretic union of space-time surfaces. Particle experiences sum of the effects caused by the classical fields at the space-time sheets (see **Fig.** `http://tgdtheory.fi/appfigures/fieldsuperpose.jpg` or **Fig. ??** in the appendix of this book).

(b) This is true also for the classical gravitational field defined by the deviation from flat Minkowski metric instandard M^4 coordinates for the space-time sheets. One can define effective metric as sum of M^4 metric and deviations. This effective metric would correspond to that of General Relativity. This resolves long standing issues relating to the interpretation of TGD.

(c) Einstein's equations could hold true for the effective metric. They are motivated by the underlying Poincare invariance which cannot be realized as global conservation laws for the effective metric. The conjecture vanishing of divergence of Khler energy momentum tensor can be seen as the microscopic justification for the claim that Einstein's equations hold true for the effective space-time.

(d) The breaking of Poincare invariance could have interpretation as effective breaking in zero energy ontology (ZEO), in which various conserved charges are length dependent and defined separately for each causal diamond (CD).

One can of course consider the possibility that Einstein's equations generalize for preferred extremals of Kähbler action. This would actually represent at space-time level the notion of QCC rather than realise QCC interpreted as EP. The condition that the energy momentum tensor for Kähler action has vanishing covariant divergence would be satisfied in GRT if Einstein's equations with cosmological term hold true. This is the case also now but one can consider also more general solutions in which one has two cosmological constants which are not genuine constants anymore [K103].

An interesting question is whether inertial-gravitational duality generalizes to the case of color gauge charges so that color gauge fluxes would correspond to "gravitational" color charges and the charges defined by the conserved currents associated with color isometries would define "inertial" color charges. Since the induced color fields are proportional to color Hamiltonians multiplied by Kähler form they vanish identically for vacuum extremals in accordance with "gravitational" color confinement.

Constraints from p-adic mass calculations and ZEO

A further important physical input comes from p-adic thermodynamics forming a core element of p-adic mass calculations.

(a) The first thing that one can get worried about relates to the extension of conformal symmetries. If the conformal symmetries generalize to $D = 4$, how can one take seriously the results of p-adic mass calculations based on 2-D conformal invariance? There is no reason to worry. The reduction of the conformal invariance to 2-D one for the preferred extremals takes care of this problem. This however requires that the fermionic contributions assignable to string world sheets and/or partonic 2-surfaces - Super- Kac-Moody contributions - should dictate the elementary particle masses. For hadrons also symplectic contributions should be present. This is a valuable hint in attempts to identify the mathematical structure in more detail.

(b) ZEO suggests that all particles, even virtual ones correspond to massless wormhole throats carrying fermions. As a consequence, twistor approach would work and the kinematical constraints to vertices would allow the cancellation of divergences. This would suggest that the p-adic thermal expectation value is for the longitudinal M^2 momentum squared (the definition of CD selects $M^1 \subset M^2 \subset M^4$ as also does number theoretic vision). Also propagator would be determined by M^2 momentum. Lorentz invariance would be obtained by integration of the moduli for CD including also Lorentz boosts of CD.

(c) In the original approach one allows states with arbitrary large values of L_0 as physical states. Usually one would require that L_0 annihilates the states. In the calculations however mass squared was assumed to be proportional L_0 apart from vacuum contribution. This is a questionable assumption. ZEO suggests that total mass squared vanishes and that one can decompose mass squared to a sum of longitudinal and transversal parts. If one can do the same decomposition to longitudinal and transverse parts also for the Super Virasoro algebra then one can calculate longitudinal mass squared as a p-adic thermal expectation in the transversal super-Virasoro algebra and only states with $L_0 = 0$ would contribute and one would have conformal invariance in the standard sense.

(d) In the original approach the assumption motivated by Lorentz invariance has been that mass squared is replaced with conformal weight in thermodynamics, and that one first calculates the thermal average of the conformal weight and then equates it with mass squared. This assumption is somewhat ad hoc. ZEO however suggests an alternative interpretation in which one has zero energy states for which longitudinal mass squared of positive energy state derive from p-adic thermodynamics. Thermodynamics - or rather, its square root - would become part of quantum theory in ZEO. M-matrix is indeed product of hermitian square root of density matrix multiplied by unitary S-matrix and defines the entanglement coefficients between positive and negative energy parts of zero energy state.

(e) The crucial constraint is that the number of super-conformal tensor factors is $N = 5$: this suggests that thermodynamics applied in Super-Kac-Moody degrees of freedom assignable to string world sheets is enough, when one is interested in the masses of fermions and gauge bosons. Super-symplectic degrees of freedom can also contribute and determine the dominant contribution to baryon masses. Should also this contribution obey p-adic thermodynamics in the case when it is present? Or does the very fact that this contribution need not be present mean that it is not thermal? The symplectic contribution should correspond to hadronic p-adic length prime rather the one assignable to (say) u quark. Hadronic p-adic mass squared and partonic p-adic mass squared cannot be summed since primes are different. If one accepts the basic rules [K47], longitudinal energy and momentum are additive as indeed assumed in perturbative QCD.

(f) Calculations work if the vacuum expectation value of the mass squared must be assumed to be tachyonic. There are two options depending on whether one whether p-adic thermodynamics gives total mass squared or longitudinal mass squared.

   i. One could argue that the total mass squared has naturally tachyonic ground state expectation since for massless extremals longitudinal momentum is light-like and transversal momentum squared is necessary present and non-vanishing by the localization to topological light ray of finite thickness of order p-adic length scale. Transversal degrees of freedom would be modeled with a particle in a box.

   ii. If longitudinal mass squared is what is calculated, the condition would require that transversal momentum squared is negative so that instead of plane wave like behavior exponential damping would be required. This would conform with the localization in transversal degrees of freedom.

4.6.4 Realization Of Space-Time SUSY In TGD

The generators of super-conformal algebras are obtained by taking fermionic currents for second quantized fermions and replacing either fermion field or its conjugate with its particular mode. The resulting super currents are conserved and define super charges. By replacing both fermion and its conjugate with modes one obtains c-number valued currents. In this manner one also obtains the analogs of super-Poincare generators labelled by the conformal weight and other spin quantum numbers as Noether charges so that space-time SUSY is suggestive.

The super-conformal invariance in spinor modes is expected to be gauge symmetry so that only the generators with vanishing string world sheet conformal weight create physical states. This would leave only the conformal quantum numbers characterizing super-symplectic generators (radial conformal weight included) under consideration and the hierarchy of its subalgebras acting as gauge symmetries giving rise to a hierarchy of criticalities having interpretation in terms of dark matter.

As found in the earlier section, the proposed anti-commutation relations for fermionic oscillator operators at the ends of string world sheets can be formulated so that they are analogous to those for Super Poincare algebra. The reason is that field equations assign a conserved 8-momentum to the light-like geodesic line defining the boundary of string at the orbit of partonic 2-surface. Octonionic representation of sigma matrices making possible generalization of twistor formalism to 8-D context is also essential. As a matter, the final justification for the analog of space-time came from the generalization of twistor approach to 8-D context.

By counting the number of spin and weak isospin components of imbedding space spinors satisfying massless algebraic Dirac equation one finds that broken $\mathcal{N}=8$ SUSY is the expected space-time SUSY. $\mathcal{N}=2$ SUSY assignable to right-handed neutrino is the least broken sub-SUSY and one is forced to consider the possibility that spartners correspond to dark matter with $h_{eff}=n\times h$ and therefore remaining undetected in recent particle physics experiments.

Super-space viz. Grassmann algebra valued fields

Standard SUSY induces super-space extending space-time by adding anti-commuting coordinates as a formal tool. Many mathematicians are not enthusiastic about this approach because of the purely formal nature of anti-commuting coordinates. Also I regard them as a non-sense geometrically and there is actually no need to introduce them as the following little argument shows.

Grassmann parameters (anti-commuting theta parameters) are generators of Grassmann algebra and the natural object replacing super-space is this Grassmann algebra with coefficients of Grassmann algebra basis appearing as ordinary real or complex coordinates. This is just an ordinary space with additional algebraic structure: the mysterious anti-commuting coordinates are not needed. To me this notion is one of the conceptual monsters created by the over-pragmatic thinking of theoreticians.

This allows allows to replace field space with super field space, which is completely well-defined object mathematically, and leave space-time untouched. Linear field space is simply replaced with its Grassmann algebra. For non-linear field space this replacement does not work. This allows to formulate the notion of linear super-field just in the same manner as it is done usually.

The generators of super-symmetries in super-space formulation reduce to super translations, which anti-commute to translations. The super generators Q_α and $\overline{Q}_{\dot{\beta}}$ of super Poincare algebra are Weyl spinors commuting with momenta and anti-commuting to momenta:

$$\{Q_\alpha,\overline{Q}_{\dot{\beta}}\} = 2\sigma^\mu_{\alpha\ \dot{beta}}P_\mu \quad . \tag{4.6.1}$$

One particular representation of super generators acting on super fields is given by

$$\begin{aligned} D_\alpha &= i\frac{\partial}{\partial_{\theta_\alpha}} \quad , \\ D_{\dot{\alpha}} &= i\frac{\partial}{\partial_{\overline{\theta}}{}_{\ \dot{alpha}}} + \theta^\beta\sigma^\mu_{\beta\dot{\alpha}}\partial_\mu \end{aligned} \tag{4.6.2}$$

Here the index raising for 2-spinors is carried out using antisymmetric 2-tensor $\epsilon^{\alpha\beta}$. Super-space interpretation is not necessary since one can interpret this action as an action on Grassmann algebra valued field mixing components with different fermion numbers.

Chiral superfields are defined as fields annihilated by $D_{\dot{\alpha}}$. Chiral fields are of form $\Psi(x^\mu + i\overline{\theta}\sigma^\mu\theta,\theta)$. The dependence on $\overline{\theta}_{\dot{\alpha}}$ comes only from its presence in the translated Minkowski coordinate annihilated by $D_{\dot{\alpha}}$. Super-space enthusiast would say that by a translation of M^4 coordinates chiral fields reduce to fields, which depend on θ only.

The space of fermionic Fock states at partonic 2-surface as TGD counterpart of chiral super field

As already noticed, another manner to realize SUSY in terms of representations the super algebra of conserved super-charges. In TGD framework these super charges are naturally associated with the modified Dirac equation, and anti-commuting coordinates and super-fields do not appear anywhere. One can however ask whether one could identify a mathematical structure replacing the notion of chiral super field.

In [K24] it was proposed that generalized chiral super-fields could effectively replace induced spinor fields and that second quantized fermionic oscillator operators define the analog of SUSY algebra. One would have $\mathcal{N} = \infty$ if all the conformal excitations of the induced spinor field restricted on 2-surface are present. For right-handed neutrino the modes are labeled by two integers and de-localized to the interior of Euclidian or Minkowskian regions of space-time sheet.

The obvious guess is that chiral super-field generalizes to the field having as its components many-fermions states at partonic 2-surfaces with theta parameters and their conjugates in one-one correspondence with fermionic creation operators and their hermitian conjugates.

(a) Fermionic creation operators - in classical theory corresponding anti-commuting Grassmann parameters - replace theta parameters. Theta parameters and their conjugates are not in one-one correspondence with spinor components but with the fermionic creation operators and their hermitian conjugates. One can say that the super-field in question is defined in the "world of classical worlds" (WCW) rather than in space-time. Fermionic Fock state at the partonic 2-surface is the value of the chiral super field at particular point of WCW .

(b) The matrix defined by the $\sigma^\mu\partial_\mu$ is replaced with a matrix defined by the Kähler-Dirac operator D between spinor modes acting in the solution space of the Kähler-Dirac equation. Since Kähler-Dirac operator annihilates the modes of the induced spinor field, super covariant derivatives reduce to ordinary derivatives with respect the theta parameters labeling the modes. Hence the chiral super field is a field that depends on θ_m or conjugates $\overline{\theta}_m$ only. In second quantization the modes of the chiral super-field are many-fermion states assigned to partonic 2-surfaces and string world sheets. Note that this is the only possibility since the notion of super-coordinate does not make sense now.

(c) It would seem that the notion of super-field does not bring anything new. This is not the case. First of all, the spinor fields are restricted to 2-surfaces. Second point is that one cannot assign to the fermions of the many-fermion states separate non-parallel or even parallel four-momenta. The many-fermion state behaves like elementary particle. This has non-trivial implications for propagators and a simple argument [K24] leads to the proposal that propagator for N-fermion partonic state is proportional to $1/p^N$. This would mean that only the states with fermion number equal to 1 or 2 behave like ordinary elementary particles.

4.6.5 Comparison Of TGD And Stringy Views About Super-Conformal Symmetries

The best manner to represent TGD based view about conformal symmetries is by comparison with the conformal symmetries of super string models.

Basic differences between the realization of super conformal symmetries in TGD and in super-string models

The realization super conformal symmetries in TGD framework differs from that in string models in several fundamental aspects.

(a) In TGD framework super-symmetry generators acting as configuration space gamma matrices carry either lepton or quark number. Majorana condition required by the hermiticity of super generators which is crucial for super string models would be in conflict with the conservation of baryon and lepton numbers and is avoided. This is made possible by the realization of bosonic generators represented as Hamiltonians of X^2-local symplectic transformations rather than vector fields generating them [K15]. This kind of representation applies also in Kac-Moody sector since the local transversal isometries localized in X^3_l and respecting light-likeness condition can be regarded as X^2 local symplectic transformations, whose Hamiltonians generate also isometries. Localization is not complete: the functions of X^2 coordinates multiplying symplectic and Kac-Moody generators are functions of the symplectic invariant $J = \epsilon^{\mu\nu} J_{\mu\nu}$ so that effective one-dimensionality results but in different sense than in conformal field theories. This realization of super symmetries is what distinguishes between TGD and super string models and leads to a totally different physical interpretation of super-conformal symmetries. The fermionic representations of super-symplectic and super Kac-Moody generators can be identified as Noether charges in standard manner.

(b) A long-standing problem of quantum TGD was that stringy propagator $1/G$ does not make sense if G carries fermion number. The progress in the understanding of second quantization of the modified Dirac operator made it however possible to identify the counterpart of G as a c-number valued operator and interpret it as different representation of G [K13].

(c) The notion of super-space is not needed at all since Hamiltonians rather than vector fields represent bosonic generators, no super-variant of geometry is needed. The distinction between Ramond and N-S representations important for $N = 1$ super-conformal symmetry and allowing only ground state weight 0 an $1/2$ disappears. Indeed, for $N = 2$ super-conformal symmetry it is already possible to generate spectral flow transforming these Ramond and N-S representations to each other (G_n is not Hermitian anymore).

(d) If Kähler action defines the Kähler-Dirac operator, the number of spinor modes could be finite. One must be here somewhat cautious since bound state in the Coulomb potential associated with electric part of induced electro-weak gauge field might give rise to an infinite number of bound states which eigenvalues converging to a fixed eigenvalue (as in the case of hydrogen atom). Finite number of generalized eigenmodes means that the representations of super-conformal algebras reduces to finite-dimensional ones in TGD framework. Also the notion of number theoretic braid indeed implies this. The physical interpretation would be in terms of finite measurement resolution. If Kähler action is complexified to include imaginary part defined by CP breaking instanton term, the number of stringy mass square eigenvalues assignable to the spinor modes becomes infinite since conformal excitations are possible. This means breakdown of exact holography and effective 2-dimensionality of 3-surfaces. It seems that the inclusion of instanton term is necessary for several reasons. The notion of finite measurement resolution forces conformal cutoff also now. There are arguments suggesting that only the modes with vanishing conformal weight contribute to the Dirac determinant defining vacuum functional identified as exponent of Kähler function in turn identified as Kähler action for its preferred extremal.

(e) What makes spinor field mode a generator of gauge super-symmetry is that is c-number and not an eigenmode of $D_K(X^2)$ and thus represents non-dynamical degrees of freedom. If the number of eigen modes of $D_K(X^2)$ is indeed finite means that most of spinor field modes represent super gauge degrees of freedom.

The super generators G are not Hermitian in TGD!

The already noticed important difference between TGD based and the usual Super Virasoro representations is that the Super Virasoro generator G cannot Hermitian in TGD. The reason is that WCW gamma matrices possess a well defined fermion number. The hermiticity of the WCW gamma matrices Γ and of the Super Virasoro current G could be achieved by posing Majorana conditions on the second quantized H-spinors. Majorana conditions can be however realized only for space-time dimension $D \ mod \ 8 = 2$ so that super string type approach does not work in TGD context. This kind of conditions would also lead to the non-conservation of baryon and lepton numbers.

An analogous situation is encountered in super-symmetric quantum mechanics, where the general situation corresponds to super symmetric operators S, $S^\dagger$, whose anti-commutator is Hamiltonian: $\{S, S^\dagger\} = H$. One can define a simpler system by considering a Hermitian operator $S_0 = S + S^\dagger$ satisfying $S_0^2 = H$: this relation is completely analogous to the ordinary Super Virasoro relation $GG = L$. On basis of this observation it is clear that one should replace ordinary Super Virasoro structure $GG = L$ with $GG^\dagger = L$ in TGD context.

It took a long time to realize the trivial fact that $N = 2$ super-symmetry is the standard physics counterpart for TGD super symmetry. $N = 2$ super-symmetry indeed involves the doubling of super generators and super generators carry $U(1)$ charge having an interpretation as fermion number in recent context. The so called short representations of $N = 2$ super-symmetry algebra can be regarded as representations of $N = 1$ super-symmetry algebra.

WCW gamma matrix Γ_n, $n > 0$ corresponds to an operator creating fermion whereas Γ_n, $n < 0$ annihilates anti-fermion. For the Hermitian conjugate $\Gamma_n^\dagger$ the roles of fermion and anti-fermion are interchanged. Only the anti-commutators of gamma matrices and their Hermitian conjugates are non-vanishing. The dynamical Kac Moody type generators are Hermitian and are constructed as bilinears of the gamma matrices and their Hermitian conjugates and, just like conserved currents of the ordinary quantum theory, contain parts proportional to $a^\dagger a$, $b^\dagger b$, $a^\dagger b^\dagger$ and ab (a and b refer to fermionic and anti-fermionic oscillator operators). The commutators between Kac Moody generators and Kac Moody generators and gamma matrices remain as such.

For a given value of m G_n, $n > 0$ creates fermions whereas G_n, $n < 0$ annihilates anti-fermions. Analogous result holds for $G_n^\dagger$. Virasoro generators remain Hermitian and decompose just like Kac Moody generators do. Thus the usual anti-commutation relations for the super Virasoro generators must be replaced with anti-commutations between G_m and $G_n^\dagger$ and one has

$$\begin{aligned} \{G_m, G_n^\dagger\} &= 2L_{m+n} + \tfrac{c}{3}(m^2 - \tfrac{1}{4})\delta_{m,-n} \ , \\ \{G_m, G_n\} &= 0 \ , \\ \{G_m^\dagger, G_n^\dagger\} &= 0 \ . \end{aligned} \tag{4.6.3}$$

The commutators of type $[L_m, L_n]$ are not changed. Same applies to the purely kinematical commutators between L_n and $G_m/G_m^\dagger$.

The Super Virasoro conditions satisfied by the physical states are as before in case of L_n whereas the conditions for G_n are doubled to those of G_n, $n < 0$ and $G_n^\dagger$, $n > 0$.

What could be the counterparts of stringy conformal fields in TGD framework?

The experience with string models would suggest the conformal symmetries associated with the complex coordinates of X^2 as a candidate for conformal super-symmetries. One can imagine two counterparts of the stringy coordinate z in TGD framework.

(a) Super-symplectic and super Kac-Moody symmetries are local with respect to X^2 in the sense that the coefficients of generators depend on the invariant $J = \epsilon^{\alpha\beta} J_{\alpha\beta}\sqrt{g_2}$ rather than being completely free [K15]. Thus the real variable J replaces complex (or hyper-complex) stringy coordinate and effective 1-dimensionality holds true also now but in different sense than for conformal field theories.

(b) The slicing of X^4 by string world sheets Y^2 and partonic 2-surfaces X^2 implied by number theoretical compactification implies string-parton duality and involves the super conformal fermionic gauge symmetries associated with the coordinates u and w in the dual dimensional reductions to stringy and partonic dynamics. These coordinates define the natural analogs of stringy coordinate. The effective reduction of X_l^3 to braid by finite measurement resolution implies the effective reduction of $X^4(X^3)$ to string world sheet. This implies quite strong resemblance with string model. The realization that spinor modes with well- define em charge must be localized at string world sheets makes the connection with strings even more explicit [K88].

One can understand how Equivalence Principle emerges in TGD framework at space-time level when many-sheeted space-time (see **Fig.** `http://tgdtheory.fi/appfigures/manysheeted.jpg` or **Fig.** 9 in the appendix of this book) is replaced with effective space-time lumping together the space-time sheets to M^4 endowed with effective metric. The quantum counterpart EP has most feasible interpretation in terms of Quantum Classical Correspondence (QCC): the conserved Kähler four-momentum equals to an eigenvalue of conserved Kähler-Dirac four-momentum acting as operator.

(c) The conformal fields of string model would reside at X^2 or Y^2 depending on which description one uses and complex (hyper-complex) string coordinate would be identified accordingly. Y^2 could be fixed as a union of stringy world sheets having the strands of number theoretic braids as its ends. The proposed definition of braids is unique and characterizes finite measurement resolution at space-time level. X^2 could be fixed uniquely as the intersection of X_l^3 (the light-like 3-surface at which induced metric of space-time surface changes its signature) with $\delta M^4_\pm \times CP_2$. Clearly, wormhole throats X_l^3 would take the role of branes and would be connected by string world sheets defined by number theoretic braids.

(d) An alternative identification for TGD parts of conformal fields is inspired by M^8-H duality. Conformal fields would be fields in WCW . The counterpart of z coordinate could be the hyper-octonionic M^8 coordinate m appearing as argument in the Laurent series of WCW Clifford algebra elements. m would characterize the position of the tip of CD and the fractal hierarchy of CDs within CDs would give a hierarchy of Clifford algebras and thus inclusions of hyper-finite factors of type II_1. Reduction to hyper-quaternionic field -that is field in M^4 center of mass degrees of freedom- would be needed to obtained associativity. The arguments m at various level might correspond to arguments of N-point function in quantum field theory.

4.7 Still about induced spinor fields and TGD counterpart for Higgs

The understanding of the modified Dirac equation and of the possible classical counterpart of Higgs field in TGD framework is not completely satisfactory. The emergence of twistor lift of Kähler action [L22] [L24] inspired a fresh approach to the problem and it turned out that a very nice understanding of the situation emerges.

More precise formulation of the Dirac equation for the induced spinor fields is the first challenge. The well-definedness of em charge has turned out to be very powerful guideline in the understanding of the details of fermionic dynamics. Although induced spinor fields have also a part assignable space-time interior, the spinor modes at string world sheets determine the fermionic dynamics in accordance with strong form of holography (SH).

The well-definedness of em charged is guaranteed if induced spinors are associated with 2-D string world sheets with vanishing classical W boson fields. It turned out that an alternative manner to satisfy the condition is to assume that induced spinors at the boundaries of string world sheets are neutrino-like and that these string world sheets carry only classical W fields. Dirac action contains 4-D interior term and 2-D term assignable to string world sheets. Strong form of holography (SH) allows to interpret 4-D spinor modes as continuations of those assignable to string world sheets so that spinors at 2-D string world sheets determine quantum dynamics.

Twistor lift combined with this picture allows to formulate the Dirac action in more detail. Well-definedness of em charge implies that charged particles are associated with string world sheets assignable to the magnetic flux tubes assignable to homologically non-trivial geodesic sphere and neutrinos with those associated with homologically trivial geodesic sphere. This explains why neutrinos are so light and why dark energy density corresponds to neutrino mass scale, and provides also a new insight about color confinement.

A further important result is that the formalism works only for imbedding space dimension $D = 8$. This is due the fact that the number of vector components is the same as the number of spinor components of fixed chirality for $D = 8$ and corresponds directly to the octonionic triality.

p-Adic thermodynamics predicts elementary particle masses in excellent accuracy without Higgs vacuum expectation: the problem is to understand fermionic Higgs couplings. The observation that CP_2 part of the modified gamma matrices gives rise to a term mixing M^4 chiralities contain derivative allows to understand the mass-proportionality of the Higgs-fermion couplings at QFT limit.

4.7.1 More precise view about modified Dirac equation

Consistency conditions demand that modified Dirac equation with modified gamma matrices Γ^α defined as contractions $\Gamma^\alpha = T^{\alpha k}\gamma_k$ of canonical momentum currents $T^{\alpha k}$ associated with the bosonic action with imbedding space gamma matrices γ_k [K88, K110]. The Dirac operator is not hermitian in the sense that the conjugation for the Dirac equation for Ψ does not give Dirac equation for $\overline{\Psi}$ unless the modified gamma matrices have vanishing covariant divergence as vector at space-time surface. This says that classical field equations are satisfied. This consistency condition holds true also for spinor modes possibly localized at string world sheets to which one can perhaps assign area action plus topological action defined by Kähler magnetic flux. The interpretation is in terms of super-conformal invariance.

The challenge is to formulate this picture more precisely and here I have not achieved a satisfactory formulation. The question has been whether interior spinor field Ψ are present at all, whether only Ψ is present and somehow becomes singular at string world sheets, or whether both stringy spinors Ψ_s and interior spinors Ψ are present. Both Ψ and Ψ_s could be present and Ψ_s could serve as source for interior spinors with the same H-chirality.

The strong form of holography (SH) suggests that interior spinor modes Ψ_n are obtained as continuations of the stringy spinor modes $\Psi_{s,n}$ and one has $\Psi = \Psi_s$ at string world sheets. Dirac action would thus have a term localized at strong world sheets and bosonic action would contain similar term by the requirement of super-conformal symmetry. Can one realize this intuition?

(a) Suppose that Dirac action has interior and stringy parts. For the twistor lift of TGD [L24] the interior part with gamma matrices given by the modified gamma matrices associated with the sum of Kähler action and volume action proportional to cosmological constant Λ. The variation with respect to the interior spinor field Ψ gives modified Dirac equation in the interior with source term from the string world sheet. The H-chiralites of Ψ and Psi_s would be same. Quark like and leptonic H-chiralities have different couplings to Kähler gauge potential and mathematical consistency strongly encourages this.

    What is important is that the string world sheet part, which is bilinear in interior and string world sheet spinor fields Ψ and Ψ_s and otherwise has the same form as Dirac action. The natural assumption is that the stringy Dirac action corresponds to the modified gamma matrices assignable to area action.

(b) String world sheet must be minimal surface: otherwise hermiticity is lost. This can be achieved either by adding to the Kähler action string world sheet area term. Whatever the correct option is, quantum criticality should determine the value of string tension. The first string model inspired guess is that the string tension is proportional to gravitational constant $1/G = 1/l_P^2$ defining the radius fo M^4 twistor sphere or to $1/R^2$, R CP_2 radius. This would however allow only strings not much longer than l_P or R. A more natural estimate is that string tension is proportional to the cosmological constant Λ and depends on p-adic length scale as $1/p$ so that the tension becomes small in long length scales. Since Λ coupling contant type parameter, this estimate looks rather reasonable.

(c) The variation of stringy Dirac action with action density

$$L = [\overline{\Psi}_s D_s^{\rightarrow}\Psi - \overline{\Psi}_s D_s^{\leftarrow}\Psi]\sqrt{g_2} + h.c. \quad (4.7.1)$$

with respect to stringy spinor field Ψ_s gives for Ψ Dirac equation $D_s\Psi = 0$ if there are no Lagrange multiplier terms (see below). The variation in interior gives $D\Psi = S = D_s\Psi_s$, where the source term S is located at string world sheets. Ψ satisfies at string world sheet the analog of 2-D massless Dirac equation associated with the induced metric. This is just what stringy picture suggests.

The stringy source term for D equals to $D_s\Psi_s$ localized at string world sheets: the construction of solutions would require the construction of propagator for D, and this does not look an attractive idea. For $D_s\Psi_s = 0$ the source term vanishes. Holomorphy for Ψ_s indeed implies $D_s\Psi = 0$.

(d) $\Psi_s = \Psi$ would realize SH as a continuation of Ψ_s from string world sheet to Ψ in the interior. Could one introduce Lagrange multiplier term

$$L_1 = \overline{\Lambda}(\Psi - \Psi_s) + h.c.$$

to realize $\Psi_s = \Psi$? Lagrange multiplier spinor field Λ would serve a source in the Dirac equation for $\Psi = \Psi_s$ and Ψ should be constructed at string world sheet in terms of stringy fermionic propagator with Λ as source. The solution for Ψ_s would require the construction of 2-D stringy propagator for Ψ_s but in principle this is not a problem since the modes can be solved by holomorphy in hypercomplex stringy coordinate. The problem of this option is that the H-chiralities of Λ and Ψ would be opposite and the coupling of opposite H-chiralities is not in spirit with H-chirality conservation.

A possible cure is to replace the Lagrange multiplier term with

$$L_1 = \overline{\Lambda}^k\gamma_k(\Psi - \Psi_s) + h.c. \ . \quad (4.7.2)$$

The variation with respect to the spin 3/2 field Λ^k would give 8 conditions - just the number of spinor components for given H-chirality - forcing $\Psi = \Psi_s$! $D = 8$ would be in crucial role! In other imbedding space dimensions the number of conditions would be too high or too low.

One would however obtain

$$D_s\Psi = D_s\Psi_s = \Lambda^k\gamma_k \ . \quad (4.7.3)$$

One could of course solve Ψ at string world sheet from $\Lambda^k\gamma_k$ by constructing the 2-D propagator associated with D_s. Conformal symmetry for the modes however implies $D_s\Psi = 0$ so that one has actually $\Lambda^k = 0$ and Λ^k remains mere formal tool to realize the constraint $\Psi = \Psi_s$ in mathematically rigorous manner for imbedding space dimension $D = 8$. This is a new very powerful argument in favor of TGD.

(e) At the string world sheets Ψ would be annihilated both by D and D_s. The simplest possibility is that the actions of D and D_s are proportional to each other at string world sheets. This poses conditions on string world sheets, which might force the CP_2 projection of string world sheet to belong to a geodesic sphere or circle of CP_2. The idea that string world sheets and also 3-D surfaces with special role in TGD could correspond to singular manifolds at which trigonometric functions representing CP_2 coordinates tend to go outside their allowed value range supports this picture. This will be discussed below.

i. For the geodesic sphere of type II induced Kähler form vanishes so that the action of 4-D Dirac massless operator would be determined by the volume term (cosmological constant). Could the action of D reduce to that of D_s at string world sheets? Does this require a reduction of the metric to an orthogonal direct sum from string world sheet tangent space and normal space and that also normal part of D annihilates the spinors at the string world sheet? The modes of Ψ at string world sheets are locally constant with respect to normal coordinates.

ii. For the geodesic sphere of type I induced Kähler form is non-vanishing and brings an additional term to D coming from CP_2 degrees of freedom. This might lead to trouble since the gamma matrix structures of D and D_s would be different. One could however add to string world sheet bosonic action a topological term as Kähler magnetic flux. Although its contribution to the field equations is trivial, the contribution to the modified gamma matrices is non-vanishing and equal to the contraction $J^{\alpha k}\gamma_k$ of half projection of the Kähler form with CP_2 gamma matrices. The presence of this term could allow the reduction of $D\Psi_s = 0$ and $D_s\Psi_s = 0$ to each other also in this case.

4.7.2 A more detailed view about string world sheets

In TGD framework gauge fields are induced and what typically occurs for the space-time surfaces is that they tend to "go out" from CP_2. Could various lower-D surfaces of space-time surface correspond to sub-manifolds of space-time surface?

(a) To get a concrete idea about the situation it is best to look what happens in the case of sphere $S^2 = CP_1$. In the case of sphere S^2 the Kähler form vanishes at South and North poles. Here the dimension is reduced by 2 since all values of ϕ correspond to the same point. $sin(\Theta)$ equals to 1 at equator - geodesic circle - and here Kähler form is non-vanishing. Here dimension is reduced by 1 unit. This picture conforms with the expectations in the case of CP_2 These two situations correspond to 1-D and 2-D geodesic sub-manifolds.

(b) CP_2 coordinates can be represented as cosines or sines of angles and the modules of cosine or sine tends to become larger than 1 (see `http://tinyurl.com/z3coqau`). In Eguchi-Hanson coordinates (r, Θ, Φ, Ψ) the coordinates r and Θ give rise to this kind of trigonometric coordinates. For the two cyclic angle coordinates (Φ, Ψ) one does not encounter this problem.

(c) In the case of CP_2 only geodesic sub-manifolds with dimensions $D = 0, 1, 2$ are possible. 1-D geodesic submanifolds carry vanishing induce spinor curvature. The impossibility of 3-D geodesic sub-manifolds would suggest that 3-D surfaces are not important. CP_2 has two geodesic spheres: S^2_I is homologically non-trivial and S^2_{II} homologically trivial (see `http://tinyurl.com/z3coqau`).

i. Let us consider S^2_I first. CP_2 has 3 poles, which obviously relates to SU(3), and in Eguchi Hanson coordinates (r, θ, Φ, Ψ) the surface $r = \infty$ is one of them and corresponds - not to a 3-sphere - but homologically non-trivial geodesic 2- sphere, which is complex sub-manifold and orbits of $SU(2)\times U(1)$ subgroup. Various values of the coordinate Ψ correspond to same point as those of Φ at the poles of S^2. The Kähler form J and classical Z^0 and γ fields are non-vanishing whereas W gauge fields vanish leaving only induced γ and Z^0 field as one learns by studying the detailed expressions for the curvature of spinor curvature and vierbein of CP_2.
String world sheet could have thus projection to S^2_I but both γ and Z^0 would be vanishing except perhaps at the boundaries of string world sheet, where Z^0 would naturally vanish in the picture provided by standard model. One can criticize the presence of Z^0 field since it would give a parity breaking term to the modified Dirac operator. SH would suggest that the reduction to electromagnetism at string boundaries might make sense as counterpart for standard model picture. Note that the original vision was that besides induced Kähler form and em field also Z^0 field could vanish at string world sheets.

ii. The homologically trivial geodesic sphere S^2_{II} is the orbit of $SO(3)$ subgroup and not a complex manifold. By looking the standard example about S^2_I, one finds that the both J, Z_0, and γ vanish and only the W components of spinor connection are non-vanishing. In this case the notion of em charge would not be well-defined for S^2_{II} without additional conditions. Partonic 2-surfaces, their light-like orbits, and boundaries of string world sheets could do so since string world sheets have 1-D intersection with with the orbits. This picture would make sense for the minimal surfaces replacing vacuum extremals in the case of twistor lift of TGD.

Since em fields are not present, the presence of classical W fields need not cause problems. The absence of classical em fields however suggests that the modes of induced spinor fields at boundaries of string worlds sheets must be em neutral and represent therefore neutrinos. The safest but probably too strong option would be right-handed neutrino having no coupling spinor connection but coupling to the CP_2 gamma matrices transforming it to left handed neutrino. Recall that ν_R represents a candidate for super-symmetry.

Neither charged leptons nor quarks would be allowed at string boundaries and classical W gauge potentials should vanish at the boundaries if also left-handed neutrinos are allowed: this can be achieved in suitable gauge. Quarks and charged leptons could reside only at string world sheets assignable to monopole flux tubes. This could relate to color confinement and also to the widely different mass scales of neutrinos and other fermions as will be found.

To sum up, the new result is that the distinction between neutrinos and other fermions could be understood in terms of the condition that em charge is well-defined. What looked originally a problem of TGD turns out to be a powerful predictive tool.

4.7.3 Classical Higgs field again

A motivation for returning back to Higgs field comes from the twistor lift of Kähler action.

(a) The twistor lift of TGD [L22] [L24] brings in cosmological constant as the coefficient of volume term resulting in dimensional reduction of 6-D Kähler action for twistor space of space-time surface realized as surface in the product of twistor space of M^4 and CP_2. The radius of the sphere of M^4 twistor bundle corresponds to Planck length. Volume term is extremely small but removes the huge vacuum degeneracy of Kähler action. Vacuum extremals are replaced by 4-D minimal surfaces and modified Dirac equation is just the analog of massless Dirac equation in complete analogy with string models.

(b) The well-definedness and conservation of fermionic em charges and SH demand the localization of fermions to string world sheets. The earlier picture assumed only em fields at string world sheets. More precise picture allows also W fields.

(c) The first guess is that string world sheets are minimal surfaces and this is supported by the previous considerations demanding also string area term and Kähler magnetic flux tube. Here gravitational constant assignable to M^4 twistor space would be the first guess for the string tension.

What one can say about the possible existence of classical Higgs field?

(a) TGD predicts both Higgs type particles and gauge bosons as bound states of fermions and antifermions and they differ only in that their polarization are in M^4 resp. CP_2 tangent space. p-adic thermodynamics [K39] gives excellent predictions for elementary particle masses in TGD framework. Higgs vacuum expectation is not needed to predict fermion or boson masses. Standard model gives only a parametrization of these masses by assuming that Higgs couplings to fermions are proportional to their masses, it does not predict them.

The experimental fact is however that the couplings of Higgs are proportional to fermion masses and TGD should be able to predict this and there is a general argument for the proportonality, which however should be deduced from basic TGD. Can one achieve this?

(b) Can one imagine any candidate for the classical Higgs field? There is no covariantly constant vector field in CP_2, whose space-time projection could define a candidate for classical Higgs field. This led years ago before the model for how bosons emerge from fermions to the wrong conclusion that TGD does not predict Higgs.

The first guess for the possibly existing classical counterpart of Higgs field would be as CP_2 part for the divergence of the space-time vector defined modified gamma matrices expressible in terms of canonical momentum currents having natural interpretation as a generalization of force for point like objects to that for extended objects. Higgs field in this sense would however vanish by above consistency conditions and would not couple to spinors at all.

Classical Higgs field should have only CP_2 part being CP_2 vector. What would be also troublesome that this proposale for classical Higgs field would involve second derivatives of imbedding space coordinates. Hence it seems that there is no hope about geometrization of classical Higgs fields.

(c) The contribution of the induced Kähler form gives to the modified gamma matrices a term expressible solely in terms of CP_2 gamma matrices. This term appears in modified Dirac equation and mixes M^4 chiralities - a signal for the massivation. This term is analogous to Higgs term expect that it contains covariant derivative.

The question that I have not posed hitherto is whether this term could at QFT limit of TGD give rise to vacuum expectation of Higgs. The crucial observation is that the presence of derivative, which in quantum theory corresponds roughly to mass proportionality of chirality mixing coupling at QFT limit. This could explain why the coupling of Higgs field to fermions is proportional to the mass of the fermion at QFT limit!

(d) For S^2_{II} type string world sheets assignable to neutrinos the contribution to the chirality mixing coupling should be of order of neutrino mass. The coefficient $1/L^4$ of the volume term defining cosmological constant [L24] separates out as over all factor in massless Dirac equation and the parameter characterizing the mass scale causing the mixing is of order $m = \omega_1\omega_2 R$. Here ω_1 characterizes the scale of gradient for CP_2 coordinates. The simplest minimal surface is that for which CP_2 projection is geodesic line with $\Phi = \omega_1 t$. ω_2 characterizes the scale of the gradient of spinor mode.

Assuming $\omega_1 = \omega_2 \equiv \omega$ the scale m is of order neutrino mass $m_\nu \simeq .1$ eV from the condition $m \sim \omega^2 R \sim m_\nu$. This gives the estimate $\omega \sim \sqrt{m_{CP_2} m_\nu} \sim 10^2 m_p$ from $m_{CP_2} \sim 10^{-4} m_P$, which is weak mass scale and therefore perfectly sensible. The reduction $\Delta c/c$ of the light velocity from maximal signal velocity due the replacement $g_{tt} = 1 - R^2\omega^2$ is $\Delta c/c \sim 10^{-34}$ and thus completely negligible. This estimate does not make sense for charged fermions, which correspond to S^2_I type string world sheets.

A possible problem is that if the value of the cosmological constant Λ evolves as $1/p$ as function of the length mass scale the mass scale of neutrinos should increase in short scales. This looks strange unless the mass scale remains below the cosmic temperature so that neutrinos would be always effectively massless.

(e) For S^2_I type string world sheets assignable to charged fermions Kähler action dominates and the mass scales are expected to be higher than for neutrinos. For S^2_I type strings the modified gamma matrices contain also Kähler term and a rough estimate is that the ratio of two contributions is the ratio of the energy density of Kähler action to vacuum energy density. As Kähler energy density exceeds the value corresponding to vacuum energy density $1/L^4$, $L \sim 40$ μm, Kähler action density begins to dominate over dark energy density.

To sum up, this picture suggest that the large difference between the mass scales of neutrinos and em charged fermions is due to the fact that neutrinos are associated with string world sheet of type II and em charged fermions with string world sheets of type I. Both strings world sheets would be accompanied by flux tubes but for charged particles the flux tubes would carry Kähler magnetic flux. Cosmological constant forced by twistor lift would make neutrinos massive and allow to understand neutrino mass scale.

Chapter 5

Recent View about Kähler Geometry and Spin Structure of "World of Classical Worlds"

5.1 Introduction

The construction of Kähler geometry of WCW ("world of classical worlds") is fundamental to TGD program. I ended up with the idea about physics as WCW geometry around 1985 and made a breakthrough around 1990, when I realized that Kähler function for WCW could correspond to Kähler action for its preferred extremals defining the analogs of Bohr orbits so that classical theory with Bohr rules would become an exact part of quantum theory and path integral would be replaced with genuine integral over WCW. The motivating construction was that for loop spaces leading to a unique Kähler geometry [A48]. The geometry for the space of 3-D objects is even more complex than that for loops and the vision still is that the geometry of WCW is unique from the mere existence of Riemann connection.

The basic idea is that WCW is union of symmetric spaces G/H labelled by zero modes which do not contribute to the WCW metric. There have been many open questions and it seems the details of the ealier approach [**?**]ust be modified at the level of detailed identifications and interpretations.

(a) A longstanding question has been whether one could assign Equivalence Principle (EP) with the coset representation formed by the super-Virasoro representation assigned to G and H in such a manner that the four-momenta associated with the representations and identified as inertial and gravitational four-momenta would be identical. This does not seem to be the case. The recent view will be that EP reduces to the view that the classical four-momentum associated with Kähler action is equivalent with that assignable to Kähler-Dirac action supersymmetrically related to Kähler action: quantum classical correspondence (QCC) would be in question. Also strong form of general coordinate invariance implying strong form of holography in turn implying that the super-symplectic representations assignable to space-like and light-like 3-surfaces are equivalent could imply EP with gravitational and inertial four-momenta assigned to these two representations.

At classical level EP follows from the interpretation of GRT space-time as effective space-time obtained by replacing many-sheeted space-time with Minkowski space with effective metric determined as a sum of Minkowski metric and sum over the deviations of the induced metrices of space-time sheets from Minkowski metric. Poincare invariance suggests strongly classical EP for the GRT limit in long length scales at least.

(b) The detailed identification of groups G and H and corresponding algebras has been a longstanding problem. Symplectic algebra associated with$\delta M^4_\pm \times CP2$ ($\delta M^4_\pm$ is light-cone boundary - or more precisely, with the boundary of causal diamond (CD) defined as Cartesian product of CP_2 with intersection of future and past direct light cones of M^4 has Kac-Moody type structure with light-like radial coordinate replacing complex coordinate z. Virasoro algebra would correspond to radial diffeomorphisms. I have also introduced Kac-Moody algebra assigned to the isometries and localized with respect to internal coordinates of the light-like 3-surfaces at which the signature of the induced metric changes from Minkowskian to Euclidian and which serve as natural correlates for elementary particles (in very general sense!). This kind of localization by force could be however argued to be rather ad hoc as opposed to the inherent localization of the symplectic algebra containing the symplectic algebra of isometries as sub-algebra. It turns out that one obtains direct sum of representations of symplectic algebra and Kac-Moody algebra of isometries naturally as required by the success of p-adic mass calculations.

(c) The dynamics of Kähler action is not visible in the earlier construction. The construction also expressed WCW Hamiltonians as 2-D integrals over partonic 2-surfaces. Although strong form of general coordinate invariance (GCI) implies strong form of holography meaning that partonic 2-surfaces and their 4-D tangent space data should code for quantum physics, this kind of outcome seems too strong. The progress in the understanding of the solutions of Kähler-Dirac equation led however to the conclusion that spinor modes other than right-handed neutrino are localized at string world sheets with strings connecting different partonic 2-surfaces. This leads to a modification of earlier construction in which WCW super-Hamiltonians are essentially integrals with integrand identified as a Noether super current for the deformations in G Each spinor mode gives rise to super current and the modes of right-handed neutrino and other fermions differ in an essential manner. Right-handed neutrino would correspond to symplectic algebra and other modes to the Kac-Moody algebra and one obtains the crucial 5 tensor factors of Super Virasoro required by p-adic mass calculations.

The matrix elements of WCW metric between Killing vectors are expressible as anti-commutators of super-Hamiltonians identifiable as contractions of WCW gamma matrices with these vectors and give Poisson brackets of corresponding Hamiltonians. The anti-commutation relates of induced spinor fields are dictated by this condition. Everything is 3-dimensional although one expects that symplectic transformations localized within interior of X^3 act as gauge symmetries so that in this sense effective 2-dimensionality is achieved. The components of WCW metric are labelled by standard model quantum numbers so that the connection with physics is extremely intimate.

(d) An open question in the earlier visions was whether finite measurement resolution is realized as discretization at the level of fundamental dynamics. This would mean that only certain string world sheets from the slicing by string world sheets and partonic 2-surfaces are possible. The requirement that anti-commutations are consistent suggests that string world sheets correspond to surfaces for which Kähler magnetic field is constant along string in well defined sense ($J_{\mu\nu}\epsilon^{\mu\nu}g^{1/2}$ remains constant along string). It however turns that by a suitable choice of coordinates of 3-surface one can guarantee that this quantity is constant so that no additional constraint results.

(e) Quantum criticality is one of the basic notions of quantum TGD and its relationship to coset construction has remained unclear. In this chapter the concrete realization of criticality in terms of symmetry breaking hierarchy of Super Virasoro algebra acting on symplectic and Kac-Moody algebras. Also a connection with finite measurement resolution - second key notion of TGD - emerges naturally.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [L12].

5.2 WCW As A Union Of Homogenous Or Symmetric Spaces

The physical interpretation and detailed mathematical understanding of super-conformal symmetries has developed rather slowly and has involved several side tracks. In the following I try to summarize the basic picture with minimal amount of formulas with the understanding that the statement "Noether charge associated with geometrically realized Kac-Moody symmetry" is enough for the reader to write down the needed formula explicitly. Formula oriented reader might deny the value of the approach giving weight to principles. My personal experience is that piles of formulas too often hide the lack of real understanding.

In the following the vision about WCW as union of coset spaces is discussed in more detail.

5.2.1 Basic Vision

The basic view about coset space construction for WCW has not changed.

(a) The idea about WCW as a union of coset spaces G/H labelled by zero modes is extremely attractive. The structure of homogenous space [A7] (`http://tinyurl.com/y7u2t8jo`) means at Lie algebra level the decomposition $g = h \oplus t$ to sub-Lie-algebra h and its complement t such that $[h, t] \subset t$ holds true. Homogeneous spaces have G as its isometries. For symmetric space the additional condition $[t, t] \subset h$ holds true and implies the existence of involution changing at the Lie algebra level the sign of elements of t and leaving the elements of h invariant. The assumption about the structure of symmetric space [A22] (`http://tinyurl.com/ycouv7uh`) implying covariantly constant curvature tensor is attractive in infinite-dimensional case since it gives hopes about calculability.

An important source of intuition is the analogy with the construction of CP_2, which is symmetric space A particular choice of h corresponds to Lie-algebra elements realized as Killing vector fields which vanish at particular point of WCW and thus leave 3-surface invariant. A preferred choice for this point is as maximum or minimum of Kähler function. For this 3-surface the Hamiltonians of h should be stationary. If symmetric space property holds true then commutators of $[t, t]$ also vanish at the minimum/maximum. Note that Euclidian signature for the metric of WCW requires that Kähler function can have only maximum or minimum for given zero modes.

(b) The basic objection against TGD is that one cannot use the powerful canonical quantization using the phase space associated with configuration space - now WCW . The reason is the extreme non-linearity of the Kähler action and its huge vacuum degeneracy, which do not allow the construction of Hamiltonian formalism. Symplectic and Kähler structure must be realized at the level of WCW . In particular, Hamiltonians must be represented in completely new manner. The key idea is to construct WCW Hamiltonians as anti-commutators of super-Hamiltonians defining the contractions of WCW gamma matrices with corresponding Killing vector fields and therefore defining the matrix elements of WCW metric in the tangent vector basis defined by Killing vector fields. Super-symmetry therefre gives hopes about constructing quantum theory in which only induced spinor fields are second quantized and imbedding space coordinates are treated purely classically.

(c) It is important to understand the difference between symmetries and isometries assigned to the Kähler function. Symmetries of Kähler function do not affect it. The symmetries of Kähler action are also symmetries of Kähler action because Kähler function is Kähler action for a preferred extremal (here there have been a lot of confusion). Isometries leave invariant only the quadratic form defined by Kähler metric $g_{M\overline{N}} = \partial_M \partial_{\overline{L}} K$ but not Kähler function in general. For G/H decomposition G represents isometries and H both isometries and symmetries of Kähler function.

CP_2 is familiar example: $SU(3)$ represents isometries and $U(2)$ leaves also Kähler function invariant since it depends on the $U(2)$ invariant radial coordinate r of CP_2. The origin $r = 0$ is left invariant by $U(2)$ but for $r > 0$ $U(2)$ performs a rotation at $r = constant$ 3-sphere. This simple picture helps to understand what happens at the level of WCW .

How to then distinguish between symmetries and isometries? A natural guess is that one obtains also for the isometries Noether charges but the vanishing of boundary terms at spatial infinity crucial in the argument leading to Noether theorem as $\Delta S = \Delta Q = 0$ does not hold true anymore and one obtains charges which are not conserved anymore. The symmetry breaking contributions would now come from effective boundaries defined by wormhole throats at which the induce metric changes its signature from Minkowskian to Euclidian. A more delicate situation is in which first order contribution to ΔS vanishes and therefore also ΔQ and the contribution to ΔS comes from second variation allowing also to define Noether charge which is not conserved.

(d) The simple picture about CP_2 as symmetric space helps to understand the general vision if one assumes that WCW has the structure of symmetric space. The decomposition $g = h+t$ corresponds to decomposition of symplectic deformations to those which vanish at 3-surface (h) and those which do not (t).

For the symmetric space option, the Poisson brackets for super generators associated with t give Hamiltonians of h identifiable as the matrix elements of WCW metric. They would not vanish although they are stationary at 3-surface meaning that Riemann connection vanishes at 3-surface. The Hamiltonians which vanish at 3-surface X^3 would correspond to t and the Hamiltonians for which Killing vectors vanish and which therefore are stationary at X^3 would correspond to h. Outside X^3 the situation would of course be different. The metric would be obtained by parallel translating the metric from the preferred point of WCW to elsewhere and symplectic transformations would make this parallel translation.

For the homogenous space option the Poisson brackets for super generators of t would still give Hamiltonians identifiable as matrix elements of WCW metric but now they would be necessary those of h. In particular, the Hamiltonians of t do not in general vanish at X^3.

5.2.2 Equivalence Principle And WCW

5.2.3 Ep At Quantum And Classical Level

Quite recently I returned to an old question concerning the meaning of Equivalence Principle (EP) in TGD framework.

Heretic would of course ask whether the question about whether EP is true or not is a pseudo problem due to uncritical assumption there really are two different four-momenta which must be identified. If even the identification of these two different momenta is difficult, the pondering of this kind of problem might be waste of time.

At operational level EP means that the scattering amplitudes mediated by graviton exchange are proportional to the product of four-momenta of particles and that the proportionality constant does not depend on any other parameters characterizing particle (except spin). The are excellent reasons to expect that the stringy picture for interactions predicts this.

(a) The old idea is that EP reduces to the coset construction for Super Virasoro algebra using the algebras associated with G and H. The four-momenta assignable to these algebras would be identical from the condition that the differences of the generators annihilate physical states and identifiable as inertial and gravitational momenta. The objection is that for the preferred 3-surface H by definition acts trivially so that time-like translations leading out from the boundary of CD cannot be contained by H unlike G. Hence four-momentum is not associated with the Super-Virasoro representations assignable to H and the idea about assigning EP to coset representations does not look promising.

(b) Another possibility is that EP corresponds to quantum classical correspondence (QCC) stating that the classical momentum assignable to Kähler action is identical with gravitational momentum assignable to Super Virasoro representations. This forced to reconsider the questions about the precise identification of the Kac-Moody algebra and about how to obtain the magic five tensor factors required by p-adic mass calculations [K79].

A more precise formulation for EP as QCC comes from the observation that one indeed obtains two four-momenta in TGD approach. The classical four-momentum assignable to the Kähler action and that assignable to the Kähler-Dirac action. This four-momentum is an operator and QCC would state that given eigenvalue of this operator must be equal to the value of classical four-momentum for the space-time surfaces assignable to the zero energy state in question. In this form EP would be highly non-trivial. It would be justified by the Abelian character of four-momentum so that all momentum components are well-defined also quantum mechanically. One can also consider the splitting of four-momentum to longitudinal and transversal parts as done in the parton model for hadrons: this kind of splitting would be very natural at the boundary of CD. The objection is that this correspondence is nothing more than QCC.

(c) A further possibility is that duality of light-like 3-surfaces and space-like 3-surfaces holds true. This is the case if the action of symplectic algebra can be defined at light-like 3-surfaces or even for the entire space-time surfaces. This could be achieved by parallel translation of light-cone boundary providing slicing of CD. The four-momenta associated with the two representations of super-symplectic algebra would be naturally identical and the interpretation would be in terms of EP.

One should also understand how General Relativity and EP emerge at classical level. The understanding comes from the realization that GRT is only an effective theory obtained by endowing M^4 with effective metric.

(a) The replacement of superposition of fields with superposition of their effects means replacing superposition of fields with the set-theoretic union of space-time surfaces. Particle experiences sum of the effects caused by the classical fields at the space-time sheets.

(b) This is true also for the classical gravitational field defined by the deviation from flat Minkowski metric instandard M^4 coordinates for the space-time sheets. One can define effective metric as sum of M^4 metric and deviations. This effective metric would correspond to that of General Relativity. This resolves long standing issues relating to the interpretation of TGD.

(c) Einstein's equations could hold true for the effective metric. They are motivated by the underlying Poincare invariance which cannot be realized as global conservation laws for the effective metric. The conjecture vanishing of divergence of Khler energy momentum tensor can be seen as the microscopic justification for the claim that Einstein's equations hold true for the effective space-time.

(d) The breaking of Poincare invariance could have interpretation as effective breaking in zero energy ontology (ZEO), in which various conserved charges are length dependent and defined separately for each causal diamond (CD).

One can of course consider the possibility that Einstein's equations generalize for preferred extremals of Kähbler action. This would actually represent at space-time level the notion of QCC rather than realise QCC interpreted as EP. The condition that the energy momentum tensor for Kähler action has vanishing covariant divergence would be satisfied in GRT if Einstein's equations with cosmological term hold true. This is the case also now but one can consider also more general solutions in which one has two cosmological constants which are not genuine constants anymore [K103].

An interesting question is whether inertial-gravitational duality generalizes to the case of color gauge charges so that color gauge fluxes would correspond to "gravitational" color charges and the charges defined by the conserved currents associated with color isometries would define "inertial" color charges. Since the induced color fields are proportional to color

Hamiltonians multiplied by Kähler form they vanish identically for vacuum extremals in accordance with "gravitational" color confinement.

5.2.4 Criticism Of The Earlier Construction

The earlier detailed realization of super-Hamiltonians and Hamiltonians can be criticized.

(a) Even after these more than twenty years it looks strange that the Hamiltonians should reduce to flux integrals over partonic 2-surfaces. The interpretation has been in terms of effective 2-dimensionality suggested strongly by strong form of general coordinate invariance stating that the descriptions based on light-like orbits of partonic 2-surfaces and space-like three surfaces at the ends of causal diamonds are dual so that only partonic 2-surfaces and 4-D tangent space data at them would matter. Strong form of holography implies effective 2-dimensionality but this should correspond gauge character for the action of symplectic generators in the interior the space-like 3-surfaces at the ends of CDs, which is something much milder.

One expects that the strings connecting partonic 2-surfaces could bring something new to the earlier simplistic picture. The guess is that imbedding space Hamiltonian assignable to a point of partonic 2-surface should be replaced with that defined as integral over string attached to the point. Therefore the earlier picture would suffer no modification at the level of general formulas.

(b) The fact that the dynamics of Kähler action and Kähler-Dirac action are not directly involved with the earlier construction raises suspicions. I have proposed that Kähler function could allow identification as Dirac determinant [K88] but one would expect more intimate connection. Here the natural question is whether super-Hamiltonians for the Kähler-Dirac action could correspond to Kähler charges constructible using Noether's theorem for corresponding deformations of the space-time surface and would also be identifiable as WCW gamma matrices.

5.2.5 Is WCW Homogenous Or Symmetric Space?

A key question is whether WCW can be symmetric space [A22] (`http://tinyurl.com/y8ojglkb`) or whether only homogenous structure is needed. The lack of covariant constancy of curvature tensor might produce problems in infinite-dimensional context.

The algebraic conditions for symmetric space are $g = h + t$, $[h, t] \subset t$, $[t, t] \subset h$. The latter condition is the difficult one.

(a) δCD Hamiltonians should induce diffeomorphisms of X^3 indeed leaving it invariant. The symplectic vector fields would be parallel to X^3. A stronger condition is that they induce symplectic transformations for which all points of X^3 remain invariant. Now symplectic vector fields vanish at preferred 3-surface (note that the symplectic transformations are r_M local symplectic transformations of $S^2 \times CP_2$).

(b) For Kac-Moody algebra inclusion $H \subset G$ for the finite-dimensional Lie-algebra induces the structure of symmetric space. If entire algebra is involved this does not look physically very attractive idea unless one believes on symmetry breaking for both $SU(3)$, $U(2)_{ew}$, and $SO(3)$ and E_2 (here complex conjugation corresponds to the involution). If one assumes only Kac-Moody algebra as critical symmetries, the number of tensor factors is 4 instead of five, and it is not clear whether one can obtain consistency with p-adic mass calculations.

Examples of 3-surfaces remaining invariant under $U(2)$ are 3-spheres of CP_2. They could correspond to intersections of deformations of CP_2 type vacuum extremals with the boundary of CD. Also geodesic spheres S^2 of CP_2 are invariant under $U(2)$ subgroup and would relate naturally to cosmic strings. The corresponding 3-surface would be $L \times S^2$, where L is a piece of light-like radial geodesic.

(c) In the case of symplectic algebra one can construct the imbedding space Hamiltonians inducing WCW Hamiltonians as products of elements of the isometry algebra of $S^2 \times CP_2$ for with parity under involution is well-defined. This would give a decomposition of the symplectic algebra satisfying the symmetric space property at the level imbedding space. This decomposition does not however look natural at the level of WCW since the only single point of CP_2 and light-like geodesic of δM^4_+ can be fixed by $SO(2) \times U(2)$ so that the 3-surfaces would reduce to pieces of light rays.

(d) A more promising involution is the inversion $r_M \rightarrow 1/r_M$ of the radial coordinate mapping the radial conformal weights to their negatives. This corresponds to the inversion in Super Virasoro algebra. t would correspond to functions which are odd functions of $u \equiv log(r_M/r_0)$ and h to even function of u. Stationary 3-surfaces would correspond to $u = 1$ surfaces for which $log(u) = 0$ holds true. This would assign criticality with Virasoro algebra as one expects on general grounds.

$r_M = constant$ surface would most naturally correspond to a maximum of Kähler function which could indeed be highly symmetric. The elements with even u-parity should define Hamiltonians, which are stationary at the maximum of Kähler function. For other 3-surfaces obtained by $/r_M$-local) symplectic transformations the situation is different: now H is replaced with its symplectic conjugate hHg^{-1} of H is acceptable and if the conjecture is true one would obtained 3-surfaces assignable to perturbation theory around given maximum as symplectic conjugates of the maximum. The condition that H leaves X^3 invariant in poin-twise manner is certainly too strong and imply that the 3-surface has single point as CP_2 projection.

(e) One can also consider the possibility that critical deformations correspond to h and non-critical ones to t for the preferred 3-surface. Criticality for given h would hold only for a preferred 3-surface so that this picture would be very similar that above. Symplectic conjugates of h would define criticality for other 3-surfaces. WCW would decompose to a union corresponding to different criticalities perhaps assignable to the hierarchy of sub-algebras of conformal algebra labelled by integer whose multiples give the allowed conformal weights. Hierarchy of breakings of conformal symmetries would characterize this hierarchy of sectors of WCW .

For sub-algebras of the conformal algebras (Kac-Moody and symplectic algebra) the condition $[t, t] \subset h$ cannot hold true so that one would obtain only the structure of homogenous space.

5.2.6 Symplectic And Kac-Moody Algebras As Basic Building Bricks

5.3 Updated View About Kähler Geometry Of WCW

During last years the understanding of the mathematical aspects of TGD and of its connection with the experimental world has developed rapidly.

TGD differs in several respects from quantum field theories and string models. The basic mathematical difference is that the mathematically poorly defined notion of path integral is replaced with the mathematically well-defined notion of functional integral defined by the Kähler function defining Kähler metric for WCW ("world of classical worlds"). Apart from quantum jump, quantum TGD is essentially theory of classical WCW spinor fields with WCW spinors represented as fermionic Fock states. One can say that Einstein's geometrization of physics program is generalized to the level of quantum theory.

It has been clear from the beginning that the gigantic super-conformal symmetries generalizing ordinary super-conformal symmetries are crucial for the existence of WCW Kähler metric. The detailed identification of Kähler function and WCW Kähler metric has however turned out to be a difficult problem. It is now clear that WCW geometry can be understood in terms of the analog of AdS/CFT duality between fermionic and space-time degrees of freedom (or between Minkowskian and Euclidian space-time regions) allowing to express Kähler metric either in terms of Kähler function or in terms of anti-commutators of WCW gamma matrices identifiable as super-conformal Noether super-charges for the symplectic algebra assignable to $\delta M^4_\pm \times CP_2$. The string model type description of gravitation emerges and also the TGD based view about dark matter becomes more precise. String tension is however dynamical rather than pregiven and the hierarchy of Planck constants is necessary in order to understand the formation of gravitationally bound states. Also the proposal that sparticles correspond to dark matter becomes much stronger: sparticles actually are dark variants of particles.

A crucial element of the construction is the assumption that super-symplectic and other super-conformal symmetries having the same structure as 2-D super-conformal groups can be seen a broken gauge symmetries such that sub-algebra with conformal weights coming as n-ples of those for full algebra act as gauge symmetries. In particular, the Noether charges of this algebra vanish for preferred extremals- this would realize the strong form of holography implied by strong form of General Coordinate Invariance. This gives rise to an infinite number of hierarchies of conformal gauge symmetry breakings with levels labelled by integers $n(i)$ such that $n(i)$ divides $n(i+1)$ interpreted as hierarchies of dark matter with levels labelled by the value of Planck constant $h_{eff} = n \times h$. These hierarchies define also hierarchies of quantum criticalities, and are proposed to give rise to inclusion hierarchies of hyperfinite factors of II_1 having interpretation in terms of finite cognitive resolution with inclusions being characterized by the integers $n(+1)/n(i)$.

These hierarchies are fundamental for the understanding of living matter. Living matter is fighting in order to stay at criticality and uses metabolic energy and homeostasis to achieve this. In the biological death of the system (self) a phase transition increasing h_{eff} finally takes place. The sub-selves of self experienced by self as mental images however die and are reborn at opposite boundary of the corresponding causal diamond (CD) and they genuinely evolve so that self can be said to become wiser even without dying! The purpose of this fighting against criticality would thus allow a possibility for sub-selves to evolve via subsequent re-incarnations. One interesting prediction is the possibility of time reversed mental images. The challenge is to understand what they do mean at the level of conscious experience.

5.3.1 Kähler Function, Kähler Action, And Connection With String Models

The definition of Kähler function in terms of Kähler action is possible because space-time regions can have also Euclidian signature of induced metric. Euclidian regions with 4-D CP_2 projection - wormhole contacts - are identified as lines of generalized Feynman diagrams - space-time correlates for basic building bricks of elementary particles. Kähler action from Minkowskian regions is imaginary and gives to the functional integrand a phase factor crucial for quantum field theoretic interpretation. The basic challenges are the precise specification of Kähler function of "world of classical worlds" (WCW) and Kähler metric.

There are two approaches concerning the definition of Kähler metric: the conjecture analogous to AdS/CFT duality is that these approaches are mathematically equivalent.

(a) The Kähler function defining Kähler metric can be identified as Kähler action for space-time regions with Euclidian signature for a preferred extremal containing 3-surface as the ends of the space-time surfaces inside causal diamond (CD). Minkowskian space-time regions give to Kähler action an imaginary contribution interpreted as the counterpart of quantum field theoretic action. The exponent of Kähler function gives rise to a mathematically well-defined functional integral in WCW . WCW metric is dictated by the Euclidian regions of space-time with 4-D CP_2 projection.

The basic question concerns the attribute "preferred". Physically the preferred extremal is analogous to Bohr orbit. What is the mathematical meaning of preferred extremal of Kähler action? The latest step of progress is the realization that the vanishing of generalized conformal charges for the ends of the space-time surface fixes the preferred extremals to high extent and is nothing but classical counterpart for generalized Virasoro and Kac-Moody conditions.

(b) Fermions are also needed. The well-definedness of electromagnetic charge led to the hypothesis that spinors are restricted at string world sheets. One could also consider associativity as basic contraint to fermionic dynamics combined with the requirement that octonionic representation for gamma matrices is equivalent with the ordinary one. The conjecture is that this leads to the same outcome. This point is highly non-trivial and will be discussed below separately.

(c) Second manner to define Kähler metric is as anticommutators of WCW gamma matrices identified as super-symplectic Noether charges for the Dirac action for induced spinors with string tension proportional to the inverse of Newton's constant. These charges are associated with the 1-D space-like ends of string world sheets connecting the wormhole throats. WCW metric contains contributions from the spinor modes associated with various string world sheets connecting the partonic 2-surfaces associated with the 3-surface.

It is clear that the information carried by WCW metric about 3-surface is rather limited and that the larger the number of string world sheets, the larger the information. This conforms with strong form of holography and the notion of measurement resolution as a property of quantums state. Duality clearly means that Kähler function is determined either by space-time dynamics inside Euclidian wormhole contacts or by the dynamics of fermionic strings in Minkowskian regions outside wormhole contacts. This duality brings strongly in mind AdS/CFT duality. One could also speak about fermionic emergence since Kähler function is dictated by the Kähler metric part from a real part of gradient of holomorphic function: a possible identification of the exponent of Kähler function is as Dirac determinant.

5.3.2 Realization Of Super-Conformal Symmetries

The detailed realization of various super-conformal symmetries has been also a long standing problem.

(a) Super-conformal symmetry requires that Dirac action for string world sheets is accompanied by string world sheet area as part of bosonic action. String world sheets are implied and can be present only in Minkowskian regions if one demands that octonionic and ordinary representations of induced spinor structure are equivalent (this requires vanishing of induced spinor curvature to achieve associativity in turn implying that CP_2 projection is 1-D). Note that 1-dimensionality of CP_2 projection is symplectically invariant property. Kähler action is not invariant under symplectic transformations. This is necessary for having non-trivial Kähler metric. Whether WCW really possesses super-symplectic isometries remains an open problem.

(b) Super-conformal symmetry also demands that Kähler action is accompanied by what I call Kähler-Dirac action with gamma matrices defined by the contractions of the canonical momentum currents with imbedding space-gamma matrices. Both the well-definedness of em charge and equivalence of octonionic spinor dynamics with ordinary one require the restriction of spinor modes to string world sheets with light-like boundaries at wormhole throats. K-D action with the localization of induced spinors at string world sheets is certainly the minimal option to consider.

(c) Strong form of holography implied by strong form of general coordinate invariance strongly suggests that super-conformal symmetry is broken gauge invariance in the sense that the clasical super-conformal charges for a sub-algebra of the symplectic algebra with conformal weights vanishing modulo some integer n vanish. The proposal is that n corresponds to the effective Planck constant as $h_{eff}/h = n$. The standard conformal symmetries for spinors modes at string world sheets is always unbroken gauge symmetry.

5.3.3 Interior Dynamics For Fermions, The Role Of Vacuum Extremals, And Dark Matter

The key role of CP_2-type and M^4-type vacuum extremals has been rather obvious from the beginning but the detailed understanding has been lacking. Both kinds of extremals are invariant under symplectic transformations of $\delta M^4 \times CP_2$, which inspires the idea that they give rise to isometries of WCW . The deformations CP_2-type extremals correspond to lines of generalized Feynman diagrams. M^4 type vacuum extremals in turn are excellent candidates for the building bricks of many-sheeted space-time giving rise to GRT space-time as approximation. For M^4 type vacuum extremals CP_2 projection is (at most 2-D) Lagrangian manifold so that the induced Kähler form vanishes and the action is fourth-order in small deformations. This implies the breakdown of the path integral approach and of canonical quantization, which led to the notion of WCW .

If the action in Minkowskian regions contains also string area, the situation changes dramatically since strings dominate the dynamics in excellent approximation and string theory should give an excellent description of the situation: this of course conforms with the dominance of gravitation.

String tension would be proportional to $1/\hbar G$ and this raises a grave classical counter argument. In string model massless particles are regarded as strings, which have contracted to a point in excellent approximation and cannot have length longer than Planck length. How this can be consistent with the formation of gravitationally bound states is however not understood since the required non-perturbative formulation of string model required by the large valued of the coupling parameter GMm is not known.

In TGD framework strings would connect even objects with macroscopic distance and would obviously serve as correlates for the formation of bound states in quantum level description. The classical energy of string connecting say the two wormhole contacts defining elementary particle is gigantic for the ordinary value of $\hbar$ so that something goes wrong.

I have however proposed [K66, K53, K109] that gravitons - at least those mediating interaction between dark matter have large value of Planck constant. I talk about gravitational Planck constant and one has $\hbar_{eff} = \hbar_{gr} = GMm/v_0$, where $v_0/c < 1$ (v_0 has dimensions of velocity). This makes possible perturbative approach to quantum gravity in the case of bound states having mass larger than Planck mass so that the parameter GMm analogous to coupling constant is very large. The velocity parameter v_0/c becomes the dimensionless coupling parameter. This reduces the string tension so that for string world sheets connecting macroscopic objects one would have $T \propto v_0/G^2Mm$. For $v_0 = GMm/\hbar$, which remains below unity for Mm/m_{Pl}^2 one would have $h_{gr}/h = 1$. Hence action remains small and its imaginary exponent does not fluctuate wildly to make the bound state forming part of gravitational interaction short ranged. This is expected to hold true for ordinary matter in elementary particle scales. The objects with size scale of large neutron (100 μm in the density of water) - probably not an accident - would have mass above Planck mass so that dark gravitons and also life would emerge as massive enough gravitational bound states are formed. $h_{gr} = h_{eff}$ hypothesis is indeed central in TGD based view about living matter.

If one assumes that for non-standard values of Planck constant only n-multiples of super-conformal algebra in interior annihilate the physical states, interior conformal gauge degrees of freedom become partly dynamical. The identification of dark matter as macroscopic quantum phases labeled by $h_{eff}/h = n$ conforms with this.

The emergence of dark matter corresponds to the emergence of interior dynamics via breaking of super-conformal symmetry. The induced spinor fields in the interior of flux tubes obeying Kähler Dirac action should be highly relevant for the understanding of dark matter. The assumption that dark particles have essentially same masses as ordinary particles suggests that dark fermions correspond to induced spinor fields at both string world sheets and in the space-time interior: the spinor fields in the interior would be responsible for the long range correlations characterizing $h_{eff}/h = n$. Magnetic flux tubes carrying dark matter are key entities in TGD inspired quantum biology. Massless extremals represent second class of M^4 type non-vacuum extremals.

This view forces once again to ask whether space-time SUSY is present in TGD and how it is realized. With a motivation coming from the observation that the mass scales of particles and sparticles most naturally have the same p-adic mass scale as particles in TGD Universe I have proposed that sparticles might be dark in TGD sense. The above argument leads to ask whether the dark variants of particles correspond to states in which one has ordinary fermion at string world sheet and 4-D fermion in the space-time interior so that dark matter in TGD sense would almost by definition correspond to sparticles!

5.3.4 Classical Number Fields And Associativity And Commutativity As Fundamental Law Of Physics

The dimensions of classical number fields appear as dimensions of basic objects in quantum TGD. Imbedding space has dimension 8, space-time has dimension 4, light-like 3-surfaces are orbits of 2-D partonic surfaces. If conformal QFT applies to 2-surfaces (this is questionable), one-dimensional structures would be the basic objects. The lowest level would correspond to discrete sets of points identifiable as intersections of real and p-adic space-time sheets. This suggests that besides p-adic number fields also classical number fields (reals, complex numbers, quaternions, octonions [A71]) are involved [K74] and the notion of geometry generalizes considerably. In the recent view about quantum TGD the dimensional hierarchy defined by classical number field indeed plays a key role. $H = M^4 \times CP_2$ has a number theoretic interpretation and standard model symmetries can be understood number theoretically as symmetries of hyper-quaternionic planes of hyper-octonionic space.

The associativity condition $A(BC) = (AB)C$ suggests itself as a fundamental physical law of both classical and quantum physics. Commutativity can be considered as an additional condition. In conformal field theories associativity condition indeed fixes the n-point functions of the theory. At the level of classical TGD space-time surfaces could be identified as maximal associative (hyper-quaternionic) sub-manifolds of the imbedding space whose points contain a preferred hyper-complex plane M^2 in their tangent space and the hierarchy finite fields-rationals-reals-complex numbers-quaternions-octonions could have direct quantum physical counterpart [K74]. This leads to the notion of number theoretic compactification analogous to the dualities of M-theory: one can interpret space-time surfaces either as hyper-quaternionic 4-surfaces of M^8 or as 4-surfaces in $M^4 \times CP_2$. As a matter fact, commutativity in number theoretic sense is a further natural condition and leads to the notion of number theoretic braid naturally as also to direct connection with super string models.

At the level of Kähler-Dirac action the identification of space-time surface as a hyper-quaternionic sub-manifold of H means that the modified gamma matrices of the space-time surface defined in terms of canonical momentum currents of Kähler action using octonionic representation for the gamma matrices of H span a hyper-quaternionic sub-space of hyper-octonions at each point of space-time surface (hyper-octonions are the subspace of complexified octonions for which imaginary units are octonionic imaginary units multiplied by commutating imaginary unit). Hyper-octonionic representation leads to a proposal for how to extend twistor program to TGD framework [K88, K76].

How to achieve associativity in the fermionic sector?

In the fermionic sector an additional complication emerges. The associativity of the tangent- or normal space of the space-time surface need not be enough to guarantee the associativity at the level of Kähler-Dirac or Dirac equation. The reason is the presence of spinor connection. A possible cure could be the vanishing of the components of spinor connection for two conjugates of quaternionic coordinates combined with holomorphy of the modes.

(a) The induced spinor connection involves sigma matrices in CP_2 degrees of freedom, which for the octonionic representation of gamma matrices are proportional to octonion units in Minkowski degrees of freedom. This corresponds to a reduction of tangent space group $SO(1,7)$ to G_2. Therefore octonionic Dirac equation identifying Dirac spinors as complexified octonions can lead to non-associativity even when space-time surface is associative or co-associative.

(b) The simplest manner to overcome these problems is to assume that spinors are localized at 2-D string world sheets with 1-D CP_2 projection and thus possible only in Minkowskian regions. Induced gauge fields would vanish. String world sheets would be minimal surfaces in $M^4 \times D^1 \subset M^4 \times CP_2$ and the theory would simplify enormously. String area would give rise to an additional term in the action assigned to the Minkowskian space-time regions and for vacuum extremals one would have only strings in the first approximation, which conforms with the success of string models and with the intuitive view that vacuum extremals of Kähler action are basic building bricks of many-sheeted space-time. Note that string world sheets would be also symplectic covariants.

Without further conditions gauge potentials would be non-vanishing but one can hope that one can gauge transform them away in associative manner. If not, one can also consider the possibility that CP_2 projection is geodesic circle S^1: symplectic invariance is considerably reduces for this option since symplectic transformations must reduce to rotations in S^1.

(c) The fist heavy objection is that action would contain Newton's constant G as a fundamental dynamical parameter: this is a standard recipe for building a non-renormalizable theory. The very idea of TGD indeed is that there is only single dimensionless parameter analogous to critical temperature. One can of coure argue that the dimensionless parameter is $\hbar G/R^2$, R CP_2 "radius".

Second heavy objection is that the Euclidian variant of string action exponentially damps out all string world sheets with area larger than $\hbar G$. Note also that the classical energy of Minkowskian string would be gigantic unless the length of string is of order Planck length. For Minkowskian signature the exponent is oscillatory and one can argue that wild oscillations have the same effect.

The hierarchy of Planck constants would allow the replacement $\hbar \rightarrow \hbar_{eff}$ but this is not enough. The area of typical string world sheet would scale as h_{eff} and the size of CD and gravitational Compton lengths of gravitationally bound objects would scale as $\sqrt{h_{eff}}$ rather than $\hbar_{eff} = GMm/v_0$, which one wants. The only way out of problem is to assume $T \propto (\hbar/h_{eff})^2 \times (1/h_{bar}G)$. This is however un-natural for genuine area action. Hence it seems that the visit of the basic assumption of superstring theory to TGD remains very short.

Is super-symmetrized Kähler-Dirac action enough?

Could one do without string area in the action and use only K-D action, which is in any case forced by the super-conformal symmetry? This option I have indeed considered hitherto. K-D Dirac equation indeed tends to reduce to a lower-dimensional one: for massless extremals the K-D operator is effectively 1-dimensional. For cosmic strings this reduction does not however take place. In any case, this leads to ask whether in some cases the solutions of Kähler-Dirac equation are localized at lower-dimensional surfaces of space-time surface.

(a) The proposal has indeed been that string world sheets carry vanishing W and possibly even Z fields: in this manner the electromagnetic charge of spinor mode could be well-defined. The vanishing conditions force in the generic case 2-dimensionality.

Besides this the canonical momentum currents for Kähler action defining 4 imbedding space vector fields must define an integrable distribution of two planes to give string world sheet. The four canonical momentum currents $\Pi_k\alpha = \partial L_K/\partial_{\partial_\alpha h^k}$ identified as imbedding 1-forms can have only two linearly independent components parallel to the string world sheet. Also the Frobenius conditions stating that the two 1-forms are proportional to gradients of two imbedding space coordinates Φ_i defining also coordinates at string world sheet, must be satisfied. These conditions are rather strong and are expected to select some discrete set of string world sheets.

(b) To construct preferred extremal one should fix the partonic 2-surfaces, their light-like orbits defining boundaries of Euclidian and Minkowskian space-time regions, and string world sheets. At string world sheets the boundary condition would be that the normal components of canonical momentum currents for Kähler action vanish. This picture brings in mind strong form of holography and this suggests that might make sense and also solution of Einstein equations with point like sources.

(c) The localization of spinor modes at 2-D surfaces would would follow from the well-definedness of em charge and one could have situation is which the localization does not occur. For instance, covariantly constant right-handed neutrinos spinor modes at cosmic strings are completely de-localized and one can wonder whether one could give up the localization inside wormhole contacts.

(d) String tension is dynamical and physical intuition suggests that induced metric at string world sheet is replaced by the anti-commutator of the K-D gamma matrices and by conformal invariance only the conformal equivalence class of this metric would matter and it could be even equivalent with the induced metric. A possible interpretation is that the energy density of Kähler action has a singularity localized at the string world sheet.

Another interpretation that I proposed for years ago but gave up is that in spirit with the TGD analog of AdS/CFT duality the Noether charges for Kähler action can be reduced to integrals over string world sheet having interpretation as area in effective metric. In the case of magnetic flux tubes carrying monopole fluxes and containing a string connecting partonic 2-surfaces at its ends this interpretation would be very natural, and string tension would characterize the density of Kähler magnetic energy. String model with dynamical string tension would certainly be a good approximation and string tension would depend on scale of CD.

(e) There is also an objection. For M^4 type vacuum extremals one would not obtain any non-vacuum string world sheets carrying fermions but the successes of string model strongly suggest that string world sheets are there. String world sheets would represent a deformation of the vacuum extremal and far from string world sheets one would have vacuum extremal in an excellent approximation. Situation would be analogous to that in general relativity with point particles.

(f) The hierarchy of conformal symmetry breakings for K-D action should make string tension proportional to $1/h_{eff}^2$ with $h_{eff} = h_{gr}$ giving correct gravitational Compton length $\Lambda_{gr} = GM/v_0$ defining the minimal size of CD associated with the system. Why the effective string tension of string world sheet should behave like $(\hbar/\hbar_{eff})^2$?

The first point to notice is that the effective metric $G^{\alpha\beta}$ defined as $h^{kl}\Pi_k^\alpha \Pi_l^\beta$, where the canonical momentum current $\Pi_k\alpha = \partial L_K/\partial_{\partial_\alpha h^k}$ has dimension $1/L^2$ as required. Kähler action density must be dimensionless and since the induced Kähler form is dimensionless the canonical momentum currents are proportional to $1/\alpha_K$.

Should one assume that α_K is fundamental coupling strength fixed by quantum criticality to $\alpha_K = 1/137$? Or should one regard g_K^2 as fundamental parameter so that one would have $1/\alpha_K = \hbar_{eff}/4\pi g_K^2$ having spectrum coming as integer multiples (recall the analogy with inverse of critical temperature)?

The latter option is the in spirit with the original idea stating that the increase of h_{eff} reduces the values of the gauge coupling strengths proportional to α_K so that perturbation series converges (Universe is theoretician friendly). The non-perturbative states would be critical states. The non-determinism of Kähler action implying that the 3-surfaces at the boundaries of CD can be connected by large number of space-time sheets forming n conformal equivalence classes. The latter option would give $G^{\alpha\beta} \propto h_{eff}^2$ and $det(G) \propto 1/h_{eff}^2$ as required.

(g) It must be emphasized that the string tension has interpretation in terms of gravitational coupling on only at the GRT limit of TGD involving the replacement of many-sheeted space-time with single sheeted one. It can have also interpretation as hadronic string tension or effective string tension associated with magnetic flux tubes and telling the density of Kähler magnetic energy per unit length.

Superstring models would describe only the perturbative Planck scale dynamics for emission and absorption of $h_{eff}/h = 1$ on mass shell gravitons whereas the quantum description of bound states would require $h_{eff}/n > 1$ when the masses. Also the effective gravitational constant associated with the strings would differ from G.

The natural condition is that the size scale of string world sheet associated with the flux tube mediating gravitational binding is $G(M+m)/v_0$, By expressing string tension in the form $1/T = n^2\hbar G_1$, $n = h_{eff}/h$, this condition gives $\hbar G_1 = \hbar^2/M_{red}^2$, $M_{red} = Mm/(M+m)$. The effective Planck length defined by the effective Newton's constant G_1 analogous to that appearing in string tension is just the Compton length associated with the reduced mass of the system and string tension equals to $T = [v_0/G(M+m)]^2$ apart from a numerical constant ($2G(M+m)$ is Schwartschild radius for the entire system). Hence the macroscopic stringy description of gravitation in terms of string differs dramatically from the perturbative one. Note that one can also understand why in the Bohr orbit model of Nottale [E1] for the planetary system and in its TGD version [K66] v_0 must be by a factor 1/5 smaller for outer planets rather than inner planets.

Are 4-D spinor modes consistent with associativity?

The condition that octonionic spinors are equivalent with ordinary spinors looks rather natural but in the case of Kähler-Dirac action the non-associativity could leak in. One could of course give up the condition that octonionic and ordinary K-D equation are equivalent in 4-D case. If so, one could see K-D action as related to non-commutative and maybe even non-associative fermion dynamics. Suppose that one does not.

(a) K-D action vanishes by K-D equation. Could this save from non-associativity? If the spinors are localized to string world sheets, one obtains just the standard stringy construction of conformal modes of spinor field. The induce spinor connection would have only the holomorphic component A_z. Spinor mode would depend only on z but K-D gamma matrix Γ^z would annihilate the spinor mode so that K-D equation would be satisfied. There are good hopes that the octonionic variant of K-D equation is equivalent with that based on ordinary gamma matrices since quaternionic coordinated reduces to complex coordinate, octonionic quaternionic gamma matrices reduce to complex gamma matrices, sigma matrices are effectively absent by holomorphy.

(b) One can consider also 4-D situation (maybe inside wormhole contacts). Could some form of quaternion holomorphy [A94] [K76] allow to realize the K-D equation just as in the case of super string models by replacing complex coordinate and its conjugate with quaternion and its 3 conjugates. Only two quaternion conjugates would appear in the spinor mode and the corresponding quaternionic gamma matrices would annihilate the spinor mode. It is essential that in a suitable gauge the spinor connection has non-vanishing components only for two quaternion conjugate coordinates. As a special case one would have a situation in which only one quaternion coordinate appears in the solution. Depending on the character of quaternionion holomorphy the modes would be labelled by one or two integers identifiable as conformal weights.

Even if these octonionic 4-D modes exists (as one expects in the case of cosmic strings), it is far from clear whether the description in terms of them is equivalent with the description using K-D equation based ordinary gamma matrices. The algebraic structure however raises hopes about this. The quaternion coordinate can be represented as sum of two complex coordinates as $q = z_1 + Jz_2$ and the dependence on two quaternion conjugates corresponds to the dependence on two complex coordinates z_1, z_2. The condition that two quaternion complexified gammas annihilate the spinors is equivalent with the corresponding condition for Dirac equation formulated using 2 complex coordinates. This for wormhole contacts. The possible generalization of this condition to Minkowskian regions would be in terms Hamilton-Jacobi structure.

Note that for cosmic strings of form $X^2 \times Y^2 \subset M^4 \times CP_2$ the associativity condition for S^2 sigma matrix and without assuming localization demands that the commutator of Y^2 imaginary units is proportional to the imaginary unit assignable to X^2 which however depends on point of X^2. This condition seems to imply correlation between Y^2 and S^2 which does not look physical.

To summarize, the minimal and mathematically most optimistic conclusion is that Kähler-Dirac action is indeed enough to understand gravitational binding without giving up the associativity of the fermionic dynamics. Conformal spinor dynamics would be associative if the spinor modes are localized at string world sheets with vanishing W (and maybe also Z) fields guaranteeing well-definedness of em charge and carrying canonical momentum currents parallel to them. It is not quite clear whether string world sheets are present also inside wormhole contacts: for CP_2 type vacuum extremals the Dirac equation would give only right-handed neutrino as a solution (could they give rise to $N = 2$ SUSY?).

The construction of preferred extremals would realize strong form of holography. By conformal symmetry the effective metric at string world sheet could be conformally equivalent with the induced metric at string world sheets.

Dynamical string tension would be proportional to $\hbar/h_{eff}^2$ due to the proportionality $\alpha_K \propto 1/h_{eff}$ and predict correctly the size scales of gravitationally bound states for $\hbar_{gr} = \hbar_{eff} = GMm/v_0$. Gravitational constant would be a prediction of the theory and be expressible in terms of α_K and R^2 and $\hbar_{eff}$ ($G \propto R^2/g_K^2$).

In fact, all bound states - elementary particles as pairs of wormhole contacts, hadronic strings, nuclei [L3], molecules, etc. - are described in the same manner quantum mechanically. This is of course nothing new since magnetic flux tubes associated with the strings provide a universal model for interactions in TGD Universe. This also conforms with the TGD counterpart of AdS/CFT duality.

The basic building bricks are symplectic algebra of δCD (this includes CP_2 besides light-cone boundary) and Kac-Moody algebra assignable to the isometries of δCD [K15]. It seems however that the longheld view about the role of Kac-Moody algebra must be modified. Also the earlier realization of super-Hamiltonians and Hamiltonians seems too naive.

(a) I have been accustomed to think that Kac-Moody algebra could be regarded as a sub-algebra of symplectic algebra. p-Adic mass calculations however requires five tensor factors for the coset representation of Super Virasoro algebra naturally assigned to the coset structure G/H of a sector of WCW with fixed zero modes. Therefore Kac-Moody algebra cannot be regarded as a sub-algebra of symplectic algebra giving only single tensor factor and thus inconsistent with interpretation of p-adic mass calculations.

(b) The localization of Kac-Moody algebra generators with respect to the internal coordinates of light-like 3-surface taking the role of complex coordinate z in conformal field theory is also questionable: the most economical option relies on localization with respect to light-like radial coordinate of light-cone boundary as in the case of symplectic algebra. Kac-Moody algebra cannot be however sub-algebra of the symplectic algebra assigned with covariantly constant right-handed neutrino in the earlier approach.

(c) Right-handed covariantly constant neutrino as a generator of super symmetries plays a key role in the earlier construction of symplectic super-Hamiltonians. What raises doubts is that other spinor modes - both those of right-handed neutrino and electro-weakly charged spinor modes - are absent. All spinor modes should be present and thus provide direct mapping from WCW geometry to WCW spinor fields in accordance with super-symmetry and the general idea that WCW geometry is coded by WCW spinor fields.

    Hence it seems that Kac-Moody algebra must be assigned with the modes of the induced spinor field which carry electroweak quantum numbers. If would be natural that the modes of right-handed neutrino having no weak and color interactions would generate the huge symplectic algebra of symmetries and that the modes of fermions with electroweak charges generate much smaller Kac-Moody algebra.

(d) The dynamics of Kähler action and Kähler-Dirac action action are invisible in the earlier construction. This suggests that the definition of WCW Hamiltonians is too simplistic.

The proposal is that the conserved super charges derivable as Noether charges and identifiable as super-Hamiltonians define WCW metric and Hamiltonians as their anti-commutators. Spinor modes would become labels of Hamiltonians and WCW geometry relates directly to the dynamics of elementary particles.

(e) Note that light-cone boundary $\delta M^4_+ = S^2 \times R_+$ allows infinite-dimensional group of isometries consisting of conformal transformation of the sphere S^2 with conformal scaling compensated by an S^2 local scaling or the light-like radial coordinate of R_+. These isometries contain as a subgroup symplectic isometries and could act as gauge symmetries of the theory.

5.4 About some unclear issues of TGD

TGD has been in the middle of palace revolution during last two years and it is almost impossible to keep the chapters of the books updated. Adelic vision and twistor lift of TGD are the newest developments and there are still many details to be understood and errors to be corrected. The description of fermions in TGD framework has contained some unclear issues. Hence the motivation for the following brief comments.

There questions about the adelic vision about symmetries. Do the cognitive representations implying number theoretic disretization of the space-time surface lead to the breaking of the basic symmetries and are preferred imbedding space coordinates actually necessary?

In the fermionic sector there are many questions deserving clarification. How quantum classical correspondence (QCC) is realized for fermions? How is SH realized for fermions and how does it lead to the reduction of dimension $D = 4$ to $D = 2$ (apart from number theoretical discretization)? Can scattering amplitudes be really formulated by using only the data at the boundaries of string sheets and what does this mean from the point of view of the modified Dirac equation? Are the spinors at light-like boundaries limiting values or sources? A long-standing issue concerns the fermionic anti-commutation relations: what motivated this article was the solution of this problem. There is also the general problem about whether statistical entanglement is “real”.

5.4.1 Adelic vision and symmetries

In the adelic TGD SH is weakened: also the points of the space-time surface having imbedding space coordinates in an extension of rationals (cognitive representation) are needed so that data are not precisely 2-D. I have believed hitherto that one must use preferred coordinates for the imbedding space H - a subset of these coordinates would define space-time coordinates. These coordinates are determined apart from isometries. Does the number theoretic discretization imply loss of general coordinate invariance and also other symmetries?

The reduction of symmetry groups to their subgroups (not only algebraic since powers of e define finite-dimensional extension of p-adic numbers since e^p is ordinary p-adic number) is genuine loss of symmetry and reflects finite cognitive resolution. The physics itself has the symmetries of real physics.

The assumption about preferred imbedding space coordinates is actually not necessary. Different choices of H-coordinates means only different and non-equivalent cognitive representations. Spherical and linear coordinates in finite accuracy do not provide equivalent representations.

5.4.2 Quantum-classical correspondence for fermions

Quantum-classical correspondence (QCC) for fermions is rather well-understood but deserves to be mentioned also here.

QCC for fermions means that the space-time surface as preferred extremal should depend on fermionic quantum numbers. This is indeed the case if one requires QCC in the sense that the fermionic representations of Noether charges in the Cartan algebras of symmetry algebras are equal to those to the classical Noether charges for preferred extremals.

Second aspect of QCC becomes visible in the representation of fermionic states as point like particles moving along the light-like curves at the light-like orbits of the partonic 2-surfaces (curve at the orbit can be locally only light-like or space-like). The number of fermions and antifermions dictates the number of string world sheets carrying the data needed to fix the preferred extremal by SH. The complexity of the space-time surface increases as the number of fermions increases.

5.4.3 Strong form of holography for fermions

It seems that scattering amplitudes can be formulated by assigning fermions with the boundaries of strings defining the lines of twistor diagrams [L22, L38]. This information theoretic dimensional reduction from $D = 4$ to $D = 2$ for the scattering amplitudes can be partially understood in terms of strong form of holography (SH): one can construct the theory by using the data at string worlds sheets and/or partonic 2-surfaces at the ends of the space-time surface at the opposite boundaries of causal diamond (CD).

4-D modified Dirac action would appear at fundamental level as supersymmetry demands but would be reduced for preferred extremals to its 2-D stringy variant serving as effective action. Also the value of the 4-D action determining the space-time dynamics would reduce to effective stringy action containing area term, 2-D Kähler action, and topological Kähler magnetic flux term. This reduction would be due to the huge gauge symmetries of preferred extremals. Sub-algebra of super-symplectic algebra with conformal weigths coming as n-multiples of those for the entire algebra and the commutators of this algebra with the entire algebra would annihilate the physical states, and thecorresponding classical Noether charges would vanish.

One still has the question why not the data at the entire string world sheets is not needed to construct scattering amplitudes. Scattering amplitudes of course need not code for the entire physics. QCC is indeed motivated by the fact that quantum experiments are always interpreted in terms of classical physics, which in TGD framework reduces to that for space-time surface.

5.4.4 The relationship between spinors in space-time interior and at boundaries between Euclidian and Minkoskian regions

Space-time surface decomposes to interiors of Minkowskian and Euclidian regions. At light-like 3-surfaces at which the four-metric changes, the 4-metric is degenerate. These metrically singular 3-surfaces - partonic orbits- carry the boundaries of string world sheets identified as carriers of fermionic quantum numbers. The boundaries define fermion lines in the twistor lift of TGD [L22, L38]. The relationship between fermions at the partonic orbits and interior of the space-time surface has however remained somewhat enigmatic.

So: What is the precise relationship between induced spinors Ψ_B at light-like partonic 3-surfaces and Ψ_I in the interior of Minkowskian and Euclidian regions? Same question can be made for the spinors Ψ_B at the boundaries of string world sheets and Ψ_I in interior of the string world sheets. There are two options to consider:

- Option I: Ψ_B is the limiting value of Ψ_I .
- Option II: Ψ_B serves as a source of Ψ_I .

For the Option I it is difficult to understand the preferred role of Ψ_B.

I have considered Option II already years ago but have not been able to decide.

(a) That scattering amplitudes could be formulated only in terms of sources only, would fit nicely with SH, twistorial amplitude construction, and also with the idea that scattering amplitudes in gauge theories can be formulated in terms of sources of boson fields assignable to vertices and propagators. Now the sources would become fermionic.

(b) One can take gauge theory as a guideline. One adds to free Dirac equation source term $_k\Psi$. Therefore the natural boundary term in the action would be of the form (forgetting overall scale factor)

$$S_B = \overline{\Psi}_I \Gamma^\alpha (C-S) A_\alpha \Psi_B + c.c \ .$$

Here the modified gamma matrix is $\Gamma^\alpha(C-S)$ (contravariant form is natural for light-like 3-surfaces) is most naturally defined by the boundary part of the action - naturally Chern-Simons term for Kähler action. A denotes the Kähler gauge potential.

(c) The variation with respect to Ψ_B gives

$$G^\alpha(C-S)A_\alpha \Psi_I = 0$$

at the boundary so that the C-S term and interaction term vanish. This does not however imply vanishing of the source term! This condition can be seen as a boundary condition.

The same argument applies also to string world sheets.

5.4.5 About second quantization of the induced spinor fields

The anti-commutation relations for the induced spinors have been a long-standing issue and during years I have considered several options. The solution of the problem looks however stupifuingly simple. The conserved fermion currents are accompanied by super-currents obtained by replacing Ψ with a mode of the induced spinor field to get $\overline{u}_n\Gamma^\alpha\Psi$ or $\overline{\Psi}\Gamma^\alpha u_n$ with the conjugate of the mode. One obtains infinite number of conserved super currents. One can also replace both Ψ and $\overline{\Psi}$ in this manner to get purely bosonic conserved currents $\overline{u}_m\Gamma^\alpha u_n$ to which one can assign a conserved bosonic charges Q_{mn}.

I noticed this years ago but did not realize that these bosonic charges define naturally anti-commutators of fermionic creation and annihilation operators! The ordinary anti-commutators of quantum field theory follow as a special case! By a suitable unitary transformation of the spinor basis one can diagonalize the hermitian matrix defined by Q_{mn} and by performing suitable scalings one can transform anti-commutation relations to the standard form. An interesting question is whether the diagonalization is needed, and whether the deviation of the diagonal elements from unity could have some meaning and possibly relate to the hierarchy $h_{eff} = n \times h$ of Planck constants - probably not.

5.4.6 Is statistical entanglement "real" entanglement?

The question about the "reality" of statistical entanglement has bothered me for years. This entanglement is maximal and it cannot be reduced by measurement so that one can argue that it is not "real". Quite recently I learned that there has been a longstanding debate about the statistical entanglement and that the issue still remains unresolved.

The idea that all electrons of the Universe are maximally entangled looks crazy. TGD provides several variants for solutions of this problem. It could be that only the fermionic oscillator operators at partonic 2-surfaces associated with the space-time surface (or its connected component) inside given CD anti-commute and the fermions are thus indistinguishable. The extremist option is that the fermionic oscillator operators belonging to a network of partonic 2-surfaces connected by string world sheets anti-commute: only the oscillator operators assignable to the same scattering diagram would anti-commute.

What about QCC in the case of entanglement. ER-EPR correspondence introduced by Maldacena and Susskind for 4 years ago proposes that blackholes (maybe even elementary particles) are connected by wormholes. In TGD the analogous statement emerged for more than decade ago - magnetic flux tubes take the role of wormholes in TGD. Magnetic flux tubes were assumed to be accompanied by string world sheets. I did not consider the question whether string world sheets are always accompanied by flux tubes.

What could be the criterion for entanglement to be "real"? "Reality" of entanglement demands some space-time correlate. Could the presence of the flux tubes make the entanglement "real"? If statistical entanglement is accompanied by string connections without magnetic flux tubes, it would not be "real": only the presence of flux tubes would make it "real". Or is the presence of strings enough to make the statistical entanglement "real". In both cases the fermions associated with disjoint space-time surfaces or with disjoint CDs would not be indistinguishable. This looks rather sensible.

The space-time correlate for the reduction of entanglement would be the splitting of a flux tube and fermionic strings inside it. The fermionic strings associated with flux tubes carrying monopole flux are closed and the return flux comes back along parallel space-time sheet. Also fermionic string has similar structure. Reconnection of this flux tube with shape of very long flattened square splitting it to two pieces would be the correlate for the state function reduction reducing the entanglement with other fermions and would indeed decouple the fermion from the network.

5.5 About The Notion Of Four-Momentum In TGD Framework

The starting point of TGD was the energy problem of General Relativity [K79]. The solution of the problem was proposed in terms of sub-manifold gravity and based on the lifting of the isometries of space-time surface to those of $M^4 \times CP_2$ in which space-times are realized as 4-surfaces so that Poincare transformations act on space-time surface as an 4-D analog of rigid body rather than moving points at space-time surface. It however turned out that the situation is not at all so simple.

There are several conceptual hurdles and I have considered several solutions for them. The basic source of problems has been Equivalence Principle (EP): what does EP mean in TGD framework [K79, K103] ? A related problem has been the interpretation of gravitational and inertial masses, or more generally the corresponding 4-momenta. In General Relativity based cosmology gravitational mass is not conserved and this seems to be in conflict with the conservation of Noether charges. The resolution is in terms of zero energy ontology (ZEO), which however forces to modify slightly the original view about the action of Poincare transformations.

A further problem has been quantum classical correspondence (QCC): are quantal four-momenta associated with super conformal representations and classical four-momenta associated as Noether charges with Kähler action for preferred extremals identical? Could inertial-gravitational duality - that is EP - be actually equivalent with QCC? Or are EP and QCC independent dualities. A powerful experimental input comes p-adic mass calculations [K100] giving excellent predictions provided the number of tensor factors of super-Virasoro representations is five, and this input together with Occam's razor strongly favors QCC=EP identification.

There is also the question about classical realization of EP and more generally, TGD-GRT correspondence.

Twistor Grassmannian approach has meant a technical revolution in quantum field theory (for attempts to understand and generalize the approach in TGD framework see [K76]. This approach seems to be extremely well suited to TGD and I have considered a generalization of this approach from $\mathcal{N} = 4$ SUSY to TGD framework by replacing point like particles with string world sheets in TGD sense and super-conformal algebra with its TGD version: the fundamental objects are now massless fermions which can be regarded as on mass shell particles also in internal lines (but with unphysical helicity). The approach solves old problems related to the realization of stringy amplitudes in TGD framework, and avoids some problems of twistorial QFT (IR divergences and the problems due to non-planar diagrams). The Yangian [A27] [B39, B30, B31] variant of 4-D conformal symmetry is crucial for the approach in $\mathcal{N} = 4$ SUSY, and implies the recently introduced notion of amplituhedron [B20]. A Yangian generalization of various super-conformal algebras seems more or less a "must" in TGD framework. As a consequence, four-momentum is expected to have characteristic multilocal contributions identifiable as multipart on contributions now and possibly relevant for the understanding of bound states such as hadrons.

5.5.1 Scale Dependent Notion Of Four-Momentum In Zero Energy Ontology

Quite generally, General Relativity does not allow to identify four-momentum as Noether charges but in GRT based cosmology one can speak of non-conserved mass [K67], which seems to be in conflict with the conservation of four-momentum in TGD framework. The solution of the problem comes in terms of zero energy ontology (ZEO) [K4, K98], which transforms four-momentum to a scale dependent notion: to each causal diamond (CD) one can assign four-momentum assigned with say positive energy part of the quantum state defined as a quantum superposition of 4-surfaces inside CD.

ZEO is necessary also for the fusion of real and various p-adic physics to single coherent whole. ZEO also allows maximal "free will" in quantum jump since every zero energy state can be created from vacuum and at the same time allows consistency with the conservation laws. ZEO has rather dramatic implications: in particular the arrow of thermodynamical time is predicted to vary so that second law must be generalized. This has especially important implications in living matter, where this kind of variation is observed.

More precisely, this superposition corresponds to a spinor field in the "world of classical worlds" (WCW) [K98]: its components - WCW spinors - correspond to elements of fermionic Fock basis for a given 4-surface - or by holography implied by general coordinate invariance (GCI) - for 3-surface having components at both ends of CD. Strong form of GGI implies strong form of holography (SH) so that partonic 2-surfaces at the ends of space-time surface plus their 4-D tangent space data are enough to fix the quantum state. The classical dynamics in the interior is necessary for the translation of the outcomes of quantum measurements to the language of physics based on classical fields, which in turn is reduced to sub-manifold geometry in the extension of the geometrization program of physics provided by TGD.

Holography is very much reminiscent of QCC suggesting trinity: GCI-holography-QCC. Strong form of holography has strongly stringy flavor: string world sheets connecting the wormhole throats appearing as basic building bricks of particles emerge from the dynamics of induced spinor fields if one requires that the fermionic mode carries well-defined electromagnetic charge [K88].

5.5.2 Are The Classical And Quantal Four-Momenta Identical?

One key question concerns the classical and quantum counterparts of four-momentum. In TGD framework classical theory is an exact part of quantum theory. Classical four-momentum corresponds to Noether charge for preferred extremals of Kähler action. Quantal four-momentum in turn is assigned with the quantum superposition of space-time sheets assigned with CD - actually WCW spinor field analogous to ordinary spinor field carrying fermionic degrees of freedom as analogs of spin. Quantal four-momentum emerges just as it does in super string models - that is as a parameter associated with the representations of super-conformal algebras. The precise action of translations in the representation remains poorly specified. Note that quantal four-momentum does not emerge as Noether charge: at at least it is not at all obvious that this could be the case.

Are these classical and quantal four-momenta identical as QCC would suggest? If so, the Noether four-momentum should be same for all space-time surfaces in the superposition. QCC suggests that also the classical correlation functions for various general coordinate invariant local quantities are same as corresponding quantal correlation functions and thus same for all 4-surfaces in quantum superposition - this at least in the measurement resolution used. This would be an extremely powerful constraint on the quantum states and to a high extend could determined the U-, M-, and S-matrices.

QCC seems to be more or less equivalent with SH stating that in some respects the descriptions based on classical physics defined by Kähler action in the interior of space-time surface and the quantal description in terms of quantum states assignable to the intersections of space-like 3-surfaces at the boundaries of CD and light-like 3-surfaces at which the signature of induced metric changes. SH means effective 2-dimensionality since the four-dimensional tangent space data at partonic 2-surfaces matters. SH could be interpreted as Kac-Mody and symplectic symmetries meaning that apart from central extension they act almost like gauge symmetries in the interiors of space-like 3-surfaces at the ends of CD and in the interiors of light-like 3-surfaces representing orbits of partonic 2-surfaces. Gauge conditions are replaced with Super Virasoro conditions. The word “almost” is of course extremely important.

5.5.3 What Equivalence Principle (EP) Means In Quantum TGD?

EP states the equivalence of gravitational and inertial masses in Newtonian theory. A possible generalization would be equivalence of gravitational and inertial four-momenta. In GRT this correspondence cannot be realized in mathematically rigorous manner since these notions are poorly defined and EP reduces to a purely local statement in terms of Einstein's equations.

What about TGD? What could EP mean in TGD framework?

(a) Is EP realized at both quantum and space-time level? This option requires the identification of inertial and gravitational four-momenta at both quantum and classical level. It is now clear that at classical level EP follows from very simple assumption that GRT space-time is obtained by lumping together the space-time sheets of the many-sheeted space-time and by the identification the effective metric as sum of M^4 metric and deviations of the induced metrics of space-time sheets from M^2 metric: the deviations indeed define the gravitational field defined by multiply topologically condensed test particle. Similar description applies to gauge fields. EP as expressed by Einstein's equations would follow from Poincare invariance at microscopic level defined by TGD space-time. The effective fields have as sources the energy momentum tensor and YM currents defined by topological inhomogenities smaller than the resolution scale.

(b) QCC would require the identification of quantal and classical counterparts of both gravitational and inertial four-momenta. This would give three independent equivalences, say $P_{I,class} = P_{I,quant}$, $P_{gr,class} = P_{gr,quant}$, $P_{gr,class} = P_{I,quant}$, which imply the remaining ones.

Consider the condition $P_{gr,class} = P_{I,class}$. At classical level the condition that the standard energy momentum tensor associated with Kähler action has a vanishing divergence is guaranteed if Einstein's equations with cosmological term are satisfied. If preferred extremals satisfy this condition they are constant curvature spaces for non-vanishing cosmological constant. A more general solution ansatz involves several functions analogous to cosmological constant corresponding to the decomposition of energy momentum tensor to terms proportional to Einstein tensor and several lower-dimensional projection operators [K103]. It must be emphasized that field equations are extremely non-linear and one must also consider preferred extremals (which could be identified in terms of space-time regions having so called Hamilton-Jacobi structure): hence these proposals are guesses motivated by what is known about exact solutions of field equations.

Consider next $P_{gr,class} = P_{I,class}$. At quantum level I have proposed coset representations for the pair of super conformal algebras g and $h \subset g$ which correspond to the coset space decomposition of a given sector of WCW with constant values of zero modes. The coset construction would state that the differences of super-Virasoro generators associated with g resp. h annhilate physical states.

The identification of the algebras g and h is not straightforward. The algebra g could be formed by the direct sum of super-symplectic and super Kac-Moody algebras and its sub-algebra h for which the generators vanish at partonic 2-surface considered. This would correspond to the idea about WCW as a coset space G/H of corresponding groups (consider as a model $CP_2 = SU(3)/U(2)$ with $U(2)$ leaving preferred point invariant). The sub-algebra h in question includes or equals to the algebra of Kac-Moody generators vanishing at the partonic 2-surface. A natural choice for the preferred WCW point would be as maximum of Kähler function in Euclidian regions: positive definiteness of Kähler function allows only single maximum for fixed values of zero modes). Coset construction states that differences of super Virasoro generators associated with g and h annihilate physical states. This implies that corresponding four-momenta are identical that is Equivalence Principle.

(c) Does EP at quantum level reduce to one aspect of QCC? This would require that classical Noether four-momentum identified as inertial momentum equals to the quantal four-momentum assignable to the states of super-conformal representations and identifiable as gravitational four-momentum. There would be only one independent condition: $P_{class} \equiv P_{I,class} = P_{gr,quant} \equiv P_{quant}$.
Holography realized as AdS/CFT correspondence states the equivalence of descriptions in terms of gravitation realized in terms of strings in 10-D space-time and gauge fields at the boundary of AdS. What is disturbing is that this picture is not completely equivalent with the proposed one. In this case the super-conformal algebra would be direct sum of super-symplectic and super Kac-Moody parts.

Which of the options looks more plausible? The success of p-adic mass calculations [K100] have motivated the use of them as a guideline in attempts to understand TGD. The basic outcome was that elementary particle spectrum can be understood if Super Virasoro algebra has five tensor factors. Can one decide the fate of the two approaches to EP using this number as an input?

This is not the case. For both options the number of tensor factors is five as required. Four tensor factors come from Super Kac-Moody and correspond to translational Kac-Moody type degrees of freedom in M^4, to color degrees of freedom and to electroweak degrees of freedom ($SU(2) \times U(1)$). One tensor factor comes from the symplectic degrees of freedom in $\Delta CD \times CP_2$ (note that Hamiltonians include also products of δCD and CP_2 Hamiltonians so that one does not have direct sum!).

The reduction of EP to the coset structure of WCW sectors is extremely beautiful property. But also the reduction of EP to QCC looks very nice and deep. It is of course possible that the two realizations of EP are equivalent and the natural conjecture is that this is the case.

For QCC option the GRT inspired interpretation of Equivalence Principle at space-time level remains to be understood. Is it needed at all? The condition that the energy momentum tensor of Kähler action has a vanishing divergence leads in General Relativity to Einstein equations with cosmological term. In TGD framework preferred extremals satisfying the analogs of Einstein's equations with several cosmological constant like parameters can be considered.

Should one give up this idea, which indeed might be wrong? Could the divergence of of energy momentum tensor vanish only asymptotically as was the original proposal? Or should one try to generalize the interpretation? QCC states that quantum physics has classical correlate at space-time level and implies EP. Could also quantum classical correspondence itself have a correlate at space-time level. If so, space-time surface would able to represent abstractions as statements about statements about.... as the many-sheeted structure and the vision about TGD physics as analog of Turing machine able to mimic any other Turing machine suggest.

5.5.4 TGD-GRT Correspondence And Equivalence Principle

One should also understand how General Relativity and EP emerge at classical level. The understanding comes from the realization that GRT is only an effective theory obtained by endowing M^4 with effective metric.

(a) The replacement of superposition of fields with superposition of their effects means replacing superposition of fields with the set-theoretic union of space-time surfaces. Particle experiences sum of the effects caused by the classical fields at the space-time sheets (see **Fig.** `http://tgdtheory.fi/appfigures/fieldsuperpose.jpg` or **Fig. ??** in the appendix of this book).

(b) This is true also for the classical gravitational field defined by the deviation from flat Minkowski metric instandard M^4 coordinates for the space-time sheets. One can define effective metric as sum of M^4 metric and deviations. This effective metric would correspond to that of General Relativity. This resolves long standing issues relating to the interpretation of TGD.

(c) Einstein's equations could hold true for the effective metric. They are motivated by the underlying Poincare invariance which cannot be realized as global conservation laws for the effective metric. The conjecture vanishing of divergence of Khler energy momentum tensor can be seen as the microscopic justification for the claim that Einstein's equations hold true for the effective space-time.

(d) The breaking of Poincare invariance could have interpretation as effective breaking in zero energy ontology (ZEO), in which various conserved charges are length dependent and defined separately for each causal diamond (CD).

One can of course consider the possibility that Einstein's equations generalize for preferred extremals of Kähbler action. This would actually represent at space-time level the notion of QCC rather than realise QCC interpreted as EP. The condition that the energy momentum tensor for Kähler action has vanishing covariant divergence would be satisfied in GRT if Einstein's equations with cosmological term hold true. This is the case also now but one can consider also more general solutions in which one has two cosmological constants which are not genuine constants anymore [K103].

5.5.5 How Translations Are Represented At The Level Of WCW ?

The four-momentum components appearing in the formulas of super conformal generators correspond to infinitesimal translations. In TGD framework one must be able to identify these infinitesimal translations precisely. As a matter of fact, finite measurement resolution implies that it is probably too much to assume infinitesimal translations. Rather, finite exponentials of translation generators are involved and translations are discretized. This does not have practical signficance since for optimal resolution the discretization step is about CP_2 length scale.

Where and how do these translations act at the level of WCW ? ZEO provides a possible answer to this question.

Discrete Lorentz transformations and time translations act in the space of CDs: inertial four-momentum

Quantum state corresponds also to wave function in moduli space of CDs. The moduli space is obtained from given CD by making all boosts for its non-fixed boundary: boosts correspond to a discrete subgroup of Lorentz group and define a lattice-like structure at the hyperboloid for which proper time distance from the second tip of CD is fixed to $T_n = n \times T(CP_2)$. The quantization of cosmic redshift for which there is evidence, could relate to this lattice generalizing ordinary 3-D lattices from Euclidian to hyperbolic space by replacing translations with boosts (velocities).

The additional degree of freedom comes from the fact that the integer $n > 0$ obtains all positive values. One has wave functions in the moduli space defined as a pile of these lattices defined at the hyperboloid with constant value of $T(CP_2)$: one can say that the points of this pile of lattices correspond to Lorentz boosts and scalings of CDs defining sub- WCW : s.

The interpretation in terms of group which is product of the group of shifts $T_n(CP_2) \rightarrow T_{n+m}(CP_2)$ and discrete Lorentz boosts is natural. This group has same Cartesian product structure as Galilean group of Newtonian mechanics. This would give a discrete rest energy and by Lorentz boosts discrete set of four-momenta giving a contribution to the four-momentum appearing in the super-conformal representation.

What is important that each state function reduction would mean localisation of either boundary of CD (that is its tip). This localization is analogous to the localization of particle in position measurement in E^3 but now discrete Lorentz boosts and discrete translations $T_n - - > T_{n+m}$ replace translations. Since the second end of CD is necessary del-ocalized in moduli space, one has kind of flip-flop: localization at second end implies de-localization at the second end. Could the localization of the second end (tip) of CD in moduli space correspond to our experience that momentum and position can be measured simultaneously? This apparent classicality would be an illusion made possible by ZEO.

The flip-flop character of state function reduction process implies also the alternation of the direction of the thermodynamical time: the asymmetry between the two ends of CDs would induce the quantum arrow of time. This picture also allows to understand what the experience growth of geometric time means in terms of CDs.

The action of translations at space-time sheets

The action of imbedding space translations on space-time surfaces possibly becoming trivial at partonic 2-surfaces or reducing to action at δCD induces action on space-time sheet which becomes ordinary translation far enough from end end of space-time surface. The four-momentum in question is very naturally that associated with Kähler action and would therefore correspond to inertial momentum for $P_{I,class} = P_{quant,gr}$ option. Indeed, one cannot assign quantal four-momentum to Kähler action as an operator since canonical quantization badly fails. In finite measurement infinitesimal translations are replaced with their exponentials for $P_{I,class} = P_{quant,gr}$ option.

What looks like a problem is that ordinary translations in the general case lead out from given CD near its boundaries. In the interior one expects that the translation acts like ordinary translation. The Lie-algebra structure of Poincare algebra including sums of translation generators with positive coefficient for time translation is preserved if only time-like superpositions if generators are allowed also the commutators of time-like translation generators with boost generators give time like translations. This defines a Lie-algebraic formulation for the arrow of geometric time. The action of time translation on preferred extremal would be ordinary translation plus continuation of the translated preferred extremal backwards in time to the boundary of CD. The transversal space-like translations could be made Kac-Moody algebra by multiplying them with functions which vanish at δCD.

A possible interpretation would be that $P_{quant,gr}$ corresponds to the momentum assignable to the moduli degrees of freedom and $P_{cl,I}$ to that assignable to the time like translations. $P_{quant,gr} = P_{cl,I}$ would code for QCC. Geometrically quantum classical correspondence would state that time-like translation shift both the interior of space-time surface and second boundary of CD to the geometric future/past while keeping the second boundary of space-time surface and CD fixed.

5.5.6 Yangian And Four-Momentum

Yangian symmetry implies the marvellous results of twistor Grassmannian approach to $\mathcal{N} = 4$ SUSY culminating in the notion of amplituhedron which promises to give a nice projective geometry interpretation for the scattering amplitudes [B20]. Yangian symmetry is a multilocal generalization of ordinary symmetry based on the notion of co-product and implies that Lie algebra generates receive also multilocal contributions. I have discussed these topics from slightly different point of view in [K76], where also references to the work of pioneers can be found.

Yangian symmetry

The notion equivalent to that of Yangian was originally introduced by Faddeev and his group in the study of integrable systems. Yangians are Hopf algebras which can be assigned with Lie algebras as the deformations of their universal enveloping algebras. The elegant but rather cryptic looking definition is in terms of the modification of the relations for generating elements [K76]. Besides ordinary product in the enveloping algebra there is co-product Δ which maps the elements of the enveloping algebra to its tensor product with itself. One can visualize product and co-product is in terms of particle reactions. Particle annihilation is analogous to annihilation of two particle so single one and co-product is analogous to the decay of particle to two. Δ allows to construct higher generators of the algebra.

Lie-algebra can mean here ordinary finite-dimensional simple Lie algebra, Kac-Moody algebra or Virasoro algebra. In the case of SUSY it means conformal algebra of M^4- or rather its super counterpart. Witten, Nappi and Dolan have described the notion of Yangian for super-conformal algebra in very elegant andconcrete manner in the article *Yangian Symmetry in D=4 superconformal Yang-Mills theory* [B30]. Also Yangians for gauge groups are discussed.

In the general case Yangian resembles Kac-Moody algebra with discrete index n replaced with a continuous one. Discrete index poses conditions on the Lie group and its representation (adjoint representation in the case of $\mathcal{N} = 4$ SUSY). One of the conditions conditions is that the tensor product $R \otimes R^*$ for representations involved contains adjoint representation only once. This condition is non-trivial. For $SU(n)$ these conditions are satisfied for any representation. In the case of $SU(2)$ the basic branching rule for the tensor product of representations implies that the condition is satisfied for the product of any representations.

Yangian algebra with a discrete basis is in many respects analogous to Kac-Moody algebra. Now however the generators are labelled by non-negative integers labeling the light-like incoming and outgoing momenta of scattering amplitude whereas in in the case of Kac-Moody algebra also negative values are allowed. Note that only the generators with non-negative conformal weight appear in the construction of states of Kac-Moody and Virasoro representations so that the extension to Yangian makes sense.

The generating elements are labelled by the generators of ordinary conformal transformations acting in M^4 and their duals acting in momentum space. These two sets of elements can be labelled by conformal weights $n = 0$ and $n = 1$ and and their mutual commutation relations are same as for Kac-Moody algebra. The commutators of $n = 1$ generators with themselves are however something different for a non-vanishing deformation parameter h. Serre's relations characterize the difference and involve the deformation parameter h. Under repeated commutations the generating elements generate infinite-dimensional symmetric algebra, the Yangian. For $h = 0$ one obtains just one half of the Virasoro algebra or Kac-Moody algebra. The generators with $n > 0$ are $n+1$-local in the sense that they involve $n+1$-forms of local generators assignable to the ordered set of incoming particles of the scattering amplitude. This non-locality generalizes the notion of local symmetry and is claimed to be powerful enough to fix the scattering amplitudes completely.

How to generalize Yangian symmetry in TGD framework?

As far as concrete calculations are considered, it is not much to say. It is however possible to keep discussion at general level and still say something interesting (as I hope!). The key question is whether it could be possible to generalize the proposed Yangian symmetry and geometric picture behind it to TGD framework.

(a) The first thing to notice is that the Yangian symmetry of $\mathcal{N} = 4$ SUSY in question is quite too limited since it allows only single representation of the gauge group and requires massless particles. One must allow all representations and massive particles so that the representation of symmetry algebra must involve states with different masses, in principle arbitrary spin and arbitrary internal quantum numbers. The candidates are obvious: Kac-Moody algebras [A9] and Virasoro algebras [A21] and their super counterparts. Yangians indeed exist for arbitrary super Lie algebras. In TGD framework conformal algebra of Minkowski space reduces to Poincare algebra and its extension to Kac-Moody allows to have also massive states.

(b) The formal generalization looks surprisingly straightforward at the formal level. In zero energy ontology one replaces point like particles with partonic two-surfaces appearing at the ends of light-like orbits of wormhole throats located to the future and past light-like boundaries of causal diamond ($CD \times CP_2$ or briefly CD). Here CD is defined as the intersection of future and past directed light-cones. The polygon with light-like momenta is naturally replaced with a polygon with more general momenta in zero energy ontology and having partonic surfaces as its vertices. Non-point-likeness forces to replace the finite-dimensional super Lie-algebra with infinite-dimensional Kac-Moody algebras and corresponding super-Virasoro algebras assignable to partonic 2-surfaces.

(c) This description replaces disjoint holomorphic surfaces in twistor space with partonic 2-surfaces at the boundaries of $CD \times CP_2$ so that there seems to be a close analogy with Cachazo-Svrcek-Witten picture. These surfaces are connected by either light-like orbits of partonic 2-surface or space-like 3-surfaces at the ends of CD so that one indeed obtains the analog of polygon.

What does this then mean concretely (if this word can be used in this kind of context)?

(a) At least it means that ordinary Super Kac-Moody and Super Virasoro algebras associated with isometries of $M^4 \times CP_2$ annihilating the scattering amplitudes must be extended to a co-algebras with a non-trivial deformation parameter. Kac-Moody group is thus the product of Poincare and color groups. This algebra acts as deformations of the light-like 3-surfaces representing the light-like orbits of particles which are extremals of Chern-Simon action with the constraint that weak form of electric-magnetic duality holds true. I know so little about the mathematical side that I cannot tell whether the condition that the product of the representations of Super-Kac-Moody and Super-Virasoro algebras contains adjoint representation only once, holds true in this case. In any case, it would allow all representations of finite-dimensional Lie group in vertices whereas $\mathcal{N} = 4$ SUSY would allow only the adjoint.

(b) Besides this ordinary kind of Kac-Moody algebra there is the analog of Super-Kac-Moody algebra associated with the light-cone boundary which is metrically 3-dimensional. The finite-dimensional Lie group is in this case replaced with infinite-dimensional group of symplectomorphisms of $\delta M^4_{+/-}$ made local with respect to the internal coordinates of the partonic 2-surface. This picture also justifies p-adic thermodynamics applied to either symplectic or isometry Super-Virasoro and giving thermal contribution to the vacuum conformal and thus to mass squared.

(c) The construction of TGD leads also to other super-conformal algebras and the natural guess is that the Yangians of all these algebras annihilate the scattering amplitudes.

(d) Obviously, already the starting point symmetries look formidable but they still act on single partonic surface only. The discrete Yangian associated with this algebra associated with the closed polygon defined by the incoming momenta and the negatives of the outgoing momenta acts in multi-local manner on scattering amplitudes. It might make sense to speak about polygons defined also by other conserved quantum numbers so that one would have generalized light-like curves in the sense that state are massless in 8-D sense.

Could Yangian symmetry provide a new view about conserved quantum numbers?

The Yangian algebra has some properties which suggest a new kind of description for bound states. The Cartan algebra generators of $n = 0$ and $n = 1$ levels of Yangian algebra commute.

Since the co-product Δ maps $n = 0$ generators to $n = 1$ generators and these in turn to generators with high value of n, it seems that they commute also with $n \geq 1$ generators. This applies to four-momentum, color isospin and color hyper charge, and also to the Virasoro generator L_0 acting on Kac-Moody algebra of isometries and defining mass squared operator.

Could one identify total four momentum and Cartan algebra quantum numbers as sum of contributions from various levels? If so, the four momentum and mass squared would involve besides the local term assignable to wormhole throats also n-local contributions. The interpretation in terms of n-parton bound states would be extremely attractive. n-local contribution would involve interaction energy. For instance, string like object would correspond to $n = 1$ level and give $n = 2$-local contribution to the momentum. For baryonic valence quarks one would have 3-local contribution corresponding to $n = 2$ level. The Yangian view about quantum numbers could give a rigorous formulation for the idea that massive particles are bound states of massless particles.

5.6 Generalization Of Ads/CFT Duality To TGD Framework

AdS/CFT duality has provided a powerful approach in the attempts to understand the non-perturbative aspects of super-string theories. The duality states that conformal field theory in n-dimensional Minkowski space M^n identifiable as a boundary of $n+1$-dimensional space AdS_{n+1} is dual to a string theory in $AdS_{n+1} \times S^{9-n}$.

As a mathematical discovery the duality is extremely interesting but it seems that it need not have much to do with physics. From TGD point of view the reason is obvious: the notion of conformal invariance is quite too limited. In TGD framework conformal invariance is extended to a super-symplectic symmetry in $\delta M^4_\pm \times CP_2$, whose Lie-algebra has the structure of conformal algebra. Also ordinary super-conformal symmetries associated with string world sheets are present as well as generalization of 2-D conformal symmetries to their analogs at light-cone boundary and light-like orbits of partonic 2-surfaces. In this framework AdS/CFT duality is expected to be modified and this seems to be the case.

The matrix elements of Kähler metric of WCW can be expressed in two manners. As contractions of the derivatives $\partial_K \partial_{\overline{L}} K$ of the Kähler function of WCW with isometry generators or as anticommutators of WCW gamma matrices identified as supersymplectic Noether super charges assignable to fermioni strings connecting partonic 2-surfaces. Kähler function is identified as Kähler action for the Euclidian space-time regions with 4-D CP_2 projection. Kähler action defines the Kähler-Dirac gamma matrices appearing in K-D action as contractions of canonical momentum currents with imbedding space gamma matrices. Bosonic and fermionic degrees of freedom are therefore dual in a well-defined sense.

This observation suggests various generalizations. There is super-symmetry between Kähler action and Kähler-Dirac action. The problem is that induced spinor fields are localized at 2-D string world sheets. Strong form of holography implying effective 2-dimensionality suggests the solution to the paradox. The paradox disappears if the Kähler action is expressible as string area for the effective metric defined by the anti-commutators of K-D gamma matrices at string world sheet. This expression allows to understand how strings connecting partonic 2-surfaces give rise to the formation of gravitationally bound states. Bound states of macroscopic size are however possible only if one allows hierarchy of Planck constants. This representation of Kähler action can be seen as one aspect of the analog of AdS/CFT duality in TGD framework.

One can imagine also other realizations. For instance, Dirac determinant for the spinors associated with string world sheets should reduce to the exponent of Kähler action.

5.6.1 Does The Exponent Of Chern-Simons Action Reduce To The Exponent Of The Area Of Minimal Surfaces?

As I scanned of hep-th I found an interesting article (see `http://tinyurl.com/ycpkrg4f`) by Giordano, Peschanski, and Seki [B48] based on AdS/CFT correspondence. What is studied is the high energy behavior of the gluon-gluon and quark-quark scattering amplitudes of $\mathcal{N} = 4$ SUSY.

(a) The proposal made earlier by Aldaya and Maldacena (see `http://tinyurl.com/ybnk6kbs`) [B18] is that gluon-gluon scattering amplitudes are proportional to the imaginary exponent of the area of a minimal surface in AdS_5 whose boundary is identified as *momentum space*. The boundary of the minimal surface would be polygon with light-like edges: this polygon and its dual are familiar from twistor approach.

(b) Giordano, Peschanski, and Seki claim that quark-quark scattering amplitude for heavy quarks corresponds to the exponent of the area for a minimal surface in the *Euclidian* version of AdS_5 which is hyperbolic space (space with a constant negative curvature): it is interpreted as a counterpart of WCW rather than momentum space and amplitudes are obtained by analytic continuation. For instance, a universal Regge behavior is obtained. For general amplitudes the exponent of the area alone is not enough since it does not depend on gluon quantum numbers and vertex operators at the edges of the boundary polygon are needed.

In the following my intention is to consider the formulation of this conjecture in quantum TGD framework. I hasten to inform that I am not a specialist in AdS/CFT and can make only general comments inspired by analogies with TGD and the generalization of AdS/CFT duality to TGD framework based on the localization of induced spinors at string world sheets, super-symmetry between bosonic and fermionic degrees of freedom at the level of WCW , and the notion of effective metric at string world sheets.

5.6.2 Does Kähler Action Reduce To The Sum Of Areas Of Minimal Surfaces In Effective Metric?

Minimal surface conjectures are highly interesting from TGD point of view. The weak form of electric magnetic duality implies the reduction of Kähler action to 3-D Chern-Simons terms. Effective 2-dimensionality implied by the strong form of General Coordinate Invariance suggests a further reduction of Chern-Simons terms to 2-D terms and the areas of string world sheet and of partonic 2-surface are the only non-topological options that one can imagine. Skeptic could of course argue that the exponent of the minimal surface area results as a characterizer of the quantum state rather than vacuum functional. In the following I end up with the proposal that the Kähler action should reduce to the sum of string world sheet areas in the effective metric defines by the anticommutators of Kähler-Dirac gamma matrices at string world sheets.

Let us look this conjecture in more detail.

(a) In zero energy ontology twistor approach is very natural since all physical states are bound states of massless particles. Also virtual particles are composites of massless states. The possibility to have both signs of energy makes possible space-like momenta for wormhole contacts. Mass shell conditions at internal lines imply extremely strong constraints on the virtual momenta and both UV and IR finiteness are expected to hold true.

(b) The weak form of electric magnetic duality [K88] implies that the exponent of Kähler action reduces to the exponent of Chern-Simons term for 3-D space-like surfaces at the ends of space-time surface inside CD and for light-like 3-surfaces. The coefficient of this term is complex since the contribution of Minkowskian regions of the space-time surface is imaginary ($\sqrt{g_4}$ is imaginary) and that of Euclidian regions (generalized Feynman diagrams) real. The Chern-Simons term from Minkowskian regions is like

Morse function and that from Euclidian regions defines Kähler function and stationary phase approximation makes sense. The two contributions are different since the space-like 3-surfaces contributing to Kähler function and Morse function are different.

(c) Electric magnetic duality [K88] leads also to the conclusion that wormhole throats carrying elementary particle quantum numbers are Kähler magnetic monopoles. This forces to identify elementary particles as string like objects with ends having opposite monopole charges. Also more complex configurations are possible.

It is not quite clear what the scale of the stringyness is. The natural first guess inspired by quantum classical correspondence is that it corresponds to the p-adic length scale of the particle characterizing its Compton length. Second possibility is that it corresponds to electroweak scale. For leptons stringyness in Compton length scale might not have any fatal implications since the second end of string contains only neutrinos neutralizing the weak isospin of the state. This kind of monopole pairs could appear even in condensed matter scales: in particular if the proposed hierarchy of Planck constants [K22] is realized.

(d) Strong form of General Coordinate Invariance requires effective 2-dimensionality. In given UV and IR resolutions either partonic 2-surfaces or string world sheets form a finite hierarchy of CDs inside CDs with given CD characterized by a discrete scale coming as an integer multiple of a fundamental scale (essentially CP_2 size). The string world sheets have boundaries consisting of either light-like curves in induced metric at light-like wormhole throats and space-like curves at the ends of CD whose M^4 projections are light-like. These braids intersect partonic 2-surfaces at discrete points carrying fermionic quantum numbers.

This implies a rather concrete analogy with $AdS_5 \times S_5$ duality, which describes gluons as open strings. In zero energy ontology (ZEO) string world sheets are indeed a fundamental notion and the natural conjecture is that these surfaces are minimal surfaces whose area by quantum classical correspondence depends on the quantum numbers of the external particles. String tension in turn should depend on gauge couplings -perhaps only Kähler coupling strength- and geometric parameters like the size scale of CD and the p-adic length scale of the particle.

(e) One can of course ask whether the metric defining the string area is induced metric or possibly the metric defined by the anti-commutators of Kähler-Dirac gamma matrices. The recent view does not actually leave any other alternative. The analog of AdS/CFT duality together with supersymmetry demands that Kähler action is proportional to the sum of the areas of string world sheets in this effective metric. Whether the vanishing of induced W fields (and possibly also Z^0) making possible well-defined em charge for the spinor nodes is realized by the condition that the string world sheet is a miniml surface in the effective metric remains an open question.

The assumption that ordinary minimal surfaces are in question is not consistent with the TGD view about the formation of gravitational bound states and if string tension is $1/\hbar G$ as in string models, only bound states with size of order Planck length are possible. This strongly favors effective metric giving string tension proportional to $1/h_{eff}^2$. How $1/h_{eff}^2$ proportionality might be understood is discussed in [K106] in terms electric-magnetic duality.

(f) One can of course still consider also the option that ordinary minimal surfaces are in question. Are the minimal surfaces in question minimal surfaces of the imbedding space $M^4 \times CP_2$ or of the space-time surface X^4? All possible 2-surfaces at the boundary of CD must be allowed so that they cannot correspond to minimal surfaces in $M^4 \times CP_2$ unless one assumes that they emerge in stationary phase approximation only. The boundary conditions at the ends of CD could however be such that *any* partonic 2-surface correspond to a minimal surfaces in X^4. Same applies to string world sheets. One might even hope that these conditions combined with the weak form of electric magnetic duality fixes completely the boundary conditions at wormhole throats and space-like ends of space-time surface.

The trace of the second fundamental form orthogonal to the string world sheet/partonic

2-surface as sub-manifold of space-time surface would vanish: this is nothing but a generalization of the geodesic motion obtained by replacing word line with a 2-D surface. It does not imply the vanishing of the trace of the second fundamental form in $M^4 \times CP_2$ having interpretation as a generalization of particle acceleration [K79]. Effective 2-dimensionality would be realized if Chern-Simons terms reduce to a sum of the areas of these minimal surfaces.

2. These arguments suggest that scattering amplitudes are proportional to the product of exponents of 2-dimensional actions which can be either imaginary or real. Imaginary exponent would be proportional to the total area of string world sheets and the imaginary unit would come naturally from $\sqrt{g_2}$, where g_2 is effective metric most naturally. Teal exponent proportional to the total area of partonic 2-surfaces. The coefficient of these areas would not in general be same.

3. The reduction of the Kähler action from Minkowskian regions to Chern-Simons terms means that Chern-Simons terms reduce to actions assignable to string world sheets. The equality of the Minkowskian and Euclidian Chern-Simons terms is suggestive but not necessarily true since there could be also other Chern-Simons contributions than those assignable to wormhole throats and the ends of space-time. The equality would imply that the total area of string world sheets equals to the total area of partonic 2-surfaces suggesting strongly a duality meaning that either Euclidian or Minkowskian regions carry the needed information.

5.6.3 Surface Area As Geometric Representation Of Entanglement Entropy?

I encountered a link to a talk by James Sully and having the title "Geometry of Compression" (see `http://tinyurl.com/ycuu8xcr`). I must admit that I understood very little about the talk. My not so educated guess is however that information is compressed: UV or IR cutoff eliminating entanglement in short length scales and describing its presence in terms of density matrix - that is thermodynamically - is another manner to say it. The TGD inspired proposal for the interpretation of the inclusions of hyper-finite factors of type II_1 (HFFs) [K87] is in spirit with this.

The space-time counterpart for the compression would be in TGD framework discretization. Discretizations using rational points (or points in algebraic extensions of rationals) make sense also p-adically and thus satisfy number theoretic universality. Discretization would be defined in terms of intersection (rational or in algebraic extension of rationals) of real and p-adic surfaces. At the level of "world of classical worlds" the discretization would correspond to - say - surfaces defined in terms of polynomials, whose coefficients are rational or in some algebraic extension of rationals. Pinary UV and IR cutoffs are involved too. The notion of p-adic manifold allows to interpret the p-adic variants of space-time surfaces as cognitive representations of real space-time surfaces.

Finite measurement resolution does not allow state function reduction reducing entanglement totally. In TGD framework also negentropic entanglement stable under Negentropy Maximixation Principle (NMP) is possible [K41]. For HFFs the projection into single ray of Hilbert space is indeed impossible: the reduction takes always to infinite-D sub-space.

The visit to the URL was however not in vain. There was a link to an article (see `http://tinyurl.com/y9h3qtr8`) [B71] discussing the geometrization of entanglement entropy inspired by the AdS/CFT hypothesis.

Quantum classical correspondence is basic guiding principle of TGD and suggests that entanglement entropy should indeed have space-time correlate, which would be the analog of Hawking-Bekenstein entropy.

Generalization of AdS/CFT to TGD context

AdS/CFT generalizes to TGD context in non-trivial manner. There are two alternative interpretations, which both could make sense. These interpretations are not mutually exclusive. The first interpretation makes sense at the level of "world of classical worlds" (WCW) with symplectic algebra and extended conformal algebra associated with $\delta M^4_\pm$ replacing ordinary conformal and Kac-Moody algebras. Second interpretation at the level of space-time surface with the extended conformal algebras of the light-likes orbits of partonic 2-surfaces replacing the conformal algebra of boundary of AdS^n.

1. First interpretation

For the first interpretation 2-D conformal invariance is generalised to 4-D conformal invariance relying crucially on the 4-dimensionality of space-time surfaces and Minkowski space.

1. One has an extension of the conformal invariance provided by the symplectic transformations of $\delta CD \times CP_2$ for which Lie algebra has the structure of conformal algebra with radial light-like coordinate of δM^4_+ replacing complex coordinate z.

2. One could see the counterpart of AdS_n as imbedding space $H = M^4 \times CP_2$ completely unique by twistorial considerations and from the condition that standard model symmetries are obtained and its causal diamonds defined as sub-sets $CD \times CP_2$, where CD is an intersection of future and past directed light-cones. I will use the shorthand CD for $CD \times CP_2$. Strings in $AdS_5 \times S^5$ are replaced with space-time surfaces inside 8-D CD.

3. For this interpretation 8-D CD replaces the 10-D space-time $AdS_5 \times S^5$. 7-D light-like boundaries of CD correspond to the boundary of say AdS_5, which is 4-D Minkowski space so that zero energy ontology (ZEO) allows rather natural formulation of the generalization of AdS/CFT correspondence since the positive and negative energy parts of zero energy states are localized at the boundaries of CD.

Second interpretation

For the second interpretation relies on the observation that string world sheets as carriers of induced spinor fields emerge in TGD framework from the condition that electromagnetic charge is well-defined for the modes of induced spinor field.

1. One could see the 4-D space-time surfaces X^4 as counterparts of AdS_4. The boundary of AdS_4 is replaced in this picture with 3-surfaces at the ends of space-time surface at opposite boundaries of CD and by strong form of holography the union of partonic 2-surfaces defining the intersections of the 3-D boundaries between Euclidian and Minkowskian regions of space-time surface with the boundaries of CD. Strong form of holography in TGD is very much like ordinary holography.

2. Note that one has a dimensional hierarchy: the ends of the boundaries of string world sheets at boundaries of CD as point-like partices, boundaries as fermion number carrying lines, string world sheets, light-like orbits of partonic 2-surfaces, 4-surfaces, imbedding space $M^4 \times CP_2$. Clearly the situation is more complex than for AdS/CFT correspondence.

3. One can restrict the consideration to 3-D sub-manifolds X^3 at either boundary of causal diamond (CD): the ends of space-time surface. In fact, the position of the other boundary is not well-defined since one has superposition of CDs with only one boundary fixed to be piece of light-cone boundary. The delocalization of the other boundary is essential for the understanding of the arrow of time. The state function reductions at fixed boundary leave positive energy part (say) of the zero energy state at that boundary invariant (in positive energy ontology entire state would remain unchanged) but affect the states associated with opposite boundaries forming a superposition which also changes between reduction: this is analog for unitary time evolution. The average for the distance between tips of CDs in the superposition increases and gives rise to the flow of time.

4. One wants an expression for the entanglement entropy between X^3 and its partner. Bekenstein area law allows to guess the general expression for the entanglement entropy: for the proposal discussed in the article the entropy would be the area of the boundary of X^3 divided by gravitational constant: $S = A/4G$. In TGD framework gravitational constant might be replaced by the square of CP_2 radius apart from numerical constant. How gravitational constant emerges in TGD framework is not completely understood although one can deduce for it an estimate using dimensional analyses. In any case, gravitational constant is a parameter

which characterizes GRT limit of TGD in which many-sheeted space-time is in long scales replaced with a piece of Minkowski space such that the classical gravitational fields and gauge potentials for sheets are summed. The physics behind this relies on the generalization of linear superposition of fields: the effects of different space-time sheets particle touching them sum up rather than fields.

5. The counterpart for the boundary of X^3 appearing in the proposal for the geometrization of the entanglement entropy naturally corresponds to partonic 2-surface or a collection of them if strong form of holography holds true.

   There is however also another candidate to be considered! Partonic 2-surfaces are basic objects, and one expects that the entanglement between fundamental fermions associated with distinct partonic 2-surfaces has string world sheets as space-time correlates. Could the area of the string world sheet in the effective metric defined by the anti-commutators of K-D gamma matrices at string world sheet provide a measure for entanglement entropy? If this conjecture is correct: the entanglement entropy would be proportional to Kähler action. Also negative values are possible for Kähler action in Minkowskian regions but in TGD framework number theoretic entanglement entropy having also negative values emerges naturally.

   Which of these guesses is correct, if any? Or are they equivalent?

With what kind of systems 3-surfaces can entangle?

With what system X^3 is entangled/can entangle? There are several options to consider and they could correspond to the two TGD variants for the AdS/CFT correspondence.

1. X^3 could correspond to a wormhole contact with Euclidian signature of induced metric. The entanglement would be between it and the exterior region with Minkowskian signature of the induced metric.

2. X^3 could correspond to single sheet of space-time surface connected by wormhole contacts to a larger space-time sheet defining its environment. More precisely, X^3 and its complement would be obtained by throwing away the wormhole contacts with Euclidian signature of induce metric. Entanglement would be between these regions. In the generalization of the formula

$$S = \frac{A}{4\hbar G}$$

   area A would be replaced by the total area of partonic 2-surfaces and G perhaps with CP_2 length scale squared.

3. In ZEO the entanglement could also correspond to time-like entanglement between the 3-D ends of the space-time surface at opposite light-like boundaries of CD. M-matrix, which can be seen as the analog of thermal S-matrix, decomposes to a product of hermitian square root of density matrix and unitary S-matrix and this hermitian matrix could also define p-adic thermodynamics. Note that in ZEO quantum theory can be regarded as square root of thermodynamics.

Minimal surface property is not favored in TGD framework

Minimal surface property for the 3-surfaces X^3 at the ends of space-time surface looks at first glance strange but a proper generalization of this condition makes sense if one assumes strong form of holography. Strong form of holography realizes General Coordinate Invariance (GCI) in strong sense meaning that light-like parton orbits and space-like 3-surfaces at the ends of space-time surfaces are equivalent physically. As a consequence, partonic 2-surfaces and their 4-D tangent space data must code for the quantum dynamics.

The mathematical realization is in terms of conformal symmetries accompanying the symplectic symmetries of $\delta M^4_\pm \times CP_2$ and conformal transformations of the light-like partonic orbit [K88]. The generalizations of ordinary conformal algebras correspond to conformal algebra, Kac-Moody algebra at the light-like parton orbits and to symplectic transformations $\delta M^4 \times CP_2$ acting as isometries of WCW and having conformal structure with respect to the light-like radial coordinate plus conformal transformations of δM^4_+, which is metrically 2-dimensional and allows extended conformal symmetries.

1. If the conformal realization of the strong form of holography works, conformal transformations act at quantum level as gauge symmetries in the sense that generators with no-vanishing conformal weight are zero or generate zero norm states. Conformal degeneracy can be eliminated by fixing the gauge somehow. Classical conformal gauge conditions analogous to Virasoro and Kac-Moody conditions satisfied by the 3-surfaces at the ends of CD are natural in this respect. Similar conditions would hold true for the light-like partonic orbits at which the signature of the induced metric changes.

2. What is also completely new is the hierarchy of conformal symmetry breakings associated with the hierarchy of Planck constants $h_{eff}/h = n$ [K22]. The deformations of the 3-surfaces which correspond to non-vanishing conformal weight in algebra or any sub-algebra with conformal weights vanishing modulo n give rise to vanishing classical charges and thus do not affect the value of the Kähler action [K88].

   The inclusion hierarchies of conformal sub-algebras are assumed to correspond to those for hyper-finite factors. There is obviously a precise analogy with quantal conformal invariance conditions for Virasoro algebra and Kac-Moody algebra. There is also hierarchy of inclusions which corresponds to hierarchy of measurement resolutions. An attractive interpretation is that singular conformal transformations relate to each other the states for broken conformal symmetry. Infinitesimal transformations for symmetry broken phase would carry fractional conformal weights coming as multiples of $1/n$.

3. Conformal gauge conditions need not reduce to minimal surface conditions holding true for all variations.

4. Note that Kähler action reduces to Chern-Simons term at the ends of CD if weak form of electric magnetic duality holds true. The conformal charges at the ends of CD cannot however reduce to Chern-Simons charges by this condition since only the charges associated with CP_2 degrees of freedom would be non-trivial.

The way out of the problem is provided by the generalization of AdS/CFT conjecture. String area is estimated in the effective metric provided by the anti-commutator of K-D gamma matrices at string world sheet.

5.6.4 Related Ideas

p-Adic mass calculations led to the introduction of the p-adic variant of Bekenstein-Hawkin law in which Planck length is replaced by p-adic length scale. This generalization is in spirit with the idea that string world sheet area is estimated in effective rather than induced metric.

p-Adic variant of Bekenstein-Hawking law

When the 3-surface corresponds to elementary particle, a direct connection with p-adic thermodynamics suggests itself and allows to answer the questions above. p-Adic thermodynamics could be interpreted as a description of the entanglement with environment. In ZEO the entanglement could also correspond to time-like entanglement between the 3-D ends of the space-time surface at opposite light-like boundaries of CD. M-matrix, which can be seen as the analog of thermal S-matrix, decomposes to a product of hermitian square root of density matrix and unitary S-matrix and this hermitian matrix could also define p-adic thermodynamics.

1. p-Adic thermodynamics [K100] would not be for energy but for mass squared (or scaling generator L_0) would describe the entanglement of the particle with environment defined by the larger space-time sheet. Conformal weights would comes as positive powers of integers (p_0^L would replace $exp(-H/T)$ to guarantee the number theoretical existence and convergence of the Boltzmann weight: note that conformal invariance that is integer spectrum of L_0 is also essential).

2. The interactions with environment would excite very massive CP_2 mass scale excitations (mass scale is about 10^{-4} times Planck mass) of the particle and give it thermal mass squared identifiable as the observed mass squared. The Boltzmann weights would be extremely small having p-adic norm about $1/p^n$, p the p-adic prime: $M_{127} = 2^{127} - 1$ for electron.

3. One of the first ideas inspired by p-adic vision was that p-adic entropy could be seen as a p-adic counterpart of Bekenstein-Hawking entropy [K49]. $S = (R^2/\hbar^2) \times M^2$ holds true identically apart from numerical constant. Note that one could interpret $R^2 M/\hbar$ as the counterpart of Schwartschild radius. Note that this radius is proportional to $1/\sqrt{p}$ so that the area A would correspond to the area defined by Compton length. This is in accordance with the third option.

What is the space-time correlate for negentropic entanglement?

The new element brought in by TGD framework is that number theoretic entanglement entropy is negative for negentropic entanglement assignable to unitary entanglement (in the sense that entanglement matrix is proportional to a unitary matrix) and NMP states that this negentropy increases [K41]. Since entropy is essentially number of energy degenerate states, a good guess is that the number $n = h_{eff}/h$ of space-time sheets associated with h_{eff} defines the negentropy. An attractive space-time correlate for the negentropic entanglement is braiding. Braiding defines unitary S-matrix between the states at the ends of braid and this entanglement is negentropic. This entanglement gives also rise to topological quantum computation.

5.6.5 The Importance Of Being Light-Like

The singular geometric objects associated with the space-time surface have become increasingly important in TGD framework. In particular, the recent progress has made clear that these objects might be crucial for the understanding of quantum TGD. The singular objects are associated not only with the induced metric but also with the effective metric defined by the anti-commutators of the Kähler-Dirac gamma matrices appearing in the Kähler-Dirac equation and determined by the Kähler action.

The singular objects associated with the induced metric

Consider first the singular objects associated with the induced metric.

1. At light-like 3-surfaces defined by wormhole throats the signature of the induced metric changes from Euclidian to Minkowskian so that 4-metric is degenerate. These surfaces are carriers of elementary particle quantum numbers and the 4-D induced metric degenerates locally to 3-D one at these surfaces.

2. Braid strands at partonic orbits - fermion lines identified as boundaries of string world sheets in the more recent terminology - are most naturally light-like curves: this correspond to the boundary condition for open strings. One can assign fermion number to the braid strands. Braid strands allow an identification as curves along which the Euclidian signature of the string world sheet in Euclidian region transforms to Minkowskian one. Number theoretic interpretation would be as a transformation of complex regions to hyper-complex regions meaning that imaginary unit i satisfying $i^2 = -1$ becomes hyper-complex unit e satisfying $e^2 = 1$. The complex coordinates $(z, \overline{z})$ become hyper-complex coordinates $(u = t + ex, v = t - ex)$ giving the standard light-like coordinates when one puts $e = 1$.

The singular objects associated with the effective metric

There are also singular objects assignable to the effective metric. According to the simple arguments already developed, string world sheets and possibly also partonic 2-surfaces are singular objects with respect to the effective metric defined by the anti-commutators of the Kähler-Dirac gamma matrices rather than induced gamma matrices. Therefore the effective metric might be more than a mere formal structure. The following is of course mere speculation and should be taken as such.

1. For instance, quaternionicity of the space-time surface *might* allow an elegant formulation in terms of the effective metric avoiding the problems due to the Minkowski signature. This is achieved if the effective metric has Euclidian signature $\epsilon \times (1,1,1,1)$, $\epsilon = \pm 1$ or a complex counterpart of the Minkowskian signature $\epsilon(1,1,-1,-1)$.

2. String word sheets and perhaps also partonic 2-surfaces might be be understood as singularities of the effective metric. What happens that the effective metric with Euclidian signature $\epsilon \times (1,1,1,1)$ transforms to the signature $\epsilon(1,1,-1,-1)$ (say) at string world sheet so that one would have the degenerate signature $\epsilon \times (1,1,0,0)$ at the string world sheet.

   What is amazing is that this works also number theoretically. It came as a total surprise to me that the notion of hyper-quaternions as a closed algebraic structure indeed exists. The hyper-quaternionic units would be given by $(1, I, iJ, iK)$, where i is a commuting imaginary unit satisfying $i^2 = -1$. Hyper-quaternionic numbers defined as combinations of these units with real coefficients do form a closed algebraic structure which however fails to be a number field just like hyper-complex numbers do. Note that the hyper-quaternions obtained with real coefficients from the basis $(1, iI, iJ, iK)$ fail to form an algebra since the product is not hyper-quaternion in this sense but belongs to the algebra of complexified quaternions. The same problem is encountered in the case of hyper-octonions defined in this manner. This has been a stone in my shoe since I feel strong disrelish towards Wick rotation as a trick for moving between different signatures.

3. Could also partonic 2-surfaces correspond to this kind of singular 2-surfaces? In principle, 2-D surfaces of 4-D space intersect at discrete points just as string world sheets and partonic 2-surfaces do so that this might make sense. By complex structure the situation is algebraically equivalent to the analog of plane with non-flat metric allowing all possible signatures (ϵ_1, ϵ_2) in various regions. At light-like curve either ϵ_1 or ϵ_2 changes sign and light-like curves for these two kinds of changes can intersect as one can easily verify by drawing what happens. At the intersection point the metric is completely degenerate and simply vanishes.

4. Replacing real 2-dimensionality with complex 2-dimensionality, one obtains by the universality of algebraic dimension the same result for partonic 2-surfaces and string world sheets. The braid ends at partonic 2-surfaces representing the intersection points of 2-surfaces of this kind would have completely degenerate effective metric so that the Kähler-Dirac gamma matrices would vanish implying that energy momentum tensor vanishes as does also the induced Kähler field.

5. The effective metric suffers a local conformal scaling in the critical deformations identified in the proposed manner. Since ordinary conformal group acts on Minkowski space and leaves the boundary of light-cone invariant, one has two conformal groups. It is not however clear whether the M^4 conformal transformations can act as symmetries in TGD, where the presence of the induced metric in Kähler action breaks M^4 conformal symmetry. As found, also in TGD framework the Kac-Moody currents assigned to the braid strands generate Yangian: this is expected to be true also for the Kac-Moody counterparts of the conformal algebra associated with quantum criticality. On the other hand, in twistor program one encounters also two conformal groups and the space in which the second conformal group acts remains somewhat mysterious object. The Lie algebras for the two conformal groups generate the conformal Yangian and the integrands of the scattering amplitudes are Yangian invariants. Twistor approach should apply in TGD if zero energy ontology is right. Does this mean a deep connection?

   What is also intriguing that twistor approach in principle works in strict mathematical sense only at signatures $\epsilon \times (1,1,-1-1)$ and the scattering amplitudes in Minkowski signature are obtained by analytic continuation. Could the effective metric give rise to the desired signature? Note that the notion of massless particle does not make sense in the signature $\epsilon \times (1,1,1,1)$.

These arguments provide genuine a support for the notion of quaternionicity and suggest a connection with the twistor approach.

5.7 Could One Define Dynamical Homotopy Groups In WCW?

Agostino Prastaro - working as professor at the University of Rome - has done highly interesting work with partial differential equations, also those assignable to geometric variational principles such as Kähler action in TGD [A32, A33]. I do not understand the mathematical details but the key idea is a simple and elegant generalization of Thom's cobordism theory, and it is difficult to avoid the idea that the application of Prastaro's idea might provide insights about the preferred extremals, whose identification is now on rather firm basis [K111].

One could also consider a definition of what one might call dynamical homotopy groups as a genuine characteristics of WCW topology. The first prediction is that the values of conserved classical Noether charges correspond to disjoint components of WCW. Could this mean that the natural topology in the parameter space of Noether charges zero modes of WCW metric) is p-adic? An analogous conjecture was made on basis of spin glass analogy long time ago. Second surprise is that the only the six lowest dynamical homotopy groups of WCW would be non-trivial. The finite number of these groups dictate by the dimension of imbedding space suggests also an interpretation as analogs of homology groups.

In the following the notion of cobordism is briefly discussed and the idea of Prastaro about assigning cobordism with partial differential equations is discussed.

5.7.1 About Cobordism As A Concept

To get some background consider first the notion of cobordism (`http://tinyurl.com/y7wdhtmv`).

1. Thom's cobordism theory [A81] is inspired by the question "When an n-manifold can be represented as a boundary of $n+1$-manifold". One can also pose additional conditions such as continuity, smoothness, orientability, one can add bundles structures and require that they are induced to n-manifold from that of $n+1$-manifold. One can also consider sub-manifolds of some higher-dimensional manifold.

   One can also fix n-manifold M and ask "What is the set of n-manifolds N with the property that there exists $n+1$-manifold W having union of $M \cup N$ as its boundary". One can also allow M to have boundary and pose the same question by allowing also the boundary of connecting $n+1$-manifold W contain also the orbits of boundaries of M and N.

   The cobordism class of M can be defined as the set of manifolds N cobordant with M - that is connectable in this manner. They have same cobordism class since cobordism is equivalence relation. The classes form also a group with respect to disjoint union. Cobordism is much rougher equivalence relation than diffeomorphy or homeomorphy since topology changes are possible. For instance, every 3-D closed un-oriented manifold is a boundary of a 4-manifold! Same is true for orientable cobordisms. Cobordism defines a category: objects are (say closed) manifolds and morphisms are cobordisms.

2. The basic result of Morse, Thom, and Milnor is that cobordism as topology changes can be engineered from elementary cobordisms. One take manifold $M \times I$ and imbeds to its other n-dimensional end the manifold $S^p \times D^q$, $n = p + q$, removes its interior and glues back $D^{p+1} \times S^{q-1}$ along its boundary to the boundary of the resulting hole. This gives n-manifold with different topology, call it N. The outcome is a cobordism connecting M and N unless there are some obstructions.

   There is a connection with Morse theory (`http://tinyurl.com/ych4chg9`) in which cobordism can be seen as a mapping of W to a unit interval such that the inverse images define a slicing of W and the inverse images at ends correspond to M and N.

3. One can generalize the abstract cobordism to that for n-sub-manifolds of a given imbedding space. This generalization is natural in TGD framework. This might give less trivial results since not all connecting manifolds are imbeddable into a given imbedding space. If connecting 4-manifolds connecting 3-manifolds with Euclidian signature (of induced metric) are assumed to have a Minkowskian signature, one obtains additional conditions, which might be too strong (the classical result of Geroch [A84] implies that non-trivial cobordism implies closed time loops - impossible in TGD).

From TGD point of view this is too strong a condition and in TGD framework space-time surfaces with both Euclidian and Minkowskian signature of the induced metric are allowed. Also cobordisms singular as 4-surfaces are analogous to 3-vertices of Feynman diagrams are allowed.

5.7.2 Prastaro's Generalization Of Cobordism Concept To The Level Of Partial Differential Equations

I am not enough mathematician in technical sense of the word to develop overall view about what Prastaro has done and I have caught only the basic idea. I have tried to understand the articles [A32, A33] with title "Geometry of PDE's. I/II: Variational PDE's and integral bordism groups" (`http://tinyurl.com/yb9wey8c` and `http://tinyurl.com/y9x55qmk`), which seem to correspond to my needs. The key idea is to generalize the cobordism concept also to partial differential equations with cobordism replaced with the time evolution defined by partial differential equation. In particular, to geometric variational principles defining as their extremals the counterparts of cobordisms.

Quite generally, and especially so in the case of the conservation of Noether charges give rise to strong selection rules since two n-surfaces with different classical charges cannot be connected by extremals of the variational principle. Note however that the values of the conserved charges depend on the normal derivatives of the imbedding space coordinates at the n-dimensional ends of cobordism. If one poses additional conditions fixing these normal derivatives, the selection rules become even stronger. In TGD framework Bohr orbit property central for the notion of WCW geometry and holography allows to hope that conserved charges depend on 3-surfaces only.

What is so beautiful in this approach that it promises to generalize the notion of cobordism and perhaps also the notions of homotopy/homology groups so that they would apply to partial differential equations quite generally, and especially so in the case of geometric variational principles giving rise to $n+1$-surfaces connecting n-surfaces characterizing the initial and final states classically. TGD with $n=3$ seems to be an ideal applications for these ideas.

Prastaro also proposes a generalization of cobordism theory to super-manifolds and quantum super-manifolds. The generalization in the case of quantum theory utilizing path integral does not not pose conditions on classical connecting field configurations. In TGD framework these generalizations are not needed since fermion number is geometrized in terms of imbedding space gamma matrices and super(-symplectic) symmetry is realized differently.

5.7.3 Why Prastaro's Idea Resonates So Strongly With TGD

Before continuing I want to make clear why Prastaro's idea resonates so strongly with TGD.

1. One of the first ideas as I started to develop TGD was that there might be selection rules analogous to those of quantum theory telling which 3-surfaces can be connected by a space-time surface. At that time I still believed in path integral formalism assuming that two 3-surfaces at different time slices with different values of Minkowski time can be connected by any space-time surface for which imbedding space coordinates have first derivatives.

   I soon learned about Thom's theory but was greatly disappointed since no selection rules were involved in the category of abstract 3-manifolds. I thought that possible selection rules should result from the imbeddability of the connecting four-manifold to $H = M^4 \times CP_2$ but my gut feeling was that these rules are more or less trivial since so many connecting 4-manifolds exist and some of them are very probably imbeddable.

   One possible source of selection rules could have been the condition that the induced metric has Minkowskian signature - one could justify it in terms of classical causality. This restricts strongly topology change in general relativity (`http://tinyurl.com/y6vuopgj`). Geroch's classical result [A84] states that non-trivial smooth Lorentz cobordism between compact 3-surfaces implies the existence of closed time loop - not possible in TGD framework. Second non-encouraging result is that scalar field propagating in trouser topology leads to an occurrence of infinite energy burst (`http://tinyurl.com/ybbuwyfj`).

In the recent formulation of TGD however also Euclidian signature of the induced metric is allowed. For space-time counterparts of 3-particle vertices three space-time surfaces are glued along their smooth 3-D ends whereas space-time surface fails to be everywhere smooth manifold. This picture fits nicely with the idea that one can engineer space-time surfaces by gluing them together along their ends.

2. At that time (before 1980) the discovery of the geometry of the "World of Classical Worlds" (WCW) as a possible solution to the failures of canonical quantization and path integral formalism was still at distance of ten years in future. Around 1985 I discovered the notion of WCW. I made some unsuccessful trials to construct its geometry, and around 1990 finally realized that 4-D general coordinate invariance is needed although basic objects are 3-D surfaces.

   This is realized if classical physics is an exact part of quantum theory - not only something resulting in a stationary phase approximation. Classical variational principle should assign to a 3-surface a physically unique space-time surface - the analog of Bohr orbit - and the action for this surface would define Kähler function defining the Kähler geometry of WCW using standard formula.

   This led to a notion of preferred extremal: absolute minimum of Kähler action was the first guess and might indeed make sense in the space-time regions with Euclidian signature of induced metric but not in Minkowskian regions, which give to the vacuum functional and exponential of Minkowskian Kähler action multiplied by imaginary unit coming from $\sqrt{g}$ - just as in quantum field theories. Euclidian regions give the analog of the free energy exponential of thermodynamics and transform path integral to mathematically well-defined functional integral.

3. After having discovered the notion of preferred extremal, I should have also realized that an interesting generalization of cobordism theory might make sense after all, and could even give rise to the classical counterparts of the selection rules! For instance, conservation of isometry charges defines equivalence classes of 3-surfaces endowed with tangent space data. Bohr orbit property could fix the tangent space data (normal derivatives of imbedding space coordinates) so that conserved classical charges would characterize 3-surfaces alone and thus cobordism equivalence classes and become analogous to topological invariants. This would be in spirit with the attribute "Topological" in TGD!

5.7.4 What Preferred Extremals Are?

The topology of WCW has remained mystery hitherto - partly due to my very limited technical skills and partly by the lack of any real physical idea. The fact, that p-adic topology seems to be natural at least as an effective topology for the maxima of Kähler function of WCW gave a hint but this was not enough.

I hope that the above summary has made clear why the idea about dynamical cobordism and even dynamical homotopy theory is so attractive in TGD framework. One could even hope that dynamics determines not only Kähler geometry but also the topology of WCW to some extend at least! To get some idea what might be involved one must however first tell about the recent situation concerning the notion of preferred extremal.

1. The recent formulation for the notion of preferred extremal relies on strong form of General Coordinate Invariance (SGCI). SGCI states that two kinds of 3-surfaces can identified as fundamental objects. Either the light-light 3-D orbits of partonic 2-surfaces defining boundaries between Minkowskian and Euclidian space-time regions or the space-like 3-D ends of space-time surfaces at boundaries of CD. Since both choices are equally good, partonic 2-surfaces and their tangent space-data at the ends of space-time should be the most economic choice.

   This eventually led to the realization that partonic 2-surfaces and string world sheets should be enough for the formulation of quantum TGD. Classical fields in the interior of space-time surface would be needed only in quantum measurement theory, which demands classical physics in order to interpret the experiments.

2. The outcome is strong form of holography (SH) stating that quantum physics should be coded by string world sheets and partonic 2-surfaces inside given causal diamond (CD). SH is very much analogous to the AdS/CFT correspondence but is much simpler: the simplicity is made possible by much larger group of conformal symmetries.

   If these 2-surfaces satisfy some consistency conditions one can continue them to 4-D space-time surface inside CD such that string world sheets are surfaces inside them satisfying the condition that charged (possibly all) weak gauge potentials identified as components of the induced spinor connection vanish at the string world sheets and also that energy momentum currents flow along these surfaces. String world sheets carry second quantized free induced spinor fields and fermionic oscillator operator basis is used to construct WCW gamma matrices.

3. The 3-surfaces at the ends of WCW must satisfy strong conditions to guarantee effective 2-dimensionality. Quantum criticality suggests the identification of these conditions. All Noether charges assignable to a sub-algebra of super-symplectic algebra isomorphic to it and having conformal weights which are n-multiples of those of entire algebra vanish/annihilate quantum states. One has infinite fractal hierarchy of broken super-conformal symmetries with the property that the sub-algebra is isomorphic with the entire algebra. This like a ball at the top of ball at the top of ....

   The speculative vision is that super-symplectic subalgebra with weights coming as n-ples of those for the entire algebra acts as an analog of conformal gauge symmetries on light-like orbits of partonic 2-surfaces, and gives rise to a pure gauge degeneracy whereas other elements of super-symplectic algebra act as dynamical symmetries. The hierarchy of quantum criticalities defines hierarchies of symmetry breakings characterized by hierarchies of sub-algebras for which one n_{i+1} is divisible by n_i. The proposal is that conformal gauge invariance means that the analogs of Bohr orbits are determined only apart from conformal gauge transformations forming to n_i conformal equivalence classes so that effectively one has n_i discrete degrees of freedom assignable to light-like partonic orbits.

4. In this framework manifolds M and N would correspond the 3-surfaces at the boundaries of CD and containing a collection strings carrying induced spinor fields. The connecting 4-surface W would contain string world sheets and the light-like orbits of partonic 2-surfaces as simultaneous boundaries for Minkowskian and Euclidian regions.

Propagator line has several meanings depending on whether one considers particles as strings, as single fermion states localizable at the ends of strings, or as Euclidian space-time regions or their light-like boundaries with singular induced metric having vanishing determinant. Vertices appear as generalizations of the stringy vertices and as generalization of the vertices of Feynman diagrams in which the incoming 4-surfaces meet along their ends.

1. Propagator line has several meanings depending on whether one considers particles as strings, as single fermion states localizable at the ends of strings, or as Euclidian space-time regions or their light-like boundaries with degenerate induced metric with vanishing determinant. Vertices appear as generalizations of the stringy vertices and as generalization of the vertices of Feynman diagrams in which the incoming 4-surfaces meet along their ends.

   (a) The lines of generalized Feynman graphs defined in topological sense are identified as slightly deformed pieces of CP_2 defining wormhole contacts connecting two Minkowskian regions and having wormhole throats identified as light-like parton orbits as boundaries. Since there is a magnetic monopole flux through the wormhole contacts they must appear as pairs (also larger number is possible) in order that magnetic field lines can close. Elementary particles correspond to pairs of wormhole contacts. At both space-time sheets the throats are connected by magnetic flux tubes carrying monopole flux so that a closed flux tube results having a shape of an extremely flattened square and having wormhole contacts at its ends. It is a matter of taste, whether to call the light-like wormhole throats or their interiors as lines of the generalized Feynman/twistor diagrams.

The light-like orbits of partonic 2-surfaces bring strongly in mind the light-like 3-surfaces along which radiation fields can be restricted - kind of shockwaves at which the signature of the induced space-time metric changes its signature.

(b) String world sheets as orbits of strings are also in an essential role and could be seen as particle like objets. String world sheets could as kind of singular solutions of field equations analogous to characteristics of hyperbolic differential equations. The isometry currents of Kähler action flow along string world sheets and field equations restricted to them are satisfied. As if one would have 2-dimensional solution. $\sqrt{g_4}$ would of course vanishes for genuinely 2-D solution but this one can argue that this is not a problem since $\sqrt{g_4}$ can be eliminated from field equations. String world sheets could serve as 2-D a analoga for a solution of hyperbolic field equations defining expanding wave front localized at 3-D light-like surface.

(c) Propagation in the third sense of word is assignable to the ends of string world sheets at the light-like orbits of partonic 2-surfaces and possibly carrying fermion number. One could say that in TGD one has both fundamental fermions serving as building bricks of elementary particles and strings characterizing interactions between particles. Fermion lines are massless in 8-D sense. By strong form of holography this quantum description has 4-D description space-time description as a classical dual.

2. The topological description of interaction vertices brings in the most important deviation from the standard picture behind cobordism: space-time surfaces are not smooth in TGD framework. One allows topological analogs of 3-vertices of Feynman diagrams realized by connecting three 4-surfaces along their smooth 3-D ends. 3-vertex is also an analog (actually much more!) for the replication in biology. This vertex is *not* the analog of stringy trouser vertex for which space-time surface is continuous whereas 3-surface at the vertex is singular (also trouser vertex could appear in TGD).

   The analog of trouser vertex for string world sheets means splitting of string and fermionic field modes decompose into superposition of modes propagating along the two branches. For instance, the propagation of photon along two paths could correspond to its geometric decay at trouser vertex not identifiable as "decay" to two separate particles.

   For the analog of 3-vertex of Feynman diagram the 3-surface at the vertex is non-singular but space-time surface is singular. The gluing along ends corresponds to genuine 3-particle vertex.

   The view about solution of PDEs generalizes dramatically but the general idea about cobordism might make sense also in the generalized context.

5.7.5 Could Dynamical Homotopy/Homology Groups Characterize WCW Topology?

The challenge is to at least formulate (with my technical background one cannot dream of much more) the analog of cobordism theory in this framework. One can actually hope even the analog of homotopy/homology theory.

1. To a given 3-surface one can assign its cobordism class as the set of 3-surfaces at the opposite boundary of CD connected by a preferred extremal. The 3-surfaces in the same cobordism class are characterized by same conserved classical Noether charges, which become analogs of topological invariants.

   One can also consider generalization of cobordisms as analogs to homotopies by allowing return from the opposite boundary of CD. This would give rise to first homotopy groupoid. One can even go back and forth several times. These dynamical cobordisms allow to divide 3-surfaces at given boundary of CD in equivalence classes characterized among other things by same values of conserved charges. One can also return to the original 3-surface. This could give rise to the analog of the first homotopy group Π_1.

2. If one takes the homotopy interpretation literally one must conclude that the 3-surfaces with different conserved Noether charges cannot be connected by any path in WCW - they belong

to disjoint components of the WCW! The zeroth dynamical homotopy group Π_0 of WCW would be non-trivial and its elements would be labelled by the conserved Noether charges defining topological invariants!

The values of the classical Noether charges would label disjoint components of WCW. The topology for the space of these parameters would be totally disconnected - no two points cannot be connected by a continuous path. p-Adic topologies are indeed totally disconnected. Could it be that p-adic topology is natural for the conserved classical Noether charges and the sectors of WCW are characetrized by p-adic number fields and their algebraic extensions?

Long time ago I noticed that the 4-D spin glass degeneracy induced by the huge vacuum degeneracy of Kähler action implies analogy between the space of maxima of Kähler function and the energy landscape of spin glass systems [K49]. Ultrametricity (`http://tinyurl.com/y6vswdoh`) is the basic property of the topology of the spin glass energy landscape. p-Adic topology is ultrametric and the proposal was that the effective topology for the space of maxima could be p-adic.

3. Isometry charges are the most important Noether charges. These Noether charges are very probably not the only conserved charges. Also the generators in the complement of the gauge sub-algebra of symplectic algebra acting as gauge conformal symmetries could be conserved. All these conserved Noether charges would define a parameter space with a natural p-adic topology.

   Since integration is problematic p-adically, one can ask whether only discrete quantum superpositions of 3-surfaces with different classical charges are allowed or whether one should even assume fixed values for the total classical Noether charges appearing in the scattering amplitudes.

   I have proposed this kind of approach for the zero modes of WCW geometry not contributing to the Kähler metric except as parameters. The integration for zero modes is also problematic because there is no metric, which would define the integration measure. Since classical charges do not correspond to quantum fluctuating degrees of freedom they should correspond to zero modes. Hence these arguments are equivalent.

The above argument led to the identification of the analogs of the homotopy group Π_0 and led to the idea about homotopy groupoid/group Π_1. The elements of Π_1 would correspond to space-time surfaces, which run arbitrary number of times fourth and back and return to the initial 3-surface at the boundary of CD. If the two preferred extremals connecting same pair of 3-surfaces can be deformed to each other, one can say that they are equivalent as dynamical homotopies (or cobordisms). What could be the allowed deformations? Are they cobordisms of cobordisms? What this could mean? Could they define the analog of homotopy groupoid Π_2 as foliations of preferred extremals connecting the same 3-surfaces?

1. The number theoretic vision about generalized Feynman diagrams suggests a possible approach. Number theoretic ideas combined with the generalization of twistor approach [K111, K76] led to the vision that generalized Feynman graphs can be identified as sequences or webs of algebraic operations in the co-algebra defined by the Yangian assignable to super-symplectic algebra [A27] [B39, B30, B31] and acting as symmetries of TGD. Generalized Feynman graphs would represent algebraic computations. Computations can be done in very many different manners and each of them corresponds to a generalized Feynman diagram. These computations transform give same final collection of "numbers" when the initial collection of "numbers" is given. Does this mean that the corresponding scattering amplitudes must be identical?

   If so, a huge generalization of the duality symmetry of the hadronic string models would suggest itself. All computations can be reduced to minimal computations. Accordingly, generalized Feynman diagrams can be reduced to trees by eliminating loops by moving the ends of the loops to same point and snipping the resulting tadpole out! The snipped of tadpole would give a mere multiplicative factor to the amplitude contributing nothing to the scattering rate - just like vacuum bubbles contribute nothing in the case of ordinary Feynman diagrams.

2. How this symmetry could be realized? Could one just assume that only the minimal generalized Feynman diagrams contribute? - not a very attractive option. Or could one hope that only tree diagrams are allowed by the classical dynamics: this was roughly the original vision? The huge vacuum degeneracy of Kähler action implying non-determinism does not encourage this option. The most attractive and most predictive realization conforming with the idea about generalized Feynman diagrammatics as arithmetics would be that all the diagrams differing by these moves give the same result. An analogous symmetry has been discovered for twistor diagrams.

3. Suppose one takes seriously the snipping of a tadpole away from diagram as a move, which does not affect the scattering amplitude. Could this move correspond to an allowed elementary cobordism of preferred extremal? If so, scattering amplitudes would have purely topological meaning as representations of the elements of cobordism classes! TGD would indeed be what it was proposed to be but in much deeper sense than I thought originally. This could also conform with the interpretation of classical charges as topological invariants, realize adelic physics at the level of WCW, and conform with the idea about TGD as almost topological QFT and perhaps generalizing it to topological QFT in generalized sense.

4. One can imagine several interpretations for the snipping operation at space-time level. TGD allows a huge classical vacuum degeneracy: all space-time surfaces having Lagrangian manifold of CP_2 as their CP_2 projection are vacuum extremals of Kähler action. Also all CP_2 extremals having 1-D light-like curve as M^4 projection are vacuum extremals but have non-vanishing Kähler action. This would not matter if one does not have superpositions since multiplicative factors are eliminated in scattering amplitudes. Could the tadpoles correspond to CP_2 type vacuum extremals at space-time level?

   There is also an alternative interpretation. In ZEO causal diamonds (CD) form a hierarchy and one can imagine that the sub-CDs of given CD correspond to quantum fluctuations. Could tadpoles be assigned to sub-CDs of CD be considered+

5. In this manner one could perhaps define elements of homotopy groupoid Π_2 as foliations preferred extremals with same ends - these would be 5-D surfaces. If one has two such 5-D foliations with the same 4-D ends, one can form the reverse of the other and form a closed surface. This would be analogous to a map of S^2 to WCW. If the two 5-D foliations cannot be transformed to each other, one would have something, which might be regarded as a non-trivial element of dynamical homotopy group Π_2.

One can ask whether one could define also the analogs of higher homology or homotopy grouppoids and groupoids Π_3 up to Π_5 - the upper bound $n = 5 = 8 - 3$ comes from the fact that foliations of foliations.. can have maximum dimension $D = 8$ and from the dimension of $D = 3$ of basic objects.

1. One could form a foliation of the foliations of preferred extremals as the element of the homotopy groupoid Π_3. Could allowed moves reduce to the snipping operation for generalized Feynman diagrams but performed along direction characterized by a new foliation parameter.

2. The topology of the zero mode sector of WCW parameterized by fixed values of conserved Noether charges as element of Π_0 could be characterized by dynamical homotopy groups Π_n, $n = 1, ..., 5$ - at least partially. These degrees of freedom could correspond to quantum fluctuating degrees of freedom. The Kähler structure of WCW and finite-D analogy suggests that all odd dynamical homotopy groups vanish so that Π_0, Π_2 and Π_4 would be the only non-trivial dynamical homotopy groups. The vanishing of Π_1 would imply that there is only single minimal generalized Feynman diagram contributing to the scattering amplitude. This also true if Feynman diagrams correspond to arithmetic operations.

3. Whether one should call these groups homotopy groups or homology groups is not obvious. The construction means that the foliations of foliations of ... can be seen as images of spheres suggesting "homotopy". The number of these groups is determined by the dimension of imbedding space, which suggests "homology".

4. Clearly, the surfaces defining the dynamical homotopy groups/groupoids would be analogs of branes of M-theory but would be obtained constructing paths of paths of paths... by starting from preferred extremals. The construction of so called n-groups (`http://tinyurl.com/yckcjcln`) brings strongly in mind this construction.

5.7.6 Appendix: About Field Equations Of TGD In Jet Bundle Formulation

Prastaro utilizes jet bundle (`http://tinyurl.com/yb2575bm`) formulation of partial differential equations (PDEs). This notion allows a very terse formulation of general PDEs as compared to the old-fashioned but much more concrete formulation that I have used. The formulation is rather formula rich and reader might lose easily his/her patience since one must do hard work to learn which formulas follow trivially from the basic definitions.

I will describe this formulation in TGD framework briefly but without explicit field equations, which can be found at [K7]. To my view a representation by using a concrete example is always more reader friendly than the general formulas derived in some reference. I explain my view about the general ideas behind jet bundle formulation with minimal number amount of formulas. The reader can find explicit formulas from the Wikipedia link above.

The basic goal is to have a geometric description of PDE. In TGD framework the geometric picture is of course present from beginning: field patterns as 4-surfaces in field space - somewhat formal geometric objects - are replaced with genuine 4-surfaces in $M^4 \times CP_2$.

Field equations as conservation laws, Frobenius integrability conditions, and a connection with quaternion analyticity

The following represents qualitative picture of field equations of TGD trying to emphasize the physical aspects. Also the possibility that Frobenius integrability conditions are satisfied and correspond to quaternion analyticity is discussed.

1. Kähler action is Maxwell action for induced Kähler form and metric expressible in terms of imbedding space coordinates and their gradients. Field equations reduce to those for imbedding space coordinates defining the primary dynamical variables. By GCI only four of them are independent dynamical variables analogous to classical fields.

2. The solution of field equations can be interpreted as a section in fiber bundle. In TGD the fiber bundle is just the Cartesian product $X^4 \times CD \times CP_2$ of space-time surface X^4 and causal diamond $CD \times CP_2$. CD is the intersection of future and past directed light-cones having two light-like boundaries, which are cone-like pieces of light-boundary $\delta M^4_\pm \times CP_2$. Space-time surface serves as base space and $CD \times CP_2$ as fiber. Bundle projection Π is the projection to the factor X^4. Section corresponds to the map $x \to h^k(x)$ giving imbedding space coordinates as functions of space-time coordinates. Bundle structure is now trivial and rather formal.

   By GCI one could also take suitably chosen 4 coordinates of $CD \times CP_2$ as space-time coordinates, and identify $CD \times CP_2$ as the fiber bundle. The choice of the base space depends on the character of space-time surface. For instance CD, CP_2 or $M^2 \times S^2$ (S^2 a geodesic sphere of CP_2), could define the base space. The bundle projection would be projection from $CD \times CP_2$ to the base space. Now the fiber bundle structure can be non-trivial and make sense only in some space-time region with same base space.

3. The field equations derived from Kähler action must be satisfied. Even more: one must have a *preferred* extremal of Kähler action. One poses boundary conditions at the 3-D ends of space-time surfaces and at the light-like boundaries of $CD \times CP_2$.

   One can fix the values of conserved Noether charges at the ends of CD (total charges are same) and require that the Noether charges associated with a sub-algebra of super-symplectic algebra isomorphic to it and having conformal weights coming as n-ples of those for the entire algebra, vanish. This would realize the effective 2-dimensionality required by SH. One must pose boundary conditions also at the light-like partonic orbits. So called weak form of electric-magnetic duality is at least part of these boundary conditions.

It seems that one must restrict the conformal weights of the entire algebra to be non-negative $r \geq 0$ and those of subalgebra to be positive: $mn > 0$. The condition that also the commutators of sub-algebra generators with those of the entire algebra give rise to vanishing Noether charges implies that all algebra generators with conformal weight $m \geq n$ vanish so the dynamical algebra becomes effectively finite-dimensional. This condition generalizes to the action of super-symplectic algebra generators to physical states.

M^4 time coordinate cannot have vanishing time derivative dm^0/dt so that four-momentum is non-vanishing for non-vacuum extremals. For CP_2 coordinates time derivatives ds^k/dt can vanish and for space-like Minkowski coordinates dm^i/dt can be assumed to be non-vanishing if M^4 projection is 4-dimensional. For CP_2 coordinates $ds^k/dt = 0$ implies the vanishing of electric parts of induced gauge fields. The non-vacuum extremals with the largest conformal gauge symmetry (very small n) would correspond to cosmic string solutions for which induced gauge fields have only magnetic parts. As n increases, also electric parts are generated. Situation becomes increasingly dynamical as conformal gauge symmetry is reduced and dynamical conformal symmetry increases.

4. The field equations involve besides imbedding space coordinates h^k also their partial derivatives up to second order. Induced Kähler form and metric involve first partial derivatives $\partial_\alpha h^k$ and second fundamental form appearing in field equations involves second order partial derivatives $\partial_\alpha \partial_\beta h^k$.

   Field equations are hydrodynamical, in other worlds represent conservation laws for the Noether currents associated with the isometries of $M^4 \times CP_2$. By GCI there are only 4 independent dynamical variables so that the conservation of $m \leq 4$ isometry currents is enough if chosen to be independent. The dimension m of the tangent space spanned by the conserved currents can be smaller than 4. For vacuum extremals one has $m = 0$ and for massless extremals (MEs) $m = 1$! The conservation of these currents can be also interpreted as an existence of $m \leq 4$ closed 3-forms defined by the duals of these currents.

5. The hydrodynamical picture suggests that in some situations it might be possible to assign to the conserved currents flow lines of currents even globally. They would define $m \leq 4$ global coordinates for some subset of conserved currents (4+8 for four-momentum and color quantum numbers). Without additional conditions the individual flow lines are well-defined but do not organize to a coherent hydrodynamic flow but are more like orbits of randomly moving gas particles. To achieve global flow the flow lines must satisfy the condition $d\phi^A/dx^\mu = k^A_B J^B_\mu$ or $d\phi^A = k^A_B J^B$ so that one can special of 3-D family of flow lines parallel to $k^A_B J^B$ at each point - I have considered this kind of possibly in [K7] at detail but the treatment is not so general as in the recent case.

   Frobenius integrability conditions (`http://tinyurl.com/yc6apam2`) follow from the condition $d^2\phi^A = 0 = dk^A_B \wedge J^B + k^A_B dJ^B = 0$ and implies that dJ^B is in the ideal of exterior algebra generated by the J^A appearing in $k^A_B J^B$. If Frobenius conditions are satisfied, the field equations can define coordinates for which the coordinate lines are along the basis elements for a sub-space of at most 4-D space defined by conserved currents. Of course, the possibility that for preferred extremals there exists $m \leq 4$ conserved currents satisfying integrability conditions is only a conjecture.

   It is quite possible to have $m < 4$. For instance for vacuum extremals the currents vanish identically For MEs various currents are parallel and light-like so that only single light-like coordinate can be defined globally as flow lines. For cosmic strings (cartesian products of minimal surfaces X^2 in M^4 and geodesic spheres S^2 in CP_2 4 independent currents exist). This is expected to be true also for the deformations of cosmic strings defining magnetic flux tubes.

6. Cauchy-Riemann conditions in 2-D situation represent a special case of Frobenius conditions. Now the gradients of real and imaginary parts of complex function $w = w(z) = u + iv$ define two conserved currents by Laplace equations. In TGD isometry currents would be gradients apart from scalar function multipliers and one would have generalization of C-R conditions. In citeallbprefextremals,twistorstory I have considered the possibility that the generalization

of Cauchy-Riemann-Fuerter conditions [A94, A72] (`http://tinyurl.com/yb8l34b5`) could define quaternion analyticity - having many non-equivalent variants - as a defining property of preferred extremals. The integrability conditions for the isometry currents would be the natural physical formulation of CRF conditions. Different variants of CRF conditions would correspond to varying number of independent conserved isometry currents.

7. The problem caused by GCI is that there is infinite number of coordinate choices. How to pick a physically preferred coordinate system? One possible manner to do this is to use coordinates for the projection of space-time surface to some preferred sub-space of imbedding - geodesic manifold is an excellent choice. Only $M^1 \times X^3$ geodesic manifolds are not possible but these correspond to vacuum extremals.

   One could also consider a philosophical principle behind integrability. The variational principle itself could give rise to at least some preferred space-time coordinates in the same manner as TGD based quantum physics would realize finite measurement resolution in terms of inclusions of HFFs in terms of hierarchy of quantum criticalities and fermionic strings connecting partonic 2-surfaces. Frobenius integrability of the isometry currents would define some preferred coordinates. Their number need not be the maximal four however.

   For instance, for massless extremals only light-like coordinate corresponding to the light-like momentum is obtained. To this one can however assign another local light-like coordinate uniquely to obtain integrable distribution of planes M^2. The solution ansatz however defines directly an integrable choice of two pairs of coordinates at imbedding space level usable also as space-time coordinates - light-like local direction defining local plane M^2 and polarization direction defining a local plane E^2. These choices define integrable distributions of orthogonal planes and local hypercomplex and complex coordinates. Pair of analogs of C-R equations is the outcome. I have called these coordinates Hamilton-Jacobi coordinates for M^4.

8. This picture allows to consider a generalization of the notion of solution of field equation to that of integral manifold (`http://tinyurl.com/yajn7cuz`. If the number of independent isometry currents is smaller than 4 (possibly locally) and the integrability conditions hold true, lower-dimensional sub-manifolds of space-time surface define integral manifolds as kind of lower-dimensional effective solutions. Genuinely lower-dimensional solutions would of course have vanishing $\sqrt{g_4}$ and vanishing Kähler action.

   String world sheets can be regarded as 2-D integral surfaces. Charged (possibly all) weak boson gauge fields vanish at them since otherwise the electromagnetic charge for spinors would not be well-defined. These conditions force string world sheets to be 2-D in the generic case. In special case 4-D space-time region as a whole can satisfy these conditions. Well-definedness of Kähler-Dirac equation [K88, K110] demands that the isometry currents of Kähler action flow along these string world sheets so that one has integral manifold. The integrability conditions would allow $2 < m \leq n$ integrable flows outside the string world sheets, and at string world sheets one or two isometry currents would vanish so that the flows would give rise 2-D independent sub-flow.

9. The method of characteristics (`http://tinyurl.com/y9dcdayt`) is used to solve hyperbolic partial differential equations by reducing them to ordinary differential equations. The (say 4-D) surface representing the solution in the field space has a foliation using 1-D characteristics. The method is especially simple for linear equations but can work also in the non-linear case. For instance, the expansion of wave front can be described in terms of characteristics representing light rays. It can happen that two characteristics intersect and a singularity results. This gives rise to physical phenomena like caustics and shock waves.

   In TGD framework the flow lines for a given isometry current in the case of an integrable flow would be analogous to characteristics, and one could also have purely geometric counterparts of shockwaves and caustics. The light-like orbits of partonic 2-surface at which the signature of the induced metric changes from Minkowskian to Euclidian might be seen as an example about the analog of wave front in induced geometry. These surfaces serve as carriers of fermion lines in generalized Feynman diagrams. Could one see the particle vertices at which the 4-D space-time surfaces intersect along their ends as analogs of intersections of characteristics -

kind of caustics? At these 3-surfaces the isometry currents should be continuous although the space-time surface has "edge".

10. The analogy with ordinary analyticity suggests that it might be possible to interpret string world sheets and partonic 2-surfaces appearing in strong form of holography (SH) as co-dimension 2 surfaces analogous to poles of analytic function in complex plane. Light-like 3-surfaces might be seen as analogs of cuts. The coding of analytic function by its singularities could be seen as analog of SH.

Jet bundle formalism

Jet bundle formalism (`http://tinyurl.com/yb2575bm`) is a modern manner to formulate PDEs in a coordinate independent manner emphasizing the local algebraic character of field equations. In TGD framework GCI of course guarantees this automatically. Beside this integrability conditions formulated in terms of Cartan's contact forms are needed.

1. The basic idea is to take the partial derivatives of imbedding space coordinates as functions of space-time coordinates as independent variables. This increases the number of independent variables. Their number depends on the degree of the jet defined and for partial differential equation of order r, for n dependent variables, and for N independent variables the number of new degrees of freedom is determined by r, n, and N -just by counting the total number of various partial derivatives from $k = 0$ to r. For $r = 1$ (first order PDE) it is $N \times (1 + n)$.

2. Jet at given space-time point is defined as a Taylor polynomial of the imbedding space coordinates as functions of space-time coordinates and is characterized by the partial derivatives at various points treated as independent coordinates analogous to imbedding space coordinate. Jet degree r is characterized by the degree of the Taylor polynomial. One can sum and multiply jets just like Taylor polynomials. Jet bundle assigns to the fiber bundle associated with the solutions of PDE corresponding jet bundle with fiber at each point consisting of jets for the independent variables ($CD \times CP_2$ coordinates) as functions of the dependent variables (space-time coordinates).

3. The field equations from the variation of Kähler action are second order partial differential equations and in terms of jet coefficients they reduce to local algebraic equations plus integrability conditions. Since TGD is very non-linear one obtains polynomial equations at each point - one for each imbedding space coordinate. Their number reduces to four by GCI. The minimum degree of jet bundle is $r = 2$ if one wants algebraic equations since field equations are second order PDEs.

4. The local algebraic conditions are not enough. One must have also conditions stating that the new independent variables associated with partial derivatives of various order reduces to appropriate multiple partial derivatives of imbedding space coordinates. These conditions can be formulated in terms of Cartan's contact forms, whose vanishing states these conditions. For instance, if dh^k is replaced by independent variable u^k, the condition $dh^k - u^k = 0$ is true for the solution surfaces.

5. In TGD framework there are good motivations to break the non-orthodoxy and use 1-jets so that algebraic equations replaced by first order PDEs plus conditions requiring vanishing of contact forms. These equations state the conservation of isometry currents implying that the 3-forms defined by the duals of isometry currents are closed. As found, this formulation reveals in TGD framework the hydrodynamic picture and suggests conditions making the system integrable in Frobenius sense.

Chapter 6

Can one apply Occam's razor as a general purpose debunking argument to TGD?

Occarm's razor have been used to debunk TGD. The following arguments provide the information needed by the reader to decide himself. Considerations are at three levels.

The level of "world of classical worlds" (WCW) defined by the space of 3-surfaces endowed with Kähler structure and spinor structure and with the identification of WCW space spinor fields as quantum states of the Universe: this is nothing but Einstein's geometrization program applied to quantum theory. Second level is space-time level.

Space-time surfaces correspond to preferred extremals of Käction in $M^4 \times CP_2$. The number of field like variables is 4 corresponding to 4 dynamically independent imbedding space coordinates. Classical gauge fields and gravitational field emerge from the dynamics of 4-surfaces. Strong form of holography reduces this dynamics to the data given at string world sheets and partonic 2-surfaces and preferred extremals are minimal surface extremals of Kähler action so that the classical dynamics in space-time interior does not depend on coupling constants at all which are visible via boundary conditions only. Continuous coupling constant evolution is replaced with a sequence of phase transitions between phases labelled by critical values of coupling constants: loop corrections vanish in given phase. Induced spinor fields are localized at string world sheets to guarantee well-definedness of em charge.

At imbedding space level the modes of imbedding space spinor fields define ground states of super-symplectic representations and appear in QFT-GRT limit. GRT involves post-Newtonian approximation involving the notion of gravitational force. In TGD framework the Newtonian force correspond to a genuine force at imbedding space level.

I was also asked for a summary about what TGD is and what it predicts. I decided to add this summary to this chapter although it is goes slightly outside of its title.

6.1 Introduction

Occam's razor argument is one the standard general purpose arguments used in debunking: the debunked theory is claimed to be hopelessly complicated. This argument is more refined that mere "You are a crackpot!" but is highly subjective and often the arguments pro or con are not given. Combined with the claim that the theory does not predict anything Occam's razor is very powerful argument unless the audience includes people who have bothered to study the debunked theory.

Let us take a closer look on this argument and compare TGD superstring models and seriously ask which of these theories is simple.

In superstring models one has strings as basic dynamical objects. They live in target space M^{10}, which in some mysterious manner (something "non-perturbative" it is) spontaneously compactifies to $M^4 \times C$, C is Calabi-Yau space. The number of them is something like 10^{500} or probably infinite: depends on the counting criterion. And this estimate leaves their metric open.

This leads to landscape and multiverse catastrophe: theory cannot predict anything. As a matter fact $M^4 \times C$:s must be allowed to deform still in Kaluza-Klein paradigm in which space-time has Calabi-Yau as small additional dimensions. An alternative manner to obtain space-time is as 3-brane. One obtains also higher-D objects. Again by some "nonperturbative" mechanisms. One does not even know what space-time is! Situation looks to me a totally hopeless mess. Reader can conclude whether to regard this as simple and elegant.

I will consider TGD at three levels. At the level of "world of classical worlds" (WCW), at space-time level, and at the level of imbedding space $H = M^4 \times CP_2$. I hope that I can convince the reader about the simplicity of the approach. The simplicity is actually quite shocking and certainly an embarrassing experience for the unhappy super string theorists meandering around in the landscape and multiverse. Behind this simplicity are however principles - something, which colleagues usually regard as unpractical philosophizing: "shut-up-and-calculate!"!

I was also asked for a summary about what TGD is and what it predicts. I decided to add this summary to this chapter although it is goes slightly outside of its title.

6.2 Simplicity at various levels

6.2.1 WCW level: a generalization of Einstein's geometrization program to entire quantum physics

I hope that the reader would read the following arguments keeping in mind the question "Is TGD really hopelessly complicated mess of pieces picked up randomly from theoretical physics?" as one debunker who told that he does not have time to read TGD formulated it.

1. Einstein's geometrization program for gravitation has been extremely successful but has failed for other classical fields, which do not have natural geometrization in the case of abstract four-manifolds with metric. One should understand standard model quantum numbers and also family replication for fermions.

   However, if space-time can be regarded surface in $H = M^4 \times CP_2$ also the classical fields find a natural geometrization as induced fields obtained basically by projecting. Also spinor structure can be induced and one avoids the problems due the fact that generic space-time as abstract 4-manifold does not allow spinor structure. The dynamics of space-time surfaces incredibly simple: only 4 field-like variables corresponding to *four* imbedding space coordinates and induced that of classical geometric fields. Nowadays one would speak of emergence. The complexity emerges from the topology of space-time surfaces giving rise to many-sheeted space-time.

2. Even this view about geometrization is generalized in TGD. Einstein's geometrization program is applied to the entire quantum physics in terms of the geometry of WCW consisting of 3-D surfaces of H. More precisely, in zero energy ontology (ZEO) it consists of pairs of 3-surfaces at opposite boundaries of causal diamond (CD) connected by a preferred extremals of a variational principle to be discussed.

   Quantum states of the Universe would correspond to the modes of formally classical WCW spinor field satisfying the analog of Dirac equation. No quantization: just the construction of WCW geometry and spinor structure. The only genuinely quantal element of quantum theory would be state function reduction and in ZEO its description leads to a quantum theory of consciousness.

To me this sounds not only simple but shockingly simple.

WCW geometry

Consider first the generalization of Einstein's program of at the level of WCW geometry [K110, K34, K15].

1. Since complex conjugation must be geometrized, WCW must allow a geometric representation of imaginary unit as an antisymmetric tensor, which is essentially square root of the negative

of the metric tensor and thus allow Kähler structure coded by Kähler function. One must have 4-D general coordinate invariance (GCI) but basic objects are 3-D surfaces. Therefore the definition of Kähler function must assign to 3-surface a unique 4-surface.

Kähler function should have physical meaning and the natural assumption is that it is Kähler action plus possibly also volume term (twistor lift implies it). Space-time surface would be a preferred extremal of this action. The interpretation is also as an analog of Bohr orbit so that Bohr orbitology would correspond exact rather than only approximate part of quantum theory in TGD framework. One could speak also of quantum classical correspondence.

2. The action principle involves coupling parameters analogous to thermodynamical parameters. Their value spectrum is fixed by the conditions that TGD is quantum critical. For instance Kähler couplings strength is analogous to critical temperature. Different values correspond to different phases. Coupling constant evolution correspond to phase transitions between these phases and loops vanish as in free field theory for $\mathcal{N} = 4$ SYM.

3. The infinite-dimensionality of WCW is a crucial element of simplicity. Already in the case of loop spaces the geometry is essentially unique: loop space is analogous to a symmetric space points of the loop space being geometrically equivalent. For loop spaces Riemann connection exists only of the metric has maximal isometries defined by Kac-Moody algebra.

   The generalization to 3-D case is compelling. In TGD Kac-Moody algebra is replaced by super-symplectic algebra, which is much larger but has same basic structure (conformal weights of two kinds) and a fractal hierarchy of isomorphic sub-algebas with conformal weights coming as multiples of those for the entire algebra is crucial. Physics is unique because of its mathematical existence. WCW decompose to a union of sectors, which are infinite-D variants of symmetric spaces labelled by zero modes whose differentials do not appear in the line element of WCW.

All this sounds to me shockingly simple.

WCW spinor structure

One must construct also spinor structure for WCW [K88, K110].

1. The modes of WCW spinor fields would correspond to the solutions of WCW Dirac equation and would define the quantum states of the Universe. WCW spinors (assignable to given 3-surface) would correspond to fermionic Fock states created by fermionic creation operators. In ZEO 3-surfaces are pairs of 3-surfaces assignable to the opposite boundaries of WCW connected by preferred extremal.

   The fermionic states are superpositions of pairs of fermion states with opposite net quantum numbers at the opposite ends of space-time surface at boundaries of CD. The entanglement coefficients define the analogs of S-matrix elements. The analog of Dirac equation is analog for super-Virasoro conditions in string models but assignable to the infinite-D supersymplectic algebra of WCW defining its isometries.

2. The construction of the geometry of WCW requires that the anticommuting gamma matrices of WCW are expressible in terms of fermionic oscillator operators assignable to the induced spinor fields at space-time surface. Fermionic anti-commutativity at space-time level is not assumed but is forced by the anticommutativity of gamma matrices to metric. Fermi statistics is geometrized.

3. The gamma matrices of WCW in the coordinates assignable to isometry generators can be regarded as generators of superconformal symmetries. They correspond to classical charges assignable to the preferred extremals and to fermionic generators. The fermionic isometry generators are fermionic bilinears and super-generators are obtained from them by replacing the second second quantized spinor field with its mode. Quantum classical correspondence between fermionic dynamics and classical dynamics (SH) requires that the eigenvalues of the fermionic Cartan charges are equal to corresponding bosonic Noether charges.

4. The outcome is that quantum TGD reduces to a theory of formally *classical* spinor fields at the level of WCW and by infinite symmetries the construction of quantum states reduces to the construction of representations of super-symplectic algebra which generalizes to Yangian algebra as twistorial picture suggests. In ZEO everything would reduce to group theory, even the construction of scattering amplitudes! In ZEO the construction of zero energy states and thus scattering amplitudes would reduce to that for the representations of Yangian variant of super-symplectic algebra [A27] [B39, B30, B31].

5. One can go to the extreme and wonder whether the scattering amplitudes as entanglement coefficients for Yangian zero energy states are just constant scalars for given values of zero modes as group invariant for isometries. This would leave only integration over zero modes and if number theoretical universality is assumed this integral reduces to sum over points with algebraic coordinates in the preferred coordinates made possible by the symmetric space property. Certainly this is one of the lines of research to be followed in future.

   Personally I find it hard to imagine anything simpler!

6.2.2 Space-time level: many-sheeted space-time and emergence of classical fields and GRT space-time

At space-time level one must consider dynamics of space-time surface and spinorial dynamics.

Dynamics of space-time surfaces

Consider first simplicity at space-time level.

1. Space-time is identified as 4-D surface in certain imbedding space required to have symmetries of special relativity - Poincare invariance. This resolves the energy problem and many other problems of GRT [K105].

   This allows also to see TGD as generalization of string models obtained by replacing strings with 3-surfaces and 2-D string world sheets with 4-D space-time surfaces. Small space-time surfaces are particles, large space-time surfaces the background space-time in which these particles "live". There are only 4 dynamical field like variables for 8-D $M^4 \times CP_2$ since GCI eliminates 4 imbedding space coordinates (they can be taken as space-tme coordinates). This should be compared with the myriads of classical fields for 10-D Einstein's theory coupled to matter fields (do not forget landscape and multiverse!)

2. Classical fields are induced at the level of single space-time sheet from their geometric counterparts in imbedding space. A more fashionable way to say the same is that they emerge. Classical gravitational field correspond to the induced metric, electroweak gauge potentials to induced spinor connection of CP_2 and color gauge potentials to projections of Killing vector fields for CP_2.

3. In TGD the space-time of GRT is replaced by many-sheeted space-time constructed from basic building bricks, which are preferred extremals of Kähler action + volume term. This action emerges in twistor lift of TGD existing only for $H = M^4 \times CP_2$: TGD is completely unique since only M^4 and CP_2 allows twistor space with Kähler structure. This also predicts Planck length as radius of twistor sphere associated with M^4. Cosmological constant appears as the coefficient of the volume term and obeys p-adic length scale evolution predicting automatically correct order of magnitude in the scale of recent cosmos. Besides this one has CP_2 size which is of same order of magnitude as GUT scale, and Kähler coupling strength. By quantum criticality the various parameters are quantized.

   Quantum criticality is basic dynamical principle [K34, L22] and discretizes coupling constant evolution: only coupling constants corresponding to quantum criticality are realized and discretized coupling constant evolution corresponds to phase transitions between these values of coupling constants. All radiative corrections vanish so that only tree diagram contribute.

4. Preferred extremals realize strong form of holography (SH) implied by strong form of GCI (SGCI) emerging naturally in TGD framework. That GCI implies SH meaning an enormous simplification at the conceptual level.

   One has two choices for fundamental 3-D objects. They could be light-like boundaries between regions of Minkowskian and Euclidian signatures of the induced metric or they could be pairs of space-time 3-surfaces at the ends of space-time surface at opposite boundaries of causal diamond (CD) (CDs for a scale hierarchy). Both options should be correct so that the intersections of these 3-surfaces consisting of partonic 2-surfaces at which light-like partonic orbits and space-like 3-surfaces intersect should carry the data making possible holography. Also data about normal space of partonic 2-surface is involved.

   SH generalizes AdS/CFT correspondence by replacing holography with what is very much like the familiar holography. String world, sheets, which are minimal surfaces carrying fermion fields and partonic 2-surfaces intersecting string world sheets at discrete points determine by SH the entire 4-D dynamics. The boundaries of string world sheets are world lines with fermion number coupling to classical Kähler force. In the interior Kähler force vanishes so that one has "dynamics of avoidance" [L19] required also by number theoretic universality satisfied if the coupling constants do not appear in the field equations at all: they are however seen in the boundary values stating vanishing of the classical super-symplectic charges (Noether's theorem) so that one obtains dependence of coupling constants via boundary conditions and coupling constant evolutions makes it manifest also classically. Hence the preferred extremals from which the space-time surfaces are engineered are extremely simple objects.

5. In twistor formulation the assumption that the inverse of Kähler coupling strength has zeros of Riemann zeta [L16] as the spectrum of its quantum critical values gives excellent prediction for the coupling constant of U(1) coupling constant of electroweak interactions. Complexity means that extremals are extremals of both Kähler action and volume term: minimal surfaces extremals of Kähler action. This would be part of preferred extremal property.

   Why α_K should be complex? If α_K is real, both bosonic and fermionic degrees of freedom for Euclidian and Minkowskian regions decouple completely. This is not physically attractive. If α_K is complex there is coupling between the two regions and the simplest assumption is that there is no Chern-Simons term in the action and one has just continuity conditions for canonical momentum current and hits super counterpart. Note the analogy with the possibility of blackhole evaporation. The presence of momentum exchange is also natural since it gives classical space-time correlates for interactions as momentum exchange.

   The conditions state that sub-algebra of super-symplectic algebra isomorphic to itself and its commutator with the entire algebra annihilate the physical states (classical Noether charges vanish). The condition could follow from minimal surface extremality or provide additional conditions reducing the degrees of freedom. In any case, 3-surfaces would be almost 2-D objects.

6. GRT space-time emerges from many-sheeted space-time as one replaces the sheets of many-sheeted space-time (4-D M^4 projection) to single slightly curved region of M^4 defining GRT space-time. Since test particle regarded as 3-surface touching the space-time sheets of many-sheeted spacetime, test particle experiences the sum of forces associated with the classical fields at the space-time sheets. Hence the classical fields of GRT space-time are sums of these fields. Disjoint union for space-time sheets maps to the sum of the induced fields. This gives standard model and GRT as long range scale limit of TGD.

How to build TGD space-time from legos?

TGD predicts shocking simplicity of both quantal and classical dynamics at space-time level. Could one imagine a construction of more complex geometric objects from basic building bricks - space-time legos?

Let us list the basic ideas.

1. Physical objects correspond to space-time surfaces of finite size - we see directly the non-trivial topology of space-time in everyday length scales.

2. There is also a fractal scale hierarchy: 3-surfaces are topologically summed to larger surfaces by connecting them with wormhole contact, which can be also carry monopole magnetic flux in which one obtains particles as pairs of these: these contacts are stable and are ideal for nailing together pieces of the structure stably.

3. In long length scales in which space-time surface tend to have 4-D M^4 projection this gives rise to what I have called many-sheeted spacetime. Sheets are deformations of canonically imbedded M^4 extremely near to each other (the maximal distance is determined by CP_2 size scale about 10^4 Planck lengths. The sheets touch each other at topological sum contacts, which can be also identified as building bricks of elementary particles if they carry monopole flux and are thus stable. In $D = 2$ it is easy to visualize this hierarchy.

What could be the simplest surfaces of this kind - the legos?

1. Assume twistor lift [L22, L24] so that action contain volume term besides Kähler action: preferred extremals can be seen as non-linear massless fields coupling to self-gravitation. They also simultaneously extremals of Kähler action. Also hydrodynamical interpretation makes sense in the sense that field equations are conservation laws. What is remarkable is that the solutions have no dependence on coupling parameters: this is crucial for realizing number theoretical universality. Boundary conditions however bring in the dependence on the values of coupling parameters having discrete spectrum by quantum criticality.

2. The simplest solutions corresponds to Lagrangian sub-manifolds of CP_2: induced Kähler form vanishes identically and one has just minimal surfaces. The energy density defined by scale dependent cosmological constant is small in cosmological scales - so that only a template of physical system is in question. In shorter scales the situation changes if the cosmological constant is proportional the inverse of p-adic prime.

   The simplest minimal surfaces are constructed from pieces of geodesic manifolds for which not only the trace of second fundamental form but the form itself vanishes. Geodesic sub-manifolds correspond to points, pieces of lines, planes, and 3-D volumes in E^3. In CP_2 one has points, circles, geodesic spheres, and CP_2 itself.

3. CP_2 type extremals defining a model for wormhole contacts, which can be used to glue basic building bricks at different scales together stably: stability follows from magnetic monopole flux going through the throat so that it cannot be split like homologically trivial contact. Elementary particles are identified as pairs of wormhole contacts and would allow to nail the legos together to from stable structures.

Amazingly, what emerges is the elementary geometry. My apologies for those who hated school geometry.

1. Geodesic minimal surfaces with vanishing induced gauge fields

Consider first *static* objects with 1-D CP_2 projection having thus *vanishing* induced gauge fields. These objects are of form $M^1 \times X^3$, $X^3 \subset E^3 \times CP_2$. M^1 corresponds to time-like or possible light-like geodesic (for CP_2 type extremals). I will consider mostly Minkowskian space-time regions in the following.

1. Quite generally, the simplest legos consist of 3-D geodesic sub-manifolds of $E^3 \times CP_2$. For E^3 their dimensions are $D = 1, 2, 3$ and for CP_2, $D = 0, 1, 2$. CP_2 allows both homologically non-trivial resp. trivial geodesic sphere S^2_I resp. S^2_{II}. The geodesic sub-manifolds cen be products $G_3 = G_{D_1} \times G_{D_2}$, $D_2 = 3 - D_1$ of geodesic manifolds G_{D_1}, $D_1 = 1, 2, 3$ for E^3 and G_{D_2}, $D_2 = 0, 1, 2$ for CP_2.

2. It is also possible to have twisted geodesic sub-manifolds G_3 having geodesic circle S^1 as CP_2 projection corresponding to the geodesic lines of $S^1 \subset CP_2$, whose projections to E^3 and CP_2 are geodesic line and geodesic circle respectively. The geodesic is characterized by S^1 wave vector. One can have this kind of geodesic lines even in $M^1 \times E^3 \times S^1$ so that the solution is characterized also by frequency and is not static in CP_2 degrees of freedom anymore.

These parameters define a four-D wave vector characterizing the warping of the space-time surface: the space-time surface remains flat but is warped. This effect distinguishes TGD from GRT. For instance, warping in time direction reduces the effective light-velocity in the sense that the time used to travel from A to B increases. One cannot exclude the possibility that the observed freezing of light in condensed matter could have this warping as space-time correlate in TGD framework.

For instance, one can start from 3-D minimal surfaces $X^2 \times D$ as local structures (thin layer in E^3). One can perform twisting by replacing D with twisted closed geodesics in $D \times S^1$: this gives valued map from D to S^1 (subset CP_2) representing geodesic line of $D \times S^1$. This geodesic sub-manifold is trivially a minimal surface and defines a two-sheeted cover of $X^2 \times D$. Wormhole contact pairs (elementary particles) between the sheets can be used to stabilize this structure.

3. Structures of form $D^2 \times S^1$, where D^2 is polygon, are perhaps the simplest building bricks for more complex structures. There are continuity conditions at vertices and edges at which polygons D_i^2 meet and one could think of assigning magnetic flux tubes with edes in the spirit of homology: edges as magnetic flux tubes, faces as 2-D geodesic sub-manifolds and interiors as 3-D geodesic sub-manifolds.

   Platonic solids as 2-D surfaces can be build are one example of this and are abundant in biology and molecular physics. An attractive idea is that molecular physics utilizes this kind of simple basic structures. Various lattices appearing in condensed matter physics represent more complex structures but could also have geodesic minimal 3-surfaces as building bricks. In cosmology the honeycomb structures having large voids as basic building bricks could serve as cosmic legos.

4. This lego construction very probably generalizes to cosmology, where Euclidian 3-space is replaced with 3-D hyperbolic space $SO(3,1)/SO(3)$. Also now one has pieces of lines, planes and 3-D volumes associated with an arbitrarily chosen point of hyperbolic space. Hyperbolic space allows infinite number of tesselations serving as analogs of 3-D lattices and the characteristic feature is quantization of redshift along line of sight for which empirical evidence is found.

5. The structures as such are still too simple to represent condensed matter systems. These basic building bricks can glued together by wormhole contact pairs defining elementary particles so that matter emerges as stabilizer of the geometry: they are the nails allowing to fix planks together, one might say.

2. Geodesic minimal surfaces with non-vanishing gauge fields

What about minimal surfaces and geodesic sub-manifolds carrying non-vanishing gauge fields - in particular em field (Kähler form identifiable as U(1) gauge field for weak hypercharge vanishes and thus also its contribution to em field)? Now one must use 2-D geodesic spheres of CP_2 combined with 1-D geodesic lines of E^2. Actually both homologically non-trivial resp. trivial geodesic spheres S_I^2 resp. S_{II}^2 can be used so that also non-vanishing Kähler forms are obtained.

The basic legos are now $D \times S_i^2$, $i = I, II$ and they can be combined with the basic legos constructed above. These legos correspond to two kinds of magnetic flux tubes in the ideal infinitely thin limit. There are good reasons to expected that these infinitely thin flux tubes can be thickened by deforming them in E^3 directions orthogonal to D. These structures could be used as basic building bricks assignable to the edges of the tensor networks in TGD.

3. Static minimal surfaces, which are not geodesic sub-manifolds

One can consider also more complex static basic building bricks by allowing bricks which are not anymore geodesic sub-manifolds. The simplest static minimal surfaces are form $M^1 \times X^2 \times S^1$, $S^1 \subset CP_2$ a geodesic line and X^2 minimal surface in E^3.

Could these structures represent higher level of self-organization emerging in living systems? Could the flexible network formed by living cells correspond to a structure involving more general minimal surfaces - also non-static ones - as basic building bricks? The Wikipedia article about minimal surfaces in E^3 suggests the role of minimal surface for instance in bio-chemistry (see http://tinyurl.com/zqlv322).

The surfaces with constant positive curvature do not allow imbedding as minimal surfaces in E^3. Corals provide an example of surface consisting of pieces of 2-D hyperbolic space H^2 immersed in E^3 (see http://tinyurl.com/ho9uvcc. Minimal surfaces have negative curvature as also H^2 but minimal surface immersions of H^2 do not exist. Note that pieces of H^2 have natural imbedding to E^3 realized as light-one proper time constant surface but this is not a solution to the problem.

Does this mean that the proposal fails?

1. One can build approximately spherical surfaces from pieces of planes. Platonic solids represents the basic example. This picture conforms with the notion of monadic manifold having as a spine a discrete set of points with coordinates in algebraic extension of rationals (preferred coordinates allowed by symmetries are in question). This seems to be the realistic option.

2. The boundaries of wormhole throats at which the signature of the induced metric changes can have arbitrarily large M^4 projection and they take the role of blackhole horizon. All physical systems have such horizon and the approximately boundaries assignable to physical objects could be horizons of this kind. In TGD one has minimal surface in $E^3 \times S^1$ rather than E^3. If 3-surface have no space-like boundaries they must be multi-sheeted and the sheets co-incide at some 2-D surface analogous to boundary. Could this 3-surface give rise to an approximately spherical boundary.

3. Could one lift the immersions of H^2 and S^2 to E^3 to minimal surfaces in $E^3 \times S^1$? The constancy of scalar curvature, which is for the immersions in question quadratic in the second fundamental form would pose one additional condition to non-linear Laplace equations expressing the minimal surface property. The analyticity of the minimal surface should make possible to check whether the hypothesis can make sense. Simple calculations lead to conditions, which very probably do not allow solution.

4. Dynamical minimal surfaces: how space-time manages to engineer itself?

At even higher level of self-organization emerge dynamical minimal surfaces. Here string world sheets as minimal surfaces represent basic example about a building block of type $X^2 \times S^2_i$. As a matter fact, S^2 can be replaced with complex sub-manifold of CP_2.

One can also ask about how to perform this building process. Also massless extremals (MEs) representing TGD view about topologically quantized classical radiation fields are minimal surfaces but now the induced Kähler form is non-vanishing. MEs can be also Lagrangian surfaces and seem to play fundamental role in morphogenesis and morphostasis as a generalization of Chladni mechanism [L27, L24]. One might say that they represent the tools to assign material and magnetic flux tube structures at the nodal surfaces of MEs. MEs are the tools of space-time engineering. Here many-sheetedness is essential for having the TGD counterparts of standing waves.

Spherically symmetry metric as minimal surface

Physical intuition and the experience with the vacuum extremals as models for GRT space-times suggests that Kähler charge is not important in the case of astrophysical objects like stars so that it might be possible to model them as minimal surfaces, which in the simplest situation have spherically symmetric metric analogous to Schwartschild solution. The vanishing of the induced Kähler form does not of course exclude the presence of electromagnetic fields. It must be of course emphasized that the assumption that single-sheeted space-time surface can model GRT-QFT limit based on many-sheeted space-time could be un-realistic.

At 90's I studied the imbeddings of Schwartschild-Nordström solution as vacuum extremals of Kähler action and found that the solution is necessarily electromagnetically charged [K79]. This property is unavoidable. The imbedding in coordinates (t, r, θ, ϕ) for X^4, (m^0, r, θ, ϕ) for M^4 and (Θ, Φ) for the trivial geodesic sphere S^2_{II} of CP_2 was not stationary as the first guess might be. m^0 relates to Schwartschild time and radial coordinate r by a shift $m^0 = \Lambda t + h(r)$. Without this shift the perihelion shift would be negligibly small.

One has $(cos(\Theta) = f(r), \Phi = \omega t + k(r))$. Also the dependence of Φ is not the first possibility to come in mind. The shifts $h(r)$ and $k(r)$ are such that the non-diagonal contribution g_{tr} to the induced metric vanishes. The question is whether one obtains spherically symmetric metric as a minimal surface.

5. General form of minimal surface equations

Consider first the minimal surface equations generally.

1. The field equations are analogous to massless wave equations for scalar fields defined by CP_2 coordinates having gravitational self coupling and also covariant derivative coupling due to the non-flatness of CP_2. One might therefore expect that the Newtonian gravitation based on Laplace equation in empty space-time regions follows as an approximation. Therefore also something analogous to Schwartschild metric is to be expected. Note that also massless extremals (MEs) are obtained as minimal surfaces so that also the topologically quantized counterparts of em and gravitational radiation emerge.

2. The general field equations can be written as vanishing of the covariant divergence for canonical momentum current $T^{k\alpha}$

$$
\begin{aligned}
D_\alpha(T^{k\alpha}\sqrt{g}) &= \partial_\alpha\left[T^{k\alpha}\sqrt{g}\right] + \left\{ {k \atop \alpha\ m} \right\} T^{m\alpha}\sqrt{g} = 0 \ , \\
T^{k\alpha} &= g^{\alpha\beta}\partial_\beta h^k \ , \\
\left\{ {k \atop \alpha\ m} \right\} &= \left\{ {k \atop l\ m} \right\}\partial_\alpha h^l \ .
\end{aligned}
\tag{6.2.1}
$$

   D_α is covariant derivative taking into account that gradient $\partial_\alpha h^k$ is imbedding space vector.

3. For isometry currents $j^{A,k}$ (Killing vector fields)

$$
T^{A,\alpha} = T^{\alpha k} h_{kl} j^{A,l} \tag{6.2.2}
$$

   the covariant divergence simplifies to ordinary divergence

$$
\partial_\alpha\left[T^{A,\alpha}\sqrt{g}\right] = 0 \ . \tag{6.2.3}
$$

   This allows to simplify the equations considerably.

6. Spherically symmetric stationary minimal surface

Consider now the spherically symmetric stationary metric representable as minimal surface.

1. In the following we consider only the region exterior to the surface defining the TGD counterpart of Schwartschild horizon and the possible horizon at which the signature of the induced metric. The first possibility is $g_{tt} = 0$ at horizon. If g_{rr} remains non-vanishing, the signature changes to Euclidian. If also $g_{rr} = 0$, both g_{tt} and g_{rr} can change sign so that one has a smooth variant of Schwartschild horizon.

   Second possibility is $g_{rr} = 0$ at radius r_E in the region below Schwartschild radius. At r_E the determinant of 4-metric would vanish and the signature of the induced metric would change to Euclidian.

2. The reduction to the conservation of isometry currents can be used for isometry current corresponding to the rotation $\Phi \rightarrow \Phi + \epsilon$ and time translation $m^0 \rightarrow m^0 + \epsilon$.

3. With the experience coming from the imbedding of Reissner-Nordström metric the ansatz is exactly the same and can be written as

$$m^0 = \Lambda t + h(r) \ , \quad \Phi = \omega t + k(r) \ , \quad u \equiv cos(\Theta) = u(r) \ , \tag{6.2.4}$$

4. The condition $g_{tr} = 0$ gives

$$\Lambda \partial_r h = R^2 \omega sin^2(\Theta) \partial_r k = 0 \ . \tag{6.2.5}$$

   This allows to integrate $h(r)$ in terms of $k(r)$.

5. The interesting components of the induced metric are

$$g_{tt} = \Lambda^2 - R^2 \omega^2 sin^2(\Theta) \ , \quad g_{rr} = -1 - R^2 (\partial_r \Theta)^2 + \Lambda^2 (\partial_r h)^2 \ . \tag{6.2.6}$$

6. The field equations reduce to conservation laws for various isometry currents. Consider energy current and the current related to the $SO(3) \subset SU(3)$ rotation acting on Φ as shift (call this current isospin current). The stationary character of the induced metric implies that the field equations reduce to the conservation of the radial current for energy current and isospin current. These two equations fix the solution together with diagonality condition. One obtains the following equations

$$\partial_r (\partial_r h \times g^{rr} \sqrt{g}) = 0 \ , \quad \partial_r (sin^2(\Theta) \partial_r k \times g^{rr} \sqrt{g}) = 0) \ . \tag{6.2.7}$$

   These two equations can be satisfied simultaneously only if one has

$$\partial_r h \times g^{rr} r^2 \sqrt{g_2} = A sin^2(\Theta) \partial_r k \times g^{rr} r^2 \sqrt{g_2} + B \ , \quad g_2 \equiv -g_{tt} g_{rr} \ . \tag{6.2.8}$$

   Note the presence of constant B.

   Second implication is

$$g^{rr} \partial_r h \sqrt{g_2} = \frac{C}{r^2} \ , \quad g^{rr} sin^2(\Theta) \partial_r k \sqrt{g_2} = \frac{D}{r^2} \ , \quad C = AD + B \ . \tag{6.2.9}$$

   By substituting the expressions for the metric one has

$$\partial_r h = \sqrt{-\frac{g_{rr}}{g_{tt}}} \times \frac{C}{r^2} \ , \quad sin^2(\Theta) \partial_r k = \sqrt{-\frac{g_{rr}}{g_{tt}}} \times \frac{D}{r^2} \ . \tag{6.2.10}$$

7. It is natural to look what one obtains in the approximation that the metric is flat expected to make sense at large distances. Putting $g_{tt} = -g_{rr} = 1$, one obtains

$$\partial_r h \simeq \frac{C}{r^2} \quad , \quad sin^2(\Theta)\partial_r k \simeq \frac{D}{r^2} \quad . \tag{6.2.11}$$

The time component of the induced metric is given by

$$g_{tt} = \Lambda^2 - R^2\omega^2 sin^2(\Theta) \simeq \Lambda^2 - \frac{D}{r^2\partial_r k} \quad . \tag{6.2.12}$$

This gives $1/r$ gravitational potential of a mass point if one has $\partial_r k \simeq E/r$ giving for $\Lambda = 1$

$$g_{tt} = 1 - \frac{r_S}{r} \quad , \quad r_S = 2GM = \frac{D}{E} \quad . \tag{6.2.13}$$

with the identification $r_S = 2GM = D/E$ inspired by the behavior of the Scwartschild metric. It seems that one can take $\Lambda = 1$ without a loss of generality.

8. Using $g_{tr} = 0$ condition this gives for h the approximate expression

$$\partial_r h \simeq \frac{D}{r^2} \quad , \quad D = \frac{R^2\omega^2}{\Lambda} \quad . \tag{6.2.14}$$

so that the field equations are consistent with the $1/r$ behavior of gravitational potential. The solution carries necessarily a non-vanishing Abelian electroweak gauge field.

9. The asymptotic behaviors of k and h would be

$$k \simeq k_0 log(\frac{r}{r_0}) \quad , \quad h \simeq h_0 - \frac{C}{r} \quad . \tag{6.2.15}$$

7. Two horizons and layered structure as basic prediction

A very interesting question is whether $g_{tt} = 0$ defines Schwartschild type horizon at which the roles of the coordinates t and r change or whether one obtains horizon at which the signature of the induced metric becomes Euclidian. The most natural option turns out to be Schwartschild like horizon at which the roles of time and radial coordinate are changed and second inner horizon at which g_{rr} changes sign again so that the induced metric has Euclidian signature below this inner horizon.

1. Unless one has $g_{tt}g_{rr} = C \neq 0$ ($C = -1$ holds true in Scwhartschild-Nordström metric) the surface $g_{tt} = 0$ - if it exists - defines a light-like 3-surface identifiable as horizon at which the signature of the induced metric changes. The conditions $g_{tt} = 0$ gives

$$\Lambda^2 - R^2\omega^2(1 - u^2) = 0 \quad . \tag{6.2.16}$$

giving

$$0 < sin^2(\Theta) = 1 - u^2 \quad = \quad \frac{\Lambda^2}{R^2\omega^2} < 1 \quad . \tag{6.2.17}$$

For $\Lambda = 1$ this condition implies that ω is a frequency of order of the inverse of CP_2 radius R. Note that $g_{tt} = 0$ need mean change of the metric signature to Euclidian if the analog of Schwarschild horizon is in question.

2. $g_{tt} = 0$ surface is light-like surface if g_{rr} has non-vanishing and finite value at it. g_{rr} could diverges at this surface guaranteeing $g_{tt}g_{rr} > 0$. The quantities $\partial_r h$ and $sin^2(\Theta)\partial_r k$ are proportional to $\sqrt{g_{rr}/g_{tt}}$, which diverges for $g_{tt} = 0$ unless also g_{rr} vanishes so that also these derivatives would diverge. The behavior of g_{rr} at this surface is

$$g_{rr} = -1 - R^2 \frac{(\partial_r u)^2}{1-u^2} + \Lambda^2 (\partial_r h)^2 \quad , \quad u \equiv cos(\Theta) \ . \tag{6.2.18}$$

   There are several options to consider.

   (a) Option I: The divergence of $(\partial_r h)^2$ as cause for the divergence of g_{rr} is out of question. If this quantity increases for small values of r, g_{rr} can change sign for with finite value of $\partial_r h$ and $u^2 < 1$ at some larger radius r_S analogous to Schwartschild radius. Since it is impossible to have two time-like directions also the sign of g_{tt} must change so that one would have the analog of Schwartschild horizon at this radius - call it r_S: $r_S = 2GM$ need not hold true. The condition $g_{tt} = 0$ at this radius fixes the value of $sin^2(\Theta)$ at this radius

$$sin^2(\Theta_S) \quad = \quad \frac{\Lambda^2}{R^2\omega^2} \ . \tag{6.2.19}$$

   If g_{rr} has finite value and is continuous, the metric has Euclidian signature in interior. If g_{rr} is discontinuous and changes sign as in the case of Schwartschild metric, one has counterpart of Scwartschild horizon without infinities. This option will be called Option I.

   (b) Second possibility giving rise to would be that u becomes equal 1. This is not consistent with $sin^2(\Theta_S) = 0$.

   (c) Option II: Voth g_{tt} and g_{rr} change their sign and vanish at r_S. This however requires both radial and time-like direction become null directions locally. Space-time surface would become locally metrically 2-dimensional at the horizon. This would conform with the idea of strong form of holography (SH) but it is not possible to have two different light-like directions simultaneously unless these directions are actually same. Mathematically it is certainly possible to have surfaces for which the dimension is locally reduced from the maximal one but it is difficult to visualize what this kind of metric reduction of local space-time dimension could mean. This option will be considered in what follows.

   To sum up, g_{rr} changes sign at horizon. For Option I g_{rr} is finite and dis-continuous. For Option II g_{rr} vanishes and is continuous. Whether g_{rr} vanishes at horizon or not, remains open.

3. For Schwartschild-Nordström metric g_{rr} becomes infinite and changes sign at horizon. The change of the roles of g_{tt} and g_{rr} could for Option II take place smoothly so that both could become zero and change their sign at r_S. This would keep $\partial_r h$ and $sin^2(\Theta)\partial_r k$ finite. One would have the analog of the interior of Schwartschild metric.

   What happens at the smaller radii? The obvious constraint is that $sin^2(\Theta)$ remains below unity. If g_{rr}/g_{tt} remains bounded, the condition for $sin^2(\Theta)\partial_k$ however suggests that $sin^2(\Theta) = 1$ is eventually achieved. This is the case also for the imbedding of Schwartschild metric. Could this horizon correspond to a surface at which the signature of the metric changes? g_{rr} should becomes zero in order to obtain light-like surface. g_{rr} contains indeed a term proportional to $1/sin^2(\Theta)$ which diverges at $u = 1$ so that g_{rr} must change sign for second time already above the radius for $sin^2(\Theta) = 1$ if h and k behaves smoothly enough. At this radius - call it r_E - g_{tt} would be finite and the signature would become Euclidian below this radius.

   One would therefore have two special radii r_S and r_E and a layer between these radii. $r_S = 2GM$ need not hold true but is expected to give a reasonable order of magnitude estimate.

Is there any empirical evidence for the existence of two horizons? There is evidence that the formation of the recently found LIGO blackhole (discussed from TGD view point in [L25]) is not fully consistent with the GRT based model (see `http://tinyurl.com/zbbz58w`). There are some indications that LIGO blackhole has a boundary layer such that the gravitational radiation is reflected forth and back between the inner and outer boundaries of the layer. In the proposed model the upper boundary would not be totally reflecting so that gravitational radiation leaks out and gave rise to echoes at times .1 sec, .2 sec, and .3 sec. It is perhaps worth of noticied that time scale .1 sec corresponds to the secondary p-adic time scale of electron (characterized by Mersenne prime $M_{127} = 2^{127} - 1$). If the minimal surface solution indeed has two horizons and a layer like structure between them, one might at least see the trouble of killing the idea that it could give rise to repeated reflections of gravitational radiation.

The proposed model (see `http://tinyurl.com/zbbz58w`) assumes that the inner horizon is Schwarstchild horizon. TGD would however suggests that the outer horizon is the TGD counterpart of Schwartschild horizon. It could have different radius since it would not be a singularity of g_{rr} (g_{tt}/g_{rr} would be finite at r_S which need not be $r_S = 2GM$ now). At r_S the tangent space of the space-time surface would become effectively 2-dimensional for $g_{rr} = 0$: the interpretation in terms of strong holography (SH) has been already mentioned.

The condition that the normal components of the canonical momentum currents for Kähler action and volume term are finite implies that $g^{nn}\sqrt{g_4}$ is finite at both sides of the horizon. Also the weak form of electric magnetic duality for Kähler form requires this. This condition can be satisfied if g_{tt} and g_{nn} approach to zero in the same manner at both sides of the horizon. Hence it seems that strong form of holography in the horizon is forced by finiteness.

One should understand why it takes rather long time $T = .1$ seconds for radiation to travel forth and back the distance $L = r_S - r_E$ between the horizons. The maximal signal velocity is reduced for the light-like geodesics of the space-time surface but the reduction should be rather large for $L \sim 20$ km (say). The effective light-velocity is measured by the coordinate time $\Delta t = \Delta m^0 + h(r_S) - h(r_E)$ needed to travel the distance from r_E to r_S. The Minkowski time Δm^0_{-+} would be the from null geodesic property and $m^0 = t + h(r)$

$$\Delta m^0_{-+} = \Delta t - h(r_S) + h(r_E) \quad , \quad \Delta t = \int_{r_E}^{r_S} \sqrt{\frac{g_{rr}}{g_{tt}}} dr \equiv \int_{r_E}^{r_S} \frac{dr}{c_\#} \quad . \tag{6.2.20}$$

Note that $c\#$ approaches zero at horizon if g_{rr} is non-vanishing at horizon.

The time needed to travel forth and back does not depend on h and would be given by

$$\Delta m^0 = 2\Delta t = 2\int_{r_E}^{r_S} \frac{dr}{c_\#} \quad . \tag{6.2.21}$$

This time cannot be shorter than the minimal time $(r_s - r_E)/c$ along light-like geodesic of M^4 since light-like geodesics at space-time surface are in general time-like curves in M^4. Since .1 sec corresponds to about 3×10^4 km, the average value of $c_\#$ should be for $L = 20$ km (just a rough guess) of order $c\# \sim 2^{-11}c$ in the interval $[r_E, r_S]$. As noticed, $T = .1$ sec is also the secondary p-adic time assignable to electron labelled by the Mersenne prime M_{127}. Since g_{rr} vanishes at r_E one has $c_\# \to \infty$. $c_\#$ is finite at r_S.

There is an intriguing connection with the notion of gravitational Planck constant. The formula for gravitational Planck constant given by $h_{gr} = GMm/v_0$ characterizing the magnetic bodies topologically for mass m topologically condensed at gravitational magnetic flux tube emanating from large mass M [K66, K53, K106, K109]. The interpretation of the velocity parameter v_0 has remained open. Could v_0 correspond to the average value of $c_\#$? For inner planets one has $v_0 \simeq 2^{-11}$ so that the order of magnitude is same as for the estimate for $c_\#$.

What about TGD inspired cosmology?

Before the discovery of the twistor lift TGD inspired cosmology has been based on the assumption that vacuum extremals provide a good estimate for the solutions of Einstein's equations at GRT limit of TGD [K79, K67] . One can find imbeddings of Robertson-Walker type metrics as vacuum extremals and the general finding is that the cosmological with super-critical and critical mass density have finite duration after which the mass density becomes infinite: this period of course ends before this. The interpretation would be in terms of the emergence of new space-time sheet at which matter represented by smaller space-time sheets suffers topological condensation. The only parameter characterizing critical cosmologies is their duration. Critical (over-critical) cosmologies having $SO3\times E^3$ ($SO(4)$) as isometry group is the duration and the CP_2 projection at homologically trivial geodesic sphere S^2: the condition that the contribution from S^2 to g_{rr} component transforms hyperbolic 3-metric to that of E^3 or S^3 metric fixes these cosmologies almost completely. Sub-critical cosmologies have one-dimensional CP_2 projection.

Do Robertson-Walker cosmologies have minimal surface representatives? Recall that minimal surface equations read as

$$\begin{aligned} D_\alpha(g^{\alpha\beta}\partial_\beta h^k\sqrt{g}) &= \partial_\alpha\left[g^{\alpha\beta}\partial_\beta h^k\sqrt{g}\right]+\left\{ {k \atop \alpha\ m} \right\}g^{\alpha\beta}\partial_\beta h^m\sqrt{g}=0 \ , \\ \left\{ {k \atop \alpha\ m} \right\} &= \left\{ {k \atop l\ m} \right\}\partial_\alpha h^l \ . \end{aligned} \tag{6.2.22}$$

Sub-critical minimal surface cosmologies would correspond to $X^4\subset M^4\times S^1$. The natural coordinates are Robertson-Walker coordinates, which co-incide with light-cone coordinates ($a=\sqrt{(m^0)^2-r_M^2}, r=r_M/a,\theta,\phi$) for light-cone M^4_+. They are related to spherical Minkowski coordinates (m^0,r_M,θ,ϕ) by ($m^0=a\sqrt{1+r^2}, r_M=ar$). $\beta=r_M/m_0=r/\sqrt{1+r^2}$ corresponds to the velocity along the line from origin (0,0) to (m^0,r_M). r corresponds to the Lorentz factor $\gamma\beta=\beta/\sqrt{1-\beta^2}$. The metric of M^4_+ is given by the diagonal form $[g_{aa}=1, g_{rr}=a^2/(1+r^2), g_{\theta\theta}=a^2r^2, g_{\phi\phi}=a^2r^2sin^2(\theta)]$. One can use the coordinates of M^4_+ also for X^4.

The ansatz for the minimal surface reads is $\Phi=f(a)$. For $f(a)=constant$ one obtains just the flat M^4_+. In non-trivial case one has $g_{aa}=1-R^2(df/da)^2$. The g^{aa} component of the metric becomes now $g^{aa}=1/(1-R^2(df/da)^2)$. Metric determinant is scaled by $\sqrt{g_{aa}}=1\rightarrow\sqrt{1-R^2(df/da)^2}$. Otherwise the field equations are same as for M^4_+. Little calculation shows that they are not satisfied unless one as $g_{aa}=1$.

Also the minimal surface imbeddings of critical and over-critical cosmologies are impossible. The reason is that the criticality alone fixes these cosmologies almost uniquely and this is too much for allowing minimal surface property.

Thus one can have only the trivial cosmology M^4_+ carrying dark energy density as a minimal surface solution! This obviously raises several questions.

1. Could $\Lambda=0$ case for which action reduces to Kähler action provide vacuum extremals provide single-sheeted model for Robertson-Walker cosmologies for the GRT limit of TGD for which many-sheeted space-time surface is replaced with a slightly curved region of M^4? Could $\Lambda=0$ correspond to a genuine phase present in TGD as formal generalization of the view of mathematicians about reals as $p=\infty$ p-adic number suggest. p-Adic length scale would be strictly infinite implying that $\Lambda\propto 1/p$ vanishes.

2. Second possibility is that TGD is quantum critical in strong sense. Not only 3-space but the entire space-time surface is flat and thus M^4_+. Only the local gravitational fields created by topologically condensed space-time surfaces would make it curved but would not cause smooth expansion. The expansion would take as quantum phase transitions reducing the value of $\Lambda\propto 1/p$ as p-adic prime p increases. p-Adic length scale hypothesis suggests that the preferred primes are near but below powers of 2 $p\simeq 2^k$ for some integers k. This led for years ago to a model for Expanding Earth [K27].

3. This picture would explain why individual astrophysical objects have not been observed to expand smoothly (except possibly in these phase transitions) but participate cosmic expansion only in the sense that the distance to other objects increase. The smaller space-time sheets glued to a given space-time sheet preserving their size would emanate from the tip of M^4_+ for given sheet.

4. RW cosmology should emerge in the idealization that the jerk-wise expansion by quantum phase transitions and reducing the value of Λ (by scalings of 2 by p-adic length scale hypothesis) can be approximated by a smooth cosmological expansion.

One should understand why Robertson-Walker cosmology is such a good approximation to this picture. Consider first cosmic redshift.

1. The cosmic recession velocity is defined from the redshift by Doppler formula.

$$z = \frac{1+\beta}{1-\beta} - 1 \simeq \beta = \frac{v}{c} \quad . \tag{6.2.23}$$

In TGD framework this should correspond to the velocity defined in terms of the coordinate r of the object.

Hubble law tells that the recession velocity is proportional to the proper distance D from the source. One has

$$v = HD \quad , \quad H = (\frac{da/dt}{a}) = \frac{1}{\sqrt{g_{aa}}a} \quad . \tag{6.2.24}$$

This brings in the dependence on the Robertson-Walker metric.

For M^4_+ one has $a = t$ and one would have $g_{aa} = 1$ and $H = 1/a$. The experimental fact is however that the value of H is larger for non-empty RW cosmologies having $g_{aa} < 1$. How to overcome this problem?

2. To understand this one must first understand the interpretation of gravitational redshift. In TGD framework the gravitational redshift is property of observer rather than source. The point is that the tangent space of the 3-surface assignable to the observer is related by a Lorent boost to that associated with the source. This implies that the four-momentum of radiation from the source is boosted by this same boost. Redshift would mean that the Lorentz boost reduces the momentum from the real one. Therefore redshift would be consistent with momentum conservation implied by Poincare symmetry.

g_{aa} for which a corresponds to the value of cosmic time for the observer should characterize the boost of observer relative to the source. The natural guess is that the boost is characterized by the value of g_{tt} in sufficiently large rest system assignable to observer with t is taken to be M^4 coordinate m^0. The value of g_{tt} fluctuates do to the presence of local gravitational fields. At the GRT limit g_{aa} would correspond to the average value of g_{tt}.

3. There is evidence that H is not same in short and long scales. This could be understood if the radiation arrives along different space-time sheets in these two situations.

4. If this picture is correct GRT description of cosmology is effective description taking into account the effect of local gravitation to the redshift, which without it would be just the M^4_+ redshift.

Einstein's equations for RW cosmology [K79, K67] should approximately code for the cosmic time dependence of mass density at given slightly deformed piece of M^4_+ representing particular sub-cosmology expanding in jerkwise manner.

1. Many-sheeted space-time implies a hierarchy of cosmologies in different p-adic length scales and with cosmological constant $\Lambda \propto 1/p$ so that vacuum energy density is smaller in long scale cosmologies and behaves on the average as $1/a^2$ where a characterizes the scale of the cosmology. In zero energy ontology given scale corresponds to causal diamond (CD) with size characterized by a defining the size scale for the distance between the tips of CD.

2. For the comoving volume with constant value of coordinate radius r the radius of the volume increases as a. The vacuum energy would increase as a^3 for comoving volume. This is in sharp conflict with the fact that the mass decreases as $1/a$ for radiation dominated cosmology, is constant for matter dominated cosmology, and is proportional to a for string dominated cosmology.

   The physical resolution of the problem is rather obvious. Space-time sheets representing topologically condensed matter have finite size. They do not expand except possibly in jerkwise manner but in this process Λ is reduced - in average manner like $1/a^2$.

   If the sheets are smaller than the cosmological space-time sheet in the scale considered and do not lose energy by radiation they represent matter dominated cosmology emanating from the vertex of M^4_+. The mass of the co-moving volume remains constant.

   If they are radiation dominated and in thermal equilibrium they lose energy by radiation and the energy of volume behaves like $1/a$.

   Cosmic strings and magnetic flux tubes have size larger than that the space-time sheet representing the cosmology. The string as linear structure has energy proportional to a for fixed value of Λ as in string dominated cosmology. The reduction of Λ decreasing on the average like $1/a^2$ implies that the contribution of given string is reduced like $1/a$ on the average as in radiation dominated cosmology.

3. GRT limit would code for these behaviours of mass density and pressure identified as scalars in GRT cosmology in terms of Einstein's equations. The time dependence of g_{aa} would code for the density of the topologically condensed matter and its pressure and for dark energy at given level of hierarchy. The vanishing of covariant divergence for energy momentum tensor would be a remnant of Poincare invariance and give Einstein's equations with cosmological term.

4. Why GRT limit would involve only the RW cosmologies allowing imbedding as vacuum extremals of Kähler action? Can one demand continuity in the sense that TGD cosmology at $p \rightarrow \infty$ limit corresponds to GRT cosmology with cosmological solutions identifiable as vacuum extremals? If this is assumed the earlier results are obtained. In particular, one obtains the critical cosmology with 2-D CP_2 projection assumed to provide a GRT model for quantum phase transitions changing the value of Λ.

If this picture is correct, TGD inspired cosmology at the level of many-sheeted space-time would be extremely simple. The new element would be many-sheetedness which would lead to more complex description provided by GRT limit. This limit would however lose the information about many-sheetedness and lead to anomalies such as two Hubble constants.

Induced spinor structure

The notion of induced spinor field deserves a more detailed discussion. Consider first induced spinor structures [K88].

1. Induced spinor field are spinors of $M^4 \times CP_2$ for which modes are characterized by chirality (quark or lepton like) and em charge and weak isospin.

2. Induced spinor spinor structure involves the projection of gamma matrices defining induced gamma matrices. This gives rise to superconformal symmetry if the action contains only volume term.

   When Kähler action is present, superconformal symmetry requires that the modified gamma matrices are contractions of canonical momentum currents with imbedding space gamma matrices. Modified gammas appear in the modified Dirac equation and action, whose solution at string world sheets trivializes by super-conformal invariance to same procedure as in the case of string models.

3. Induced spinor fields correspond to two chiralities carrying quark number and lepton number. Quark chirality does not carry color as spin-like quantum number but it corresponds to a

color partial wave in CP_2 degrees of freedom: color is analogous to angular momentum. This reduces to spinor harmonics of CP_2 describing the ground states of the representations of super-symplectic algebra.

The harmonics do not satisfy correct correlation between color and electroweak quantum numbers although the triality t=0 for leptonic waves and t=1 for quark waves. There are two manners to solve the problem.

(a) Super-symplectic generators applied to the ground state to get vanishing ground states weight instead of the tachyonic one carry color and would give for the physical states correct correlation: leptons/quarks correspond to the same triality zero(one partial wave irrespective of charge state. This option is assumed in p-adic mass calculations [K39].

(b) Since in TGD elementary particles correspond to pairs of wormhole contacts with weak isospin vanishing for the entire pair, one must have pair of left and right-handed neutrinos at the second wormhole throat. It is possible that the anomalous color quantum numbers for the entire state vanish and one obtains the experimental correlation between color and weak quantum numbers. This option is less plausible since the cancellation of anomalous color is not local as assume in p-adic mass calculations.

The understanding of the details of the fermionic and actually also geometric dynamics has taken a long time. Super-conformal symmetry assigning to the geometric action of an object with given dimension an analog of Dirac action allows however to fix the dynamics uniquely and there is indeed dimensional hierarchy resembling brane hierarchy.

1. The basic observation was following. The condition that the spinor modes have well-defined em charge implies that they are localized to 2-D string world sheets with vanishing W boson gauge fields which would mix different charge states. At string boundaries classical induced W boson gauge potentials guarantee this. Super-conformal symmetry requires that this 2-surface gives rise to 2-D action which is area term plus topological term defined by the flux of Kähler form.

2. The most plausible assumption is that induced spinor fields have also interior component but that the contribution from these 2-surfaces gives additional delta function like contribution: this would be analogous to the situation for branes. Fermionic action would be accompanied by an area term by supersymmetry fixing modified Dirac action completely once the bosonic actions for geometric object is known. This is nothing but super-conformal symmetry.

   One would actually have the analog of brane-hierarchy consisting of surfaces with dimension D= 4,3,2,1 carrying induced spinor fields which can be regarded as independent dynamical variables and characterized by geometric action which is D-dimensional analog of the action for Kähler charged point particle. This fermionic hierarchy would accompany the hierarchy of geometric objects with these dimensions and the modified Dirac action would be uniquely determined by the corresponding geometric action principle (Kähler charged point like particle, string world sheet with area term plus Kähler flux, light-like 3-surface with Chern-Simons term, 4-D space-time surface with Kähler action).

3. This hierarchy of dynamics is consistent with SH only if the dynamics for higher dimensional objects is induced from that for lower dimensional objects - string world sheets or maybe even their boundaries orbits of point like fermions. Number theoretic vision [K111] suggests that this induction relies algebraic continuation for preferred extremals. Note that quaternion analyticity [L22] means that quaternion analytic function is determined by its values at 1-D curves.

4. Quantum-classical correspondences (QCI) requires that the classical Noether charges are equal to the eigenvalues of the fermionic charges for surfaces of dimension $D = 0, 1, 2, 3$ at the ends of the CDs. These charges would not be separately conserved. Charges could flow between objects of dimension $D + 1$ and D - from interior to boundary and vice versa. Four-momenta and also other charges would be complex as in twistor approach: could complex values relate somehow to the finite life-time of the state?

If quantum theory is square root of thermodynamics as zero energy ontology suggests, the idea that particle state would carry information also about its life-time or the time scale of CD to which is associated could make sense. For complex values of α_K there would be also flow of canonical and super-canonical momentum currents between Euclidian and Minkowskian regions crucial for understand gravitational interaction as momentum exchange at imbedding space level.

5. What could be the physical interpretation of the bosonic and fermionic charges associated with objects of given dimension? Condensed matter physicists assign routinely physical states to objects of various dimensions: is this assignment much more than a practical approximation or could condensed matter physics already be probing many-sheeted physics?

SUSY and TGD

From this one ends up to the possibility of identifying the counterpart of SUSY in TGD framework [K95].

1. In TGD the generalization of much larger super-conformal symmetry emerges from the super-symplectic symmetries of WCW. The mathematically questionable notion of super-space is not needed: only the realization of super-algebra in terms of WCW gamma matrices defining super-symplectic generators is necessary to construct quantum states. As a matter of fact, also in QFT approach one could use only the Clifford algebra structure for super-multiplets. No Majorana condition on fermions is needed as for $\mathcal{N} = 1$ space-time SUSY and one avoids problems with fermion number non-conservation.

2. In TGD the construction of sparticles means quite concretely adding fermions to the state. In QFT it corresponds to transformation of states of integer and half-odd integer spin to each other. This difference comes from the fact that in TGD particles are replaced with point like particles.

3. The analog of $\mathcal{N} = 2$ space-time SUSY could be generated by covariantly constant right handed neutrino and antineutrino. Quite generally the mixing of fermionic chiralities implied by the mixing of M^4 and CP_2 gamma matrices implies SUSY breaking at the level of particle masses (particles are massless in 8-D sense). This breaking is purely geometrical unlike the analog of Higgs mechanism proposed in standard SUSY.

There are several options to consider.

1. The analog of brane hierarchy is realized also in TGD. Geometric action has parts assignable to 4-surface, 3-D light like regions between Minkowskian and Euclidian regions, 2-D string world sheets, and their 1-D boundaries. They are fixed uniquely. Also their fermionic counterparts - analogs of Dirac action - are fixed by super-conformal symmetry. Elementary particles reduce so composites consisting of point-like fermions at boundaries of wormhole throats of a pair of wormhole contacts.

   This forces to consider 3 kinds of SUSYs! The SUSYs associated with string world sheets and space-time interiors would certainly be broken since there is a mixing between M^4 chiralities in the modified Dirac action. The mass scale of the broken SUSY would correspond to the length scale of these geometric objects and one might argue that the decoupling between the degrees of freedom considered occurs at high energies and explains why no evidence for SUSY has been observed at LHC. Also the fact that the addition of massive fermions at these dimensions can be interpreted differently. 3-D light-like 3-surfaces could be however an exception.

2. For 3-D light-like surfaces the modified Dirac action associated with the Chern-Simons term does not mix M^4 chiralities (signature of massivation) at all since modified gamma matrices have only CP_2 part in this case. All fermions can have well-defined chirality. Even more: the modified gamma matrices have no M^4 part in this case so that these modes carry no four-momentum - only electroweak quantum numbers and spin. Obviously, the excitation of these fermionic modes would be an ideal manner to create spartners of ordinary particles consting

of fermion at the fermion lines. SUSY would be present if the spin of these excitations couples - to various interactions and would be exact in absence of coupling to interior spinor fields.

What would be these excitations? Chern-Simons action and its fermionic counterpart are non-vanishing only if the CP_2 projection is 3-D so that one can use CP_2 coordinates. This strongly suggests that the modified Dirac equation demands that the spinor modes are co-variantly constant and correspond to covariantly constant right-handed neutrino providing only spin.

If the spin of the right-handed neutrino adds to the spin of the particle and the net spin couples to dynamics, $\mathcal{N}=2$ SUSY is in question. One would have just action with unbroken SUSY at QFT limit? But why also right-handed neutrino spin would couple to dynamics if only CP_2 gamma matrices appear in Chern-Simons-Dirac action? It would seem that it is independent degree of freedom having no electroweak and color nor even gravitational couplings by its covariant constancy. I have ended up with just the same SUSY-or-no-SUSY that I have had earlier.

3. Can the geometric action for light-like 3-surfaces contain Chern-Simons term?

   (a) Since the volume term vanishes identically in this case, one could indeed argue that also the counterpart of Kähler action is excluded. Moreover, for so called massless extremals of Kähler action reduces to Chern-Simons terms in Minkowskian regions and this could happen quite generally: TGD with only Kähler action would be almost topological QFT as I have proposed. Volume term however changes the situation via the cosmological constant. Kähler-Dirac action in the interior does not reduce to its Chern-Simons analog at light-like 3-surface.

   (b) The problem is that the Chern-Simons term at the two sides of the light-like 3-surface differs by factor $\sqrt{-1}$ coming from the ratio of $\sqrt{g_4}$ factors which themselves approach to zero: oOne would have the analog of dipole layer. This strongly suggests that one should not include Chern-Simons term at all.

   Suppose however that Chern-Simons terms are present at the two sides and α_K is real so that nothing goes through the horizon forming the analog of dipole layer. Both bosonic and fermionic degrees of freedom for Euclidian and Minkowskian regions would decouple completely but currents would flow to the analog of dipole layer. This is not physically attractive.

   The canonical momentum current and its super counterpart would give fermionic source term $\Gamma^n\Psi_{int,\pm}$ in the modified Dirac equation defined by Chern-Simons term at given side $\pm$: $\pm$ refers to Minkowskian/Euclidian part of the interior. The source term is proportional to $\Gamma^n\Psi_{int,\pm}$ and Γ^n is in principle mixture of M^4 and CP_2 gamma matrices and therefore induces mixing of M^4 chiralities and therefore also 3-D SUSY breaking. It must be however emphasized that Γ^n is singular and one must be consider the limit carefully also in the case that one has only continuity conditions. The limit is not completely understood.

   (c) If α_K is complex there is coupling between the two regions and the simplest assumption has been that there is no Chern-Simons term as action and one has just continuity conditions for canonical momentum current and hits super counterpart.

   The cautious conclusion is that 3-D Chern-Simons term and its fermionic counterpart are absent.

4. What about the addition of fermions at string world sheets and interior of space-time surface ($D=2$ and $D=4$). For instance, in the case of hadrons $D=2$ excitations could correspond to addition of quark in the interior of hadronic string implying additional states besides the states obtained assuming only quarks at string ends. Let us consider the interior ($D=4$). For instance, inn the case of hadrons $D=2$ excitations could correspond to addition of quark in the interior of hadronic string implying additional states besides the states obtained assuming only quarks at string ends. The smallness of cosmological constant implies that the contribution to the four-momentum from interior should be rather small so that an

interpretation in terms of broken SUSY might make sense. There would be mass $m \sim .03$ eV per volume with size defined by the Compton scale $\hbar/m$. Note however that cosmological constant has spectrum coming as inverse powers of prime so that also higher mass scales are possible.

This interpretation might allow to understand the failure to find SUSY at LHC. Sparticles could be obtained by adding interior right-handed neutrinos and antineutrinos to the particle state. They could be also associated with the magnetic body of the particle. Since they do not have color and weak interactions, SUSY is not badly broken. If the mass difference between particle and sparticle is of order $m = .03$ eV characterizing dark energy density ρ_{vac}, particle and sparticle could not be distinguished in higher energy physics at LHC since it probes much shorter scales and sees only the particle. I have already earlier proposed a variant of this mechanism but without SUSY breaking.

To discover SUSY one should do very low energy physics in the energy range $m \sim .03$ eV having same order of magnitude as thermal energy $kT = 2.6 \times 10^{-2}$ eV at room temperature 25 °C. One should be able to demonstrate experimentally the existence of sparticle with mass differing by about $m \sim .03$ eV from the mass of the particle (one cannot exclude higher mass scales since Λ is expected to have spectrum). An interesting question is whether the sparticles associated with standard fermions could give rise to Bose-Einstein condensates whose existence in the length scale of large neutron is strongly suggested by TGD view about living matter.

6.2.3 Imbedding space level

In GRT the description of gravitation involve only space-time and gravitational force is eliminated. In TGD also imbedding space level is involved with the description [L22].

1. The incoming and outgoing states of particle reaction are labelled by the quantum numbers associated with the isometries of the imbedding space and by the contributions of super-symplectic generators and isometry generators to the quantum numbers. This follows from the fact that the ground states of super-symplectic representations correspond to the modes of imbedding space spinors fields. These quantum numbers appear in the S-matrix of QFT limit too. In particular, color quantum numbers as angular momentum like quantum numbers at fundamental level are transformed to spin-like quantum numbers at QFT limit.

2. In GRT the applications rely on Post-Newtonian approximation (PNA). This means that the notion of gravitational force is brought to the theory although it has been eliminated from the basic GRT. This is not simple. One could argue that there is genuine physics behind this PNA and TGD suggests what this physics is.

   At the level of space-time surfaces particles move along geodesic lines and in TGD minimal surface equation states the generalization of the geodesic line property for 3-D particles. At the imbedding space level gravitational interaction involves exchanges of four-momentum and in principle of color quantum numbers too. Indeed, there is an exchange of classical charges through the light-like 3-surfaces defining the boundaries of Euclidian regions defining Euclidian regions as "lines" of generalized scattering diagrams. This however requires that Kähler coupling strength is allowed to be complex (say correspond to zero of Riemann Zeta). Hence in TGD also Newtonian view would be correct and needed.

6.3 Some questions about TGD

In Face Book I was made a question about general aspects of TGD. It was impossible to answer the question with few lines and I decided to write a blog posting, which then gave rise to this section. This text talks from different perspective about same topics as the article Can one apply Occams razor as a general purpose debunking argument to TGD? [L20] trying o emphasize the simplicity of the basic principles of TGD and of the resulting theory.

6.3.1 In what aspects TGD extends other theory/theories of physics?

I will replace "extends" with "modifies" since TGD also simplifies in many respects. I shall restrict the considerations to the ontological level which to my view is the really important level.

1. Space-time level is where TGD started from. Space-time as an abstract 4-geometry is replaced as space-time as 4-surface in $M^4 \times CP_2$. In GRT space-time is small deformation of Minkowski space.

   In TGD both Relativity Principle (RP) of Special Relativity (SRT) and General Coordinate Invariance (GCI) and Equivalence Principle (EP) of General Relativity hold true. In GRT RP is given up and leads to the loss of conservation laws since Noether theorem cannot be applied anymore: this is what led to the idea about space-time as surface in H. Strong form of holography (SH) is a further principle reducing to strong form of GCI (SGCI).

2. TGD as a physical theory extends to a theory of consciousness and cognition. Observer as something external to the Universe becomes part of physical system - the notion of self - and quantum measurement theory which is the black sheet of quantum theory extends to a theory of consciousness and also of cognition relying of p-adic physics as correlate for cognition. Also quantum biology becomes part of fundamental physics and consciousness and life are seen as basic elements of physical existence rather than something limited to brain.

   One important aspect is a new view about time: experienced time and geometric time are not one and same thing anymore although closely related. ZEO explains how the experienced flow and its direction emerges. The prediction is that both arrows of time are possible and that this plays central role in living matter.

3. p-Adic physics is a new element and an excellent candidate for a correlate of cognition. For instance, imagination could be understood in terms of non-determinism of p-adic partial differential equations for p-adic variants of space-time surfaces. p-Adic physics and fusion of real and various p-adic physics to adelic physics provides fusion of physics of matter with that of cognition in TGD inspired theory of cognition. This means a dramatic extension of ordinary physics. Number Theoretical Universality states that in certain sense various p-adic physics and real physics can be seen as extensions of physics based on algebraic extensions of rationals (and also those generated by roots of e inducing finite-D extensions of p-adics).

4. Zero energy ontology (ZEO) in which so called causal diamonds (CDs, analogs Penrose diagrams) can be seen as being forced by very simple condition: the volume action forced by twistor lift of TGD must be finite. CD would represent the perceptive field defined by finite volume of imbedding space $H = M^4 \times CP_2$.

   ZEO implies that conservation laws formulated only in the scale of given CD do not anymore fix select just single solution of field equations as in classical theory. Theories are strictly speaking impossible to test in the old classical ontology. In ZEO testing is possible be sequence of state function reductions giving information about zero energy states.

   In principle transition between any two zero energy states - analogous to events specified by the initial and final states of event - is in principle possible but Negentropy Maximization Principle (NMP) as basic variational principle of state function reduction and of consciousness restricts the possibilities by forcing generation of negentropy: the notion of negentropy requires p-adic physics.

   Zero energy states are quantum superpositions of classical time evolutions for 3-surfaces and classical physics becomes exact part of quantum physics: in QFTs this is only the outcome of stationary phase approximation. Path integral is replaced with well-defined functional integral- not over all possible space-time surface but pairs of 3-surfaces at the ends of space-time at opposite boundaries of CD.

   ZEO leads to a theory of consciousness as quantum measurement theory in which observer ceases to be outsider to the physical world. One also gets rid of the basic problem caused by the conflict of the non-determinism of state function reduction with the determinism of the unitary evolution. This is obviously an extension of ordinary physics.

5. Hierarchy of Planck constants represents also an extension of quantum mechanics at QFT limit. At fundamental level one actually has the standard value of h but at QFT limit one has effective Planck constant $h_{eff}/h = n$, $n = 1, 2, ...$ This generalizes quantum theory. This scaling of h has a simple topological interpretation: space-time surface becomes n-fold covering of itself and the action becomes n-multiple of the original which can be interpreted as $h_{eff}/h = n$.

   The most important applications are to biology, where quantum coherence could be understood in terms of a large value of h_{eff}/h. The large n phases resembles the large N limit of gauge theories with gauge couplings behaving as $\alpha \propto 1/N$ used as a kind of mathematical trick. Also gravitation is involved: h_{eff} is associated with the flux tubes mediating various interactions (being analogs to wormholes in ER-EPR correspondence). In particular, one can speak about h_{gr}, which Nottale introduced originally and $h_{eff} = h_{gr}$ plays key role in quantum biology according to TGD.

6.3.2 In what sense TGD is simplification/extension of existing theory?

1. Classical level: Space-time as 4-surface of H means a huge reduction in degrees of freedom. There are only 4 field like variables - suitably chosen 4 coordinates of $H = M^4 \times CP_2$. All classical gauge fields and gravitational field are fixed by the surface dynamics. There are no primary gauge fields or gravitational fields nor any other fields in TGD Universe and they appear only at the QFT limit [K7, K112, L24].

   GRT limit would mean that many-sheeted space-time is replaced by single slightly curved region of M^4. The test particle - small particle like 3-surface - touching the sheets simultaneously experience sum of gravitational forces and gauge forces. It is natural to assume that this superposition corresponds at QFT limit to the sum for the deviations of induced metrics of space-time sheets from flat metric and sum of induce gauge potentials. These would define the fields in standard model + GRT. At fundamental level effects rather than fields would superpose. This is absolutely essential for the possibility of reducing huge number field like degrees of freedom. One can obviously speak of emergence of various fields.

   A further simplification is that only preferred extremals for which data coding for them are reduced by SH to 2-D string like world sheets and partonic 2-surfaces are allowed. TGD is almost like string model but space-time surfaces are necessary for understanding the fact that experiments must be analyzed using classical 4-D physics. Things are extremely simple at the level of single space-time sheet.

   Complexity emerges from many-sheetedness. From these simple basic building bricks - minimal surface extremals of Kähler action (not the extremal property with respect to Kähler action and volume term strongly suggested by the number theoretical vision plus analogs of Super Virasoro conditions in initial data) - one can engineer space-time surfaces with arbitrarily complex topology - in all length scales. An extension of existing space-time concept emerges. Extremely simple locally, extremely complex globally with topological information added to the Maxwellian notion of fields (topological field quantization allowing to talk about field identify of system/field body/magnetic body.

   Another new element is the possibility of space-time regions with Euclidian signature of the induced metric. These regions correspond to 4-D "lines" of general scattering diagrams. Scattering diagrams has interpretation in terms of space-time geometry and topology.

2. The construction of quantum TGD using canonical quantization or path integral formalism failed completely for Kähler action by its huge vacuum degeneracy. The presence of volume term still suffers from complete failure of perturbation theory and extreme non-linearity. This led to the notion of world of classical worlds (WCW) - roughly the space of 3-surfaces. Essentially pairs of 3-surfaces at the boundaries of given CD connected by preferred extremals of action realizing SH and SGCI.

   The key principle is geometrization of the entire quantum theory, not only of classical fields geometrized by space-time as surface vision. This requires geometrization of hermitian conjugation and representation of imaginary unit geometrically. Kähler geometry for

WCW [K34, K15, K110] makes this possible and is fixed once Kähler function defining Kähler metric is known. Kähler action for a preferred extremal of Kähler action defining space-time surface as an analog of Bohr orbit was the first guess but twistor lift forced to add volume term having interpretation in terms of cosmological constant.

Already the geometrization of loop spaces demonstrated that the geometry - if it exists - must have maximal symmetries (isometries). There are excellent reasons to expect that this is true also in $D = 3$. Physics would be unique from its mathematical existence!

3. WCW has also spinor structure [K88, K110]. WCW spinors correspond to fermionic Fock states using oscillator operators assignable to the induced spinor fields - free spinor fiels. WCW gamma matrices are linear combinations of these oscillator operators and Fermi statistics reduces to spinor geometry.

4. There is **no quantization** in TGD framework at the level of WCW [K14, L22]. The construction of quantum states and S-matrix reduces to group theory by the huge symmetries of WCW. Therefore zero energy states of Universe (or CD) correspond formally to **classical** WCW spinor fields satisfying WCW Dirac equation analogous to Super Virasoro conditions and defining representations for the Yangian generalization of the isometries of WCW (so called super-symplectic group assignable to $\delta M^4_+ \times CP_2$. In ZEO stated are analogous to pairs of initial and final states and the entanglement coefficients between positive and negative energy parts of zero energy states expected to be fixed by Yangian symmetry define scattering matrix and have purely group theoretic interpretation. If this is true, entire dynamics would reduce to group theory in ZEO.

6.3.3 What is the hypothetical applicability of the extension - in energies, sizes, masses etc?

TGD is a unified theory and is meant to apply in all scales. Usually the unifications rely on reductionistic philosophy and try to reduce physics to Planck scale. Also super string models tried this and failed: what happens at long length scales was completely unpredictable (landscape catastrophe).

Many-sheeted space-time however forces to adopt fractal view. Universe would be analogous to Mandelbrot fractal down to CP_2 scale. This predicts scaled variants of say hadron physics and electroweak physics. p-Adic length scale hypothesis and hierarchy of phases of matter with $h_{eff}/h = n$ interpreted as dark matter gives a quantitative realization of this view.

1. p-Adic physics shows itself also at the level of real physics [K100]. One ends up to the vision that particle mass squared has thermal origin: the p-adic variant of particle mass square is given as thermal mass squared given by p-adic thermodynamics mappable to real mass squared by what I call canonical identification. p-Adic length scale hypothesis states that preferred p-adic primes characterizing elementary particles correspond to primes near to power of 2: $p \simeq 2^k$. p-Adic length scale is proportional to $p^{1/2}$.

   This hypothesis is testable and it turns out that one can predict particle mass rather accurately. This is highly non-trivial since the sensitivity to the integer k is exponential. So called Mersenne primes turn out to be especially favoured. This part of theory was originally inspired by the regularities of particle mass spectrum. I have developed arguments for why the crucial p-adic length scale hypothesis - actually its generalization - should hold true. A possible interpretation is that particles provide cognitive representations of themselves by p-adic thermodynamics.

2. p-Adic length scale hypothesis leads also to consider the idea that particles could appear as different p-adically scaled up variants. For instance, ordinary hadrons to which one can assign Mersenne prime $M_{107} = 2^{107} - 1$ could have fractally scaled variants. M_{89} and $M_{G,107}$ (Gaussian prime) would be two examples and there are indications at LHC for these scaled up variants of hadron physics [K42, K43]. These fractal copies of hadron physics and also of electroweak physics would correspond to extension of standard model.

3. Dark matter hierarchy predicts zoomed up copies of various particles. The simplest assumption is that masses are not changed in the zooming up. One can however consider that binding energy scale scales non-trivially. The dark phases would emerge are quantum criticality and give rise to the associated long range correlations (quantum lengths are typically scaled up by $h_{eff}/h = n$).

6.3.4 What is the leading correction/contribution to physical effects due to TGD onto particles, interactions, gravitation, cosmology?

1. Concerning particles I already mentioned the key predictions.
   (a) The existence of scaled variants of various particles and entire branches of physics. The fundamental quantum numbers are just standard model quantum numbers code by CP_2 geometry.
   (b) Particle families have topological description meaning that space-time topology would be an essential element of particle physics [K12]. The genus of partonic 2-surfaces (number of handles attached to sphere) is $g = 0, 1, 2, ...$ and would give rise to family replication. $g < 2$ partonic 2-surfaces have always global conformal symmetry Z_2 and this suggests that they give rise to elementary particles identifiable as bound states of g handles. For $g > 2$ this symmetry is absent in the generic case which suggests that they can be regarded as many-handle states with mass continuum rather than elementary particles. 2-D anyonic systems could represent an example of this.
   (c) A hierarchy of dynamical symmetries as remnants of super-symplectic symmetry however suggests itself [K14, K110]. The super-symplectic algebra possess infinite hierarchy of isomorphic sub-algebras with conformal weights being n-multiples of for those for the full algebra (fractal structure again possess also by ordinary conformal algebras). The hypothesis is that sub-algebra specified by n and its commutator with full algebra annihilate physical states and that corresponding classical Noether charges vanish. This would imply that super-symplectic algebra reduces to finite-D Kac-Moody algebra acting as dynamical symmetries. The connection with ADE hierarchy of Kac-Moody algebras suggests itself. This would predict new physics. Condensed matter physics comes in mind.
   (d) Number theoretic vision suggests that Galois groups for the algebraic extensions of rationals act as dynamical symmetry groups. They would act on algebraic discretizations of 3-surfaces and space-time surfaces necessary to realize number theoretical universality. This would be completely new physics.

2. Interactions would be mediated at QFT limit by standard model gauge fields and gravitons. QFT limit however loses all information about many-sheetedness and there would be anomalies reflecting this information loss. In many-sheeted space-time light can propagate along several paths and the time taken to travel along light-like geodesic from A to B depends on space-time sheet since the sheet is curved and warped. Neutrinos and gamma rays from SN1987A arriving at different times would represent a possible example of this. It is quite possible that the outer boundaries of even macroscopic objects correspond to boundaries between Euclidian and Minkowskian regions at the space-time sheet of the object.

   The failure of QFTs to describe bound states of say hydrogen atom could be second example: many-sheetedness and identification of bound states as single connected surface formed by proton and electron would be essential and taken into account in wave mechanical description but not in QFT description.

3. Concerning gravitation the basic outcome is that by number theoretical vision all preferred extremals are extremals of both Kähler action and volume term. This is true for all known extremals what happens if one introduces the analog of Kähler form in M^4 is an open question) [L24].

   Minimal surfaces carrying no Kähler field would be the basic model for gravitating system. Minimal surface equation are non-linear generalization of d'Alembert equation with gravitational self-coupling to induce gravitational metric. In static case one has analog for the

Laplace equation of Newtonian gravity. One obtains analog of gravitational radiation as "massless extremals" and also the analog of spherically symmetric stationary metric.

Blackholes would be modified. Besides Schwartschild horizon which would differ from its GRT version there would be horizon where signature changes. This would give rise to a layer structure at the surface of blackhole [L24].

4. Concerning cosmology the hypothesis has been that RW cosmologies at QFT limit can be modelled as vacuum extremals of Kä hler action. This is admittedly ad hoc assumption inspired by the idea that one has infinitely long p-adic length scale so that cosmological constant behaving like $1/p$ as function of p-adic length scale assignable with volume term in action vanishes and leaves only Kähler action [**?**]grprebio. This would predict that cosmology with critical is specified by a single parameter - its duration as also over-critical cosmology [K67]. Only sub-critical cosmologies have infinite duration.

   One can look at the situation also at the fundamental level. The addition of volume term implies that the only RW cosmology realizable as minimal surface is future light-cone of M^4. Empty cosmology which predicts non-trivial slightly too small redshift just due to the fact that linear Minkowski time is replaced with light-cone proper time constant for the hyperboloids of M^4_+. Locally these space-time surfaces are however deformed by the addition of topologically condensed 3-surfaces representing matter. This gives rise to additional gravitational redshift and the net cosmological redshift. This also explains why astrophysical objects do not participate in cosmic expansion but only comove. They would have finite size and almost Minkowski metric.

   The gravitational redshift would be basically a kinematical effect. The energy and momentum of photons arriving from source would be conserved but the tangent space of observer would be Lorentz-boosted with respect to source and this would course redshift.

   The very early cosmology could be seen as gas of arbitrarily long cosmic strings in H (or M^4) with 2-D M^4 projection [K67, K108]. Horizon would be infinite and TGD suggests strongly that large values of h_{eff}/h makes possible long range quantum correlations. The phase transition leading to generation of space-time sheets with 4-D M^4 projection would generate many-sheeted space-time giving rise to GRT space-time at QFT limit. This phase transition would be the counterpart of the inflationary period and radiation would be generated in the decay of cosmic string energy to particles.

Part II

GENERAL THEORY

Chapter 7

Construction of Quantum Theory: Symmetries

7.1 Introduction

This chapter provides a summary about the role of symmetries in the construction of quantum TGD. The discussions are based on the general vision that quantum states of the Universe correspond to the modes of classical spinor fields in the configuration space - "world of the classical worlds" (WCW) - identified as the infinite-dimensional WCW of light-like 3-surfaces of $H = M^4 \times CP_2$ (more or less-equivalently, the corresponding 4-surfaces defining generalized Bohr orbits). The following topics are discussed on basis of this vision.

7.1.1 Physics As Infinite-Dimensional Kähler Geometry

1. The basic idea is that it is possible to reduce quantum theory to WCW geometry and spinor structure. The geometrization of loop spaces inspires the idea that the mere existence of Riemann connection fixes WCW Kähler geometry uniquely. Accordingly, WCW can be regarded as a union of infinite-dimensional symmetric spaces labeled by zero modes labeling classical non-quantum fluctuating degrees of freedom.

   The huge symmetries of the WCW geometry deriving from the light-likeness of 3-surfaces and from the special conformal properties of the boundary of 4-D light-cone would guarantee the maximal isometry group necessary for the symmetric space property. Quantum criticality is the fundamental hypothesis allowing to fix the Kähler function and thus dynamics of TGD uniquely. Quantum criticality leads to surprisingly strong predictions about the evolution of coupling constants.

2. WCW spinors correspond to Fock states and anti-commutation relations for fermionic oscillator operators correspond to anti-commutation relations for the gamma matrices of the WCW. WCW gamma matrices contracted with Killing vector fields give rise to a super-symplectic algebra which together with Hamiltonians of the WCW forms what I have used to call super-symplectic algebra.

   Super-symplectic degrees of freedom represent completely new degrees of freedom and have no electroweak couplings. In the case of hadrons super-symplectic quanta correspond to what has been identified as non-perturbative sector of QCD: they define TGD correlate for the degrees of freedom assignable to hadronic strings. They are responsible for the most of the mass of hadron and resolve spin puzzle of proton.

3. Besides super-symplectic symmetries there are Super-Kac Moody symmetries assignable to light-like 3-surfaces and together these algebras extend the conformal symmetries of string models to dynamical conformal symmetries instead of mere gauge symmetries. The construction of the representations of these symmetries is one of the main challenges of quantum TGD.

4. Modular invariance is one aspect of conformal symmetries and plays a key role in the understanding of elementary particle vacuum functionals and the description of family replication phenomenon in terms of the topology of partonic 2-surfaces.

5. Kähler-Dirac equation gives also rise to a hierarchy super-conformal algebras assignable to zero modes. These algebras follow from the existence of conserved fermionic currents. The corresponding deformations of the space-time surface correspond to vanishing second variations of Kähler action and provide a realization of quantum criticality. This led to a breakthrough in the understanding of the Kähler-Dirac action via the addition of a measurement interaction term to the action allowing to obtain among other things stringy propagator and the coding of quantum numbers of super-conformal representations to the geometry of space-time surfaces required by quantum classical correspondence.

   A second breakthrough came from the realization that the well-definedness of em charge forces in the generic situation localization of the modes to 2- surfaces at which induced W fields and also Z^0 fields above weak scale vanish.

6. The effective 2-dimensionality of the space-like 3-surfaces realizing quantum holography can be formulated as a symmetry stating that the replacement of wormhole throat by any light-like 3-surfaces parallel to it in the slicing of the space-time sheet induces only a gauge transformation of WCW Kähler function adding to it a real part of a holomorphic function of complex coordinate of WCW depending also on zero modes. This means that the Kähler metric of WCW remains invariant. It is also postulated that measurement interaction added to the Kähler-Dirac action induces similar gauge symmetry.

7. The study of the Kähler-Dirac equation leads to a detailed identification of super charges of the super-conformal algebras relevant for TGD [K97]: these results represent the most recent layer in the development of ideas about supersymmetry in TGD Universe. Whereas many considerations related to supersymmetry represented earlier rely on general arguments, the results deriving from the Kähler-Dirac equation are rather concrete and clarify the crucial role of the right-handed neutrino in TGD based realization of super-conformal symmetries. $\mathcal{N} = 1$ SUSY- now almost excluded at LHC - is not possible in TGD because it requires Majorana spinors. Also $\mathcal{N} = 2$ variant of the standard space-time SUSY seems to be excluded in TGD Universe. Fermionic oscillator operators for the induced spinor fields restricted to 2-D surfaces however generate large $\mathcal{N}$ SUSY and super-conformal algebra and the modes of right-handed neutrino its 4-D version.

7.1.2 P-Adic Physics As Physics Of Cognition

p-Adic mass calculations relying on p-adic length scale hypothesis led to an understanding of elementary particle masses using only super-conformal symmetries and p-adic thermodynamics. The need to fuse real physics and various p-adic physics to single coherent whole led to a generalization of the notion of number obtained by gluing together reals and p-adics together along common rationals and algebraics. The interpretation of p-adic space-time sheets is as correlates for cognition. p-Adic and real space-time sheets intersect along common rationals and algebraics and the subset of these points defines what I call number theoretic braid in terms of which both WCW geometry and S-matrix elements should be expressible. Thus one would obtain number theoretical discretization, which involves no ad hoc elements and is inherent to the physics of TGD.

The original idea was that the notion of number theoretic braid could pose strong number theoretic conditions on physics just as p-adic thermodynamics poses on elementary particle mass spectrum. A practically oriented physicist would argue that general braids must be allowed if one wants to calculate something and that number theoretic braids represent only the intersection between the real and various p-adic physics. He could also insist that at the level of WCW various sectors must be realized in a more abstract manner - say as hierarchies of polynomials with coefficients belonging to various extensions or rationals so that one can speak about surfaces common to real and various p-adic sectors. In this view the fusion of various physics would be analogous to the completion of rationals to various number fields.

Perhaps the most dramatic implication relates to the fact that points, which are p-adically infinitesimally close to each other, are infinitely distant in the real sense (recall that real and p-adic imbedding spaces are glued together along rational imbedding space points). This means that any open set of p-adic space-time sheet is discrete and of infinite extension in the real sense. This means that cognition is a cosmic phenomenon and involves always discretization from the point of view of the real topology. The testable physical implication of effective p-adic topology of real space-time sheets is p-adic fractality meaning characteristic long range correlations combined with short range chaos.

Also a given real space-time sheets should correspond to a well-defined prime or possibly several of them. The classical non-determinism of Kähler action should correspond to p-adic non-determinism for some prime(s) p in the sense that the effective topology of the real space-time sheet is p-adic in some length scale range. p-Adic space-time sheets with same prime should have many common rational points with the real space-time and be easily transformable to the real space-time sheet in quantum jump representing intention-to-action transformation. The concrete model for the transformation of intention to action leads to a series of highly non-trivial number theoretical conjectures assuming that the extensions of p-adics involved are finite-dimensional and can contain also transcendentals.

An ideal realization of the space-time sheet as a cognitive representation results if the CP_2 coordinates as functions of M^4_+ coordinates have the same functional form for reals and various p-adic number fields and that these surfaces have discrete subset of rational numbers with upper and lower length scale cutoffs as common. The hierarchical structure of cognition inspires the idea that S-matrices form a hierarchy labeled by primes p and the dimensions of algebraic extensions.

The number-theoretic hierarchy of extensions of rationals appears also at the level of WCW spinor fields and allows to replace the notion of entanglement entropy based on Shannon entropy with its number theoretic counterpart having also negative values in which case one can speak about genuine information. In this case case entanglement is stable against Negentropy Maximization Principle stating that entanglement entropy is minimized in the self measurement and can be regarded as bound state entanglement. Bound state entanglement makes possible macro-temporal quantum coherence. One can say that rationals and their finite-dimensional extensions define islands of order in the chaos of continua and that life and intelligence correspond to these islands.

TGD inspired theory of consciousness and number theoretic considerations inspired for years ago the notion of infinite primes [K72]. It came as a surprise, that this notion might have direct relevance for the understanding of mathematical cognition. The idea is very simple. There is infinite hierarchy of infinite rationals having real norm one but different but finite p-adic norms. Thus single real number (complex number, (hyper-)quaternion, (hyper-)octonion) corresponds to an algebraically infinite-dimensional space of numbers equivalent in the sense of real topology. Space-time and imbedding space points become infinitely structured and single space-time point would represent the Platonia of mathematical ideas. This structure would be completely invisible at the level of real physics but would be crucial for mathematical cognition and explain why we are able to imagine also those mathematical structures which do not exist physically. Space-time could be also regarded as an algebraic hologram. The connection with Brahman=Atman idea is also obvious.

7.1.3 Hierarchy Of Planck Constants And Dark Matter Hierarchy

The realization for the hierarchy of Planck constants proposed as a solution to the dark matter puzzles leads to a profound generalization of quantum TGD through a generalization of the notion of imbedding space to characterize quantum criticality. The resulting space has a book like structure with various almost-copies of the imbedding space representing the pages of the book meeting at quantum critical sub-manifolds. A particular page of the book can be seen as an n-fold singular covering or factor space of CP_2 or of a causal diamond (CD) of M^4 defined as an intersection of the future and past directed light-cones. Therefore the cyclic groups Z_n appear as discrete symmetry groups.

The original intuition was the the space-time would be n-sheeted for $h_{eff} = n$. Quantum criticality expected on basis of the vacuum degeneracy of Kähler action suggests that conformal symmetries act as critical deformations respecting the light-likeness of partonic orbits at which the signature of the induced metric changes from Minkowskian to Euclidian. Therefore one would have n conformal equivalence classes of physically equivalent space-time sheets. A hierarchy of breakings of conformal symmetry is expected on basis of ordinary catastrophe theory. These breakings would correspond to the hierarchy defined by the sub-algebras of conformal algebra or associated algebra for which conformal weights are divisible by n. This defines infinite number of inclusion hierarchies $.. \subset C(n_1) \subset C(n_3)...$ such that n_{i+1} divides n_i. These hierarchies could correspond to inclusion hierarchies of hyper-finite factors and conformal algebra acting as gauge transformations would naturally define the notion of finite measurement resolution.

This topic will not be discussed in this chapter since it is discussed in earlier chapter [K102].

7.1.4 Number Theoretical Symmetries

TGD as a generalized number theory vision leads to the idea that also number theoretical symmetries are important for physics.

1. There are good reasons to believe that the strands of number theoretical braids can be assigned with the roots of a polynomial with suggests the interpretation corresponding Galois groups as purely number theoretical symmetries of quantum TGD. Galois groups are subgroups of the permutation group S_∞ of infinitely manner objects acting as the Galois group of algebraic numbers. The group algebra of S_∞ is HFF which can be mapped to the HFF defined by WCW spinors. This picture suggest a number theoretical gauge invariance stating that S_∞ acts as a gauge group of the theory and that global gauge transformations in its completion correspond to the elements of finite Galois groups represented as diagonal groups of $G \times G \times ....$ of the completion of S_∞.

2. HFFs inspire also an idea about how entire TGD emerges from classical number fields, actually their complexifications. In particular, SU(3) acts as subgroup of octonion automorphisms leaving invariant preferred imaginary unit. If space-time surfaces are hyper-quaternionic (meaning that the octonionic counterparts of the Kähler-Dirac gamma matrices span complex quaternionic sub-algebra of octonions) and contain at each point a preferred plane M^2 of M^4, one ends up with $M^8 - H$ duality stating that space-time surfaces can be equivalently regarded as surfaces in M^8 or $M^4 \times CP_2$. One can actually generalize M^2 to a two-dimensional Minkowskian sub-manifold of M^4. One ends up with quantum TGD by considering associative sub-algebras of the local octonionic Clifford algebra of M^8 or H. so that TGD could be seen as a generalized number theory.

This idea will not be discussed in this chapter since it has better place in the book about physics as generalized number theory [K71].

The appendix of the book gives a summary about basic concepts of TGD with illustrations. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [L12].

7.2 Symmetries

The most general expectation is that WCW can be regarded as a union of coset spaces which are infinite-dimensional symmetric spaces with Kähler structure: $C(H) = \cup_i G/H(i)$.

Index i labels 3-topology and zero modes. The group G, which can depend on 3-surface, can be identified as a subgroup of diffeomorphisms of $\delta M^4_+ \times CP_2$ and H must contain as its subgroup a group, whose action reduces to $Diff(X^3)$ so that these transformations leave 3-surface invariant.

The task is to identify plausible candidate for G and H and to show that the tangent space of WCW allows Kähler structure, in other words that the Lie-algebras of G and $H(i)$ allow complexification. One must also identify the zero modes and construct integration measure for the functional integral in these degrees of freedom. Besides this one must deduce information about the explicit form of WCW metric from symmetry considerations combined with the hypothesis that Kähler function is Kähler action for a preferred extremal of Kähler action. One must of course understand what “preferred” means.

7.2.1 General Coordinate Invariance And Generalized Quantum Gravitational Holography

The basic motivation for the construction of WCW geometry is the vision that physics reduces to the geometry of classical spinor fields in the infinite-dimensional configuration space of 3-surfaces of $M^4_+ \times CP_2$ or of $M^4 \times CP_2$. Hermitian conjugation is the basic operation in quantum theory and its geometrization requires that WCW possesses Kähler geometry. Kähler geometry is coded into Kähler function.

The original belief was that the four-dimensional general coordinate invariance of Kähler function reduces the construction of the geometry to that for the boundary of configuration space consisting of 3-surfaces on $\delta M^4_+ \times CP_2$, the moment of big bang. The proposal was that Kähler function $K(Y^3)$ could be defined as a preferred extremal of so called Kähler action for the unique space-time surface $X^4(Y^3)$ going through given 3-surface Y^3 at $\delta M^4_+ \times CP_2$. For Diff^4 transforms of Y^3 at $X^4(Y^3)$ Kähler function would have the same value so that Diff^4 invariance and degeneracy would be the outcome. The proposal was that the preferred extremals are absolute minima of Kähler action.

This picture turned out to be too simple.

1. I have already described the recent view about light-like 3-surfaces as generalized Feynman diagrams and space-time surfaces as preferred extremals of Kähler action and will not repeat what has been said.

2. It has also become obvious that the gigantic symmetries associated with $\delta M^4_\pm \times CP_2 \subset CD \times CP_2$ manifest themselves as the properties of propagators and vertices. Cosmological considerations, Poincare invariance, and the new view about energy favor the decomposition of WCW to a union of configuration spaces assignable to causal diamonds CDs defined as intersections of future and past directed light-cones. The minimum assumption is that CDs label the sectors of CH: the nice feature of this option is that the considerations of this chapter restricted to $\delta M^4_+ \times CP_2$ generalize almost trivially. This option is beautiful because the center of mass degrees of freedom associated with the different sectors of CH would correspond to M^4 itself and its Cartesian powers.

The definition of the Kähler function requires that the many-to-one correspondence $X^3 \rightarrow X^4(X^3)$ must be replaced by a bijective correspondence in the sense that X^3_l as light-like 3-surface is unique among all its Diff^4 translates. This also allows physically preferred "gauge fixing" allowing to get rid of the mathematical complications due to Diff4 degeneracy. The internal geometry of the space-time sheet must define the preferred 3-surface X^3_l.

The realization of this vision means a considerable mathematical challenge. The effective metric 2-dimensionality of 3-dimensional light-like surfaces X^3_l of M^4 implies generalized conformal and symplectic symmetries allowing to generalize quantum gravitational holography from light like boundary so that the complexities due to the non-determinism can be taken into account properly.

7.2.2 Light Like 3-D Causal Determinants And Effective2-Dimensionality

The light like 3-surfaces X^3_l of space-time surface appear as 3-D causal determinants. Basic examples are boundaries and elementary particle horizons at which Minkowskian signature of the induced metric transforms to Euclidian one. This brings in a second conformal symmetry (see **Fig. 7.1**) related to the metric 2-dimensionality of the 3-D light-like 3-surface. This symmetry is identifiable as TGD counterpart of the Kac Moody symmetry of string models. The challenge is to understand the relationship of this symmetry to WCW geometry and the interaction between the two conformal symmetries.

1. Field-particle duality is realized. Light-like 3-surfaces X^3_l -generalized Feynman diagrams - correspond to the particle aspect of field-particle duality whereas the physics in the interior of space-time surface $X^4(X^3_l)$ would correspond to the field aspect. Generalized Feynman diagrams in 4-D sense could be identified as regions of space-time surface having Euclidian signature.

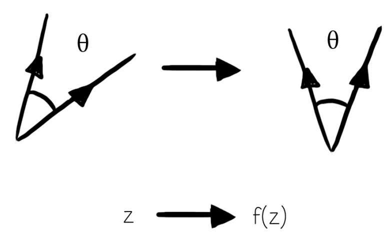

Figure 7.1: Conformal symmetry preserves angles in complex plane

2. One could also say that light-like 3-surfaces X_l^3 and the space-like 3-surfaces X^3 in the intersections of $X^4(X_l^3) \cap CD \times CP_2$ where the causal diamond CD is defined as the intersections of future and past directed light-cones provide dual descriptions.

3. Generalized coset construction implies that the differences of super-symplectic and Super Kac-Moody type Super Virasoro generators annihilated physical states. This construction in turn led to the realization that WCW for fixed values of zero modes - in particular the values of the induced Kähler form of $\delta M_\pm^4 \times CP_2$ - allows identification as a coset space obtained by dividing the symplectic group of $\delta M_\pm^4 \times CP_2$ with Kac-Moody group, whose generators vanish at $X^2 = X_l^3 \times \delta M_\pm^4 \times CP_2$. One can say that quantum fluctuating degrees of freedom in a very concrete sense correspond to the local variant of $S^2 \times CP_2$.

The analog of conformal invariance in the light-like direction of X_l^3 and in the light-like radial direction of $\delta M_\pm^4$ implies that the data at either X^3 or X_l^3 should be enough to determine WCW geometry. This implies that the relevant data is contained to their intersection X^2 at least for finite regions of X^3. This is the case if the deformations of X_l^3 not affecting X^2 and preserving light-likeness corresponding to zero modes or gauge degrees of freedom and induce deformations of X^3 also acting as zero modes. The outcome is effective 2-dimensionality. One must be however cautious in order to not make over-statements. The reduction to 2-D theory in global sense would trivialize the theory and the reduction to 2-D theory must takes places for finite region of X^3 only so one has in well defined sense three-dimensionality in discrete sense. A more precise formulation of this vision is in terms of hierarchy of CDs containing CDs containing.... The introduction of sub-CD: s brings in improved measurement resolution and means also that effective 2-dimensionality is realized in the scale of sub-CD only.

One cannot over-emphasize the importance of the effective 2-dimensionality. What was regarded originally as a victory was that it simplifies dramatically the earlier formulas for WCW metric involving 3-dimensional integrals over $X^3 \subset M_+^4 \times CP_2$ reducing now to 2-dimensional integrals. One can of course criticize so strong form of effective 2-dimensionality as unphysical. As often happens, the later progress led to the comeback of the formulation involving 3-surfaces! The stringy picture implied by the solutions of Kähler-Dirac action led to the 3-D picture with effective 2-dimensionality realized in terms of super conformal symmetries.

7.2.3 Magic Properties Of Light Cone Boundary And Isometries OfWCW

The special conformal, metric and symplectic properties of the light cone of four-dimensional Minkowski space: δM_+^4, the boundary of four-dimensional light cone is metrically 2-dimensional(!) sphere allowing infinite-dimensional group of conformal transformations and isometries(!) as well as Kähler structure. Kähler structure is not unique: possible Kähler structures of light cone boundary are paramet3rized by Lobatchevski space $SO(3,1)/SO(3)$. The requirement that the isotropy group $SO(3)$ of S^2 corresponds to the isotropy group of the unique classical 3-momentum assigned to $X^4(Y^3)$ defined as a preferred extremum of Kähler action, fixes the choice of the complex structure uniquely. Therefore group theoretical approach and the approach based on Kähler action complement each other.

1. The allowance of an infinite-dimensional group of isometries isomorphic to the group of conformal transformations of 2-sphere is completely unique feature of the 4-dimensional light cone boundary. Even more, in case of $\delta M^4_+ \times CP_2$ the isometry group of δM^4_+ becomes localized with respect to CP_2! Furthermore, the Kähler structure of δM^4_+ defines also symplectic structure.

   Hence any function of $\delta M^4_+ \times CP_2$ would serve as a Hamiltonian transformation acting in both CP_2 and δM^4_+ degrees of freedom. These transformations obviously differ from ordinary local gauge transformations. This group leaves the symplectic form of $\delta M^4_+ \times CP_2$, defined as the sum of light cone and CP_2 symplectic forms, invariant. The group of symplectic transformations of $\delta M^4_+ \times CP_2$ is a good candidate for the isometry group of WCW .

2. The approximate symplectic invariance of Kähler action is broken only by gravitational effects and is exact for vacuum extremals. If Kähler function were exactly invariant under the symplectic transformations of CP_2, CP_2 symplectic transformations wiykd correspond to zero modes having zero norm in the Kähler metric of WCW . This does not make sense since symplectic transformations of $\delta M^4 \times CP_2$ actually parameterize the quantum fluctuation degrees of freedom.

3. The groups G and H, and thus WCW itself, should inherit the complex structure of the light cone boundary. The diffeomorphims of M^4 act as dynamical symmetries of vacuum extremals. The radial Virasoro localized with respect to $S^2 \times CP_2$ could in turn act in zero modes perhaps inducing conformal transformations: note that these transformations lead out from the symmetric space associated with given values of zero modes.

7.2.4 Symplectic Transformations Of $\Delta M^4_+ \times CP_2$ As Isometries Of WCW

The symplectic transformations of $\delta M^4_+ \times CP_2$ are excellent candidates for inducing symplectic transformations of the WCW acting as isometries. There are however deep differences with respect to the Kac Moody algebras.

1. The conformal algebra of WCW is gigantic when compared with the Virasoro + Kac Moody algebras of string models as is clear from the fact that the Lie-algebra generator of a symplectic transformation of $\delta M^4_+ \times CP_2$ corresponding to a Hamiltonian which is product of functions defined in δM^4_+ and CP_2 is sum of generator of δM^4_+-local symplectic transformation of CP_2 and CP_2-local symplectic transformations of δM^4_+. This means also that the notion of local gauge transformation generalizes.

2. The physical interpretation is also quite different: the relevant quantum numbers label the unitary representations of Lorentz group and color group, and the four-momentum labeling the states of Kac Moody representations is not present. Physical states carrying no energy and momentum at quantum level are predicted. The appearance of a new kind of angular momentum not assignable to elementary particles might shed some light to the longstanding problem of baryonic spin (quarks are not responsible for the entire spin of proton). The possibility of a new kind of color might have implications even in macroscopic length scales.

3. The central extension induced from the natural central extension associated with $\delta M^4_+ \times CP_2$ Poisson brackets is anti-symmetric with respect to the generators of the symplectic algebra rather than symmetric as in the case of Kac Moody algebras associated with loop spaces. At first this seems to mean a dramatic difference. For instance, in the case of CP_2 symplectic transformations localized with respect to δM^4_+ the central extension would vanish for Cartan algebra, which means a profound physical difference. For $\delta M^4_+ \times CP_2$ symplectic algebra a generalization of the Kac Moody type structure however emerges naturally.

   The point is that δM^4_+-local CP_2 symplectic transformations are accompanied by CP_2 local δM^4_+ symplectic transformations. Therefore the Poisson bracket of two δM^4_+ local CP_2 Hamiltonians involves a term analogous to a central extension term symmetric with respect to CP_2 Hamiltonians, and resulting from the δM^4_+ bracket of functions multiplying the Hamiltonians. This additional term could give the entire bracket of the WCW Hamiltonians at the maximum of the Kähler function where one expects that CP_2 Hamiltonians vanish and have a form essentially identical with Kac Moody central extension because it is indeed symmetric with respect to indices of the symplectic group.

7.2.5 Does The Symmetric Space Property Correspond To Coset Construction For Super Virasoro Algebras?

The idea about symmetric space is extremely beautiful but it took a long time and several false alarms before the time was ripe for identifying the precise form of the Cartan decomposition $g = t + h$ satisfying the defining conditions

$$g = t + h \ , \quad [t,t] \subset h \ , \quad [h,t] \subset t \ . \tag{7.2.1}$$

The ultimate solution of the puzzle turned out to be amazingly simple and came only after quantum TGD was understood well enough.

1. WCW geometry allows two super-conformal symmetries. The first one corresponds to super-symplectic transformations acting at the level of imbedding space. The second one corresponds to super Kac-Moody symmetry acting as deformations of light-like 3-surfaces respecting their light-likeness.

2. It took considerable amount of trials and errors to realize that both symplectic and Kac-Moody algebras are needed to generate the entire isometry algebra g. h is sub-algebra of this extended algebra. In general case the elements of both algebras are non-vanishing at the prefered partonic 2-surfaces considered.

3. Strong form of holography implies that transformations located to the interior of space-like 3-surface and light-like partonic orbit define zero modes and act like gauge symmetries. The physically non-trivial transformations correspond to transformations acting non-trivially at space-like 3-surfaces. g corresponds to the algebra generated by these transformations. For preferred p3-surface - identified as (say) maximum of Kähler function - h corresponds to the elements of this algebra reducing to infinitesimal diffeomorphisms.

4. Coset representation has five tensor factors as required by p-adic mass calculations and they correspond to color algebra, to two factors from electroweak $U(2)$, to one factor from transversal M^4 translations and one factor from symplectic algebra (note that also Hamiltonians which are products of δM^4_+ and CP_2 Hamiltonians are possible.

5. The realization of WCW sectors with fixed values of zero modes as symmetric spaces G/H (analogous to $CP_2 = SU(3)/U(2)$) suggests that one can assign super-Virasoro algebras with G and H as a generalized coset representation for g and h so that the differences of the generators of two super Virasoro algebras annihilate the physical states for coset representations. This obviously generalizes Goddard-Olive-Kent construction [A61]. It however does not imply Equivalence Principle as believed for a long time.

7.2.6 Symplectic And Kac-Moody Algebras As Basic Building Bricks

Concerning the interpretation of the relationship between symplectic and Kac-Moody algebra there are some poorly understood points, which directly relate to what one means with precise interpretation of strong form of holography.

The basic building bricks are symplectic algebra of δCD (this includes CP_2 besides light-cone boundary) and Kac-Moody algebra assignable to the isometries of δCD [K15]. It seems however that the longheld view about the role of Kac-Moody algebra must be modified. Also the earlier realization of super-Hamiltonians and Hamiltonians seems too naive.

1. I have been accustomed to think that Kac-Moody algebra could be regarded as a sub-algebra of symplectic algebra. p-Adic mass calculations however requires five tensor factors for the coset representation of Super Virasoro algebra naturally assigned to the coset structure G/H of a sector of WCW with fixed zero modes. Therefore Kac-Moody algebra cannot be regarded as a sub-algebra of symplectic algebra giving only single tensor factor and thus inconsistent with interpretation of p-adic mass calculations.

2. The localization of Kac-Moody algebra generators with respect to the internal coordinates of light-like 3-surface taking the role of complex coordinate z in conformal field theory is also questionable: the most economical option relies on localization with respect to light-like radial coordinate of light-cone boundary as in the case of symplectic algebra. Kac-Moody algebra cannot be however sub-algebra of the symplectic algebra assigned with covariantly constant right-handed neutrino in the earlier approach.

3. Right-handed covariantly constant neutrino as a generator of super symmetries plays a key role in the earlier construction of symplectic super-Hamiltonians. What raises doubts is that other spinor modes - both those of right-handed neutrino and electro-weakly charged spinor modes - are absent. All spinor modes should be present and thus provide direct mapping from WCW geometry to WCW spinor fields in accordance with super-symmetry and the general idea that WCW geometry is coded by WCW spinor fields.

   Hence it seems that Kac-Moody algebra must be assigned with the modes of the induced spinor field which carry electroweak quantum numbers. If would be natural that the modes of right-handed neutrino having no weak and color interactions would generate the huge symplectic algebra of symmetries and that the modes of fermions with electroweak charges generate much smaller Kac-Moody algebra.

4. The dynamics of Kähler action and Kähler-Dirac action action are invisible in the earlier construction. This suggests that the definition of WCW Hamiltonians is too simplistic. The proposal is that the conserved super charges derivable as Noether charges and identifiable as super-Hamiltonians define WCW metric and Hamiltonians as their anti-commutators. Spinor modes would become labels of Hamiltonians and WCW geometry relates directly to the dynamics of elementary particles.

5. Note that light-cone boundary $\delta M^4_+ = S^2\times R_+$ allows infinite-dimensional group of isometries consisting of conformal transformation of the sphere S^2 with conformal scaling compensated by an S^2 local scaling or the light-like radial coordinate of R_+. These isometries contain as a subgroup symplectic isometries and could act as gauge symmetries of the theory.

Gauge symmetry property means that the Kähler metric of the WCW is same for all choices of preferred X^3. Kähler function would however differ by a real part of a holomorphic function of WCW coordinates for different choices of preferred X^3.

Strong form of holography (or strong form of GCI) implies that one can take either space-like or light-like 3-surfaces as basic objects and consider the action the super-symplectic algebra also for the light-like 3-surfaces. This is possible by just parallelly translating the light-like boundary of CD so that one obtains slicing of CD by these light-like 3-surfaces. The equality of four-momenta associated with the two super-conformal representations might allow interpretation in terms of equivalence of gravitational and inertial four-momenta.

7.2.7 Comparison Of TGD And Stringy Views About Super-Conformal Symmetries

The best manner to represent TGD based view about conformal symmetries is by comparison with the conformal symmetries of super string models.

Basic differences between the realization of super conformal symmetries in TGD and in super-string models

The realization super conformal symmetries in TGD framework differs from that in string models in several fundamental aspects.

1. In TGD framework super-symmetry generators acting as configuration space gamma matrices carry either lepton or quark number. Majorana condition required by the hermiticity of super generators which is crucial for super string models would be in conflict with the conservation of baryon and lepton numbers and is avoided. This is made possible by the realization of bosonic generators represented as Hamiltonians of X^2-local symplectic transformations rather than vector fields generating them [K15]. This kind of representation

applies also in Kac-Moody sector since the local transversal isometries localized in X_l^3 and respecting light-likeness condition can be regarded as X^2 local symplectic transformations, whose Hamiltonians generate also isometries. Localization is not complete: the functions of X^2 coordinates multiplying symplectic and Kac-Moody generators are functions of the symplectic invariant $J = \epsilon^{\mu\nu} J_{\mu\nu}$ so that effective one-dimensionality results but in different sense than in conformal field theories. This realization of super symmetries is what distinguishes between TGD and super string models and leads to a totally different physical interpretation of super-conformal symmetries. The fermionic representations of super-symplectic and super Kac-Moody generators can be identified as Noether charges in standard manner.

2. A long-standing problem of quantum TGD was that stringy propagator $1/G$ does not make sense if G carries fermion number. The progress in the understanding of second quantization of the modified Dirac operator made it however possible to identify the counterpart of G as a c-number valued operator and interpret it as different representation of G [K13].

3. The notion of super-space is not needed at all since Hamiltonians rather than vector fields represent bosonic generators, no super-variant of geometry is needed. The distinction between Ramond and N-S representations important for $N = 1$ super-conformal symmetry and allowing only ground state weight 0 an $1/2$ disappears. Indeed, for $N = 2$ super-conformal symmetry it is already possible to generate spectral flow transforming these Ramond and N-S representations to each other (G_n is not Hermitian anymore).

4. If Kähler action defines the Kähler-Dirac operator, the number of spinor modes could be finite. One must be here somewhat cautious since bound state in the Coulomb potential associated with electric part of induced electro-weak gauge field might give rise to an infinite number of bound states which eigenvalues converging to a fixed eigenvalue (as in the case of hydrogen atom). Finite number of generalized eigenmodes means that the representations of super-conformal algebras reduces to finite-dimensional ones in TGD framework. Also the notion of number theoretic braid indeed implies this. The physical interpretation would be in terms of finite measurement resolution. If Kähler action is complexified to include imaginary part defined by CP breaking instanton term, the number of stringy mass square eigenvalues assignable to the spinor modes becomes infinite since conformal excitations are possible. This means breakdown of exact holography and effective 2-dimensionality of 3-surfaces. It seems that the inclusion of instanton term is necessary for several reasons. The notion of finite measurement resolution forces conformal cutoff also now. There are arguments suggesting that only the modes with vanishing conformal weight contribute to the Dirac determinant defining vacuum functional identified as exponent of Kähler function in turn identified as Kähler action for its preferred extremal.

5. What makes spinor field mode a generator of gauge super-symmetry is that is c-number and not an eigenmode of $D_K(X^2)$ and thus represents non-dynamical degrees of freedom. If the number of eigen modes of $D_K(X^2)$ is indeed finite means that most of spinor field modes represent super gauge degrees of freedom.

The super generators G are not Hermitian in TGD!

The already noticed important difference between TGD based and the usual Super Virasoro representations is that the Super Virasoro generator G cannot Hermitian in TGD. The reason is that WCW gamma matrices possess a well defined fermion number. The hermiticity of the WCW gamma matrices Γ and of the Super Virasoro current G could be achieved by posing Majorana conditions on the second quantized H-spinors. Majorana conditions can be however realized only for space-time dimension $D \ mod\ 8 = 2$ so that super string type approach does not work in TGD context. This kind of conditions would also lead to the non-conservation of baryon and lepton numbers.

An analogous situation is encountered in super-symmetric quantum mechanics, where the general situation corresponds to super symmetric operators S, $S^\dagger$, whose anti-commutator is Hamiltonian: $\{S, S^\dagger\} = H$. One can define a simpler system by considering a Hermitian operator

$S_0 = S + S^\dagger$ satisfying $S_0^2 = H$: this relation is completely analogous to the ordinary Super Virasoro relation $GG = L$. On basis of this observation it is clear that one should replace ordinary Super Virasoro structure $GG = L$ with $GG^\dagger = L$ in TGD context.

It took a long time to realize the trivial fact that $N = 2$ super-symmetry is the standard physics counterpart for TGD super symmetry. $N = 2$ super-symmetry indeed involves the doubling of super generators and super generators carry $U(1)$ charge having an interpretation as fermion number in recent context. The so called short representations of $N = 2$ super-symmetry algebra can be regarded as representations of $N = 1$ super-symmetry algebra.

WCW gamma matrix Γ_n, $n > 0$ corresponds to an operator creating fermion whereas Γ_n, $n < 0$ annihilates anti-fermion. For the Hermitian conjugate $\Gamma_n^\dagger$ the roles of fermion and anti-fermion are interchanged. Only the anti-commutators of gamma matrices and their Hermitian conjugates are non-vanishing. The dynamical Kac Moody type generators are Hermitian and are constructed as bilinears of the gamma matrices and their Hermitian conjugates and, just like conserved currents of the ordinary quantum theory, contain parts proportional to $a^\dagger a$, $b^\dagger b$, $a^\dagger b^\dagger$ and ab (a and b refer to fermionic and anti-fermionic oscillator operators). The commutators between Kac Moody generators and Kac Moody generators and gamma matrices remain as such.

For a given value of m G_n, $n > 0$ creates fermions whereas G_n, $n < 0$ annihilates anti-fermions. Analogous result holds for $G_n^\dagger$. Virasoro generators remain Hermitian and decompose just like Kac Moody generators do. Thus the usual anti-commutation relations for the super Virasoro generators must be replaced with anti-commutations between G_m and $G_n^\dagger$ and one has

$$\begin{aligned} \{G_m, G_n^\dagger\} &= 2L_{m+n} + \frac{c}{3}(m^2 - \frac{1}{4})\delta_{m,-n} \ , \\ \{G_m, G_n\} &= 0 \ , \\ \{G_m^\dagger, G_n^\dagger\} &= 0 \ . \end{aligned} \qquad (7.2.2)$$

The commutators of type $[L_m, L_n]$ are not changed. Same applies to the purely kinematical commutators between L_n and $G_m/G_m^\dagger$.

The Super Virasoro conditions satisfied by the physical states are as before in case of L_n whereas the conditions for G_n are doubled to those of G_n, $n < 0$ and $G_n^\dagger$, $n > 0$.

What could be the counterparts of stringy conformal fields in TGD framework?

The experience with string models would suggest the conformal symmetries associated with the complex coordinates of X^2 as a candidate for conformal super-symmetries. One can imagine two counterparts of the stringy coordinate z in TGD framework.

1. Super-symplectic and super Kac-Moody symmetries are local with respect to X^2 in the sense that the coefficients of generators depend on the invariant $J = \epsilon^{\alpha\beta} J_{\alpha\beta}\sqrt{g_2}$ rather than being completely free [K15]. Thus the real variable J replaces complex (or hyper-complex) stringy coordinate and effective 1-dimensionality holds true also now but in different sense than for conformal field theories.

2. The slicing of X^4 by string world sheets Y^2 and partonic 2-surfaces X^2 implied by number theoretical compactification implies string-parton duality and involves the super conformal fermionic gauge symmetries associated with the coordinates u and w in the dual dimensional reductions to stringy and partonic dynamics. These coordinates define the natural analogs of stringy coordinate. The effective reduction of X_l^3 to braid by finite measurement resolution implies the effective reduction of $X^4(X^3)$ to string world sheet. This implies quite strong resemblance with string model. The realization that spinor modes with well- define em charge must be localized at string world sheets makes the connection with strings even more explicit [K88].

   One can understand how Equivalence Principle emerges in TGD framework at space-time level when many-sheeted space-time (see **Fig.** `http://tgdtheory.fi/appfigures/manysheeted.jpg` or **Fig.** 9 in the appendix of this book) is replaced with effective space-time lumping together the space-time sheets to M^4 endowed with effective metric. The quantum counterpart EP has most feasible interpretation in terms of Quantum Classical Correspondence (QCC): the conserved Kähler four-momentum equals to an eigenvalue of conserved Kähler-Dirac four-momentum acting as operator.

3. The conformal fields of string model would reside at X^2 or Y^2 depending on which description one uses and complex (hyper-complex) string coordinate would be identified accordingly. Y^2 could be fixed as a union of stringy world sheets having the strands of number theoretic braids as its ends. The proposed definition of braids is unique and characterizes finite measurement resolution at space-time level. X^2 could be fixed uniquely as the intersection of X^3_l (the light-like 3-surface at which induced metric of space-time surface changes its signature) with $\delta M^4_\pm \times CP_2$. Clearly, wormhole throats X^3_l would take the role of branes and would be connected by string world sheets defined by number theoretic braids.

4. An alternative identification for TGD parts of conformal fields is inspired by $M^8 - H$ duality. Conformal fields would be fields in WCW . The counterpart of z coordinate could be the hyper-octonionic M^8 coordinate m appearing as argument in the Laurent series of WCW Clifford algebra elements. m would characterize the position of the tip of CD and the fractal hierarchy of CDs within CDs would give a hierarchy of Clifford algebras and thus inclusions of hyper-finite factors of type II_1. Reduction to hyper-quaternionic field -that is field in M^4 center of mass degrees of freedom- would be needed to obtained associativity. The arguments m at various level might correspond to arguments of N-point function in quantum field theory.

7.3 WCW As A Union Of Homogenous Or Symmetric Spaces

The physical interpretation and detailed mathematical understanding of super-conformal symmetries has developed rather slowly and has involved several side tracks. In the following I try to summarize the basic picture with minimal amount of formulas with the understanding that the statement "Noether charge associated with geometrically realized Kac-Moody symmetry" is enough for the reader to write down the needed formula explicitly. Formula oriented reader might deny the value of the approach giving weight to principles. My personal experience is that piles of formulas too often hide the lack of real understanding.

In the following the vision about WCW as union of coset spaces is discussed in more detail.

7.3.1 Basic Vision

The basic view about coset space construction for WCW has not changed.

1. The idea about WCW as a union of coset spaces G/H labelled by zero modes is extremely attractive. The structure of homogenous space [A7] (`http://tinyurl.com/y7u2t8jo`) means at Lie algebra level the decomposition $g = h \oplus t$ to sub-Lie-algebra h and its complement t such that $[h,t] \subset t$ holds true. Homogeneous spaces have G as its isometries. For symmetric space the additional condition $[t,t] \subset h$ holds true and implies the existence of involution changing at the Lie algebra level the sign of elements of t and leaving the elements of h invariant. The assumption about the structure of symmetric space [A22] (`http://tinyurl.com/ycouv7uh`) implying covariantly constant curvature tensor is attractive in infinite-dimensional case since it gives hopes about calculability.

   An important source of intuition is the analogy with the construction of CP_2, which is symmetric space A particular choice of h corresponds to Lie-algebra elements realized as Killing vector fields which vanish at particular point of WCW and thus leave 3-surface invariant. A preferred choice for this point is as maximum or minimum of Kähler function. For this 3-surface the Hamiltonians of h should be stationary. If symmetric space property holds true then commutators of $[t,t]$ also vanish at the minimum/maximum. Note that Euclidian signature for the metric of WCW requires that Kähler function can have only maximum or minimum for given zero modes.

2. The basic objection against TGD is that one cannot use the powerful canonical quantization using the phase space associated with configuration space - now WCW . The reason is the extreme non-linearity of the Kähler action and its huge vacuum degeneracy, which do not allow the construction of Hamiltonian formalism. Symplectic and Kähler structure must be realized at the level of WCW . In particular, Hamiltonians must be represented in completely

new manner. The key idea is to construct WCW Hamiltonians as anti-commutators of super-Hamiltonians defining the contractions of WCW gamma matrices with corresponding Killing vector fields and therefore defining the matrix elements of WCW metric in the tangent vector basis defined by Killing vector fields. Super-symmetry therefre gives hopes about constructing quantum theory in which only induced spinor fields are second quantized and imbedding space coordinates are treated purely classically.

3. It is important to understand the difference between symmetries and isometries assigned to the Kähler function. Symmetries of Kähler function do not affect it. The symmetries of Kähler action are also symmetries of Kähler action because Kähler function is Kähler action for a preferred extremal (here there have been a lot of confusion). Isometries leave invariant only the quadratic form defined by Kähler metric $g_{M\overline{N}} = \partial_M \partial_{\overline{L}} K$ but not Kähler function in general. For G/H decomposition G represents isometries and H both isometries and symmetries of Kähler function.

   CP_2 is familiar example: $SU(3)$ represents isometries and $U(2)$ leaves also Kähler function invariant since it depends on the $U(2)$ invariant radial coordinate r of CP_2. The origin $r=0$ is left invariant by $U(2)$ but for $r>0$ $U(2)$ performs a rotation at $r = constant$ 3-sphere. This simple picture helps to understand what happens at the level of WCW .

   How to then distinguish between symmetries and isometries? A natural guess is that one obtains also for the isometries Noether charges but the vanishing of boundary terms at spatial infinity crucial in the argument leading to Noether theorem as $\Delta S = \Delta Q = 0$ does not hold true anymore and one obtains charges which are not conserved anymore. The symmetry breaking contributions would now come from effective boundaries defined by wormhole throats at which the induce metric changes its signature from Minkowskian to Euclidian. A more delicate situation is in which first order contribution to ΔS vanishes and therefore also ΔQ and the contribution to ΔS comes from second variation allowing also to define Noether charge which is not conserved.

4. The simple picture about CP_2 as symmetric space helps to understand the general vision if one assumes that WCW has the structure of symmetric space. The decomposition $g = h + t$ corresponds to decomposition of symplectic deformations to those which vanish at 3-surface (h) and those which do not (t).

   For the symmetric space option, the Poisson brackets for super generators associated with t give Hamiltonians of h identifiable as the matrix elements of WCW metric. They would not vanish although they are stationary at 3-surface meaning that Riemann connection vanishes at 3-surface. The Hamiltonians which vanish at 3-surface X^3 would correspond to t and the Hamiltonians for which Killing vectors vanish and which therefore are stationary at X^3 would correspond to h. Outside X^3 the situation would of course be different. The metric would be obtained by parallel translating the metric from the preferred point of WCW to elsewhere and symplectic transformations would make this parallel translation.

   For the homogenous space option the Poisson brackets for super generators of t would still give Hamiltonians identifiable as matrix elements of WCW metric but now they would be necessary those of h. In particular, the Hamiltonians of t do not in general vanish at X^3.

7.3.2 Equivalence Principle And WCW

7.3.3 Ep At Quantum And Classical Level

Quite recently I returned to an old question concerning the meaning of Equivalence Principle (EP) in TGD framework.

Heretic would of course ask whether the question about whether EP is true or not is a pseudo problem due to uncritical assumption there really are two different four-momenta which must be identified. If even the identification of these two different momenta is difficult, the pondering of this kind of problem might be waste of time.

At operational level EP means that the scattering amplitudes mediated by graviton exchange are proportional to the product of four-momenta of particles and that the proportionality constant does not depend on any other parameters characterizing particle (except spin). The are excellent reasons to expect that the stringy picture for interactions predicts this.

1. The old idea is that EP reduces to the coset construction for Super Virasoro algebra using the algebras associated with G and H. The four-momenta assignable to these algebras would be identical from the condition that the differences of the generators annihilate physical states and identifiable as inertial and gravitational momenta. The objection is that for the preferred 3-surface H by definition acts trivially so that time-like translations leading out from the boundary of CD cannot be contained by H unlike G. Hence four-momentum is not associated with the Super-Virasoro representations assignable to H and the idea about assigning EP to coset representations does not look promising.

2. Another possibility is that EP corresponds to quantum classical correspondence (QCC) stating that the classical momentum assignable to Kähler action is identical with gravitational momentum assignable to Super Virasoro representations. This forced to reconsider the questions about the precise identification of the Kac-Moody algebra and about how to obtain the magic five tensor factors required by p-adic mass calculations [K79].

   A more precise formulation for EP as QCC comes from the observation that one indeed obtains two four-momenta in TGD approach. The classical four-momentum assignable to the Kähler action and that assignable to the Kähler-Dirac action. This four-momentum is an operator and QCC would state that given eigenvalue of this operator must be equal to the value of classical four-momentum for the space-time surfaces assignable to the zero energy state in question. In this form EP would be highly non-trivial. It would be justified by the Abelian character of four-momentum so that all momentum components are well-defined also quantum mechanically. One can also consider the splitting of four-momentum to longitudinal and transversal parts as done in the parton model for hadrons: this kind of splitting would be very natural at the boundary of CD. The objection is that this correspondence is nothing more than QCC.

3. A further possibility is that duality of light-like 3-surfaces and space-like 3-surfaces holds true. This is the case if the action of symplectic algebra can be defined at light-like 3-surfaces or even for the entire space-time surfaces. This could be achieved by parallel translation of light-cone boundary providing slicing of CD. The four-momenta associated with the two representations of super-symplectic algebra would be naturally identical and the interpretation would be in terms of EP.

One should also understand how General Relativity and EP emerge at classical level. The understanding comes from the realization that GRT is only an effective theory obtained by endowing M^4 with effective metric.

1. The replacement of superposition of fields with superposition of their effects means replacing superposition of fields with the set-theoretic union of space-time surfaces. Particle experiences sum of the effects caused by the classical fields at the space-time sheets.

2. This is true also for the classical gravitational field defined by the deviation from flat Minkowski metric instandard M^4 coordinates for the space-time sheets. One can define effective metric as sum of M^4 metric and deviations. This effective metric would correspond to that of General Relativity. This resolves long standing issues relating to the interpretation of TGD.

3. Einstein's equations could hold true for the effective metric. They are motivated by the underlying Poincare invariance which cannot be realized as global conservation laws for the effective metric. The conjecture vanishing of divergence of Khler energy momentum tensor can be seen as the microscopic justification for the claim that Einstein's equations hold true for the effective space-time.

4. The breaking of Poincare invariance could have interpretation as effective breaking in zero energy ontology (ZEO), in which various conserved charges are length dependent and defined separately for each causal diamond (CD).

One can of course consider the possibility that Einstein's equations generalize for preferred extremals of Kähbler action. This would actually represent at space-time level the notion of QCC rather than realise QCC interpreted as EP. The condition that the energy momentum tensor for Kähler action has vanishing covariant divergence would be satisfied in GRT if Einstein's equations with cosmological term hold true. This is the case also now but one can consider also more general solutions in which one has two cosmological constants which are not genuine constants anymore [K103].

An interesting question is whether inertial-gravitational duality generalizes to the case of color gauge charges so that color gauge fluxes would correspond to "gravitational" color charges and the charges defined by the conserved currents associated with color isometries would define "inertial" color charges. Since the induced color fields are proportional to color Hamiltonians multiplied by Kähler form they vanish identically for vacuum extremals in accordance with "gravitational" color confinement.

7.3.4 Criticism Of The Earlier Construction

The earlier detailed realization of super-Hamiltonians and Hamiltonians can be criticized.

1. Even after these more than twenty years it looks strange that the Hamiltonians should reduce to flux integrals over partonic 2-surfaces. The interpretation has been in terms of effective 2-dimensionality suggested strongly by strong form of general coordinate invariance stating that the descriptions based on light-like orbits of partonic 2-surfaces and space-like three surfaces at the ends of causal diamonds are dual so that only partonic 2-surfaces and 4-D tangent space data at them would matter. Strong form of holography implies effective 2-dimensionality but this should correspond gauge character for the action of symplectic generators in the interior the space-like 3-surfaces at the ends of CDs, which is something much milder.

   One expects that the strings connecting partonic 2-surfaces could bring something new to the earlier simplistic picture. The guess is that imbedding space Hamiltonian assignable to a point of partonic 2-surface should be replaced with that defined as integral over string attached to the point. Therefore the earlier picture would suffer no modification at the level of general formulas.

2. The fact that the dynamics of Kähler action and Kähler-Dirac action are not directly involved with the earlier construction raises suspicions. I have proposed that Kähler function could allow identification as Dirac determinant [K88] but one would expect more intimate connection. Here the natural question is whether super-Hamiltonians for the Kähler-Dirac action could correspond to Kähler charges constructible using Noether's theorem for corresponding deformations of the space-time surface and would also be identifiable as WCW gamma matrices.

7.3.5 Is WCW Homogenous Or Symmetric Space?

A key question is whether WCW can be symmetric space [A22] (`http://tinyurl.com/y8ojglkb`) or whether only homogenous structure is needed. The lack of covariant constancy of curvature tensor might produce problems in infinite-dimensional context.

The algebraic conditions for symmetric space are $g = h + t$, $[h,t] \subset t$, $[t,t] \subset h$. The latter condition is the difficult one.

1. δCD Hamiltonians should induce diffeomorphisms of X^3 indeed leaving it invariant. The symplectic vector fields would be parallel to X^3. A stronger condition is that they induce symplectic transformations for which all points of X^3 remain invariant. Now symplectic vector fields vanish at preferred 3-surface (note that the symplectic transformations are r_M local symplectic transformations of $S^2 \times CP_2$).

2. For Kac-Moody algebra inclusion $H \subset G$ for the finite-dimensional Lie-algebra induces the structure of symmetric space. If entire algebra is involved this does not look physically very attractive idea unless one believes on symmetry breaking for both $SU(3)$, $U(2)_{ew}$, and $SO(3)$

and E_2 (here complex conjugation corresponds to the involution). If one assumes only Kac-Moody algebra as critical symmetries, the number of tensor factors is 4 instead of five, and it is not clear whether one can obtain consistency with p-adic mass calculations.

Examples of 3-surfaces remaining invariant under $U(2)$ are 3-spheres of CP_2. They could correspond to intersections of deformations of CP_2 type vacuum extremals with the boundary of CD. Also geodesic spheres S^2 of CP_2 are invariant under $U(2)$ subgroup and would relate naturally to cosmic strings. The corresponding 3-surface would be $L \times S^2$, where L is a piece of light-like radial geodesic.

3. In the case of symplectic algebra one can construct the imbedding space Hamiltonians inducing WCW Hamiltonians as products of elements of the isometry algebra of $S^2 \times CP_2$ for with parity under involution is well-defined. This would give a decomposition of the symplectic algebra satisfying the symmetric space property at the level imbedding space. This decomposition does not however look natural at the level of WCW since the only single point of CP_2 and light-like geodesic of δM^4_+ can be fixed by $SO(2) \times U(2)$ so that the 3-surfaces would reduce to pieces of light rays.

4. A more promising involution is the inversion $r_M \rightarrow 1/r_M$ of the radial coordinate mapping the radial conformal weights to their negatives. This corresponds to the inversion in Super Virasoro algebra. t would correspond to functions which are odd functions of $u \equiv log(r_M/r_0)$ and h to even function of u. Stationary 3-surfaces would correspond to $u = 1$ surfaces for which $log(u) = 0$ holds true. This would assign criticality with Virasoro algebra as one expects on general grounds.

   $r_M = constant$ surface would most naturally correspond to a maximum of Kähler function which could indeed be highly symmetric. The elements with even u-parity should define Hamiltonians, which are stationary at the maximum of Kähler function. For other 3-surfaces obtained by $/r_M$-local) symplectic transformations the situation is different: now H is replaced with its symplectic conjugate hHg^{-1} of H is acceptable and if the conjecture is true one would obtained 3-surfaces assignable to perturbation theory around given maximum as symplectic conjugates of the maximum. The condition that H leaves X^3 invariant in pointwise manner is certainly too strong and imply that the 3-surface has single point as CP_2 projection.

5. One can also consider the possibility that critical deformations correspond to h and non-critical ones to t for the preferred 3-surface. Criticality for given h would hold only for a preferred 3-surface so that this picture would be very similar that above. Symplectic conjugates of h would define criticality for other 3-surfaces. WCW would decompose to a union corresponding to different criticalities perhaps assignable to the hierarchy of sub-algebras of conformal algebra labelled by integer whose multiples give the allowed conformal weights. Hierarchy of breakings of conformal symmetries would characterize this hierarchy of sectors of WCW .

   For sub-algebras of the conformal algebras (Kac-Moody and symplectic algebra) the condition $[t, t] \subset h$ cannot hold true so that one would obtain only the structure of homogenous space.

7.3.6 Symplectic And Kac-Moody Algebras As Basic Building Bricks

7.3.7 WCW As A Union Of Symmetric Spaces

In finite-dimensional context globally symmetric spaces are of form G/H and connection and curvature are independent of the metric, provided it is left invariant under G. The hope is that same holds true in infinite-dimensional context. The most one can hope of obtaining is the decomposition $C(H) = \cup_i G/H_i$ over orbits of G. One could allow also symmetry breaking in the sense that G and H depend on the orbit: $C(H) = \cup_i G_i/H_i$ but it seems that G can be chosen to be same for all orbits. What is essential is that these groups are infinite-dimensional. The basic properties of the coset space decomposition give very strong constraints on the group H, which certainly contains the subgroup of G, whose action reduces to diffeomorphisms of X^3.

If G is symplectic group of $\delta M^4_\pm \times CP_2$ then H is its subgroup, and one can wonder whether this is really consistent with the identification of H as Kac-Moody algebra assignable to light-like

3-surfaces. This raises the possibility that SKM acts as pure gauge symmetries and has nothing to do with the coset decomposition.

The improved understanding of solutions of the Kähler-Dirac equation [K88] also leads to the realization that the direct sum of super-symplectic algebra and isometry algebra is more natural spectrum generating algebra. For super-symplectic algebra super-generators are represented in terms of contractions of covariantly constant right-handed neutrino mode with second quantized spinor field. For isometry sub-algebra super generators have representation in terms of contractions of modes of induced spinor field localized at string world sheets is a more natural identification of the fundamental conformal algebra and gives five tensor factors as required by p-adic mass calculations.

Consequences of the decomposition

If the decomposition to a union of coset spaces indeed occurs, the consequences for the calculability of the theory are enormous since it suffices to find metric and curvature tensor for single representative 3-surface on a given orbit (contravariant form of metric gives propagator in perturbative calculation of matrix elements as functional integrals over the WCW). The representative surface can be chosen to correspond to the maximum of Kähler function on a given orbit and one obtains perturbation theory around this maximum (Kähler function is not isometry invariant).

The task is to identify the infinite-dimensional groups G and H and to understand the zero mode structure of the WCW . Almost twenty (seven according to long held belief!) years after the discovery of the candidate for the Kähler function defining the metric, it became finally clear that these identifications follow quite nicely from $Diff^4$ invariance and $Diff^4$ degeneracy as well as special properties of the Kähler action.

The guess (not the first one!) would be following. G corresponds to the symplectic transformations of $\delta M^4_\pm \times CP_2$ leaving the induced Kähler form invariant. If G acts as isometries the values of Kähler form at partonic 2-surfaces (remember effective 2-dimensionality) are zero modes and WCW allows slicing to symplectic orbits of the partonic 2-surface with fixed induced Kähler form. Quantum fluctuating degrees of freedom would correspond to symplectic group and to the fluctuations of the induced metric. The group H dividing G would in turn correspond to the symplectic isometries reducing to diffeomorphisms at the 3-surfaces or possibly at partonic 2-surfaces only.

H could but not not need to corresponds to the Kac-Moody symmetries respecting light-likeness of X^3_l and acting in X^3_l but trivially at the partonic 2-surface X^2. The action of course extends also to the interior of space-like 3-surface X^3 at the boundary of CD. This coset structure was originally suggested via coset construction for super Virasoro algebras of super-symplectic and super Kac-Moody algebras.

WCW isometries as a subgroup of $Diff(\delta M^4_+ \times CP_2)$

The reduction to light cone boundary leads to the identification of the isometry group as some subgroup of for the group G for the diffeomorphisms of $\delta M^4_+ \times CP_2$. These diffeomorphisms indeed act in a natural manner in δCH, the space of 3-surfaces in $\delta M^4_+ \times CP_2$. WCW is expected to decompose to a union of the coset spaces G/H_i, where H_i corresponds to some subgroup of G containing the transformations of G acting as diffeomorphisms for given X^3. Geometrically the vector fields acting as diffeomorphisms of X^3 are tangential to the 3-surface. H_i could depend on the topology of X^3 and since G does not change the topology of 3-surface each 3-topology defines separate orbit of G. Therefore, the union involves sum over all on topologies of X^3 plus possibly other "zero modes". Different topologies are naturally glued together since singular 3-surfaces intermediate between two 3-topologies correspond to points common to the two sectors with different topologies.

7.3.8 Isometries Of WCW Geometry As SymplecticTransformations Of $\Delta M^4_+ \times CP_2$

During last decade I have considered several candidates for the group G of isometries of WCW as the sub-algebra of the subalgebra of $Diff(\delta M^4_+ \times CP_2)$. To begin with let us write the general decomposition of $diff(\delta M^4_+ \times CP_2)$:

$$diff(\delta M^4_+ \times CP_2) = S(CP_2) \times diff(\delta M^4_+) \oplus S(\delta M^4_+) \times diff(CP_2) \ . \tag{7.3.1}$$

Here $S(X)$ denotes the scalar function basis of space X. This Lie-algebra is the direct sum of light cone diffeomorphisms made local with respect to CP_2 and CP_2 diffeomorphisms made local with respect to light cone boundary.

The idea that entire diffeomorphism group would act as isometries looks unrealistic since the theory should be more or less equivalent with topological field theory in this case. Consider now the various candidates for G.

1. The fact that symplectic transformations of CP_2 and M^4_+ diffeomorphisms are dynamical symmetries of the vacuum extremals suggests the possibility that the diffeomorphisms of the light cone boundary and symplectic transformations of CP_2 could leave Kähler function invariant and thus correspond to zero modes. The symplectic transformations of CP_2 localized with respect to light cone boundary acting as symplectic transformations of CP_2 have interpretation as local color transformations and are a good candidate for the isometries. The fact that local color transformations are not even approximate symmetries of Kähler action is not a problem: if they were exact symmetries, Kähler function would be invariant and zero modes would be in question.

2. CP_2 local conformal transformations of the light cone boundary act as isometries of δM^4_+. Besides this there is a huge group of the symplectic symmetries of $\delta M^4_+ \times CP_2$ if light cone boundary is provided with the symplectic structure. Both groups must be considered as candidates for groups of isometries. $\delta M^4_+ \times CP_2$ option exploits fully the special properties of $\delta M^4_+ \times CP_2$, and one can develop simple argument demonstrating that $\delta M^4_+ \times CP_2$ symplectic invariance is the correct option. Also the construction of WCW gamma matrices as super-symplectic charges supports $\delta M^4_+ \times CP_2$ option.

7.3.9 SUSY Algebra Defined By The Anti-Commutation Relations Of Fermionic Oscillator Operators And WCW Local Clifford Algebra Elements As Chiral Super-Fields

Whether TGD allows space-time supersymmetry has been a long-standing question. Majorana spinors appear in $N = 1$ super-symmetric QFTs- in particular minimally super-symmetric standard model (MSSM). Majorana-Weyl spinors appear in M-theory and super string models. An undesirable consequence is chiral anomaly in the case that the numbers of left and right handed spinors are not same. For $D = 11$ and $D = 10$ these anomalies cancel which led to the breakthrough of string models and later to M-theory. The probable reason for considering these dimensions is that standard model does not predict right-handed neutrino (although neutrino mass suggests that right handed neutrino exists) so that the numbers of left and right handed Weyl-spinors are not the same.

In TGD framework the situation is different. Covariantly constant right-handed neutrino spinor acts as a super-symmetry in CP_2. One might think that right-handed neutrino in a well-defined sense disappears from the spectrum as a zero mode so that the number of right and left handed chiralities in $M^4 \times CP_2$ would not be same. For light-like 3-surfaces covariantly constant right-handed neutrino does not however solve the counterpart of Dirac equation for a non-vanishing four-momentum and color quantum numbers of the physical state. Therefore it does not disappear from the spectrum anymore and one expects the same number of right and left handed chiralities.

In TGD framework the separate conservation of baryon and lepton numbers excludes Majorana spinors and also the the Minkowski signature of $M^4 \times CP_2$ makes them impossible. The conclusion that TGD does not allow super-symmetry is however wrong. For $\mathcal{N} = 2N$ Weyl spinors are indeed possible and if the number of right and left handed Weyl spinors is same super-symmetry is possible. In 8-D context right and left-handed fermions correspond to quarks and leptons and since color in TGD framework corresponds to CP_2 partial waves rather than spin like quantum number, also the numbers of quark and lepton-like spinors are same.

The physical picture suggest a new kind of approach to super-symmetry in the sense that the anti-commutations of fermionic oscillator operators associated with the modes of the induced spinor fields define a structure analogous to SUSY algebra. This means that $\mathcal{N} = 2N$ SUSY with large N is in question allowing spins higher than two and also large fermion numbers. Recall that $\mathcal{N} \leq 32$ is implied by the absence of spins higher than two and the number of real spinor components is $N = 32$ also in TGD. The situation clearly differs from that encountered in super-string models and SUSYs and the large value of N allows to expect very powerful constraints on dynamics irrespective of the fact that SUSY is broken. Right handed neutrino modes define a sub-algebra for which the SUSY is only slightly broken by the absence of weak interactions and one could also consider a theory containing a large number of $\mathcal{N} = 2$ super-multiplets corresponding to the addition of right-handed neutrinos and antineutrinos at the wormhole throat.

Masslessness condition is essential for super-symmetry and at the fundamental level it could be formulated in terms of Kähler-Dirac gamma matrices using octonionic representation and assuming that they span local quaternionic sub-algebra at each point of the space-time sheet. SUSY algebra has standard interpretation with respect to spin and isospin indices only at the partonic 2-surfaces so that the basic algebra should be formulated at these surfaces. Effective 2-dimensionality would require that partonic 2-surfaces can be taken to be ends of any light-like 3-surface Y_l^3 in the slicing of the region surrounding a given wormhole throat.

Super-algebra associated with the Kähler-Dirac gamma matrices

Anti-commutation relations for fermionic oscillator operators associated with the induced spinor fields are naturally formulated in terms of the Kähler-Dirac gamma matrices. Super-conformal symmetry suggests that the anti-commutation relations for the fermionic oscillator operators at light-like 3-surfaces or at their ends are most naturally formulated as anti-commutation relations for SUSY algebra. The resulting anti-commutation relations would fix the quantum TGD.

$$\begin{aligned} \{a_{n\alpha}^\dagger, a_{n\beta}\} &= D_{mn}D_{\alpha\beta} \ , \\ D &= (p^\mu + \sum_a Q_a^\mu)\hat{\sigma}^\mu \ . \end{aligned} \tag{7.3.2}$$

Here p^μ and Q_a^μ are space-time projections of momentum and color charges in Cartan algebra. Their action is purely algebraic. The anti-commutations are nothing but a generalization of the ordinary equal-time anti-commutation relations for fermionic oscillator operators to a manifestly covariant form. The matrix $D_{m,n}$ is expected to reduce to a diagonal form with a proper normalization of the oscillator operators. The experience with extended SUSY algebra suggest that the anti-commutators could contain additional central term proportional to $\delta_{\alpha\beta}$.

One can consider basically two different options concerning the definition of the super-algebra.

1. If the super-algebra is defined at the 3-D ends of the intersection of X^4 with the boundaries of CD, the modified gamma matrices appearing in the operator D appearing in the anti-commutator are associated with Kähler action. If the generalized masslessness condition $D^2 = 0$ holds true -as suggested already earlier- one can hope that no explicit breaking of super-symmetry takes place and elegant description of massive states as effectively massless states making also possible generalization of twistor is possible. One must however notice that also massive representatives of SUSY exist.

2. SUSY algebra could be also defined at 2-D ends of light-like 3-surfaces.

According to considerations of [K88] these options are equivalent for a large class of space-time sheets. If the effective 3-dimensionality realized in the sense that the effective metric defined by the Kähler-Dirac gamma matrices is degenerate, propagation takes place along 3-D light-like 3-surfaces. This condition definitely fails for string like objects.

One can realize the local Clifford algebra also by introducing theta parameters in the standard manner and the expressing a collection of local Clifford algebra element with varying values of fermion numbers (function of CD and CP_2 coordinates) as a chiral super-field. The definition of a chiral super field requires the introduction of super-covariant derivatives. Standard form for the anti-commutators of super-covariant derivatives D_α make sense only if they do not affect the Kähler-Dirac gamma matrices. This is achieved if p_k acts on the position of the tip of CD (rather than internal coordinates of the space-time sheet). Q_a in turn must act on CP_2 coordinates of the tip.

Super-fields associated with WCW Clifford algebra

WCW local Clifford algebra elements possess definite fermion numbers and it is not physically sensible to super-pose local Clifford algebra elements with different fermion numbers. The extremely elegant formulation of super-symmetric theories in terms of super-fields encourages to ask whether the local Clifford algebra elements could allow expansion in terms of complex theta parameters assigned to various fermionic oscillator operator in order to obtain formal superposition of elements with different fermion numbers. One can also ask whether the notion of chiral super field might make sense.

The obvious question is whether it makes sense to assign super-fields with the Kähler-Dirac gamma matrices.

1. Kähler-Dirac gamma matrices are not covariantly constant but this is not a problem since the action of momentum generators and color generators on space-time coordinates is purely algebraic.

2. One can define the notion of chiral super-field also at the fundamental level. Chiral super-field would be continuation of the local Clifford algebra of associated with CD to a local Clifford algebra element associated with the union of CDs. This would allow elegant description of cm degrees of freedom, which are the most interesting as far as QFT limit is considered.

3. Kähler function of WCW as a function of complex coordinates could be extended to a chiral super-field defined in quantum fluctuation degrees of freedom. It would depend on zero modes too. Does also the latter dependence allow super-space continuation? Coefficients of powers of theta would correspond to fermionic oscillator operators. Does this function define the propagators of various states associated with light-like 3-surface? WCW complex coordinates would correspond to the modes of induced spinor field so that super-symmetry would be realized very concretely.

7.3.10 Identification Of Kac-Moody Symmetries

The Kac-Moody algebra of symmetries acting as symmetries respecting the light-likeness of 3-surfaces plays a crucial role in the identification of quantum fluctuating WCW degrees of freedom contributing to the metric. The recent vision looks like follows.

1. The recent interpretation is that these symmetries are due to the non-determinism of Kähler action and transform to each other preferred extremals with same space-like surfaces as their ends at the boundaries of causal diamond. These space-time surfaces have same Kähler action and possess same conserved quantities.

2. The sub-algebra of conformal symmetries acts as gauge transformations of these infinite set of degenerate preferred extremals and there is finite number n of gauge equivalence classes. n corresponds to the effective (or real depending on interpretation) value of Planck constant $h_{eff} = n \times h$. The further conjecture is that the sub-algebra of conformal algebra for which conformal weights are integers divisible by n act as genuine gauge symmetries. If Kähler action reduces to a sum of 3-D Chern-Simons terms for preferred extremals, it is enough to consider the action on light-like 3-surfaces. For gauge part of algebra the algebra acts trivially at space-like 3-surfaces.

3. A good guess is that the Kac-Moody type algebra corresponds to the sub-algebra of symplectic isometries of $\delta M^4_\pm \times CP_2$ acting on light-like 3-surfaces and having continuation to the interior.

A stronger assumption is that isometries are in question. For CP_2 nothing would change but light-cone boundary $\delta M^4_\pm = S^2 \times R_+$ has conformal transformations of S^2 as isometries. The conformal scaling is compensated by S^2-local scaling of the light like radial coordinate of R_+.

4. This super-conformal algebra realized in terms of spinor modes and second quantized induced spinor fields would define the Super Kac-Moody algebra. The generators of this Kac-Moody type algebra have continuation from the light-like boundaries to deformations of preferred extremals and at least the generators of sub-algebra act trivially at space-like 3-surfaces.

The following is an attempt to achieve a more detailed identification of the Kac-Moody algebra is considered.

Identification of Kac-Moody algebra

The generators of bosonic super Kac-Moody algebra leave the light-likeness condition $\sqrt{g_3} = 0$ invariant. This gives the condition

$$\delta g_{\alpha\beta} Cof(g^{\alpha\beta}) = 0 \ , \tag{7.3.3}$$

Here Cof refers to matrix cofactor of $g_{\alpha\beta}$ and summation over indices is understood. The conditions can be satisfied if the symmetries act as combinations of infinitesimal diffeomorphisms $x^\mu \rightarrow x^\mu + \xi^\mu$ of X^3 and of infinitesimal conformal symmetries of the induced metric

$$\delta g_{\alpha\beta} = \lambda(x) g_{\alpha\beta} + \partial_\mu g_{\alpha\beta}\xi^\mu + g_{\mu\beta}\partial_\alpha \xi^\mu + g_{\alpha\mu}\partial_\beta \xi^\mu \ . \tag{7.3.4}$$

Ansatz as an X^3-local conformal transformation of imbedding space

Write δh^k as a super-position of X^3-local infinitesimal diffeomorphisms of the imbedding space generated by vector fields $J^A = j^{A,k}\partial_k$:

$$\delta h^k = c_A(x) j^{A,k} \ . \tag{7.3.5}$$

This gives

$$\begin{aligned} c_A(x) \quad & \left[D_k j^A_l + D_l j^A_k\right] \partial_\alpha h^k \partial_\beta h^l + 2\partial_\alpha c_A h_{kl} j^{A,k}\partial_\beta h^l \\ = \quad & \lambda(x) g_{\alpha\beta} + \partial_\mu g_{\alpha\beta}\xi^\mu + g_{\mu\beta}\partial_\alpha \xi^\mu + g_{\alpha\mu}\partial_\beta \xi^\mu \ . \end{aligned} \tag{7.3.6}$$

If an X^3-local variant of a conformal transformation of the imbedding space is in question, the first term is proportional to the metric since one has

$$D_k j^A_l + D_l j^A_k = 2h_{kl} \ . \tag{7.3.7}$$

The transformations in question includes conformal transformations of $H_\pm$ and isometries of the imbedding space H.

The contribution of the second term must correspond to an infinitesimal diffeomorphism of X^3 reducible to infinitesimal conformal transformation ψ^μ:

$$2\partial_\alpha c_A h_{kl} j^{A,k}\partial_\beta h^l = \xi^\mu \partial_\mu g_{\alpha\beta} + g_{\mu\beta}\partial_\alpha \xi^\mu + g_{\alpha\mu}\partial_\beta \xi^\mu \ . \tag{7.3.8}$$

A rough analysis of the conditions

One could consider a strategy of fixing c_A and solving solving ξ^μ from the differential equations. In order to simplify the situation one could assume that $g_{ir} = g_{rr} = 0$. The possibility to cast the metric in this form is plausible since generic 3-manifold allows coordinates in which the metric is diagonal.

1. The equation for g_{rr} gives

$$\partial_r c_A h_{kl} j^{Ak} \partial_r h^k \quad = \quad 0 \ . \tag{7.3.9}$$

   The radial derivative of the transformation is orthogonal to X^3. No condition on ξ^α results. If c_A has common multiplicative dependence on $c_A = f(r) d_A$ by a one obtains

$$d_A h_{kl} j^{Ak} \partial_r h^k \quad = \quad 0 \ . \tag{7.3.10}$$

   so that J^A is orthogonal to the light-like tangent vector $\partial_r h^k$ X^3 which is the counterpart for the condition that Kac-Moody algebra acts in the transversal degrees of freedom only. The condition also states that the components g_{ri} is not changed in the infinitesimal transformation.

   It is possible to choose $f(r)$ freely so that one can perform the choice $f(r) = r^n$ and the notion of radial conformal weight makes sense. The dependence of c_A on transversal coordinates is constrained by the transversality condition only. In particular, a common scale factor having free dependence on the transversal coordinates is possible meaning that X^3- local conformal transformations of H are in question.

2. The equation for g_{ri} gives

$$\partial_r \xi^i \quad = \quad \partial_r c_A h_{kl} j^{Ak} h^{ij} \partial_j h^k \ . \tag{7.3.11}$$

   The equation states that g_{ri} are not affected by the symmetry. The radial dependence of ξ^i is fixed by this differential equation. No condition on ξ^r results. These conditions imply that the local gauge transformations are dynamical with the light-like radial coordinate r playing the role of the time variable. One should be able to fix the transformation more or less arbitrarily at the partonic 2-surface X^2.

3. The three independent equations for g_{ij} give

$$\xi^\alpha \partial_\alpha g_{ij} + g_{kj} \partial_i \xi^k + g_{ki} \partial_j \xi^k \quad = \quad \partial_i c_A h_{kl} j^{Ak} \partial_j h^l \ . \tag{7.3.12}$$

   These are 3 differential equations for 3 functions ξ^α on 2 independent variables x^i with r appearing as a parameter. Note however that the derivatives of ξ^r do not appear in the equation. At least formally equations are not over-determined so that solutions should exist for arbitrary choices of c_A as functions of X^3 coordinates satisfying the orthogonality conditions. If this is the case, the Kac-Moody algebra can be regarded as a local algebra in X^3 subject to the orthogonality constraint.

   This algebra contains as a subalgebra the analog of Kac-Moody algebra for which all c_A except the one associated with time translation and fixed by the orthogonality condition depends on the radial coordinate r only. The larger algebra decomposes into a direct sum of representations of this algebra.

Commutators of infinitesimal symmetries

The commutators of infinitesimal symmetries need not be what one might expect since the vector fields ξ^μ are functionals c_A and of the induced metric and also c_A depends on induced metric via the orthogonality condition. What this means that $j^{A,k}$ in principle acts also to ϕ_B in the commutator $[c_A J^A, c_B J^B]$.

$$\left[c_A J^A, c_B J^B\right] = c_A c_B J^{[A,B]} + J^A \circ c_B J^B - J^B \circ c_A J^A \ , \tag{7.3.13}$$

where $\circ$ is a short hand notation for the change of c_B induced by the effect of the conformal transformation J^A on the induced metric.

Luckily, the conditions in the case $g_{rr} = g_{ir} = 0$ state that the components g_{rr} and g_{ir} of the induced metric are unchanged in the transformation so that the condition for c_A resulting from g_{rr} component of the metric is not affected. Also the conditions coming from $g_{ir} = 0$ remain unchanged. Therefore the commutation relations of local algebra apart from constraint from transversality result.

The commutator algebra of infinitesimal symmetries should also close in some sense. The orthogonality to the light-like tangent vector creates here a problem since the commutator does not obviously satisfy this condition automatically. The problem can be solved by following the recipes of non-covariant quantization of string model.

1. Make a choice of gauge by choosing time translation P^0 in a preferred M^4 coordinate frame to be the preferred generator $J^{A_0} \equiv P^0$, whose coefficient $\Phi_{A_0} \equiv \Psi(P^0)$ is solved from the orthogonality condition. This assumption is analogous with the assumption that time coordinate is non-dynamical in the quantization of strings. The natural basis for the algebra is obtained by allowing only a single generator J^A besides P^0 and putting $d_A = 1$.

2. This prescription must be consistent with the well-defined radial conformal weight for the $J^A \neq P^0$ in the sense that the proportionality of d_A to r^n for $J^A \neq P^0$ must be consistent with commutators. SU(3) part of the algebra is of course not a problem. From the Lorentz vector property of P^k it is clear that the commutators resulting in a repeated commutation have well-defined radial conformal weights only if one restricts $SO(3,1)$ to $SO(3)$ commuting with P^0. Also D could be allowed without losing well-defined radial conformal weights but the argument below excludes it. This picture conforms with the earlier identification of the Kac-Moody algebra.

   Conformal algebra contains besides Poincare algebra and the dilation $D = m^k\partial_{m^k}$ the mutually commuting generators $K^k = (m^r m_r \partial_{m^k} - 2m^k m^l \partial_{m^l})/2$. The commutators involving added generators are

$$\begin{array}{ll} \left[D, K^k\right] = -K^k \ , & \left[D, P^k\right] = P^k \ , \\ \left[K^k, K^l\right] = 0 \ , & \left[K^k, P^l\right] = m^{kl}D - M^{kl} \ . \end{array} \tag{7.3.14}$$

   From the last commutation relation it is clear that the inclusion of K^k would mean loss of well-defined radial conformal weights.

3. The coefficient dm^0/dr of $\Psi(P^0)$ in the equation

$$\Psi(P^0)\frac{dm^0}{dr} = -J^{Ak} h_{kl} \partial_r h^l$$

   is always non-vanishing due to the light-likeness of r. Since P^0 commutes with generators of $SO(3)$ (but not with D so that it is excluded!), one can *define* the commutator of two generators as a commutator of the remaining part and identify $\Psi(P^0)$ from the condition above.

4. Of course, also the more general transformations act as Kac-Moody type symmetries but the interpretation would be that the sub-algebra plays the same role as $SO(3)$ in the case of Lorentz group: that is gives rise to generalized spin degrees of freedom whereas the entire algebra divided by this sub-algebra would define the coset space playing the role of orbital degrees of freedom. In fact, also the Kac-Moody type symmetries for which c_A depends on the transversal coordinates of X^3 would correspond to orbital degrees of freedom. The presence of these orbital degrees of freedom arranging super Kac-Moody representations into infinite multiplets labeled by function basis for X^2 means that the number of degrees of freedom is much larger than in string models.

5. It is possible to replace the preferred time coordinate m^0 with a preferred light-like coordinate. There are good reasons to believe that orbifold singularity for phases of matter involving non-standard value of Planck constant corresponds to a preferred light-ray going through the tip of $\delta M^4_\pm$. Thus it would be natural to assume that the preferred M^4 coordinate varies along this light ray or its dual. The Kac-Moody group $SO(3)\times E^3$ respecting the radial conformal weights would reduce to $SO(2)\times E^2$ as in string models. E^2 would act in tangent plane of $S^2_\pm$ along this ray defining also $SO(2)$ rotation axis.

Hamiltonians

The action of these transformations on Kähler action is well-defined and one can deduce the conserved quantities having identification as WCW Hamiltonians. Hamiltonians also correspond to closed 2-forms. The condition that the Hamiltonian reduces to a dual of closed 2-form is satisfied because X^2-local conformal transformations of $M^4_\pm \times CP_2$ are in question (X^2-locality does not imply any additional conditions).

The action of Kac-Moody algebra on spinors and fermionic representations of Kac-Moody algebra

One can imagine two interpretations for the action of generalized Kac-Moody transformations on spinors.

1. The basic goal is to deduce the fermionic Noether charge associated with the bosonic Kac-Moody symmetry and this can be done by a standard recipe. The first contribution to the charge comes from the transformation of Kähler-Dirac gamma matrices appearing in the Kähler-Dirac action associated with fermions. Second contribution comes from spinor rotation.

2. Both SO(3) and SU(3) rotations have a standard action as spin rotation and electro-weak rotation allowing to define the action of the Kac-Moody algebra J^A on spinors.

How central extension term could emerge?

The central extension term of Kac-Moody algebra could correspond to a symplectic extension which can emerge from the freedom to add a constant term to Hamiltonians as in the case of super-symplectic algebra. The expression of the Hamiltonians as closed forms could allow to understand how the central extension term emerges.

In principle one can construct a representation for the action of Kac-Moody algebra on fermions a representations as a fermionic bilinear and the central extension of Kac-Moody algebra could emerge in this construction just as it appears in Sugawara construction.

About the interpretation of super Kac-Moody symmetries

Also the light like 3-surfaces X^3_l of H defining elementary particle horizons at which Minkowskian signature of the metric is changed to Euclidian and boundaries of space-time sheets can act as causal determinants, and thus contribute to WCW metric. In this case the symmetries correspond to the isometries of the imbedding space localized with respect to the complex coordinate of the 2-surface X^2 determining the light like 3-surface X^3_l so that Kac-Moody type symmetry results.

Also the condition $\sqrt{g_3} = 0$ for the determinant of the induced metric seems to define a conformal symmetry associated with the light like direction.

If is enough to localize only the H-isometries with respect to X^3_l, the purely bosonic part of the Kac-Moody algebra corresponds to the isometry group $M^4 \times SO(3,1) \times SU(3)$. The physical interpretation of these symmetries is not so obvious as one might think. The point is that one can generalize the formulas characterizing the action of infinitesimal isometries on spinor fields of finite-dimensional Kähler manifold to the level of the configuration space. This gives rise to bosonic generators containing also a sigma-matrix term bilinear in fermionic oscillator operators. This representation need not be equivalent with the purely fermionic representations provided by induced Dirac action. Thus one has two groups of local color charges and the challenge is to find a physical interpretation for them.

The following arguments support one possible identification.

1. The hint comes from the fact that $U(2)$ in the decomposition $CP_2 = SU(3)/U(2)$ corresponds in a well-defined sense electro-weak algebra identified as a holonomy algebra of the spinor connection. Hence one could argue that the $U(2)$ generators of either $SU(3)$ algebra might be identifiable as generators of local $U(2)_{ew}$ gauge transformations whereas non-diagonal generators would correspond to Higgs field. This interpretation would conform with the idea that Higgs field is a genuine scalar field rather than a composite of fermions.

2. Since X^3_l-local $SU(3)$ transformations represented by fermionic currents are characterized by central extension they would naturally correspond to the electro-weak gauge algebra and Higgs bosons. This is also consistent with the fact that both leptons and quarks define fermionic Kac Moody currents.

3. The fact that only quarks appear in the gamma matrices of the WCW supports the view that action of the generators of X^3_l-local color transformations on WCW spinor fields represents local color transformations. If the action of X^3_l-local $SU(3)$ transformations on WCW spinor fields has trivial central extension term the identification as a representation of local color symmetries is possible.

The topological explanation of the family replication phenomenon is based on an assignment of 2-dimensional boundary to a 3-surface characterizing the elementary particle. The precise identification of this surface has remained open and one possibility is that the 2-surface X^2 defining the light light-like surface associated with an elementary particle horizon is in question. This assumption would conform with the notion of elementary particle vacuum functionals defined in the zero modes characterizing different conformal equivalences classes for X^2.

The relationship of the Super-Kac Moody symmetry to the standard super-conformal invariance

Super-Kac Moody symmetry can be regarded as $N = 4$ complex super-symmetry with complex H-spinor modes of H representing the 4 physical helicities of 8-component leptonic and quark like spinors acting as generators of complex dynamical super-symmetries. The super-symmetries generated by the covariantly constant right handed neutrino appear with *both* M^4 helicities: it however seems that covariantly constant neutrino does not generate any global super-symmetry in the sense of particle-sparticle mass degeneracy. Only right-handed neutrino spinor modes (apart from covariantly constant mode) appear in the expressions of WCW gamma matrices forming a subalgebra of the full super-algebra.

$N = 2$ real super-conformal algebra is generated by the energy momentum tensor $T(z)$, $U(1)$ current $J(z)$, and super generators $G^{\pm}(z)$ carrying $U(1)$ charge. Now $U(1)$ current would correspond to right-handed neutrino number and super generators would involve contraction of covariantly constant neutrino spinor with second quantized induced spinor field. The further facts that $N = 2$ algebra is associated naturally with Kähler geometry, that the partition functions associated with $N = 2$ super-conformal representations are modular invariant, and that $N = 2$ algebra defines so called chiral ring defining a topological quantum field theory [A59], lend a further support for the belief that $N = 2$ super-conformal algebra acts in super-symplectic degrees of freedom.

The values of c and conformal weights for $N = 2$ super-conformal field theories are given by

$$\begin{aligned} c &= \frac{3k}{k+2} \ , \\ \Delta_{l,m}(NS) &= \frac{l(l+2)-m^2}{4(k+2)} \ , \ l = 0,1,...,k \ , \\ q_m &= \frac{m}{k+2} \ , \ m = -l,-l+2,....,l-2,l \ . \end{aligned} \tag{7.3.15}$$

q_m is the fractional value of the $U(1)$ charge, which would now correspond to a fractional fermion number. For $k = 1$ one would have $q = 0, 1/3, -1/3$, which brings in mind anyons. $\Delta_{l=0,m=0} = 0$ state would correspond to a massless state with a vanishing fermion number. Note that $SU(2)_k$ Wess-Zumino model has the same value of c but different conformal weights. More information about conformal algebras can be found from the appendix of [A59].

For Ramond representation $L_0 - c/24$ or equivalently G_0 must annihilate the massless states. This occurs for $\Delta = c/24$ giving the condition $k = 2\left[l(l+2) - m^2\right]$ (note that k must be even and that $(k, l, m) = (4, 1, 1)$ is the simplest non-trivial solution to the condition). Note the appearance of a fractional vacuum fermion number $q_{vac} = \pm c/12 = \pm k/4(k+2)$. I have proposed that NS and Ramond algebras could combine to a larger algebra containing also lepto-quark type generators but this not necessary.

The conformal algebra defined as a direct sum of Ramond and NS $N = 4$ complex sub-algebras associated with quarks and leptons might further extend to a larger algebra if lepto-quark generators acting effectively as half odd-integer Virasoro generators can be allowed. The algebra would contain spin and electro-weak spin as fermionic indices. Poincare and color Kac-Moody generators would act as symplectically extended isometry generators on WCW Hamiltonians expressible in terms of Hamiltonians of $X^3_l \times CP_2$. Electro-weak and color Kac-Moody currents have conformal weight $h = 1$ whereas T and G have conformal weights $h = 2$ and $h = 3/2$.

The experience with $N = 4$ complex super-conformal invariance suggests that the extended algebra requires the inclusion of also second quantized induced spinor fields with $h = 1/2$ and their super-partners with $h = 0$ and realized as fermion-anti-fermion bilinears. Since G and Ψ are labeled by 2×4 spinor indices, super-partners would correspond to $2 \times (3 + 1) = 8$ massless electro-weak gauge boson states with polarization included. Their inclusion would make the theory highly predictive since induced spinor and electro-weak fields are the fundamental fields in TGD.

7.3.11 Coset Space Structure For WCW As A SymmetricSpace

The key ingredient in the theory of symmetric spaces is that the Lie-algebra of G has the following decomposition

$$\begin{aligned} &g = h + t \ , \\ &[h, h] \subset h \ , \quad [h, t] \subset t \ , \quad [t, t] \subset h \ . \end{aligned}$$

In present case this has highly nontrivial consequences. The commutator of *any* two infinitesimal generators generating nontrivial deformation of 3-surface belongs to h and thus vanishing norm in the WCW metric at the point which is left invariant by H. In fact, this same condition follows from Ricci flatness requirement and guarantees also that G acts as isometries of WCW . This generalization is supported by the properties of the unitary representations of Lorentz group at the light cone boundary and by number theoretical considerations.

The algebras suggesting themselves as candidates are symplectic algebra of $\delta M^{\pm} \times CP_2$ and Kac-Moody algebra mapping light-like 3-surfaces to light-like 3-surfaces to be discussed in the next section.

The identification of the precise form of the coset space structure is however somewhat delicate.

1. The essential point is that both symplectic and Kac-Moody algebras allow representation in terms of X^3_l-local Hamiltonians. The general expression for the Hamilton of Kac-Moody algebra is

$$H = \sum \Phi_A(x) H^A \ . \tag{7.3.16}$$

Here H^A are Hamiltonians of $SO(3) \times SU(3)$ acting in $\delta X_l^3 \times CP_2$. For symplectic algebra any Hamiltonian is allowed. If x corresponds to any point of X_l^3, one must assume a slicing of the causal diamond CD by translates of $\delta M_\pm^4$.

2. For symplectic generators the dependence of form on r^Δ on light-like coordinate of $\delta X_l^3 \times CP_2$ is allowed. Δ is complex parameter whose modulus squared is interpreted as conformal weight. Δ is identified as analogous quantum number labeling the modes of induced spinor field.

3. One can wonder whether the choices of the $r_M = constant$ sphere S^2 is the only choice. The Hamiltonin-Jacobi coordinate for $X^4_{X_l^3}$ suggest an alternative choice as E^2 in the decomposition of $M^4 = M^2(x) \times E^2(x)$ required by number theoretical compactification and present for known extremals of Kähler action with Minkowskian signature of induced metric. In this case $SO(3)$ would be replaced with $SO(2)$. It however seems that the radial light-like coordinate u of $X^4(X_l^3)$ would remain the same since any other curve along light-like boundary would be space-like.

4. The vector fields for representing Kac-Moody algebra must vanish at the partonic 2-surface $X^2 \subset \delta M_\pm^4 \times CP_2$. The corresponding vector field must vanish at each point of X^2:

$$j^k = \sum \Phi_A(x) J^{kl} H_l^A = 0 \ . \tag{7.3.17}$$

This means that the vector field corresponds to $SO(2) \times U(2)$ defining the isotropy group of the point of $S^2 \times CP_2$.

This expression could be deduced from the idea that the surfaces X^2 are analogous to origin of CP_2 at which $U(2)$ vector fields vanish. WCW at X^2 could be also regarded as the analog of the origin of local $S^2 \times CP_2$. This interpretation is in accordance with the original idea which however was given up in the lack of proper realization. The same picture can be deduced from braiding in which case the Kac-Moody algebra corresponds to local $SO(2) \times U(2)$ for each point of the braid at X^2. The condition that Kac-Moody generators with positive conformal weight annihilate physical states could be interpreted by stating effective 2-dimensionality in the sense that the deformations of X_l^3 preserving its light-likeness do not affect the physics. Note however that Kac-Moody type Virasoro generators do not annihilate physical states.

5. Kac-Moody algebra generator must leave induced Kähler form invariant at X^2. This is of course trivial since the action leaves each point invariant. The conditions of Cartan decomposition are satisfied. The commutators of the Kac-Moody vector fields with symplectic generators are non-vanishing since the action of symplectic generator on Kac-Moody generator restricted to X^2 gives a non-vanishing result belonging to the symplectic algebra. Also the commutators of Kac-Moody generators are Kac-Moody generators.

7.3.12 The Relationship Between Super-Symplectic And SuperKac-Moody Algebras, Equivalence Principle, And Justification Of P-Adic Thermodynamics

The relationship between super-symplectic algebra (SS) acting at light-cone boundary and Super Kac-Moody algebra (SKM) assumed to act on light-like 3-surfaces and by continuation of the action also to the space-like 3-surfaces at the boundaries of CD has remained somewhat enigmatic due to the lack of physical insights.

Corresponding to the coset decomposition G/H of WCW there is also the sub-algebra SD of SS acting as diffeomorphisms of given 3-surface. This algebra acts as gauge algebra. It seems that SKM and SD cannot be the same algebra.

The construction of WCW gamma matrices and study of the solutions of Kähler-Dirac equation support strongly the conclusion that the construction of physical states involves the direct sum of two algebras SS and SI. The super-generators of SS are realized using only covariantly constant mode for the right-handed neutrino. The isometry sub-algebra SI is realized using all spinor modes. The direct sum $SS \oplus SI$ has the 5 tensor factors required by p-adic mass calculations. SI is Kac-Moody algebra and could be a natural identification for SKM. This forces to give up the construction of coset representation for the Super-Virasoro algebras.

This is not the only problem. The question to precisely what extent Equivalence Principle (EP) remains true in TGD framework and what might be the precise mathematical realization of EP and to wait for an answer for rather long time. Also the justification of p-adic thermodynamics for the scaling generator L_0 of Virasoro algebra - in obvious conflict with the basic wisdom that this generator should annihilate physical states - remained lacking.

One cannot still exclude the possibility that these three problems could have a common solution in terms of an appropriate coset representation. Quantum variant of EP cannot not follow from the coset representation for SS and SD. The coset representation of SS and $SI = SKM$ could however make sense and would be realized in the tensor product for the representations of SS and SI and would have the five tensor factors. Physical states would correspond to those for the direct sum $SS \oplus SI$. Since $SS \oplus SI$ acts as a spectrum generating algebra rather than gauge algebras, the condition that L_0 annihilates the physical states is not necessary. The coset representation would differ from the representation for $SS \oplus SI$ only that the states would be annihilated by the differences of the SV generators rather than their sums.

New vision about the relationship between various algebras

Consider now the new vision about the relationship between SSV, its sub-algebra acting as diffeomorphisms of 3-surface and $SKMV$.

1. The isometries G of sub- WCW associted with given CD are symplectic transformations of $\delta CD \times CP_2$ [K15] (note that I have used the attribute "canonical" instead of "symplectic" in some contexts) reducing to diffeomorphisms at partonic 2-surfaces or at the entire 3-surfaces at the boundaries of CD. H acts a symplectic subgroup acting as diffeomorphisms of X^3 or partonic 2-surfaces. It should annihilate physical states so that SD associated with $H \subset G$ is not interesting as far as coset representations are considered.

   Only the sub-algebra SI associated with symplectic isometries can provide coset representation. The representation space would be generated by the action of $SS \oplus SI$ in terms of fermionic oscillator operators and WCW isometry algebra. The same representation space allows also the representation of sums of super generators so that one has two options. $SS \oplus SI$ and $SS - SI$.

2. Consider first the $SS \oplus SI$ option. In this case the number of tensor factors in Super-Virasoro algebra is five as required by the p-adic mass calculations. L_n annihilated physical states but there is no need for L_0 to annihilate them since symplectic algebra is not gauge algebra.

3. Consider next the $SS - SI$ obtain, the coset representation. A generalization of the coset construction obtained by replacing finite-dimensional Lie group with infinite-dimensional symplectic group suggests itself. The differences of Super-Virasor algebra elements for SS and SI would annihilate physical states. Also the generators O_n, $n > 0$, for both algebras would annihilate the physical states so that the differences of the elements would annihilate automatically physical states for $n > 0$. For coset representation one could even require that the difference of the scaling generators L_0 annihilates the physical states.

   The problem is however that the Super Virasoro algebra generators do note reduce to the sums of generators assignable to SS and SI so that one does not obtain the five tensor factors.

The coset representation motivated the proposal was that identical action of the Dirac operators assignable to G and H in coset representation could provide the long sought-for precise realization of Equivalence Principle (EP) in TGD framework. EP would state that the total inertial four-momentum and color quantum numbers assignable to G are equal to the gravitational four-momentum and color quantum numbers assignable to H. One can argue that since super-symplectic transformations correspond to the isometries of the "world of classical worlds", the assignment of the attribute "inertial" to them is natural.

This interpretation is not feasible if H corresponds acts as diffeomorphisms: the four-momentum associated with SD most naturally vanishes since it represents diffeomorphisms. If H corresponds to SI, one has the problem with the number of tensor factors. Therefore $SS \oplus SI$ seems to be the only working option.

A more feasible realization of EP quantum level is as Quantum Classical Correspondence (QCC) stating that the conserved four-momentum associated with Kähler action equals to an eigenvalue of the conserved Kähler-Dirac four-momentum having natural interpretation as gravitational four-momentum due the fact that well-defined em charge for spinor modes forces them in the generic case to string world sheets. At classical level EP follows at GRT limit obtained by lumping many-sheeted space-time to M^4 with effective metric satisfying Einstein's equations as a reflection of the underlying Poincare invariance.

Consistency with p-adic thermodynamics

The consistency with p-adic thermodynamics provides a strong reality test and has been already used as a constraint in attempts to understand the super-conformal symmetries in partonic level.

1. The hope was that for SS/SI coset representations the p-adic thermal expectation values of the SS and SI conformal weights would be non-vanishing and identical and mass squared could be identified equivalently either as the expectation value of SI or SS scaling generator L_0. There would be no need to give up Super Virasoro conditions for $SS - SI$.

2. There seems consistency with p-adic mass calculations for hadrons [K47] since the non-perturbative SS contributions and perturbative SKM contributions to the mass correspond to space-time sheets labeled by different p-adic primes. The earlier statement that SS is responsible for the dominating non-perturbative contributions to the hadron mass transforms to a statement reflecting $SS - SI$ duality. The perturbative quark contributions to hadron masses can be calculated most conveniently by using p-adic thermodynamics for SI whereas non-perturbative contributions to hadron masses can be calculated most conveniently by using p-adic thermodynamics for SS. Also the proposal that the exotic analogs of baryons resulting when baryon looses its valence quarks [K42] remains intact in this framework.

3. The results of p-adic mass calculations depend crucially on the number N of tensor factors contributing to the Super-Virasoro algebra. The required number is $N = 5$ and during years I have proposed several explanations for this number. This excldes the coset representation SS/SI. $SS \oplus SI$ however survives. It indeed seems that holonomic contributions related to spinor modes other than covariantly constant right-handed neutrino- that is electro-weak and spin contributions- must be regarded as contributions separate from those coming from isometries. SKM algebras in electro-weak degrees and spin degrees of of freedom, would give 2+1=3 tensor factors corresponding to $U(2)_{ew} \times SU(2)$. $SU(3)$ and $SO(3)$ (or $SO(2) \subset SO(3)$ leaving the intersection of light-like ray with S^2 invariant) would give 2 additional tensor factors. Altogether one would indeed have 5 tensor factors.

There are some further questions which pop up in mind immediately.

1. In positive energy ontology Lorentz invariance requires the interpretation of mass squared as thermal expectation value of the conformal weight assignable to vibrational degrees of freedom. In Zero Energy Ontology (ZEO) quantum theory can be formally regarded as a square root of thermodynamics and it is possible to speak about thermal expectation value of mass squared without losing Lorentz invariance since the zero energy state corresponds to a square root of density matrix expressible as product of hermitian and unitary matrices.

This implies that one can speak about thermal expectation value of mass squared rather than conformal weight. This might have some non-trivial experimental consequences since the energies of states with the same free momentum contributing to the thermal expectation value are different.

2. The coefficient of proportionality can be however deduced from the observation that the mass squared values for CP_2 Dirac operator correspond to definite values of conformal weight in p-adic mass calculations. It is indeed possible to assign to partonic 2-surface X^2 CP_2 partial waves correlating strongly with the net electro-weak quantum numbers of the parton so that the assignment of ground state conformal weight to CP_2 partial waves makes sense. The identification of the spinor partial waves is in terms of ground states of super-conformal representations.

3. In the case of M^4 degrees of freedom it is strictly speaking not possible to talk about momentum eigen states since translations take parton out of δH_+. This would suggests that 4-momentum must be assigned with the tip of the light-cone containing the particle but this is not consistent with zero energy ontology. Hence it seems that one must restrict the translations of X_l^3 to time like translations in the direction of geometric future at $\delta M_+^4 \times CP_2$. The decomposition of the partonic 3-surface X_l^3 to regions $X_{l,i}^3$ carrying non-vanishing induced Kähler form and the possibility to assign $M^2(x) \subset M^4$ to the tangent space of $X^4(X_l^3)$ at points of X_l^3 suggests that the points of number theoretic braid to which oscillator operators can be assigned can carry four-momentum in the plane defined by $M^2(x)$. One could assume that the four-momenta assigned with points in given region X_i^3 are collinear but even this restriction is not necessary.

4. The additivity of conformal weight means additivity of mass squared at parton level and this has been indeed used in p-adic mass calculations. This implies the conditions

$$(\sum_i p_i)^2 = \sum_i m_i^2 \tag{7.3.18}$$

The assumption $p_i^2 = m_i^2$ makes sense only for massless partons moving collinearly. In the QCD based model of hadrons only longitudinal momenta and transverse momentum squared are used as labels of parton states, which together with the presence of preferred plane M^2 would suggest that one has

$$\begin{aligned} p_{i,||}^2 &= m_i^2 \ , \\ -\sum_i p_{i,\perp}^2 + 2\sum_{i,j} p_i \cdot p_j &= 0 \ . \end{aligned} \tag{7.3.19}$$

The masses would be reduced in bound states: $m_i^2 \to m_i^2 - (p_T^2)_i$. This could explain why massive quarks can behave as nearly massless quarks inside hadrons.

How it is possible to have negative conformal weights for ground states?

p-Adic mass calculations require negative conformal weights for ground states [K39]. The only elegant solution of the problems caused by this requirement seems to be p-adic: the conformal weights are positive in the real sense but as p-adic numbers their dominating part is negative integer (in the real sense), which can be compensated by the conformal weights of Super Virasoro generators.

1. If $\pm\lambda_i^2$ as such corresponds to a ground state conformal weight and if λ_i is real the ground state conformal weight positive in the real sense. In complex case (instanton term) the most natural formula is $h = \pm|\lambda|^2$.

2. The first option is based on the understanding of conformal excitations in terms of CP breaking instanton term added to the modified Dirac operator. In this case the conformal weights are identified as $h = n - |\lambda_k|^2$ and the minus sign comes from the Euclidian signature of the effective metric for the Kähler-Dirac operator. Ground state conformal weight would be non-vanishing for non-zero modes of $D(X_l^3)$. Massless bosons produce difficulties unless one has $h = |\lambda_i(1) - \lambda_i(2)|^2$, where $i = 1, 2$ refers to the two wormhole throats. In this case the difference can vanish and its non-vanishing would be due to the symmetric breaking. This scenario is assumed in p-adic mass calculations. Fermions are predicted to be always massive since zero modes of $D(X^2)$ represent super gauge degrees of freedom.

3. In the context of p-adic thermodynamics a loop hole opens allowing λ_i to be real. In spirit of rational physics suppose that one has in natural units $h = \lambda_i^2 = xp^2 - n$, where x is integer. This number is positive and large in the real sense. In p-adic sense the dominating part of this number is $-n$ and can be compensated by the net conformal weight n of Super Virasoro generators acting on the ground state. xp^2 represents the small Higgs contribution to the mass squared proportional to $(xp^2)_R \simeq x/p^2$ ($_R$ refers to canonical identification). By the basic features of the canonical identification $p > x \simeq p$ should hold true for gauge bosons for which Higgs contribution dominates. For fermions x should be small since p-adic mass calculations are consistent with the vanishing of Higgs contribution to the fermion mass. This would lead to the earlier conclusion that xp^2 and hence B_K is large for bosons and small for fermions and that the size of fermionic (bosonic) wormhole throat is large (small). This kind of picture is consistent with the p-adic modular arithmetics and suggests by the cutoff for conformal weights implied by the fact that both the number of fermionic oscillator operators and the number of points of number theoretic braid are finite. This solution is however tricky and does not conform with number theoretical universality.

7.4 Are Both Symplectic And Conformal Field Theories Needed?

Symplectic (or canonical as I have called them) symmetries of $\delta M^4_+ \times CP_2$ (light-cone boundary briefly) act as isometries of the "world of classical worlds". One can see these symmetries as analogs of Kac-Moody type symmetries with symplectic transformations of $S^2 \times CP_2$, where S^2 is $r_M = constant$ sphere of light-cone boundary, made local with respect to the light-like radial coordinate r_M taking the role of complex coordinate. Thus finite-dimensional Lie group G is replaced with infinite-dimensional group of symplectic transformations. This inspires the question whether a symplectic analog of conformal field theory at $\delta M^4_+ \times CP_2$ could be relevant for the construction of n-point functions in quantum TGD and what general properties these n-point functions would have. This section appears already in the previous chapter about symmetries of quantum TGD [K14] but because the results of the section provide the first concrete construction recipe of M-matrix in zero energy ontology, it is included also in this chapter.

7.4.1 Symplectic QFT At Sphere

Actually the notion of symplectic QFT emerged as I tried to understand the properties of cosmic microwave background which comes from the sphere of last scattering which corresponds roughly to the age of 5×10^5 years [K53]. In this situation vacuum extremals of Kähler action around almost unique critical Robertson-Walker cosmology imbeddable in $M^4 \times S^2$, where there is homologically trivial geodesic sphere of CP_2. Vacuum extremal property is satisfied for any space-time surface which is surface in $M^4 \times Y^2$, Y^2 a Lagrangian sub-manifold of CP_2 with vanishing induced Kähler form. Symplectic transformations of CP_2 and general coordinate transformations of M^4 are dynamical symmetries of the vacuum extremals so that the idea of symplectic QFT emerges natural. Therefore I shall consider first symplectic QFT at the sphere S^2 of last scattering with temperature fluctuation $\Delta T/T$ proportional to the fluctuation of the metric component g_{aa} in Robertson-Walker coordinates.

1. In quantum TGD the symplectic transformation of the light-cone boundary would induce action in the "world of classical worlds" (light-like 3-surfaces). In the recent situation it is

convenient to regard perturbations of CP_2 coordinates as fields at the sphere of last scattering (call it S^2) so that symplectic transformations of CP_2 would act in the field space whereas those of S^2 would act in the coordinate space just like conformal transformations. The deformation of the metric would be a symplectic field in S^2. The symplectic dimension would be induced by the tensor properties of R-W metric in R-W coordinates: every S^2 coordinate index would correspond to one unit of symplectic dimension. The symplectic invariance in CP_2 degrees of freedom is guaranteed if the integration measure over the vacuum deformations is symplectic invariant. This symmetry does not play any role in the sequel.

2. For a symplectic scalar field $n \geq 3$-point functions with a vanishing anomalous dimension would be functions of the symplectic invariants defined by the areas of geodesic polygons defined by subsets of the arguments as points of S^2. Since n-polygon can be constructed from 3-polygons these invariants can be expressed as sums of the areas of 3-polygons expressible in terms of symplectic form. n-point functions would be constant if arguments are along geodesic circle since the areas of all sub-polygons would vanish in this case. The decomposition of n-polygon to 3-polygons brings in mind the decomposition of the n-point function of conformal field theory to products of 2-point functions by using the fusion algebra of conformal fields (very symbolically $\Phi_k\Phi_l = c_{kl}^m\Phi_m$). This intuition seems to be correct.

3. Fusion rules stating the associativity of the products of fields at different points should generalize. In the recent case it is natural to assume a non-local form of fusion rules given in the case of symplectic scalars by the equation

$$\Phi_k(s_1)\Phi_l(s_2) = \int c_{kl}^m f(A(s_1,s_2,s_3))\Phi_m(s)d\mu_s \quad . \tag{7.4.1}$$

   Here the coefficients c_{kl}^m are constants and $A(s_1,s_2,s_3)$ is the area of the geodesic triangle of S^2 defined by the sympletic measure and integration is over S^2 with symplectically invariant measure $d\mu_s$ defined by symplectic form of S^2. Fusion rules pose powerful conditions on n-point functions and one can hope that the coefficients are fixed completely.

4. The application of fusion rules gives at the last step an expectation value of 1-point function of the product of the fields involves unit operator term $\int c_{kl} f(A(s_1,s_2,s))Idd\mu_s$ so that one has

$$\langle\Phi_k(s_1)\Phi_l(s_2)\rangle = \int c_{kl} f(A(s_1,s_2,s))d\mu_s \quad . \tag{7.4.2}$$

   Hence 2-point function is average of a 3-point function over the third argument. The absence of non-trivial symplectic invariants for 1-point function means that $n = 1$- an are constant, most naturally vanishing, unless some kind of spontaneous symmetry breaking occurs. Since the function $f(A(s_1,s_2,s_3))$ is arbitrary, 2-point correlation function can have both signs. 2-point correlation function is invariant under rotations and reflections.

7.4.2 Symplectic QFT With Spontaneous Breaking Of Rotational And Reflection Symmetries

CMB data suggest breaking of rotational and reflection symmetries of S^2. A possible mechanism of spontaneous symmetry breaking is based on the observation that in TGD framework the hierarchy of Planck constants assigns to each sector of the generalized imbedding space a preferred quantization axes. The selection of the quantization axis is coded also to the geometry of "world of classical worlds", and to the quantum fluctuations of the metric in particular. Clearly, symplectic QFT with spontaneous symmetry breaking would provide the sought-for really deep reason for the quantization of Planck constant in the proposed manner.

1. The coding of angular momentum quantization axis to the generalized imbedding space geometry allows to select South and North poles as preferred points of S^2. To the three arguments s_1, s_2, s_3 of the 3-point function one can assign two squares with the added point being either North or South pole. The difference

$$\Delta A(s_1, s_2, s_3) \equiv A(s_1, s_2, s_3, N) - A(s_1, s_2, s_3, S) \tag{7.4.3}$$

of the corresponding areas defines a simple symplectic invariant breaking the reflection symmetry with respect to the equatorial plane. Note that ΔA vanishes if arguments lie along a geodesic line or if any two arguments co-incide. Quite generally, symplectic QFT differs from conformal QFT in that correlation functions do not possess singularities.

2. The reduction to 2-point correlation function gives a consistency conditions on the 3-point functions

$$\langle (\Phi_k(s_1)\Phi_l(s_2))\Phi_m(s_3)\rangle = c_{kl}^r \int f(\Delta A(s_1, s_2, s))\langle \Phi_r(s)\Phi_m(s_3)\rangle d\mu_s$$
$$= \tag{7.4.4}$$
$$c_{kl}^r c_{rm} \int f(\Delta A(s_1, s_2, s)) f(\Delta A(s, s_3, t)) d\mu_s d\mu_t \ . \tag{7.4.5}$$

Associativity requires that this expression equals to $\langle \Phi_k(s_1)(\Phi_l(s_2)\Phi_m(s_3))\rangle$ and this gives additional conditions. Associativity conditions apply to $f(\Delta A)$ and could fix it highly uniquely.

3. 2-point correlation function would be given by

$$\langle \Phi_k(s_1)\Phi_l(s_2)\rangle = c_{kl} \int f(\Delta A(s_1, s_2, s)) d\mu_s \tag{7.4.6}$$

4. There is a clear difference between $n > 3$ and $n = 3$ cases: for $n > 3$ also non-convex polygons are possible: this means that the interior angle associated with some vertices of the polygon is larger than π. $n = 4$ theory is certainly well-defined, but one can argue that so are also $n > 4$ theories and skeptic would argue that this leads to an inflation of theories. TGD however allows only finite number of preferred points and fusion rules could eliminate the hierarchy of theories.

5. To sum up, the general predictions are following. Quite generally, for $f(0) = 0$ n-point correlation functions vanish if any two arguments co-incide which conforms with the spectrum of temperature fluctuations. It also implies that symplectic QFT is free of the usual singularities. For symmetry breaking scenario 3-point functions and thus also 2-point functions vanish also if s_1 and s_2 are at equator. All these are testable predictions using ensemble of CMB spectra.

7.4.3 Generalization To Quantum TGD

Since number theoretic braids are the basic objects of quantum TGD, one can hope that the n-point functions assignable to them could code the properties of ground states and that one could separate from n-point functions the parts which correspond to the symplectic degrees of freedom acting as symmetries of vacuum extremals and isometries of the "world of classical worlds".

1. This approach indeed seems to generalize also to quantum TGD proper and the n-point functions associated with partonic 2-surfaces can be decomposed in such a manner that one obtains coefficients which are symplectic invariants associated with both S^2 and CP_2 Kähler form.

2. Fusion rules imply that the gauge fluxes of respective Kähler forms over geodesic triangles associated with the S^2 and CP_2 projections of the arguments of 3-point function serve basic building blocks of the correlation functions. The North and South poles of S^2 and three poles of CP_2 can be used to construct symmetry breaking n-point functions as symplectic invariants. Non-trivial 1-point functions vanish also now.

3. The important implication is that n-point functions vanish when some of the arguments co-incide. This might play a crucial role in taming of the singularities: the basic general prediction of TGD is that standard infinities of local field theories should be absent and this mechanism might realize this expectation.

Next some more technical but elementary first guesses about what might be involved.

1. It is natural to introduce the moduli space for n-tuples of points of the symplectic manifold as the space of symplectic equivalence classes of n-tuples. In the case of sphere S^2 convex n-polygon allows $n+1$ 3-sub-polygons and the areas of these provide symplectically invariant coordinates for the moduli space of symplectic equivalence classes of n-polygons (2^n-D space of polygons is reduced to $n+1$-D space). For non-convex polygons the number of 3-sub-polygons is reduced so that they seem to correspond to lower-dimensional sub-space. In the case of CP_2 n-polygon allows besides the areas of 3-polygons also 4-volumes of 5-polygons as fundamental symplectic invariants. The number of independent 5-polygons for n-polygon can be obtained by using induction: once the numbers $N(k,n)$ of independent $k \leq n$-simplices are known for n-simplex, the numbers of $k \leq n+1$-simplices for $n+1$-polygon are obtained by adding one vertex so that by little visual gymnastics the numbers $N(k,n+1)$ are given by $N(k,n+1) = N(k-1,n)+N(k,n)$. In the case of CP_2 the allowance of 3 analogs $\{N,S,T\}$ of North and South poles of S^2 means that besides the areas of polygons (s_1,s_2,s_3), (s_1,s_2,s_3,X), (s_1,s_2,s_3,X,Y), and (s_1,s_2,s_3,N,S,T) also the 4-volumes of 5-polygons (s_1,s_2,s_3,X,Y), and of 6-polygon (s_1,s_2,s_3,N,S,T), $X,Y \in \{N,S,T\}$ can appear as additional arguments in the definition of 3-point function.

2. What one really means with symplectic tensor is not clear since the naive first guess for the n-point function of tensor fields is not manifestly general coordinate invariant. For instance, in the model of CMB, the components of the metric deformation involving S^2 indices would be symplectic tensors. Tensorial n-point functions could be reduced to those for scalars obtained as inner products of tensors with Killing vector fields of $SO(3)$ at S^2. Again a preferred choice of quantization axis would be introduced and special points would correspond to the singularities of the Killing vector fields.

   The decomposition of Hamiltonians of the "world of classical worlds" expressible in terms of Hamiltonians of $S^2 \times CP_2$ to irreps of $SO(3)$ and $SU(3)$ could define the notion of symplectic tensor as the analog of spherical harmonic at the level of WCW . Spin and gluon color would have natural interpretation as symplectic spin and color. The infinitesimal action of various Hamiltonians on n-point functions defined by Hamiltonians and their super counterparts is well-defined and group theoretical arguments allow to deduce general form of n-point functions in terms of symplectic invariants.

3. The need to unify p-adic and real physics by requiring them to be completions of rational physics, and the notion of finite measurement resolution suggest that discretization of also fusion algebra is necessary. The set of points appearing as arguments of n-point functions could be finite in a given resolution so that the p-adically troublesome integrals in the formulas for the fusion rules would be replaced with sums. Perhaps rational/algebraic variants of $S^2 \times CP_2 = SO(3)/SO(2) \times SU(3)/U(2)$ obtained by replacing these groups with their rational/algebraic variants are involved. Tedrahedra, octahedra, and dodecahedra suggest themselves as simplest candidates for these discretized spaces. Also the symplectic moduli space would be discretized to contain only n-tuples for which the symplectic invariants are numbers in the allowed algebraic extension of rationals. This would provide an abstract looking but actually very concrete operational approach to the discretization involving only areas of n-tuples as internal coordinates of symplectic equivalence classes of n-tuples. The best that one could achieve would be a formulation involving nothing below measurement resolution.

4. This picture based on elementary geometry might make sense also in the case of conformal symmetries. The angles associated with the vertices of the S^2 projection of n-polygon could define conformal invariants appearing in n-point functions and the algebraization of the corresponding phases would be an operational manner to introduce the space-time correlates for the roots of unity introduced at quantum level. In CP_2 degrees of freedom the projections of n-tuples to the homologically trivial geodesic sphere S^2 associated with the particular sector of CH would allow to define similar conformal invariants. This framework gives dimensionless areas (unit sphere is considered). p-Adic length scale hypothesis and hierarchy of Planck constants would bring in the fundamental units of length and time in terms of CP_2 length.

The recent view about M-matrix described is something almost unique determined by Connes tensor product providing a formal realization for the statement that complex rays of state space are replaced with $\mathcal{N}$ rays where $\mathcal{N}$ defines the hyper-finite sub-factor of type II_1 defining the measurement resolution. M-matrix defines time-like entanglement coefficients between positive and negative energy parts of the zero energy state and need not be unitary. It is identified as square root of density matrix with real expressible as product of of real and positive square root and unitary S-matrix. This S-matrix is what is measured in laboratory. There is also a general vision about how vertices are realized: they correspond to light-like partonic 3-surfaces obtained by gluing incoming and outgoing partonic 3-surfaces along their ends together just like lines of Feynman diagrams. Note that in string models string world sheets are non-singular as 2-manifolds whereas 1-dimensional vertices are singular as 1-manifolds. These ingredients we should be able to fuse together. So we try once again!

1. *Iteration* starting from vertices and propagators is the basic approach in the construction of n-point function in standard QFT. This approach does not work in quantum TGD. Symplectic and conformal field theories suggest that *recursion* replaces iteration in the construction. One starts from an n-point function and reduces it step by step to a vacuum expectation value of a 2-point function using fusion rules. Associativity becomes the fundamental dynamical principle in this process. Associativity in the sense of classical number fields has already shown its power and led to a hyper-octoninic formulation of quantum TGD promising a unification of various visions about quantum TGD [K74].

2. Let us start from the representation of a zero energy state in terms of a causal diamond defined by future and past directed light-cones. Zero energy state corresponds to a quantum superposition of light-like partonic 3-surfaces each of them representing possible particle reaction. These 3-surfaces are very much like generalized Feynman diagrams with lines replaced by light-like 3-surfaces coming from the upper and lower light-cone boundaries and glued together along their ends at smooth 2-dimensional surfaces defining the generalized vertices.

3. It must be emphasized that the generalization of ordinary Feynman diagrammatics arises and conformal and symplectic QFTs appear only in the calculation of single generalized Feynman diagram. Therefore one could still worry about loop corrections. The fact that no integration over loop momenta is involved and there is always finite cutoff due to discretization together with recursive instead of iterative approach gives however good hopes that everything works. Note that this picture is in conflict with one of the earlier approaches based on positive energy ontology in which the hope was that only single generalized Feynman diagram could define the U-matrix thought to correspond to physical S-matrix at that time.

4. One can actually simplify things by identifying generalized Feynman diagrams as maxima of Kähler function with functional integration carried over perturbations around it. Thus one would have conformal field theory in both fermionic and WCW degrees of freedom. The light-like time coordinate along light-like 3-surface is analogous to the complex coordinate of conformal field theories restricted to some curve. If it is possible continue the light-like time coordinate to a hyper-complex coordinate in the interior of 4-D space-time sheet, the correspondence with conformal field theories becomes rather concrete. Same applies to the light-like radial coordinates associated with the light-cone boundaries. At light-cone

boundaries one can apply fusion rules of a symplectic QFT to the remaining coordinates. Conformal fusion rules are applied only to point pairs which are at different ends of the partonic surface and there are no conformal singularities since arguments of n-point functions do not co-incide. By applying the conformal and symplectic fusion rules one can eventually reduce the n-point function defined by the various fermionic and bosonic operators appearing at the ends of the generalized Feynman diagram to something calculable.

5. Finite measurement resolution defining the Connes tensor product is realized by the discretization applied to the choice of the arguments of n-point functions so that discretion is not only a space-time correlate of finite resolution but actually defines it. No explicit realization of the measurement resolution algebra $\mathcal{N}$ seems to be needed. Everything should boil down to the fusion rules and integration measure over different 3-surfaces defined by exponent of Kähler function and by imaginary exponent of Chern-Simons action. The continuation of WCW Clifford algebra for 3-surfaces with cm degrees of freedom fixed to a hyper-octonionic variant of gamma matrix field of super-string models defined in M^8 (hyper-octonionic space) and $M^8 \leftrightarrow M^4 \times CP_2$ duality leads to a unique choice of the points, which can contribute to n-point functions as intersection of M^4 subspace of M^8 with the counterparts of partonic 2-surfaces at the boundaries of light-cones of M^8. Therefore there are hopes that the resulting theory is highly unique. Symplectic fusion algebra reduces to a finite algebra for each space-time surface if this picture is correct.

6. Consider next some of the details of how the light-like 3-surface codes for the fusion rules associated with it. The intermediate partonic 2- surfaces must be involved since otherwise the construction would carry no information about the properties of the light-like 3-surface, and one would not obtain perturbation series in terms of the relevant coupling constants. The natural assumption is that partonic 2-surfaces belong to future/past directed light-cone boundary depending on whether they are on lower/upper half of the causal diamond. Hyper-octonionic conformal field approach fixes the n_{int} points at intermediate partonic two-sphere for a given light-like 3-surface representing generalized Feynman diagram, and this means that the contribution is just N-point function with $N = n_{out} + n_{int} + n_{in}$ calculable by the basic fusion rules. Coupling constant strengths would emerge through the fusion coefficients, and at least in the case of gauge interactions they must be proportional to Kähler coupling strength since n-point functions are obtained by averaging over small deformations with vacuum functional given by the exponent of Kähler function. The first guess is that one can identify the spheres $S^2 \subset \delta M^4_\pm$ associated with initial, final and, and intermediate states so that symplectic n-points functions could be calculated using single sphere.

These findings raise the hope that quantum TGD is indeed a solvable theory. Even if one is not willing to swallow any bit of TGD, the classification of the symplectic QFTs remains a fascinating mathematical challenge in itself. A further challenge is the fusion of conformal QFT and symplectic QFT in the construction of n-point functions. One might hope that conformal and symplectic fusion rules can be treated separately. This separation indeed happens since conformal degrees of freedom correspond to quantum fluctuations contributing to the WCW metric and affecting the induced metric whereas symplectic invariants correspond to non-quantum fluctuating zero modes defining the part of quantum state not affected by quantum fluctuations parameterized by the symplectic group of $\delta M^4_\pm \times CP_2$. Also the dream about symplectic fusion rules have been realized. An explicit construction of symplectic fusion algebras is represented in [K10].

Chapter 8

Zero Energy Ontology and Matrices

8.1 Introduction

Zero energy ontology has become gradually one of the corner stones of quantum TGD. Quantum criticality has been the key idea from beginning but its understanding has grown rather slowly. Now it can be understood in terms of several hierarchies: hierarchy of Planck constants, hierarchy of breakings of super-symplectic symmetry represented as gauge symmetry, hierarchy of CDs, even hierarchy of conscious entities. Hyperfinite factors of type II_1 are highly suggestive candidates for the mathematical realization of these hierarchies. This motivate the discussion of ZEO and HFFs in the same chapter. Only general identifications for M and U matrices generalizing S-matrix to TGD framework are given but concrete proposals are left to later chapters.

8.1.1 Zero Energy Ontology And Interpretation Of Light-Like 3-Surfaces As Generalized Feynman Diagrams

1. Zero energy ontology (ZEO) is the cornerstone of the construction. Zero energy states have vanishing net quantum numbers and consist of positive and negative energy parts, which can be thought of as being localized at the boundaries of light-like 3-surface X_l^3 connecting the light-like boundaries of a causal diamond CD identified as intersection of future and past directed light-cones. There is entire hierarchy of CDs, whose scales are suggested to come as powers of 2. A more general proposal is that prime powers of fundamental size scale are possible and would conform with the most general form of p-adic length scale hypothesis. The hierarchy of size scales assignable to CDs corresponds to a hierarchy of length scales and code for a hierarchy of radiative corrections to generalized Feynman diagrams.

2. Either space-like 3-surfaces at the boundaries of CDs or light-like 3-surfaces connecting the boundaries of CDs can be seen as the basic dynamical objects of quantum TGD and have interpretation as generalized Feynman diagrams having light-like 3-surfaces as lines glued together along their ends defining vertices as 2-surfaces. By effective 2-dimensionality (holography) of light-like 3-surfaces the interiors of light-like 3-surfaces are analogous to gauge degrees of freedom and partially parameterized by Kac-Moody group respecting the light-likeness of 3-surfaces. This picture differs dramatically from that of string models since light-like 3-surfaces replacing stringy diagrams are singular as manifolds whereas 2-surfaces representing vertices are not.

3. String word sheets and partonic 2-surfaces however appear also in TGD as carriers of spinor modes: this follows from the condition that em charge is well defined for the modes. The condition follows also from number theoretic arguments and is assumed quite generally. This has far reaching consequences for the understanding of gravitation in TGD framework and profound deviations from string models are predicted due to the hierarchy of Planck constants absolutely essential for the description of gravitational bound states in terms of strings

connecting partonic 2-surfaces. Macroscopic quantum coherence in even astrophysical scales is predicted [K106, K109].

8.1.2 Identification Of The Counterpart Of M-Matrix As Time-Like Entanglement Coefficients

1. The TGD counterpart of S-matrix -call it M-matrix- defines time-like entanglement coefficients between positive and negative energy parts of zero energy state located at the light-like boundaries of CD. One can also assign to quantum jump between zero energy states a matrix-call it U-matrix - which is unitary and assumed to be expressible in terms of M-matrices. M-matrix need not be unitary unlike the U-matrix characterizing the unitary process forming part of quantum jump. There are several arguments suggesting that M-matrix cannot be unitary but can be regarded as thermal S-matrix so that thermodynamics would become an essential part of quantum theory. In fact, M-matrix can be decomposed to a product of positive diagonal matrix identifiable as square root of density matrix and unitary matrix so that quantum theory would be kind of square root of thermodynamics. Path integral formalism is given up although functional integral over the 3-surfaces is present.

2. In the general case only thermal M-matrix defines a normalizable zero energy state so that thermodynamics or at least formalism resembling thermodynamics becomes part of quantum theory. One can assign to M-matrix a complex parameter whose real part has interpretation as interaction time and imaginary part as the inverse temperature.

8.1.3 Topics Of The Chapter

The goal is to provide some conceptual background for the attempts to identify scattering amplitudes in TGD framework.

First the basic ideas and implications of ZEO are described. I will represent motivations for ZEO in TGD framework, compare ZEO with the positive energy ontology, and try to make clear the implications of ZEO for quantum measurement theory since they relate also directly to the notion of conscious observer as it is understood in TGD inspired theory of consciousness. After that the definitions of M-matrix and U-matrix are discussed.

The notion of hyper-finite factor expected to play central role in the mathematical description of finite measurement resolution, in the realization of the hierarchy of Planck constants [K22, K106], the hierarchy quantum criticalities, and the hierarchy of gauge symmetry breakings for the super-symplectic algebra. This motivates the discussion of the basic results and ideas are about HFFs. The views about M-matrix as a characterizer of time-like entanglement and M-matrix as a functor are analyzed. The role of hyper-finite factors in the construction of M-matrix is considered. One section is devoted to the possibility that Connes tensor product could define fundamental vertices. A more detailed discussion can be found in the book [K99], in particular in chapter [K87].

I do not pretend of having handle about the huge technical complexities and can only recommend the works of von Neumann [A78, A92, A82, A67]. Tomita [A76]. [B50, B22, B52]. the work of Powers and Araki and Woods which served as starting point for the work of Connes [A29, A28]. The work of Jones [A55], and other leading figures in the field. What is may main contribution is fresh physical interpretation of this mathematics which also helps to make mathematical conjectures. The book of Connes [A29] available in web provides an excellent overall view about von Neumann algebras and non-commutative geometry.

In the last section some general speculations about U-matrix are represented. The negative and positive energy parts of zero energy state can contain zero energy parts in shorter scales - quantum field theorist might talk about quantum fluctuations. One can have also U-matrix and M-matrix elements between this kind of states and even between zero energy states and a hierarchy suggests itself. Since fermions could be seen as correlates of Boolean cognition and zero energy states in fermion sectors as quantal Boolean statements, one can ask whether these matrices could define Boolean hierarchies: statements about statements about...

8.2 Zero Energy Ontology

Zero energy ontology has changed profoundly the views about the construction of S-matrix and forced to introduce the separate notions of M-matrix and U-matrix. M-matrix generalizes the notion of S-matrix as used in particle physics. The unitary U-matrix is something new having a natural place in TGD inspired theory of consciousness. Therefore it it best to begin the discussion with a brief summary of zero energy ontology.

8.2.1 Motivations For Zero Energy Ontology

Zero energy ontology was first forced by the finding that the imbeddings of Robertson-Walker cosmologies to $M^4 \times CP_2$ are vacuum extremals. The interpretation is that positive and negative energy parts of states compensate each other so that all quantum states have vanishing net quantum numbers. One can however assign to state quantum numbers as those of the positive energy part of the state. At space-time level zero energy state can be visualized as having positive energy part in geometric past and negative energy part in geometric future. In time scales shorter than the temporal distance between states positive energy ontology works. In longer time scales the state is analogous to a quantum fluctuation.

Zero energy ontology gives rise to a profound distinction between TGD and standard QFT. Physical states are identified as states with vanishing net quantum numbers, in particular energy. Everything is creatable from vacuum - and one could add- by intentional action so that zero energy ontology is profoundly Eastern. Positive *resp.* negative energy parts of states can be identified as states associated with 2-D partonic surfaces at the boundaries of future *resp.* past directed light-cones, whose tips correspond to the arguments of n-point functions. Each incoming/outgoing particle would define a mini-cosmology corresponding to not so big bang/crunch. If the time scale of perception is much shorter than time interval between positive and zero energy states, the ontology looks like the Western positive energy ontology. Bras and kets correspond naturally to the positive and negative energy states and phase conjugation for laser photons making them indeed something which seems to travel in opposite time direction is counterpart for bra-ket duality.

8.2.2 Zero Energy Ontology

Zero energy ontology (ZEO) is one of the cornerstones of TGD and has become part of TGD during last six years. Zero energy states are identified as superpositions of pairs of positive and negative energy states assigned with the future and past boundaries of causal diamonds (CDs) and correspond in ordinary ontology to physical events with positive and negative energy parts of the state identified as counterparts for the initial and final states of the event. Effective 2-dimensionality allows a further reduction to the level of partonic 2-surfaces: also their 4-D tangent space data matter. Symmetry considerations lead to a beautiful view about generalizations S-matrix to U-matrix in terms o forthogonal M-matrices which in turn are expressible as products of orthogonal basis of hermitian square roots of density matrices and unitary S-matrix [K91]. One can say that quantum theory is "complex" square root of thermodynamics.

Therefore one should try to find tests for ZEO.

The hierarchy of CDs

The basic assumption is that the sizes of CDs come as integer multiples of CP_2 scale R and for prime multiples of R correspond to secondary p-adic length scales $L_{p,2} = L_{p,1}\sqrt{p}$, $L_{p_1} = R\sqrt{p}$, where R denotes CP_2 scale. For electron with $p = M_{127} = 2^{127} - 1$ one has $T_{p_2} = .1$ seconds and defines a fundamental bio-rhythm. This time scale should have preferred role in physics. More generally the secondary p-adic time scales assignable to elementary particles should define time scales relevant to macroscopic physics. The corresponding size scale can be assigned to the magnetic body of the elementary particle. Also it should be possible to assign to quark mass scales special biological time scales as has been indeed done [K5]. h predictions could be tested.

Generalization of standard conservation laws in ZEO

ZEO together with sub-manifold geometry provides a new view about conservation laws and resolves the problem posed by the fact that gravitational interactions do not seem to respect energy conservation in cosmological time scales. Conservation laws holds true only in the scale associated with given CD, not universally (this would allow only single infinitely large CD).

Superconducting coherent states involve quantum superposition of states with different numbers of Cooper pairs and therefore break the super-selection rule associated with fermion number in ordinary ontology. In ZEO they could be understood without giving up the superselection rule associated with fermion number.

Experimental tests should try to prove that quantum number conservation is a length scale dependent notion. For instance, creation of matter from vacuum is possible in ZEO, and one might hope that its occurrenece could be in some scale for CDs aritficially.

Breaking of second law in standard form

In standard physics second law states that all systems are entropic but a system can reduce its entropy by feeding its entropy to the environment. Negentropic entanglement carries genuine information and life can be seen as islands of negentropy in the sea of entropy. This forces to generalized second law. The proposed generalization (see `http://tinyurl.com/ybg8qypx`) [L9] [K41] can be characterized as maximally pessimistic.

The generation of negentropic entanglement is assumed to be accompanied by generation of compensating entropic entanglement. The modified form of second law is suggested by the mechanism of directed attention based on negentropic entanglement assignable to magnetic flux tube connecting selfandtarget. Negentropic entanglement prevails during the attention but disappears after state function reduction giving rise to entropy at the level of ensemble. Second law would hold true above time scale assignab le to the duration of negentropic entanglement.

There are also other reasons to reconsider second law. The breaking of second law in standard form since the arrow of geometric time can change locally. Living systems are indeed accompanied by syntropic effects as realized by Italian quantum physicist Fantappie [J3, J4]. These effects could be understood as entropic effects but with a reversed arrow of geometric time. The mechanism would be based on negative energy signals. Phase conjugate laser waves are known to obey second law in reversed direction of geometric time. Cooling effects due to the absorption of negative energy signals inducing the breaking of the standard form of the second law are predicted to be possible. One can also imagine a spontaneous excitation of atoms generating radiation in the return to ground state in a situation when there is a target able to receive negative energy signals emitted in spontaneous excitation.

Standard form of second law assumes that quantum coherence is absent in the scales in which it is applied. Both the hierarchy of Planck constants and negentropic entanglement however make possible macroscopic quantum coherence characterized by the scale involved and the natural guess is that the time scale associated with causal diamond in question defines the scale above which one can expect second law to hold. There is evidence for the breaking of second law in time scale of.1 seconds [D2].

Negative energy signals

Zero energy ontology allows to assign to zero energy states an arrow of time naturally since one can require that states have well defined single particle quantum numbers at either upper or lower boundary of CD. Also the spontaneous change of the arrow of geometric time is possible. The simplest possible description for U-process is that U-matrix relates to each other these two kinds of states and state function reductions can occur at upper and lower boundaries of CD meaning reduction to single particle states with well defined quantum numbers. The precise correlates for the generation of geometric arrow of time are not completely understood.

Negative energy signals to geometric past would serve as counterparts for time reversed states in the case of radiation and phase conjugate laser waves are natural counterparts for them. The signal property requires a dissipative process proceeding in preferred time direction and this kind of process has been assigned to sub-CDs and should proceed as state function reduction sequence in preferred direction of time determined by the quantum arrow of time for the zero energy state.

This process would be essential for the experience of flow of time in preferred direction and for generation of arrow of geometric time as explain in previous chapter and also in [K4]. For phase conjugate laser beams the reversed time direction for dissipation is observed.

Negative energy signals make possible remote metabolism as sucking of energy from remote energy source provided resonance conditions for transitions are satisfied. The counterpart of population inverted laser could serve as ideal source and the negative energy signal could serve as a control switch inducing phase transition like process taking the excited atom like systems to ground state (induce emission). This process should occur in living matter. Anomalous excitation of atomic state by absorbing energy by remote metabolism and subsequent generation of radiation could also serve as a signature. It could also lead to cooling effects breaking second law.

Negative energy signals would also make possible realization of intentional action by initiating the activity already in geometric past. This would be very desirable in rapidly changing circumstances. The time anomalies of Libet for active aspect of consciousness could be interpreted in terms of time mirror mechanism [J1] and further experiments in longer time scales might be perhaps carried out.

Negative energy signals could be also essential for the mechanism of long term memory. They would induce a breakdown for a system analogous to population reversed laser via induced emission meaning generation of strong positive energy signal [K61].

Definition of energy in zero energy ontology

The approach relying on the two super conformal structures of quantum TGD gives hopes of defining the notion of energy for positive and negative energy parts of the state.

1. CD allows translational invariance only in its interior and since partonic two surfaces are located to the boundary of CD, one can argue that translations assigned to them lead out from CD. One can however argue that if it is enough to assign eigenstates of four-momentum to partons and require that only the total four-momentum generators acts on the physical state by shifting CD. Since total four-momentum vanishes for CD this would mean that wave function in cm degrees of CD is just constant plane wave. Super-conformal invariance would indeed allow to assign momentum eigenstates to the super-conformal representations.

2. A more stringent condition would be that four-momentum generators act as translation like operators on partons themselves. Since light-like 3-surfaces assignable to incoming and outgoing legs of the generalized Feynman diagrams are the basic objects, one can hope of having enough translational invariance to define the notion of energy. If translations are restricted to time-like translations acting in the direction of the future (past) then one has local translation invariance of dynamics for classical field equations inside $\delta M^4_\pm$ as a kind of semigroup. Also the M^4 translations leading to interior of X^4 from the light-like 2-surfaces surfaces act as translations. Classically these restrictions correspond to non-tachyonic momenta defining the allowed directions of translations realizable as particle motions. These two kinds of translations can be assigned to super-symplectic conformal symmetries at $\delta M^4_\pm \times CP_2$ and and super Super-Kac-Moody type conformal symmetries acting as super-symplectic isometries. Super-symplectic algebra is realized in terms of second quantized spinor fields and covariantly constant modes of right-handed neutrino. Symplectic group has as sub-group symplectic isometries and the Super-Kac-Moody algebra associated with this group and represented in terms of spinor modes localized to string world sheets plays also a key role in TGD.

   Finite M^4 translations to the interior of CD do not respect the shape of the partonic 2-surface. Local M^4 translations vanishing at the boundary of CD however act as Kac-Moody symmetries of the light-like 3-surfaces and reduce physically to gauge transformations: hence one could allow also the deformations of the partonic 2-surface in the interior of the light-like 3-surface. This corresponds to the effective metric 2-dimensionality stating that all information both about the geometry of WCW and quantum physics is carried by the partonic 2-surfaces X^2 resulting as intersections of the light-like 3-surfaces X^3_l and space-like 3-D surfaces X^3 at the boundaries of CD and the distribution of 4-D tangent planes of X^2.

3. The condition selecting preferred extremals of Kähler action is induced by a global selection of $M^2 \subset M^4$ as a plane belonging to the tangent space of X^4 at all its points [K14] and interpreted as a plane of nonphysical polarizations so that direct connection with number theory and gauge symmetries emerges. The M^4 translations of X^4 as a whole in general respect the form of this condition in the interior. Furthermore, if M^4 translations are restricted to M^2, also the condition itself - rather than only its general form - is respected. This observation, the earlier experience with p-adic mass calculations, and also the treatment of quarks and gluons in QCD encourage to consider the possibility that translational invariance should be restricted to M^2 translations so that mass squared, longitudinal momentum and transversal mass squared would be well defined quantum numbers. This would be enough to realize zero energy ontology. Encouragingly, M^2 appears also in the generalization of the causal diamond to a book-like structure forced by the realization of the hierarchy of Planck constant at the level of the imbedding space.

4. That the cm degrees of freedom for CD would be gauge like degrees of freedom sounds strange. The paradoxical feeling disappears as one realizes that this is not the case for sub-CDs, which indeed can have non-trivial correlation functions with either upper or lower tip of the CD playing a role analogous to that of an argument of n-point function in QFT description. One can also say that largest CD in the hierarchy defines infrared cutoff.

Objection against zero energy ontology and quantum classical correspondence

The motivation for requiring geometry and topology of space-time as correlates for quantum states is the belief that quantum measurement theory requires the representability of the outcome of quantum measurement in terms of classical physics -and if one believes in geometrization- one ends up with generalization of Einstein's vision.

There is however a counter argument against this view and second one against zero energy ontology in which one assigns eigenstates of four-momentum with causal diamonds (CDs).

1. One can argue that momentum eigenstates for which particle regarded as a topological inhomogenuity of space-time surface, which is non-localized cannot allow a space-time correlate.

2. Even worse, CDs have finite size so that strict four-momentum eigenstates strictly are not possible.

On the other hand, the paradoxical fact is that we are able to perceive momentum eigenstates and they look localized to us. This cannot be understood in the framework of standard Poincare symmetry.

The resolution of the objections and of the apparent paradox could rely on conformal symmetry assignable to light-like 3-surfaces implying a generalization of Poincare symmetry and other symmetries with their Kac-Moody variants for which symmetry transformations become local.

1. Poincare group is replaced by its Kac-Moody variant so that all non-constant translations act as gauge symmetries. Translations which are constant in the interior of CD and trivial at the boundaries of CDs are physically equivalent with constant translations. Hence the latter objection can be circumvented.

2. The same argument allows also a localization of momentum eigenstates at the boundaries of CD. In the interior the state is non-local. Classically the momentum eigenstate assigned with the partonic 2-surface is characterized by its 4-D tangent space data coding for momentum classically. The Kähler-Dirac equation and Kähhler action indeed contain and additional term representing coupling to four-momenta of particles. Formally this corresponds only to a gauge transform linear in momentum but Kahler gauge potential has U(1) gauge symmetry only as a spin glass like degeneracy, not as a gauge symmetry so that space-time surface depends on momenta.

3. Conscious observer corresponds in TGD inspired theory of consciousness to CD and the sensory data of the observer come from partonic 2-surfaces at the boundaries of CD and its sub-CDs. This implies classicality of sensory experience and momentum eigenstates look classical for conscious perceiver.

The usual argument resolving the paradox is based on the notion of wave packet and also this notion could be involved. The notion of finite measurement resolution is key notion of TGD and it is quite possible that one can require the localization of momentum eigenstates at the boundaries of CDs only modulo finite measurement resolution for the position of the partonic 2-surfaces.

8.2.3 The Anatomy Of Quantum Jump In Zero Energy Ontology (ZEO)

Zero energy ontology (ZEO) emerged around 2005 and has had profound consequences for the understanding of quantum TGD. The basic implication is that state function reductions occur at the opposite light-like boundaries of causal diamonds (CDs) forming a hierarchy, and produce zero energy states with opposite arrows of time. Also concerning the identification of quantum jump as moment of consciousness ZEO encourages rather far reaching conclusions. In ZEO the only difference between motor action and sensory representations is that the arrows of imbedding space time (CDs) are opposite for them. Furthermore, sensory perception followed by motor action corresponds to a basic structure in the sequence of state function reductions and it seems that these processes occur fractally for CDs of various size scales.

1. State function reduction can be performed to either boundary of CD but not both simultaneously. State function reduction at either boundary is equivalent to state preparation giving rise to a state with well defined quantum numbers (particle numbers, charges, four-momentum, etc...) at this boundary of CD. At the other boundary single particle quantum numbers are not well defined although total conserved quantum numbers at boundaries are opposite by the zero energy property for every pair of positive and negative energy states in the superposition. State pairs with different total energy, fermion number, etc.. for other boundary are possible: for instance, the coherent states of super-conductor for which fermion number is ill defined are possible in zero energy ontology and do not break the super-selection rules.

2. The basic objects coding for physics are U-matrix, M-matrices and S-matrix. M-matrices correspond to hermitian square roots of density matrices multiplied by a universal S-matrix which depends on the scale n of CD in very simple manner: $S(n) = S^n$ giving thus a unitary representation for scalings. The explicit construction of a unitary U-matrix in terms of M-matrices is carried out in [K91]: U-matrix elements are essentially inner products of M-matrices associated with CDs with various size scales. One can say that quantum theory is formally a square root of thermodynamics. The thermodynamics in question would however relate more naturally to NMP rather than second law, which at ensemble level and for ordinary entanglement can be seen as a consequence of NMP.

   The non-triviality of M-matrix requires that for given state reduced at say the "lower" boundary of CD there is entire distribution of states at "upper boundary" (given initial state can lead to a continuum of final states). Even more, all size scales of CDs are possible since the position of only the "lower" boundary of CD is localized in quantum jump whereas the location of upper boundary of CD can vary so that one has distribution over CDs with different size scales and over their Lorentz boots and translates.

3. The quantum arrow of time follows from the asymmetry between positive and negative energy parts of the state: the other is prepared and the other corresponds to the superposition of the final states resulting when interactions are turned on: also quantum superposition over CDs of different sizes with second boundary belonging to the same fixed $\delta M^4_\pm$ is possible. What is remarkable that the arrow of time at imbedding space level (at least) changes direction as quantum jump occurs to opposite boundary.

   It is however possible to have sequences of quantum jumps occurring at the same boundary: these periods are counterparts for repeated state function reductions, which do not change the state at all in standard quantum measurement theory. During these periods the superposition of opposite boundaries of CDs and states at them change, and the average distance between the tips of CDs tends to increase, hence the flow of subjective time and its arrow.

   NMP dictates when the first quantum jumps to the opposite boundary of CD takes place. The sequence of state function reduction at the same boundary defines self as a conscious entity and the increase of the average distance between the tips of CD defines the life-time of self.

This brings strongly in mind the old proposal of Fantappie [J3] that in living matter the arrow of time is not fixed and that entropy and its diametric opposite syntropy apply to the two arrows of the imbedding space time. The arrow of subjective time assignable to second law would hold true but the increase of syntropy would be basically a reflection of second law since only the arrow of the geometric time at imbedding space level has changed direction. The arrow of geometric at space-time level which conscious observer experiences directly could be always the same if quantum classical correspondence holds true in the sense that the arrow of time for zero energy states corresponds to arrow of time for preferred extremals. The failure of strict non-determinism making possible phenomena analogous to multi-furcations makes this possible.

4. This picture differs radically from the standard view and if quantum jump represents a fundamental algorithm, this variation of the arrow of geometric time should manifest itself in the functioning of brain and living organisms. The basic building brick in the functioning of brain is the formation of sensory representation followed by motor action/volition realized as the first reduction at the opposite boundary.

   These processes look very much like temporal mirror images of each other such as the state function reductions to opposite boundaries of CD look like. The fundamental process could correspond to a sequences of these two kinds of state function reductions at opposite boundaries of CDs and maybe independently for CDs of different size scales in a "many-particle" state defined by a union of CDs.

How the formation of cognitive and sensory representations could relate to quantum jump?

1. The earlier view was based on the idea that p-adic space-time sheets can transform to real ones and vice versa in quantum jump and these process correspond to a realization of intention as action and formation of though. This view is mathematically awkward and has been replaced with the adelic vision in which all systems have both sensory (real space-time sheets) and cognitive (p-adic space-time sheets) space-time correlates. The real and p-adic number fields form a book like structure - adele- with an algebraic extension of rationals as its back. Same applieds at the level of imbedding space, space-time surfaces, and WCW. In this framedwork holography makes it possible to understand real and p-adic space-time surfaces as continuations of string world sheets and partonic 2-surfaces to space-time surfaces, either real or p-adic. The string world sheets themselves are in the intersection of reality and various p-adicities in the sense that the parameters characterizing them belong to an extension of rational numbers.

2. Self having the mental image about intention can be be seen as the agent transforming intention to action. By NMP negentropy is typically generated in this transition tending to increase the value of Planck constant $h_{eff} = n \times h$ and thus reducing quantum criticality and occurring therefore spontaneously. Negentropy Maximization Principle eventually forces the occurrence of volitional action - self experiences the urge to perform the action so strong that cannot resist. Subself representing the mental image about intention tries to prevent it as long as possible because it means death: all living systems try to stay at the existing level of criticality and avoid the fatal final state function reduction by practicing homeostasis and using metabolic energy. Weak form of NMP states that self has freedom to decide whether it performs the reduction producing maximal entanglement negentropy. It can also perform ordinary quantum jump reducing entanglement entropy to zero and destroying entanglement. The outcome is isolation from the external world. The motivation for the weak form of NMP is that we do not live in the best possible world and have free will to choose between Good and Evil. Strong form of NMP would produce always mazimal negentropy gain and would mean best possible world.ur in various length scales in fractal manner.

8.2.4 Conscious Entities And Arrow Of Time In TGD Universe

"Fractality from your blog" posed an interesting question about possible asymmetry between boundaries of causal diamond CD. The answer to the question led to recall once again the incomplete understanding of details about how the arrow of time emerges in zero energy ontology (ZEO).

The basic vision is following.

1. CDs form a fractal scale hierarchy. Zero energy states possess a wave function in moduli degrees of freedom characterizing sizes of CDs as well telling what Lorentz boost leaving boundary invariant are allowed for them. Boosts form by number theoretic constraints a discrete subgroup of Lorentz group defining analogs of lattices generated by boosts instead of translations.

2. The arrow of subjective time maps to that of geometric time somehow. The origin of arrow comes from the fact that state function reductions can occur to either boundary of given CD and reduction creates time-asymmetric state since second boundary of CD is in a quantum superposition of different sizes and there is a superposition of many-particle states with different particles numbers and quantum number distributions. It is possible that each state function reduction leaving the passive boundary intact, involves localization in the moduli space of CDs with second boundary fixed.

3. Subjective existence corresponds to a sequence *of moments of consciousness*: state function reductions at opposite boundaries of CDs. State function reduction localizes either boundary but the second boundary is in a quantum superposition of several locations and size scales for CD. This predicts that the arrow of time is not constant. In fact, there is considerable evidence for the variation of the arrow of time in living systems and Fantappie [J3] introduced long time ago the notion of syntropy to describe his view about the situation.

4. The first very naive proposal was that state function reductions occur *alternately* to the two boundaries of CD. This assumption would be indeed natural if one considered single fixed CD rather than superposition CDs with different size and state function reduction localizing their either boundary: restriction to single CD was what I indeed did first.

5. This assumption leads to the question about why do we do not observe this alternation of the arrow of time all the time in our personal experience. Some people actually claim to have actually experienced a temporary change of the arrow of time: I belong to them and I can tell that the experience is frightening. But why do we experience the arrow of time as stable in the standard state of consciousness?

One possible way to solve the problem - perhaps the simplest one - is that state function reduction to the same boundary of CD can occur many times repeatedly. This solution is so absolutely trivial that I could perhaps use this triviality to defend myself for not realizing it immediately!

I made this totally trivial observation only after I had realized that also in this process the wave function in the moduli space of CDs change in these reductions. Zeno effect in ordinary measurement theory relies on the possibility of repeated state function reductions. In the ordinary quantum measurement theory repeated state function reductions do not affect the state in this kind of sequence but in ZEO the wave function in the moduli space labelling different CDs with the same boundary could change in each quantum jump. It would be natural that this sequence of quantum jumps give rise to the experience about flow of time? This option would allow the size scale of CD associated with human consciousness be rather short, say.1 seconds. It would allow to understand why we do not observe continual change of arrow of time.

Maybe living systems are working hardly to keep the personal arrow of time un-changed - living creatures try to prevent kettle from boiling by staring at it intensely. Maybe it would be extremely difficult to live against the collective arrow of time.

An objection against this picture as compared to the original one assuming alternate reductions to the opposite boundaries of CD is that is that one can understand state preparation as state function reduction to the opposite boundary. This interpetation makes sense almost as such also in the new picture if the average time period for which the reductions occur to a given boundary is shorter in elementary particles scales than in macroscopic scales characteristic for human consciousness. The approximate reversibility in elementary particle scales can be understood as summing up of the two arrows of time to no arrow at all.

This picture allows also to identify self as a continuous entity as the sequence of state function reductions occurring at the same boundary of CD. The average increase of the temporal distance between the tips of cD defines the life-time of self. The number of reductions would give a measure for the subjectively experienced of life-time of self.

In elementary particle time scales reversibility is a good approximation and this suggests that in elementary particle scales the number of state function reductions at the same boundary of CD is small so that the effects due to the change of the arrow of time cancel on the average.

NMP would eventually force "death" of self since the state function reduction at opposite boundary would generate more negentropy. "Death" of self would mean birth of self asssociated with the opposite boundary of CD. The age of self identified as the proper time distance between the tips would increase in statistical sense even when its arrow can change. The act of volition would have a natural identification as the first state function reduction at the opposite boundary of CD.

This picture raises a series of questions. Do our wake-up periods correspond to sequences of state function reductions for self and are sleeping periods wake-up periods of the self at the opposite boundary of CD? The arrow of geometric time should change at some space-time sheet associated with the self hierarchy. How could one demonstrate this? Are the memories of the "other" self predictions of future from our point of view? Do we sleep in order to get information from future, to remember what the future will be?

How the hierarchy of Planck constants defining a hierarchy of quantum criticalities does relate to this picture? The ageing of self having has as a correlate the increase of the size scale of CD. Could this increase be due to the increase of h_{eff} expected to occur spontaneously since it corresponds to a reduction of criticality and therefore to the appearance of new physical degrees of freedom as symplectic gauge degrees of freedom transform to physical ones in gauge symmetry breaking. This is not the case. The time evolution must be analogous to shift in time rather than scaling. This of course corresponds to the QFT view about time evolution.

In the first state function reduction to the opposite boundary of CD however scaling of CD is possible and would correspond to the scaling of CD represented by exponent of infinitesimal scaling operator as in conformal field theories. The emergence of bew physical degrees of freedom suggest increasing perceptive and cognitive capabilities. The increase of h_{eff} could be seen as evolution as also the associated increase of resources of negentropic entanglement suggests. The total increase of h_{eff} measured by the ratio $h_{eff}(final)/h_{eff}(initial)$ could be seen as a measure for the progress per single life period of self.

8.3 A Vision About The Role Of HFFs In TGD

It is clear that at least the hyper-finite factors of type II_1 assignable to WCW spinors must have a profound role in TGD. Whether also HFFS of type III_1 appearing also in relativistic quantum field theories emerge when WCW spinors are replaced with spinor fields is not completely clear. I have proposed several ideas about the role of hyper-finite factors in TGD framework. In particular, Connes tensor product is an excellent candidate for defining the notion of measurement resolution.

In the following this topic is discussed from the perspective made possible by zero energy ontology and the recent advances in the understanding of M-matrix using the notion of bosonic emergence. The conclusion is that the notion of state as it appears in the theory of factors is not enough for the purposes of quantum TGD. The reason is that state in this sense is essentially the counterpart of thermodynamical state. The construction of M-matrix might be understood in the framework of factors if one replaces state with its "complex square root" natural if quantum theory is regarded as a "complex square root" of thermodynamics. It is also found that the idea that Connes tensor product could fix M-matrix is too optimistic but an elegant formulation in terms of partial trace for the notion of M-matrix modulo measurement resolution exists and Connes tensor product allows interpretation as entanglement between sub-spaces consisting of states not distinguishable in the measurement resolution used. The partial trace also gives rise to non-pure states naturally.

The newest element in the vision is the proposal that quantum criticality of TGD Universe is realized as hierarchies of inclusions of super-conformal algebras with conformal weights coming as multiples of integer n, where n varies. If n_1 divides n_2 then various super-conformal algebras C_{n_2} are contained in C_{n_1}. This would define naturally the inclusion.

8.3.1 Basic Facts About Factors

In this section basic facts about factors are discussed. My hope that the discussion is more mature than or at least complementary to the summary that I could afford when I started the work with factors for more than half decade ago. I of course admit that this just a humble attempt of a physicist to express physical vision in terms of only superficially understood mathematical notions.

Basic notions

First some standard notations. Let $\mathcal{B}(\mathcal{H})$ denote the algebra of linear operators of Hilbert space $\mathcal{H}$ bounded in the norm topology with norm defined by the supremum for the length of the image of a point of unit sphere $\mathcal{H}$. This algebra has a lot of common with complex numbers in that the counterparts of complex conjugation, order structure and metric structure determined by the algebraic structure exist. This means the existence involution -that is *- algebra property. The order structure determined by algebraic structure means following: $A \geq 0$ defined as the condition $(A\xi, \xi) \geq 0$ is equivalent with $A = B^*B$. The algebra has also metric structure $||AB|| \leq ||A|| ||B|$ (Banach algebra property) determined by the algebraic structure. The algebra is also C^* algebra: $||A^*A|| = ||A||^2$ meaning that the norm is algebraically like that for complex numbers.

A von Neumann algebra $\mathcal{M}$ [A26] is defined as a weakly closed non-degenerate *-subalgebra of $\mathcal{B}(\mathcal{H})$ and has therefore all the above mentioned properties. From the point of view of physicist it is important that a sub-algebra is in question.

In order to define factors one must introduce additional structure.

1. Let $\mathcal{M}$ be subalgebra of $\mathcal{B}(\mathcal{H})$ and denote by $\mathcal{M}'$ its commutant $(\mathcal{H})$ commuting with it and allowing to express $\mathcal{B}(\mathcal{H})$ as $\mathcal{B}(\mathcal{H}) = \mathcal{M} \vee \mathcal{M}'$.

2. A factor is defined as a von Neumann algebra satisfying $\mathcal{M}'' = \mathcal{M}$ $\mathcal{M}$ is called factor. The equality of double commutant with the original algebra is thus the defining condition so that also the commutant is a factor. An equivalent definition for factor is as the condition that the intersection of the algebra and its commutant reduces to a complex line spanned by a unit operator. The condition that the only operator commuting with all operators of the factor is unit operator corresponds to irreducibility in representation theory.

3. Some further basic definitions are needed. $\Omega \in \mathcal{H}$ is cyclic if the closure of $\mathcal{M}\Omega$ is $\mathcal{H}$ and separating if the only element of $\mathcal{M}$ annihilating Ω is zero. Ω is cyclic for $\mathcal{M}$ if and only if it is separating for its commutant. In so called standard representation Ω is both cyclic and separating.

4. For hyperfinite factors an inclusion hierarchy of finite-dimensional algebras whose union is dense in the factor exists. This roughly means that one can approximate the algebra in arbitrary accuracy with a finite-dimensional sub-algebra.

The definition of the factor might look somewhat artificial unless one is aware of the underlying physical motivations. The motivating question is what the decomposition of a physical system to non-interacting sub-systems could mean. The decomposition of $\mathcal{B}(\mathcal{H})$ to $\vee$ product realizes this decomposition.

1. Tensor product $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$ is the decomposition according to the standard quantum measurement theory and means the decomposition of operators in $\mathcal{B}(\mathcal{H})$ to tensor products of mutually commuting operators in $\mathcal{M} = \mathcal{B}(\mathcal{H}_1)$ and $\mathcal{M}' = \mathcal{B}(\mathcal{H}_2)$. The information about $\mathcal{M}$ can be coded in terms of projection operators. In this case projection operators projecting to a complex ray of Hilbert space exist and arbitrary compact operator can be expressed as a sum of these projectors. For factors of type I minimal projectors exist. Factors of type I_n

correspond to sub-algebras of $\mathcal{B}(\mathcal{H})$ associated with infinite-dimensional Hilbert space and I_∞ to $\mathcal{B}(\mathcal{H})$ itself. These factors appear in the standard quantum measurement theory where state function reduction can lead to a ray of Hilbert space.

2. For factors of type II no minimal projectors exists whereas finite projectors exist. For factors of type II_1 all projectors have trace not larger than one and the trace varies in the range $(0,1]$. In this case cyclic vectors Ω exist. State function reduction can lead only to an infinite-dimensional subspace characterized by a projector with trace smaller than 1 but larger than zero. The natural interpretation would be in terms of finite measurement resolution. The tensor product of II_1 factor and I_∞ is II_∞ factor for which the trace for a projector can have arbitrarily large values. II_1 factor has a unique finite tracial state and the set of traces of projections spans unit interval. There is uncountable number of factors of type II but hyper-finite factors of type II_1 are the exceptional ones and physically most interesting.

3. Factors of type III correspond to an extreme situation. In this case the projection operators E spanning the factor have either infinite or vanishing trace and there exists an isometry mapping $E\mathcal{H}$ to $\mathcal{H}$ meaning that the projection operator spans almost all of $\mathcal{H}$. All projectors are also related to each other by isometry. Factors of type III are smallest if the factors are regarded as sub-algebras of a fixed $\mathcal{B}(\mathcal{H})$ where $\mathcal{H}$ corresponds to isomorphism class of Hilbert spaces. Situation changes when one speaks about concrete representations. Also now hyper-finite factors are exceptional.

4. Von Neumann algebras define a non-commutative measure theory. Commutative von Neumann algebras indeed reduce to $L^\infty(X)$ for some measure space (X,μ) and vice versa.

Weights, states and traces

The notions of weight, state, and trace are standard notions in the theory of von Neumann algebras.

1. A weight of von Neumann algebra is a linear map from the set of positive elements (those of form a^*a) to non-negative reals.

2. A positive linear functional is weight with $\omega(1)$ finite.

3. A state is a weight with $\omega(1) = 1$.

4. A trace is a weight with $\omega(aa^*) = \omega(a^*a)$ for all a.

5. A tracial state is a weight with $\omega(1) = 1$.

A factor has a trace such that the trace of a non-zero projector is non-zero and the trace of projection is infinite only if the projection is infinite. The trace is unique up to a rescaling. For factors that are separable or finite, two projections are equivalent if and only if they have the same trace. Factors of type I_n the values of trace are equal to multiples of $1/n$. For a factor of type I_∞ the value of trace are $0,1,2,....$ For factors of type II_1 the values span the range $[0,1]$ and for factors of type II_∞ n the range $[0,\infty)$. For factors of type III the values of the trace are 0, and ∞.

Tomita-Takesaki theory

Tomita-Takesaki theory is a vital part of the theory of factors. First some definitions.

1. Let $\omega(x)$ be a faithful state of von Neumann algebra so that one has $\omega(xx^*) > 0$ for $x > 0$. Assume by Riesz lemma the representation of ω as a vacuum expectation value: $\omega = (\cdot\Omega,\Omega)$, where Ω is cyclic and separating state.

2. Let

$$L^\infty(\mathcal{M}) \equiv \mathcal{M} \ , \quad L^2(\mathcal{M}) = \mathcal{H} \ , \quad L^1(\mathcal{M}) = \mathcal{M}_* \ , \tag{8.3.1}$$

where $\mathcal{M}_*$ is the pre-dual of $\mathcal{M}$ defined by linear functionals in $\mathcal{M}$. One has $\mathcal{M}_*{}^* = \mathcal{M}$.

3. The conjugation $x \to x^*$ is isometric in $\mathcal{M}$ and defines a map $\mathcal{M} \to L^2(\mathcal{M})$ via $x \to x\Omega$. The map $S_0; x\Omega \to x^*\Omega$ is however non-isometric.

4. Denote by S the closure of the anti-linear operator S_0 and by $S = J\Delta^{1/2}$ its polar decomposition analogous that for complex number and generalizing polar decomposition of linear operators by replacing (almost) unitary operator with anti-unitary J. Therefore $\Delta = S^*S > 0$ is positive self-adjoint and J an anti-unitary involution. The non-triviality of Δ reflects the fact that the state is not trace so that hermitian conjugation represented by S in the state space brings in additional factor $\Delta^{1/2}$.

5. What x can be is puzzling to physicists. The restriction fermionic Fock space and thus to creation operators would imply that Δ would act non-trivially only vacuum state so that $\Delta > 0$ condition would not hold true. The resolution of puzzle is the allowance of tensor product of Fock spaces for which vacua are conjugates: only this gives cyclic and separating state. This is natural in ZEO.

The basic results of Tomita-Takesaki theory are following.

1. The basic result can be summarized through the following formulas

$$\Delta^{it} M \Delta^{-it} = \mathcal{M} \ \ , J\mathcal{M}J = \mathcal{M}' \ \ .$$

2. The latter formula implies that $\mathcal{M}$ and $\mathcal{M}'$ are isomorphic algebras. The first formula implies that a one parameter group of modular automorphisms characterizes partially the factor. The physical meaning of modular automorphisms is discussed in [A45, A70] Δ is Hermitian and positive definite so that the eigenvalues of $log(\Delta)$ are real but can be negative. Δ^{it} is however not unitary for factors of type II and III. Physically the non-unitarity must relate to the fact that the flow is contracting so that hermiticity as a local condition is not enough to guarantee unitarity.

3. $\omega \to \sigma_t^\omega = Ad\Delta^{it}$ defines a canonical evolution -modular automorphism- associated with ω and depending on it. The Δ:s associated with different ω:s are related by a unitary inner automorphism so that their equivalence classes define an invariant of the factor.

Tomita-Takesaki theory gives rise to a non-commutative measure theory which is highly non-trivial. In particular the spectrum of Δ can be used to classify the factors of type II and III.

Modular automorphisms

Modular automorphisms of factors are central for their classification.

1. One can divide the automorphisms to inner and outer ones. Inner automorphisms correspond to unitary operators obtained by exponentiating Hermitian Hamiltonian belonging to the factor and connected to identity by a flow. Outer automorphisms do not allow a representation as a unitary transformations although $log(\Delta)$ is formally a Hermitian operator.

2. The fundamental group of the type II_1 factor defined as fundamental group group of corresponding II_∞ factor characterizes partially a factor of type II_1. This group consists real numbers λ such that there is an automorphism scaling the trace by λ. Fundamental group typically contains all reals but it can be also discrete and even trivial.

3. Factors of type III allow a one-parameter group of modular automorphisms, which can be used to achieve a partial classification of these factors. These automorphisms define a flow in the center of the factor known as flow of weights. The set of parameter values λ for which ω is mapped to itself and the center of the factor defined by the identity operator (projector to the factor as a sub-algebra of $\mathcal{B}(\mathcal{H})$) is mapped to itself in the modular automorphism defines the Connes spectrum of the factor. For factors of type III_λ this set consists of powers of $\lambda < 1$. For factors of type III_0 this set contains only identity automorphism so that there is no periodicity. For factors of type III_1 Connes spectrum contains all real numbers so that the automorphisms do not affect the identity operator of the factor at all.

The modules over a factor correspond to separable Hilbert spaces that the factor acts on. These modules can be characterized by M-dimension. The idea is roughly that complex rays are replaced by the sub-spaces defined by the action of $\mathcal{M}$ as basic units. M-dimension is not integer valued in general. The so called standard module has a cyclic separating vector and each factor has a standard representation possessing antilinear involution J such that $\mathcal{M}' = J\mathcal{M}J$ holds true (note that J changes the order of the operators in conjugation). The inclusions of factors define modules having interpretation in terms of a finite measurement resolution defined by $\mathcal{M}$.

Crossed product as a manner to construct factors of type III

By using so called crossed product crossedproduct for a group G acting in algebra A one can obtain new von Neumann algebras. One ends up with crossed product by a two-step generalization by starting from the semidirect product $G \triangleleft H$ for groups defined as $(g_1, h_1)(g_2, h_2) = (g_1h_1(g_2), h_1h_2)$ (note that Poincare group has interpretation as a semidirect product $M^4 \triangleleft SO(3,1)$ of Lorentz and translation groups). At the first step one replaces the group H with its group algebra. At the second step the the group algebra is replaced with a more general algebra. What is formed is the semidirect product $A \triangleleft G$ which is sum of algebras Ag. The product is given by $(a_1, g_1)(a_2, g_2) = (a_1g_1(a_2), g_1g_2)$. This construction works for both locally compact groups and quantum groups. A not too highly educated guess is that the construction in the case of quantum groups gives the factor $\mathcal{M}$ as a crossed product of the included factor $\mathcal{N}$ and quantum group defined by the factor space $\mathcal{M}/\mathcal{N}$.

The construction allows to express factors of type III as crossed products of factors of type II_∞ and the 1-parameter group G of modular automorphisms assignable to any vector which is cyclic for both factor and its commutant. The ergodic flow θ_λ scales the trace of projector in II_∞ factor by $\lambda > 0$. The dual flow defined by G restricted to the center of II_∞ factor does not depend on the choice of cyclic vector.

The Connes spectrum - a closed subgroup of positive reals - is obtained as the exponent of the kernel of the dual flow defined as set of values of flow parameter λ for which the flow in the center is trivial. Kernel equals to $\{0\}$ for III_0, contains numbers of form $log(\lambda)Z$ for factors of type III_λ and contains all real numbers for factors of type III_1 meaning that the flow does not affect the center.

Inclusions and Connes tensor product

Inclusions $\mathcal{N} \subset \mathcal{M}$ of von Neumann algebras have physical interpretation as a mathematical description for sub-system-system relation. In [K87] there is more extensive TGD colored description of inclusions and their role in TGD. Here only basic facts are listed and the Connes tensor product is explained.

For type I algebras the inclusions are trivial and tensor product description applies as such. For factors of II_1 and III the inclusions are highly non-trivial. The inclusion of type II_1 factors were understood by Vaughan Jones [A1] and those of factors of type III by Alain Connes [A28] .

Formally sub-factor $\mathcal{N}$ of $\mathcal{M}$ is defined as a closed *-stable C-subalgebra of $\mathcal{M}$. Let $\mathcal{N}$ be a sub-factor of type II_1 factor $\mathcal{M}$. Jones index $\mathcal{M} : \mathcal{N}$ for the inclusion $\mathcal{N} \subset \mathcal{M}$ can be defined as $\mathcal{M} : \mathcal{N} = dim_N(L^2(\mathcal{M})) = Tr_{N'}(id_{L^2(\mathcal{M})})$. One can say that the dimension of completion of $\mathcal{M}$ as $\mathcal{N}$ module is in question.

Basic findings about inclusions

What makes the inclusions non-trivial is that the position of $\mathcal{N}$ in $\mathcal{M}$ matters. This position is characterized in case of hyper-finite II_1 factors by index $\mathcal{M} : \mathcal{N}$ which can be said to the dimension of $\mathcal{M}$ as $\mathcal{N}$ module and also as the inverse of the dimension defined by the trace of the projector from $\mathcal{M}$ to $\mathcal{N}$. It is important to notice that $\mathcal{M} : \mathcal{N}$ does not characterize either $\mathcal{M}$ or $\mathcal{M}$, only the imbedding.

The basic facts proved by Jones are following [A1] .

1. For pairs $\mathcal{N} \subset \mathcal{M}$ with a finite principal graph the values of $\mathcal{M} : \mathcal{N}$ are given by

$$
\begin{aligned}
&a)\ \mathcal{M}:\mathcal{N} = 4cos^2(\pi/h)\ \ ,\quad h\geq 3\ \ ,\\
&b)\ \mathcal{M}:\mathcal{N}\geq 4\ \ .
\end{aligned}
\tag{8.3.2}
$$

the numbers at right hand side are known as Beraha numbers [A60] . The comments below give a rough idea about what finiteness of principal graph means.

2. As explained in [B53] , for $\mathcal{M}:\mathcal{N}<4$ one can assign to the inclusion Dynkin graph of ADE type Lie-algebra g with h equal to the Coxeter number h of the Lie algebra given in terms of its dimension and dimension r of Cartan algebra r as $h=(dimg(g)-r)/r$. The Lie algebras of $SU(n)$, E_7 and D_{2n+1} are however not allowed. For $\mathcal{M}:\mathcal{N}=4$ one can assign to the inclusion an extended Dynkin graph of type ADE characterizing Kac Moody algebra. Extended ADE diagrams characterize also the subgroups of SU(2) and the interpretation proposed in [A91] is following. The ADE diagrams are associated with the $n=\infty$ case having $\mathcal{M}:\mathcal{N}\geq 4$. There are diagrams corresponding to infinite subgroups: SU(2) itself, circle group U(1), and infinite dihedral groups (generated by a rotation by a non-rational angle and reflection. The diagrams corresponding to finite subgroups are extension of A_n for cyclic groups, of D_n dihedral groups, and of E_n with n=6,7,8 for tetrahedron, cube, dodecahedron. For $\mathcal{M}:\mathcal{N}<4$ ordinary Dynkin graphs of D_{2n} and E_6,E_8 are allowed.

Connes tensor product

The inclusions The basic idea of Connes tensor product is that a sub-space generated sub-factor $\mathcal{N}$ takes the role of the complex ray of Hilbert space. The physical interpretation is in terms of finite measurement resolution: it is not possible to distinguish between states obtained by applying elements of $\mathcal{N}$.

Intuitively it is clear that it should be possible to decompose $\mathcal{M}$ to a tensor product of factor space $\mathcal{M}/\mathcal{N}$ and $\mathcal{N}$:

$$
\mathcal{M}\ \ =\ \ \mathcal{M}/\mathcal{N}\otimes\mathcal{N}\ \ . \tag{8.3.3}
$$

One could regard the factor space $\mathcal{M}/\mathcal{N}$ as a non-commutative space in which each point corresponds to a particular representative in the equivalence class of points defined by $\mathcal{N}$. The connections between quantum groups and Jones inclusions suggest that this space closely relates to quantum groups. An alternative interpretation is as an ordinary linear space obtained by mapping $\mathcal{N}$ rays to ordinary complex rays. These spaces appear in the representations of quantum groups. Similar procedure makes sense also for the Hilbert spaces in which $\mathcal{M}$ acts.

Connes tensor product can be defined in the space $\mathcal{M}\otimes\mathcal{M}$ as entanglement which effectively reduces to entanglement between $\mathcal{N}$ sub-spaces. This is achieved if $\mathcal{N}$ multiplication from right is equivalent with $\mathcal{N}$ multiplication from left so that $\mathcal{N}$ acts like complex numbers on states. One can imagine variants of the Connes tensor product and in TGD framework one particular variant appears naturally as will be found.

In the finite-dimensional case Connes tensor product of Hilbert spaces has a rather simple representation. If the matrix algebra N of $n\times n$ matrices acts on V from right, V can be regarded as a space formed by $m\times n$ matrices for some value of m. If N acts from left on W, W can be regarded as space of $n\times r$ matrices.

1. In the first representation the Connes tensor product of spaces V and W consists of $m\times r$ matrices and Connes tensor product is represented as the product VW of matrices as $(VW)_{mr}e^{mr}$. In this representation the information about N disappears completely as the interpretation in terms of measurement resolution suggests. The sum over intermediate states defined by N brings in mind path integral.

2. An alternative and more physical representation is as a state

$$\sum_n V_{mn} W_{nr} e^{mn} \otimes e^{nr}$$

in the tensor product $V \otimes W$.

3. One can also consider two spaces V and W in which N acts from right and define Connes tensor product for $A^\dagger \otimes_N B$ or its tensor product counterpart. This case corresponds to the modification of the Connes tensor product of positive and negative energy states. Since Hermitian conjugation is involved, matrix product does not define the Connes tensor product now. For $m = r$ case entanglement coefficients should define a unitary matrix commuting with the action of the Hermitian matrices of N and interpretation would be in terms of symmetry. HFF property would encourage to think that this representation has an analog in the case of HFFs of type II_1.

4. Also type I_n factors are possible and for them Connes tensor product makes sense if one can assign the inclusion of finite-D matrix algebras to a measurement resolution.

Factors in quantum field theory and thermodynamics

Factors arise in thermodynamics and in quantum field theories [A80, A45, A70] . There are good arguments showing that in HFFs of III_1 appear are relativistic quantum field theories. In non-relativistic QFTs the factors of type I appear so that the non-compactness of Lorentz group is essential. Factors of type III_1 and III_λ appear also in relativistic thermodynamics.

The geometric picture about factors is based on open subsets of Minkowski space. The basic intuitive view is that for two subsets of M^4, which cannot be connected by a classical signal moving with at most light velocity, the von Neumann algebras commute with each other so that $\vee$ product should make sense.

Some basic mathematical results of algebraic quantum field theory [A70] deserve to be listed since they are suggestive also from the point of view of TGD.

1. Let $\mathcal{O}$ be a bounded region of R^4 and define the region of M^4 as a union $\cup_{|x|<\epsilon}(\mathcal{O} + x)$ where $(\mathcal{O} + x)$ is the translate of O and $|x|$ denotes Minkowski norm. Then every projection $E \in \mathcal{M}(\mathcal{O})$ can be written as WW^* with $W \in \mathcal{M}(\mathcal{O}_\epsilon)$ and $W^*W = 1$. Note that the union is not a bounded set of M^4. This almost establishes the type III property.

2. Both the complement of light-cone and double light-cone define HFF of type III_1. Lorentz boosts induce modular automorphisms.

3. The so called split property suggested by the description of two systems of this kind as a tensor product in relativistic QFTs is believed to hold true. This means that the HFFs of type III_1 associated with causally disjoint regions are sub-factors of factor of type I_∞. This means

$$\mathcal{M}_1 \subset \mathcal{B}(\mathcal{H}_1) \times 1 \ \ , \ \ \mathcal{M}_2 \subset 1 \otimes \mathcal{B}(\mathcal{H}_2) \ \ .$$

An infinite hierarchy of inclusions of HFFs of type III_1s is induced by set theoretic inclusions.

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8.3.2 TGD And Factors

The following vision about TGD and factors relies heavily on zero energy ontology, TGD inspired quantum measurement theory, basic vision about quantum TGD, and bosonic emergence.

The problems

Concerning the role of factors in TGD framework there are several problems of both conceptual and technical character.

1. Conceptual problems

It is safest to start from the conceptual problems and take a role of skeptic.

1. Under what conditions the assumptions of Tomita-Takesaki formula stating the existence of modular automorphism and isomorphy of the factor and its commutant hold true? What is the physical interpretation of the formula $\mathcal{M}' = J\mathcal{M}J$ relating factor and its commutant in TGD framework?

2. Is the identification $M = \Delta^{it}$ sensible is quantum TGD and ZEO, where M-matrix is "complex square root" of exponent of Hamiltonian defining thermodynamical state and the notion of unitary time evolution is given up? The notion of state ω leading to Δ is essentially thermodynamical and one can wonder whether one should take also a "complex square root" of ω to get M-matrix giving rise to a genuine quantum theory.

3. TGD based quantum measurement theory involves both quantum fluctuating degrees of freedom assignable to light-like 3-surfaces and zero modes identifiable as classical degrees of freedom assignable to interior of the space-time sheet. Zero modes have also fermionic counterparts. State preparation should generate entanglement between the quantal and classical states. What this means at the level of von Neumann algebras?

4. What is the TGD counterpart for causal disjointness. At space-time level different space-time sheets could correspond to such regions whereas at imbedding space level causally disjoint CDs would represent such regions.

2. Technical problems

There are also more technical questions.

1. What is the von Neumann algebra needed in TGD framework? Does one have a a direct integral over factors? Which factors appear in it? Can one construct the factor as a crossed product of some group G with direct physical interpretation and of naturally appearing factor A? Is A a HFF of type II_∞? assignable to a fixed CD? What is the natural Hilbert space $\mathcal{H}$ in which A acts?

2. What are the geometric transformations inducing modular automorphisms of II_∞ inducing the scaling down of the trace? Is the action of G induced by the boosts in Lorentz group. Could also translations and scalings induce the action? What is the factor associated with the union of Poincare transforms of CD? $log(\Delta)$ is Hermitian algebraically: what does the non-unitarity of $exp(log(\Delta)it)$ mean physically?

3. Could Ω correspond to a vacuum which in conformal degrees of freedom depends on the choice of the sphere S^2 defining the radial coordinate playing the role of complex variable in the case of the radial conformal algebra. Does *-operation in $\mathcal{M}$ correspond to Hermitian conjugation for fermionic oscillator operators and change of sign of super conformal weights?

The exponent of the Kähler-Dirac action gives rise to the exponent of Kähler function as Dirac determinant and fermionic inner product defined by fermionic Feynman rules. It is implausible that this exponent could as such correspond to ω or Δ^{it} having conceptual roots in thermodynamics rather than QFT. If one assumes that the exponent of the Kähler-Dirac action defines a "complex square root" of ω the situation changes. This raises technical questions relating to the notion of square root of ω.

1. Does the complex square root of ω have a polar decomposition to a product of positive definite matrix (square root of the density matrix) and unitary matrix and does $\omega^{1/2}$ correspond to the modulus in the decomposition? Does the square root of Δ have similar decomposition with modulus equal equal to $\Delta^{1/2}$ in standard picture so that modular automorphism, which is inherent property of von Neumann algebra, would not be affected?

2. Δ^{it} or rather its generalization is defined modulo a unitary operator defined by some Hamiltonian and is therefore highly non-unique as such. This non-uniqueness applies also to $|\Delta|$. Could this non-uniqueness correspond to the thermodynamical degrees of freedom?

ZEO and factors

The first question concerns the identification of the Hilbert space associated with the factors in ZEO. As the positive or negative energy part of the zero energy state space or as the entire space of zero energy states? The latter option would look more natural physically and is forced by the condition that the vacuum state is cyclic and separating.

1. The commutant of HFF given as $\mathcal{M}' = J\mathcal{M}J$, where J is involution transforming fermionic oscillator operators and bosonic vector fields to their Hermitian conjugates. Also conformal weights would change sign in the map which conforms with the view that the light-like boundaries of CD are analogous to upper and lower hemispheres of S^2 in conformal field theory. The presence of J representing essentially Hermitian conjugation would suggest that positive and zero energy parts of zero energy states are related by this formula so that state space decomposes to a tensor product of positive and negative energy states and M-matrix can be regarded as a map between these two sub-spaces.

2. The fact that HFF of type II_1 has the algebra of fermionic oscillator operators as a canonical representation makes the situation puzzling for a novice. The assumption that the vacuum is cyclic and separating means that neither creation nor annihilation operators can annihilate it. Therefore Fermionic Fock space cannot appear as the Hilbert space in the Tomita-Takesaki theorem. The paradox is circumvented if the action of * transforms creation operators acting on the positive energy part of the state to annihilation operators acting on negative energy part of the state. If J permutes the two Fock vacuums in their tensor product, the action of S indeed maps permutes the tensor factors associated with $\mathcal{M}$ and $\mathcal{M}'$.

It is far from obvious whether the identification $M = \Delta^{it}$ makes sense in ZEO.

1. In ZEO M-matrix defines time-like entanglement coefficients between positive and negative energy parts of the state. M-matrix is essentially "complex square root" of the density matrix and quantum theory similar square root of thermodynamics. The notion of state as it appears in the theory of HFFs is however essentially thermodynamical. Therefore it is good to ask whether the "complex square root of state" could make sense in the theory of factors.

2. Quantum field theory suggests an obvious proposal concerning the meaning of the square root: one replaces exponent of Hamiltonian with imaginary exponential of action at $T \rightarrow 0$ limit. In quantum TGD the exponent of Kähler-Dirac action giving exponent of Kähler function as real exponent could be the manner to take this complex square root. Kähler-Dirac action can therefore be regarded as a "square root" of Kähler action.

3. The identification $M = \Delta^{it}$ relies on the idea of unitary time evolution which is given up in ZEO based on CDs? Is the reduction of the quantum dynamics to a flow a realistic idea? As will be found this automorphism could correspond to a time translation or scaling for either upper or lower light-cone defining CD and can ask whether Δ^{it} corresponds to the exponent of scaling operator L_0 defining single particle propagator as one integrates over t. Its complex square root would correspond to fermionic propagator.

4. In this framework $J\Delta^{it}$ would map the positive energy and negative energy sectors to each other. If the positive and negative energy state spaces can identified by isometry then $M = J\Delta^{it}$ identification can be considered but seems unrealistic. $S = J\Delta^{1/2}$ maps positive and negative energy states to each other: could S or its generalization appear in M-matrix as a part which gives thermodynamics? The exponent of the Kähler-Dirac action does not seem to provide thermodynamical aspect and p-adic thermodynamics suggests strongly the presence exponent of $exp(-L_0/T_p)$ with T_p chose in such manner that consistency with p-adic thermodynamics is obtained. Could the generalization of $J\Delta^{n/2}$ with Δ replaced with its "square root" give rise to padic thermodynamics and also ordinary thermodynamics at the level of density matrix? The minimal option would be that power of Δ^{it} which imaginary value of t is responsible for thermodynamical degrees of freedom whereas everything else is dictated by the unitary S-matrix appearing as phase of the "square root" of ω.

Zero modes and factors

The presence of zero modes justifies quantum measurement theory in TGD framework and the relationship between zero modes and HFFs involves further conceptual problems.

1. The presence of zero modes means that one has a direct integral over HFFs labeled by zero modes which by definition do not contribute to WCW line element. The realization of quantum criticality in terms of Kähler-Dirac action [K88] suggests that also fermionic zero mode degrees of freedom are present and correspond to conserved charges assignable to the critical deformations of the pace-time sheets. Induced Kähler form characterizes the values of zero modes for a given space-time sheet and the symplectic group of light-cone boundary characterizes the quantum fluctuating degrees of freedom. The entanglement between zero modes and quantum fluctuating degrees of freedom is essential for quantum measurement theory. One should understand this entanglement.

2. Physical intuition suggests that classical observables should correspond to longer length scale than quantal ones. Hence it would seem that the interior degrees of freedom outside CD should correspond to classical degrees of freedom correlating with quantum fluctuating degrees of freedom of CD.

3. Quantum criticality means that Kähler-Dirac action allows an infinite number of conserved charges which correspond to deformations leaving metric invariant and therefore act on zero modes. Does this super-conformal algebra commute with the super-conformal algebra associated with quantum fluctuating degrees of freedom? Could the restriction of elements of quantum fluctuating currents to 3-D light-like 3-surfaces actually imply this commutativity. Quantum holography would suggest a duality between these algebras. Quantum measurement theory suggests even 1-1 correspondence between the elements of the two superconformal algebras. The entanglement between classical and quantum degrees of freedom would mean that prepared quantum states are created by operators for which the operators in the two algebras are entangled in diagonal manner.

4. The notion of finite measurement resolution has become key element of quantum TGD and one should understand how finite measurement resolution is realized in terms of inclusions

of hyper-finite factors for which sub-factor defines the resolution in the sense that its action creates states not distinguishable from each other in the resolution used. The notion of finite measurement resolution suggests that one should speak about entanglement between sub-factors and corresponding sub-spaces rather than between states. Connes tensor product would code for the idea that the action of sub-factors is analogous to that of complex numbers and tracing over sub-factor realizes this idea.

5. Just for fun one can ask whether the duality between zero modes and quantum fluctuating degrees of freedom representing quantum holography could correspond to $\mathcal{M}' = J\mathcal{M}J$? This interpretation must be consistent with the interpretation forced by zero energy ontology. If this crazy guess is correct (very probably not!), both positive and negative energy states would be observed in quantum measurement but in totally different manner. Since this identity would simplify enormously the structure of the theory, it deserves therefore to be shown wrong.

Crossed product construction in TGD framework

The identification of the von Neumann algebra by crossed product construction is the basic challenge. Consider first the question how HFFs of type II_∞ emerge, how modular automorphisms act on them, and how one can understand the non-unitary character of the Δ^{it} in an apparent conflict with the hermiticity and positivity of Δ.

1. The Clifford algebra at a given point of WCW(CD) (light-like 3-surfaces with ends at the boundaries of CD) defines HFF of type II_1 or possibly a direct integral of them. For a given CD having compact isotropy group $SO(3)$ leaving the rest frame defined by the tips of CD invariant the factor defined by Clifford algebra valued fields in WCW(CD) is most naturally HFF of type II_∞. The Hilbert space in which this Clifford algebra acts, consists of spinor fields in WCW(CD). Also the symplectic transformations of light-cone boundary leaving light-like 3-surfaces inside CD can be included to G. In fact all conformal algebras leaving CD invariant could be included in CD.

2. The downwards scalings of the radial coordinate r_M of the light-cone boundary applied to the basis of WCW (CD) spinor fields could induce modular automorphism. These scalings reduce the size of the portion of light-cone in which the WCW spinor fields are non-vanishing and effectively scale down the size of CD. $exp(iL_0)$ as algebraic operator acts as a phase multiplication on eigen states of conformal weight and therefore as apparently unitary operator. The geometric flow however contracts the CD so that the interpretation of $exp(itL_0)$ as a unitary modular automorphism is not possible. The scaling down of CD reduces the value of the trace if it involves integral over the boundary of CD. A similar reduction is implied by the downward shift of the upper boundary of CD so that also time translations would induce modular automorphism. These shifts seem to be necessary to define rest energies of positive and negative energy parts of the zero energy state.

3. The non-triviality of the modular automorphisms of II_∞ factor reflects different choices of ω. The degeneracy of ω could be due to the non-uniqueness of conformal vacuum which is part of the definition of ω. The radial Virasoro algebra of light-cone boundary is generated by $L_n = L^*_{-n}$, $n \neq 0$ and $L_0 = L_0^*$ and negative and positive frequencies are in asymmetric position. The conformal gauge is fixed by the choice of $SO(3)$ subgroup of Lorentz group defining the slicing of light-cone boundary by spheres and the tips of CD fix $SO(3)$ uniquely. One can however consider also alternative choices of $SO(3)$ and each corresponds to a slicing of the light-cone boundary by spheres but in general the sphere defining the intersection of the two light-cone does not belong to the slicing. Hence the action of Lorentz transformation inducing different choice of $SO(3)$ can lead out from the preferred state space so that its representation must be non-unitary unless Virasoro generators annihilate the physical states. The non-vanishing of the conformal central charge c and vacuum weight h seems to be necessary and indeed can take place for super-symplectic algebra and Super Kac-Moody algebra since only the differences of the algebra elements are assumed to annihilate physical states.

Modular automorphism of HFFs type III_1 can be induced by several geometric transformations for HFFs of type III_1 obtained using the crossed product construction from II_∞ factor by extending CD to a union of its Lorentz transforms.

1. The crossed product would correspond to an extension of II_∞ by allowing a union of some geometric transforms of CD. If one assumes that only CDs for which the distance between tips is quantized in powers of 2, then scalings of either upper or lower boundary of CD cannot correspond to these transformations. Same applies to time translations acting on either boundary but not to ordinary translations. As found, the modular automorphisms reducing the size of CD could act in HFF of type II_∞.

2. The geometric counterparts of the modular transformations would most naturally correspond to any non-compact one parameter sub-group of Lorentz group as also QFT suggests. The Lorentz boosts would replace the radial coordinate r_M of the light-cone boundary associated with the radial Virasoro algebra with a new one so that the slicing of light-cone boundary with spheres would be affected and one could speak of a new conformal gauge. The temporal distance between tips of CD in the rest frame would not be affected. The effect would seem to be however unitary because the transformation does not only modify the states but also transforms CD.

3. Since Lorentz boosts affect the isotropy group $SO(3)$ of CD and thus also the conformal gauge defining the radial coordinate of the light-cone boundary, they affect also the definition of the conformal vacuum so that also ω is affected so that the interpretation as a modular automorphism makes sense. The simplistic intuition of the novice suggests that if one allows wave functions in the space of Lorentz transforms of CD, unitarity of Δ^{it} is possible. Note that the hierarchy of Planck constants assigns to CD preferred M^2 and thus direction of quantization axes of angular momentum and boosts in this direction would be in preferred role.

4. One can also consider the HFF of type III_λ if the radial scalings by negative powers of 2 correspond to the automorphism group of II_∞ factor as the vision about allowed CDs suggests. $\lambda = 1/2$ would naturally hold true for the factor obtained by allowing only the radial scalings. Lorentz boosts would expand the factor to HFF of type III_1. Why scalings by powers of 2 would give rise to periodicity should be understood.

The identification of M-matrix as modular automorphism Δ^{it}, where t is complex number having as its real part the temporal distance between tips of CD quantized as 2^n and temperature as imaginary part, looks at first highly attractive, since it would mean that M-matrix indeed exists mathematically. The proposed interpretations of modular automorphisms do not support the idea that they could define the S-matrix of the theory. In any case, the identification as modular automorphism would not lead to a magic universal formula since arbitrary unitary transformation is involved.

Quantum criticality and inclusions of factors

Quantum criticality fixes the value of Kähler coupling strength but is expected to have also an interpretation in terms of a hierarchies of broken conformal gauge symmetries suggesting hierarchies of inclusions.

1. In ZEO 3-surfaces are unions of space-like 3-surfaces at the ends of causal diamond (CD). Space-time surfaces connect 3-surfaces at the boundaries of CD. The non-determinism of Kähler action allows the possibility of having several space-time sheets connecting the ends of space-time surface but the conditions that classical charges are same for them reduces this number so that it could be finite. Quantum criticality in this sense implies non-determinism analogous to that of critical systems since preferred extremals can co-incide and suffer this kind of bifurcation in the interior of CD. This quantum criticality can be assigned to the hierarchy of Planck constants and the integer n in $h_{eff} = n \times h$ [K22] corresponds to the number of degenerate space-time sheets with same Kähler action and conserved classical charges.

2. Also now one expects a hierarchy of criticalities and since criticality and conformal invariance are closely related, a natural conjecture is that the fractal hierarchy of sub-algebras of conformal algebra isomorphic to conformal algebra itself and having conformal weights coming as multiples of n corresponds to the hierarchy of Planck constants. This hierarchy would define a hierarchy of symmetry breakings in the sense that only the sub-algebra would act as gauge symmetries.

3. The assignment of this hierarchy with super-symplectic algebra having conformal structure with respect to the light-like radial coordinate of light-cone boundary looks very attractive. An interesting question is what is the role of the super-conformal algebra associated with the isometries of light-cone boundary $R_+ \times S^2$ which are conformal transformations of sphere S^2 with a scaling of radial coordinate compensating the scaling induced by the conformal transformation. Does it act as dynamical or gauge symmetries?

4. The natural proposal is that the inclusions of various superconformal algebras in the hierarchy define inclusions of hyper-finite factors which would be thus labelled by integers. Any sequences of integers for which n_i divides n_{i+1} would define a hierarchy of inclusions proceeding in reverse direction. Physically inclusion hierarchy would correspond to an infinite hierarchy of criticalities within criticalities.

8.3.3 Can One Identify M-Matrix From Physical Arguments?

Consider next the identification of M-matrix from physical arguments from the point of view of factors.

A proposal for M-matrix

The proposed general picture reduces the core of U-matrix to the construction of S-matrix possibly having the real square roots of density matrices as symmetry algebra. This structure can be taken as a template as one tries to to imagine how the construction of M-matrix could proceed in quantum TGD proper.

1. At the bosonic sector one would have converging functional integral over WCW . This is analogous to the path integral over bosonic fields in QFTs. The presence of Kähler function would make this integral well-defined and would not encounter the difficulties met in the case of path integrals.

2. In fermionic sector 1-D Dirac action and its bosonic counterpart imply that spinors modes localized at string world sheets are eigenstates of induced Dirac operator with generalized eigenvalue $p^k\gamma_k$ defining light-like 8-D momentum so that one would obtain fermionic propagators massless in 8-D sense at light-light geodesics of imbedding space. The 8-D generalization of twistor Grassmann approach is suggestive and would mean that the residue integral over fermionic virtual momenta gives only integral over massless momenta and virtual fermions differ from real fermions only in that they have non-physical polarizations so that massless Dirac operator replacing the propagator does not annihilate the spinors at the other end of the line.

3. Fundamental bosons (not elementary particles) correspond to wormhole contacts having fermion and antifermion at opposite throats and bosonic propagators are composite of massless fermion propagators. The directions of virtual momenta are obviously strongly correlated so that the approximation as a gauge theory with gauge symmetry breaking in almost massless sector is natural. Massivation follows necessary from the fact that also elementary particles are bound states of two wormhole contacts.

4. Physical fermions and bosons correspond to pairs of wormhole contacts with throats carrying Kähler magnetic charge equal to Kähler electric charge (dyon). The absence of Dirac monopoles (as opposed to homological magnetic monopoles due to CP_2 topology) implies that wormhole contacts must appear as pairs (also large numbers of them are possible and 3

valence quarks inside baryons could form Kähler magnetic tripole). Hence elementary particles would correspond to pairs of monopoles and are accompanied by Kähler magnetic flux loop running along the two space-time sheets involved as well as fermionic strings connecting the monopole throats.

There seems to be no specific need to assign string to the wormhole contact and if is a piece of deformed CP_2 type vacuum extremal this might not be even possible: the Kähler-Dirac gamma matrices would not span 2-D space in this case since the CP_2 projection is 4-D. Hence massless fermion propagators would be assigned only with the boundaries of string world sheets at Minkowskian regions of space-time surface. One could say that physical particles are bound states of massless fundamental fermions and the non-collinearity of their four-momenta can make them massive. Therefore the breaking of conformal invariance would be due to the bound state formation and this would also resolve the infrared divergence problems plaguing Grassmann twistor approach by introducing natural length scale assignable to the size of particles defined by the string like flux tube connecting the wormhole contacts. This point is discussed in more detail in [K76].

The bound states would form representations of super-conformal algebras so that stringy mass formula would emerge naturally. p-Adic mass calculations indeed assume conformal invariance in $CP2$ length scale assignable to wormhole contacts. Also the long flux tube strings contribute to the particle masses and would explain gauge boson masses.

5. The interaction vertices would correspond topologically to decays of 3-surface by splitting in complete analogy with ordinary Feynman diagrams. At the level of orbits of partonic 2-surface the vertices would be represented by partonic 2-surfaces. In [K76] the interpretation of scattering ampiltudes as sequences of algebraic operations for the Yangian of super-symplectic algebra is proposed: product and co-product would define time 3-vertex and its time reversal. At the level of fermions the diagrams reduce to braid diagrams since fermions are "free". At vertices fermions can however reflect in time direction so that fermion-antifermion annihilations in classical fields can be said to appear in the vertices.

   The Yangian is generated by super-symplectic fermionic Noether charges assignable to the strings connecting partonic 2-surfaces. The interpretation of vertices as algebraic operations implies that all sequences of operations connecting given collections of elements of Yangian at the opposite boundaries of CD give rise to the same amplitude. This means a huge generalization of the duality symmetry of hadronic string models that I have proposed already earlier: the chapter [K6] is a remnant of an "idea that came too early". The propagators are associated with the fermionic lines identifiable as boundaries of string world sheets. These lines are light-like geodesics of H and fermion lines correspond topartial wave in the space S^3 of light like 8-momenta with fixed M^4 momentum. For external lines M^8 momentum corresponds to the $M^4 \times CP_2$ quantum numbers of a spinor harmonic.

   The amplitudes can be formulated using only partonic 2-surfaces and string world sheets and the algebraic continuation to achieve number theoretic Universality should be rather straightforward: the parameters characterizing 2-surfaces - by conformal invariance various conformal moduli - in the algebraic extension of rationals are replaced with real and various p-adic numbers.

6. Wormhole contacts represent fundamental interaction vertex pairs and propagators between them and one has stringy super-conformal invariance. Therefore there are excellent reasons to expect that the perturbation theory is free of divergences. Without stringy contributions for massive conformal excitations of wormhole contacts one would obtain the usual logarithmic UV divergences of massless gauge theories. The fact that physical particles are bound states of massless particles, gives good hopes of avoiding IR divergences of massless theories.

The figures **??**, **??** (`http://tgdtheory.fi/appfigures/elparticletgd.jpg` `http://tgdtheory.fi/appfigures/tgdgrpahs.jpg`) in the appendix of this book illustrate the relationship between TGD diagrammatics, QFT diagrammatics and stringy diagrammatics. In [K76] a more detailed construction based on the generalization of twistor approach and the idea that scattering amplitudes represent sequences of algebraic operation in the Yangian of super-symplectic algebra, is considered.

Quantum TGD as square root of thermodynamics

ZEO (ZEO) suggests strongly that quantum TGD corresponds to what might be called square root of thermodynamics. Since fermionic sector of TGD corresponds naturally to a hyper-finite factor of type II_1, and super-conformal sector relates fermionic and bosonic sectors (WCW degrees of freedom), there is a temptation to suggest that the mathematics of von Neumann algebras generalizes: in other worlds it is possible to speak about the complex square root of ω defining a state of von Neumann algebra [A80] [K87]. This square root would bring in also the fermionic sector and realized super-conformal symmetry. The reduction of determinant with WCW vacuum functional would be one manifestation of this supersymmetry.

The exponent of Kähler function identified as real part of Kähler action for preferred extremals coming from Euclidian space-time regions defines the modulus of the bosonic vacuum functional appearing in the functional integral over WCW. The imaginary part of Kähler action coming from the Minkowskian regions is analogous to action of quantum field theories and would give rise to interference effects distinguishing thermodynamics from quantum theory. This would be something new from the point of view of the canonical theory of von Neumann algebra. The saddle points of the imaginary part appear in stationary phase approximation and the imaginary part serves the role of Morse function for WCW.

The exponent of Kähler function depends on the real part of t identified as Minkowski distance between the tips of CD. This dependence is not consistent with the dependence of the canonical unitary automorphism Δ^{it} of von Neumann algebra on t [A80], [K87] and the natural interpretation is that the vacuum functional can be included in the definition of the inner product for spinors fields of WCW. More formally, the exponent of Kähler function would define ω in bosonic degrees of freedom.

Note that the imaginary exponent is more natural for the imaginary part of Kähler action coming from Minkowskian region. In any case, one has combination of thermodynamics and QFT and the presence of thermodynamics makes the functional integral mathematically well-defined.

Number theoretic vision requiring number theoretical universality suggests that the value of CD size scales as defined by the distance between the tips is expected to come as integer multiples of CP_2 length scale - at least in the intersection of real and p-adic worlds. If this is the case the continuous faimily of modular automorphisms would be replaced with a discretize family.

Quantum criticality and hierarchy of inclusions

Quantum criticality and related fractal hierarchies of breakings of conformal symmetry could allow to understand the inclusion hierarchies for hyper-finite factors. Quantum criticality - implied by the condition that the Kähler-Dirac action gives rise to conserved currents assignable to the deformations of the space-time surface - means the vanishing of the second variation of Kähler action for these deformations. Preferred extremals correspond to these 4-surfaces and $M^8 - M^4 \times CP_2$ duality would allow to identify them also as associative (co-associative) space-time surfaces.

Quantum criticality is basically due to the failure of strict determinism for Kähler action and leads to the hierarchy of dark matter phases labelled by the effective value of Planck constant $h_{eff} = n \times h$. These phases correspond to space-time surfaces connecting 3-surfaces at the ends of CD which are multi-sheeted having n conformal equivalence classes.

Conformal invariance indeed relates naturally to quantum criticality. This brings in n discrete degrees of freedom and one can technically describe the situation by using n-fold singular covering of the imbedding space [K22]. One can say that there is hierarchy of broken conformal symmetries in the sense that for $h_{eff} = n \times h$ the sub-algebra of conformal algebras with conformal weights coming as multiples of n act as gauge symmetries. This implies that classical symplectic Noether charges vanish for this sub-algebra. The quantal conformal charges associated with induced spinor fields annihilate the physical states. Therefore it seems that the measured quantities are the symplectic charges and there is not need to introduce any measurement interaction term and the formalism simplifies dramatically.

The resolution increases with $h_{eff}/h = n$. Also the number of of strings connecting partonic 2-surfaces (in practice elementary particles and their dark counterparts plus bound states generated by connecting dark strings) characterizes physically the finite measurement resolution. Their presence is also visible in the geometry of the space-time surfaces through the conditions that induced W fields vanish at them (well-definedness of em charge), and by the condition that the canonical momentum currents for Kähler action define an integrable distribution of planes parallel to the string world sheet. In spirit with holography, preferred extremal is constructed by fixing string world sheets and partonic 2-surfaces and possibly also their light-like orbits (should one fix wormhole contacts is not quite clear). If the analog of AdS/CFT correspondence holds true, the value of Kähler function is expressible as the energy of string defined by area in the effective metric defined by the anti-commutators of K-D gamma matrices.

Super-symplectic algebra, whose charges are represented by Noether charges associated with strings connecting partonic 2-surfaces extends to a Yangian algebra with multi-stringy generators [K76]. The better the measurement resolution, the larger the maximal number of strings associated with the multilocal generator.

Kac-Moody type transformations preserving light-likeness of partonic orbits and possibly also the light-like character of the boundaries of string world sheets carrying modes of induced spinor field underlie the conformal gauge symmetry. The minimal option is that only the light-likeness of the string end world line is preserved by the conformal symmetries. In fact, conformal symmetries was originally deduced from the light-likeness condition for the M^4 projection of CP_2 type vacuum extremals.

The inclusions of super-symplectic Yangians form a hierarchy and would naturally correspond to inclusions of hyperfinite factors of type II_1. Conformal symmetries acting as gauge transformations would naturally correspond to degrees of freedom below measurement resolution and would correspond to included subalgebra. As h_{eff} increases, infinite number of these gauge degrees of freedom become dynamical and measurement resolution is increased. This picture is definitely in conflict with the original view but the reduction of criticality in the increase of h_{eff} forces it.

Summarizing

On basis of above considerations it seems that the idea about "complex square root" of the state ω of von Neumann algebras might make sense in quantum TGD. Also the discretized versions of modular automorphism assignable to the hierarchy of CDs would make sense and because of its non-uniqueness the generator Δ of the canonical automorphism could bring in the flexibility needed one wants thermodynamics. Stringy picture forces to ask whether Δ could in some situation be proportional $exp(L_0)$, where L_0 represents as the infinitesimal scaling generator of either super-symplectic algebra or super Kac-Moody algebra (the choice does not matter since the differences of the generators annihilate physical states in coset construction). This would allow to reproduce real thermodynamics consistent with p-adic thermodynamics. Note that also p-adic thermodynamics would be replaced by its square root in ZEO.

8.3.4 Finite Measurement Resolution And HFFs

The finite resolution of quantum measurement leads in TGD framework naturally to the notion of quantum M-matrix for which elements have values in sub-factor $\mathcal{N}$ of HFF rather than being complex numbers. M-matrix in the factor space $\mathcal{M}/\mathcal{N}$ is obtained by tracing over $\mathcal{N}$. The condition that $\mathcal{N}$ acts like complex numbers in the tracing implies that M-matrix elements are proportional to maximal projectors to $\mathcal{N}$ so that M-matrix is effectively a matrix in $\mathcal{M}/\mathcal{N}$ and situation becomes finite-dimensional. It is still possible to satisfy generalized unitarity conditions but in general case tracing gives a weighted sum of unitary M-matrices defining what can be regarded as a square root of density matrix.

About the notion of observable in ZEO

Some clarifications concerning the notion of observable in zero energy ontology are in order.

1. As in standard quantum theory observables correspond to hermitian operators acting on either positive or negative energy part of the state. One can indeed define hermitian conjugation for positive and negative energy parts of the states in standard manner.

2. Also the conjugation $A \rightarrow JAJ$ is analogous to hermitian conjugation. It exchanges the positive and negative energy parts of the states also maps the light-like 3-surfaces at the upper boundary of CD to the lower boundary and vice versa. The map is induced by time reflection in the rest frame of CD with respect to the origin at the center of CD and has a well defined action on light-like 3-surfaces and space-time surfaces. This operation cannot correspond to the sought for hermitian conjugation since JAJ and A commute.

3. In order to obtain non-trivial fermion propagator one must add to Dirac action 1-D Dirac action in induced metric with the boundaries of string world sheets at the light-like parton orbits. Its bosonic counterpart is line-length in induced metric. Field equations imply that the boundaries are light-like geodesics and fermion has light-like 8-momentum. This suggests strongly a connection with quantum field theory and an 8-D generalization of twistor Grassmannian approach. By field equations the bosonic part of this action does not contribute to the Kähler action. Chern-Simons Dirac terms to which Kähler action reduces could be responsible for the breaking of CP and T symmetries as they appear in CKM matrix.

4. ZEO gives Cartan sub-algebra of the Lie algebra of symmetries a special status. Only Cartan algebra acting on either positive or negative states respects the zero energy property but this is enough to define quantum numbers of the state. In absence of symmetry breaking positive and negative energy parts of the state combine to form a state in a singlet representation of group. Since only the net quantum numbers must vanish ZEO allows a symmetry breaking respecting a chosen Cartan algebra.

5. In order to speak about four-momenta for positive and negative energy parts of the states one must be able to define how the translations act on CDs. The most natural action is a shift of the upper (lower) tip of CD. In the scale of entire CD this transformation induced Lorentz boost fixing the other tip. The value of mass squared is identified as proportional to the average of conformal weight in p-adic thermodynamics for the scaling generator L_0 for either super-symplectic or Super Kac-Moody algebra.

Inclusion of HFFs as characterizer of finite measurement resolution at the level of S-matrix

The inclusion $\mathcal{N} \subset \mathcal{M}$ of factors characterizes naturally finite measurement resolution. This means following things.

1. Complex rays of state space resulting usually in an ideal state function reduction are replaced by $\mathcal{N}$-rays since $\mathcal{N}$ defines the measurement resolution and takes the role of complex numbers in ordinary quantum theory so that non-commutative quantum theory results. Non-commutativity corresponds to a finite measurement resolution rather than something exotic occurring in Planck length scales. The quantum Clifford algebra $\mathcal{M}/\mathcal{N}$ creates physical states modulo resolution. The fact that $\mathcal{N}$ takes the role of gauge algebra suggests that it might be necessary to fix a gauge by assigning to each element of $\mathcal{M}/\mathcal{N}$ a unique element of $\mathcal{M}$. Quantum Clifford algebra with fractal dimension $\beta = \mathcal{M} : \mathcal{N}$ creates physical states having interpretation as quantum spinors of fractal dimension $d = \sqrt{\beta}$. Hence direct connection with quantum groups emerges.

2. The notions of unitarity, hermiticity, and eigenvalue generalize. The elements of unitary and hermitian matrices and $\mathcal{N}$-valued. Eigenvalues are Hermitian elements of $\mathcal{N}$ and thus correspond entire spectra of Hermitian operators. The mutual non-commutativity of eigenvalues guarantees that it is possible to speak about state function reduction for quantum spinors. In the simplest case of a 2-component quantum spinor this means that second component of quantum spinor vanishes in the sense that second component of spinor annihilates physical state and second acts as element of $\mathcal{N}$ on it. The non-commutativity of spinor components implies correlations between then and thus fractal dimension is smaller than 2.

3. The intuition about ordinary tensor products suggests that one can decompose Tr in $\mathcal{M}$ as

$$Tr_{\mathcal{M}}(X) = Tr_{\mathcal{M}/\mathcal{N}} \times Tr_{\mathcal{N}}(X) \ . \tag{8.3.4}$$

Suppose one has fixed gauge by selecting basis $|r_k\rangle$ for $\mathcal{M}/\mathcal{N}$. In this case one expects that operator in $\mathcal{M}$ defines an operator in $\mathcal{M}/\mathcal{N}$ by a projection to the preferred elements of $\mathcal{M}$.

$$\langle r_1|X|r_2\rangle \quad = \quad \langle r_1|Tr_{\mathcal{N}}(X)|r_2\rangle \ . \tag{8.3.5}$$

4. Scattering probabilities in the resolution defined by $\mathcal{N}$ are obtained in the following manner. The scattering probability between states $|r_1\rangle$ and $|r_2\rangle$ is obtained by summing over the final states obtained by the action of $\mathcal{N}$ from $|r_2\rangle$ and taking the analog of spin average over the states created in the similar from $|r_1\rangle$. $\mathcal{N}$ average requires a division by $Tr(P_{\mathcal{N}}) = 1/\mathcal{M}:\mathcal{N}$ defining fractal dimension of $\mathcal{N}$. This gives

$$p(r_1 \rightarrow r_2) \quad = \quad \mathcal{M}:\mathcal{N} \times \langle r_1|Tr_{\mathcal{N}}(SP_{\mathcal{N}}S^{\dagger})|r_2\rangle \ . \tag{8.3.6}$$

This formula is consistent with probability conservation since one has

$$\sum_{r_2} p(r_1 \rightarrow r_2) \quad = \quad \mathcal{M}:\mathcal{N} \times Tr_N(SS^{\dagger}) = \mathcal{M}:\mathcal{N} \times Tr(P_N) = 1 \ . \tag{8.3.7}$$

5. Unitarity at the level of $\mathcal{M}/\mathcal{N}$ can be achieved if the unit operator Id for $\mathcal{M}$ can be decomposed into an analog of tensor product for the unit operators of $\mathcal{M}/\mathcal{N}$ and $\mathcal{N}$ and M decomposes to a tensor product of unitary M-matrices in $\mathcal{M}/\mathcal{N}$ and $\mathcal{N}$. For HFFs of type II projection operators of $\mathcal{N}$ with varying traces are present and one expects a weighted sum of unitary M-matrices to result from the tracing having interpretation in terms of square root of thermodynamics.

6. This argument assumes that $\mathcal{N}$ is HFF of type II_1 with finite trace. For HFFs of type III_1 this assumption must be given up. This might be possible if one compensates the trace over $\mathcal{N}$ by dividing with the trace of the infinite trace of the projection operator to $\mathcal{N}$. This probably requires a limiting procedure which indeed makes sense for HFFs.

Quantum M-matrix

The description of finite measurement resolution in terms of inclusion $\mathcal{N} \subset \mathcal{M}$ seems to boil down to a simple rule. Replace ordinary quantum mechanics in complex number field C with that in $\mathcal{N}$. This means that the notions of unitarity, hermiticity, Hilbert space ray, etc.. are replaced with their $\mathcal{N}$ counterparts.

The full M-matrix in $\mathcal{M}$ should be reducible to a finite-dimensional quantum M-matrix in the state space generated by quantum Clifford algebra $\mathcal{M}/\mathcal{N}$ which can be regarded as a finite-dimensional matrix algebra with non-commuting $\mathcal{N}$-valued matrix elements. This suggests that full M-matrix can be expressed as M-matrix with $\mathcal{N}$-valued elements satisfying $\mathcal{N}$-unitarity conditions.

Physical intuition also suggests that the transition probabilities defined by quantum S-matrix must be commuting hermitian $\mathcal{N}$-valued operators inside every row and column. The traces of these operators give $\mathcal{N}$-averaged transition probabilities. The eigenvalue spectrum of these Hermitian matrices gives more detailed information about details below experimental resolution. $\mathcal{N}$-hermicity and commutativity pose powerful additional restrictions on the M-matrix.

Quantum M-matrix defines $\mathcal{N}$-valued entanglement coefficients between quantum states with $\mathcal{N}$-valued coefficients. How this affects the situation? The non-commutativity of quantum spinors has a natural interpretation in terms of fuzzy state function reduction meaning that quantum spinor corresponds effectively to a statistical ensemble which cannot correspond to pure state. Does this mean that predictions for transition probabilities must be averaged over the ensemble defined by "quantum quantum states"?

Quantum fluctuations and inclusions

Inclusions $\mathcal{N} \subset \mathcal{M}$ of factors provide also a first principle description of quantum fluctuations since quantum fluctuations are by definition quantum dynamics below the measurement resolution. This gives hopes for articulating precisely what the important phrase "long range quantum fluctuations around quantum criticality" really means mathematically.

1. Phase transitions involve a change of symmetry. One might hope that the change of the symmetry group $G_a \times G_b$ could universally code this aspect of phase transitions. This need not always mean a change of Planck constant but it means always a leakage between sectors of imbedding space. At quantum criticality 3-surfaces would have regions belonging to at least two sectors of H.

2. The long range of quantum fluctuations would naturally relate to a partial or total leakage of the 3-surface to a sector of imbedding space with larger Planck constant meaning zooming up of various quantal lengths.

3. For M-matrix in $\mathcal{M}/\mathcal{N}$ regarded as $calN$ module quantum criticality would mean a special kind of eigen state for the transition probability operator defined by the M-matrix. The properties of the number theoretic braids contributing to the M-matrix should characterize this state. The strands of the critical braids would correspond to fixed points for $G_a \times G_b$ or its subgroup.

M-matrix in finite measurement resolution

The following arguments relying on the proposed identification of the space of zero energy states give a precise formulation for M-matrix in finite measurement resolution and the Connes tensor product involved. The original expectation that Connes tensor product could lead to a unique M-matrix is wrong. The replacement of ω with its complex square root could lead to a unique hierarchy of M-matrices with finite measurement resolution and allow completely finite theory despite the fact that projectors have infinite trace for HFFs of type III_1.

1. In ZEO the counterpart of Hermitian conjugation for operator is replaced with $\mathcal{M} \rightarrow J\mathcal{M}J$ permuting the factors. Therefore $N \in \mathcal{N}$ acting to positive (negative) energy part of state corresponds to $N \rightarrow N' = JNJ$ acting on negative (positive) energy part of the state.

2. The allowed elements of N much be such that zero energy state remains zero energy state. The superposition of zero energy states involved can however change. Hence one must have that the counterparts of complex numbers are of form $N = JN_1J \vee N_2$, where N_1 and N_2 have same quantum numbers. A superposition of terms of this kind with varying quantum numbers for positive energy part of the state is possible.

3. The condition that N_{1i} and N_{2i} act like complex numbers in $\mathcal{N}$-trace means that the effect of $JN_{1i}J \vee N_{2i}$ and $JN_{2i}Ji \vee N_{1i}$ to the trace are identical and correspond to a multiplication by a constant. If $\mathcal{N}$ is HFF of type II_1 this follows from the decomposition $\mathcal{M} = \mathcal{M}/\mathcal{N} \otimes \mathcal{N}$ and from $Tr(AB) = Tr(BA)$ assuming that M is of form $M = M_{\mathcal{M}/\mathcal{N}} \times P_{\mathcal{N}}$. Contrary to the original hopes that Connes tensor product could fix the M-matrix there are no conditions on $M_{\mathcal{M}/\mathcal{N}}$ which would give rise to a finite-dimensional M-matrix for Jones inclusions. One can replaced the projector $P_{\mathcal{N}}$ with a more general state if one takes this into account in * operation.

4. In the case of HFFs of type III_1 the trace is infinite so that the replacement of Tr_N with a state ω_N in the sense of factors looks more natural. This means that the counterpart of * operation exchanging N_1 and N_2 represented as $SA\Omega = A^*\Omega$ involves Δ via $S = J\Delta^{1/2}$. The exchange of N_1 and N_2 gives altogether Δ. In this case the KMS condition $\omega_{\mathcal{N}}(AB) = \omega_{\mathcal{N}}\Delta A)$ guarantees the effective complex number property [A10] .

5. Quantum TGD more or less requires the replacement of ω with its "complex square root" so that also a unitary matrix U multiplying Δ is expected to appear in the formula for S and guarantee the symmetry. One could speak of a square root of KMS condition [A10] in

this case. The QFT counterpart would be a cutoff involving path integral over the degrees of freedom below the measurement resolution. In TGD framework it would mean a cutoff in the functional integral over WCW and for the modes of the second quantized induced spinor fields and also cutoff in sizes of causal diamonds. Discretization in terms of braids replacing light-like 3-surfaces should be the counterpart for the cutoff.

6. If one has M-matrix in $\mathcal{M}$ expressible as a sum of M-matrices of form $M_{\mathcal{M}/\mathcal{N}} \times M_{\mathcal{N}}$ with coefficients which correspond to the square roots of probabilities defining density matrix the tracing operation gives rise to square root of density matrix in M.

Is universal M-matrix possible?

The realization of the finite measurement resolution could apply only to transition probabilities in which $\mathcal{N}$-trace or its generalization in terms of state ω_N is needed. One might however dream of something more.

1. Maybe there exists a universal M-matrix in the sense that the same M-matrix gives the M-matrices in finite measurement resolution for all inclusions $\mathcal{N} \subset \mathcal{M}$. This would mean that one can write

$$M = M_{\mathcal{M}/\mathcal{N}} \otimes M_{\mathcal{N}} \tag{8.3.8}$$

for any physically reasonable choice of $\mathcal{N}$. This would formally express the idea that M is as near as possible to M-matrix of free theory. Also fractality suggests itself in the sense that $M_{\mathcal{N}}$ is essentially the same as $M_{\mathcal{M}}$ in the same sense as $\mathcal{N}$ is same as $\mathcal{M}$. It might be that the trivial solution $M = 1$ is the only possible solution to the condition.

2. $M_{\mathcal{M}/\mathcal{N}}$ would be obtained by the analog of $Tr_{\mathcal{N}}$ or ω_N operation involving the "complex square root" of the state ω in case of HFFs of type III_1. The QFT counterpart would be path integration over the degrees of freedom below cutoff to get effective action.

3. Universality probably requires assumptions about the thermodynamical part of the universal M-matrix. A possible alternative form of the condition is that it holds true only for canonical choice of "complex square root" of ω or for the S-matrix part of M:

$$S = S_{\mathcal{M}/\mathcal{N}} \otimes S_{\mathcal{N}} \tag{8.3.9}$$

for any physically reasonable choice $\mathcal{N}$.

4. In TGD framework the condition would say that the M-matrix defined by the Kähler-Dirac action gives M-matrices in finite measurement resolution via the counterpart of integration over the degrees of freedom below the measurement resolution.

An obvious counter argument against the universality is that if the M-matrix is "complex square root of state" cannot be unique and there are infinitely many choices related by a unitary transformation induced by the flows representing modular automorphism giving rise to new choices. This would actually be a well-come result and make possible quantum measurement theory.

In the section "Handful of problems with a common resolution" it was found that one can add to both Kähler action and Kähler-Dirac action a measurement interaction term characterizing the values of measured observables. The measurement interaction term in Kähler action is Lagrange multiplier term at the space-like ends of space-time surface fixing the value of classical charges for the space-time sheets in the quantum superposition to be equal with corresponding quantum charges. The term in Kähler-Dirac action is obtained from this by assigning to this term canonical momentum densities and contracting them with gamma matrices to obtain Kähler-Dirac gamma matrices appearing in 3-D analog of Dirac action. The constraint terms would leave Kähler function and Kähler metric invariant but would restrict the vacuum functional to the subset of 3-surfaces with fixed classical conserved charges (in Cartan algebra) equal to their quantum counterparts.

Connes tensor product and space-like entanglement

Ordinary linear Connes tensor product makes sense also in positive/negative energy sector and also now it makes sense to speak about measurement resolution. Hence one can ask whether Connes tensor product should be posed as a constraint on space-like entanglement. The interpretation could be in terms of the formation of bound states. The reducibility of HFFs and inclusions means that the tensor product is not uniquely fixed and ordinary entanglement could correspond to this kind of entanglement.

Also the counterpart of p-adic coupling constant evolution would makes sense. The interpretation of Connes tensor product would be as the variance of the states with respect to some subgroup of $U(n)$ associated with the measurement resolution: the analog of color confinement would be in question.

2-vector spaces and entanglement modulo measurement resolution

John Baez and collaborators [A64] are playing with very formal looking formal structures obtained by replacing vectors with vector spaces. Direct sum and tensor product serve as the basic arithmetic operations for the vector spaces and one can define category of n-tuples of vectors spaces with morphisms defined by linear maps between vectors spaces of the tuple. n-tuples allow also element-wise product and sum. They obtain results which make them happy. For instance, the category of linear representations of a given group forms 2-vector spaces since direct sums and tensor products of representations as well as n-tuples make sense. The 2-vector space however looks more or less trivial from the point of physics.

The situation could become more interesting in quantum measurement theory with finite measurement resolution described in terms of inclusions of hyper-finite factors of type II_1. The reason is that Connes tensor product replaces ordinary tensor product and brings in interactions via irreducible entanglement as a representation of finite measurement resolution. The category in question could give Connes tensor products of quantum state spaces and describing interactions. For instance, one could multiply M-matrices via Connes tensor product to obtain category of M-matrices having also the structure of 2-operator algebra.

1. The included algebra represents measurement resolution and this means that the infinite-D sub-Hilbert spaces obtained by the action of this algebra replace the rays. Sub-factor takes the role of complex numbers in generalized QM so that one obtains non-commutative quantum mechanics. For instance, quantum entanglement for two systems of this kind would not be between rays but between infinite-D subspaces corresponding to sub-factors. One could build a generalization of QM by replacing rays with sub-spaces and it would seem that quantum group concept does more or less this: the states in representations of quantum groups could be seen as infinite-dimensional Hilbert spaces.

2. One could speak about both operator algebras and corresponding state spaces modulo finite measurement resolution as quantum operator algebras and quantum state spaces with fractal dimension defined as factor space like entities obtained from HFF by dividing with the action of included HFF. Possible values of the fractal dimension are fixed completely for Jones inclusions. Maybe these quantum state spaces could define the notions of quantum 2-Hilbert space and 2-operator algebra via direct sum and tensor production operations. Fractal dimensions would make the situation interesting both mathematically and physically.

Suppose one takes the fractal factor spaces as the basic structures and keeps the information about inclusion.

1. Direct sums for quantum vectors spaces would be just ordinary direct sums with HFF containing included algebras replaced with direct sum of included HFFs.

2. The tensor products for quantum state spaces and quantum operator algebras are not anymore trivial. The condition that measurement algebras act effectively like complex numbers would require Connes tensor product involving irreducible entanglement between elements belonging to the two HFFs. This would have direct physical relevance since this entanglement cannot be reduced in state function reduction. The category would defined interactions in terms of Connes tensor product and finite measurement resolution.

3. The sequences of super-conformal symmetry breakings identifiable in terms of inclusions of super-conformal algebras and corresponding HFFs could have a natural description using the 2-Hilbert spaces and quantum 2-operator algebras.

8.3.5 Questions About Quantum Measurement Theory In Zero Energy Ontology

The following summary about quantum measurement theory in ZEO is somewhat out-of-date and somewhat sketchy. For more detailed view see [K41, K81, K4].

Fractal hierarchy of state function reductions

In accordance with fractality, the conditions for the Connes tensor product at a given time scale imply the conditions at shorter time scales. On the other hand, in shorter time scales the inclusion would be deeper and would give rise to a larger reducibility of the representation of $\mathcal{N}$ in $\mathcal{M}$. Formally, as $\mathcal{N}$ approaches to a trivial algebra, one would have a square root of density matrix and trivial S-matrix in accordance with the idea about asymptotic freedom.

M-matrix would give rise to a matrix of probabilities via the expression $P(P_+ \to P_-) = Tr[P_+ M^\dagger P_- M]$, where P_+ and P_- are projectors to positive and negative energy energy $\mathcal{N}$-rays. The projectors give rise to the averaging over the initial and final states inside $\mathcal{N}$ ray. The reduction could continue step by step to shorter length scales so that one would obtain a sequence of inclusions. If the U-process of the next quantum jump can return the M-matrix associated with $\mathcal{M}$ or some larger HFF, U process would be kind of reversal for state function reduction.

Analytic thinking proceeding from vision to details; human life cycle proceeding from dreams and wild actions to the age when most decisions relate to the routine daily activities; the progress of science from macroscopic to microscopic scales; even biological decay processes: all these have an intriguing resemblance to the fractal state function reduction process proceeding to to shorter and shorter time scales. Since this means increasing thermality of M-matrix, U process as a reversal of state function reduction might break the second law of thermodynamics.

The conservative option would be that only the transformation of intentions to action by U process giving rise to new zero energy states can bring in something new and is responsible for evolution. The non-conservative option is that the biological death is the U-process of the next quantum jump leading to a new life cycle. Breathing would become a universal metaphor for what happens in quantum Universe. The 4-D body would be lived again and again.

How quantum classical correspondence is realized at parton level?

Quantum classical correspondence must assign to a given quantum state the most probable space-time sheet depending on its quantum numbers. The space-time sheet $X^4(X^3)$ defined by the Kähler function depends however only on the partonic 3-surface X^3, and one must be able to assign to a given quantum state the most probable X^3 - call it X^3_{max} - depending on its quantum numbers.

$X^4(X^3_{max})$ should carry the gauge fields created by classical gauge charges associated with the Cartan algebra of the gauge group (color isospin and hypercharge and electromagnetic and Z^0 charge) as well as classical gravitational fields created by the partons. This picture is very similar to that of quantum field theories relying on path integral except that the path integral is restricted to 3-surfaces X^3 with exponent of Kähler function bringing in genuine convergence and that 4-D dynamics is deterministic apart from the delicacies due to the 4-D spin glass type vacuum degeneracy of Kähler action.

Stationary phase approximation selects X^3_{max} if the quantum state contains a phase factor depending not only on X^3 but also on the quantum numbers of the state. A good guess is that the needed phase factor corresponds to either Chern-Simons type action or an action describing the interaction of the induced gauge field with the charges associated with the braid strand. This action would be defined for the induced gauge fields. YM action seems to be excluded since it is singular for light-like 3-surfaces associated with the light-like wormhole throats (not only $\sqrt{det(g_3)}$ but also $\sqrt{det(g_4)}$ vanishes).

The challenge is to show that this is enough to guarantee that $X^4(X^3_{max})$ carries correct gauge charges. Kind of electric-magnetic duality should relate the normal components F_{ni} of the gauge fields in $X^4(X^3_{max})$ to the gauge fields F_{ij} induced at X^3. An alternative interpretation is in terms of quantum gravitational holography.

One is forced to introduce gauge couplings and also electro-weak symmetry breaking via the phase factor. This is in apparent conflict with the idea that all couplings are predictable. The essential uniqueness of M-matrix in the case of HFFs of type II_1 (at least) however means that their values as a function of measurement resolution time scale are fixed by internal consistency. Also quantum criticality leads to the same conclusion. Obviously a kind of bootstrap approach suggests itself.

Quantum measurements in ZEO

ZEO based quantum measurement theory leads directly to a theory of conscious entities. The basic idea is that state function reduction localizes the second boundary of CD so that it becomes a piece of light-cone boundary (more precisely $\delta M^4_\pm \times CP_2$).

Repeated reductions are possible as in standard quantum measurement theory and leave the passive boundary of CD. Repeated reduction begins with U process generating a superposition of CDs with the active boundary of CD being de-localized in the moduli space of CDs, and is followed by a localization in this moduli space so that single CD is the outcome. This process tends to increase the distance between the ends of the CD and has interpretation as a space-time correlate for the flow of subjective time.

Self as a conscious entity corresponds to this sequence of repeated reductions on passive boundary of CD. The first reduction at opposite boundary means death of self and its re-incarnation at the opposite boundary of CD. Also the increase of Planck constant and generation of negentropic entanglement is expected to be associated with this state function reduction.

Weak form of NMP is the most plausible variational principle to characterize the state function reduction. It does not require maximal negentropy gain for state function reductions but allows it. In other words, the outcome of reduction is n-dimensional eigen space of density matrix space but this space need not have maximum possible dimension and even 1-D ray is possible in which case the entanglement negentropy vanishes for the final state and system becomes isolated from the rest of the world. Weak form of NMP brings in free will and can allow also larger negentropy gain than the strong form if n is a product of primes. The beauty of this option is that one can understand how the generalization of p-adic length scale hypothesis emerges.

8.3.6 Miscellaneous

The following considerations are somewhat out-of-date: hence the title "Miscellaneous".

Connes tensor product and fusion rules

One should demonstrate that Connes tensor product indeed produces an M-matrix with physically acceptable properties.

The reduction of the construction of vertices to that for n-point functions of a conformal field theory suggest that Connes tensor product is essentially equivalent with the fusion rules for conformal fields defined by the Clifford algebra elements of $CH(CD)$ (4-surfaces associated with 3-surfaces at the boundary of causal diamond CD in M^4), extended to local fields in M^4 with gamma matrices acting on WCW spinor s assignable to the partonic boundary components.

Jones speculates that the fusion rules of conformal field theories can be understood in terms of Connes tensor product [A91] and refers to the work of Wassermann about the fusion of loop group representations as a demonstration of the possibility to formula the fusion rules in terms of Connes tensor product [A34] .

Fusion rules are indeed something more intricate that the naive product of free fields expanded using oscillator operators. By its very definition Connes tensor product means a dramatic reduction of degrees of freedom and this indeed happens also in conformal field theories.

1. For non-vanishing n-point functions the tensor product of representations of Kac Moody group associated with the conformal fields must give singlet representation.

2. The ordinary tensor product of Kac Moody representations characterized by given value of central extension parameter k is not possible since k would be additive.

3. A much stronger restriction comes from the fact that the allowed representations must define integrable representations of Kac-Moody group [A42] . For instance, in case of $SU(2)_k$ Kac Moody algebra only spins $j \leq k/2$ are allowed. In this case the quantum phase corresponds to $n = k+2$. $SU(2)$ is indeed very natural in TGD framework since it corresponds to both electro-weak $SU(2)_L$ and isotropy group of particle at rest.

Fusion rules for localized Clifford algebra elements representing operators creating physical states would replace naive tensor product with something more intricate. The naivest approach would start from M^4 local variants of gamma matrices since gamma matrices generate the Clifford algebra Cl associated with $CH(CD)$. This is certainly too naive an approach. The next step would be the localization of more general products of Clifford algebra elements elements of Kac Moody algebras creating physical states and defining free on mass shell quantum fields. In standard quantum field theory the next step would be the introduction of purely local interaction vertices leading to divergence difficulties. In the recent case one transfers the partonic states assignable to the light-cone boundaries $\delta M^4_\pm(m_i) \times CP_2$ to the common partonic 2-surfaces X^2_V along $X^3_{L,i}$ so that the products of field operators at the same space-time point do not appear and one avoids infinities.

The remaining problem would be the construction an explicit realization of Connes tensor product. The formal definition states that left and right $\mathcal{N}$ actions in the Connes tensor product $\mathcal{M} \otimes_{\mathcal{N}} \mathcal{M}$ are identical so that the elements $nm_1 \otimes m_2$ and $m_1 \otimes m_2 n$ are identified. This implies a reduction of degrees of freedom so that free tensor product is not in question. One might hope that at least in the simplest choices for $\mathcal{N}$ characterizing the limitations of quantum measurement this reduction is equivalent with the reduction of degrees of freedom caused by the integrability constraints for Kac-Moody representations and dropping away of higher spins from the ordinary tensor product for the representations of quantum groups. If fusion rules are equivalent with Connes tensor product, each type of quantum measurement would be characterized by its own conformal field theory.

In practice it seems safest to utilize as much as possible the physical intuition provided by quantum field theories. In [K13] a rather precise vision about generalized Feynman diagrams is developed and the challenge is to relate this vision to Connes tensor product.

Connection with topological quantum field theories defined by Chern-Simons action

There is also connection with topological quantum field theories (TQFTs) defined by Chern- Simons action [A50] .

1. The light-like 3-surfaces X^3_l defining propagators can contain unitary matrix characterizing the braiding of the lines connecting fermions at the ends of the propagator line. Therefore the modular S-matrix representing the braiding would become part of propagator line. Also incoming particle lines can contain similar S-matrices but they should not be visible in the M-matrix. Also entanglement between different partonic boundary components of a given incoming 3-surface by a modular S-matrix is possible.

2. Besides CP_2 type extremals MEs with light-like momenta can appear as brehmstrahlung like exchanges always accompanied by exchanges of CP_2 type extremals making possible momentum conservation. Also light-like boundaries of magnetic flux tubes having macroscopic size could carry light-like momenta and represent similar brehmstrahlung like exchanges. In this case the modular S-matrix could make possible topological quantum computations in $q \neq 1$ phase [K85] . Notice the somewhat counter intuitive implication that magnetic flux tubes of macroscopic size would represent change in quantum jump rather than quantum state. These quantum jumps can have an arbitrary long geometric duration in macroscopic quantum phases with large Planck constant [K18] .

There is also a connection with topological QFT defined by Chern-Simons action allowing to assign topological invariants to the 3-manifolds [A50] . If the light-like CDs $X^3_{L,i}$ are boundary components, the 3-surfaces associated with particles are glued together somewhat like they are glued in the process allowing to construct 3-manifold by gluing them together along boundaries. All 3-manifold topologies can be constructed by using only torus like boundary components.

This would suggest a connection with 2+1-dimensional topological quantum field theory defined by Chern-Simons action allowing to define invariants for knots, links, and braids and 3-manifolds using surgery along links in terms of Wilson lines. In these theories one consider gluing of two 3-manifolds, say three-spheres S^3 along a link to obtain a topologically non-trivial 3-manifold. The replacement of link with Wilson lines in $S^3\#S^3 = S^3$ reduces the calculation of link invariants defined in this manner to Chern-Simons theory in S^3.

In the recent situation more general structures are possible since arbitrary number of 3-manifolds are glued together along link so that a singular 3-manifolds with a book like structure are possible. The allowance of CDs which are not boundaries, typically 3-D light-like throats of wormhole contacts at which induced metric transforms from Minkowskian to Euclidian, brings in additional richness of structure. If the scaling factor of CP_2 metric can be arbitrary large as the quantization of Planck constant predicts, this kind of structure could be macroscopic and could be also linked and knotted. In fact, topological condensation could be seen as a process in which two 4-manifolds are glued together by drilling light-like CDs and connected by a piece of CP_2 type extremal.

8.4 The Relation Between U-Matrix And M-Matrices

S-matrix is the key notion in quantum field theories. In Zero Energy Ontology (ZEO) this notion must be replaced with the triplet U-matrix, M-matrix, and S-matrix. U-matrix realizes unitary time evolution in the space for zero energy states realized geometrically as dispersion in the moduli space of causal diamonds (CDs) leaving second boundary (passive boundary) of CD and states at it fixed.

This process can be seen as the TGD counterpart of repeated state function reductions leaving the states at passive boundary unaffected and affecting only the member of state pair at active boundary (Zeno effect) [K41]. In TGD inspired theory of consciousness self corresponds to the sequence of these state function reductions [K81, K4, K65]. M-matrix describes the entanglement between positive and negative energy parts of zero energy states and is expressible as a hermitian square root H of density matrix multiplied by a unitary matrix S, which corresponds to ordinary S-matrix, which is universal and depends only the size scale n of CD through the formula $S(n) = S^n$. M-matrices and H-matrices form an orthonormal basis at given CD and H-matrices would naturally correspond to the generators of super-symplectic algebra.

The first state function reduction to the opposite boundary corresponds to what happens in quantum physics experiments. The relationship between U- and S-matrices has remained poorly understood.

The original view about the relationship was a purely formal guess: M-matrices would define the orthonormal rows of U-matrix. This guess is not correct physically and one must consider in detail what U-matrix really means.

1. First about the geometry of CD [K91]. The boundaries of CD will be called passive and active: passive boundary correspond to the boundary at which repeated state function reductions take place and give rise to a sequence of unitary time evolutions U followed by localization in the moduli of CD each. Active boundary corresponds to the boundary for which U induces delocalization and modifies the states at it.

   The moduli space for the CDs consists of a discrete subgroup of scalings for the size of CD characterized by the proper time distance between the tips and the sub-group of Lorentz boosts leaving passive boundary and its tip invariant and acting on the active boundary only. This group is assumed to be represented unitarily by matrices Λ forming the same group for all values of n.

   The proper time distance between the tips of CDs is quantized as integer multiples of the minimal distance defined by CP_2 time: $T = nT_0$. Also in quantum jump in which the size scale n of CD increases the increase corresponds to integer multiple of T_0. Using the logarithm of proper time, one can interpret this in terms of a scaling parametrized by an integer. The possibility to interpret proper time translation as a scaling is essential for having a manifest Lorentz invariance: the ordinary definition of S-matrix introduces preferred rest system.

2. The physical interpretation would be roughly as follows. M-matrix for a given CD codes for the physics as we usually understand it. M-matrix is product of square root of density matrix and S-matrix depending on the size scale of CD and is the analog of thermal S-matrix. State function at the opposite boundary of CD corresponds to what happens in the state function reduction in particle physics experiments. The repeated state function reductions at same boundary of CD correspond to TGD version of Zeno effect crucial for understanding consciousness. Unitary U-matrix describes the time evolution zero energy states due to the increase of the size scale of CD (at least in statistical sense). This process is dispersion in the moduli space of CDs: all possible scalings are allowed and localization in the space of moduli of CD localizes the active boundary of CD after each unitary evolution.

In the following I will proceed by making questions. One ends up to formulas allowing to understand the architecture of U-matrix and to reduce its construction to that for S-matrix having interpretation as exponential of the generator L_1 of the Virasoro algebra associated with the super-symplectic algebra.

8.4.1 What One Can Say About M-Matrices?

1. The first thing to be kept in mind is that M-matrices act in the space of zero energy states rather than in the space of positive or negative energy states. For a given CD M-matrices are products of hermitian square roots of hermitian density matrices acting in the space of zero energy states and universal unitary S-matrix $S(CD)$ acting on states at the active end of CD (this is also very important to notice) depending on the scale of CD:

$$M^i = H^i \circ S(CD) \ .$$

Here "$\circ$" emphasizes the fact that S acts on zero energy states at active boundary only. H^i is hermitian square root of density matrix and the matrices H^i must be orthogonal for given CD from the orthonormality of zero energy states associated with the same CD. The zero energy states associated with different CDs are not orthogonal and this makes the unitary time evolution operator U non-trivial.

2. Could quantum measurement be seen as a measurement of the observables defined by the Hermitian generators H^i? This is not quite clear since their action is on zero energy states. One might actually argue that the action of this kind of observables on zero energy states does not affect their vanishing net quantum numbers. This suggests that H^i carry no net quantum numbers and belong to the Cartan algebra. The action of S is restricted at the active boundary of CD and therefore it does not commute with H^i unless the action is in a separate tensor factor. Therefore the idea that S would be an exponential of generators H^i and thus commute with them so that H^i would correspond to sub-spaces remaining invariant under S acting unitarily inside them does not make sense.

3. In TGD framework symplectic algebra actings as isometries of WCW is analogous to a Kac-Moody algebra with finite-dimensional Lie-algebra replaced with the infinite-dimensional symplectic algebra with elements characterized by conformal weights [K15, K14]. There is a temptation to think that the H^i could be seen as a representation for this algebra or its sub-algebra. This algebra allows an infinite fractal hierarchy of sub-algebras of the super-symplectic algebra isomorphic to the full algebra and with conformal weights coming as n-ples of those for the full algebra. In the proposed realization of quantum criticality the elements of the sub-algebra characterized by n act as a gauge algebra. An interesting question is whether this sub-algebra is involved with the realization of M-matrices for CD with size scale n. The natural expectation is that n defines a cutoff for conformal weights relating to finite measurement resolution.

8.4.2 How Does The Size Scale Of CD Affect M-Matrices?

1. In standard quantum field theory (QFT) S-matrix represents time translation. The obvious generalization is that now scaling characterized by integer n is represented by a unitary S-matrix that is as n:th power of some unitary matrix S assignable to a CD with minimal size: $S(CD) = S^n$. $S(CD)$ is a discrete analog of the ordinary unitary time evolution operator with n replacing the continuous time parameter.

2. One can see M-matrices also as a generalization of Kac-Moody type algebra. Also this suggests $S(CD) = S^n$, where S is the S-matrix associated with the minimal CD. S becomes representative of phase $exp(i\phi)$. The inner product between CDs of different size scales can n_1 and n_2 can be defined as

$$\begin{aligned} &\langle M^i(m), M^j(n)\rangle = Tr(S^{-m} \circ H^i H^j \circ S^n) \times \theta(n-m) \ , \\ &\theta(n) = 1 \ \text{ for } \ n \geq 0 \ , \ \ \theta(n) = 0 \ \text{ for } \ n < 0 \ . \end{aligned} \tag{8.4.1}$$

Here I have denoted the action of S-matrix at the active end of CD by "$\circ$" in order to distinguish it from the action of matrices on zero energy states which could be seen as belonging to the tensor product of states at active and passive boundary.

It turns out that unitarity conditions for U-matrix are invariant under the translations of n if one assumes that the transitions obey strict arrow of time expressed by $n_j - n_i \geq 0$. This simplifies dramatically unitarity conditions. This gives orthonormality for M-matrices associated with identical CDs. This inner product could be used to identify U-matrix.

3. How do the discrete Lorentz boosts affecting the moduli for CD with a fixed passive boundary affect the M-matrices? The natural assumption is that the discrete Lorentz group is represented by unitary matrices λ: the matrices M^i are transformed to $M^i \circ \lambda$ for a given Lorentz boost acting on states at active boundary only.

One cannot completely exclude the possibility that S acts unitarily at both ends of zero energy states. In this case the scaling would be interpreted as acting on zero energy states rather than those at active boundary only. The zero energy state basis defined by M_i would depend on the size scale of CD in more complex manner. This would not affect the above formulas except by dropping away the "$\circ$".

Unitary U must characterize the transitions in which the moduli of the active boundary of causal diamond (CD) change and also states at the active boundary (paired with unchanging states at the passive boundary) change. The arrow of the experienced flow of time emerges during the period as state function reductions take place to the fixed ("passive") boundary of CD and do not affect the states at it. Note that these states form correlated pairs with the changing states at the active boundary. The physically motivated question is whether the arrow of time emerges statistically from the fact that the size of CD tends to increase in average sense in repeated state function reductions or whether the arrow of geometric time is strict. It turns out that unitarity conditions simplify dramatically if the arrow of time is strict.

8.4.3 What Can One Say About U-Matrix?

1. Just from the basic definitions the elements of a unitary matrix, the elements of U are between zero energy states (M-matrices) between two CDs with possibly different moduli of the active boundary. Given matrix element of U should be proportional to an inner product of two M-matrices associated with these CDs. The obvious guess is as the inner product between M-matrices

$$\begin{aligned} U^{ij}_{m,n} &= \langle M^i(m,\lambda_1), M^j(n,\lambda_2)\rangle \\ &= Tr(\lambda_1^\dagger S^{-m}\circ H^iH^j\circ S^n\lambda_2) \\ &= Tr(S^{-m}\circ H^iH^j\circ S^n\lambda_2\lambda_1^{-1})\theta(n-m) \ . \end{aligned} \tag{8.4.2}$$

Here the usual properties of the trace are assumed. The justification is that the operators acting at the active boundary of CD are special case of operators acting non-trivially at both boundaries.

2. Unitarity conditions must be satisfied. These conditions relate S and the hermitian generators H^i serving as square roots of density matrices. Unitarity conditions $UU^\dagger = U^\dagger U = 1$ is defined in the space of zero energy states and read as

$$\sum_{j_1n_1} U^{ij_1}_{mn_1}(U^\dagger)^{j_1j}_{n_1n} = \delta^{i,j}\delta_{m,n}\delta_{\lambda_1,\lambda_2} \tag{8.4.3}$$

To simplify the situation let us make the plausible hypothesis contribution of Lorentz boosts in unitary conditions is trivial by the unitarity of the representation of discrete boosts and the independence on n.

3. In the remaining degrees of freedom one would have

$$\sum_{j_1,k\geq Max(0,n-m)} Tr(S^k\circ H^iH^{j_1})Tr(H^{j_1}H^j\circ S^{n-m-k}) = \delta^{i,j}\delta_{m,n} \ . \tag{8.4.4}$$

The condition $k\geq Max(0,n-m)$ reflects the assumption about a strict arrow of time and implies that unitarity conditions are invariant under the proper time translation $(n,m)\rightarrow(n+r,m+r)$. Without this condition n back-wards translations (or rather scalings) to the direction of geometric past would be possible for CDs of size scale n and this would break the translational invariance and it would be very difficult to see how unitarity could be achieved. Stating it in a general manner: time translations act as semigroup rather than group.

4. Irreversibility reduces dramatically the number of the conditions. Despite this their number is infinite and correlates the Hermitian basis and the unitary matrix S. There is an obvious analogy with a Kac-Moody algebra at circle with S replacing the phase factor $exp(in\phi)$ and H^i replacing the finite-dimensional Lie-algebra. The conditions could be seen as analogs for the orthogonality conditions for the inner product. The unitarity condition for the analog situation would involve phases $exp(ik\phi_1)\leftrightarrow S^k$ and $exp(i(n-m-k)\phi_2)\leftrightarrow S^{n-m-k}$ and trace would correspond to integration $\int d\phi_1$ over ϕ_1 in accordance with the basic idea of non-commutative geometry that trace corresponds to integral. The integration of ϕ_i would give $\delta_{k,0}$ and $\delta_{m,n}$. Hence there are hopes that the conditions might be satisfied. There is however a clear distinction to the Kac-Moody case since S^n does not in general act in the orthogonal complement of the space spanned by H^i.

5. The idea about reduction of the action of S to a phase multiplication is highly attractive and one could consider the possibility that the basis of H^i can be chosen in such a manner that H^i are eigenstates of of S. This would reduce the unitarity constraint to a form in which the summation over k can be separated from the summation over j_1.

$$\sum_{k\geq Max(0,n-m)} exp(iks_i-(n-m-k)s_j)\sum_{j_1} Tr(H^iH^{j_1})Tr(H^{j_1}H^j) = \delta^{i,j}\delta_{m,n} \ . \tag{8.4.5}$$

The summation over k should gives a factor proportional to δ_{s_i,s_j}. If the correspondence between H^i and eigenvalues s_i is one-to-one, one obtains something proportional to $\delta(i,j)$ apart from a normalization factor. Using the orthonormality $Tr(H^iH^j) = \delta^{i,j}$ one obtains for the left hand side of the unitarity condition

$$exp(is_i(n-m))\sum_{j_1} Tr(H^iH^{j_1})Tr(H^{j_1}H^j) = exp(is_i(n-m))\delta_{i,j} \quad . \tag{8.4.6}$$

Clearly, the phase factor $exp(is_i(n-m))$ is the problem. One should have Kronecker delta $\delta_{m,n}$ instead. One should obtain behavior resembling Kac-Moody generators. H^i should be analogs of Kac-Moody generators and include the analog of a phase factor coming visible by the action of S.

8.4.4 How To Obtain Unitarity Correctly?

It seems that the simple picture is not quite correct yet. One should obtain somehow an integration over angle in order to obtain Kronecker delta.

1. A generalization based on replacement of real numbers with function field on circle suggests itself. The idea is to the identify eigenvalues of generalized Hermitian/unitary operators as Hermitian/unitary operators with a spectrum of eigenvalues, which can be continuous. In the recent case S would have as eigenvalues functions $\lambda_i(\phi) = exp(is_i\phi)$. For a discretized version ϕ would have has discrete spectrum $\phi(n) = 2\pi k/n$. The spectrum of λ_i would have n as cutoff. Trace operation would include integration over ϕ and one would have analogs of Kac-Moody generators on circle.

2. One possible interpretation for ϕ is as an angle parameter associated with a fermionic string connecting partonic 2-surface. For the super-symplectic generators suitable normalized radial light-like coordinate r_M of the light-cone boundary (containing boundary of CD) would be the counterpart of angle variable if periodic boundary conditions are assumed.

   The eigenvalues could have interpretation as analogs of conformal weights. Usually conformal weights are real and integer valued and in this case it is necessary to have generalization of the notion of eigenvalues since otherwise the exponentials $exp(is_i)$ would be trivial. In the case of super-symplectic algebra I have proposed that the generating elements of the algebra have conformal weights given by the zeros of Riemann zeta. The spectrum of conformal weights for the generators would consist of linear combinations of the zeros of zeta with integer coefficients. The imaginary parts of the conformal weights could appear as eigenvalues of S.

3. It is best to return to the definition of the U-matrix element to check whether the trace operation appearing in it can already contain the angle integration. If one includes to the trace operation appearing the integration over ϕ it gives $\delta_{m,n}$ factor and U-matrix has elements only between states assignable to the same causal diamond. Hence one must interpret U-matrix elements as functions of ϕ realized factors $exp(i(s_n - s_m)\phi)$. This brings strongly in mind operators defined as distributions of operators on line encountered in the theory of representations of non-compact groups such as Lorentz group. In fact, the unitary representations of discrete Lorentz groups are involved now.

4. The unitarity condition contains besides the trace also the integrations over the two angle parameters ϕ_i associated with the two U-matrix elements involved. The left hand side of the unitarity condition reads as

$$\begin{aligned} \sum_{k\geq Max(0,n-m)} & I(ks_i)I((n-m-k)s_j)\times \sum_{j_1} Tr(H^iH^{j_1})Tr(H^{j_1}H^j) \\ & = \delta^{i,j}\delta_{m,n} \quad , \\ I(s) = \frac{1}{2\pi}\times \int d\phi exp(is\phi) = \delta_{s,0} \quad . \end{aligned} \tag{8.4.7}$$

Integrations give the factor $\delta_{k,0}$ eliminating the infinite sum obtained otherwise plus the factor $\delta_{n,m}$. Traces give Kronecker deltas since the projectors are orthonormal. The left hand side equals to the right hand side and one achieves unitarity. It seems that the proposed ansatz works and the U-matrix can be reduced by a general ansatz to S-matrix.

5. It should be made clear that the use of eigenstates of S is only a technical trick, the physical states need not be eigenstates. If the active parts of zero energy states where eigenstates of S, U-matrix would not have matrix elements between different H^i and projection operator could not change during time evolution.

8.4.5 What About The Identification Of S?

1. S should be exponential of time the scaling operator whose action reduces to a time translation operator along the time axis connecting the tips of CD and realized as scaling. In other words, the shift $t/T_0 = m \rightarrow m+n$ corresponds to a scaling $t/T_0 = m \rightarrow km$ giving $m+n = km$ in turn giving $k = 1+n/m$. At the limit of large shifts one obtains $k \simeq n/m \rightarrow \infty$, which corresponds to QFT limit. nS corresponds to $(nT_0)\times(S/T_0) = TH$ and one can ask whether QFT Hamiltonian could corresponds to $H = S/T_0$.

2. It is natural to assume that the operators H^i are eigenstates of radial scaling generator $L_0 = ir_Md/dr_M$ at both boundaries of CD and have thus well-defined conformal weights. As noticed the spectrum for super-symplectic algebra could also be given in terms of zeros of Riemann zeta.

3. The boundaries of CD are given by the equations $r_M = m^0$ and $r_M = T - m_0$, m_0 is Minkowski time coordinate along the line between the tips of CD and T is the distance between the tips. From the relationship between r_M and m_0 the action of the infinitesimal translation $H \equiv i\partial/\partial_{m^0}$ can be expressed as conformal generator $L_{-1} = i\partial/\partial_{rM} = r_M^{-1}L_0$. Hence the action is non-diagonal in the eigenbasis of L_0 and multiplies with the conformal weights and reduces the conformal weight by one unit. Hence the action of U can change the projection operator. For large values of conformal weight the action is classically near to that of L_0: multiplication by L_0 plus small relative change of conformal weight.

4. Could the spectrum of H be identified as energy spectrum expressible in terms of zeros of zeta defining a good candidate for the super-symplectic radial conformal weights. This certainly means maximal complexity since the number of generators of the conformal algebra would be infinite. This identification might make sense in chaotic or critical systems. The functions $(r_M/r_0)^{1/2+iy}$ and $(r_M/r_0)^{-2n}$, $n>0$, are eigenmodes of r_M/dr_M with eigenvalues $(1/2+iy)$ and $-2n$ corresponding to non-trivial and trivial zeros of zeta.

   There are two options to consider. Either L_0 or iL_0 could be realized as a hermitian operator. These options would correspond to the identification of mass squared operator as L_0 and approximation identification of Hamiltonian as iL_1 as iL_0 making sense for large conformal weights.

   (a) Suppose that $L_0 = r_Md/dr_M$ realized as a hermitian operator would give harmonic oscillator spectrum for conformal confinement. In p-adic mass calculations the string model mass formula implies that L_0 acts essentially as mass squared operator with integer spectrum. I have proposed conformal confinent for the physical states net conformal weight is real and integer valued and corresponds to the sum over negative integer valued conformal weights corresponding to the trivial zeros and sum over real parts of non-trivial zeros with conformal weight equal to 1/2. Imaginary parts of zeta would sum up to zero.

(b) The counterpart of Hamiltonian as a time translation is represented by $H = iL_0 = ir_M d/dr_M$. Conformal confinement is now realized as the vanishing of the sum for the real parts of the zeros of zeta: this can be achieved. As a matter fact the integration measure dr_M/r_M brings implies that the net conformal weight must be 1/2. This is achieved if the number of non-trivial zeros is odd with a judicious choice of trivial zeros. The eigenvalues of Hamiltonian acting as time translation operator could correspond to the linear combination of imaginary part of zeros of zeta with integer coefficients. This is an attractive hypothesis in critical systems and TGD Universe is indeed quantum critical.

8.4.6 What About Quantum Classical Correspondence?

Quantum classical correspondence realized as one-to-one map between quantum states and zero modes has not been discussed yet.

1. M-matrices would act in the tensor product of quantum fluctuating degrees of freedom and zero modes. The assumption that zero energy states form an orthogonal basis implies that the hermitian square roots of the density matrices form an orthonormal basis. This condition generalizes the usual orthonormality condition.

2. The dependence on zero modes at given boundary of CD would be trivial and induced by 1-1 correspondence $|m\rangle \rightarrow z(m)$ between states and zero modes assignable to the state basis $|m_\pm$ at the boundaries of CD, and would mean the presence of factors $\delta_{z_+,f(m_+)} \times \delta_{z_-,f(n_-)}$ multiplying M-matrix $M^i_{m,n}$.

To sum up, it seems that the architecture of the U-matrix and its relationship to the S-matrix is now understood and in accordance with the intuitive expectations the construction of U-matrix reduces to that for S-matrix and one can see S-matrix as discretized counterpart of ordinary unitary time evolution operator with time translation represented as scaling: this allows to circumvent problems with loss of manifest Poincare symmetry encountered in quantum field theories and allows Lorentz invariance although CD has finite size. What came as surprise was the connection with stringy picture: strings are necessary in order to satisfy the unitary conditions for U-matrix. Second outcome was that the connection with super-symplectic algebra suggests itself strongly. The identification of hermitian square roots of density matrices with Hermitian symmetry algebra is very elegant aspect discovered already earlier. A further unexpected result was that U-matrix is unitary only for strict arrow of time (which changes in the state function reduction to opposite boundary of CD).

Chapter 9

What Scattering Amplitudes Should Look Like?

9.1 Introduction

During years I have spent a lot of time and effort in attempts to imagine various options for the construction of S-matrix - in Zero Energy Ontology (ZEO) M- and U-matrices - and it seems that there are quite many strong constraints, which might lead to a more or less unique final result if some young analytically blessed brain decided to transform these assumptions to concrete calculational recipes.

The realization that WCW spinors correspond to von Neumann algebras known as hyper-finite factors of type II_1 meant [K87, K22] a turning point also in the attempts to construct S-matrix. A sequence of trials and errors led rapidly to the generalization of the quantum measurement theory and re-interpretation of S-matrix elements as entanglement coefficients of zero energy states in accordance with the zero energy ontology applied already earlier in TGD inspired cosmology [K16]. ZEO motivated the replacement of the term "S-matrix" with "M-matrix". This led to the discovery that rather stringy formulas for M-matrix elements emerge in TGD framework.

The purpose of this chapter is to collect to single chapter various general ideas about the construction of M-matrix scattered in the chapters of books about TGD and often drowned into details and plagued by side tracks and give a brief summary about intuitive picture behind various matrices. Also a general vision about generalized Feynman diagrams is formulated. A more detailed construction is suggested in the chapters about twistors and TGD.

My hope is that this chapter might provide a kind of bird's eye of view and help the reader to realize how fascinating and profound and near to physics the mathematics of hyper-finite factors is.

The goal is to sketch an overall view about the ideas which have led to the recent view about the construction of M-matrix. First the basic philosophical ideas are discussed. These include the basic ideas behind TGD inspired theory of consciousness [K75]. the identification of p-adic physics as physics of cognition forcing the central idea of number theoretic universality, quantum classical correspondence, and the crucial notion of zero energy ontology.

The understanding of the fundamental variational principles of TGD is so detailed that one can sketch a rather concrete formulation for the generalized Feynman rules. The generalized Feynman diagrams correspond to Euclidian regions of 4-D surfaces - preferred extremals - defined by orbits of wormhole contacts plus the string world sheets connecting them and carrying spinor modes. Fermioaction contains also a part associated with the boundaries of string world sheets at partonic orbits. As a consequence, fundamental fermions propagate as particles with momenta which are light-like in 8-D sense along the light-like geodesics defined by the boundaries of string world sheets at which spinor modes are localized. This strongly suggests 8-D generalization of twistor approach.

The topological identification of the basic interaction vertices is as partonic 2-surfaces at which the orbits of partonic 2-surfaces meet. Fermions behave like free massless (in 8-D sense) particles during propagation along boundaries of string world sheets but interact at partonic surfaces and associated wormhole contacts by classical induced gauge fields. The naive guess would be that the conformal scaling generator L_0 for super-symplectic algebra could serving as propagator mediating the interaction between fermions at opposite wormhole throats.

The notion of preferred extremal does not favor ordinary Feynman diagrammatics resulting from path integral approach. The picture suggested by twistorialization looks more natural. Scattering amplitudes would be analogous to a minimal sequences of calculations transforming a given initial state to a given final state located at boundaries of CD. I proposed this vision for many years ago in terms of bi-algebras and related structures but gave it up as too speculative, and the only remnant of the enthusiasism period is a little appendix [K6]. The basic operations would be product and co-product in the Yangian associated with the super-symplectic algebra. Interaction vertices would correspond product and co-product for the generators of the Yangian algebra. The generators of this algebra would be Noether super charges associated with strings connecting partonic two surfaces.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [L12].

9.2 General Vision Behind Matrices

In the following I summarize the basic notions and ideas discussed in previous chapters.

9.2.1 Basic Principles

My original intention was to summarize the basic principles of Quantum TGD first. The problem is however where to start from since everything is so tightly interwoven that linear representation proceeding from principles to consequences seems impossible. Therefore it might be a good idea to try to give a summary with emphasis on what has happened during the few months in turn of 2008 to 2009 assuming that the reader is familiar with the basic concepts discussed in previous chapters. This summary gives also a bird's eye of view about what I believe M-matrix to be. Later this picture is used to answer the questions raised in the earlier version of this chapter.

Zero energy ontology

One of the key notions underlying the recent developments is zero energy ontology.

1. Zero energy ontology leads naturally to the identification of light-like 3-surfaces interpreted as a generalization of Feynman diagrams as the most natural dynamical objects (equivalent with space-like 3-surface by holography).

2. The fractal hierarchy of causal diamonds (CD) with light like boundaries of CD interpreted as carriers of positive and negative energy parts of zero energy state emerges naturally. If the scales of CDs come as powers of 2, p-adic length scale hypothesis follows as a consequence.

3. The identification of M-matrix as time-like entanglement coefficients between zero energy states identified as the product of positive square root of the density matrix and unitary S-matrix emerges naturally and leads to the unification of thermodynamics and quantum theory.

4. The identification of M-matrix in terms of Connes tensor product means that the included algebra $\mathcal{N} \subset \mathcal{M}$ acts effectively like complex numbers and does not affect the physical state. The interpretation is that $\mathcal{N}$ corresponds to zero energy states in size scales smaller than the measurement resolution and thus the insertion of this kind of zero energy state should not have any observable effects. The uniqueness of Connes tensor product gives excellent hopes that the M-matrix could be unique apart from the square root of of density matrix.

5. The unitary U-matrix between zero energy states assignable to quantum jump has nothing to do with S-matrix measured in particle physics experiments. A possible interpretation

is in terms of consciousness theory. For instance, U-matrix could make sense even for p-adic-to-real transitions interpreted as transformations of intentions to actions making sense since zero energy state is generated ("Everything is creatable from vacuum" is the basic principle of zero energy ontology) [K41] . One can express U-matrix as a collection of M-matrices labeled by zero energy states and unitaritity conditions for U-matrix boil down to orthogonality conditions for the zero energy states defined by M-matrices.

The notion of finite measurement resolution

The notion of finite measurement resolution as a basic dynamical principle of quantum TGD might be seen by a philosophically minded reader as the epistemological counterpart of zero energy ontology.

1. As far as length scale resolution is considered, finite measurement resolution implies that only CDs above some size scale are allowed. This is not an approximation but a property of zero energy state so that zero energy states realize finite measurement resolution in their structure. One might perhaps say that quantum states represent only the information that we can becomes conscious of.

2. In the case of angle resolution the hierarchy of Planck constants accompanied by a hierarchy of algebraic extensions of rationals by roots of unity, and realized in terms of the book like structures assigned with CD and CP_2, is a natural outcome of this thinking.

3. Number theoretic braids implying discretization at parton level can be seen as a space-time correlate for the finite measurement resolution. Zero energy states should contain in their construction only information assignable to the points of the braids. Note however that there is also information about tangent space of space-time surface at these points so that the theory does not reduce to a genuinely discrete theory. Each choice of M^2 and geodesic spheres defines a selection of quantization axis and different choice of the number theoretic braid. Hence discreteness does not reduce to that resulting from the assumption that space-time as the arena of dynamics is discrete but reflects the limits to what we can measure, perceive, and cognize in continuous space-time. Zero energy state corresponds to wave-function in the space of these choices realized as the union of copies of the page $CD \times CP_2$. Quantum measurement must induce a localization to single point in this space unless one is ready to take seriously the notion of quantum multiverse.

4. Finite measurement resolution allows a realization in terms of inclusions $\mathcal{N} \subset \mathcal{M}$ of hyper-finite factors of type II_1 (HFFs) about which the WCW Clifford algebra provides standard example. Also the factor spaces $\mathcal{M}/\mathcal{N}$ are suggestive and should correspond to quantum variants of HFFs with a finite quantum dimension. p-Adic coupling constant evolution can be understood in this framework and corresponds to the inclusions of HFFs realized as inclusions of spaces of zero energy states with two different scale cutoffs.

Number theoretical compactification and $M^8 - H$ duality

The closely related notions of number theoretical compactification and $M^8 - H$ duality have had a decisive impact on the understanding of the mathematical structure of quantum TGD.

1. The hypothesis is that TGD allows two equivalent descriptions using either M^8- the space of hyper-octonions- or $H = M^4 \times CP_2$ as imbedding space so that standard model symmetries have a number theoretic interpretation. The underlying philosophy is that the world of classical worlds and thus H is unique so that the symmetries of H should be something very special. Number theoretical symmetries indeed fulfil this criterion.

2. In M^8 description space-time surfaces decompose to hyper-quaternionic and co-hyperquaternionic regions. The map assigning to $X^4 \subset M^8$ the image in $X^4 \subset H$ must be a isometry and also preserve the induced Kähler form so that the Kähler action has same value in the two spaces. The isometry groups of E^4 and CP_2 are different, and the interpretation is that the low energy description of hadrons in terms of $SO(4)$ symmetry and high energy description in terms of $SU(3)$ gauge group reflect this duality.

3. Number theoretic compactification implies very detailed conjectures about the preferred extremals of Kähler action implying dual slicings of the M^4 projection of space-time surface to string world sheets Y^2 and partonic 2-surfaces X^2 for Minkowskian signature of induced metric. This occurs for the known extremals of Kähler action of this kind [K7, K74, K111]. These slicings allow to understand how Equivalence Principle emerges via its stringy variant in TGD framework through dimensional reduction. The tangent spaces of Y^2 and X^2 define local planes of physical and un-physical polarizations and M^2 defines also the plane for the four-momentum assignable to the braid strand so that gauge symmetries are purely number theoretical interpretation.

4. Also a slicing of $X^4(X_l^3)$ to light-like 3-surfaces Y_l^3 parallel to X_l^3 giving equivalent space-time representations of partonic dynamics is predicted. This implies holography meaning an effective reduction of space-like 3-surfaces to 2-D surfaces. Number theoretical compactification leads also to a dramatic progress in the construction of quantum TGD in terms of the second quantized induced spinor fields. The holography seems however to be not quite simple as one might think first. Kac-Moody symmetries respecting the light-likeness of X_l^3 and leaving X^2 fixed act as gauge transformations and all light-like 3-surfaces with fixed ends and related by Kac-Moody symmetries would be geometrically equivalent in the sense that WCW Kähler metric is identical for them. These transformations would also act as zero modes of Kähler action.

5. A physically attractive realization of the braids - and more generally- of slicings of space-time surface by 3-surfaces and string world sheets, is discussed in [K35] by starting from the observation that TGD defines an almost topological QFT of braids, braid cobordisms, and 2-knots. The boundaries of the string world sheets at the space-like 3-surfaces at boundaries of CDs and wormhole throats would define space-like and time-like braids uniquely.

   The idea relies on a rather direct translation of the notions of singular surfaces and surface operators used in gauge theory approach to knots [A51] to TGD framework. It leads to the identification of slicing by three-surfaces as that induced by the inverse images of $r = constant$ surfaces of CP_2, where r is $U(2)$ invariant radial coordinate of CP_2 playing the role of Higgs field vacuum expectation value in gauge theories. $r = \infty$ surfaces correspond to geodesic spheres and define analogs of fractionally magnetically charged Dirac strings identifiable as preferred string world sheets. The union of these sheets labelled by subgroups $U(2) \subset SU(3)$ would define the slicing of space-time surface by string world sheets. The choice of $U(2)$ relates directly to the choice of quantization axes for color quantum numbers characterizing CD and would have the choice of braids and string world sheets as a space-time correlate.

WCW spinor structure

The construction of WCW ("world of classical worlds", configuration space) spinor structure in terms of second quantized induced spinor fields is certainly the most important step made hitherto towards explicit formulas for M-matrix elements.

1. Number theoretical compactification ($M^8 - H$ duality) states that space-time surfaces can be equivalently regarded as 4-dimensional surfaces of either $H = M^4 \times CP_2$ or of 8-D Minkowski space M^8, and consisting of hyper-quaternionic and co-hyper-quaternionic regions identified as regions with Minkowskian and Euclidian signatures of induced metric. Duality preserves induced metric and Kähler form. This duality poses very strong constraints on the geometry of the preferred extremals of Kähler action implying dual slicings of the space-time surface by string worlds sheets and partonic 2-surfaces as also by light-like 1-surfaces and light-like 3-surfaces. These predictions are consistent what is known about the extremals of Kähler action. The predictions of number theoretical compactification lead to dramatic progress in the construction of configurations space spinor structure and geometry.

2. The construction of WCW geometry and spinor structure in terms of induced spinor fields leads to the conclusion that finite measurement resolution is an intrinsic property of quantum states basically due to the vacuum degeneracy of Kähler action. This gives a justification for the notion of number theoretic braid effectively replacing light-like 3-surfaces. Hence the

infinite-dimensional WCW is replaced with a finite-dimensional space $(\delta M^4_\pm \times CP_2)^n/S_n$. A possible interpretation is that the finite fermionic oscillator algebra for given partonic 2-surface X^2 represents the factor space $\mathcal{M}/\mathcal{N}$ identifiable as quantum variant of Clifford algebra. $(\delta M^4_\pm \times CP_2)^n/S_n$ would represent its bosonic analog.

3. The isometries of the WCW corresponds to X^2 local symplectic transformations $\delta M^4_\pm \times CP_2$ depending only on the value of the invariant $\epsilon^{\mu\nu} J_{\mu\nu}$, where $J_{\mu\nu}$ can correspond to the Kähler form induced from $\delta M^4_\pm$ or CP_2. This group parameterizes quantum fluctuating degrees of freedom. Zero modes correspond to coordinates which cannot be made complex, in particular to the values of the induced symplectic form which thus behaves as a classical field so that WCW allows a slicing by the classical field patterns $J_{\mu\nu}(x)$ representing zero modes.

4. By the effective 2-dimensionality of light-like 3-surfaces X^3_l (holography) the interiors of light-like 3-surfaces are analogous to gauge degrees of freedom and partially parameterized by Kac-Moody group respecting the light-likeness of 3-surfaces. Quantum classical correspondence suggests that gauge fixing in Kac-Moody degrees of freedom takes place and implies correlation between the quantum numbers of the physical state and X^3_l or equivalently any light-like 3-surface Y^3_l parallel to X^3_l. There would be no path integral over X^3_l and only functional integral defined by WCW geometry over partonic 2-surfaces.

5. The condition that the Noether currents assignable to the modified Dirac equation are conserved requires that space-time surfaces correspond to extremals for which second variation of Kähler action vanishes. A milder condition is that the rank of the matrix defined by the second variation of Kähler action is less than maximal. Preferred extremals of Kähler action can be identified as this kind of 4-surface and the interpretation is in terms of quantum criticality.

   For given preferred extremal one expects the existence of an infinite number of deformations with a vanishing second variation of Kähler action. These deformations act as conformal gauge symmetries realizing quantum criticality at space-time level. The natural assumption is that the number, call it n, of conformal gauge equivalence classes of space-time surfaces with fixed 3-surfaces at their ends at the boundaries of CD is finite. This integer would characterize the effective value of Planck constant $h_{eff} = n \times h$.

6. The physically most transparent formulation of criticality as a hierarchy of broken super-symplectic conformal symmetries emerged rather recently. Super-symplectic algebra has an infinite fractal hierarchy of isomorphic sub-algebras with conformal weights coming as multiple of integer n for a given sub-algebra. The natural hypothesis is that the sub-algebra labelled by n acts as a conformal gauge algebra. This gives rise to infinite number of hierarchies of super-symplectic breakings labelled by sequences of integers $n_{i+1} = \prod_{k<i+1} m_k$. In a given symmetry breaking criticality is reduced as gauge degrees of freedom transform to physical ones. At quantum level the gauge sub-algebra labelled by n annihilates the physical states. At space-time level the corresponding super-symplectic Noether charges vanish. This defines precisely what it means to be a preferred extremal in zero energy ontology (ZEO).

Hierarchy of Planck constants

The hierarchy of Planck constants realized as a replacement of CD and CP_2 of $CD \times CP_2$ with book like structures labeled by finite subgroups of $SU(2)$ assignable to Jones inclusions is now relatively well understood as also its connection to dark matter, charge fractionization, and anyons [K22, K55].

1. This notion leads also to a unique identification of number theoretical braids as intersections of CD (CP_2) projection of X^3_l and the back M^2 (the backs S^2_I and S^2_{II}) of M^4 (CP_2) book. The spheres S^2_I and $S^2_{II}$are geodesic spheres of CP_2 orthogonal to each other).

2. The formulation of M-matrix should involve the local data from the points of number theoretic braids at partonic 2-surfaces. This data involves information about tangent space of $X^4(X^3)$ so that the theory does not reduce to 2-D theory. The hierarchy of CDs within CDs means that the improvement of measurement resolution brings in new CDs with smaller size.

3. The points of number theoretical braids are by definition quantum critical with respect to the phase transitions changing Planck constant and meaning leakage between different pages of the books in question. This quantum criticality need not be equivalent with the quantum criticality in the sense of the degeneracy of the matrix like entity defined by the second variation of Kähler action. Note that the entire partonic 2-surface at the boundary of CD cannot be quantum critical unless it corresponds to vacuum state with only topological degrees of freedom excited (that is have as its CD (CP_2) projection at the back of CD (CP_2) book or both) since Planck constant would be ill-defined in this kind of situation.

Super-conformal symmetries

The attempts to understand super-conformal symmetries has been unavoidably a guess work and produced several alternative scenarios. The consistency with p-adic mass calculations requiring five tensor factors to Super-Virasoro algebra has been the basic experimental constraint. The work with Kähler-Dirac equation has helped dramatically in the attempts to understand of super-conformal symmetries. Also the understanding of Super-Kac-Moody symmetries acting as gauge symmetries and made possible by the non-determinism of Kähler action has helped a lot.

There have been a considerable progress also in the understanding of super-conformal symmetries [K88, K14].

1. Super-symplectic algebra corresponds to the isometries of WCW constructed in terms covariantly constant right handed neutrino mode and second quantized induced spinor field Ψ and the corresponding Super-Kac-Moody algebra restricted to symplectic isometries and realized in terms of all spinor modes and Ψ is the most plausible identification of the superconformal algebras when the constraints from p-adic mass calculations are taken into account. These algebras act as dynamical rather than gauge algebras and related to the isometries of WCW .

2. One expects also gauge symmetries due to the non-determinism of Kähler action. They transform to each other preferred extremals having fixed 3-surfaces as ends at the boundaries of the causal diamond. They preserve the value of Kähler action and those of conserved charges. The assumption is that there are n gauge equivalence classes of these surfaces and that n defines the value of the effective Planck constant $h_{eff} = n \times h$ in the effective GRT type description replacing many-sheeted space-time with single sheeted one.

3. An interesting question is whether the symplectic isometries of $\delta M^4_\pm \times CP_2$ should be extended to include all isometries of $\delta M^4_\pm = S^2 \times R_+$ in one-one correspondence with conformal transformations of S^2.The S^2 local scaling of the light-like radial coordinate r_M of R_+ compensates the conformal scaling of the metric coming from the conformal transformation of S^2. Also light-like 3-surfaces allow the analogs of these isometries.

4. A further step of progress relates to the understanding of the fusion rules of symplectic field theory [K10]. These fusion rules makes sense only if one allows discretization that is number theoretic braids. An infinite hierarchy of symplectic fusion algebras can be identified with nice number theoretic properties (only roots of unity appear in structure constants). Hence there are good hopes that symplecto-conformal N-point functions defining the vertices of generalized Feynman diagrams can be constructed exactly.

5. The possible reduction of the fermionic Clifford algebra to a finite-dimensional one means that super-conformal algebras must have a cutoff in conformal weights. These algebras must reduce to finite dimensional ones and the replacement of integers with finite field is what comes first in mind.

6. The conserved fermionic currents implied by vanishing second variations of Kähler action for preferred extremal define a hierarchy of super-conformal algebras assignable to zero modes. These currents are appear in the expression of measurement interactions added to the Kähler-Dirac action in order to obtain stringy propagators and the coding of super-conformal quantum numbers to space-time geometry.

9.2.2 Various Inputs To The Construction Of M-Matrix

It is perhaps wise to summarize briefly the vision about M-matrix.

Zero energy ontology and interpretation of light-like 3-surfaces as generalized Feynman diagrams

1. Zero energy ontology is the cornerstone of the construction. Zero energy states have vanishing net quantum numbers and consist of positive and negative energy parts, which can be thought of as being localized at the boundaries of light-like 3-surface X_l^3 connecting the light-like boundaries of a causal diamond CD identified as intersection of future and past directed light-cones. There is entire hierarchy of CDs, whose scales are suggested to come as powers of 2. A more general proposal is that prime powers of fundamental size scale are possible and would conform with the most general form of p-adic length scale hypothesis. The hierarchy of size scales assignable to CDs corresponds to a hierarchy of length scales and code for a hierarchy of radiative corrections to generalized Feynman diagrams.

2. Light-like 3-surfaces are the basic dynamical objects of quantum TGD and have interpretation as generalized Feynman diagrams having light-like 3-surfaces as lines glued together along their ends defining vertices as 2-surfaces. By effective 2-dimensionality (holography) of light-like 3-surfaces the interiors of light-like 3-surfaces are analogous to gauge degrees of freedom and partially parameterized by Kac-Moody group respecting the light-likeness of 3-surfaces. This picture differs dramatically from that of string models since light-like 3-surfaces replacing stringy diagrams are singular as manifolds whereas 2-surfaces representing vertices are not.

Identification of TGD counterpart of S-matrix as time-like entanglement coefficients

1. The TGD counterpart of S-matrix -call it M-matrix- defines time-like entanglement coefficients between positive and negative energy parts of zero energy state located at the light-like boundaries of CD. One can also assign to quantum jump between zero energy states a matrix- call it U-matrix - which is unitary and assumed to be expressible in terms of M-matrices. M-matrix need not be unitary unlike the U-matrix characterizing the unitary process forming part of quantum jump. There are several good arguments suggesting that that M-matrix cannot be unitary but can be regarded as thermal S-matrix so that thermodynamics would become an essential part of quantum theory. In fact, M-matrix can be decomposed to a product of positive diagonal matrix identifiable as square root of density matrix and unitary matrix so that quantum theory would be kind of square root of thermodynamics. Path integral formalism is given up although functional integral over the 3-surfaces is present.

2. In the general case only thermal M-matrix defines a normalizable zero energy state so that thermodynamics becomes part of quantum theory. One can assign to M-matrix a complex parameter whose real part has interpretation as interaction time and imaginary part as the inverse temperature.

Hyper-finite factors and M-matrix

HFFs of type III_1 provide a general vision about M-matrix.

1. The factors of type III allow unique modular automorphism Δ^{it} (fixed apart from unitary inner automorphism). This raises the question whether the modular automorphism could be used to define the M-matrix of quantum TGD. This is not the case as is obvious already from the fact that unitary time evolution is not a sensible concept in zero energy ontology.

2. Concerning the identification of M-matrix the notion of state as it is used in theory of factors is a more appropriate starting point than the notion modular automorphism but as a generalization of thermodynamical state is certainly not enough for the purposes of quantum TGD and quantum field theories (algebraic quantum field theorists might disagree!). Zero energy ontology requires that the notion of thermodynamical state should be replaced with its "complex square root" abstracting the idea about M-matrix as a product of positive

square root of a diagonal density matrix and a unitary S-matrix. This generalization of thermodynamical state -if it exists- would provide a firm mathematical basis for the notion of M-matrix and for the fuzzy notion of path integral.

3. The existence of the modular automorphisms relies on Tomita-Takesaki theorem, which assumes that the Hilbert space in which HFF acts allows cyclic and separable vector serving as ground state for both HFF and its commutant. The translation to the language of physicists states that the vacuum is a tensor product of two vacua annihilated by annihilation oscillator type algebra elements of HFF and creation operator type algebra elements of its commutant isomorphic to it. Note however that these algebras commute so that the two algebras are not hermitian conjugates of each other. This kind of situation is exactly what emerges in zero energy ontology: the two vacua can be assigned with the positive and negative energy parts of the zero energy states entangled by M-matrix.

4. There exists infinite number of thermodynamical states related by modular automorphisms. This must be true also for their possibly existing "complex square roots". Physically they would correspond to different measurement interactions giving rise to Kähler functions of WCW differing only by a real part of holomorphic function of complex coordinates of WCW and arbitrary function of zero mode coordinates and giving rise to the same Kähler metric of WCW .

Connes tensor product as a realization of finite measurement resolution

The inclusions $\mathcal{N} \subset \mathcal{M}$ of factors allow an attractive mathematical description of finite measurement resolution in terms of Connes tensor product but do not fix M-matrix as was the original optimistic belief.

1. In zero energy ontology $\mathcal{N}$ would create states experimentally indistinguishable from the original one. Therefore $\mathcal{N}$ takes the role of complex numbers in non-commutative quantum theory. The space $\mathcal{M}/\mathcal{N}$ would correspond to the operators creating physical states modulo measurement resolution and has typically fractal dimension given as the index of the inclusion. The corresponding spinor spaces have an identification as quantum spaces with non-commutative $\mathcal{N}$-valued coordinates.

2. This leads to an elegant description of finite measurement resolution. Suppose that a universal M-matrix describing the situation for an ideal measurement resolution exists as the idea about square root of state encourages to think. Finite measurement resolution forces to replace the probabilities defined by the M-matrix with their $\mathcal{N}$ "averaged" counterparts. The "averaging" would be in terms of the complex square root of $\mathcal{N}$-state and a direct analog of functionally or path integral over the degrees of freedom below measurement resolution defined by (say) length scale cutoff.

3. One can construct also directly M-matrices satisfying the measurement resolution constraint. The condition that $\mathcal{N}$ acts like complex numbers on M-matrix elements as far as $\mathcal{N}$-"averaged" probabilities are considered is satisfied if M-matrix is a tensor product of M-matrix in $\mathcal{M}(\mathcal{N}$ interpreted as finite-dimensional space with a projection operator to $\mathcal{N}$. The condition that $\mathcal{N}$ averaging in terms of a complex square root of $\mathcal{N}$ state produces this kind of M-matrix poses a very strong constraint on M-matrix if it is assumed to be universal (apart from variants corresponding to different measurement interactions).

Conformal symmetries and stringy diagrammatics

The Kähler-Dirac equation has rich super-conformal symmetries helping to achieve concrete vision about the structure of M-matrix in terms of generalized Feynman diagrammatics.

Both super-conformal symmetries and the effective reduction of space-time sheet to string world sheets at Minkowskian regions as a consequence of finite measurement resolution suggest that the generalized Feynman diagrams have as vertices N-point functions of a conformal field theory assignable to the partonic 2-surfaces at which the lines of Feynman diagram meet. The vertices can be assigned with wormhole contacts with Euclidian signture of induced metric. In

Minkowskian regions fundamental fermions propagate like massless particles along boundaries of string world sheets. One can say that a hybrid of Feynman and stringy diagrammatics results.

Finite measurement resolution means that this conformal theory is defined in the discrete set defined by the intersections of braids defined by boundaries of string worlds sheets with partonic two-surfaces. The presence of symplectic invariants in turn suggest a symplectic variant of conformal field theory leading to a concrete construction of symplectic fusion rules relying in crucial manner to discretization.

TGD as almost topological QFT

The idea that TGD could be regarded as almost topological QFT has been very fruitful although the hypothesis that Chern-Simons term for induced Kähler gauge potential assignable to light-like 3-surfaces identified as regions of space-time where the Euclidian signature of induced metric assignable to the interior or generalized Feynman diagram changes to Minkowskian one turned out to be too strong. The reduction of WCW and its Clifford algebra to finite dimensional structures due to finite measurement resolution however realizes this idea but in different manner.

1. There is functional integral over the small deformations of Feynman cobordisms corresponding to the maxima of Kähler function which is finite-dimensional if finite measurement resolution is taken into account. Almost topological QFT property of quantum suggests the identification of M-matrix as a functor from the category of generalized Feynman cobordisms (generalized Feynman diagrams) to the category of operators mapping the Hilbert space of positive energy states to that for negative energy states: these Hilbert spaces are assignable to partonic 2-surfaces.

2. The limit at which momenta vanish is well-defined for M-matrix since the Kähler-Dirac action contains measurement interaction term and at this limit one indeed obtains topological QFT.

3. Almost TQFT property suggests that braiding S-matrices should have important role in the construction. It is indeed possible to assign the with the lines of the generalized Feynman diagram. The reduction of quantum TGD to topological QFT should occur at quantum criticality with respect to the change of Planck constant since in this situation the M-matrix should not depend at all on Planck constant. Factoring QFTs in 1+1 dimensions give examples of this kind of theories.

Heuristic picture about generalized Feynman rules

Concerning the understanding of the relationship between HFFs and M-matrix the basic implications are following.

1. General visions do not allow to provide explicit expressions for M-matrix elements. Therefore one must be humble and try to feed in all understanding about quantum TGD and from the quantum field theoretic picture. In particular, the dependence of M-matrix on Planck constant should be such that the addition of loop corrections as sub- CDs corresponds to an expansion in powers of $1/\hbar$ as in quantum field theory whereas for tree diagrams there is no dependence on $\hbar$.

2. The vacuum degeneracy of Kähler action and the identification of Kähler function as Dirac determinant strongly suggest that fermionic oscillator operators define what could be interpreted as a finite quantum-dimensional Clifford algebra identifiable as a factor space $\mathcal{M}/\mathcal{N}$, $\mathcal{N} \subset \mathcal{M}$. One must be however very cautious since also an alternative option in which excitations of labeled by conformal weight are present cannot be excluded. Finite-dimensionality would mean an enormous simplification, and together with the unique identification of number theoretic braids as orbits of the end points of string world sheets this means that the dynamics is finite-quantum-dimensional conforming with the fact effective finite-dimensionality is the defining property of HFFs. Physical states would realize finite measurement resolution in their structure so that approximation would cease to be an approximation.

3. An interesting question is whether this means that M-matrix must be replaced with quantum M-matrix with operator valued matrix elements and whether the probabilities should be determined by taking traces of these operators having interpretation as averaging over $\mathcal{N}$ defining the degrees of freedom below measurement resolution. This kind of picture would conform with the basic properties of HFFs.

4. To the strands of number theoretic braids one would attach fermionic propagators. Since bosons correspond to fermion pairs at the throats of wormhole contact, all propagators reduce to fermionic ones. As found, the addition of measurement interaction term fixes fermionic propagator completely and gives it a stringy character.

5. Similar correlation function in WCW degrees of freedom would be given in lowest order - and perhaps exact - approximation in terms of the contravariant metric of the configuration space proportional to g_K^2. Besides this the exponent of Kähler action would be involved. For elementary particles it would be the exponent of Kähler action for CP_2 type vacuum extremal. In this manner something combinatorially very similar to standard perturbation theory would result and there are excellent hopes that p-adic coupling constant evolution in powers of 2 is consistent with the standard coupling constant evolution.

6. Vertices correspond to n-point functions. The contribution depending on fermionic fields defines the quantum number dependent part of the vertices and comes from the fermion field and their conjugats attached to the ends of propagator lines identified as braid strands. Besides this there is a symplecto-conformal contribution to the vertex.

7. The stringy variant of twistor Grassmannian approach is highly suggestive since the necessary conditions are satisfied. In particular, the fundamental fermions propagate in the internal lines effectively as massless on-mass shell states but with non-physical polarization. M^4 *resp.*CP_2 is the unique 4-D manifold *resp.* compact manifold with Minkowskian *resp.* Euclidian signature of metric allowing twistor space with Kähler structure. This suggests that a generalization of twistorialization to 8-D context makes sense. The twistor space for CP_2 is 6-dimensional flag manifold $SU(3)/U(1) \times U(1)$ parameterizing the choice of color quantization axes and has popped up earlier in TGD inspired theory of consciousness.

The expansion of M-matrix in powers of $\hbar$

One should understand how the proportionality of gauge couplings to g_K^2 emerges and how loops give rise to powers of α_K. In zero energy ontology one does not calculate M-matrix but tries to construct zero energy state in the hope that QFT wisdom yields cold help to construct Connes tensor product correctly.

1. The basic rule of quantum field theory is that each loop gives $\alpha = g^2/4\pi$ and thus $1/\hbar$ factor whereas in tree diagrams only g^2 appears so that they correspond to the semiclassical approximation.

2. This rule is obtained if one assumes loops correspond to a hierarchy of sub- CDs and that in loop one can distinguish one line as "base line" and other lines as radiative corrections. To each internal line one must one must assign the factor $r^{-1/2} = (\hbar_0/\hbar)^{1/2}$ and factor g_K^2 except to the portion of base line appearing in loop since otherwise double counting would result. This dictates the expansion of M-matrix in powers of $r^{-1/2}$. It would not be too surprising to have this kind of expansion.

3. g_K^2 factor comes from the functional integral over the partonic 2-surface selected by stationary phase approximation using the exponent of Kähler action. The functional integral over the WCW degrees of freedom is carried out using contravariant Kähler metric as a propagator and this gives g_K^2 factor in the lowest non-trivial order since one must develop a perturbation theory with respect to the deformations at the partonic 2-surfaces at the ends of line.

   If the analogs of radiative corrections to this functional integral vanish - as suggested by quantum criticality and required by number theoretic universality - the resulting dependence on g_K^2 is exact and completely analogous to the free field theory propagator. The numerical factors give the appropriate gauge coupling squared.

4. Besides this one must assign to the ends of the propagator line positive and negative energy parts of quantum state representing the particle in question. These give a contribution which is zeroth order in $\hbar$. For instance, gauge bosons correspond to fermionic bilinears. Essentially fermion currents formed from spinor fields at the two light-like wormhole throats of the wormhole contact at which the signature of the induced metric changes are in question. Correct dimension requires the presence of $1/\hbar$ factor in boson state and $1/\sqrt{\hbar}$ factor in fermion state. The correlators between fermionic fields at the end points of the line are proportional to $\hbar$ so that normalization factors cancel the $\hbar$ dependence. Besides this one would expect N-points function of symplecto-conformal QFT with $N = N_{in} + N_{out}$ having no dependence on $\hbar$.

9.2.3 But What About The Concrete Feynman Rules?

The skeptic reader can say that all this is just an endless list of general principles. I dare however claim that the only manner to proceed is to try to identify the general principles first. At this moment the understanding of the fundamental variational principled of TGD understood at such level of detail that one can indeed sketch a rather concrete formulation for the generalized Feynman rules. The generalized Feynman diagrams correspond to the 4-D surfaces defined by the Euclidian regions defined by wormhole contacts plus the string world sheets connecting them and carrying spinor modes. One might also talk about combination of Feynman diagrams and stringy diagrams or even about generalization of Wilson loops. The lines of these diagrams form also braids.

1. The boundaries of string world sheets at which the modes of induced spinor field are localized (by well-definedness of em charge) carry fermion number and are identifiable as braid strands within partonic orbits at which the signature of the induced metric changes from Minkowskian to Euclidian. 1-D Dirac action for induced metric and its bosonic counterpart - must be assigned with partonic orbits in order to obtain non-trivial fermionic propagator. Massles fermion propagator emerges if light-like portions of string world sheet boundary contain 1-D Dirac action in induced metric. The bosonic part of this action implied by supersymmetry implies that light-like geodesic of imbedding space is in question and there is a conserved light-like four-momentum associated with the fermion line.

2. The fundamental interaction is the scattering of fermions at opposite wormhole throats of wormhole contact. With string model based intuition one can argue that this interaction must correspond essentially to the stringy propagator $1/L_0$ so that one would obtain a combination of Feynman rules and stringy rules. The vertices correspond topologically to a fusion of 4-D lines along the 3-surfaces at their ends and this means deviation from string model picture: stringy diagrams correspond at topological level to what happens when particle travels between A and B along two different routes and has nothing to do with particle decay.

   One can criticize this idea about ad hoc character. Furthermore, super-symmetry requires also the presence of super-generator G and its hermitian conjugate. In TGD however these operators carry baryon or lepton number and cannot appear as propagators unless they appear as pairs $GG^{dagger} \propto L_0$.

   The vision about scattering amplitudes as sequences of algebraic operations with 3-vertices identified as product and co-products in super-sympelectic Yangian of super-symplectic algebra looks much more feasible option [K76].

3. Physical particles are bound states of massless fundamental fermions and correspond to pairs of wormhole contacts: a pair is required since wormhole throats behave effectively as magnetic monopoles and closed flux tube consisting of pieces at the two space-time sheets and wormhole contacts is required. This resolves the infrared difficulties of twistor approach. Twistor Grassmann approach strongly suggests that the residue integral over the virtual four-momenta reduces the propagators of fundamental fermions to their inverses at mass-shell so that only non-physical fermion helicities appear as virtual fermions.

The reader wishing for a brief summary of TGD might find the three articles about TGD, TGD inspired theory of consciousness, and TGD based view about quantum biology helpful [L6, L5, L4].

9.3 How To Define Generalized Feynman Diagrams?

S-matrix codes to a high degree the predictions of quantum theories. The longstanding challenge of TGD has been to construct or at least demonstrate the mathematical existence of S-matrix- or actually M-matrix which generalizes this notion in ZEO (ZEO) [K63] . This work has led to the notion of generalized Feynman diagram and the challenge is to give a precise mathematical meaning for this object. The attempt to understand the counterpart of twistors in TGD framework [K76] has inspired several key ideas in this respect but it turned out that twistors themselves need not be absolutely necessary in TGD framework.

1. The notion of generalized Feyman diagram defined by replacing lines of ordinary Feynman diagram with light-like 3-surfaces (elementary particle sized wormhole contacts with throats carrying quantum numbers) and vertices identified as their 2-D ends - I call them partonic 2-surfaces is central. Speaking somewhat loosely, generalized Feynman diagrams (plus background space-time sheets) define the "world of classical worlds" (WCW). These diagrams involve the analogs of stringy diagrams but the interpretation is different: the analogs of stringy loop diagrams have interpretation in terms of particle propagating via two different routes simultaneously (as in the classical double slit experiment) rather than as a decay of particle to two particles. For stringy diagrams the counterparts of vertices are singular as manifolds whereas the entire diagrams are smooth. For generalized Feynman diagrams vertices are smooth but entire diagrams represent singular manifolds just like ordinary Feynman diagrams do. String like objects however emerge in TGD and even ordinary elementary particles are predicted to be magnetic flux tubes of length of order weak gauge boson Compton length with monopoles at their ends as shown in accompanying article. This stringy character should become visible at LHC energies.

2. ZEO (ZEO) and causal diamonds (intersections of future and past directed light-cones) define second key ingredient. The crucial observation is that in ZEO it is possible to identify off mass shell particles as pairs of on mass shell fermions at throats of wormhole contact since both positive and negative signs of energy are possible and one obtains also space-like total momenta for wormhole contact behaving as a boson. The localization of fermions to string world sheets and the fact that super-conformal generator G carries fermion number combined with twistorial consideration support the view that the propagators at fermionic lines are of form $(1/G)ip^k\gamma_k(1/G^\dagger + h.c.$ and thus hermitian. In strong models $1/G$ would serve as a propagator and this requires Majorana condition fixing the dimension of the target space to 10 or 11.

3. A powerful constraint is number theoretic universality requiring the existence of Feynman amplitudes in all number fields when one allows suitable algebraic extensions: roots of unity are certainly required in order to realize p-adic counterparts of plane waves. Also imbedding space, partonic 2-surfaces and WCW must exist in all number fields and their extensions. These constraints are enormously powerful and the attempts to realize this vision have dominated quantum TGD for last two decades.

4. Representation of 8-D gamma matrices in terms of octonionic units and 2-D sigma matrices is a further important element as far as twistors are considered [K76] . Kähler-Dirac gamma matrices at space-time surfaces are quaternionic/associative and allow a genuine matrix representation. As a matter fact, TGD and WCW could be formulated as study of associative local sub-algebras of the local Clifford algebra of 8-D imbedding space parameterized by quaternionic space-time surfaces.

5. A central conjecture has been that associative (co-associative) 4-surfaces correspond to preferred extremals of Kähler action [K88]. It took long time to realize that in ZEO the notion of preferred extremal might be un-necessary! The reason is that 3-surfaces are now pairs of

3-surfaces at boundaries of causal diamonds and for deterministic dynamics the space-time surface connecting them is expected to be more or less unique. Now the action principle is non-deterministic but the non-determinism would give rise to additional discrete dynamical degrees of freedom naturally assignable to the hierarchy of Planck constants $h_{eff} = n \times h$, n the number of space-time surface with same fixed ends at boundaries of CD and with same values of Kähler action and of conserved quantities. One must be however cautions: this leaves the possibility that there is a gauge symmetry present so that the n sheets correspond to gauge equivalence classes of sheets. Conformal invariance is associated with criticality and is expected to be present also now.

One can of course also ask whether one can assume that the pairs of 3-surfaces at the ends of CD are totally un-correlated. If this assumption is not made then preferred extremal property would make sense also in ZEO and imply additional correlation between the members of these pairs. This kind of correlations would correspond to the Bohr orbit property, which is very attractive space-time correlate for quantum states. This kind of correlates are also expected as space-time counterpart for the correlations between initial and final state in quantum dynamics.

6. A further conjecture has been that preferred extremals are in some sense critical (second variation of Kähler action could vanish for infinite number of deformations defining a super-conformal algebra). The non-determinism of Kähler action implies this property for $n > 0$ in $h_{eff} = nh$. If the criticality is present, it could correspond to conformal gauge invariance defined by sub-algebras of conformal algebra with conformal weights coming as multiples of n and isomorphic to the conformal algebra itself.

7. As far as twistors are considered, the first key element is the reduction of the octonionic twistor structure to quaternionic one at space-time surfaces and giving effectively 4-D spinor and twistor structure for quaternionic surfaces.

Quite recently quite a dramatic progress took place in this approach [K88, K76] .

1. The progress was stimulated by the simple observation that on mass shell property puts enormously strong kinematic restrictions on the loop integrations. With mild restrictions on the number of parallel fermion lines appearing in vertices (there can be several since fermionic oscillator operator algebra defining SUSY algebra generates the parton states)- all loops are manifestly finite and if particles has always mass -say small p-adic thermal mass also in case of massless particles and due to IR cutoff due to the presence largest CD- the number of diagrams is finite. Unitarity reduces to Cutkosky rules [B70] automatically satisfied as in the case of ordinary Feynman diagrams.

2. Ironically, twistors which stimulated all these development do not seem to be absolutely necessary in this approach although they are of course possible. Situation changes if one does not assume small p-adically thermal mass due to the presence of massless particles and one must sum infinite number of diagrams. Here a potential problem is whether the infinite sum respects the algebraic extension in question.

This is about fermionic and momentum space aspects of Feynman diagrams but not yet about the functional (not path-) integral over small deformations of the partonic 2-surfaces. The basic challenges are following.

1. One should perform the functional integral over WCW degrees of freedom for fixed values of on mass shell momenta appearing in the internal lines. After this one must perform integral or summation over loop momenta. Note that the order is important since the space-time surface assigned to the line carries information about the quantum numbers associated with the line by quantum classical correspondence realized in terms of Kähler-Dirac operator.

2. One must define the functional integral also in the p-adic context. p-Adic Fourier analysis relying on algebraic continuation raises hopes in this respect. p-Adicity suggests strongly that the loop momenta are discretized and ZEO predicts this kind of discretization naturally.

It indeed seems that the functional integrals over WCW could be carried out at general level both in real and p-adic context. This is due to the symmetric space property (maximal number of isometries) of WCW required by the mere mathematical existence of Kähler geometry [K34] in infinite-dimensional context already in the case of much simpler loop spaces [A48] .

1. The p-adic generalization of Fourier analysis allows to algebraize integration- the horrible looking technical challenge of p-adic physics- for symmetric spaces for functions allowing the analog of discrete Fourier decomposition. Symmetric space property is indeed essential also for the existence of Kähler geometry for infinite-D spaces as was learned already from the case of loop spaces. Plane waves and exponential functions expressible as roots of unity and powers of p multiplied by the direct analogs of corresponding exponent functions are the basic building bricks and key functions in harmonic analysis in symmetric spaces. The physically unavoidable finite measurement resolution corresponds to algebraically unavoidable finite algebraic dimension of algebraic extension of p-adics (at least some roots of unity are needed). The cutoff in roots of unity is very reminiscent to that occurring for the representations of quantum groups and is certainly very closely related to these as also to the inclusions of hyper-finite factors of type II_1 defining the finite measurement resolution.

2. WCW geometrization reduces to that for a single line of the generalized Feynman diagram defining the basic building brick for WCW. Kähler function decomposes to a sum of "kinetic" terms associated with its ends and interaction term associated with the line itself. p-Adicization boils down to the condition that Kähler function, matrix elements of Kähler form, WCW Hamiltonians and their super counterparts, are rational functions of complex WCW coordinates just as they are for those symmetric spaces that I know of. This would allow a continuation to p-adic context.

In the following this vision about generalized Feynman diagrams is discussed in more detail.

9.3.1 Questions

The goal is a proposal for how to perform the integral over WCW for generalized Feynman digrams and the best manner to proceed to to this goal is by making questions.

What does finite measurement resolution mean?

The first question is what finite measurement resolution means.

1. One expects that the algebraic continuation makes sense only for a finite measurement resolution in which case one obtains only finite sums of what one might hope to be algebraic functions. The finiteness of the algebraic extension would be in fact equivalent with the finite measurement resolution.

2. Finite measurement resolution means a discretization in terms of number theoretic braids. p-Adicization condition suggests that that one must allow only the number theoretic braids. For these the ends of braid at boundary of CD are algebraic points of the imbedding space. This would be true at least in the intersection of real and p-adic worlds.

3. The question is whether one can localize the points of the braid. The necessity to use momentum eigenstates to achieve quantum classical correspondence in the Kähler-Dirac action [K88] suggests however a de-localization of braid points, that is wave function in space of braid points. In real context one could allow all possible choices for braid points but in p-adic context only algebraic points are possible if one wants to replace integrals with sums. This implies finite measurement resolution analogous to that in lattice. This is also the only possibility in the intersection of real and p-adic worlds.

   A non-trivial prediction giving a strong correlation between the geometry of the partonic 2-surface and quantum numbers is that the total number $n_F + n_{\overline{F}}$ of fermions and anti-fermions is bounded above by the number n_{alg} of algebraic points for a given partonic 2-surface: $n_F + n_{\overline{F}} \leq n_{alg}$. Outside the intersection of real and p-adic worlds the problematic aspect of this definition is that small deformations of the partonic 2-surface can radically change the number of algebraic points unless one assumes that the finite measurement resolution means restriction of WCW to a sub-space of algebraic partonic surfaces.

4. Braids defining propagator lines for fundamental fermions (to be distinguished from observer particles) emerges naturally. Braid strands correspond to the boundaries of string world sheets at which the modes of induced spinor fields are localized from the condition that em charge is well-defined: induced W field and above weak scale also Z^0 field vanish at them.

   In order to obtain non-trivial fermion propagator one must add to Dirac action 1-D Dirac action in induced metric with the boundaries of string world sheets at the light-like parton orbits. Its bosonic counterpart is line-length in induced metric. Field equations imply that the boundaries are light-like geodesics and fermion has light-like 8-momentum. This suggests strongly a connection with quantum field theory and an 8-D generalization of twistor Grassmannian approach. By field equations the bosonic part of this action does not contribute to the Kähler action. The light-like 8-momenta p^k have same M^4 and CP_2 mass squared and latter correspond to the the eigenvalues of the CP_2 spinor d'Alembertian by quantum-classical correspondence.

5. One has also discretization of the relative position of the second tip of CD at the hyperboloid isometric with mass shell. Only the number of braid points and their momenta would matter, not their positions.

6. The quantum numbers characterizing positive and negative energy parts of zero energy states couple directly to space-time geometry via the measurement interaction terms in Kähler action expressing the equality of classical conserved charges in Cartan algebra with their quantal counterparts for space-time surfaces in quantum superposition. This makes sense if classical charges parametrize zero modes. The localization in zero modes in state function reduction would be the WCW counterpart of state function collapse.

How to define integration in WCW degrees of freedom?

The basic question is how to define the integration over WCW degrees of freedom.

1. What comes mind first is Gaussian perturbation theory around the maxima of Kähler function. Gaussian and metric determinants cancel each other and only algebraic expressions remain. Finiteness is not a problem since the Kähler function is non-local functional of 3-surface so that no local interaction vertices are present. One should however assume the vanishing of loops required also by algebraic universality and this assumption look unrealistic when one considers more general functional integrals than that of vacuum functional since free field theory is not in question. The construction of the inverse of the WCW metric defining the propagator is also a very difficult challenge. Duistermaat-Hecke theorem states that something like this known as localization might be possible and one can also argue that something analogous to localization results from a generalization of mean value theorem.

2. Symmetric space property is more promising since it might reduce the integrations to group theory using the generalization of Fourier analysis for group representations so that there would be no need for perturbation theory in the proposed sense. In finite measurement resolution the symmetric spaces involved would be finite-dimensional. Symmetric space structure of WCW could also allow to define p-adic integration in terms of p-adic Fourier analysis for symmetric spaces. Essentially algebraic continuation of the integration from the real case would be in question with additional constraints coming from the fact that only phase factors corresponding to finite algebraic extensions of rationals are used. Cutoff would emerge automatically from the cutoff for the dimension of the algebraic extension.

How to define generalized Feynman diagrams?

Integration in symmetric spaces could serve as a model at the level of WCW and allow both the understanding of WCW integration and p-adicization as algebraic continuation. In order to get a more realistic view about the problem one must define more precisely what the calculation of the generalized Feynman diagrams means.

1. WCW integration must be carried out separately for all values of the momenta associated with the internal lines. The reason is that the spectrum of eigenvalues λ_i of the Kähler-Dirac operator D depends on the momentum of line and momentum conservation in vertices translates to a correlation of the spectra of D at internal lines.

2. For tree diagrams algebraic continuation to the p-adic context if the expression involves only the replacement of the generalized eigenvalues of D as functions of momenta with their p-adic counterparts besides vertices. If these functions are algebraically universal and expressible in terms of harmonics of symmetric space , there should be no problems.

3. If loops are involved, one must integrate/sum over loop momenta. In p-adic context difficulties are encountered if the spectrum of the momenta is continuous. The integration over on mass shell loop momenta is analogous to the integration over sub-CDs, which suggests that internal line corresponds to a $sub - CD$ in which it is at rest. There are excellent reasons to believe that the moduli space for the positions of the upper tip is a discrete subset of hyperboloid of future light-cone. If this is the case, the loop integration indeed reduces to a sum over discrete positions of the tip. p-Adizication would thus give a further good reason why for ZEO.

4. Propagator is expressible in terms of the inverse of generalized eigenvalue and there is a sum over these for each propagator line. At vertices one has products of WCW harmonics assignable to the incoming lines. The product must have vanishing quantum numbers associated with the phase angle variables of WCW. Non-trivial quantum numbers of the WCW harmonic correspond to WCW quantum numbers assignable to excitations of ordinary elementary particles. WCW harmonics are products of functions depending on the "radial" coordinates and phase factors and the integral over the angles leaves the product of the first ones analogous to Legendre polynomials $P_{l,m}$, These functions are expected to be rational functions or at least algebraic functions involving only square roots.

5. In ordinary QFT incoming and outgoing lines correspond to propagator poles. In the recent case this would mean that incoming stringy lines at the ends of CD correspond to fermions satisfying the stringy mass formula serving as a generalization of masslessness condition.

9.3.2 Generalized Feynman Diagrams At Fermionic And Momentum SpaceLevel

Negative energy ontology has already led to the idea of interpreting the virtual particles as pairs of positive and negative energy wormhole throats. Hitherto I have taken it as granted that ordinary Feynman diagrammatics generalizes more or less as such. It is however far from clear what really happens in the verties of the generalized Feynman diagrams. The safest approach relies on the requirement that unitarity realized in terms of Cutkosky rules in ordinary Feynman diagrammatics allows a generalization. This requires loop diagrams. In particular, photon-photon scattering can take place only via a fermionic square loop so that it seems that loops must be present at least in the topological sense.

One must be however ready for the possibility that something unexpectedly simple might emerge. For instance, the vision about algebraic physics allows naturally only finite sums for diagrams and does not favor infinite perturbative expansions. Hence the true believer on algebraic physics might dream about finite number of diagrams for a given reaction type. For simplicity generalized Feynman diagrams without the complications brought by the magnetic confinement since by the previous arguments the generalization need not bring in anything essentially new.

The basic idea of duality in early hadronic models was that the lines of the dual diagram representing particles are only re-arranged in the vertices. This however does not allow to get rid of off mass shell momenta. ZEO encourages to consider a stronger form of this principle in the sense that the virtual momenta of particles could correspond to pairs of on mass shell momenta of particles. If also interacting fermions are pairs of positive and negative energy throats in the interaction region the idea about reducing the construction of Feynman diagrams to some kind of lego rules might work.

Virtual particles as pairs of on mass shell particles in ZEO

The first thing is to try to define more precisely what generalized Feynman diagrams are. The direct generalization of Feynman diagrams implies that both wormhole throats and wormhole contacts join at vertices.

1. A simple intuitive picture about what happens is provided by diagrams obtained by replacing the points of Feynman diagrams (wormhole contacts) with short lines and imagining that the throats correspond to the ends of the line. At vertices where the lines meet the incoming on mass shell quantum numbers would sum up to zero. This approach leads to a straightforward generalization of Feynman diagrams with virtual particles replaced with pairs of on mass shell throat states of type $++$, $--$, and $+-$. Incoming lines correspond to $++$ type lines and outgoing ones to $--$ type lines. The first two line pairs allow only time like net momenta whereas $+-$ line pairs allow also space-like virtual momenta. The sign assigned to a given throat is dictated by the sign of the on mass shell momentum on the line. The condition that Cutkosky rules generalize as such requires $++$ and $--$ type virtual lines since the cut of the diagram in Cutkosky rules corresponds to on mass shell outgoing or incoming states and must therefore correspond to $++$ or $--$ type lines.

2. The basic difference as compared to the ordinary Feynman diagrammatics is that loop integrals are integrals over mass shell momenta and that all throats carry on mass shell momenta. In each vertex of the loop mass incoming on mass shell momenta must sum up to on mass shell momentum. These constraints improve the behavior of loop integrals dramatically and give excellent hopes about finiteness. It does not however seem that only a finite number of diagrams contribute to the scattering amplitude besides tree diagrams. The point is that if a the reactions $N_1 \rightarrow N_2$ and $N_2 \rightarrow N_3$,, where N_i denote particle numbers, are possible in a common kinematical region for N_2-particle states then also the diagrams $N_1 \rightarrow N_2 \rightarrow N_2 \rightarrow N_3$ are possible. The virtual states N_2 include all all states in the intersection of kinematically allow regions for $N_1 \rightarrow N_2$ and $N_2 \rightarrow N_3$. Hence the dream about finite number possible diagrams is not fulfilled if one allows massless particles. If all particles are massive then the particle number N_2 for given N_1 is limited from above and the dream is realized.

3. For instance, loops are not possible in the massless case or are highly singular (bringing in mind twistor diagrams) since the conservation laws at vertices imply that the momenta are parallel. In the massive case and allowing mass spectrum the situation is not so simple. As a first example one can consider a loop with three vertices and thus three internal lines. Three on mass shell conditions are present so that the four-momentum can vary in 1-D subspace only. For a loop involving four vertices there are four internal lines and four mass shell conditions so that loop integrals would reduce to discrete sums. Loops involving more than four vertices are expected to be impossible.

4. The proposed replacement of the elementary fermions with bound states of elementary fermions and monopoles $X_\pm$ brings in the analog of stringy diagrammatics. The 2-particle wave functions in the momentum degrees of freedom of fermion and $X_\pm$ might allow more flexibility and allow more loops. Note however that there are excellent hopes about the finiteness of the theory also in this case.

Loop integrals are manifestly finite

One can make also more detailed observations about loops.

1. The simplest situation is obtained if only 3-vertices are allowed. In this case conservation of momentum however allows only collinear momenta although the signs of energy need not be the same. Particle creation and annihilation is possible and momentum exchange is possible but is always light-like in the massless case. The scattering matrices of supersymmetric YM theories would suggest something less trivial and this raises the question whether something is missing. Magnetic monopoles are an essential element of also these theories as also massivation and symmetry breaking and this encourages to think that the formation of massive states as fermion $X_\pm$ pairs is needed. Of course, in TGD framework one has also high mass excitations of the massless states making the scattering matrix non-trivial.

2. In YM theories on mass shell lines would be singular. In TGD framework this is not the case since the propagator is defined as the inverse of the 3-D dimensional reduction of the Kähler-Dirac operator D containing also coupling to four-momentum (this is required by quantum classical correspondence and guarantees stringy propagators),

$$\begin{aligned} D &= i\hat{\Gamma}^{\alpha}p_{\alpha} + \hat{\Gamma}^{\alpha}D_{\alpha} \ , \\ p_{\alpha} &= p_k\partial_{\alpha}h^k \ . \end{aligned} \tag{9.3.1}$$

   The propagator does not diverge for on mass shell massless momenta and the propagator lines are well-defined. This is of course of essential importance also in general case. Only for the incoming lines one can consider the possibility that 3-D Dirac operator annihilates the induced spinor fields. All lines correspond to generalized eigenstates of the propagator in the sense that one has $D_3\Psi = \lambda\gamma\Psi$, where γ is Kähler-Dirac gamma matrix in the direction of the stringy coordinate emanating from light-like surface and D_3 is the 3-dimensional dimensional reduction of the 4-D Kähler-Dirac operator. The eigenvalue λ is analogous to energy. Note that the eigenvalue spectrum depends on 4-momentum as a parameter.

3. Massless incoming momenta can decay to massless momenta with both signs of energy. The integration measure $d^2k/2E$ reduces to dx/x where $x \geq 0$ is the scaling factor of massless momentum. Only light-like momentum exchanges are however possible and scattering matrix is essentially trivial. The loop integrals are finite apart from the possible delicacies related to poles since the loop integrands for given massless wormhole contact are proportional to dx/x^3 for large values of x.

4. Irrespective of whether the particles are massless or not, the divergences are obtained only if one allows too high vertices as self energy loops for which the number of momentum degrees of freedom is $3N-4$ for N-vertex. The construction of SUSY limit of TGD in [K24] led to the conclusion that the parallelly propagating N fermions for given wormhole throat correspond to a product of N fermion propagators with same four-momentum so that for fermions and ordinary bosons one has the standard behavior but for $N > 2$ non-standard so that these excitations are not seen as ordinary particles. Higher vertices are finite only if the total number N_F of fermions propagating in the loop satisfies $N_F > 3N-4$. For instance, a 4-vertex from which $N = 2$ states emanate is finite.

Taking into account magnetic confinement

What has been said above is not quite enough. The weak form of electric-magnetic duality [B5] leads to the picture about elementary particles as pairs of magnetic monopoles inspiring the notions of weak confinement based on magnetic monopole force. Also color confinement would have magnetic counterpart. This means that elementary particles would behave like string like objects in weak boson length scale. Therefore one must also consider the stringy case with wormhole throats replaced with fermion-$X_{\pm}$ pairs ($X_{\pm}$ is electromagnetically neutral and $\pm$ refers to the sign of the weak isospin opposite to that of fermion) and their super partners.

1. The simplest assumption in the stringy case is that fermion-$X_{\pm}$ pairs behave as coherent objects, that is scatter elastically. In more general case only their higher excitations identifiable in terms of stringy degrees of freedom would be created in vertices. The massivation of these states makes possible non-collinear vertices. An open question is how the massivation fermion-$X_{\pm}$ pairs relates to the existing TGD based description of massivation in terms of Higgs mechanism and Kähler-Dirac operator.

2. Mass renormalization could come from self energy loops with negative energy lines as also vertex normalization. By very general arguments supersymmetry implies the cancellation of the self energy loops but would allow non-trivial vertex renormalization [K24] .

3. If only 3-vertices are allowed, the loops containing only positive energy lines are possible if on mass shell fermion-$X_\pm$ pair (or its superpartner) can decay to a pair of positive energy pair particles of same kind. Whether this is possible depends on the masses involved. For ordinary particles these decays are not kinematically possible below intermediate boson mass scale (the decays $F_1 \rightarrow F_2 + \gamma$ are forbidden kinematically or by the absence of flavor changing neutral currents whereas intermediate gauge bosons can decay to on mass shell fermion-anti-fermion pair).

4. The introduction of IR cutoff for 3-momentum in the rest system associated with the largest CD (causal diamond) looks natural as scale parameter of coupling constant evolution and p-adic length scale hypothesis favors the inverse of the size scale of CD coming in powers of two. This parameter would define the momentum resolution as a discrete parameter of the p-adic coupling constant evolution. This scale does not have any counterpart in standard physics. For electron, d quark, and u quark the proper time distance between the tips of CD corresponds to frequency of 10 Hz, 1280 Hz, and 160 Hz: all these frequencies define fundamental bio-rhythms [K18] .

These considerations have left completely untouched one important aspect of generalized Feynman diagrams: the necessity to perform a functional integral over the deformations of the partonic 2-surfaces at the ends of the lines- that is integration over WCW. Number theoretical universality requires that WCW and these integrals make sense also p-adically and in the following these aspects of generalized Feynman diagrams are discussed.

9.3.3 Harmonic Analysis In WCW As A Manner To Calculate WCW-Functional Integrals

Previous examples suggest that symmetric space property, Kähler and symplectic structure and the use of symplectic coordinates consisting of canonically conjugate pairs of phase angles and corresponding "radial" coordinates are essential for WCW integration and p-adicization. Kähler function, the components of the metric, and therefore also metric determinant and Kähler function depend on the "radial" coordinates only and the possible generalization involves the identification the counterparts of the "radial" coordinates in the case of WCW.

Conditions guaranteeing the reduction to harmonic analysis

The basic idea is that harmonic analysis in symmetric space allows to calculate the functional integral over WCW.

1. Each propagator line corresponds to a symmetric space defined as a coset space G/H of the symplectic group and Kac-Moody group and one might hope that the proposed p-adicization works for it- at least when one considers the hierarchy of measurement resolutions forced by the finiteness of algebraic extensions. This coset space is as a manifold Cartesian product $(G/H) \times (G/H)$ of symmetric spaces G/H associated with ends of the line. Kähler metric contains also an interaction term between the factors of the Cartesian product so that Kähler function can be said to reduce to a sum of "kinetic" terms and interaction term.

2. Effective 2-dimensionality and ZEO allow to treat the ends of the propagator line independently. This means an enormous simplification. Each line contributes besides propagator a piece to the exponent of Kähler action identifiable as interaction term in action and depending on the propagator momentum. This contribution should be expressible in terms of generalized spherical harmonics. Essentially a sum over the products of pairs of harmonics associated with the ends of the line multiplied by coefficients analogous to $1/(p^2 - m^2)$ in the case of the ordinary propagator would be in question. The optimal situation is that the pairs are harmonics and their conjugates appear so that one has invariance under G analogous to momentum conservation for the lines of ordinary Feynman diagrams.

3. Momentum conservation correlates the eigenvalue spectra of the Kähler-Dirac operator D at propagator lines [K88] . G-invariance at vertex dictates the vertex as the singlet part of the product of WCW harmonics associated with the vertex and one sums over the harmonics for

each internal line. p-Adicization means only the algebraic continuation to real formulas to p-adic context.

4. The exponent of Kähler function depends on both ends of the line and this means that the geometries at the ends are correlated in the sense that that Kähler form contains interaction terms between the line ends. It is however not quite clear whether it contains separate "kinetic" or self interaction terms assignable to the line ends. For Kähler function the kinetic and interaction terms should have the following general expressions as functions of complex WCW coordinates:

$$\begin{aligned} K_{kin,i} &= \sum_n f_{i,n}(Z_i)\overline{f_{i,n}(Z_i)} + c.c \ , \\ K_{int} &= \sum_n g_{1,n}(Z_1)\overline{g_{2,n}(Z_2)} + c.c \ , i = 1,2 \ . \end{aligned} \quad (9.3.2)$$

Here $K_{kin,i}$ define "kinetic" terms and K_{int} defines interaction term. One would have what might be called holomorphic factorization suggesting a connection with conformal field theories.

Symmetric space property -that is isometry invariance- suggests that one has

$$f_{i,n} = f_{2,n} \equiv f_n \ , \ g_{1,n} = g_{2,n} \equiv g_n \quad (9.3.3)$$

such that the products are invariant under the group H appearing in G/H and therefore have opposite H quantum numbers. The exponent of Kähler function does not factorize although the terms in its Taylor expansion factorize to products whose factors are products of holomorphic and antiholomorphic functions.

5. If one assumes that the exponent of Kähler function reduces to a product of eigenvalues of the Kähler-Dirac operator eigenvalues must have the decomposition

$$\lambda_k = \prod_{i=1,2} exp\left[\sum_n c_{k,n} g_n(Z_i)\overline{g_n(Z_i)} + c.c\right] \times exp\left[\sum_n d_{k,n} g_n(Z_1)\overline{g_n(Z_2)} + c.c\right] \quad (9.3.4)$$

Hence also the eigenvalues coming from the Dirac propagators have also expansion in terms of G/H harmonics so that in principle WCW integration would reduce to Fourier analysis in symmetric space.

Generalization of WCW Hamiltonians

This picture requires a generalization of the view about configuration space Hamiltonians since also the interaction term between the ends of the line is present not taken into account in the previous approach.

1. The proposed representation of WCW Hamiltonians as flux Hamiltonians [K15, K88]

$$\begin{aligned} Q(H_A) &= \int H_A(1+K)Jd^2x \ , \\ J &= \epsilon^{\alpha\beta}J_{\alpha\beta} \ , \ J^{03}\sqrt{g_4} = KJ_{12} \ . \end{aligned} \quad (9.3.5)$$

works for the kinetic terms only since J cannot be the same at the ends of the line. The formula defining K assumes weak form of self-duality (^{03} refers to the coordinates in the complement of X^2 tangent plane in the 4-D tangent plane). K is assumed to be symplectic invariant and constant for given X^2. The condition that the flux of $F^{03} = (\hbar/g_K)J^{03}$ defining the counterpart of Kähler electric field equals to the Kähler charge g_K gives the condition $K = g_K^2/\hbar$, where g_K is Kähler coupling constant. Within experimental uncertainties one has $\alpha_K = g_K^/ 4\pi\hbar_0 = \alpha_{em} \simeq 1/137$, where α_{em} is finite structure constant in electron length scale and $\hbar_0$ is the standard value of Planck constant.

The assumption that Poisson bracket of WCW Hamiltonians reduces to the level of imbedding space - in other words $\{Q(H_A), Q(H_B\} = Q(\{H_A, H_B\})$ - can be justified. One starts from the representation in terms of say flux Hamiltonians $Q(H_A)$ and *defines* $J_{A,B}$ as $J_{A,B} \equiv Q(\{H_A, H_B\})$. One has $\partial H_A/\partial t_B = \{H_B, H_A\}$, where t_B is the parameter associated with the exponentiation of H_B. The inverse J^{AB} of $J_{A,B} = \partial H_B/\partial t_A$ is expressible as $J^{A,B} = \partial t_A/\partial H_B$. From these formulas one can deduce by using chain rule that the bracket $\{Q(H_A), Q(H_B\} = \partial t_C Q(H_A) J^{CD} \partial t_D Q(H_B)$ of flux Hamiltonians equals to the flux Hamiltonian $Q(\{H_A, H_B\})$.

2. One should be able to assign to WCW Hamiltonians also a part corresponding to the interaction term. The symplectic conjugation associated with the interaction term permutes the WCW coordinates assignable to the ends of the line. One should reduce this apparently non-local symplectic conjugation (if one thinks the ends of line as separate objects) to a non-local symplectic conjugation for $\delta CD \times CP_2$ by identifying the points of lower and upper end of CD related by time reflection and assuming that conjugation corresponds to time reflection. Formally this gives a well defined generalization of the local Poisson brackets between time reflected points at the boundaries of CD. The connection of Hermitian conjugation and time reflection in quantum field theories is is in accordance with this picture.

3. The only manner to proceed is to assign to the flux Hamiltonian also a part obtained by the replacement of the flux integral over X^2 with an integral over the projection of X^2 to a sphere S^2 assignable to the light-cone boundary or to a geodesic sphere of CP_2, which come as two varieties corresponding to homologically trivial and non-trivial spheres. The projection is defined as by the geodesic line orthogonal to S^2 and going through the point of X^2. The hierarchy of Planck constants assigns to CD a preferred geodesic sphere of CP_2 as well as a unique sphere S^2 as a sphere for which the radial coordinate r_M or the light-cone boundary defined uniquely is constant: this radial coordinate corresponds to spherical coordinate in the rest system defined by the time-like vector connecting the tips of CD. Either spheres or possibly both of them could be relevant.

   Recall that also the construction of number theoretic braids and symplectic QFT [K13] led to the proposal that braid diagrams and symplectic triangulations could be defined in terms of projections of braid strands to one of these spheres. One could also consider a weakening for the condition that the points of the number theoretic braid are algebraic by requiring only that the S^2 coordinates of the projection are algebraic and that these coordinates correspond to the discretization of S^2 in terms of the phase angles associated with θ and ϕ.

   This gives for the corresponding contribution of the WCW Hamiltonian the expression

$$Q(H_A)_{int} = \int_{S^2_\pm} H_A X \delta^2(s_+, s_-) d^2 s_\pm = \int_{P(X^2_+)\cap P(X^2_-)} \frac{\partial(s^1, s^2)}{\partial(x^1_\pm, x^2_\pm)} d^2 x_\pm \ . \tag{9.3.6}$$

   Here the Poisson brackets between ends of the line using the rules involve delta function $\delta^2(s_+, s_-)$ at S^2 and the resulting Hamiltonians can be expressed as a similar integral of $H_{[A,B]}$ over the upper or lower end since the integral is over the intersection of S^2 projections.

   The expression must vanish when the induced Kähler form vanishes for either end. This is achieved by identifying the scalar X in the following manner:

$$\begin{aligned} X &= J_+^{kl} J_{kl}^- \ , \\ J_\pm^{kl} &= (1 + K_\pm)\partial_\alpha s^k \partial_\beta s^l J_\pm^{\alpha\beta} \ . \end{aligned} \tag{9.3.7}$$

The tensors are lifts of the induced Kähler form of $X^2_\pm$ to S^2 (not CP_2).

4. One could of course ask why these Hamiltonians could not contribute also to the kinetic terms and why the brackets with flux Hamiltonians should vanish. This relate to how one *defines* the Kähler form. It was shown above that in case of flux Hamiltonians the definition of Kähler form as brackets gives the basic formula $\{Q(H_A), Q(H_B)\} = Q(\{H_A, H_B\}$ and same should hold true now. In the recent case $J_{A,B}$ would contain an interaction term defined in terms of flux Hamiltonians and the previous argument should go through also now by identifying Hamiltonians as sums of two contributions and by introducing the doubling of the coordinates t_A.

5. The quantization of the Kähler-Dirac operator must be reconsidered. It would seem that one must add to the super-Hamiltonian completely analogous term obtained by replacing $(1+K)J$ with $X\partial(s^1,s^2)/\partial(x^1_\pm,x^2_\pm)$. Besides the anti-commutation relations defining correct anti-commutators to flux Hamiltonians, one should pose anti-commutation relations consistent with the anti-commutation relations of super Hamiltonians. In these anti-commutation relations $(1+K)J\delta^2(x,y)$ would be replaced with $X\delta^2(s^+,s^-)$. This would guarantee that the oscillator operators at the ends of the line are not independent and that the resulting Hamiltonian reduces to integral over either end for $H_{[A,B]}$.

6. In the case of CP_2 the Hamiltonians generating isometries are rational functions. This should hold true also now so that p-adic variants of Hamiltonians as functions in WCW would make sense. This in turn would imply that the components of the WCW Kähler form are rational functions. Also the exponentiation of Hamiltonians make sense p-adically if one allows the exponents of group parameters to be functions $Exp_p(t)$.

Does the expansion in terms of partial harmonics converge?

The individual terms in the partial wave expansion seem to be finite but it is not at all clear whether the expansion in powers of K actually converges.

1. In the proposed scenario one performs the expansion of the vacuum functional $exp(K)$ in powers of K and therefore in negative powers of α_K. In principle an infinite number of terms can be present. This is analogous to the perturbative expansion based on using magnetic monopoles as basic objects whereas the expansion using the contravariant Kähler metric as a propagator would be in positive powers of α_K and analogous to the expansion in terms of magnetically bound states of wormhole throats with vanishing net value of magnetic charge. At this moment one can only suggest various approaches to how one could understand the situation.

2. Weak form of self-duality and magnetic confinement could change the situation. Performing the perturbation around magnetic flux tubes together with the assumed slicing of the space-time sheet by stringy world sheets and partonic 2-surfaces could mean that the perturbation corresponds to the action assignable to the electric part of Kähler form proportional to α_K by the weak self-duality. Hence by $K = 4\pi\alpha_K$ relating Kähler electric field to Kähler magnetic field the expansion would come in powers of a term containing sum of terms proportional to α_K^0 and α_K. This would leave to the scattering amplitudes the exponents of Kähler function at the maximum of Kähler function so that the non-analytic dependence on α_K would not disappear.

A further reason to be worried about is that the expansion containing infinite number of terms proportional to α_K^0 could fail to converge.

1. This could be also seen as a reason for why magnetic singlets are unavoidable except perhaps for $\hbar < \hbar_0$. By the holomorphic factorization the powers of the interaction part of Kähler action in powers of $1/\alpha_K$ would naturally correspond to increasing and opposite net values of the quantum numbers assignable to the WCW phase coordinates at the ends of the propagator line. The magnetic bound states could have similar expansion in powers of α_K as pairs of states with arbitrarily high but opposite values of quantum numbers. In the functional

integral these quantum numbers would compensate each other. The functional integral would leave only an expansion containing powers of α_K starting from some finite possibly negative (unless one assumes the weak form of self-duality) power. Various gauge coupling strengths are expected to be proportional to α_K and these expansions should reduce to those in powers of α_K.

2. Since the number of terms in the fermionic propagator expansion is finite, one might hope on basis of super-symmetry that the same is true in the case of the functional integral expansion. By the holomorphic factorization the expansion in powers of K means the appearance of terms with increasingly higher quantum numbers. Quantum number conservation at vertices would leave only a finite number of terms to tree diagrams. In the case of loop diagrams pairs of particles with opposite and arbitrarily high values of quantum numbers could be generated at the vertex and magnetic confinement might be necessary to guarantee the convergence. Also super-symmetry could imply cancellations in loops.

Could one do without flux Hamiltonians?

The fact that the Kähler functions associated with the propagator lines can be regarded as interaction terms inspires the question whether the Kähler function could contain only the interaction terms so that Kähler form and Kähler metric would have components only between the ends of the lines.

1. The basic objection is that flux Hamiltonians too beautiful objects to be left without any role in the theory. One could also argue that the WCW metric would not be positive definite if only the non-diagonal interaction term is present. The simplest example is Hermitian 2×2-matrix with vanishing diagonal for which eigenvalues are real but of opposite sign.

2. One could of course argue that the expansions of $exp(K)$ and λ_k give in the general powers $(f_n\overline{f_n})^m$ analogous to diverging tadpole diagrams of quantum field theories due to local interaction vertices. These terms do not produce divergences now but the possibility that the exponential series of this kind of terms could diverge cannot be excluded. The absence of the kinetic terms would allow to get rid of these terms and might be argued to be the symmetric space counterpart for the vanishing of loops in WCW integral.

3. In ZEO this idea does not look completely non-sensical since physical states are pairs of positive and negative energy states. Note also that in quantum theory only creation operators are used to create positive energy states. The manifest non-locality of the interaction terms and absence of the counterparts of kinetic terms would provide a trivial manner to get rid of infinities due to the presence of local interactions. The safest option is however to keep both terms.

Summary

The discussion suggests that one must treat the entire Feynman graph as single geometric object with Kähler geometry in which the symmetric space is defined as product of what could be regarded as analogs of symmetric spaces with interaction terms of the metric coming from the propagator lines. The exponent of Kähler function would be the product of exponents associated with all lines and contributions to lines depend on quantum numbers (momentum and color quantum numbers) propagating in line via the coupling to the Kähler-Dirac operator. The conformal factorization would allow the reduction of integrations to Fourier analysis in symmetric space. What is of decisive importance is that the entire Feynman diagrammatics at WCW level would reduce to the construction of WCW geometry for a single propagator line as a function of quantum numbers propagating on the line.

9.4 A More Detailed View About The Construction Of Scattering Amplitudes

The following represents an update view about construction of scattering amplitudes at the level of "world of classical worlds" (WCW).

9.4.1 Basic Principles

In order to facilitate the challenge of the reader I summarize basic ideas behind the construction of scattering amplitudes.

Construction of scattering amplitudes as functional integrals in WCW

The decomposition of space-time surface to Minkowskian and Eucldian regions is the basic distinction from ordinary quantum field theories since it replaces path integral with mathematically well-defined functional integral over WCW .

1. Space-time surface decomposes to regions with Minkowskian or Euclidian signature of the induced metric. The regions with Euclidian metric are identified as lines of generalized Feynman diagrams. The boundaries between two kinds of regions - to be called parton orbits - can be regarded as carriers of elementary particle quantum numbers such as fermion number assignable to the boundaries of string world sheets at them. Induced spinor fields are localized at them from the well-definedness of electromagnetic charge requiring that induced W boson fields vanish. Hence strings emerge from TGD. Note that at boundary between Euclidian and Minkowskian regions the metric determinant vanishes. Unlike the name would suggest, generalized Feynman diagrams are analogous to twistor diagrams, and instead of infinite number of superposed diagrams there might just single diagram.

2. Weak form of electric magnetic duality together with the assumption that the term $j^\alpha A_\alpha$ in Kähler action vanishes imply that Kähler action reduces to 3-D Chern-Simons term. This hypothesis is inspired by TGD as almost topological quantum field theory conjecture. In Minkowskian regions this conjecture is very natural. In the Euclidian region the contribution to Kähler action need not reduce to a mere Chern-Simons term associated with its boundary. This would be due to the non-triviality of the U(1) bundle defined by Kähler form giving also Chern-Simons terms inside the CP_2 type vacuum extremal.

3. Scattering amplitude is a functional integral over space-time surfaces: the data about these space-time surfaces are coded by their ends about the opposite light-like boundaries of causal diamond (CD) of given scale. The weight function in the functional integral is exponential of Kähler function of "world of classical worlds" coming from Euclidian regions of the space-time surface representing lines of generalized Feynman diagram and being deformation of CP_2 type vacuum extremals representing wormhole contacts connecting two space-time sheets with Minkowskian signature of induced metric. Kähler function is the exponent of Kähler action from Euclidian regions. The real exponent takes care that the functional integral is obtained instead of path integral so that the outcome is mathematically well-defined.

4. Euclidian region would give only the analog of thermodynamics but there is also an imaginary exponential coming from the exponential of the imaginary Kähler action from Minkowskian regions. Space-time surfaces are extremals of Kähler action and for very general ansatz Minkowskian contribution to Kähler action reduces to imaginary Chern-Simons term at the light-like 3-D boundary between regions at which the 4-D metric is degenerate. This term makes possible interference of different contributions to the functional integral which is absolutely essential in quantum field theory.

5. The details of the theory in fermionic sector have turned out to be crucial. From the well-definedness of the electric charge for the modes of the induced spinor field - and also by number theoretic arguments - spinor modes are localized at 2-D string world sheets carrying vanishing W gauge fields. Preferred extremals can be constructed by fixing first partonic 2-surfaces, string world sheets, and possibly also the light-like orbits of partonic 2-surfaces and

posing the condition that the canonical momentum densities have no components normal to string world sheets. Also the condition that a sub-algebra of super-symplectic algebra gives rise to vanishing Noether charges at the space-like ends of preferred extremal is natural.

This construction would conform with the strong form of holography. The boundaries of string world sheets at the light-like orbits of partonic 2-surfaces carry 1-D Dirac action for induced gamma matrices. The bosonic counterpart of this action gives as solutions light-like geodesics of imbedding space - light-likeness in 8-D sense. 1-D Dirac equation for induced gamma matrices is satisfied. A very twistorial picture emerges and suggests 8-D generalization of twistor approach. M^4 and CP_2 are indeed twistorially completely unique.

6. The generators of super-symplectic algebra can be represented as Noether charges for the fermionic strings and the supercharges identifiable as WCW gamma matrices are natural identification for fermionic oscillator operators. Since one expects that a given partonic 2-surface is connected to a large number of partonic 2-surfaces a generalization to Yangian [A27] [B39, B30, B31] of super-symplectic algebra seems necessary and is in spirit with twistorialization. It seems possible to identify the fundamental vertices assignable to partonic 2-surfaces at which three lines of diagram meet in terms of product and co-product for Yangian so that there are hopes about realizing the already forgotten TGD inspired dream about reduction of scattering amplitudes to sequences of algebraic operations of Yangian with minimal length and connecting chosen initial and final states at the boundaries of CD. Universe would be Yangian algebraist!

So what one expects vertices and propagators to be? Fermionic propagators would be massless in 8-D sense and they should be contracted with the legs of the vertices defined by product tor co-product involving three Yangian generators. Structure constants would define the coupling constants. Each Yangian generator would involve a collection of fermions fields associated with strings and with each fermion field propagator would contract. The only modification of the ordinary vertex is that partonic 2-surfaces carry many-fermion states and the vertices involve 3 multi- fermion states. Fermion lines can also turn backwards in time: this gives rise to virtual bosons.

Why it might work?

There are many reasons encouraging the hopes about calculable theory.

1. The theory has huge super-conformal symmetries dramatically reducing the dynamical degrees of freedom by the choice of conformal gauge. This implies that both the space-like 3-surfaces at the ends of space-time surface and partonic orbits satisfy classical Super conformal conditions for generalizations of ordinary super-conformal algebras perhaps extending to multilocal Yangian with loci identified as strings connecting partonic 2-surfaces at the light-like boundary of CD. This algebra extends also to include both boundaries of CD. Fermionic anticommutation relations which allow by 2-dimensionality of string world sheet also quantum group variant determine the anticommutations between all generators.

   Yangian symmetry in turn gives excellent hopes about twistorialization: in fact, $M^4 \times CP_2$ is completely unique choice for the imbedding space by twistorial considerations and the product of the twistor spaces of M^4 and CP_2 allows to constructed the twistor spaces of space-time surfaces as liftings of the extremals of Kähler action to 6-D sphere bundles over space-time surface.

2. The integrand in the functional integral represents the analog of ordinary Feynman diagrams involving only fermions and 1-D lines. Indeed, by bosonic emergence all bosons (in fact all elementary particles) can be regarded as composites of fundamental fermions. The only purely fermionic vertices are 2-fermion vertices. 3-vertices correspond to space-time surfaces meeting along common 3-surface and are thus purely topological, and as already mentioned could correspond to product and co-product for Yangian. This is of course excellent news from the point of view of finiteness. The fermionic vertices are represented by the discontinuity of the Kähler-Dirac operator associated with the string boundary line at partonic 2-surface so that there are no coupling constants involved. The only fundamental coupling parameter

is Kähler strength whose value is dictated by quantum criticality as the analog of critical temperature.

One must have a view about what elementary particles - as opposed to fundamental fermions - are, how the ordinary view about scattering based on exchanges of elementary particles emerges from this picture and how say BFF vertex reduces to a diagram at for fundamental fermions involving only 2-fermion vertices.

9.4.2 Elementary Particles In TGD Framework

The notion of elementary particles involves two aspects: elementary particles as space-time surfaces and elementary particles as many-fermion states with fundamental fermions localized at the wormhole throats and defining elementary particles as their bound states (including physical fermions).

Let us first summarize what kind of picture ZEO suggests about elementary particles.

1. Kähler magnetically charged wormhole throats are the basic building bricks of elementary particles. The lines of generalized Feynman diagrams are identified as the Euclidian regions of space-time surface. The weak form of electric magnetic duality forces magnetic monopoles and gives classical quantization of the Kähler electric charge. Wormhole throat is a carrier of many-fermion state with parallel momenta and the fermionic oscillator algebra gives rise to a badly broken large $\mathcal{N}$ SUSY [K24].

2. The first guess would be that elementary fermions correspond to wormhole throats with unit fermion number and bosons to wormhole contacts carrying fermion and anti-fermion at opposite throats. The magnetic charges of wormhole throats do not however allow this option. The reason is that the field lines of Kähler magnetic monopole field must close. Both in the case of fermions and bosons one must have a pair of wormhole contacts (see **Fig.** `http://tgdtheory.fi/appfigures/wormholecontact.jpg` or **Fig. ??** in the appendix of this book) connected by flux tubes. The most general option is that net quantum numbers are distributed amongst the four wormhole throats. A simpler option is that quantum numbers are carried by the second wormhole: fermion quantum numbers would be carried by its second throat and bosonic quantum numbers by fermion and anti-fermion at the opposite throats. All elementary particles would therefore be accompanied by parallel flux tubes and string world sheets.

3. A cautious proposal in its original form was that the throats of the other wormhole contact could carry weak isospin represented in terms of neutrinos and neutralizing the weak isospin of the fermion at second end. This would imply weak neutrality and weak confinement above length scales longer than the length of the flux tube. This condition might be un-necessarily strong.

   The realization of the weak neutrality using pair of left handed neutrino and right handed antineutrino or a conjugate of this state is possible if one allows right-handed neutrino to have also unphysical helicity. The weak screening of a fermion at wormhole throat is possible if ν_R is a constant spinor since in this case Dirac equation trivializes and allows both helicities as solutions. The new element from the solution of the Kähler-Dirac equation is that ν_R would be interior mode de-localized either to the other wormhole contact or to the Minkowskian flux tube. The state at the other end of the flux tube is sparticle of left-handed neutrino.

   It must be emphasized that weak confinement is just a proposal and looks somewhat complex: Nature is perhaps not so complex at the basic level. To understand this better, one can think about how M_{89} mesons having quark and antiquark at the ends of long flux tube returning back along second space-time sheet could decay to ordinary quark and antiquark.

9.4.3 Scattering Amplitudes

The basic challenge is to introduce vertices and fermionic propagators. The recent based on stringy realization of Yangian algebra allows to do this.

Fermionic propagators

How fermionic propagators emerge? The first explanation coming in mind is based on the discontinuity associated with the Dirac operator at the partonic 2-surfaces defining vertices.

Discontinuities can be of two different types. Fermionic lines has discontinuous tangent at the partonic 2-surfaces meaning local non-conservation of light-like 8-momentum. Also second kind of discontinuity in which two lines belonging to orbits of distinct partonic 2-surfaces emerge at single point. Their 8-momenta need not be opposite if one requires only global momentum conservation. If it is assumed one can say that fermionic line turns backwards in time. These kind of pairs of lines forming closed curves with peaks at ends are associated with bosonic propagators-say those describing boson exchange between two fermions.

The discontinuities of the induced spinor along the fermionic line making a turn at the partonic 2-surface give rise to delta function singularities under the action of 1-D Dirac operator. This would give Dirac equation with a source term and its solution would be given by Dirac propagator convoluted with the discontinuity.

Vertices

Vertices can be considered at both space-time level and fermionic level.

1. At space-time level vertices correspond to the fusion of space-surfaces representing particles along common 3-surface defining the vertex. At the parton level 3-light-like parton orbits fuse together along partonic 2-surface. In these vertices particle number changes this change correspond the change of particle number for elementary particles.

2. At fermion level vertices are localized at the partonic 2-surfaces. The above argument would suggest that vertices corresponds to the discontinuity of the Kähler Dirac operator at the corner of the line representing the boundary of string world sheet. The creation of fermion pair from vacuum corresponds to an corner of string boundary at which the boundaries of string world sheets associated with two outgoing or incoming sheets meet. The creation/annihilation of a fermion pair is essential for the realization of say tree diagrams describing fermion scattering by virtual boson exchange.

The identification of vertex as a product or co-product in Yangian looks the most promising approach. The charges of the super-symplectic Yangian are associated with strings and are either linear or bilinear in the fermion field. The fermion fields associated with the partonic 2-surface defining the vertex are contracted with fermion fields associated with other partonic 2-surfaces using the same rule as in Wick expansion in quantum field theories. The contraction gives fermion propagator at each leg plus vertex factor. Vertex factor is proportional to the contraction of spinor modes with the operators defining the Noether charge or super charge - essentially Kähler-Dirac gamma matrix and the representation of the action of the symplectic generator on fermion realizable in terms of sigma matrices. This is very much like the corresponding expression in gauge theories but with gauge algebra replaced with symplectic algebra. The possibility of contractions of creation and annihilation operator for fermion lines associated with opposite wormfhole throats at the same partonic 2-surface (for Noether charge bilinear in fermion field) gives bosonic exchanges as lines in which the fermion lines turns in time direction: otherwise only regroupings of fermions would take place. One obtains integration of the light-like 8-momenta of fermions in natural manner and something resembling very strongly standard QFT. The integration interpreted as residue integral should give only inverse of the propagator actin on on mass shell states with wrong helicity. Virtual fermions would have wrong helicity unlikes incoming ones.

9.4.4 What One Should Obtain At QFT Limit?

After functional integration over WCW of one should obtain a scattering amplitude in which the fermionic 2- vertices defined as discontinuities of the Kähler-Dirac operator at partonic 2-surfaces should boil down to a contraction of an M^8 vector with gamma matrices of M^8. This vector has dimension of mass. This basic parameter should characterize many different physical situations. Consider only the description of massivation of elementary particles regarded as bound states of fundamental massless fermions and the mixing of left and right-handed fermions. Also CKM mixing should involve this parameter. These vectors should also appear in Higgs couplings, which in QFT description contain Higgs vacuum expectation as a factor.

In twistor approach virtual particles have complex light-like 8-momenta. Fundamental fermions have most naturally real and light-like momenta. $\mathcal{N} = 4$ SUSY describes gauge bosons which correspond to bound states of fundamental fermions in TGD. This suggests that the four-momenta of bound states of massless fermions - be they hadrons, leptons, or gauge bosons - can be taken to be complex.

There is an intriguing connection with TGD based notion of space-time. In TGD one obtains at space-time level complexified light-like 8-momenta since the 8-momentum from Minkowskian/Euclidian region is real/imaginary. In the case of physical particle necessary involving two wormhole contacts and two flux tubes connecting them the total complexifies four momentum would be sum of two real and two imaginary contributions. Every elementary particle should have also imaginary part in its 8-momentum and would be massless in complexified sense allowing mass in real sense given by the length of the imaginary four-momentum. In twistor approach complex light-like momenta indeed appear in BCFW bridge.

TGD predicts Higgs boson although Higgs expectation does not have any role in quantum TGD proper. Higgs vacuum expectation is however a necessary part of QFT limit (Higgs decays to WW pairs require that vacuum expectation is non-vanishing). Higgs vacuum expectation must correspond in TGD framework to a quantity with dimensions of mass. In TGD Higgs cannot be scalar but a vector in CP_2 degrees of freedom. The problem is that CP_2 does not allow covariantly constant vectors. The imaginary part of classical four-momentum gives a parameter which has interpretation as a vector in the tangent space of which is same as that of $M^4 \times CP_2$. Could $M^8 - H$ duality be realized at the level of tangent space and for relate four-momentum and color quantum numbers to the E^4 part of 8-momentum?

Elementary particles of course need not be eigenstates of the CP_2 part of 8-momentum. For a fixed mass one can have wave functions in the space of CP_3 part of 8-momentum analogous to S^3 spherical harmonics at the sphere of E^4 with radius defined by the length of imaginary four-momentum (mass). These harmonics are characterized by $SO(4)$ quantum numbers. Could one interpret this complexification in terms of M^8-duality and say that SO(4) defines the symmetries for the low energy dual of WCW defining high energy description of QCD based on SU(3) symmetry. SO(4) would corresponds to the symmetry group assigned to hadrons in the approach based on conserved vector currents and partially conserved axial currents. SO(4) would be much more general and associated also with leptons.

The anomalous color hyper-charge of leptonic spinors would imply that one can have also in the case of leptons a wave function in S^3. Higher harmonics would correspond to color excitations of leptons and quarks. If one considers gamma matrices, complexification of M^4 means introduction of gamma matrix algebra of complexified M^4 requiring 8 gamma matrices. This suggests a connection with $M^8 - H$ duality. All elementary particles have also imaginary part of four-momentum and the 8-momentum can be interpreted as M^8-momentum combining the four-momentum and color quantum numbers together.

Chapter 10

Does Riemann Zeta Code for Generic Coupling Constant Evolution?

10.1 Introduction

During years I have made several attempts to understand coupling evolution in TGD framework.

1. The first idea dates back to the discovery of WCW Kähler geometry defined by Kähler function defined by Kähler action (this happened around 1990) [K34]. The only free parameter of the theory is Kähler coupling strength α_K analogous to temperature parameter α_K postulated to be is analogous to critical temperature. Whether only single value or entire spectrum of of values α_K is possible, remained an open question.

   About decade ago I realized that Kähler action is *complex* receiving a real contribution from space-time regions of Euclidian signature of metric and imaginary contribution from the Minkoswkian regions. Euclidian region would give Kähler function and Minkowskian regions analog of QFT action of path integral approach defining also Morse function. Zero energy ontology (ZEO) [K105] led to the interpretation of quantum TGD as complex square root of thermodynamics so that the vacuum functional as exponent of Kähler action could be identified as a complex square root of the ordinary partition function. Kähler function would correspond to the real contribution Kähler action from Euclidian space-time regions. This led to ask whether also Kähler coupling strength might be complex: in analogy with the complexification of gauge coupling strength in theories allowing magnetic monopoles. Complex α_K could allow to explain CP breaking. I proposed that instanton term also reducing to Chern-Simons term could be behind CP breaking

2. p-Adic mass calculations for 2 decades ago [K39] inspired the idea that length scale evolution is discretized so that the real version of p-adic coupling constant would have discrete set of values labelled by p-adic primes. The simple working hypothesis was that Kähler coupling strength is renormalization group (RG) invariant and only the weak and color coupling strengths depend on the p-adic length scale. The alternative ad hoc hypothesis considered was that gravitational constant is RG invariant. I made several number theoretically motivated ad hoc guesses about coupling constant evolution, in particular a guess for the formula for gravitational coupling in terms of Kähler coupling strength, action for CP_2 type vacuum extremal, p-adic length scale as dimensional quantity [K3]. Needless to say these attempts were premature and a hoc.

3. The vision about hierarchy of Planck constants $h_{eff} = n \times h$ and the connection $h_{eff} = h_{gr} = GMm/v_0$, where $v_0 < c = 1$ has dimensions of velocity [K106] forced to consider very seriously the hypothesis that Kähler coupling strength has a spectrum of values in one-one correspondence with p-adic length scales. A separate coupling constant evolution

associated with h_{eff} induced by $\alpha_K \propto 1/\hbar_{eff} \propto 1/n$ looks natural and was motivated by the idea that Nature is theoretician friendly: when the situation becomes non-perturbative, Mother Nature comes in rescue and an h_{eff} increasing phase transition makes the situation perturbative again.

Quite recently the number theoretic interpretation of coupling constant evolution [K111] [L15] in terms of a hierarchy of algebraic extensions of rational numbers inducing those of p-adic number fields encouraged to think that $1/\alpha_K$ has spectrum labelled by primes and values of h_{eff}. Two coupling constant evolutions suggest themselves: they could be assigned to length scales and angles which are in p-adic sectors necessarily discretized and describable using only algebraic extensions involve roots of unity replacing angles with discrete phases.

4. Few years ago the relationship of TGD and GRT was finally understood [K79]. GRT space-time is obtained as an approximation as the sheets of the many-sheeted space-time of TGD are replaced with single region of space-time. The gravitational and gauge potential of sheets add together so that linear superposition corresponds to set theoretic union geometrically. This forced to consider the possibility that gauge coupling evolution takes place only at the level of the QFT approximation and α_K has only single value. This is nice but if true, one does not have much to say about the evolution of gauge coupling strengths.

5. The analogy of Riemann zeta function with the partition function of complex square root of thermodynamics suggests that the zeros of zeta have interpretation as inverses of complex temperatures $s = 1/\beta$. Also $1/\alpha_K$ is analogous to temperature. This led to a radical idea to be discussed in detail in the sequel.

   Could the spectrum of $1/\alpha_K$ reduce to that for the zeros of Riemann zeta or - more plausibly - to the spectrum of poles of fermionic zeta $\zeta_F(ks) = \zeta(ks)/\zeta(2ks)$ giving for $k = 1/2$ poles as zeros of zeta and as point $s = 2$? $\zeta_F$is motivated by the fact that fermions are the only fundamental particles in TGD and by the fact that poles of the partition function are naturally associated with quantum criticality whereas the vanishing of ζ and varying sign allow no natural physical interpretation.

   The poles of $\zeta_F(s/2)$ define the spectrum of $1/\alpha_K$ and correspond to zeros of $\zeta(s)$ and to the pole of $\zeta(s/2)$ at $s = 2$. The trivial poles for $s = 2n$, $n = 1, 2, ..$ correspond naturally to the values of $1/\alpha_K$ for different values of $h_{eff} = n \times h$ with n even integer. Complex poles would correspond to ordinary QFT coupling constant evolution. The zeros of zeta in increasing order would correspond to p-adic primes in increasing order and UV limit to smallest value of poles at critical line. One can distinguish the pole $s = 2$ as extreme UV limit at which QFT approximation fails totally. CP_2 length scale indeed corresponds to GUT scale.

6. One can test this hypothesis. $1/\alpha_K$corresponds to the electroweak U(1) coupling strength so that the identification $1/\alpha_K = 1/\alpha_{U(1)}$ makes sense. One also knows a lot about the evolutions of $1/\alpha_{U(1)}$ and of electromagnetic coupling strength $1/\alpha_{em} = 1/[cos^2(\theta_W)\alpha_{U(1)}$. What does this predict?

   It turns out that at p-adic length scale $k = 131$ ($p \simeq 2^k$ by p-adic length scale hypothesis, which now can be understood number theoretically [K111]) fine structure constant is predicted with .7 per cent accuracy if Weinberg angle is assumed to have its value at atomic scale! It is difficult to believe that this could be a mere accident because also the prediction evolution of $\alpha_{U(1)}$ is correct qualitatively. Note however that for $k = 127$ labelling electron one can reproduce fine structure constant with Weinberg angle deviating about 10 per cent from the measured value of Weinberg angle. Both models will be considered.

7. What about the evolution of weak, color and gravitational coupling strengths? Quantum criticality suggests that the evolution of these couplings strengths is universal and independent of the details of the dynamics. Since one must be able to compare various evolutions and combine them together, the only possibility seems to be that the spectra of gauge coupling strengths are given by the poles of $\zeta_F(w)$ but with argument $w = w(s)$ obtained by a global conformal transformation of upper half plane - that is Möbius transformation (see `https://en.wikipedia.org/wiki/Mbius_transformation`) with real coefficients (element of $GL(2, R)$) so that one as $\zeta_F((as+b)/(cs+d))$. Rather general arguments force it to be and

element of $GL(2, Q)$, $GL(2, Z)$ or maybe even $SL(2, Z)$ $(ad - bc = 1)$ satisfying additional constraints. Since TGD predicts several scaled variants of weak and color interactions, these copies could be perhaps parameterized by some elements of $SL(2, Z)$ and by a scaling factor K.

Could one understand the general qualitative features of color and weak coupling contant evolutions from the properties of corresponding Möbius transformation? At the critical line there can be no poles or zeros but could asymptotic freedom be assigned with a pole of $cs+d$ and color confinement with the zero of $as + b$ at real axes? Pole makes sense only if Kähler action for the preferred extremal vanishes. Vanishing can occur and does so for massless extremals characterizing conformally invariant phase. For zero of $as + b$ vacuum function would be equal to one unless Kähler action is allowed to be infinite: does this make sense?. One can however hope that the values of parameters allow to distinguish between weak and color interactions. It is certainly possible to get an idea about the values of the parameters of the transformation and one ends up with a general model predicting the entire electroweak coupling constant evolution successfully.

To sum up, the big idea is the identification of the spectra of coupling constant strengths as poles of $\zeta_F((as + b/)(cs + d))$ identified as a complex square root of partition function with motivation coming from ZEO, quantum criticality, and super-conformal symmetry; the discretization of the RG flow made possible by the p-adic length scale hypothesis $p \simeq k^k$, k prime; and the assignment of complex zeros of ζ with p-adic primes in increasing order. These assumptions reduce the coupling constant evolution to four real rational or integer valued parameters (a, b, c, d). In the sequel this vision is discussed in more detail.

10.2 Fermionic Zeta As Partition Function And Quantum Criticality

Riemann zeta has formal interpretation as a partition function $\zeta = Z_B = \prod 1/(1-p^s)$ for a gas of bosons with energies coming as integer multiples of $log(p)$, for given mode labelled by prime p. I have proposed different interpretation based on the fermionic zeta ζ_F based on its representation as a product

$$\zeta_F = \prod_p (1 + p^s)$$

of single fermion partition functions associated with fermions with energy $log(p)$ (by Fermi statistics the fermion number is 0 or 1). In this framework the *poles* (not zeros!) of the fermionic zeta $\zeta_F(ks) = \zeta(ks)/\zeta(2ks)$ (the value of k turns out to be $k = 1/2$) (this identity is trivial to deduce) correspond to $s/2$, where s is either trivial or non-trivial zero of zeta (denominator), or the pole of zeta at $s = 1$ (numerator). Trivial poles are negative integers $s = -1 - 2, -3...$ suggesting an interpretation as conformal weights. This interpretation is proposed also for the nontrivial poles.

ζ_F emerges naturally in TGD, where the only fundamental (to be distinguished from elementary) particles are fermions. The assignment of physics to *poles* rather than zeros of ζ_F is also natural. The interpretation inspired by the structure of super-symplectic algebra is as conformal weights associated with the representations of extended super-conformal symmetry associated with super-symplectic algebra defining symmetries of TGD at the level of "World of Classical Worlds" (WCW).

"Conformal confinement" states that the sum of conformal weights of particles in given state is real. I discovered the idea for decade ago but gave it up to end up with it again. The fractal structure of superconformal algebra conforms with quantum criticality: infinite hierarchy of symmetry breakings to sub-symmetry isomorphic to original one! The conformal structure is infinitely richer than the ordinary one since the algebra in question has infinite number of generating elements labelled by all zeros of zeta rather than a handful of conformal weights ($n = -2, ... +2$ for Virasoro algebra). Kind of Mandelbrot fractal is in question. There is however deviation from the ordinary conformal symmetry since real conformal weights can have only one sign (for generating elements all negative conformal weights $n = -1, -2, -.$ are realized as poles of $1/\zeta(2s)$ but $n = 1$

realized as pole of $\zeta(s)$ is the only positive conformal weight). Situation is therefore not quite identical with that in conformal field theories although also conformal field theories realizes only positive conformal weights (positivity is a convention) and have also some tachyonic conformal weights which are negative.

The problem of all attempts to interpret zeros of zeta relates to the fact that zeros are *not* purely imaginary but possess the troublesome real part $Re(s) = 1/2$. This led me to consider coherent states instead of eigenstates of Hamiltonian in my proposal, which I christened a strategy for proving Riemann hypothesis [K64], [L1]. Zeta has phase at the critical line so the interpretation as a partition function can be only formal. So called Z function defined at critical line and obtained by extracting the phase of zeta out, is real at critical line.

In TGD framework the solution of these problems is provided by zero energy ontology (ZEO). Quantum theory is "complex square root" of thermodynamics and means that partition function becomes a complex entity having also a phase. The well-known function

$$\xi(s) = \frac{1}{2}\pi^{-s/2}s(s-1)\Gamma(s/2)(\zeta(s)$$

assignable to Riemann zeta having same zeros and basic symmetries has at critical line phase equal ± 1 except at zeros where the phase can be defined only as a limit depending the direction from which the zero is approached. Fermionic partion function $\zeta_F(s)$ has a complex phase and it is not clear whether it makes sense to assign with it the analog of $\xi(s)$. Ordinary partition function is modulus squared for the generalized partition function.

Why does the partition function interpretation does demand poles?

1. In ordinary thermodynamics the vanishing of partition function makes sense only at the limit of zero temperature when all Boltzmann weights approach to zero. By subtracting the energy of the lowest energy state from the energies the partition function becomes non-vanishing also in this case. Hence the idea that partition function vanishes does not look very attractive. The varying sign is even worse problem.

2. Since the temperature interpreted as $1/s$ in the partition function is not infinite could mean that one has analog of Hagedorn temperature (see `http://tinyurl.com/pvkbrum`): the degeneracy of states increases exponentially with temperature and at Hagedorn temperature compensates the s exponential decreases of Boltzmann weights so that partition function is sum of infinite number of terms approaching to unity. Hagedorn temperature relates by strong form of holography to magnetic flux tubes behaving as strings with infinite number of degrees of freedom. One would have quantum critical system possessing supersymplectic symmetry and other superconformal symmetries predicted by TGD [K15, K14, K76].

3. The temperature is complex for non-trivial zeros. This requires a generalization of thermodynamics by making partition function complex. Modulus squared of this function takes the role of an ordinary partition function. One can allow in the case of Kähler action the replacement of argument s with $ks+b$ without giving up the basic features of U(1) coupling constant evolution. Here one can allow rational numbers k and b. The inverse temperature for $\zeta_F(ks+b)$ is identified as $\beta = 1/T = k(s+b)$. It turns out that in the model for coupling constant evolution the scaling factor $k = 1/2$ is required. b is not completely fixed.

   Complex temperature is indeed the natural quantity to consider in ZEO. The real part of temperature at critical line equals to $Re(\beta) = (s+b)/4k$, with b rational or integer for $\zeta_F(w = k(s+b))$ at poles assignable with the zeros of $\zeta(2k(s+b))$ in denominator. Imaginary part

$$Im\,[\beta] = \frac{1}{T} = \frac{1}{2k}(b + \ frac12 + iy) \tag{10.2.1}$$

   of the inverse temperature does not depend on b. Infinite number of critical temperatures is predicted and a discrete coupling constant evolution takes place already at the level of basic quantum TGD rather than emerging only at the QFT limit - I have also considered the possibility that coupling constant evolution emerges at the QFT limit only [K3]. One could even allow Möbius transformation with real coefficients in the argument of ζ_F and that this could allow the understanding of the evolutions of weak and colour coupling constants.

$\zeta_F(w)$ at $s = -(n-b)/k$ are also present. For $s = 1/T$ they would correspond to negative temperatures $\beta = (-n+b)/k$? In the real context and for Hamiltonian with a fixed sign this looks weird. Preferred extremals can be however dominated by either electric or magnetic fields and the sign of the action density depends on this.

4. Interestingly, in p-adic thermodynamics p-adic temperatures has just the values $T = -1/n$ if one defines p-adic Boltzmann weight as $exp(-E/T) \rightarrow p^{-E/T}$, with $E = n \geq 0$ conformal weight. The condition that weight approaches zero requires that T identified in this is as real integer negative for p-adic thermodynamics! Trivial poles would correspond to p-adic thermodynamics and non-trivial poles to ordinary real thermodynamics! Note that the earlier convention is that $T = 1/n$ is positive: the change of the sign is just a convention. Could the hierarchy of p-adic thermodynamics labelled by p-adic primes corresponds to the sequence of critical zeros of zeta? Number theoretic vision indeed leads to this proposal [L15], [K111].

   The factor $1/(1-p^n)$ at the real poles $s = -2n$ would exist p-adically in p-adic number field Q_p so that the factors of zeta would correspond to adelic decomposition of the partition function. At critical line in turn $1/1 + p^{1/2+iy}$ would exist for zeros y for which p^{iy} is root of unity (note that $p^{1/2}$ is somewhat problematic for Q_p: does it make sense to speak about an extension of Q_p containing $sqrtp$ or is the extension just the same p-adic number field but with different definition of norm?). That p^{iy} is root of unity for some set $C(p)$ of zeros y associated with p was proposed in [L15], [K111]. Now $C(p)$ would consist of single zero $y = y(p)$.

10.2.1 Could The Spectrum Of Kähler Couplings Strength Correspond To Poles Of $\zeta_F(s/2)$?

The idea that the spectrum of conformal weights for supersymplectic algebra is given by the poles of ζ_F is not new [L15].

Poles of $\zeta_F(ks)$ ($k = /2$ turns out to be the correct choice) have also interpretation as complexified temperatures. Kähler action can be interpreted as a complexified partition function and the inverse $1/\alpha_K$ of Kähler coupling appears in the role of critical inverse temperature β. The original hypothesis was that Kähler coupling strength has only single value. The hierarchy of quantum criticalities and its assignment with number theoretical hierarchy of algebraic extensions of rationals led to consider the possibility that Kähler coupling strength has a spectrum corresponding to a hierarchy of critical temperatures. Quantum criticality and Hagedorn temperature for magnetic flux tubes as string like objects are indeed key elements of TGD.

The hypothesis to be studied is that the values $1/\alpha_K$ correspond to poles of

$$\zeta_F(ks) = \zeta(ks)/\zeta(2ks)$$

with the identification $1/\alpha_K = ks$. The model for coupling constant evolution however favors $k = 1/2$ predicting that poles correspond to zeros of zeta in the denominator of ζ_F and $s = 2$ in its numerator. For $k = 1/2$ only even negative integers would appear in the spectrum and there would be pole at $s = 2$. Here one onr also allow the sift $ks \rightarrow ks + b$, b integer without shifting the imaginary parts of poles crucial for the coupling constant evolution. This induces a shift $Re[s] \rightarrow kRe[s] + b$ for the real parts of poles.

For nontrivial poles this requires the replacement of temperature with a complex temperature. Therefore also $1/\alpha_K$ becomes complex. This is just what the ZEO inspired idea about quantum theory as complex square root of thermodynamics suggests. Kähler action is also complex already for real values of $1/\alpha_K$ since Euclidian *resp.* Minkowskian regions give real/imaginary contribution to the Kähler action.

The poles of ζ_F would appear both as spectrum of complex critical temperatures $\beta = 1/T = 1/\alpha_K$ and as spectrum of supersymplectic conformal weights. ζ_F is complex along the critical line containing the complex poles. This makes sense only in ZEO. ξ function associated with ζ is real at critical line but the problems are vanishing at finite temperature, indefinite sign, and also the fact that partition function interpretation fails at positive real axis. This does not conform with the intuitive picture about partition function defined in terms of Boltzmann weights.

10.2.2 The Identification Of $1/\alpha_K$ As Inverse Temperature Identified As Pole Of ζ_F

Let us list the general assumptions of the model based on the identification of $1/\alpha_K$ as a complexified inverse temperature in turn identified as zero of ζ_F.

1. I have earlier considered the number theoretical vision based on the assumption that vacuum functional identified as exponent of Kähler action receiving real/imaginary contributions from Euclidian/Minkowskian space-time regions exists simultaneously in all number fields. This is in spirit with the idea of integrability meaning that functional integral reduces to a sum over exponents of Kähler action associated with stationary points. What is nice that by the Kähler property of WCW metric Gaussian and metric determinants cancel [K34, K111] and one indeed obtains a discrete sum over exponentials making sense also in p-adic sectors, where ordinary integration does not make sense. Number theoretic universality is realized if one allows the extension of rationals containing also some roots of e if the exponent reduces to a product of root of unity and product of rational powers of e (e^p is ordinary p-adic number) and integer powers of primes p. It is perhaps needless to emphasize the importance of this result.

   The criticism is obvious: how does one know, which preferred extremals have a number theoretically universal action exponent? For calculational purposes it might not be necessary to know this. The easy option would be that all preferred extremals are number theoretically universal: this cannot be however the case if the values of $1/\alpha_K$ correspond to zeros of ζ. Second option is that in the sum over preferred extremals those which do not have a number theoretically universal exponent give a vanishing net contribution and are effectively absent. The situation brings in mind the reduction of momentum spectrum of a particle in a box to momenta equal to $k = n2\pi/L$, L the length of the box. The contributions of other plane waves in integrals vanish since they are dropped away by boundary conditions.

   Strong form of number theoretic universality requires that the exponent of Kähler action reduces to a product of rational power of some prime p or $e^{m/n}$ and a root of unity [K111], [L15]. This might be too strong a condition and weaker condition allows also powers of p mapped to real sector and vice versa by canonical identification. One could pose root of unity condition for the phase of $exp(S_K)$ as a boundary condition at the ends of causal diamond (CD) stating that some integer power of the exponent of Kähler action for the given value of α_K is real. If $exp(K)$ contains $e^{m/n}$ factor but no p^n factors, the reality of the n^{th} power of $exp(i\pi K)$ would reveal this. Single p^n factor in absence of $e^{m/n}$ factor could be detected by requiring that the exponent $exp(iyK)$ is real for some y (imaginary part of zero of zeta with p^{iy} a root of unity).

2. The assumption that $1/\alpha_K$ corresponds to a nontrivial zero of zeta has strong constraints on the values of the reduced Kähler action $S_{K,red} = \alpha_K S_K$ for which the classical field equations do not depend on α_K at all. The reason is that the S_K must be proposal to $1/\alpha_K$ to achieve number theoretical universality. Number theoretical universality thus implies that preferred extremals depend on $1/\alpha_K$ - this is something very quantal. The proportionality $1/\alpha_K$ to $h_{eff} = n \times h$ is highly suggestive. It does not destroy number theoretical universality for given preferred extremal.

3. $1/\alpha_K$ has form $1/\alpha_K = s = a+ib = (1/2k)(1/2+iy/2)$ for nontrivial poles, $1/\alpha_K = s = -n/k$ for trivial poles of $1/\zeta(2s)$, and $1/\alpha_K = s = 1/k$ for the pole of ζ. $k = 1/2$ is the physically preferred choice.

   Kähler action can be written as a sum of Euclidian and Minkowskian contributions: $K = K_E + iK_M$. For non-trivial poles in the case of $1/\alpha_K = ks$ one has

$$K = s \times (K_E + iK_M) = \frac{1}{k} \times \left[\frac{K_E}{2} - yK_M + i(\frac{K_M}{2} + yK_E)\right] \quad . \tag{10.2.2}$$

Here $K_{red} = K_E + iK_M$ is *reduced* Kähler action. This option generalizes directly the original proposal.

4. For trivial poles $s = -n/k$ and $s = 1/k$ one has

$$K = \frac{s}{k} \times K_{red} = \frac{s}{k} \times (K_E + iK_M) \ . \tag{10.2.3}$$

5. For real poles universality holds true without additional conditions since the multiplication of $1/\alpha_K$ by the scaling factor $-n_2/n_1$ does not spoil number theoretical universality. One can of course consider this condition. It predicts that the K_{red} is scaled by n_1/n_2 in the transition $n_2 \to n_1$. For nontrivial poles K_{red} is scaled by the complex ratio s_2/s_1.

   An attractive possibility is that the hierarchy of Planck constants corresponds to this RG evolution. n would correspond to the number of sheets in the n-sheeted covering for which sheets co-incide at the ends of space-time at the boundaries of CD. Therefore p-adic and $h_{eff} = n \times h$ hierarchies would find a natural interpretation in terms of zeros of ζ_F. To avoid confusion let us make clear that the values of $n = h_{eff}/h$ would not correspond to trivial poles.

Number theoretical universality could be realized in terms of RG invariance leaving the vacuum functional invariant but deforming the vacuum extremal. The hierarchy of Planck constants and p-adic length scale hierarchy could be interpreted as RG flows along real axis and critical line.

1. The grouping of poles to 4 RG orbits corresponding to non-trivial poles $y > 0$ and $y < 0$, to poles $s = -n/k < 0$, and $s = 1/k$ looks natural. The differential equations for RG evolution of Kähler action would be replaced with a difference equation relating the values of Kähler action for two subsequent critical poles of ζ_F.

2. Number theoretical universality allows to relate Minkowskian and Euclidian contributions K_M and K_E to each other. Earlier I have not even tried to deduce any correlation between them although the boundary conditions at light-like wormhole throats at which the signature of the induced metric changes, probably give strong constraints.

   The strongest form of the number theoretical universality condition assumes

$$K_{red} = K_{red,E} + iK_{red,M} = \alpha_K K_1 = \frac{K_1}{s} = K(\alpha_K = 1) \ , \quad s = \frac{1}{\alpha_K} \ . \tag{10.2.4}$$

   K_1 satisfies the number theoretic universality meaning that $exp(K_1) = expK(\alpha_K = 1)$ reduces to a product of powers primes, root of e and root of unity.

   This ansatz has the very remarkable property that α_K disappears from the vacuum functional completely so that the RG action can be regarded as a symmetry leaving vacuum function invariant. This operation however changes the preferred extremal and reduced Kähler action so that the situation is non-classical. RG orbit would start from the pole $s = 1$ and contain complex poles.

3. The large CP breaking suggested by complexity of α_K would disappear at the level of vacuum functional and appears only at the level of preferred extremals. If this is to conform with the quantum classical correspondence, correlation functions, which must break CP symmetry receive this breaking from preferred extremals. $s = 1/2k$ and complex poles belong to the same orbit. This ansatz is not necessary for poles $s = 1/k$ and $s = -n/k$ for which number theoretic universality conditions are satisfied irrespective of the value of s.

4. A more realistic looking solution is obtained by assuming that complex poles correspond to separate orbit or even that positive and negative values of y correspond to separate orbits. RG flow would begin from the lowest zero of zeta at either side of real axis. This gives

$$K_{red} = \frac{\alpha_K}{\alpha_{K,0}} \times K_{red}(\alpha_{K,0}) \ . \tag{10.2.5}$$

   Also now the vacuum functional is invariant and preferred extremal changes in RG evolution. In accordance with quantum classical correspondence one has however a breaking of CP symmetry also at the level of vacuum functional due to the complexity of $\alpha_{K,0}$ unless $K_{red}(\alpha_{K,0})$ is proportional to $\alpha_{K,0}$.

Remark: The above arguments must be modified if one includes to the action cosmological volume term strongly suggested by twistor lift of TGD.

10.3 About Coupling Constant Evolution

p-Adic mass calculations inspired the hypothesis that the continuous coupling constant evolution in QFTs reduces in TGD framework to a discrete p-adic coupling constant evolution but assuming that α_K is absolute RG invariant. Therefore the hypothesis that the evolution of $1/\alpha_K$ defined by the non-trivial poles of ζ_F corresponds to the p-adic coupling constant evolution deserves a serious consideration.

1. p-Adic length scale hypothesis in the strong form states that primes $p \simeq 2^k$, k prime, correspond to physically preferred p-adic length scales. This would suggest that non-trivial zeros $s_1, s_2, s_3, ..$ taken in increasing order for magnitude correspond to primes $k = 2, 3, 5, 7...$ as suggested also in [L15], [K111]. This allows to assign to each zero s_n a unique prime: $p \leftrightarrow y(p)$ and this suggests more precise of the earlier hypothesis to state that $p^{iy(p)}$ is root of unity. The class of zeros associated with p would contain single zero.

   Discrete p-adic length scale evolution would thus correspond to the evolution of non-trivial zeros. The evolution associated with the hierarchy of Planck constants could only multiple Kähler action with integer. To make this more concrete one must consider detailed physical interpretation.

2. $1/\alpha_K$ corresponds to $U(1)$ coupling of standard model: $\alpha_K = \alpha(U(1)) \equiv 1/\alpha_1$. Kähler action could be seen as analogous to a Hamiltonian associated with electroweak U(1) symmetry. $U(1)$ gauge theory is not asymptotically free and this correspond to the fact that $Im(1/\alpha_K) = y$ approaches in UV to the lowest zero $y = 14.12...$ In IR y diverges, which conforms with $U(1)$ gauge theory symmetry.

   Electromagnetic coupling corresponds to

$$\frac{1}{\alpha_{em}} = \frac{1}{\alpha_K cos^2(\theta_W)} \ . \tag{10.3.1}$$

   The challenge is to understand also the evolution of $cos^2(\theta_W)$ allowing in turn to understand the entire electroweak evolution.

3. The values of electroweak couplings at the length scale of electron ($k = 127$ or at 4 times longer length scale $k = 131$ ($L(131) = .1$ Angstrom) are well-known and this provides a killer test for the model. Depending on whether one assumes fine structure constant to correspond to $L(127)$ associated with electron or to 4 times long length scale $L(131)$ one has too options. $L(131)$ allows to reproduce fine structure constant with a value of $p = sin^2(\theta_W)$ deviating only .7 per cent from its measured value in this length scale! If this is not a mere nasty

accident, Riemann zeta might code the entire electroweak physics and perhaps even strong interactions!

The first guess is that UV asymptotia for the Weinberg angle is same as for GUTS: $p \to 3/8$ for $p = 2$ giving $1/\alpha_{em} \to 22.61556016$. IR asymptotia corresponds to $p \to 0$ implying $1/\alpha_{em} = 1/\alpha_K$. Notice that the evolution is rather fast in extreme UV. In extreme IR it becomes slow. It turns out that the UV behavior of Weinberg angle does not conform with this naive expectation.

4. Since p-adic length scale is proportional to $1/p^{1/2}$ it is enough to obtain RG evolution for coupling constnt as function of p. One obtains reasonably accurate understanding about the evolution by deducing an estimate for pdy/dp . This is obtained as $pdy/dp = (dy/dN)(dN/dk)p(dk/dp)$.

   - $p \simeq 2^k$ implies $k \simeq log(p)/log(2)$ and $pdk/dp \simeq 1/log(2)$.
   - The approximate formula for the number $N(y)$ of zeros smaller than y is given by

   $$N(y) \sim u \times log(u) \ \ , \ \ u = \frac{y}{2\pi}$$

   giving

   $$\frac{dN}{dy} \sim \frac{1}{2\pi} \times (log(u) - 1), \ \ u = \frac{y}{2\pi} \ \ .$$

   - The number $\pi(k)$ of primes smaller than k is given by

   $$N(k) \sim \frac{k}{log(k)}$$

   giving

   $$\frac{dN(y)}{dk} \sim \frac{1}{log(k)} - \frac{1}{log(k)^2} \ \ .$$

   By combining the formulas, one obtains

   $$p\frac{dy}{dp} = \beta = \frac{2\pi}{log(2)} \times (\frac{1}{log(y/2\pi)} - 1) \times (\frac{1}{log(k)} - \frac{1}{log(k)^2}) \ \ , \ \ k = \frac{log(p)}{log(2)} \ \ . \tag{10.3.2}$$

   The beta function for the evolution as function of p-adic length scale differs by factor 2 from this one. Note that also double logarithms appear in the formula. Note that beta function depends on y logarithmically making the equation rather nonlinear. This dependence can be shifted to the left hand side and by replacing y which appropriation chosen function of it one obtains

   $$p\frac{dN(y)}{dp} = \beta_1 = \frac{1}{log(k)} - \frac{1}{log(k)^2} \ \ , \ \ k = \frac{log(p)}{log(2)} \ \ . \tag{10.3.3}$$

5. Coupling constant evolution would take place at the level of single space-time sheet. Observations involve averaging over space-time sheet sizes characterized by p-adic length scales so that a direct comparison with experimental facts is not quite easy and requires a concrete statistical model.

The entire electroweak $U(1)$ coupling constant evolution would be predicted exactly from number theory. Physics would represent mathematics rather than vice versa. Concerning experimental testing a couple of remarks are in order.

1. An open question is how much many-sheetedness of space-time affects situation: one expects kind of statistical average of say Weinberg angles over p-adic length scales coming from a superposition over space-time sheets of many-sheeted space-time. Space-time with single sheet is not easy to construct experimentally although mathematically it is extremely simple system as compared to the space-time of GRT.

2. The discreteness of the coupling constant evolution at fundamental level is one testable prediction. There is no continuous flow but sequence of phases with fixed point behavior with discrete phase transitions between them. At QFT limit one expects that continuous coupling constant evolution emerges is statistical average.

3. Later it will be found that the entire electroweak evolution can be predicted and this prediction is certainly testable.

10.3.1 General Description Of Coupling Strengths In Terms Of Complex Square Root Of Thermodynamics

The above picture is unsatisfactory in the sense that it says nothing about the evolution of other electroweak couplings and of color coupling strength. Does number theory fix also them rather than only $U(1)$ coupling? And what about color coupling strength α_s?

Here quantum TGD as a complex square root of thermodynamics vision helps.

1. Kähler action reduces for preferred extremals to Abelian Chern-Simons action localized at the ends of space-time surfaces at boundaries of causal diamond (CD) and possibly contains terms also at light-like orbits of partonic 2-surfaces. This corresponds to almost topological QFT property of TGD.

2. Kähler action contains additional boundary terms which serve as analogs for Lagrangian multipler terms fixing the numbers of various particles in thermodynamics. Now they fix the values of isometry charges for instance, or force the symplectic charges for a sub-algebra to vanish.

   Lagrangian multiplies can be written in the form μ_i/T in ordinary thermodynamics: μ_i denotes the chemical potentials assignable to particle of type i. Number theoretical universality strongly favors similar representation now. For instance, this would give

$$\frac{1}{\alpha_{em}} = \frac{\mu_{em}}{\alpha_K} \ , \ \ \mu_{em} = \frac{1}{cos^2(\theta_W)} \ . \tag{10.3.4}$$

   In the same manner SU(2) coupling strength given by

$$\frac{1}{\alpha_W} = \frac{\mu_W}{\alpha_K} = \frac{cot^2(\theta_W)}{\alpha_K} \tag{10.3.5}$$

   would define $cot^2(\theta_W)$ as analog of chemical potential.

3. In the case of weak interactions Chern-Simons term for induced SU(2) gauge potentials as a boundary term would be the analog of Kähler action having interpretation as Lagrangian multiplier term. In color degrees of freedom also an analog of Chern-Simons term would be in question and would be associated with the classical color gauge field defined by H_AJ, where H_A is Hamiltonian of color isometry in CP_2 and J is induced Kähler form.

4. The conditions for number theoretical universality would become more complex as also RG invariance interpreted in terms of number theoretical universality.

This picture assuming a linear relationship between generic coupling strength α and α_K in terms of chemical potential is not yet general enough.

10.3.2 Does ζ_F With $GL(2,Q)$ Transformed Argument Dictate The Evolution Of Other Couplings?

It seems that one cannot avoid dynamics totally. The dynamics at (quantum) criticality is however universal. This raises the hope that the evolution of coupling constant is universal and does not depend on the details of the dynamics at all. This could also explain the marvellous successes of QED and standard model

At criticality the dynamics reduces to conformal invariance by quantum criticality, and this inspires the idea about the values of coupling constant strength as poles of a meromorphic function obtained from ζ_F by a conformal transformation of the argument. After all, what one must understand is the relationship between $1/\alpha_W$ and $1/\alpha_K$, and the linear relationship between them can be seen as a simplifying assumption and an approximation.

The values of generic coupling strength - call it just α (to be not confused with α_{em}) without specifying the interaction - would still correspond to poles of $\zeta_F(s)$ but with a transformed argument s. Conformal transformation would relate various coupling constant evolutions to each other and allow to combine them together in a unique manner. Discreteness is of course absolutely essential. The analysis of the situation leads to a surprisingly simple picture about the coupling constant evolutions for weak and color coupling strengths.

1. By the symmetry of ζ_F under the reflection with respect to x-axis one can restrict the consideration to globally defined conformal transformations of the upper half plane identifiable as Möbius tranformations $w = (as+b)/(cs+d)$ with the real matrix coefficients (a,b,c,d). One can express the transformation as a product of an overall scaling by factor k and $GL(2,R)$ transformation with $ad - bc = 1$. Number theoretical universality demands that k and the coefficients a,b,c,d of $GL(2,R)$ matrix are real rationals. Number theoretically $GL(2,Q)$ is attractive and one can consider also the possibility that the transformation matrix $GL(2,Z)$ matrix with a,b,c,d integers. $SL(2,Z)$ is probably too restrictive option.

2. The Möbius transformation $w = (as + b)/(cs + d)$ acts on zeros of ζ mapping the discrete coupling constant evolution for $1/\alpha_K$ to that for $1/\alpha_W$ or $1/\alpha_s$. The transformed coupling constant depends logarithmically on p-adic length scale via $1/\alpha_K$ supporting the interpretation in terms of RG flow induced by that for $1/\alpha_K$ - something very natural since Kähler action is in special role in TGD framework since it determines the dynamics of preferred extremals.

3. Asymptotically (long length scales) one has $w \rightarrow a/c$ for $a \neq 0$ so that both at critical line and real axis one has accumulation of critical points to $w = a/c$! Thus for the option allowing only very large value of coupling strength in IR one has

$$w = K \times \frac{as+b}{cs+d} \ , \ ad - bc = 1 \ \ \text{(Option 1)} \ . \tag{10.3.6}$$

$a/c = 0$ $(a = 0)$ corresponds to a diverging coupling strength (for color interactions and for weak interactions for vanishing Weinberg angle) and corresponds to $w = K \times b/cs + d$. $ad - bc = 1$ gives $b = -c = 1$ and if one accepts the IR divergence of coupling constant, one has

$$w = \frac{K}{-s+d} \ \ \text{(Option 2)} \ . \tag{10.3.7}$$

The only free parameters are the rational $K > 0$ and integer d. w has pole at $s = d$ mapped to 1 by ζ_F.

To gain physical insight consider the situation at real axes.

1. The real poles $s = -n/k$ and $s = 1/k$ are mapped to poles on real axes and the reflection symmetry with respect to x-axis is respected. Negative poles would be thus mapped to negative poles for $d \in 0, 1$ and $k < 0$. One could also require that the pole $s = 1$ is mapped to positive pole. For option 2 it is mapped to $w = +\infty$.

2. For option 1 this is true if one has $cs + d < 0$ and $as + b > 0$. The other manner to satisfy the conditions is $cs + d > 0$ and $as + b < 0$ for $s = -1, -2, ...$ By replacing the (a, b, c, d) with $(-a, -b, -c, -d)$ these conditions can be transformed to each other so that it is enough to consider the first conditions. The first form of the condition requires $c > 0$ and $a < 0$.

   The condition that $s = 1/k$ goes to a positive pole gives $c/k + d > 0$ and $a/k + b > 0$. Altogether this gives for the two Options the conditions

$$\begin{aligned} w &= K \times \frac{as+b}{cs+d} < 0 \ , \\ k &> 0 \ , a < 0 \ , \ c > 0 \ , \ \frac{c}{k} + d > 0 \ , \ \frac{a}{k} + b > 0 \ . \ \text{(Option 1)} \ , \end{aligned} \tag{10.3.8}$$

   and

$$w = \frac{K}{-s + \frac{1}{k}} < 0 \ , \ k > 0 \ . \ \text{(Option 2)} \tag{10.3.9}$$

3. For option 2 $s = 1/k$ phase is mapped to $w = +\infty$. Coupling strength vanishes in this phase: this brings in mind the asymptotic freedom for QCD realized at extreme UV? In long scales α would behave like $1/\alpha_K$ and diverge suggesting that Option 2 provides at least an idealized description of QCD. The scaling parameter K would remain the only free parameter.

   For option 1 α can become arbitrary large in long scales but remains finite. The analog of asymptotically free phase is replaced with that having non-vanishing inverse coupling strength $w = (a+b)/(c+d)$. The interpretation could be in terms of weak coupling constant evolution. The non-vanishing of the parameter a would distinguish between weak and strong coupling constant evolution.

   By feeding in information about the evolution of weak and color coupling strengths, one can deduce information about the values of K and a.

Whether the analogs of weak and Chern-Simons actions can satisfy the number theoretical universality, when the transformation is non-linear is far from obvious since the induced gauge fields are not independent.

10.3.3 Questions About Coupling Constant Evolution

The simplest hypothesis conforming with the general form of Yang-Mills action is $1/\alpha_K = s$, where s is zero of zeta. With the identification $1/\alpha_K = 1/\alpha_{U(1)}$ this predicts the evolution of U(1) coupling and one obtains excellent prediction in p-adic length scale $k = 131$ ($L(131 \simeq 10^{-11}$ meters).

How general is the formula for $1/\alpha_K$?

Is the simplest linear form for $1/\alpha_K$ general enough? Consider first the most general form of $2\pi/\alpha_K$ taking as input the fact that its imaginary is equal to $1/\alpha_{U(1)}$ and corresponds to imaginary part y of zero of zeta at critical line.

Linear Möbius transformations $w = (as + b)/d$ with real coefficients do not affect $Im[s]$ and therfore the inverse of the imaginary part of the Kähler coupling strength which corresponds to th inverse of the measured U(1) coupling strength. The general formula for complex Kähler coupling strength would be

$$w = s + \frac{b}{d} \tag{10.3.10}$$

in the case of $SL(2, Q)$ giving $Re[1/\alpha_K] = 1/2 + b/d$. This would correspond to the analog of the inverse temperature appearing in the real exponent of Kähler function. For $SL(2, Z)$ on obtains

$$w = s + b \ , \ \ b \in Z \ . \tag{10.3.11}$$

This gives $Re[1/\alpha_K] = 1/2 + b$.

Does the reduction to Chern-Simons term give constraints

The coefficient of non-Abelian Chern-Simons action is quantized to integer and one can wonder whether this has any implications in TGD framework.

1. The Minkowskian term in Kähler action reduces to Abelian Chern-Simons term for Kähler action. In non-Abelian case the coefficient of Chern-Simons action (see `http://tinyurl.com/y7nfaj67`) is $k_1/4\pi$, where k_1 is integer.

   In Abelian case the triviality of gauge transformations does not give any condition on the phase factor so that in principle no conditions are obtained. One can however look what this condition gives. The coefficient of Chern-Simons term coming from in Kähler action is $1/(8\pi\alpha_K)$. For non-Abelian Chern-Simons theory with n fermions one obtains action $k \to k - n/2$. Depending on gauge group k_1 can vanish modulo 2 or 4. For the zeros at the real axes this would give the condition

   $$\frac{s}{2} = s + \frac{b}{d} = Re[\frac{1}{\alpha_K}] = 2k_1 \ , \ \ s = -2n < 0 \ \text{ or } \ s = 2 \ , \tag{10.3.12}$$

   which is identically satisfied for integer valued b/d. Thus it seems that $SL(2, Z)$ is forced by the Chern-Simons argument in the case of Kähler action, which is however not too convincing for U(1).

   For non-trivial zeros it is not at all clear whether one certainly cannot apply the condition since there is also a contribution yS_E to the imaginary part. In any case, the condition would be

   $$\frac{Re[s]}{2} = 1/2 + \frac{b}{d} = Re[\frac{1}{\alpha_K}] = 2k_1 \ . \tag{10.3.13}$$

   b/d must be half odd integer to satisfy the condition so that one would have $SL(2, Z)$ instead of $SL(2, Q)$. This is however in conflict with the Chern-Simons condition at real axis.

2. $w = s + b/d$ implies that the trivial poles $s = -2n$, $n > 0$, at the real axes are shifted to $s = -2n + b/d$ and become fractional. The poles at $s = 2$ is shifted to $2 + b/d$.

In the non-Abelian case one expects also Chern-Simons term but now emerging as an analog of Lagrange multiplier term rather than fundamental action reducing to Chern-Simons term. For $w = (as + b)(cs + d)$ the poles at real axis are mapped to rational numbers $w = (am + b)/(cm + d)$, $m = -2n$ or $m = 2$. Chern-Simons action would suggest integers. Gauge transformations would transform the action by a phase which is a root of unity. Vacuum functional is ZEO an analog of wave function as a square root of action exponential. Can one allow the wave function to be a finitely-many valued section in bundle?

Does the evolution along real axis corresponds to a confining or topological phase?

At real axis the imaginary part of s vanishes. Since it corresponds to the inverse of the gauge coupling strength, one can ask whether the proper interpretation is in terms of confining phase in which gauge coupling is literally infinite and it does not make sense to speak of perturbation theory. Instead one would have a phase in which Minkowski part of the Kähler action contributes only to the imaginary Chern-Simons term but not to the real part of the action. Topological QFT also based on Chern-Simons action also suggests itself.

The vanishing of gauge coupling strength is not a catastrophe now since the real part is non-vanishing. What looks strange that this phase is obtained also for Kähler coupling strength. Could this interpreted in terms of the fact that induced gauge potentials are not independent dynamical degrees of freedom but expressible in terms of CP_2 coordinates.

The spectrum of $1/\alpha_K$ at real axis has the $-2n + \frac{b}{d}$ and $2 + \frac{b}{d}$ and is integer or half-odd integer valued by the conditions on Chern-Simons action. One could make the entire spectrum integer value by a proper choice of b/d.

Integer valuedness forced by Chern-Simons condition leads to ask whether the situation could relate to hierarchy of Planck constants. This cannot be the case. One can assign to each value of y p-adic coupling constant labelled by prime k ($p \simeq 2^k$) a hierarchy of Planck constants $h_{eff} = n \times h$. If number theoretical universality is realized for $n = 1$, it is realized for all values of n and one can say that one has $1/\alpha = n/\alpha$ fora generic coupling strength α.

p-Adic temperature $T = 1/n$ using $log(p)$ as a unit correspond to the temperature parameter defined by α_K: the values of both are inverse integers. p-Adic thermodynamics might therefore provide a proper description for the confining phase as also the success of p-adic mass calculations encourages to think.

The sign of $1/\alpha_K$ is not fixed for the simplest option. The shift by $\frac{b}{d}$ could fix the sign to be negative. There is however no absolute need for a fixed sign since in Minkowskian regions the sign of Kähler action density depends on whether magnetic or electric fields dominate. In Euclidian regions the sign is always positive. Since the real part of Kähler action receives contributions from both Euclidian and Minkowskian regions it can can well have both signs so that for preferred extremals the signs of the real part of Kähler coupling strength and proper Kähler action compensate each other.

10.4 A Model For Electroweak Coupling Constant Evolution

In the following a model for electroweak coupling constant evolution using as inputs Weinberg angle at p-adic length scale $k = 127$ of electron or at four times longer scale $k = 131$ and in weak length scale $k = 89$ is developed.

10.4.1 Evolution Of Weinberg Angle

Concerning the electroweak theory, a key question is whether the notion of Weinberg angle still makes sense or whether one must somehow generalize the notion. Experimental data plus the prediction for $1/\alpha_{U(1)}$ as zero of zeta suggest that Weinberg angle varies. For instance, the value of$1/\alpha_{U(1)}$ for $k = 89$ corresponds to weak length scale and is 87.4 whereas fine structure constant is around 127. This gives $sin^2(\theta_W) \sim .312$, which is larger than standard model value.

1. Assume that the coupling constant evolutions for $1/\alpha_{em}$ and $1/\alpha_W$ correspond to different Möbius transformations acting in a nonlinear manner to s. Tangent of Weinberg angle is defined as the ratio of weak and U(1) coupling constants: $tan(\theta_W) = g_W/g_{U(1)}$ and it expresses the vectorial character of electromagnetic coupling. One can write

$$sin^2(\theta_W) = \frac{1}{1+X} \quad , \quad X = \frac{\alpha_{U(1)}}{\alpha_W} \quad . \tag{10.4.1}$$

One can write the ansätze for for the coupling strengths as imaginary parts of complexified ones:

$$\begin{aligned}
\frac{1}{\alpha_{U(1)}} &= Im[s+b] = y \ , \ s = \frac{1}{2} + iy \\
\frac{1}{\alpha_W} &= Im[\frac{a_W s + b_W}{c_W s + d_W}] = \frac{Dy}{c^2(\frac{1}{4}+y^2)+cd+d^2)} \ , \\
D &= ad - bc \ .
\end{aligned} \tag{10.4.2}$$

Here $GL(2,Q)$ matrices are assumed and determinant $D = ad - bc$ is allowed to differ from unity. From this one obtains for the Weinberg angle the expression

$$sin^2(\theta_W(y)) = \ = \frac{1}{1+[\frac{c^2}{D}(y^2+\frac{1}{4})+\frac{d}{c}+(\frac{d}{c})^2]} \ , \ D = ad - bc \ .$$

As the physical intuition suggests, Weinberg angle approaches zero at long length scales ($y \to \infty$). The value at short distance limit (the lowest zero $y_0 = 14.13$ at critical line) assignable to $p = 2$ is given by

$$sin^2(\theta_W(y_1)) = \frac{1}{1+\frac{c^2}{D}[(y_1^2+\frac{1}{4}+\frac{d}{c}+\frac{d}{c})^2]} \ .$$

Note that Weinberg angle decreases monotonically with y. The choices for which c^2/D are equivalent but the parameters (a, b, c, d) can be chosen nearer to integers for large enough D.

2. How to fix the parameters D, c, d?

   (a) The first guess $D = ad - bc = 1$ would reduces the unknown parameters to c, d. This does not however allow even approximately integer valued parameters a, b, cd.

   (b) The GUT value of Weinberg angle at this limit is $sin^2(\theta_W) = 3/8$. TGD suggests that the values of Weinberg angle correspond to Pythagorean triangles (see `http://tinyurl.com/o7c4pkt`). The lowest primitive Pythagorean triangle (side lengths are coprimes, (see `http://tinyurl.com/j6ojlko`) corresponds to the triplet (3,4,9) forming the trunk of the 3-tree formed by the primitive Pythagorean triangles with 3 triangles emanating at each node) and to $sin^2(\theta_W) = 9/25$ slightly smaller than the GUT value. The problem is that y_0 is not a rational number and for rational values of c, d the equation for Weinberg angule cannot be satisfied.

   (c) An alternative more reliable option is to use as input Weinberg angle at intermediate boson length scale $k = 89$ which corresponds to $y(24) = 87.4252746$. The value of fine structure constant at Z^0 boson length scale is about $1/\alpha_{em}(89) \simeq 127$. From this one would obtain

   $$sin^2(\theta_W(k=89)) = 1 - \frac{y_{24}}{\alpha_{em}(89)} = 1 - \frac{\alpha_{U(1)}(24)}{\alpha_{em}(89)} \simeq 0.3116, \ . \tag{10.4.3}$$

   One can obviously criticize the rather large value of the Weinberg angle forced by the value of $y(24)$ as being smaller than the experimental value. Experiments however suggests that Weinberg angle starts to increase after Z^0 pole. Gauge theory limit corresponds to a limit at which the sheets of many-sheeted are lumped together and one obtains a statistical average and the contributions of longer scale might increase the value of $1/\alpha_{U(1)}(24)$ and therefore reduce the value of the effective Weinberg angle.

(d) Another input is the value of fine structure constant either at $k = 127$ corresponding to electron's p-adic length scale or at $k = 131$ ($L(131) = 10^{-11}$ meters and four times the p-adic length scale of electron) fixed by the condition that fine structure constant $\alpha_{em} = \alpha_{(U(1)}cos^2(\theta_W)$ corresponds its low energy value $1/\alpha_{em} = 137.035999139$ assigned often to electron length scale. From $y(32(= 1/\alpha_{U(1)} = 105.446623$ or $y(31) = 103.725538$ and $1/\alpha_{em}(131) = 137.035999139$ one can estimate the value of Weinberg angle as

$$\begin{aligned} sin^2(\theta_W(k=131)) &= 1 - \frac{y_{32}}{\alpha_{em}(131)} \simeq 0.23052 \text{ or} \\ sin^2(\theta_W(k=130)) &= 1 - \frac{y_{32}}{\alpha_{em}(127)} \ . \end{aligned} \tag{10.4.4}$$

It turns out that the first option does not work unless one assumes $1/alpha_{em}(k = 89) \leq 125.5263$ rather than $1/alpha_{em}(k = 89) \simeq 127$. The deviation is about 1-2 per cent. Second option works with a minimal modification for $1/alpha_{em}(k = 89) \simeq 127$.

(e) The value of $y(1)$ is $y_1 = 14.13472$. The two latter conditions give rise to the following series of equations

$$\begin{aligned} X(k) &= cot^2(\theta_W)(k) = \frac{c^2}{D}(y^2(k) + A) \ , \ A = \frac{1}{4} + \frac{d}{c} + (\frac{d}{c})^2 \ , \\ \frac{X(24)}{X(K)} &\equiv Y = \frac{cot^2(\theta_W)(24)}{cot^2(\theta_W)(K)} = \frac{y^2(24) + A}{y^2(K) + A} \ , \\ A &= \frac{Y(y^2(K) - y^2(24))}{1 - Y} \ . \end{aligned} \tag{10.4.5}$$

Here K is either $K = 31$ or $K = 32$ corresponding to the p-adic length scale $k = 127$ or 131. It turns out that only $K = 31$ works fo $1/\alpha_{em}(89) = 127$.

Also following parameters can be expressed in terms of the data.

$$\begin{aligned} \frac{c^2}{D} &= \frac{cot^2(\theta_W)(K)}{y^2(K) + A} \ , \\ \frac{d}{c} &= \frac{1}{2}\left(-1 + \sqrt{A}\right) \ , \\ sin^2(\theta_W)(1) &= \frac{1}{1 + X(1)} \ , \ X(1) = \frac{c^2}{D}\left(y^2(1) + A\right) \ . \end{aligned} \tag{10.4.6}$$

If the parameters a, b, c, d are integers, the equations cannot be satisfied exactly. For $K = 32$ it turns out that parameter A is negative for $1/alpha_{em}(k = 89) \leq 125.5263$. For $K = 31$ still negative and small so that $A = 0$ is the natural choice breaking slightly the conditions. **Table 10.1** represent both options.

(f) For $D = 1$ one has $c^2 \simeq 0.0002894$, which is very near to zero and not an integer. c must be non-vanishing to obtain a running Weinberg angle. For the general value of D the role c is taken by c^2D as an invariant fixed by the input data. $c \to c = 2$ requires $D = 1 \to int(4/c^2) = 138$. $D = 139$ almost equally good. One has $d/c = -0.5$ for $A = 0$ so that one would have $d = -1, c = 2$ for mimimum option. The condition $ad - bc = -a - 2b = D$ allows to estimate the values of the integer valued parameters a and b and get additional constraint on integer D. The values are not completely unique without additional conditions, say $b = 1$. This would give $a = -D + 2 = -137$ for $D = 139$ (one cannot avoid association with the famous "137"!).

3. Consider now the physical predictions. The evolution of Weinberg angle is depicted in the tables **10.1** and **10.2** for $k = 127$ model whereas tables **10.3** and **10.4** give the predictions of $k = 131$ model. The value of Weinberg angle at electron scale $k = 127$ is predicted to be $sin^2(\theta_w) \simeq 0.2430$ deviating from its measured value by 5 per cent. For $k = 131$ the Weinberg angle deviates .7 per cent from the measured value but the value of $1/\alpha_{em}(k = 89)$ is about 1 per cent too small.

   The expression for the predicted value of Weinberg angle at p-adic length scale $p = 2$ is $sin^2(\theta_W)_{p=2} \simeq 0.9453368487$, which is near to its maximal value and much larger than the $sin^2(\theta_W)_{p=2} \simeq 0.375$ of GUTs. This prediction was a total surprise but could be consistent with the new physics predicted by TGD predicting several scaled up copies of hadron physics above weak scale.

   A related surprise at the high energy end was that $1/\alpha_{em}$ begins to increase again at $k = 13$ and is near to fine structure constant at $k = 11$! As if asymptotic freedom would apply to all couplings except $U(1)$ coupling. This behavior is due to the approach of $cos^2(\theta_W)$ to zero. One can of course ask whether $sin^2(\theta_W) = 1$ could be obtained for a suitable choice of the parameters. This can be achieved only for $y(1) = 0$ which is not possible since A the parameter A cannot be negative.

To sum up, experimental input allows to fix electroweak coupling constant evolution completely. The problematic feature of $k = 127$ model is the possibly too large value of Weinberg theta at low energies. The predicted scaled up copies of hadron physics could explain why Weinberg angle must increase at high energies. At electron length scale the 5 per cent too high value is somewhat disturbing. The many-sheeted space-time requiring lumping together of sheets to get space-time of General Relativity might help to understand why measured Weinberg angle is smaller than predicted. Average over sheets of different sizes could be in question.

10.4.2 Test For The Model Of Electroweak Coupling Constant Evolution

One can check whether the values of 100 lowest non-trivial zeros are consistent with their assignment with primes k in $p \simeq 2^k$ and whether the model is consistent with the value of fine structure constant $1/\alpha_{em} = 137.035999139$ and experimental value $P = .2312$ of Weinberg angle assigned either with electron's p-adic length scale $k = 127$ or $k = 131$ (0.1 Angstroms).

The tables below summarize the values of $1/\alpha_K$ identified as imaginary part of Riemann zero and $\alpha_{em} = \alpha_K(1 - P)$ for the model already discussed. P is .7 per cent smaller than the experimental value $P = .2312$ for $k = 131$. This agreement is excellent but it turns out that the model works only if fine structure constant corresponds to $\alpha_{em}(k)$ in electron length scale $k = 127$.

For $k = 127$ one obtains fine structure constant correctly for $P = 0.243078179077$ about 10 per cent larger than the experimental value. The predicted value of α_K at scale $k = 127$ changes from $\alpha_K = \alpha_{em}$ to $\alpha(U(1))$ due the presence of $cos^2(\theta_W) = .77$. One can wonder whether this is consistent with the p-adic mass calculations and the condition on CP_2 coming from the string tension of cosmic strings.

The predicted value of α_K changes at electron length scale by the introduction of $cos(theta_W)$ factor. The formula for the p-adic mass squared involves second order contribution which cannot be predicted with certainty. This contribution is 20 per cent at maximum so that the change of α_K by 10 per cent can be tolerated.

Galactic rotation velocity spectrum gives also constraint on the string tension of cosmic strings and in this manner also to the value of the inverse $1/R$ of CP_2 radius to which p-adic mass scales are proportional. The size scale or large voids corresponds roughly to $k = 293$. From **Table 10.2** one has $1/\alpha_K = 167.2$. If the condition $\alpha_K \simeq \alpha_{em}$ holds true in long length scales, the scaling of $1/\alpha_K = 1/\alpha_{em}$ used earlier would be given by $r \simeq 167/137$ and would increase the string tension of cosmic strings by factor 1.2. This could be compensated by scaling $R^2_{CP_2}$ by the same factor. CP_2 mass scale would be scaled by factor $1/\sqrt{1.2} \simeq .9$. Also this can be tolerated. Note that maximal value cosmic string tension is assumed making sense only for the ideal cosmic strings with 2-D M^4 projection. Thickening of cosmic strings reduces their tension since magnetic energy per length is reduced.

Table 10.1: Table represents the first 35 zeros of zeta identified as values of $\alpha_K = \alpha(U(1))$, the corresponding primes k ($p \simeq 2^k$), the predicted values of both Weinberg angle and of $\alpha_{em} = \alpha(U(1))cos^2(\theta_W)$ assuming the proposed model for $sin^2(\theta_W)$.

n	y	k	$sin^2(\theta_W)$	$1/\alpha_{em}$
hline 1	14.1347251	2	0.945336	258.5784
2	21.0220396	3	0.886600	185.3802
3	25.0108575	5	0.846706	163.1566
4	30.4248761	7	0.788698	143.9880
5	32.9350615	11	0.761068	137.8428
6	37.5861781	13	0.709786	129.5121
7	40.9187190	17	0.673584	125.3579
8	43.3270732	19	0.647955	123.0727
9	48.0051508	23	0.599889	119.9796
10	49.7738324	29	0.582401	119.1907
11	52.9703214	31	0.551851	118.1982
12	56.4462476	37	0.520249	117.6574
13	59.3470440	41	0.495203	117.5663
14	60.8317785	43	0.482855	117.6301
15	65.1125440	47	0.449024	118.1767
16	67.0798105	53	0.434344	118.5877
17	69.5464017	59	0.416691	119.2275
18	72.0671576	61	0.399493	120.0105
19	75.7046906	67	0.376117	121.3444
20	77.1448400	71	0.367315	121.9326
21	79.3373750	73	0.354389	122.8874
22	82.9103808	79	0.334500	124.5836
23	84.7354929	83	0.324876	125.5111
24	87.4252746	89	0.311321	126.9464
25	88.8091112	97	0.304627	127.7144
26	92.4918992	101	0.287691	129.8480
27	94.6513440	103	0.278326	131.1552
28	95.8706342	107	0.273213	131.9102
29	98.8311942	109	0.261303	133.7912
30	101.317851	113	0.251824	135.4198
31	103.725538	127	0.243078	137.0359
32	105.446623	131	0.237073	138.2133
33	107.168611	137	0.231264	139.4088
34	111.029535	139	0.218919	142.1486
35	111.874659	149	0.216337	142.7587

Table 10.2: Table represents the zeros y_n of zeta in the range $n \in [35, 70]$ identified as values of $\alpha_K = \alpha(U(1))$, the corresponding primes k ($p \simeq 2^k$), the predicted values of both Weinberg angle and of $\alpha_{em} = \alpha(U(1))cos^2(\theta_W)$ assuming the proposed model for $sin^2(\theta_W)$.

n	y	k	$sin^2(\theta_W)$	$1/\alpha_{em}$
hline 36	114.320220	151	0.209095	144.5436
37	116.226680	157	0.203677	145.9543
38	118.790782	163	0.196690	147.8767
39	121.370125	167	0.189990	149.8379
40	122.946829	173	0.186049	151.0495
41	124.256818	179	0.182861	152.0633
42	127.516683	181	0.175248	154.6123
43	129.578704	191	0.170659	156.2431
44	131.087688	193	0.167407	157.4452
45	133.497737	197	0.162390	159.3794
46	134.756509	199	0.159853	160.3964
47	138.116042	211	0.153349	163.1322
48	139.736208	223	0.150345	164.4624
49	141.123707	227	0.147838	165.6068
50	143.111845	229	0.144348	167.2548
51	146.000982	233	0.139481	169.6662
52	147.422765	239	0.137170	170.8597
53	150.053520	241	0.133037	173.0796
54	150.925257	251	0.131706	173.8183
55	153.024693	257	0.128579	175.6036
56	156.112909	263	0.124167	178.2452
57	157.597591	269	0.122123	179.5214
58	158.849988	271	0.120436	180.6009
59	161.188964	277	0.117374	182.6243
60	163.030709	281	0.115040	184.2239
61	165.537069	283	0.111970	186.4094
62	167.184439	293	0.110016	187.8511
63	169.094515	307	0.107811	189.5277
64	169.911976	311	0.106886	190.2468
65	173.411536	313	0.103056	193.3360
66	174.754191	317	0.101639	194.5256
67	176.441434	331	0.099898	196.0238
68	178.377407	337	0.097952	197.7472
69	179.916484	347	0.096444	199.1206
70	182.207078	349	0.094262	201.1698

Table 10.3: Table represents the first 35 zeros of zeta identified as values of $\alpha_K = \alpha(U(1))$, the corresponding primes k ($p \simeq 2^k$), the predicted values of both Weinberg angle and of $\alpha_{em} = \alpha(U(1))cos^2(\theta_W)$ assuming the $k = 131$ model for $sin^2(\theta_W)$.

n	y	k	$sin^2(\theta_W)$	$1/\alpha_{em}$
hline 1	14.1347251	2	0.943414	249.7949
2	21.0220396	3	0.882868	179.4744
3	25.0108575	5	0.841896	158.1927
4	30.4248761	7	0.782535	139.9074
5	32.9350615	11	0.754350	134.0732
6	37.5861781	13	0.702190	126.2089
7	40.9187190	17	0.665488	122.3238
8	43.3270732	19	0.639563	120.2072
9	48.0051508	23	0.591074	117.3933
10	49.7738324	29	0.573475	116.6964
11	52.9703214	31	0.542785	115.8544
12	56.4462476	37	0.511110	115.4580
13	59.3470440	41	0.486058	115.4744
14	60.8317785	43	0.473724	115.5892
15	65.1125440	47	0.439988	116.2700
16	67.0798105	53	0.425376	116.7369
17	69.5464017	59	0.407825	117.4423
18	72.0671576	61	0.390747	118.2878
19	75.7046906	67	0.367570	119.7045
20	77.1448400	71	0.358853	120.3232
21	79.3373750	73	0.346062	121.3225
22	82.9103808	79	0.326403	123.0862
23	84.7354929	83	0.316902	124.0459
24	87.4252746	89	0.303530	125.5263
25	88.8091112	97	0.296931	126.3164
26	92.4918992	101	0.280251	128.5057
27	94.6513440	103	0.271035	129.8435
28	95.8706342	107	0.266007	130.6152
29	98.8311942	109	0.254301	132.5350
30	101.317851	113	0.244992	134.1945
31	103.725538	127	0.236408	135.8390
32	105.446623	131	0.230518	137.0359
33	107.168611	137	0.224822	138.2504
34	111.029535	139	0.212726	141.0304
35	111.874659	149	0.210197	141.6489

Table 10.4: Table represents the zeros y_n of zeta in the range $n \in [35, 70]$ identified as values of $\alpha_K = \alpha(U(1))$, the corresponding primes k ($p \simeq 2^k$), the predicted values of both Weinberg angle and of $\alpha_{em} = \alpha(U(1))cos^2(\theta_W)$ assuming the $k = 131$ model for $sin^2(\theta_W)$.

n	y	k	$sin^2(\theta_W)$	$1/\alpha_{em}$
hline 36	114.320220	151	0.203108	143.4576
37	116.226680	157	0.197806	144.8861
38	118.790782	163	0.190972	146.8316
39	121.370125	167	0.184423	148.8150
40	122.946829	173	0.180571	150.0397
41	124.256818	179	0.177456	151.0641
42	127.516683	181	0.170022	153.6387
43	129.578704	191	0.165542	155.2850
44	131.087688	193	0.162368	156.4981
45	133.497737	197	0.157474	158.4494
46	134.756509	199	0.154999	159.4751
47	138.116042	211	0.148658	162.2333
48	139.736208	223	0.145730	163.5739
49	141.123707	227	0.143287	164.7270
50	143.111845	229	0.139887	166.3872
51	146.000982	233	0.135146	168.8158
52	147.422765	239	0.132897	170.0175
53	150.053520	241	0.128873	172.2522
54	150.925257	251	0.127578	172.9957
55	153.024693	257	0.124534	174.7923
56	156.112909	263	0.120242	177.4499
57	157.597591	269	0.118254	178.7336
58	158.849988	271	0.116613	179.8194
59	161.188964	277	0.113635	181.8541
60	163.030709	281	0.111367	183.4623
61	165.537069	283	0.108383	185.6594
62	167.184439	293	0.106483	187.1085
63	169.094515	307	0.104341	188.7935
64	169.911976	311	0.103443	189.5162
65	173.411536	313	0.099722	192.6201
66	174.754191	317	0.098346	193.8152
67	176.441434	331	0.096655	195.3201
68	178.377407	337	0.094766	197.0512
69	179.916484	347	0.093302	198.4305
70	182.207078	349	0.091184	200.4884

Chapter 11

Could $\mathcal{N} = 2$ Super-conformal Theories Be Relevant For TGD?

11.1 Introduction

The concrete realization of the super-conformal symmetry (SCS) in TGD framework has remained poorly understood. In particular, the question how SCS relates to super-conformal field theories (SCFTs) has remained an open question. The most general super-conformal algebra assignable to string world sheets by strong form of holography has $\mathcal{N}$ equal to the number of 4+4 =8 spin states of leptonic and quark type fundamental spinors but the space-time SUSY is badly broken for it. Covariant constancy of the generating spinor modes is replaced with holomorphy - kind of "half covariant constancy". I have considered earlier a proposal that $\mathcal{N} = 4$ SCA could be realized in TGD framework but given up this idea. Right-handed neutrino and antineutrino are excellent candidates for generating $\mathcal{N} = 2$ SCS with a minimal breaking of the corresponding space-time SUSY. Covariant constant neutrino is an excellent candidate for the generator of $\mathcal{N} = 2$ SCS. The possibility of this SCS in TGD framework will be considered in the sequel.

11.1.1 Questions about SCS in TGD framework

This work was inspired by questions not related to $\mathcal{N} = 2$ SCS, and it is good to consider first these questions.

Could the super-conformal generators have conformal weights given by poles of fermionic zeta?

The conjecture [L16] is that the conformal weights for the generators super-symplectic representation correspond to the negatives of $h = -ks_k$ of the poles s_k fermionic partition function $\zeta_F(ks) = \zeta(ks)/\zeta(2ks)$ defining fermionic partition function. Here k is constant, whose value must be fixed from the condition that the spectrum is physical. $\zeta(ks)$ defines bosonic partition function for particles whos energies are given by $log(p)$, p prime. These partition functions require complex temperature but is completely sensible in Zero Energy Ontology (ZEO), where thermodynamics is replaced with its complex square root.

For non-trivial zeros $2ks = 1/2 + iy$ of $\zeta(2ks)$ s would correspond pole $s = (1/2 + iy)/2k$ of $z_F(ks)$. The corresponding conformal weights would be $h = (-1/2 - iy)/2k$. For trivial zeros $2ks = -2n$, $n = 1, 2, ..$ $s = -n/k$ would correspond to conformal weights $h = n/k > 0$. Conformal confinement is assumed meaning that the sum of imaginary parts of of generators creating the state vanishes.

What can one say about the value of k? The pole of $\zeta(ks)$ at $s = 1/k$ would correspond to pole and conformal weight $h = -1/k$. For $k = 1$ the trivial conformal weights would be positive integers $h = 1, 2, ...$: this certainly makes sense. This gives for the real part for non-trivial conformal weights $h = -1/4$. By conformal confinement both pole and its conjugate belong to the state so that this contribution to conformal weight is negative half integers: this is consistent with the facts about super-conformal representations. For the ground state of super-conformal representation the conformal weight for conformally confined state would be $h = -K/2$. In p-adic mass calculations one would have $K = 6$ [K39].

The negative ground state conformal weights of particles look strange but p-adic mass calculations require that the ground state conformal weights of particles are negative: $h = -3$ is required.

What could be the origin of negative ground state conformal weights?

Super-symplectic conformal symmetries are realized at light-cone boundary and various Hamiltonians defined analogs of Kac-Moody generators are proportional functions $f(r_M)H_{J,m}H_A$, where $H_{J,m}$ correspond to spherical harmonics at the 2-sphere $R_M = constant$ and H_A is color partial wave in CP_2, $f(r_M)$ is a partial wave in radial light-like coordinate which is eigenstate of scaling operator $L_0 = r_M d/dR_M$ and has the form $(r_M/r_0)^{-h}$, where h is conformal weight which must be of form $h = -1/2 + iy$.

To get plane wave normalization for the amplitudes

$$(\frac{r_M}{r_0})^{-h} = (\frac{r_M}{r_0})^{-1/2} exp(iyx) \ \ , \ \ x = log(\frac{r_M}{r_0}) \ \ ,$$

one must assume $h = -1/2+iy$. Together with the invariant integration measure dr_M this gives for the inner product of two conformal plane waves $exp(iy_i x)$, $x = log(r_M/r_0)$ the desired expression $\int exp[iy_1 - y_2)x]dx = \delta(y_1 - y_2)$, where $dx = dr_M/r_M$ is scaling invariance integration measure. This is just the usual inner product of plane waves labelled by momenta y_i.

If r_M/r_0 can be identified as a coordinate along fermionic string (this need not be always the case) one can interpret it as real or imaginary part of a hypercomplex coordinate at string world sheet and continue these wave functions to the entire string world sheets. This would be very elegant realization of conformal invariance.

How to relate degenerate representations with $h > 0$ to the massless states constructed from tachyonic ground states with negative conformal weight?

This realization would however suggest that there must be also an interpretation in which ground states with negative conformal weight $h_{vac} = -k/2$ are replaced with ground states having vanishing conformal weights $h_{vac} = 0$ as in minimal SCAs and what is regarded as massless states have conformal weights $h = -h_{vac} > 0$ of the lowest physical state in minimal SCAs.

One could indeed start directly from the scaling invariant measure dr_M/r_M rather than allowing it to emerge from dr_M. This would require in the case of p-adic mass calculations that has representations satisfying Virasoro conditions for weight $h = -h_{vac} > 0$. p-Adic mass squared would be now shifted downwards and proportional to $L_0 + h_{vac}$. There seems to be no fundamental reason preventing this interpretation. One can also modify scaling generator L_0 by an additive constant term and this does not affect the value of c. This operation corresponds to replacing basis $\{z^n\}$ with basis $\{z^{n+1/2}\}$.

What makes this interpretation worth of discussing is that the entire machinery of conformal field theories with non-vanishing central charge and non-vanishing but positive ground state conformal weight becomes accessible allowing to determine not only the spectrum for these theories but also to determine the partition functions and even to construct n-point functions in turn serving as basic building bricks of S-matrix elements [L22].

ADE classification of these CFTs in turn suggests at connection with the inclusions of hyperfinite factors and hierarchy of Planck constants. The fractal hierarchy of broken conformal symmetries with sub-algebra defining gauge algebra isomorphic to entire algebra would give rise to dynamic symmetries and inclusions for HFFs suggest that ADE groups define Kac-Moody type symmetry algebras for the non-gauge part of the symmetry algebra.

11.1.2 Questions about $\mathcal{N} = 2$ SCS

$\mathcal{N} = 2$ SCFTs has some inherent problems. For instance, it has been claimed that they reduce to topological QFTs. Whether $\mathcal{N} = 2$ can be applied in TGD framework is questionable: they have critical space-time dimension $D = 4$ but since the required metric signature of space-time is wrong.

Inherent problems of $\mathcal{N} = 2$ SCS

$\mathcal{N} = 2$ SCS has some severe inherent problems.

1. $\mathcal{N} = 2$ SCS has critical space-time dimension $D = 4$, which is extremely nice. On the other, $\mathcal{N} = 2$ requires that space-time should have complex structure and thus metric signature (4,0), (0,4) or (2,2) rather than Minkowski signature. Similar problem is encountered in twistorialization and TGD proposal is Hamilton-Jacobi structure (se the appendix of [K112]), which is hybrid of hypercomplex structure and Kähler structure. There is also an old proposal by Pope et al [B59]that one can obtain by a procedure analogous to dimensional reduction $\mathcal{N} = 2$ SCS from a 6-D theory with signature (3,3). The lifting of Kähler action to twistor space level allows the twistor space of M^4 to have this signature and the degrees of freedom of the sphere S^2 are indeed frozen.

2. There is also an argument by Eguchi that $\mathcal{N} = 2$ SCFTs reduce under some conditions to mere topological QFTs [B34]. Thi looks bad but there is a more refined argument that $\mathcal{N} = 2$ SCFT transforms to a topological CFT only by a suitable twist [B32, B57]. This is a highly attractive feature since TGD can be indeed regarded as almost topological QF. For instance, Kähler action in Minkowskian regions could reduce to Chern-Simons term for a very general solution ansatz. Only the volume term having interpretation in terms of cosmological constant [L22] (extremely small in recent cosmology) would not allow this kind of reduction. The topological description of particle reactions based on generalized Feynman diagrams identifiable in terms of space-time regions with Euclidian signature of the induced metric would allow to build n-point functions in the fermionic sector as those of a free field theory. Topological QFT in bosonic degrees of freedom would correspond naturally to the braiding of fermion lines.

Can one really apply $\mathcal{N} = 2$ SCFTs to TGD?

TGD version of SCA is gigantic as compared to the ordinary SCA. This SCA involves super-symplectic algebra associated with metrically 2-dimensional light-cone boundary (light-like boundaries of causal diamonds) and the corresponding extended conformal algebra (light-like boundary is metrically sphere S^2). Both these algebras have conformal structure with respect to the light-like radial coordinate r_M and conformal algebra also with respect to the complex coordinate of S^2. Symplectic algebra replaces finite-dimensional Lie algebra as the analog of Kac-Moody algebra. Also light-like orbits of partonic 2-surfaces possess this SCA but now Kac-Moody algebra is defined by isometries of imbedding space. String world sheets possess an ordinary SCA assignable to isometries of the imbedding space. An attractive interpretation is that r_M at light-cone boundary corresponds to a coordinate along fermionic string extendable to a hypercomplex coordinate at string world sheet.

$\mathcal{N} = 8$ SCS seems to be the most natural candidate for SCS behind TGD: all fermion spin states would correspond to generators of this symmetry. Since the modes generating the symmetry are however only half-covariantly constant (holomorphic) this SUSY is badly broken at space-time level and the minimal breaking occurs for $\mathcal{N} = 2$ SCS generated by right-handed neutrino and antineutrino.

The key motivation for the application of minimal $\mathcal{N} = 2$ SCFTs to TGD is that SCAs for them have a non-vanishing central charge c and vacuum weight $h \geq 0$ and the degenerate character of ground state allows to deduce differential equations for n-point functions so that these theories are exactly solvable. It would be extremely nice is scattering amplitudes were basically determined by n-point functions for minimal SCFTs.

A further motivation comes from the following insight. ADE classification of $\mathcal{N} = 2$ SCFTs is extremely powerful result and there is connection with the hierarchy of inclusions of hyperfinite factors of type II_1, which is central for quantum TGD. The hierarchy of Planck constants assignable to the hierarchy of isomorphic sub-algebras of the super-symplectic and related algebras suggest interpretation in terms of ADE hierarchy a rather detailed view about a hierarchy of conformal field theories and even the identification of primary fields in terms of critical deformations.

The application $\mathcal{N} = 2$ SCFTs in TGD framework can be however challenged. The problem caused by the negative value of vacuum conformal weight has been already discussed but there are also other problems.

1. One can argue that covariantly constant right-handed neutrino - call it ν_R - defines a pure gauge super-symmetry and it has taken along time to decide whether this is the case or not. Taking at face value the lacking evidence for space-time SUSY from LHC would be easy but too light-hearted manner to get rid of the problem.

   Could it be that at space-time level covariantly constant right-handed neutrino (ν_R) and its antiparticle ($\overline{\nu_R}$) generates pure gauge symmetry so that the resulting sfermions correspond to zero norm states? The oscillator operators for ν_R at imbedding space level have commutator proportional to $p^k\gamma_k$ vanishing at the limit of vanishing massless four-momentum. This would imply that they generate sfermions as zero norm states. This argument is however formulated at the level of imbedding space: induced spinor modes reside at string world sheets and covariant constancy is replaced by holomorphy!

   At the level of induced spinor modes located at string world sheets the situation is indeed different. The anti-commutators are not proportional to $p^k\gamma_k$ but in Zero Energy Ontology (ZEO) can be taken to be proportional to $n^k\gamma_k$ where n_k is light-like vector dual to the light-like radial vector of the point of the light-like boundary of causal diamond CD (part of light-one boundary) considered. Therefore also constant ν_R and $\overline{\nu_R}$ are allowed as non-zero norm states and the 3 sfermions are physical particles. Both ZEO and strong form of holography (SH) would play crucial role in making the SCS dynamical symmetry.

2. Second objection is that LHC has failed to detect sparticles. In TGD framework this objection cannot be taken seriously. The breaking of $\mathcal{N} = 2$ SUSY would be most naturally realized as different p-adic length scales for particle and sparticle. The mass formula would be the same apart from different p-adic mass scale. Sparticles could emerge at short p-adic length scale than those studied at LHC (labelled by Mersenne primes M_{89} and $M_{G,79} = (1+i)^79)$.

   One the other hand, one could argue that since covariantly constant right-handed neutrino has no electroweak-, color- nor gravitational interactions, its addition to the state should not change its mass. Again the point is however that one considers only neutrinos at string world sheet so that covariant constancy is replaced with holomorphy and all modes of right-handed neutrino are involved. Kähler Dirac equation brings in mixing of left and right-handed neutrinos serving as signature for massivation in turn leading to SUSY breaking. One can of course ask whether the p-adic mass scales could be identical after all. Could the sparticles be dark having non-standard value of Planck constant $h_{eff} = n \times h$ and be created only at quantum criticality [K106].

This is a brief overall view about the most obvious problems and proposed solution of them in TGD framework and in the following I will discuss the details. I am of course not a SCFT professional. I however dare to trust my physical intuition since experience has taught to me that it is better to concentrate on physics rather than get drowned in poorly understood mathematical technicalities.

11.2 Some CFT backround

The construction of CFTs involves as the first step construction of irreducible unitary representations of conformal algebras. They are completely known for the central charge $0 \leq c \leq 1$. One can also construct modular invariant partition functions for tensor products possibly serving as partition functions of CFTs. Already Belavin, Polyakov and Zhamolodzhikov [B23] discovered in their pioneering paper so called minimal models with the defining property that the state space realizes only finite number of irreducible representations.

11.2.1 Modular invariant partition functions

The classification of modular functions leads to the ADE scheme [B28] (http://tinyurl.com/h9val5g). The physical picture is that the primary fields of minimal CFT correspond to deformations of a critical system in some configuration space. One can construct all minimal orbifold CFTs in orbifolds $G\backslash C^2$ of C^2 in which the discrete subgroup G of $SU(2)$ acts linearly [B62]. This is a minimal realization. ADE scheme enters via the ADE classification for the discrete subgroups of $SU(2)$ (see http://tinyurl.com/jyjplzc).

ADE classification gives an amazingly detailed view about the spectrum of minimal models and also about their partition functions [B28] (see http://tinyurl.com/zlhk3wu). More general rational CFTs can possess infinite families of Virasoro representations, which an be however organized to representations of W-algebra. So called WZW models provide an important example constructible for any semi-simple Lie algebra.

The decomposition of RCFT Hilbert space to sum over tensor products of spaces carrying irreducible unitary representation conformal algebra and is conjugate can be written as

$$H = \oplus_{j\overline{j}} N_{j\overline{j}} H_j \otimes H_{\overline{j}} \quad . \tag{11.2.1}$$

There are consistency conditions on the coefficients due to the conditions that the CFT must exist on any Riemann surface. Verlinde algebra (see http://tinyurl.com/y8p9muj6) expresses the fusion rules. The associative Verlinde algebra is finite-dimensional and has as its elements primary fields and its structure constants code for the fusion rules. Especially interesting primary fields are those which are simple in the sense that the product of two primary fields contains only one prime field.

It is good to understand how one ends up with the expression of partition function in conformal field theories.

1. Start from the fact that conformal invariance fixes the complex function by data at 1-dimensional curve and one can speak about analog of time evolution in direction orthogonal to this curve. Introduce Hamiltonian for the Euclidian "time" evolution in finite "time" interval defining an annulus at 2-D surface with boundaries identified as initial and final times. Assume periodic boundary conditions in Euclidian "time" direction so that the annulus effectively closes to a torus. The outcome is a conformal field theory at torus although one starts from conformal invariance at sphere or even Riemann surface with higher genus.

2. Torus has several conformally inequivalent variants since it can be obtained from complex plane by identifying the points differing by a translation generated by real unit 1 and complex number τ. The possible values of τ defines the moduli space for conformal equivalence classes of torus since the angle angle between the sides of this elementary cell and the ratio of the lengths of homologically non-trivial geodesics of torus are conformal invariants. Modular invariance however implies that the values of τ differing by PSL(2,Z) transformation are equivalent.

3. What happens if one applies this procedure at higher genus surface? If the annulus is around the handle of this kind of surface, one might have a problem since it is not clear whether periodic boundary conditions can be identified in terms of a compactification to torus - this kind of annulus cannot be physically compactified to a torus. One can also consider a Hamiltonian evolution associated with any curve characterized by homology class telling how many times the curve winds around various handles. Can one just use the parameter τ or should one take into account the homology class of the annulus.

   One can challenge the idea about Hamiltonian time evolution as a formal trick and consider the possibility that partition functions is defined for the entire 2-surface in moduli space. In this kind of situation it would be trivial for sphere.

4. One can write explicitly the expression for the Euclidian "time" evolution operator between the ends of annulus as an exponential:

$$exp(-H_{cycl}L) = exp\left[2\pi i\tau(L_0 - \frac{c}{24}) - 2\pi i\overline{tau}(\overline{L}_0 - \frac{c}{24}\right] \quad . \tag{11.2.2}$$

Partition function is defined as the trace

$$Z(\tau) = Tr\left[exp(-H_{cycl}L)\right] \quad . \tag{11.2.3}$$

$$\chi_j(q) = Tr\left[exp\left[2\pi i\tau(L_0 - \tfrac{c}{24})\right]\right] = q^{h_j - \frac{c}{24}} \sum m_n q^n \quad , \quad q = exp(i2\pi\tau) \quad , \quad \overline{q} = exp(-i2\pi\overline{\tau}) \tag{11.2.4}$$

5. The decomposition of Hilbert space translates to a decomposition of the partition function as

$$Z(\tau) = \sum_{j\overline{j}} N_{j\overline{j}}\chi_j(q) \times \chi_{\overline{j}}(\overline{q}) \quad . \tag{11.2.5}$$

   Here one can wonder whether one could give up the interpretation in terms of Hamiltonian time evolution and consider just partition function in the moduli space of torus (or higher genus surface).

Modular invariance poses strong conditions of the expression of partition function of system as sum over products $chi_j\overline{\chi_{\overline{j}}}$ of characters assignable to irreducible unitary representations of Virasoro algebra. In the case of torus moduli correspond to complex plane whose points differing by a transformations by the discrete group SL(2,Z) are identified. The resulting moduli space has topology of torus. The generators of modular transformations are unit shift T: $\tau \rightarrow \tau + 1$ and inversion S: $\tau \rightarrow -1/\tau$ and it is enough to demand that the partition function is invariant under these transformations. The action of these transformations on characters induce an unitary automorphisms of the matrix $N_{j\overline{j}}$ and the condition is that the actions of S and T are trivial

$$TNT^{\dagger} = SNS^{\dagger} = N \quad . \tag{11.2.6}$$

It is interesting to relate this picture to TGD framework where one has string world sheets and partonic 2-surfaces.

1. The annulus picture applies to string world sheets. At the ends space-time surface at boundaries of CD one has fermionic strings connecting wormhole throat to another one along the first space-time sheet and returning back along second space-time sheet and forming thus a closed string, whose time evolution defines string space-time sheet as a cylindrical object. The strings at the ends of CD can get knotted and braided. They can also reconnect - the interpretation is in terms of standard stringy vertices. In fact this gives rise to 2-braiding possible because space-time dimension is 4.

   One can also consider loops as handles attached to these annuli: since the induced metric is allowed to have Euclidian signature, they are in principle possible but involve always Euclidian regions around points, where the time direction of closed homologically trivial time loop defined by the time coordinate of Minkowski space changes. Preferred extremal property might forbid loop corrections in Minkowskian space-time regions but allow them inside Euclidian regions representing lines of scattering diagrams.

2. The moduli space for the conformal equivalence classes of partonic 2-surfaces is important in the TGD based model for family replication phenomenon [K12]. In TGD context one must construct modular invariant partition functions in these higher-dimensional moduli spaces - I call them elementary particle vacuum functionals. These partition functions do not allow interpretation in terms of Hamiltonian time evolution.

11.2.2 Degenerate conformal representations and minimal models

So called degenerate representations allow to construct minimal models with finite number of primary fields and derive also differential equations for their correlation functions. Degeneracy condition fixes the spectrum of so called minimal conformal field theories.

1. The conformal weight the ground state is fixed to $h \geq 0$. Virasoro conditions must be satisfied: it is enough that the generators L_1 and L_2 annihilate the ground state. The defining feature of degenerate representations is that they possess states with zero norm created by generates with negative conformal weights from the ground state.

2. Degenerate states are obtained as linear combinations of states constructible using products $\prod_k L_{-k}^{-n_k}$, $N = \sum_k n_k k$ of generators with total conformal weight $-N$ operating on ground state with weight h. Degeneracy means that some combination of the generators with total weight $-N$ annihilates the state. Besides this ordinary Virasoro conditions for generators with positive weight are satisfied. The existence of the degenerate state means that the metric of this sub-state space is degenerate so that its determinant - so called Kac determinant vanishes. This brings strongly in mind criticality: at criticality sub-representation is isolated from the larger representation and defines zero norm states. These would correspond to zero modes appearing at criticality and not contributing to the potential function.

3. Vanishing of Kac determinant gives a condition allowing to deduce a general formula for the allowed values of the central charge c defining the central extension of conformal algebra. One can factorize Kac determinant to a product form $\prod_n (h - h_n)$ and the eigenvalues h_n defined the ground state weights allowing the degeneracy. Unitarity gives a further condition on the representation and for $c < 1$ this dictates the spectrum of vacuum conformal weights completely.

One can deduce an explicit expression for the Kac determinant as function of c and h and this gives rise to the following fundamental formulas [B28] (see `http://tinyurl.com/h9val5g`) for the values of central charge c and ground state conformal weight h for which the determinant vanishes. For $c > 1$ the determinant does not vanish and is positive. For $c < 1$ situation is different.

$$\begin{array}{llll} c = c_{p,q} = 1 - \frac{6(p-q)^2}{pq} \ , & p \ and \ q \ coprime \ , & p,q = 1,2,3,... \\ h = h_{r,s}(p,q) = \frac{[pr-qs]^2-(p-q)^2}{4pq} \ , & 1 \leq r \leq q-1 , & 1 \leq s \leq p-1 \ . \end{array} \tag{11.2.7}$$

For these values of c and h the representation defined by dividing away zero norm states is irreducible and unitary. So called minimal models forming a special case of them and possessing finite number of primary fields correspond to these representations.

Why the degeneracy is so important? Suppose that primary conformal fields Φ_k have conformal weight h and satisfy the degeneracy condition. Then n-point functions satisfy also the appropriate form of the degeneracy condition being annihilated by the combination of Virasoro generators with total weight $-N$. This gives rise to n partial differential equations of order N for $\langle \Phi(z_1)...\Phi(z_n)\rangle$ allowing to solve the conformal field theory exactly. In TGD this generalizes would give a powerful tool to determine the correlation functions at string world sheets.

The standard example is provided by the $N = 2$ case. The operator $O = L_{-2} - \frac{3}{2h+1} L_{-1}^2$ generates from the ground state with conformal weight h zero norm state provided the condition $c = 2h(5-8h)/(2h+1)$ is satisfied. For $h = 1/2$ this gives $c = 1/2$. Primary fields of the CFT are annihilated by this operator as also n-point functions and this gives second order differential equations for the n-point functions.

If the proposed interpretation of negative conformal weights in TGD framework is correct then one can add the condition $h = K/2$ to the conditions fixing c and h. Although SCFT rather than CFT is expected to be interesting from TGD point of view, one can just for fun see the above conditions for c and h allow $h = K/2$. Direct calculation for $p = m, q = m+1$ shows that for $m = 4$ ($c = 1/2$), $x = 1$ and $x = 1/2$ are realized for ($r = 3, s = 1$) and ($r = 3, s = 2$) respectively. For $m = 5$ one obtains $x = 3$ corresponding to $r = 4$ and $s = 1$. For $m = 6$ one obtains $x = 5$. It is not clear $(p,q) = (m, m+1)$ allows to realize $h = K/2$ or even $h = 5/2$ and $h = 2$.

11.2.3 Minimal $\mathcal{N}=2$ SCFTs

$\mathcal{N}=2$ SCA

$\mathcal{N}=2$ SCA is spanned by Virasoro generators L_n and their super counterparts G_r, where r is either integer (Ramond) or half-odd integer (Neveu-Schwartz) plus generators of conserved U(1) current J (se `http://tinyurl.com/yblzbovb`). Ramond and Neveu-Scwartz and these representations can be mapped to each other by spectral automorphism.

The commutation/anticommutation relations for $\mathcal{N}=2$ algebra are given by

$$\begin{array}{l} [L_m,L_n]=(m-n)L_{m+n}+\frac{c}{12}(m^3-m)\delta_{m+n,0} \ , \\ [L_m,J_n]=-nJ_{m+n} \ , \\ [J_m,J_n]=\frac{c}{3}m\delta_{m+n,0} \ , \\ \{G_r^+,G_s^-\}=L_{r+s}+\frac{1}{2}(r-s)J_{r+s}+\frac{c}{6}(r^2-\frac{1}{4})\delta_{r+s,0} \ , \\ \{G_r^+,G_s^+\}=0=\{G_r^-,G_s^-\} \ , \\ [L_m,G_r^{\pm}]=(\frac{m}{2}-r)G^{\pm}_{r+m} \ , \\ [J_m,G_r^{\pm}]=\pm G^{\pm}_{m+r} \ . \end{array} \tag{11.2.8}$$

Also in the case of SCFTs one it is natural to search for sub-representations with ground state weight h and annihilated by some generator of conformal weight $-N$. In this case the operatiors would be monomials of Virasoro generators and their super counterparts and also now the vanishing of Kac-determinant [B25], whose expression was deduced by Boucher, Friedan and Kent, would allow to deduce information about allowed values of c and h. Also in this case the n-point functions $\langle\Phi(z_1)...\Phi(z_n)$ satisfy N:th order the differential equations implied by the condition that the generator in question annihilates the primary fields.

Spectral automorphism mapping Ramond and N-S representations to each other

Spectral automorphism maps both the algebra and its representations to new ones. The spectral automorphism mapping Ramond representation to N-S representation is given by

$$\begin{array}{l} \alpha(L_n)=L_n+\theta J_n+\frac{\theta^2}{6}\delta_{n,0} \ , \\ \alpha(J_n)=J_n+\frac{\theta}{3}\delta_{n,0} \ , \\ \alpha(G_r^{\pm})=G^{\pm}_{r\pm\theta} \ . \end{array} \tag{11.2.9}$$

The inverse of the automorphism is given by

$$\begin{array}{l} \alpha^{-1}(L_n)=L_n-\theta J_n+\frac{\theta^2}{6}\delta_{n,0} \ , \\ \alpha^{-1}(J_n)=J_n-\frac{c}{3}\theta\delta_{n,0} \ , \\ \alpha^{-1}(G_r^{\pm})=G^{\pm}_{r\mp\theta} \ . \end{array} \tag{11.2.10}$$

For $\theta=1/2$ one obtains Ramond-NS spectral mapping.

Central extension term contains par linear in m. This is changed as one finds by calculating the commutators of the transformed Virasoro generators and expressing it in in terms of transformed generators. This does not affect the value of c. No change occurs for $k=2$ minimal representations with $Q=k/2(k+2)-1/4=0$. Also the term linear in m remains unaffected if the $\theta=1/2$ flow is modified to

$$\alpha(L_n) \quad = \quad L_n+\frac{1}{2}J_n+(\frac{1}{24}-\frac{Q_{N-S}}{2})\delta_{n,0} \ . \tag{11.2.11}$$

Also the ground state is changed in the spectral flow and Q_{N-S} labels the ground state charge for the resulting N-S representation. For minimal SCAs the flow must label $(h,Q)_R$ to Ramond state to $(h,Q)_{N-S}$.

If the linear term of central extension is unaffected in the flow, the values of h and Q change as follows:

$$
\begin{array}{l}
h_R \rightarrow h_{new,R} + \frac{c}{24} = h_{N-S} \ , \\
Q \rightarrow Q_{new,R} + \frac{c}{6} = Q_{N-S} \ .
\end{array}
\tag{11.2.12}
$$

The simplest guess is that the change leaves (a,b) unchanged and just drops the 1/8 term from h and Q. This condition determines the values of $h_{new,R}$ and $Q_{new,R}$ for minimal representations to

$$
\begin{array}{l}
h_{new,R} = \frac{1}{8} - \frac{c}{24} = \frac{1}{8} - \frac{k}{8(k+2)} \ , \\
Q_{new,R} = \frac{1}{4} - \frac{1}{2k(k+2)} \ .
\end{array}
\tag{11.2.13}
$$

Degenerate representations

The classification of unitary minimal super-conformal field theories is surprisingly well-understood [B65] (see `http://tinyurl.com/yctvyk2o`). ADE patterns are involved also in the classification of minimal SCFTs. The good news is that $\mathcal{N}=2$ superstrings have critical dimension $D=4$. The bad news is that the signature of the space-time metric is either (0,4), (2,2) or (4,0) rather than Minkowkian (1,3). This problem will be considered later in more detail.

I am not specialist and can only list the results. It is to be emphasized that not only the spectrum of basic parameters but also the partition functions are known, and correlation functions can be constructed.

1. The values of the central charge are given by

$$
c \ = \ \frac{3k}{k+2} \ , \ \ k=0,1,2...
\tag{11.2.14}
$$

   Central charge has values $c = 0, 1, 3/2, 9/5, ...$ and approaches $c=3$ for large values of k.

2. The vacuum conformal weights and U(1) charges depend on two integer valued parameters a,b besides k

$$
\begin{array}{lcl}
h_{ab} & = & \frac{a(a+2)-b^2}{4(k+2)} + \frac{(a+b)_2^2}{8} \ , \\
Q_{ab} & = & \frac{b}{2(k+2)} - \frac{(a+b)_2^2}{4} \ .
\end{array}
\tag{11.2.15}
$$

   Here the conditions

$$
a = 0,...,k \ , \quad |b-(a+b)_2| \leq a \ , \quad (a+b)_2 \equiv a+b \mod 2
\tag{11.2.16}
$$

   are satisfied. For Ramond type representations $(a+b)_2 = 1$ ($a+b$ is odd) is satisfied and for N-S type representations $(a+b)_2 = 0$ ($a+b$ is even) is satisfied. Note that $(h,Q) = (0,0)$ is possible only for $(a,b) = 0$ in the case of $N-S$ representation. For Ramond representation this would give $(h,Q) = (1/8, -1/4)$.

11.3 Could $\mathcal{N} = 2$ super-conformal algebra be relevant for TGD?

Despite various objections already discussed in the introduction there are good reasons to pose the question of the title.

11.3.1 How does the ADE picture about SCFTs and criticality emerge in TGD?

The crucial question in TGD framework is how the ADE picture relates to criticality and SCFTs in 2 dimensions. That the SCFT would be defined in 2 dimensions follows from SH.

1. The connection of ADE with inclusions of hyperfinite factors and with the hierarchy of Planck constants defining a hierarchy of dark matters are basic conjectures of TGD.

2. Finite number of degrees of freedom is left when a H_+ sub-algebra of super-symplectic or some other conformal algebra isomorphic to the entire algebra G_+ and the commutator $[H_+, G_+]$ ("+" refers to non-negative conformal weights) annihilate the states. The conjecture is that this gives rise to a finite-dimensional ADE type algebra defining Kac-Moody algebra or gauge algebra whose constant generators however act non-trivially. Denote the resulting finite-D ADE group by A_+. The Kac-Moody algebra might act on fermionic strings whereas the super-symplectic algebra would at at the boundary of CD.

3. At criticality a phase transition changing the value of Planck constant and thus H_+ and A_+ take place. These phase transitions would have a natural description in ZEO: the group ADE group A_+ would be smaller or larger an the other end of space-time surface at the opposite boundary of CD.

4. If the groups $A_{+,i}$ and $A_{+,f}$ satisfy $A_{+,i} \subset A_{+,f}$, new degrees of freedom appear. They correspond to the coset space $A_{+,f}/A_{+,i}$. Coset spaces typically form orbifolds: in fact the term orbi-fold comes from the identification of orbifold as the space of orbits, now those of $A_{+,i}$ in $A_{+,f}$. One would have orbifolds of ADE groups belonging associated with the hierarchy of inclusions labelled perhaps by Planck constants.

5. The orbifolds $O = A_{+,f}/A_{+,i}$ are however orbifolds of ADE groups, which are in 1-1 correspondence with the finite ADE subgroups G of SU(2). Does this mean that the orbifold $O = A_{+,f}/A_{+,i}$ is somehow determined by orbifold $G\backslash SU(2)$? As far as orbifold property is considered, $A_{+,i}$ would be effectively finite-D $G \subset SU(2)$. Mathematician could probably answer this question immediately.

   This kind of reduction of relevant degrees of freedom takes place in catastrophe theory, where only very few degrees of freedom determine the type of catastrophe: also in this case criticality is involved and catastrophes correspond to a hierarchy of criticalities.

6. The hierarchy of Planck constants corresponds to a hierarchy of coverings of space-time surface determined by strong form of holography by those for string world sheets. Could the discrete ADE groups G act in both the fibers and bases of these coverings?

Orbifoldings correspond to pairs of ADE groups appearing in the tensor product of representations. The first guess is that this is due to pairing of Ramond and N-S representations but ADE pairs appear also for conformal minimal models without super-symmetry. Second guess is that the tensor product pairing in TGD framework reflects the fact that one has always a pair of wormhole throats associated with the wormhole contact.

Concluding, it would be very natural to identify the orbifold degrees of in $O = A_{+,f}/A_{+,i}$ primary fields of minimal SCFT. This makes sense if the orbifolding reduces effectively to that for SU(2) by finite discrete subgroup.

11.3.2 Degrees of freedom and dynamics

$N = 2$SCA or should be generated by the addition of right-handed neutrino or antineutrino to one-fermion state. The interpretation as a pure gauge symmetry seems plausible. Instead of trying to make ad hoc guesses by searching the enormous highly technical literature on the subject, it is better to try to build the physical picture first and hope that professionals could get motivated to perform detailed constructions.

Consider first the degrees of freedom involved.

1. In bosonic sector one has at the fundamental level deformations of string world sheets (possibly of partonic 2-surfaces too). There are also deformations of string world sheets in CP_2 degrees of freedom: the latter could be assigned with electroweak gauge bosons and SU(3) Killing vectors related to color gauge potentials defining representation spaces for Kac-Moody algebras involved. $\mathcal{N} = 2$ SCA should determine correlation functions for these. At higher abstraction level the dynamical variables would correspond to representations of ADE groups assignable to inclusions of HFFs and primary fields would correspond to orbifolds of groups assignable to the hierarchy of Planck constants.

2. In M^4 degrees of freedom there are 2 degrees of freedom orthogonal to string world sheets which correspond to complex coordinate. They would give rise to 2 additional tensor factors to the super Virasoro algebra, which should have 5 tensor factors if p-adic mass calculations are taken at face value. $N = 2$SCA should have this number of tensor factors.

3. There are also fermionic degrees of freedom associated with the induced spinors at string world sheets and they would contribute to SCA too.

What one can say about the dynamics?

1. The dynamics at the level of physical particles would be essentially due to the non-trivial topological vertex in which 3 light-like 3-surfaces join along their ends. This dynamics would have huge symmetry generalizing the duality symmetry of hadronic string models: scattering diagram would be analogous to a computation with vertices having identification as algebraic operations and all computations connecting given sets of objects in initial and final state would be equivalent. This symmetry would allow to move the ends of internal lines so that loops could be transformed to tadpoles and snipped away giving a braided tree diagram as minimal scattering diagram. Something analogous to this happens for twistor Grassmann diagrams.

2. To the lines meeting at vertices defined by partonic 2-surfaces one can assign the fundamental four-fermion vertex [L22] defining second dynamics. This vertex does not however correspond to ordinary fermion vertex involving quartic term in fermion fields but corresponds to redistribution of fermion lines between the 3-legs. Therefore fermion dynamics would be free and this would allow to avoid divergences. The tensor net construction [L22] suggests for a very elegant description of these computations in terms of so called perfect tensors defining the nodes of the net and defining isometries between any leg and its complement with each leg involving unitary braiding operation.

3. The third dynamics would be at the level of Kähler action defined by the functional integral for the exponent of Kähler action. Quantum criticality motivates the proposal is that it is RG invariant in the sense that loop corrections vanish since Kähler coupling strength is analogous to critical temperature and is piecewise constant so that coupling constant evolution is discrete and the values fo α_K are labelled by a subset of p-adic primes.

11.3.3 Covariantly constant right-handed neutrinos as generators of super-conformal symmetries

As explained in the introduction, holomorphic right-handed neutrinos could generate the super-conformal symmetries with minimal breaking. Also other fermionic spin states (at imbedding base level) would generate super-conformal symmetries but they would be badly broken.

1. At imbedding space level massless modes of right-handed neutrino are covariantly constant in CP_2 and do not mix with left handed neutrinos. On the other hand, *induced* (as opposed to imbedding space -) right-handed neutrino spinors, which are not constant, mix with the left handed neutrino spinor modes and they are physical degrees of freedom. This follows from the mixing of the M^4 and CP_2 contributions to modified gamma matrices determined by the Kähler action and are essentially contractions of canonical momentum currents with imbedding space gamma matrices.

2. Induced spinor modes at string world sheets must carry vanishing weak W and possibly also Z fields to guarantee that em charge is well-defined. SH implies that the data at string world sheets are enough to construct the quantum theory. The assumption about localization is thus natural but not actually necessary, and it is not even clear whether Kähler-Dirac equation is really consistent with the localization at string world sheets although the special properties of Kähler Dirac gamma matrices (in particular, the degenerate character of the effective space-time metric defined by their anti-commutators) suggests this.

3. One must not forget that the conformal structure of solutions is extremely powerful and makes the situation almost independent of the Dirac action used. Dirac equation reduces essentially to holomorphy and to the condition that other half of the modified gamma matrices annihilate the spinor mode. One can therefore ask whether string world sheets could be minimal surfaces and whether Dirac equation in the induced metric could be satisfied at string world sheets. The trace of the second fundamental form giving rise to a term mixing M^4 chiralities vanishes in this case but there is still the mixing of gamma matrices inducing mixing of M^4 chiralities serving as a signal for massivation in M^4 sense.

4. The interpretation of $\mathcal{N} = 2$ supersymmetry possibly generated by right-handed neutrino has remained unresolved. As explained in the introduction, this problem disappears in ZEO since the boundary of CD allows anti-commutators of holomorphic ν_R oscillator operators to be non-vanishing also for constant mode and one obtains constant modes with non-vanishing norm to which space-time $\mathcal{N} = 2$ SUSY can be assigned.

5. A further complication is brought by the recent progress in twistorialization of Kähler action [L22]. It adds to the Kähler action extremely small volume term, and this term could spoil the idea about localization of the modes at string world sheets. Again the conformal structure of the solutions would save the situation if one does not require localization to string world sheets. The picture would be in accordance with SH.

11.3.4 Is $\mathcal{N} = 2$ SCS possible?

Could one assign $\mathcal{N} = 2$SCA with these degrees of freedom?

1. $\mathcal{N} = 2$SCA can be associated with any Super-Kac Moody algebra defined by simple Lie group by coset construction (see `http://tinyurl.com/yd2zqjvz`), in particular for $CP_2 = SU(3)/SU(2) \times U(1)$. The Kac-Moody algebra defined by the product of color group and electroweak group is not simple, but the fact that electroweak group holonomy group of CP_2 strongly suggests that $\mathcal{N} = 2$ SCA is possible. This would take care of color and electroweak degrees of freedom.

2. There are also 2 degrees of freedom corresponding to M^4 deformations of string world sheet orthogonal to the sheet. Free field construction would assign $\mathcal{N} = 2$ to the degrees of freedom orthogonal to the string world sheet but the central charge is $c = 3 > 3k/(k + 2)$ for the unitary $\mathcal{N} = 2$ SCFTs. Personally I do not see any reason why one could not have tensor product of several $\mathcal{N} = 2$SCAs with different central charges.

There are some objections against the idea of understanding the correlation functions of this dynamics in terms of $\mathcal{N} = 2$ SCA.

1. $\mathcal{N} = 2$ SCA is claimed to require (2,2) signature for the metric of the target space in stringy realization: in Minkowskian resp. Euclidian space-time regions the induced metric

has signature (1,-1,-1,-1) resp.(-1,-1,-1,-1). To my best understanding the target space is associated with one particular realization so that this objection need not be crucial. Note that also in twistor Grassmann approach (2,2) signature plays also special role making things well-defined whereas in other signature one must apply Wick-rotation.

2. There is also an argument that $\mathcal{N}=2$ SCFTs reduce to topological QFTs. TGD is indeed almost topological QFT and inside the string world sheets one expects the S-matrix to reduce to braiding S-matrix. The non-triviality of the scattering amplitudes would come from topology: one could assign the points of n-points functions to the ends of different legs of the diagrams.

The minimal models seem however to have the same symmetries as TGD and could therefore give some idea about what might be expected. $h=K/2$ condition for the representations of degenerate representations of $\mathcal{N}=2$ SCA follows if h corresponds to the actual conformal weight of a massless state shifted to zero by redefinition of the scaling generator L_0 by shift $L_0 \rightarrow L_0-h$. In the alternative picture this shift would map vacuum state with vanishing conformal weight to that with negative conformal weight $-h$. If $-h$ is sum over conformal weights $-1/2$ for the "wave functions" at light-cone boundary are proportional to $r_M^{-1/2}$ factor then it must be negative half integer and one has $h=K/2$.

This picture conforms also with the hypothesis that the poles of fermionic zeta determine the conformal weights for the generators of super-conformal symmetry with physical states assumed to satisfy conformal confinement implying that the imaginary parts of generators of SCA remain hidden. Note that the number of generators for the SCAs would be infinite unlike for ordinary SCAs: this would be also due to the fact that symplectic group is infinite-dimensional. Conformal confinement allows how the reduction of the conformal algebra at string world sheets to the ordinary super-conformal algebra. Also thermalization would occur only for this algebra.

For these reasons it is interesting to look what one obtains now by applying $h=K/2$ condition

1. $\mathcal{N}=2$ super-conformal symmetry algebra (see `http://tinyurl.com/yd2zqjvz`) involving besides Virasoro generators also generators for U(1) current and their super-counterparts is a reasonable candidate in TGD framework where classical Kähler current is conserved. The addition of right-handed neutrino or its antiparticle is an excellent candidate for generating exact $\mathcal{N}=2$ space-time supersymmetry as super-gauge symmetry as already explained. The conservation of quark and lepton numbers however allows to consider badly broken conformal SUSY algebra with larger value of $\mathcal{N}$.

2. The infinite-D symplectic algebra replaces the Kac-Moody algebra at light-cone boundary. At the light-like orbits of partons one obtains the counterpart of Kac-Moody algebra associated with the isometries of H and holonomies of CP_2. One might hope that p-adic thermodynamics involving only super-Virasoro generators is not affected at all by these complications. The states of additional algebras would only define the ground states of the Kac-Moody typ Super-Virasoro representations assignable to string world sheets (no thermalization in super-symplectic nor Kac-Moody degrees of freedom would occur), and the quantum numbers in question would correspond to quantum numbers of massless particles with massive excitations having mass scale defined by CP_2 mass scale.

11.3.5 How to circumvent the signature objection against $\mathcal{N}=2$ SCFT?

As already noticed $\mathcal{N}=2$ SCA is claimed to require (2,2) signature for the metric of the target space in the stringy realization. The problem is that $\mathcal{N}=2$ super-conformal symmetry requires space-time to have complex structure. Could one circumvent this objection?

The first attempt is based on the observation that the notion of Kähler structure generalizes in TGD framework to what I have called Hamilton-Jacobi structure. This means that the complex structure is hybrid of hypercomplex structure in longitudinal tangent space M^2 and of ordinary complex structure in transversal space E^2. The signature poses also problem in the definition of twistor structure and is circumvented using this construction.

The second attempt is based on the twistor lift of Kähler action.

1. Pope et al [B59] (see http://tinyurl.com/jnon4fh) propose that one might start from 6-D theory space-time signature (1,1,1-1,1,-1) with $\mathcal{N}=2$ supersymmetry and perform kind of dimensional reduction freezing 2 time coordinates of a 6-D space to obtain $\mathcal{N}=2$ superstrings in the resulting effectively 4-dimensional space-time with signature (1,-1,-1,-1).

2. The twistor lift of TGD replaces space-time surface with its 6-D twistor space. One can choose the metric signature of the sphere S^2 having radius of order Planck constant defining the fiber of twistor space $M^4\times S^2$ to be (1,1) or (-1,-1). For (1,1) one obtains signature (1,1,1,-1,-1,-1). Dimensional reduction is involved and the analog for the freezing of S^2 time dimensions takes place. This suggests that one could have $\mathcal{N}=2$ symmetry at the level of twistor spaces of space-time surfaces.

3. These two approaches seem to be very closely related in TGD framework.

Third trial would be based on the idea that the signature of the effective metric defined by the anticommutators of the modified gamma matrices appearing in modified Dirac action takes care of the problem by giving signature (1,1,-1,-1) for the effective metric. The following argument does not support this option.

1. In Kähler-Dirac action the modified gamma matrices define effective space-time metric $G^{\alpha\beta}$ via their anticommutators. The physical role of $G^{\alpha\beta}$ has remained obscure. One has $G^{\alpha\beta}=T^{\alpha}_kT^{beta}_lh^{kl}$, where T^{α}_k is the canonical momentum current.

2. There are two contributions to T^{α}_k corresponding to Kähler action and extremely small volume term suggested by the twistor lift of Kähler action having interpretation in terms of cosmological constant. Let us write Kähler action density as $L_K=kJ^{\mu\nu}J_{mu\nu}\sqrt{g}/2$ and volume action density as $L_{vol}=K\sqrt{g}$. One can write T^{α}_k as

$$\begin{aligned} T^{\alpha}_{k} &= [T^{\alpha\beta}[g]g_{k\beta}+T^{\alpha\beta}[J]J_{k,\beta} \ , \\ g_{k\beta} &= h_{kl}\partial_\beta h^l \ \ , \ \ J_{k,\beta}=J_{kl}\partial_\beta h^l \ , \end{aligned} \tag{11.3.1}$$

The tensors appearing in this formula can be expressed in a concise notation as

$$\begin{aligned} T[g] &= T[K,g]+T[vol,g] \ , \\ T[K,g] &= \frac{\partial L_K}{\partial g}\equiv k[J\circ J-\frac{1}{4}Tr(J\circ J)\sqrt{g}\equiv T_{K,1}+T_{K,2} \ , \\ T[vol,g] &= \frac{\partial L_{vol}}{\partial g}=\frac{K}{2}g \ , \\ T[J] &= \frac{\partial L_K}{\partial J}=kJ\sqrt{g} \ , \end{aligned} \tag{11.3.2}$$

$\circ$ denotes product of tensors defined by contraction. $T^{\alpha\beta}[g]$ is energy momentum tensor and $T^{\alpha\beta}[J]=kJ^{\alpha\beta}$ is its analog coming from variations with respect to induced Kähler form. The following formulas will be used.

$$g_{k\mu}g^k_{\ \nu}=g_{\mu\nu} \ \ , \ \ g_{k\mu}J^k_{\ \nu}=J_{\mu\nu} \ \ , \ \ J_{k\mu}J^k_{\ \nu}=-s_{\mu\nu} \tag{11.3.3}$$

Here s refers to CP_2 metric. G can be written in compact notation as

$$
\begin{aligned}
&G = G[g,g] + G[J,J] + 2G[g,J] \ \ , \\
&G[g,g] = T \circ T \ \ , \\
&G[J,J] = -T[J] \circ s \circ T[J] = -k^2 J \circ s \circ J \times det(g) \ \ , \\
&G[g,J] = T \circ J \circ T[J] = kT \circ J \circ J \times \sqrt{g} = T \circ T_{K,1} \ \ .
\end{aligned}
\tag{11.3.4}
$$

The expression for Gboils down to

$$
\begin{aligned}
G &= 4T_{K,1} \circ T_{K,1} + 4T_{K,1} \circ T_{K,2} + T_{K,2} \circ T_{K,2} \\
&- kJ \circ s \circ J + KT_{K,1} + \frac{kK}{2} T_{1K} \\
&+ \frac{K^2}{4} g \ \ .
\end{aligned}
\tag{11.3.5}
$$

The terms are quartic, quadratic, and zeroth order in J. One should disentangle these terms and be able to see whether the signature of G could be (1,1,-1,-1) in the vicinity of string world sheets. I have not been able to identify any obvious mechanism.

11.3.6 The necessity of Kac-Moody algebra of $SU(2) \times U(1)$

An interesting observation [B62] (see `http://tinyurl.com/hdy66lt`) is that the central charge $c = 3k/(k+2)$ emerges by Sugawara construction of the (Super-)Virasoro algebra for $SU(2)$ for (Super-)Kac-Moody algebra with central charge k.

1. In the general case one has following expressions for the central charge c and ground state weight h of the Super Virasoro algebra associated with Super-Kac-Moody algebra

$$
\begin{aligned}
c &= \frac{kdim(G)}{k+g} \ \ , \\
h(\lambda) &= \frac{C(\lambda)}{2(k+g)} \ \ .
\end{aligned}
\tag{11.3.6}
$$

   C is Casimir operator in representation λ of G and g is the dual Coxeter number (half of the value of Casimir in fundamental representation).

2. If one accepts these formulas for c and h, the $\mathcal{N} = 2$ SUSY fixes Kac-Moody group to be $SU(2)$ or possibly electroweak $SU(2) \times U(1)$ as physical intuition suggests. The value $c = 3k_1/(k_1+1)$ requires $k = 2k_1$ and $h = K/2$ gives $C(\lambda) = j(j+1) = 2K(k_1+1)$.

3. What is the interpretation of $SU(2)$? Electroweak $SU(2)$ operating in fermionic electro-weak spin degrees of freedom is a natural candidate and would require and also allow the inclusion of also U(1) factor naturally identifiable as the U(1) charge of the $\mathcal{N} = 2$ SCFT. In fact, the detailed study of Ramond representations show that U(1) factor must contribute to the ground state conformal weight in order to satisfy $h = K/2$ condition.

11.3.7 $h=K/2$ condition for Ramond representations

The question is whether $h=K/2$ suggested by the conformal invariance for the radial coordinate at light-like boundary can be achieved for these representations. Consider first Ramond type representations.

1. The condition on the allowed values $h=K/2$ of the ground state conformal weight gives

$$
\begin{array}{l}
h_{ab} = \frac{a(a+2)-b^2}{4(k+2)} + \frac{1}{8} = \frac{K}{2} \ , \quad 0 \leq a \leq k \ , \quad b \leq a+1 \ , \\
Q_{ab} = \frac{b}{2(k+2)} - \frac{1}{4} \ .
\end{array}
\tag{11.3.7}
$$

   Also the value of U(1) charge is given.

2. A possible manner to get rid of the problematic 1/8 term is to assume

$$
-\frac{b^2}{4(k+2)} + \frac{1}{8} = 0
\tag{11.3.8}
$$

   satisfied under the conditions

$$
k = 2k_1 \ , \quad b^2 = k_1+1 \ .
\tag{11.3.9}
$$

   This fixes the spectrum of k_1 to values $0,3,8,15,24,35,...$ and non-negative integer b satisfying $|b-1|<a$ determines the value of k_1.

3. As a consequence, one obtains the condition

$$
\frac{a(a+2)}{4(k+2)} = \frac{K}{2} \ .
\tag{11.3.10}
$$

   This condition can be satisfied if one has

$$
a = k = K \ .
\tag{11.3.11}
$$

   Second option $a=k+2=K-2$ does not satisfy the condition $a \leq k$.

4. Altogether one obtains

$$
\begin{array}{l}
k = 2k_1 \ , \quad k_1 = b^2-1 \ , \quad a = k = K \leq k \ , \\
c = \frac{3k_1}{k_1+1} \ , \quad Q = \frac{1}{4}(\frac{1}{b}-1) \ .
\end{array}
\tag{11.3.12}
$$

   U(1) charge is quantized unless one as $b=1$ giving $k_1=0$ so that one has also $k=0$. One can ask whether the fractionization of U(1) charge could relate to the charge fractionization possibly related to the hierarchy of Planck constants and/or to the braid statistics. Should one require that physical states have integer charge? Could conformal confinement imply vanishing of ground state U(1) charge automatically? This is is true if complex conjugate conformal weights correspond to opposite U(1) charges.

It is interesting to see whether this picture is consistent with the predictions of the $SU(2)\times U(1)$ Kac-Moody algebra option.

1. Ramond option corresponds naturally to the half-odd integers spin for the Super-Kac-Moody associated with SU(2) as will be found. For physical reasons one can expect that also U(1) tensor factor is present and adds to the vacuum conformal weight. From the general expression of the conformal weight one expects that the term 1/8 is this contribution.

   This would suggests the condition in SU(2) degrees of freedom in terms of half odd integer spin $j=(2r+1)/2$

$$\frac{a(a+2)-b^2}{4(k+2)} = \frac{a(a+2)}{4(k+2)}-\frac{1}{8} = \frac{(2r+1)(2r+3)}{8(k+2)} = \frac{K}{2}-\frac{1}{8} \ . \tag{11.3.13}$$

   This gives the conditions

$$2a(a+2)-k+2 = (2r+1)(2r+3) \ , \quad \frac{(2r+1)(2r+3)}{k+2} = 4K-1 \ . \tag{11.3.14}$$

   This condition can be satisfied if $k+2$ divides the numerator - say $(2r+1)$ or $(2r+3)$. The conclusion is that the U(1) factor must be present, which in turn supports the interpretation in terms of gauge group of electroweak interactions and extended holonomy group of CP_2 needed to obtain respectable spinor structure.

11.3.8 $h=K/2$ condition for N-S type representations

One can look the situation also for the N-S type representations. In this case one expects that spin is even. It is rather clear that the interpretation is in terms of sfermions is not correct. Spin for N-S states is even, which encourages the interpretation as bosonic states involving fermion and antifermion at same or opposite throats of wormhole contact.

1. The values of ground state conformal weight and U(1) charge are assumed to be given by

$$\begin{aligned} h_{ab} &= \tfrac{a(a+2)-b^2}{4(k+2)} = \tfrac{K}{4} \quad , \\ Q_{ab} &= \tfrac{b}{2(k+2)} \ . \end{aligned} \tag{11.3.15}$$

2. In the case of SU(2) Kac-Moody algebra one would have $h_{ab}=j(j+1)/2(k+2)$, which would give

$$a(a+2)-b^2 = 2j(j+1) \ , \quad \tfrac{j(j+1)}{k+2} = K \ . \tag{11.3.16}$$

   Two solutions of the latter equation are

   - $j=k+2$ giving $k=K-3$ and $j=K-1$
   - $j+1=k+2$ given $k=K-1$ and $j=K$.

   The values of $j$are integers as expected.

3. The condition $a(a+2) - b^2 = j(j+1)$ gives a further number theoretic constraint. Special solutions are $a = j - 1, b = 0$ and $a = j = b^2$.

To sum up, $\mathcal{N} = 2$ superconformal theories provide an attractive approach in attempts to gain a more detailed understanding of the super-conformal invariance at string world sheets. The fermionic n-point functions as restricted to string world sheets in turn could correspond to n-point functions for a CFT assignable to partonic 2-surfaces and one should understand the relationship between these two CFTs. More generally, strong form of holography allows to except CFT description for both the spin and orbital degrees of freedom of WCW and one should understand their relationship. It must be however emphasized that the actual SCA in TGD corresponds to the number $\mathcal{N} = \forall$ of spin states for H-spinors. The corresponding space-time SUSY is expected to be badly broken.

Part III

TWISTORS AND TGD

Chapter 12

TGD variant of Twistor Story

12.1 Introduction

Twistor Grassmannian formalism has made a breakthrough in $\mathcal{N} = 4$ supersymmetric gauge theories and the Yangian symmetry suggests that much more than mere technical breakthrough is in question. Twistors seem to be tailor made for TGD but it seems that the generalization of twistor structure to that for 8-D imbedding space $H = M^4 \times CP_2$ is necessary. M^4 (and S^4 as its Euclidian counterpart) and CP_2 are indeed unique in the sense that they are the only 4-D spaces allowing twistor space with Kähler structure.

The Cartesian product of twistor spaces $P_3 = SU(2,2)/SU(2,1) \times U(1)$ and F_3 defines twistor space for the imbedding space H and one can ask whether this generalized twistor structure could allow to understand both quantum TGD [K62, K71, K101] and classical TGD [K50] defined by the extremals of Kähler action.

In the following I summarize first the basic results and problems of the twistor approach. After that I describe some of the mathematical background and develop a proposal for how to construct extremals of Kähler action in terms of the generalized twistor structure. One ends up with a scenario in which space-time surfaces are lifted to twistor spaces by adding CP_1 fiber so that the twistor spaces give an alternative representation for generalized Feynman diagrams having as lines space-time surfaces with Euclidian signature of induced metric and having wormhole contacts as basic building bricks.

There is also a very close analogy with superstring models. Twistor spaces replace Calabi-Yau manifolds [A2, A90] and the modification recipe for Calabi-Yau manifolds by removal of singularities can be applied to remove self-intersections of twistor spaces and mirror symmetry [B21]emerges naturally. The overall important implication is that the methods of algebraic geometry used in super-string theories should apply in TGD framework.

The physical interpretation is totally different in TGD. Twistor space has space-time as base-space rather than forming with it Cartesian factors of a 10-D space-time. The Calabi-Yau landscape is replaced with the space of twistor spaces of space-time surfaces having interpretation as generalized Feynman diagrams and twistor spaces as sub-manifolds of $P_3 \times F_3$ replace Witten's twistor strings [B33]. The space of twistor spaces is the lift of the "world of classical worlds" (WCW) by adding the CP_1 fiber to the space-time surfaces so that the analog of landscape has beautiful geometrization.

The classical view about twistorialization of TGD makes possible a more detailed formulation of the previous ideas about the relationship between TGD and Witten's theory and twistor Grassmann approach.

1. The notion of quaternion analyticity extending the notion of ordinary analyticity to 4-D context is highly attractive but has remained one of the long-standing ideas difficult to take quite seriously but equally difficult to throw to paper basked. Four-manifolds possess almost quaternion structure. In twistor space context the formulation of quaternion analyticity becomes possible and relies on an old notion of tri-holomorphy about which I had not been aware earlier. The natural formulation for the preferred extremal property is as a condition stating that various charges associated with generalized conformal algebras vanish for preferred ex-

tremals. This leads to ask whether Euclidian space-time regions could be quaternion-Kähler manifolds for which twistor spaces are so called Fano spaces. In Minkowskian regions so called Hamilton-Jacobi property would apply.

2. The generalization of Witten's twistor theory to TGD framework is a natural challenge and the 2-surfaces studied defining scattering amplitudes in Witten's theory could correspond to partonic 2-surfaces identified as algebraic surfaces characterized by degree and genus. Besides this also string world sheets are needed. String worlds have 1-D lines at the light-like orbits of partonic 2-surfaces as their boundaries serving as carriers of fermions. This leads to a rather detailed generalization of Witten's approach using the generalization of twistors to 8-D context.

3. The generalization of the twistor Grassmannian approach to 8-D context is second fascinating challenge. If one requires that the basic formulas relating twistors and four-momentum generalize one must consider the situation in tangent space M^8 of imbedding space ($M^8 - H$ duality) and replace the usual sigma matrices having interpretation in terms of complexified quaternions with octonionic sigma matrices.

   The condition that octonionic spinors are are equivalent with ordinary spinors has strong consequences. Induced spinors must be localized to 2-D string world sheets, which are (co-)commutative sub-manifolds of (co-)quaternionic space-time surface. Also the gauge fields should vanish since they induce a breaking of associativity even for quaternionic and complex surface so that CP_2 projection of string world sheet must be 1-D. If one requires also the vanishing of gauge potentials, the projection is geodesic circle of CP_2 so that string world sheets are restricted to Minkowskian space-time regions. Although the theory would be free in fermionic degrees of freedom, the scattering amplitudes are non-trivial since vertices correspond to partonic 2-surfaces at which partonic orbits are glued together along common ends. The classical light-like 8-momentum associated with the boundaries of string world sheets defines the gravitational dual for 4-D momentum and color quantum numbers associated with imbedding space spinor harmonics. This leads to a more detailed formulation of Equivalence Principle which would reduce to $M^8 - H$ duality basically.

   Number theoretic interpretation of the positivity of Grassmannians is highly suggestive since the canonical identification maps p-adic numbers to non-negative real numbers. A possible generalization is obtained by replacing positive real axis with upper half plane defining hyperbolic space having key role in the theory of Riemann surfaces. The interpretation of scattering amplitudes as representations of permutations generalizes to interpretation as braidings at surfaces formed by the generalized Feynman diagrams having as lines the light-like orbits of partonic surfaces. This because 2-fermion vertex is the only interaction vertex and induced by the non-continuity of the induced Dirac operator at partonic 2-surfaces. OZI rule generalizes and implies an interpretation in terms of braiding consistent with the TGD as almost topological QFT vision. This suggests that non-planar twistor amplitudes are constructible as analogs of knot and braid invariants by a recursive procedure giving as an outcome planar amplitudes.

4. Yangian symmetry is associated with twistor amplitudes and emerges in TGD from completely different idea interpreting scattering amplitudes as representations of algebraic manipulation sequences of minimal length (preferred extremal instead of path integral over space-time surfaces) connecting given initial and final states at boundaries of causal diamond. The algebraic manipulations are carried out in Yangian using product and co-product defining the basic 3-vertices analogous to gauge boson absorption and emission. 3-surface representing elementary particle splits into two or vice versa such that second copy carries quantum numbers of gauge boson or its super counterpart. This would fix the scattering amplitude for given 3-surface and leave only the functional integral over 3-surfaces.

12.2 Background And Motivations

In the following some background plus basic facts and definitions related to twistor spaces are summarized. Also reasons for why twistor are so relevant for TGD is considered at general level.

12.2.1 Basic Results And Problems Of Twistor Approach

The author describes both the basic ideas and results of twistor approach as well as the problems.

Basic results

There are three deep results of twistor approach besides the impressive results which have emerged after the twistor resolution.

1. Massless fields of arbitrary helicity in 4-D Minkowski space are in 1-1 correspondence with elements of Dolbeault cohomology in the twistor space CP_3. This was already the discovery of Penrose..The connection comes from Penrose transform. The light-like geodesics of M^4 correspond to points of 5-D sub-manifold of CP_3 analogous to light-cone boundary. The points of M^4 correspond to complex lines (Riemann spheres) of the twistor space CP_3: one can imagine that the point of M^4 corresponds to all light-like geodesics emanating from it and thus to a 2-D surface (sphere) of CP_3. Twistor transform represents the value of a massless field at point of M^4 as a weighted average of its values at sphere of CP_3. This correspondence is formulated between open sets of M^4 and of CP_3. This fits very nicely with the needs of TGD since causal diamonds which can be regarded as open sets of M^4 are the basic objects in zero energy ontology (ZEO).

2. Self-dual instantons of non-Abelian gauge theories for SU(n) gauge group are in one-one correspondence with holomorphic rank-N vector bundles in twistor space satisfying some additional conditions. This generalizes the correspondence of Penrose to the non-Abelian case. Instantons are also usually formulated using classical field theory at four-sphere S^4 having Euclidian signature.

3. Non-linear gravitons having self-dual geometry are in one-one correspondence with spaces obtained as complex deformations of twistor space satisfying certain additional conditions. This is a generalization of Penrose's discovery to the gravitational sector.

Complexification of M^4 emerges unavoidably in twistorial approach and Minkowski space identified as a particular real slice of complexified M^4 corresponds to the 5-D subspace of twistor space in which the quadratic form defined by the SU(2,2) invariant metric of the 8-dimensional space giving twistor space as its projectivization vanishes. The quadratic form has also positive and negative values with its sign defining a projective invariant, and this correspond to complex continuations of M^4 in which positive/negative energy parts of fields approach to zero for large values of imaginary part of M^4 time coordinate.

Interestgingly, this complexification of M^4 is also unavoidable in the number theoretic approach to TGD: what one must do is to replace 4-D Minkowski space with a 4-D slice of 8-D complexified quaternions. What is interesting is that real M^4 appears as a projective invariant consisting of light-like projective vectors of C^4 with metric signature (4,4). Equivalently, the points of M^4 represented as linear combinations of sigma matrices define hermitian matrices.

Basic problems of twistor approach

The best manner to learn something essential about a new idea is to learn about its problems. Difficulties are often put under the rug but the thesis is however an exception in this respect. It starts directly from the problems of twistor approach. There are two basic challenges.

1. Twistor approach works as such only in the case of Minkowski space. The basic condition for its applicability is that the Weyl tensor is self-dual. For Minkowskian signature this leaves only Minkowski space under consideration. For Euclidian signature the conditions are not quite so restrictive. This looks a fatal restriction if one wants to generalize the result of Penrose to a general space-time geometry. This difficulty is known as "googly" problem.

   According to the thesis MHV construction of tree amplitudes of $\mathcal{N}=4$ SYM based on topological twistor strings in CP_3 meant a breakthrough and one can indeed understand also have analogs of non-self-dual amplitudes. The problem is however that the gravitational theory assignable to topological twistor strings is conformal gravity, which is generally regarded as non-physical. There have been several attempts to construct the on-shell scattering amplitudes of Einstein's gravity theory as subset of amplitudes of conformal gravity and also thesis considers this problem.

2. The construction of quantum theory based on twistor approach represents second challenge. In this respect the development of twistor approach to $\mathcal{N} = 4$ SYM meant a revolution and one can indeed construct twistorial scattering amplitudes in M^4.

12.2.2 Results About Twistors Relevant For TGD

First some background.

1. The twistors originally introduced by Penrose (1967) have made breakthrough during last decade. First came the twistor string theory of Edward Witten [B33] proposed twistor string theory and the work of Nima-Arkani Hamed and collaborators [B38] led to a revolution in the understanding of the scattering amplitudes of scattering amplitudes of gauge theories [B27, B26, B39]. Twistors do not only provide an extremely effective calculational method giving even hopes about explicit formulas for the scattering amplitudes of $\mathcal{N} = 4$ supersymmetric gauge theories but also lead to an identification of a new symmetry: Yangian symmetry [A27], [B30, B31], which can be seen as multilocal generalization of local symmetries.

   This approach, if suitably generalized, is tailor-made also for the needs of TGD. This is why I got seriously interested on whether and how the twistor approach in empty Minkowski space M^4 could generalize to the case of $H = M^4 \times CP_2$. The twistor space associated with H should be just the cartesian product of those associated with its Cartesian factors. Can one assign a twistor space with CP_2?

2. First a general result [A63] deserves to be mentioned: any oriented manifold X with Riemann metric allows 6-dimensional twistor space Z as an almost complex space. If this structure is integrable, Z becomes a complex manifold, whose geometry describes the conformal geometry of X. In general relativity framework the problem is that field equations do not imply conformal geometry and twistor Grassmann approach certainly requires conformal structure.

3. One can consider also a stronger condition: what if the twistor space allows also Kähler structure? The twistor space of empty Minkowski space M^4 (and its Euclidian counterpart S^4 is the Minkowskian variant of $P_3 = SU(2,2)/SU(2,1) \times U(1)$ of 3-D complex projective space $CP_3 = SU(4)/SU(3) \times U(1)$ and indeed allows Kähler structure.

   The points of the Euclidian twistor space $CP_3 = SU(4)/SU(3) \times U(1)$ can be represented by any column of the 4×4 matrix representing element of $SU(4)$ with columns differing by phase multiplication being identified. One has four coordinate charts corresponding to four different choices of the column. The points of its Minkowskian variant $CP_{2,1} = SU(2,2)/SU(2,1) \times U(1)$ can be represented in similar manner as U(1) gauge equivalence classes for given column of SU(3,1) matrices, whose rows and columns satisfy orthonormality conditions with respect to the hermitian inner product defined by Minkowskian metric $\epsilon = (1, 1, -1, -1)$.

   Rather remarkably, there are *no other space-times* with Minkowski signature allowing twistor space with Kähler structure [A63]. Does this mean that the empty Minkowski space of special relativity is much more than a limit at which space-time is empty?

   This also means a problem for GRT. Twistor space with Kähler structure seems to be needed but general relativity does not allow it. Besides twistor problem GRT also has energy problem: matter makes space-time curved and the conservation laws and even the definition of energy and momentum are lost since the underlying symmetries giving rise to the conservation laws through Noether's theorem are lost. GRT has therefore two bad mathematical problems which might explain why the quantization of GRT fails. This would not be surprising since quantum theory is to high extent representation theory for symmetries and symmetries are lost. Twistors would extend these symmetries to Yangian symmetry but GRT does not allow them.

4. What about twistor structure in CP_2? CP_2 allows complex structure (Weyl tensor is self-dual), Kähler structure plus accompanying symplectic structure, and also quaternion structure. One of the really big personal surprises of the last years has been that CP_2 twistor space indeed allows Kähler structure meaning the existence of antisymmetric tensor representing imaginary unit whose tensor square is the negative of metric in turn representing real unit.

   The article by Nigel Hitchin, a famous mathematical physicist, describes a detailed argument identifying S^4 and CP_2 as the only compact Riemann manifolds allowing Kählerian twistor space [A63]. Hitchin sent his discovery for publication 1979. An amusing co-incidence is that I discovered CP_2 just this year after having worked with S^2 and found that it does not really allow to understand standard model quantum numbers and gauge fields. It is difficult to avoid thinking that maybe synchrony indeed a real phenomenon as TGD inspired theory of consciousness predicts to be possible but its creator cannot quite believe. Brains at different side of globe discover simultaneously something closely related to what some conscious self at the higher level of hierarchy using us as instruments of thinking just as we use nerve cells is intensely pondering.

   Although 4-sphere S^4 allows twistor space with Kähler structure, it does not allow Kähler structure and cannot serve as candidate for S in $H = M^4 \times S$. As a matter of fact, S^4 can be seen as a Wick rotation of M^4 and indeed its twistor space is CP_3.

   In TGD framework a slightly different interpretation suggests itself. The Cartesian products of the intersections of future and past light-cones - causal diamonds (CDs) - with CP_2 - play a key role in ZEO (ZEO) [K4]. Sectors of "world of classical worlds" (WCW) [K34, K15] correspond to 4-surfaces inside $CD \times CP_2$ defining a the region about which conscious observer can gain conscious information: state function reductions - quantum measurements - take place at its light-like boundaries in accordance with holography. To be more precise, wave functions in the moduli space of CDs are involved and in state function reductions come as sequences taking place at a given fixed boundary. This kind of sequence is identifiable as self and give rise to the experience about flow of time. When one replaces Minkowski metric with Euclidian metric, the light-like boundaries of CD are contracted to a point and one obtains topology of 4-sphere S^4.

5. Another really big personal surprise was that there are *no other* compact 4-manifolds with Euclidian signature of metric allowing twistor space with Kähler structure! The imbedding space $H = M^4 \times CP_2$ is not only physically unique since it predicts the quantum number spectrum and classical gauge potentials consistent with standard model but also mathematically unique!

   After this I dared to predict that TGD will be the theory next to GRT since TGD generalizes string model by bringing in 4-D space-time. The reasons are many-fold: TGD is the only known solution to the two big problems of GRT: energy problem and twistor problem. TGD is consistent with standard model physics and leads to a revolution concerning the identification of space-time at microscopic level: at macroscopic level it leads to GRT but explains some of its anomalies for which there is empirical evidence (for instance, the observation that neutrinos arrived from SN1987A at two different speeds different from light velocity [**?**] has natural explanation in terms of many-sheeted space-time). TGD avoids the landscape problem of M-theory and anthropic non-sense. I could continue the list but I think that this is enough.

6. The twistor space of CP_2 is 3-complex dimensional flag manifold $F_3 = SU(3)/U(1) \times U(1)$ having interpretation as the space for the choices of quantization axes for the color hypercharge and isospin. This choice is made in quantum measurement of these quantum numbers and a means localization to single point in F_3. The localization in F_3 could be higher level measurement leading to the choice of quantizations for the measurement of color quantum numbers.

   F_3 is symmetric space meaning that besides being a coset space with $SU(3)$ invariant metric it also has involutions acting as a reflection at geodesics through a point remaining fixed under the involution. As a symmetric space with Fubini-Study metric F_3 is positive constant curvature space having thus positive constant sectional curvatures. This implies Einstein space property. This also conforms with the fact that F_3 is CP_1 bundle over CP_2 as base space (for more details see `http://tinyurl.com/ychdeqjz`).

The points of flag manifold $SU(3)/U(1)\times U(1)$ can be represented locally by identifying $SU(3)$ matrices for which columns differ by multiplication from left with exponential $e^{i(aY+bI_3)}$, a and b arbitrary real numbers. This transformation allows what might be called a "gauge choice". For instance, first two elements of the first row can be made real in this manner. These coordinates are not global.

7. Analogous interpretation could make sense for M^4 twistors represented as points of P_3. Twistor corresponds to a light-like line going through some point of M^4 being labelled by 4 position coordinates and 2 direction angles: what higher level quantum measurement could involve a choice of light-like line going through a point of M^4? Could the associated spatial direction specify spin quantization axes? Could the associated time direction specify preferred rest frame? Does the choice of position mean localization in the measurement of position? Do momentum twistors relate to the localization in momentum space? These questions remain fascinating open questions and I hope that they will lead to a considerable progress in the understanding of quantum TGD.

8. It must be added that the twistor space of CP_2 popped up much earlier in a rather unexpected context [K31]: I did not of course realize that it was twistor space. Topologist Barbara Shipman [A37] has proposed a model for the honeybee dance leading to the emergence of F_3. The model led her to propose that quarks and gluons might have something to do with biology. Because of her position and specialization the proposal was forgiven and forgotten by community. TGD however suggests both dark matter hierarchies and p-adic hierarchies of physics [K22, K106]. For dark hierarchies the masses of particles would be the standard ones but the Compton scales would be scaled up by $h_{eff}/h = n$ [K106]. Below the Compton scale one would have effectively massless gauge boson: this could mean free quarks and massless gluons even in cell length scales. For p-adic hierarchy mass scales would be scaled up or down from their standard values depending on the value of the p-adic prime.

12.2.3 Basic Definitions Related To Twistor Spaces

One can find from web several articles explaining the basic notions related to twistor spaces and Calabi-Yau manifolds. At the first look the notions of twistor as it appears in the writings of physicists and mathematicians don't seem to have much common with each other and it requires effort to build the bridge between these views. The bridge comes from the association of points of Minkowski space with the spheres of twistor space: this clearly corresponds to a bundle projection from the fiber to the base space, now Minkowski space. The connection of the mathematician's formulation with spinors remains still somewhat unclear to me although one can understand CP_1 as projective space associated with spinors with 2 complex components. Minkowski signature poses additional challenges. In the following I try my best to summarize the mathematician's view, which is very natural in classical TGD.

There are many variants of the notion of twistor depending on whether how powerful assumptions one is willing to make. The weakest definition of twistor space is as CP_1 bundle of almost complex structures in the tangent spaces of an orientable 4-manifold. Complex structure at given point means selection of antisymmetric form J whose natural action on vector rotates a vector in the plane defined by it by $\pi/2$ and thus represents the action of imaginary unit. One must perform this kind of choice also in normal plane and the direct sum of the two choices defines the full J. If one chooses J to be self-dual or anti-self-dual (eigenstate of Hodge star operation), one can fix J uniquely. Orientability makes possible the Hodge star operation involving 4-dimensional permutation tensor.

The condition $i^1 = -1$ is translated to the condition that the tensor square of J equals to $J^2 = -g$. The possible choices of J span sphere S^2 defining the fiber of the twistor spaces. This is not quite the complex sphere CP_1, which can be thought of as a projective space of spinors with two complex components. Complexification must be performed in both the tangent space of X^4 and of S^2. Note that in the standard approach to twistors the entire 6-D space is projective space P_3 associated with the C^8 having interpretation in terms of spinors with 4 complex components.

One can introduce almost complex structure also to the twistor space itself by extending the almost complex structure in the 6-D tangent space obtained by a preferred choice of J by identifying it as a point of S^2 and acting in other points of S^2 identified as antisymmetric tensors. If these points are interpreted as imaginary quaternion units, the action is commutator action divided by 2. The existence of quaternion structure of space-time surfaces in the sense as I have proposed in TGD framework might be closely related to the twistor structure.

Twistor structure as bundle of almost complex structures having itself almost complex structure is characterized by a hermitian Kähler form ω defining the almost complex structure of the twistor space. Three basic objects are involved: the hermitian form h, metric g and Kähler form ω satisfying $h = g + i\omega$, $g(X,Y) = \omega(X, JY)$.

In the base space the metric of twistor space is the metric of the base space and in the tangent space of fibre the natural metric in the space of antisymmetric tensors induced by the metric of the base space. Hence the properties of the twistor structure depend on the metric of the base space.

The relationship to the spinors requires clarification. For 2-spinors one has natural Lorentz invariant antisymmetric bilinear form and this seems to be the counterpart for J?

One can consider various additional conditions on the definition of twistor space.

1. Kähler form ω is not closed in general. If it is, it defines symplectic structure and Kähler structure. S^4 and CP_2 are the only compact spaces allowing twistor space with Kähler structure [A63].

2. Almost complex structure is not integrable in general. In the general case integrability requires that each point of space belongs to an open set in which vector fields of type (1, 0) or (0, 1) having basis ∂/∂_{z^k} and $\partial/\partial_{\overline{z}^k}$ expressible as linear combinations of real vector fields with complex coefficients commute to vector fields of same type. This is non-trivial conditions since the leading names for the vector field for the partial derivatives does not yet guarantee these conditions.

   This necessary condition is also enough for integrability as Newlander and Nirenberg have demonstrated. An explicit formulation for the integrability is as the vanishing of Nijenhuis tensor associated with the antisymmetric form J (see (`http://tinyurl.com/ybp9vsa5` and `http://tinyurl.com/y8j36p4m`). Nijenhuis tensor characterizes Nijenhuis bracket generalizing ordinary Lie bracket of vector fields (for detailed formula see `http://tinyurl.com/y83mbnso`).

3. In the case of twistor spaces there is an alternative formulation for the integrability. Curvature tensor maps in a natural manner 2-forms to 2-forms and one can decompose the Weyl tensor W identified as the traceless part of the curvature tensor to self-dual and anti-self-dual parts W^+ and W^-, whose actions are restricted to self-dual resp. antiself-dual forms (self-dual and anti-self-dual parts correspond to eigenvalue + 1 and -1 under the action of Hodge * operation: for more details see `http://tinyurl.com/ybkhj4np`). If W^+ or W^- vanishes - in other worlds W is self-dual or anti-self-dual - the assumption that J is self-dual or anti-self-dual guarantees integrability. One says that the metric is anti-self-dual (ASD). Note that the vanishing of Weyl tensor implies local conformal flatness (M^4 and sphere are obviously conformally flat). One might think that ASD condition guarantees that the parallel translation leaves J invariant.

   ASD property has a nice implication: the metric is balanced. In other words one has $d(\omega \wedge \omega) = 2\omega \wedge d\omega = 0$.

4. If the existence of complex structure is taken as a part of definition of twistor structure, one encounters difficulties in general relativity. The failure of spin structure to exist is similar difficulty: for CP_2 one must indeed generalize the spin structure by coupling Kähler gauge potential to the spinors suitably so that one obtains gauge group of electroweak interactions.

5. One could also give up the global existence of complex structure and require symplectic structure globally: this would give $d\omega = 0$. A general result is that hyperbolic 4-manifolds allow symplectic structure and ASD manifolds allow complex structure and hence balanced metric.

12.2.4 Why Twistor Spaces With Kähler Structure?

I have not yet even tried to answer an obvious question. Why the fact that M^4 and CP_2 have twistor spaces with Kähler structure could be so important that it could fix the entire physics? Let us consider a less general question. Why they would be so important for the classical TGD - exact part of quantum TGD - defined by the extremals of Kähler action [K7] ?

1. Properly generalized conformal symmetries are crucial for the mathematical structure of TGD [K15, K88, K14, K76]. Twistor spaces have almost complex structure and in these two special cases also complex, Kähler, and symplectic structures (note that the integrability of the almost complex structure to complex structure requires the self-duality of the Weyl tensor of the 4-D manifold).

   For years ago I considered the possibility that complex 3-manifolds of $CP_3 \times CP_3$ could have the structure of S^2 fiber space and have space-time surfaces as base space. I did not realize that these spaces could be twistor spaces nor did I realize that CP_2 allows twistor space with Kähler structure so that $CP_3 \times F_3$ looks a more plausible choice.

   The expectation was that the Cartesian product $CP_3 \times F_3$ of the two twistor spaces with Kähler structure is fundamental for TGD. The obvious wishful thought is that this space makes possible the construction of the extremals of Kähler action in terms of holomorphic surfaces defining 6-D twistor sub-spaces of $CP_3 \times F_3$ allowing to circumvent the technical problems due to the signature of M^4 encountered at the level of $M^4 \times CP_2$. It would also make the magnificent machinery of the algebraic geometry so powerful in string theories a tool of TGD. Here CP_3 could be replaced with its non-compact form and the problem is that one can have only compactification of M^4 for which metric is defined only modulo conformal scaling. There is however a problem: the compactified Minkowski space or its complexification has a metric defined only modulo conformal factor. This is not a problem in conformally invariant theories but becomes a problem if one wants to speak of induced metric.

   The next realization was that M^4 allows twistor bundle also in purely geometric sense and this bundle is just $T(M^4) = M^4 \times CP_2$. The two variants of twistor space would naturally apply at the level of momentum space and imbedding space.

2. Every 4-D orientable Riemann manifold allows a twistor space as 6-D bundle with CP_1 as fiber and possessing almost complex structure. Metric and various gauge potentials are obtained by inducing the corresponding bundle structures. Hence the natural guess is that the twistor structure of space-time surface defined by the induced metric is obtained by induction from that for $T(M^4) \times F_3$ by restricting its twistor structure to a 6-D (in real sense) surface of $T(M^4) \times F_3$ with a structure of twistor space having at least almost complex structure with CP_1 as a fiber. For the imbedding of the twistor space of space-time this requires the identification of S^2 fibers of $T(M^4)$ and F_3. If so then one can indeed identify the base space as 4-D space-time surface in $M^4 \times CP_2$ using bundle projections in the factors $T(M^4)$ and F_3.

3. There might be also a connection to the number theoretic vision about the extremals of Kähler action. At space-time level however complexified quaternions and octonions could allow alternative formulation. I have indeed proposed that space-time surfaces have associative of co-associative meaning that the tangent space or normal space at a given point belongs to quaternionic subspace of complexified octonions.

12.3 The Identification Of 6-D Twistor Spaces As Sub-Manifolds Of 12-D Twistor Space

How to identify the 6-D sub-manifolds with the structure of twistor space? Is this property all that is needed? Can one find a simple solution to this condition? What is the relationship of twistor spaces to the Calabi-Yau manifolds of super string models? In the following intuitive considerations of a simple minded physicist. Mathematician could probably make much more interesting comments.

12.3.1 Conditions For Twistor Spaces As Sub-Manifolds

Consider the conditions that must be satisfied using local trivializations of the twistor spaces. It will be assumed that the twistor space $T(M^4)$ is CP_3 or its Minkowskian variant. It has turned out that a more reasonable option $T(M^4) = M^4 \times CP_1$ is possible. The following consideration is however for CP_3 option. Before continuing let us introduce complex coordinates $z_i = x_i + iy_i$ resp. $w_i = u_i + iv_i$ for CP_3 resp. F_3.

1. 6 conditions are required and they must give rise by bundle projection to 4 conditions relating the coordinates in the Cartesian product of the base spaces of the two bundles involved and thus defining 4-D surface in the Cartesian product of compactified M^4 and CP_2.

2. One has Cartesian product of two fiber spaces with fiber CP_1 giving fiber space with fiber $CP_1^1 \times CP_1^2$. For the 6-D surface the fiber must be CP_1. It seems that one must identify the two spheres CP_1^i. Since holomorphy is essential, holomorphic identification $w_1 = f(z_1)$ or $z_1 = f(w_1)$ is the first guess. A stronger condition is that the function f is meromorphic having thus only finite numbers of poles and zeros of finite order so that a given point of CP_1^i is covered by CP_1^{i+1}. Even stronger and very natural condition is that the identification is bijection so that only Möbius transformations parametrized by SL(2, C) are possible.

3. Could the Möbius transformation $f : CP_1^1 \to CP_1^2$ depend parametrically on the coordinates z_2, z_3 so that one would have $w_1 = f_1(z_1, z_2, z_3)$, where the complex parameters a, b, c, d ($ad - bc = 1$) of Möbius transformation depend on z_2 and z_3 holomorphically? Does this mean the analog of local SL(2, C) gauge invariance posing additional conditions? Does this mean that the twistor space as surface is determined up to SL(2, C) gauge transformation?

   What conditions can one pose on the dependence of the parameters a, b, c, d of the Möbius transformation on (z_2, z_3)? The spheres CP_1 defined by the conditions $w_1 = f(z_1, z_2, z_3)$ and $z_1 = g(w_1, w_2, w_3)$ must be identical. Inverting the first condition one obtains $z_1 = f^{-1}(w_1, z_2, z_3)$. If one requires that his allows an expression as $z_1 = g(w_1, w_2, w_3)$, one must assume that z_2 and z_3 can be expressed as holomorphic functions of (w_2, w_3): $z_i = f_i(w_k)$, $i = 2, 3$, $k = 2, 3$. Of course, non-holomorphic correspondence cannot be excluded.

4. Further conditions are obtained by demanding that the known extremals - at least non-vacuum extremals - are allowed. The known extremals [K7] can be classified into CP_2 type vacuum extremals with 1-D light-like curve as M^4 projection, to vacuum extremals with CP_2 projection, which is Lagrangian sub-manifold and thus at most 2-dimensional, to massless extremals with 2-D CP_2 projection such that CP_2 coordinates depend on arbitrary manner on light-like coordinate defining local propagation direction and space-like coordinate defining a local polarization direction, and to string like objects with string world sheet as M^4 projection (minimal surface) and 2-D complex sub-manifold of CP_2 as CP_2 projection, . There are certainly also other extremals such as magnetic flux tubes resulting as deformations of string like objects. Number theoretic vision relying on classical number fields suggest a very general construction based on the notion of associativity of tangent space or co-tangent space.

5. The conditions coming from these extremals reduce to 4 conditions expressible in the holomorphic case in terms of the base space coordinates (z_2, z_3) and (w_2, w_3) and in the more general case in terms of the corresponding real coordinates. It seems that holomorphic ansatz is not consistent with the existence of vacuum extremals, which however give vanishing contribution to transition amplitudes since WCW ("world of classical worlds") metric is completely degenerate for them.

   The mere condition that one has CP_1 fiber bundle structure does not force field equations since it leaves the dependence between real coordinates of the base spaces free. Of course, CP_1 bundle structure alone does not imply twistor space structure. One can ask whether non-vacuum extremals could correspond to holomorphic constraints between (z_2, z_3) and (w_2, w_3).

6. The metric of twistor space is not Kähler in the general case. However, if it allows complex structure there is a Hermitian form ω, which defines what is called balanced Kähler form [A89]

satisfying $d(\omega \wedge \omega) = 2\omega \wedge d\omega = 0$: ordinary Kähler form satisfying $d\omega = 0$ is special case about this. The natural metric of compact 6-dimensional twistor space is therefore balanced. Clearly, mere CP_1 bundle structure is not enough for the twistor structure. If the Kähler and symplectic forms are induced from those of $CP_3 \times Y_3$, highly non-trivial conditions are obtained for the imbedding of the twistor space, and one might hope that they are equivalent with those implied by Kähler action at the level of base space.

7. Pessimist could argue that field equations are additional conditions completely independent of the conditions realizing the bundle structure! One cannot exclude this possibility. Mathematician could easily answer the question about whether the proposed CP_1 bundle structure with some added conditions is enough to produce twistor space or not and whether field equations could be the additional condition and realized using the holomorphic ansatz.

12.3.2 Twistor Spaces By Adding CP_1 Fiber To Space-Time Surfaces

The physical picture behind TGD is the safest starting point in an attempt to gain some idea about what the twistor spaces look like.

1. Canonical imbeddings of M^4 and CP_2 and their disjoint unions are certainly the natural starting point and correspond to canonical imbeddings of CP_3 and F_3 to $CP_3 \times F_3$.

2. Deformations of M^4 correspond to space-time sheets with Minkowskian signature of the induced metric and those of CP_2 to the lines of generalized Feynman diagrams. The simplest deformations of M^4 are vacuum extremals with CP_2 projection which is Lagrangian manifold.

   Massless extremals represent non-vacuum deformations with 2-D CP_2 projection. CP_2 coordinates depend on local light-like direction defining the analog of wave vector and local polarization direction orthogonal to it.

   The simplest deformations of CP_2 are CP_2 type extremals with light-like curve as M^4 projection and have same Kähler form and metric as CP_2. These space-time regions have Euclidian signature of metric and light-like 3-surfaces separating Euclidian and Minkowskian regions define parton orbits.

   String like objects are extremals of type $X^2 \times Y^2$, X^2 minimal surface in M^4 and Y^2 a complex sub-manifold of CP_2. Magnetic flux tubes carrying monopole flux are deformations of these.

   Elementary particles are important piece of picture. They have as building bricks wormhole contacts connecting space-time sheets and the contacts carry monopole flux. This requires at least two wormhole contacts connected by flux tubes with opposite flux at the parallel sheets.

3. Space-time surfaces are constructed using as building bricks space-time sheets, in particular massless exrremals, deformed pieces of CP_2 defining lines of generalized Feynman diagrams as orbits of wormhole contacts, and magnetic flux tubes connecting the lines. Space-time surfaces have in the generic case discrete set of self intersections and it is natural to remove them by connected sum operation. Same applies to twistor spaces as sub-manifolds of $CP_3 \times F_3$ and this leads to a construction analogous to that used to remove singularities of Calabi-Yau spaces [A89].

Physical intuition suggests that it is possible to find twistor spaces associated with the basic building bricks and to lift this engineering procedure to the level of twistor space in the sense that the twistor projections of twistor spaces would give these structure. Lifting would essentially mean assigning CP_1 fiber to the space-time surfaces.

1. Twistor spaces should decompose to regions for which the metric induced from the $CP_3 \times F_3$ metric has different signature. In particular, light-like 5-surfaces should replace the light-like 3-surfaces as causal horizons. The signature of the Hermitian metric of 4-D (in complex sense) twistor space is (1, 1, -1, -1). Minkowskian variant of CP_3 is defined as projective space $SU(2,2)/SU(2,1) \times U(1)$. The causal diamond (CD) (intersection of future and past directed light-cones) is the key geometric object in ZEO (ZEO) and the generalization to the intersection of twistorial light-cones is suggestive.

2. Projective twistor space has regions of positive and negative projective norm, which are 3-D complex manifolds. It has also a 5-dimensional sub-space consisting of null twistors analogous to light-cone and has one null direction in the induced metric. This light-cone has conic singularity analogous to the tip of the light-cone of M^4.

   These conic singularities are important in the mathematical theory of Calabi-You manifolds since topology change of Calabi-Yau manifolds via the elimination of the singularity can be associated with them. The S^2 bundle character implies the structure of S^2 bundle for the base of the singularity (analogous to the base of the ordinary cone).

3. Null twistor space corresponds at the level of M^4 to the light-cone boundary (causal diamond has two light-like boundaries). What about the light-like orbits of partonic 2-surfaces whose light-likeness is due to the presence of CP_2 contribution in the induced metric? For them the determinant of induced 4-metric vanishes so that they are genuine singularities in metric sense. The deformations for the canonical imbeddings of this sub-space (F_3 coordinates constant) leaving its metric degenerate should define the lifts of the light-like orbits of partonic 2-surface. The singularity in this case separates regions of different signature of induced metric.

   It would seem that if partonic 2-surface begins at the boundary of CD, conical singularity is not necessary. On the other hand the vertices of generalized Feynman diagrams are 3-surfaces at which 3-lines of generalized Feynman digram are glued together. This singularity is completely analogous to that of ordinary vertex of Feynman diagram. These singularities should correspond to gluing together 3 deformed F_3 along their ends.

4. These considerations suggest that the construction of twistor spaces is a lift of construction space-time surfaces and generalized Feynman diagrammatics should generalize to the level of twistor spaces. What is added is CP_1 fiber so that the correspondence would rather concrete.

5. For instance, elementary particles consisting of pairs of monopole throats connected buy flux tubes at the two space-time sheets involved should allow lifting to the twistor level. This means double connected sum and this double connected sum should appear also for deformations of F_3 associated with the lines of generalized Feynman diagrams. Lifts for the deformations of magnetic flux tubes to which one can assign CP_3 in turn would connect the two F_3s.

6. A natural conjecture inspired by number theoretic vision is that Minkowskian and Euclidian space-time regions correspond to associative and co-associative space-time regions. At the level of twistor space these two kinds of regions would correspond to deformations of CP_3 and F_3. The signature of the twistor norm would be different in this regions just as the signature of induced metric is different in corresponding space-time regions.

   These two regions of space-time surface should correspond to deformations for disjoint unions of CP_3s and F_3s and multiple connected sum form them should project to multiple connected sum (wormhole contacts with Euclidian signature of induced metric) for deformed CP_3s. Wormhole contacts could have deformed pieces of F_3 as counterparts.

There are interesting questions related to the detailed realization of the twistor spaces of space-time surfaces.

1. In the case of CP_2 J would naturally correspond to the Kähler form of CP_2. Could one identify J for the twistor space associated with space-time surface as the projection of J? For deformations of CP_2 type vacuum extremals the normalization of J would allow to satisfy the condition $J^2 = -g$. For general extremals this is not possible. Should one be ready to modify the notion of twistor space by allowing this?

2. Or could the associativity/co-associativity condition realized in terms of quaternionicity of the tangent or normal space of the space-time surface guaranteeing the existence of quaternion units solve the problem and J could be identified as a representation of unit quaternion? In this case J would be replaced with vielbein vector and the decomposition 1+3 of the tangent space implied by the quaternion structure allows to use 3-dimensional permutation symbol

to assign antisymmetric tensors to the vielbein vectors. Also the triviality of the tangent bundle of 3-D space allowing global choices of the 3 imaginary units could be essential.

3. Does associativity/co-associativity imply twistor space property or could it provide alternative manner to realize this notion? Or could one see quaternionic structure as an extension of almost complex structure. Instead of single J three orthogonal J: s (3 almost complex structures) are introduced and obey the multiplication table of quaternionic units? Instead of S^2 the fiber of the bundle would be $SO(3) = S^3$. This option is not attractive. A manifold with quaternionic tangent space with metric representing the real unit is known as quaternionic Riemann manifold and CP_2 with holonomy U(2) is example of it. A more restrictive condition is that all quaternion units define closed forms: one has quaternion Kähler manifold, which is Ricci flat and has in 4-D case Sp(1)=SU(2) holonomy. (see `http://tinyurl.com/y9qtoebe`).

4. Anti-self-dual property (ASD) of metric guaranteeing the integrability of almost complex structure of the twistor space implies the condition $\omega \wedge d\omega = 0$ for the twistor space. What does this condition mean physically for the twistor spaces associated with the extremals of Kähler action? For the 4-D base space this property is of course identically true. ASD property need of course not be realized.

12.3.3 Twistor Spaces As Analogs Of Calabi-Yau Spaces Of Super String Models

CP_3 is also a Calabi-Yau manifold in the strong sense that it allows Kähler structure and complex structure. Witten's twistor string theory considers 2-D (in real sense) complex surfaces in twistor space CP_3 or its Minkowskian variant. This choice does not however seem to be natural from the point of view of the induced geometry although it looks natural at the level of momentum space. It is less well-known that M^4 allows also second twistor space $T(M^4) = M^4 \times CP_1$, and this looks very natural concerning twistor lift of TGD replacing space-time surfaces in H with their twistor spaces in $T(H) = T(M^4) \times T(CP_2)$.

The original identification $T(M^4)$ with CP_3 or its Minkowskian variant has been given up bit it inspired some questions discussed in the sequel.

1. Could TGD in twistor space formulation be seen as a generalization of this theory?

2. General twistor space is not Calabi-Yau manifold because it does does not have Kähler structure. Do twistor spaces replace Calabi-Yaus in TGD framework?

3. Could twistor spaces be Calabi-Yau manifolds in some weaker sense so that one would have a closer connection with super string models.

Consider the last question.

1. One can indeed define non-Kähler Calabi-Yau manifolds by keeping the hermitian metric and giving up symplectic structure or by keeping the symplectic structure and giving up hermitian metric (almost complex structure is enough). Construction recipes for non-Kähler Calabi-Yau manifold are discussed in [A89]. It is shown that these two manners to give up Kähler structure correspond to duals under so called mirror symmetry [B21] which maps complex and symplectic structures to each other. This construction applies also to the twistor spaces.

2. For the modification giving up symplectic structure, one starts from a smooth Kähler Calabi-Yau 3-fold Y, such as CP_3. One assumes a discrete set of disjoint rational curves diffeomorphic to CP_1. In TGD framework work they would correspond to special fibers of twistor space.

   One has singularities in which some rational curves are contracted to point - in twistorial case the fiber of twistor space would contract to a point - this produces double point singularity which one can visualize as the vertex at which two cones meet (sundial should give an idea about what is involved). One deforms the singularity to a smooth complex manifold. One could interpret this as throwing away the common point and replacing it with connected sum contact: a tube connecting the holes drilled to the vertices of the two cones. In TGD one would talk about wormhole contact.

3. Suppose the topology looks locally like $S^3 \times S^2 \times R_{\pm}$ near the singularity, such that two copies analogous to the two halves of a cone (sundial) meet at single point defining double point singularity. In the recent case S^2 would correspond to the fiber of the twistor space. S^3 would correspond to 3-surface and $R_{\pm}$ would correspond to time coordinate in past/future direction. S^3 could be replaced with something else.

   The copies of $S^3 \times S^2$ contract to a point at the common end of R_+ and R_- so that both the based and fiber contracts to a point. Space-time surface would look like the pair of future and past directed light-cones meeting at their tips.

   For the first modification giving up symplectic structure only the fiber S^2 is contracted to a point and $S^2 \times D$ is therefore replaced with the smooth "bottom" of S^3. Instead of sundial one has two balls touching. Drill small holes two the two S^3s and connect them by connected sum contact (wormhole contact). Locally one obtains $S^3 \times S^3$ with k connected sum contacts.

   For the modification giving up Hermitian structure one contracts only S^3 to a point instead of S^2. In this case one has locally two CP_3: s touching (one can think that CP_n is obtained by replacing the points of C^n at infinity with the sphere CP_1). Again one drills holes and connects them by a connected sum contact to get k-connected sum of CP_3.

   For k CP_1s the outcome looks locally like to a k-connected sum of $S^3 \times S^3$ or CP_3 with $k \geq 2$. In the first case one loses symplectic structure and in the second case hermitian structure. The conjecture is that the two manifolds form a mirror pair.

   The general conjecture is that all Calabi-Yau manifolds are obtained using these two modifications. One can ask whether this conjecture could apply also the construction of twistor spaces representable as surfaces in $CP_3 \times F_3$ so that it would give mirror pairs of twistor spaces.

4. This smoothing out procedures isa actually unavoidable in TGD because twistor space is sub-manifold. The 6-D twistor spaces in 12-D $T(M^4) \times F_3$ have in the generic case self intersections consisting of discrete points. Since the fibers CP_1 cannot intersect and since the intersection is point, it seems that the fibers must contract to a point. In the similar manner the 4-D base spaces should have local foliation by spheres or some other 3-D objects with contract to a point. One has just the situation described above.

   One can remove these singularities by drilling small holes around the shared point at the two sheets of the twistor space and connected the resulting boundaries by connected sum contact. The preservation of fiber structure might force to perform the process in such a manner that local modification of the topology contracts either the 3-D base (S^3 in previous example or fiber CP_1 to a point.

The interpretation of twistor spaces is of course totally different from the interpretation of Calabi-Yaus in superstring models. The landscape problem of superstring models is avoided and the multiverse of string models is replaced with generalized Feynman diagrams! Different twistor spaces correspond to different space-time surfaces and one can interpret them in terms of generalized Feynman diagrams since bundle projection gives the space-time picture. Mirror symmetry means that there are two different Calabi-Yaus giving the same physics. Also now twistor space for a given space-time surface can have several imbeddings - perhaps mirror pairs define this kind of imbeddings.

To sum up, the construction of space-times as surfaces of H lifted to those of (almost) complex sub-manifolds in $T(M^4) times F_3$ with induced twistor structure shares the spirit of the vision that induction procedure is the key element of classical and quantum TGD. It also gives deep connection with the mathematical methods applied in super string models and these methods should be of direct use in TGD.

12.4 Witten's Twistor String Approach And TGD

The twistor Grassmann approach has led to a phenomenal progress in the understanding of the scattering amplitudes of gauge theories, in particular the $\mathcal{N} = 4$ SUSY.

As a non-specialist I have been frustrated about the lack of concrete picture, which would help to see how twistorial amplitudes might generalize to TGD framework. A pleasant surprise in this respect was the proposal of a particle interpretation for the twistor amplitudes by Nima Arkani Hamed et al in the article "Unification of Residues and Grassmannian Dualities" [B40] (see `http://tinyurl.com/y86mad5n`)

In this interpretation incoming particles correspond to spheres CP_1 so that n-particle state corresponds to $(CP_1)^n/Gl(2)$ (the modding by $Gl(2)$ might be seen as a kind of formal generalization of particle identity by replacing permutation group S_2 with $Gl(2)$ of 2×2 matrices). If the number of "wrong" helicities in twistor diagram is k, this space is imbedded to $CP^n_{k-1}/Gl(k)$ as a surface having degree $k-1$ using Veronese map to achieve the imbedding. The imbedding space can be identified as Grassmannian $G(k,n)$. This surface defines the locus of the multiple residue integral defining the twistorial amplitude.

The particle interpretation brings in mind the extension of single particle configuration space E^3 to its Cartesian power E^{3n}/S_n for n-particle system in wave mechanics. This description could make sense when point-like particle is replaced with 3-surface or partonic 2-surface: one would have Cartesian product of WCWs divided my S_n. The generalization might be an excellent idea as far calculations are considered but is not in spirit with the very idea of string models and TGD that many-particle states correspond to unions of 3-surfaces in H (or light-like boundaries of causal diamond (CD) in Zero Energy Ontology (ZEO).

Witten's twistor string theory [B33] is more in spirit with TGD at fundamental level since it is based on the identification of generalization of vertices as 2-surfaces in twistor space.

1. There are several kinds of twistors involved. For massless external particles in eigenstates of momentum and helicity null twistors code the momentum and helicity and are pairs of 2-spinor and its conjugate. More general momenta correspond to two independent 2-spinors.

   One can perform twistor Fourier transform for the conjugate 2-spinor to obtain twistors coding for the points of compactified Minkowski space. Wave functions in this twistor space characterized by massless momentum and helicity appear in the construction of twistor amplitudes. BCFW recursion relation [B26] allows to construct more complex amplitudes assuming that intermediate states are on mass shells massless states with complex momenta.

   One can perform twistor Fourier transformation (there are some technical problems in Minkowski signature) also for the second 2-spinor to get what are called momentum twistors providing in some aspects simpler description of twistor amplitudes. These code for the four-momenta propagating between vertices at which the incoming particles arrive and the differences if two subsequent momenta are equal to massless external momenta.

2. In Witten's theory the interactions of incoming particles correspond to amplitudes in which the twistors appearing as arguments of the twistor space wave functions characterized by momentum and helicity are localized to complex curves X^2 of twistor space CP_3 or its Minkowskian counterpart. This can be seen as a non-local twistor space variant of local interactions in Minkowski space.

   The surfaces X^2 are characterized by their degree d (of the polynomial of complex coordinates defining the algebraic 2-surface) the genus g of the algebraic surface, by the number k of "wrong" (helicity violating) helicities, and by the number of loops of corresponding diagram of SUSY amplitude: one has $d = k - 1 + l$, $g \leq l$. The interaction vertex in twistor space is not anymore completely local but the n particles are at points of the twistorial surface X^2.

In the following a proposal generalizing Witten's approach to TGD is discussed.

1. The fundamental challenge is the generalization of the notion of twistor associated with massless particle to 8-D context, first for $M^4 = M^4 \times E^4$ and then for $H = M^4 \times CP_2$. The notion of twistor space solves this question at geometric level. As far as construction of the TGD variant of Witten's twistor string is considered, this might be quite enough.

2. $M^8 - H$ duality and quantum-classical correspondence however suggest that M^8 twistors might allow tangent space description of four-momentum, spin, color quantum numbers and electroweak numbers and that this is needed. What comes in mind is the identification of fermion lines as light-like geodesics possessing M^8 valued 8-momentum, which would define the long sought gravitational counterparts of four-momentum and color quantum numbers at classical point-particle level. The M^8 part of this four-momentum would be equal to that associated with imbedding space spinor mode characterizing the ground state of super-conformal representation for fundamental fermion.

   Hence one might also think of starting from the 4-D condition relating Minkowski coordinates to twistors and looking what it could mean in the case of M^8. The generalization is indeed possible in $M^8 = M^4 \times E^4$ by its flatness if one replaces gamma matrices with octonionic gamma matrices.

   In the case of $M^4 \times CP_2$ situation is different since for octonionic gamma matrices $SO(1,7)$ is replaced with G_2, and the induced gauge fields have different holonomy structure than for ordinary gamma matrices and octonionic sigma matrides appearing as charge matrices bring in also an additional source of non-associativity. Certainly the notion of the twistor Fourier transform fails since CP_2 Dirac operator cannot be algebraized.

   Algebraic twistorialization however works for the light-like fermion lines at which the ordinary and octonionic representations for the induced Dirac operator are equivalent. One can indeed assign 8-D counterpart of twistor to the particle classically as a representation of light-like hyper-octonionic four-momentum having massive M^4 and CP_2 projections and CP_2 part perhaps having interpretation in terms of classical tangent space representation for color and electroweak quantum numbers at fermionic lines.

   If all induced electroweak gauge fields - rather than only charged ones as assumed hitherto - vanish at string world sheets, the octonionic representation is equivalent with the ordinary one. The CP_2 projection of string world sheet should be 1-dimensional: inside CP_2 type vacuum extremals this is impossible, and one could even consider the possibility that the projection corresponds to CP_2 geodesic circle. This would be enormous technical simplification. What is important that this would not prevent obtaining non-trivial scattering amplitudes at elementary particle level since vertices would correspond to re-arrangement of fermion lines between the generalized lines of Feynman diagram meeting at the vertices (partonic 2-surfaces).

3. In the fermionic sector one is forced to reconsider the notion of the induced spinor field. The modes of the imbedding space spinor field should co-incide in some region of the space-time surface with those of the induced spinor fields. The light-like fermionic lines defined by the boundaries of string world sheets or their ends are the obvious candidates in this respect. String world sheets is perhaps too much to require.

   The only reasonable identification of string world sheet gamma matrices is as induced gamma matrices and super-conformal symmetry requires that the action contains string world sheet area as an additional term just as in string models. String tension would correspond to gravitational constant and its value - that is ratio to the CP_2 radius squared, would be fixed by quantum criticality.

4. The generalization of the Witten's geometric construction of scattering amplitudes relying on the induction of the twistor structure of the imbedding space to that associated with space-time surface looks surprisingly straight-forward and would provide more precise formulation of the notion of generalized Feynman diagrams forcing to correct some wrong details. One of the nice outcomes is that the genus appearing in Witten's formulation naturally corresponds to family replication in TGD framework.

12.4.1 Basic Ideas About Twistorialization Of TGD

The recent advances in understanding of TGD motive the attempt to look again for how twistor amplitudes could be realized in TGD framework. There have been several highly non-trivial steps of progress leading to a new more profound understanding of basic TGD.

1. $M^4 \times CP_2$ is twistorially unique [K76] in the sense that its factors are the only 4-D geometries allowing twistor space with Kähler structure (M^4 corresponds to S^4 in Euclidian signature) [A63]. The twistor spaces in question are CP_3 for S^4 and its Minkowskian variant for M^4 (I will use P^3 as short hand for both twistor spaces) and the flag manifold $F = SU(3)/U(1) \times U(1)$ parametrizing the choices of quantization axes for color group $SU(3)$ in the case of CP_2. Recall that twistor spaces are S^2 bundles over the base space and that all orientable four-manifolds have twistor space in this sense. Note that space-time surfaces allow always almost quaternionic structure and that that preferred extremals are suggested to be quaternionic [K76].

2. The light-likeness condition for twistors in M^4 is fundamental in the ordinary twistor approach. In 8-D context light-likeness holds in generalized sense for the spinor harmonics of H: the square of the Dirac operator annihilates spinor modes. In the case M^8 one can indeed define twistors by generalizing the standard approach by replacing ordinary gamma matrices with octonionic ones [**?**] For H octonionic and ordinary gamma matrices are equivalent at the fermionic lines defined by the light-like boundaries of string world sheets and at string world sheets if they carry vanishing induced electro-weak gauge fields that is have 1-D CP_2 projection.

3. Twistor spaces emerge in TGD framework as lifts of space-time surfaces to corresponding twistor spaces realized as 6-surfaces in the lift of $M^4 \times CP_2$ to $T(H) = P^3 \times F$ having as base spaces space-time surfaces. This implies that that generalized Feynman diagrams and also generalized twistor diagrams can be lifted to diagrams in T and that the construction of twistor spaces as surfaces of T has very concrete particle interpretation.

   The modes of the imbedding space spinor field defining ground states of the extended conformal algebras for which classical conformal charges vanish at the ends of the space-time surface (this defines gauge conditions realizing strong form of holography [K88]) are lifted to the products of modes of spinor fields in $T(H)$ characterized by four-momentum and helicity in M^4 degrees of freedom and by color quantum numbers and electroweak quantum numbers in F degrees of freedom. Thus twistorialization provides a purely geometric representation of spin and electro-weak spin and it seems that twistorialization allows to a formulation without H-spinors.

   What is especially nice, that twistorialization extends to from spin to include also electroweak spin. These two spins correspond correspond to M^4 and CP_2 helicities for the twistor space amplitude, and are non-local properties of these amplitudes. In TGD framework only twistor amplitudes for which helicities correspond to that for massless fermion and antifermion are possible and by fermion number conservation the numbers of positive and negative helicities are identical and equal to the fermion number (or antifermion number). Separate lepton and baryon number conservation realizing 8-D chiral symmetry implies that M^4 and CP_2 helicities are completely correlated.

   For massless fermions in M^4 sense helicity is opposite for fermion and antifermion and conserved. The contributions of initial and final states to k are same and equal to $k_i = k_f = 2(n(F) - n(\overline{F})$. This means restriction to amplitudes with $k = 2(n(F) - n(\overline{F})$. If fermions are massless only in M^8 sense, chirality mixing occurs and this rule does not hold anymore. This holds true in quark and lepton sector separately.

4. All generalized Feynman graphs defined in terms of Euclidian regions of space-time surface are lifted to twistor spaces [K14]. Incoming particles correspond quantum mechanically to twistor space amplitudes defined by their momenta and helicities and and classically to the entire twistor space of space-time surface as a surface in the twistor space of H. Of course, also the Minkowskian regions have this lift. The vertices of Feynman diagrams correspond to regions of twistor space in which the incoming twistor spaces meet along their 5-D ends having also S^2 bundle structure over space-like 3-surfaces. These space-like 3-surfaces correspond to ends of Euclidian and Minkowskian space-time regions separated from each other by light-like 3-surfaces at which the signature of the metric changes from Minkowskian to Euclidian. These "partonic orbits" have as their ends 2-D partonic surfaces. By strong form of General Coordinate Invariance implying strong of holography, these 2-D partonic surfaces and their

4-D tangent space data should code for quantum physics. Their lifts to twistor space are 4-D S^2 bundles having partonic 2-surface X^2 as base.

5. The well-definedness of em charge for the spinor modes demands that they are localized at 2-D string world sheets [K88] and also these world sheets are lifted to sub-spaces of twistor space of space-time surface. If one demands that octonionic Dirac operator makes sense at string world sheets, they must carry vanishing induced electro-weak gauge fields and string world sheets could be minimal surfaces in $M^4 \times S^1$, $S^1 \subset CP_2$ a geodesic circle.

   The boundaries of string world sheets at partonic orbits define light-like curves identifiable as carriers of fermion number and they define the analogs of lines of Feynman diagrams in ordinary sense. The only purely fermionic vertices are 2-fermion vertices at the partonic 2-surfaces at which the end of space-time surface meet. As already explained, the string world sheets can be seen as correlates for the correlations between fermion vertices at different wormhole throats giving rise to the counterpart of bosonic propagator in quantum field theories.

   The localization of spinor fields to 2-D string world sheets corresponds to the localization of twistor amplitudes to their 4-D lifts, which are S^2 bundles and the boundaries of string world sheets are lifted to 3-D twistor lifts of fermion lines. Clearly, the localization of spinors to string world sheets would be absolutely essential for the emergence of twistor description.

6. All elementary particles are many particle bound states of massless fundamental fermions: the non-collinearity (and possible complex character) of massless momenta explains massivation. The fundamental fermions are localized at wormhole throats defining the light-like orbits of partonic 2-surfaces. Throats are associated with wormhole contacts connecting two space-time sheets. Stability of the contact is guaranteed by non-vanishing monopole magnetic flux through it and this requires the presence of second wormhole contact so that a closed magnetic flux tube carrying monopole flux and involving the two space-time sheets is formed. The net fermionic quantum numbers of the second throat correspond to particle's quantum numbers and above weak scale the weak isospins of the throats sum up to zero.

7. Fermionic 2-vertex is the only *local* many-fermion vertex [K14] being analogous to a mass insertion. The non-triviality of fundamental 4-fermion vertex is due to classical interactions between fermions at opposite throats of worm-hole. The non-triviality of the theory involves also the analog of OZI mechanism: the fermionic lines inside partonic orbits are redistributed in vertices. Lines can also turn around in time direction which corresponds to creation or annihilation of a pair. 3-particle vertices are obtained only in topological sense as 3 space-time surfaces are glued together at their ends. The interaction between fermions at different wormhole throats is described in terms of string world sheets.

8. The earlier proposal was that the fermions in the internal fermion lines are massless in M^4 sense but have non-physical helicity so that the algebraic M^4 Dirac operator emerging from the residue integration over internal four-momentum does not annihilate the state at the end of the propagator line. Now the algebraic induced Dirac operator defines the propagator at fermion lines. Should one assume generalization of non-physical helicity also now?

9. All this stuff must be lifted to twistorial level and one expects that the lift to S^2 bundle allows an alternative description of fermions and spinor structure so that one can speak of induced twistor structure instead of induced spinor structure. This approach allows also a realization of M^4 conformal symmetries in terms of globally well-defined linear transformations so that it might be that twistorialization is not a mere reformulation but provides a profound unification of bosonic and fermionic degrees of freedom.

12.4.2 The Emergence Of The Fundamental 4-Fermion Vertex And Of Boson Exchanges

The emergence of the fundamental 4-fermion vertex and of boson exchanges deserves a more detailed discussion.

1. I have proposed that the discontinuity of the Dirac operator at partonic two-surface (corner of fermion line) defines both the fermion boson vertex and TGD analog of mass insertion (not scalar but imbedding space vector) giving rise to mass parameter having interpretation as Higgs vacuum expectation and various fermionic mixing parameters at QFT limit of TGD obtained by approximating many-sheeted space-time of TGD with the single sheeted region of M^4 such that gravitational field and gauge potentials are obtained as sums of those associated with the sheets.

2. Non-trivial scattering requires also correlations between fermions at different partonic 2-surfaces. Both partonic 2-surfaces and string world sheets are needed to describe these correlations. Therefore the string world sheets and partonic 2-surfaces cannot be dual: both are needed and this means deviation from Witten's theory. Fermion vertex corresponds to a "corner" of a fermion line at partonic 2-surface at which generalized 4-D lines of Feynman diagram meet and light-like fermion line changes to space-like one. String world sheet with its corners at partonic 2-surfaces (wormhole throats) describes the momentum exchange between fermions. The space-like string curve connecting two wormhole throats serves as the analog of the exchanged gauge boson.

3. Two kinds of 4-fermion amplitudes can be considered depending on whether the string connects throats of single wormhole contact (CP_2 scale) or of two wormhole contacts (p-adic length scale - typically of order elementary particle Compton length). If string worlds sheets have 1-D CP_2 projection, only Minkowskian string world sheets are possible. The exchange in Compton scale should be assignable to the TGD counterpart of gauge boson exchange and the fundamental 4-fermion amplitude should correspond to single wormhole contact: string need not to be involved now. Interaction is basically classical interaction assignable to single wormhole contact generalizing the point like vertex.

4. The possible TGD counterparts of BCFW recursion relations [B26] should use the twistorial representations of fundamental 4-fermion scattering amplitude as seeds. Yangian invariance poses very strong conditions on the form of these amplitudes and the exchange of massless bosons is suggestive for the general form of amplitude.

   The 4-fermion amplitude assignable to two wormhole throats defines the analog of gauge boson exchange and is expressible as fusion of two fundamental 4-fermion amplitudes such that the 8-momenta assignable to the fermion and anti-fermion at the opposite throats of exchanged wormhole contact are complex by BCFW shift acting on them to make the exchanged momenta massless but complex. This entity could be called fundamental boson (not elementary particle).

5. Can one assume that the fundamental 4-fermion amplitude allows a purely formal composition to a product of $F\overline{F}B_v$ amplitudes, B_v a purely fictive boson? Two 8-momenta at both $F\overline{F}B_v$ vertices must be complex so that at least two external fermionic momenta must be complex. These external momenta are naturally associated with the throats of the a wormhole contact defining virtual fundamental boson. Rather remarkably, without the assumption about product representation one would have general four-fermion vertex rather than boson exchange. Hence gauge theory structure is not put in by hand but emerges.

12.4.3 What About SUSY In TGD?

Extended super-conformal symmetry is crucial for TGD and extends to quaternion-super-conformal symmetry giving excellent hopes about calculability of the theory. $\mathcal{N} = 4$ space-time supersymmetry is in the key role in the approach of Witten and others.

In TGD framework space-time SUSY could be present as an approximate symmetry.

1. The many fermion states at partonic surfaces are created by oscillator operators of fermionic Clifford algebra having also interpretation as a supersymmetric algebra but in principle having $\mathcal{N} = \infty$. This SUSY is broken since the generators of SUSY carry four-momentum.

2. More concrete picture would be that various SUSY multiplets correspond to collinear many-fermion states at the same wormhole throat. By fermionic statistics only the collinear states

for which internal quantum numbers are different are realized: other states should have antisymmetric wave function in spatial degrees of freedom implying wiggling in CP_2 scale so that the mass of the state would be very high. In both quark and lepton sectors one would have $\mathcal{N} = 4$ SUSY so that one would have the analog $\mathcal{N} = \forall$ SUSY (color is not spin-like quantum number in TGD).

At the level of diagrammatics single line would be replaced with "line bundle" representing the fermions making the many-fermion state at the light-like orbit of the partonic 2-surface. The fusion of neighboring collinear multifermion stats in the twistor diagrams could correspond to the process in which partonic 2-surfaces and single and many-fermion states fuse.

3. Right handed neutrino modes, which are not covariantly constant, are also localized at the fermionic lines and defines the least broken $\mathcal{N} = 2$ SUSY. The covariantly constant mode seems to be a pure gauge degree of freedom since it carriers no quantum numbers and the SUSY norm associated with it vanishes. The breaking would be smallest for $\mathcal{N} = 2$ variant assignable to right-handed neutrino having no weak and color interactions with other particles but whose mixing with left-handed neutrino already induces SUSY breaking.

Why this SUSY has not been observed? One can consider two scenarios [K95].

1. The first scenario relies on the absence of weak and color interactions: one can argue that the bound states of fermions with right-handed neutrino are highly unstable since only gravitational interaction so that sparticle decays very rapidly to particle and right-handed or left-handed neutrino. By Uncertainty Principle this makes sparticle very massive, maybe having mass of order CP_2 mass. Neutrino mixing caused by the mixing of M^4 and CP_2 gamma matrices in induced gamma matrices is the weak point of this argument.

2. The mixing of left and right-handed neutrinos could be characterized by the p-adic mass scale of neutrinos and be long. Sparticles would have same p-adic mass scale as particles and would be dark having non-standard value of Planck constant $h_{eff} = n \times h$: this would scale up the lifetime by factor n and correlate with breaking of conformal symmetry assignable to the mixing [K95].

What implications the approximate SUSY would have for scattering amplitudes?

1. $k = 2(n(F) - n(\overline{F})$ condition reduces the number of amplitudes dramatically if the fermions are massless in M^4 sense but the presence of weak iso-spin implies that the number of amplitudes is 2^n as in non-supersymmetric gauge theories. One however expects broken SUSY with generators consisting of fermionic oscillator operators at partonic 2-surfaces with symmetry breaking taking place only at the level of physical particles identifiable as many particle bound states of massless (in 8-D sense) particles. This motivates the guess that the formal $F\overline{F}B_v$ amplitudes defining fundamental 4-fermion vertex are expressible as those associated with $\mathcal{N} = 4$ SUSY in quark and lepton sectors respectively. This would reduce the number of independent amplitudes to just one.

2. Since SUSY and its breaking emerge automatically in TGD framework, super-space can provide a useful technical tool but is not fundamental.

   Side note: The number of external fermions is always even suggesting that the super-conformal anomalies plaguing the amplitudes with odd n (`http://tinyurl.com/yb85tnvc`) [B64] are absent.

12.4.4 What Does One Really Mean With The Induction Of Imbedding Space Spinors?

The induction of spinor structure is a central notion of TGD but its detailed definition has remained somewhat obscure. The attempt to generalize Witten's approach has made it clear that the mere restriction of spinor fields to space-time surfaces is not enough and that one must understand in detail the correspondence between the modes of imbedding space spinor fields and those of induced spinor fields.

Even the identification of space-time gamma matrices is far from obvious at string world sheets.

1. The simplest notion of the space-time gamma matrices is as projections of imbedding space gamma matrices to the space-time surface - induced gamma matrices. If one assumes that induced spinor fields are defined at the entire space-time surfaces, this notion fails to be consistent with fermionic super-conformal symmetry unless one replaces Kähler action by space-time volume. This option is certainly unphysical.

2. The notion of Kähler-Dirac matrices in the interior of space as gamma matrices defined as contractions of canonical momentum densities of Kähler with imbedding space gamma matrices allows to have conformal super-symmetry with fermionic super charges assignable to the modes of the induced spinor field. Also Chern-Simons action could define gamma matrices in the same manner at the light-like 3-surfaces between Minkowskian and Euclidian space-time regions and at space-like 3-surfaces at the ends of space-time surface. Chern-Simons-Dirac matrices would involve only CP_2 gamma matrices.

It is however not quite clear whether the spinor fields in the interior of space-time surface are needed at all in the twistorial approach and they seem to be only an un-necessary complication. For instance, their modes would have well-defined electromagnetic charge only when induced W gauge fields vanish, which implies that CP_2 projection is 2-dimensional. This forces to consider very seriously the possibility that induced spinor fields reside at string world sheets only (their ends are at partonic 2-surfaces). This option supported also by strong form of holography and number theoretic universality.

What about the space-time gamma matrices at string world sheets and their boundaries?

1. The first option would be reduction of Kähler-Dirac gamma matrices by requiring that they are parallel to the string world sheets. This however poses additional conditions besides the vanishing of W fields already restricting the dimension to two in the generic case. The conditions state that the imbedding space 1-forms defined by the canonical momentum densities of Kähler action involve only 2 linearly independent ones and that they are proportional to imbedding space coordinate gradients: this gives Frobenius conditions. These conditions look first too strong but one can also think that one fixes first string world sheets, partonic 2-surfaces, and perhaps also their light-like orbits, requires that the normal components of canonical momentum currents at string world sheets vanish, and deduces space-time surface from this data. This would be nothing but strong form of holography!

   For this option the string world sheets could emerge in the sense that it would be possible to express Kähler action as an area of string world sheet in the effective metric defined by the anticommutator of K-D gamma matrices appearing also in the expressions for the matrix elements of WCW metric. Gravitational constant would be a prediction of the theory.

2. Second possibility is to use induced gamma matrices automatically parallel to the string world sheet so that no additional conditions would result. This would also conform with the ordinary view about string world sheets and spinors.

   Supersymmetry would require the addition of the area of string world sheet to the action defining Kähler function in Euclidian regions and its counterpart in Minkowskian regions. This would bring in gravitational constant, which otherwise remains a prediction. Quantum criticality could fix the ratio of $\hbar G/R^2$ (R is CP_2 radius). The vanishing of induced weak gauge fields requires that string world sheets have 1-D CP_2 projection and are thus restricted to Minkowskian regions with at most 3-D CP_2 projection. Even stronger condition would be that string world sheets are minimal surfaces in $M^4 \times S^1$, S^1 a geodesic sphere of CP_2.

   There are however grave objections. The presence of a dimensional parameter G as fundamental coupling parameter does not encourage hopes about the renomalizibility of the theory. The idea that strings connecting partonic 2-surfaces gives rise to the formation of gravitationally bound states is suggested by AdS/CFT correspondence. The problem is that the string tension defined by gravitational constant is so large that only Planck length sized bound states are feasible. Even the replacement $\hbar \to \hbar_{eff}$ fails to allow gravitationally bound states with length scale of order Schwartschild radius. For the K-D option the string tension behaves like $1/\hbar^2$ and there are no problems in this respect.

At this moment my feeling is that the first option - essentially the original view - is the correct one. The short belief that the second option is the correct choice was a sidetrack, which however helped to become convinced that the original vision is indeed correct, and to understand the general mechanism for the formation of bound states in terms of strings connection partonic 2-surfaces (in the earlier picture I talked about magnetic flux tubes carrying monopole flux: the views are equivalent).

Both options have the following nice features.

1. Induced gammas at the light-like string boundaries would be light-like. Massless Dirac equation would assign to spinors at these lines a light-like space-time four-momentum and twistorialize it. This four-momentum would be essentially the tangent vector of the light-like curve and would not have a constant direction. Light-likeness for it means light-likeness in 8-D sense since light-like curves in H correspond to non-like momenta in M^4. Both M^4 mass squared and CP_2 mass would be conserved. Even four-momentum could be conserved if M^4 projection of stringy curve is geodesic line of M^4.

2. A new connection with Equivalence Principle (EP) would emerge. One could interpret the induced four-momentum as gravitational four-momentum which would be massless in 4-D sense and correspond to inertial 8-momentum. EP wold state in the weakest form that only the M^4 masses associated with the two momenta are identical. Stronger condition would be that that the Minkowski parts of the two momenta co-incide at the ends of fermion lines at partonic 2-surfaces. Even stronger condition is that the 8-momentum is 8-momentum is conserved along fermion line. This is certainly consistent with the ordinary view about Feynman graphs. This is guaranteed if the light-like curve is light-like geodesic of imbedding space.

The induction of spinor fields has also remained somewhat imprecise notion. It how seems that quantum-classical correspondence forces a unique picture.

1. Does the induced spinor field co-incide with imbedding space spinor harmonic in some domain? This domain would certainly include the ends of fermionic lines at partonic 2-surfaces associated with the incoming particles and vertices. Could it include also the boundaries of string world sheets and perhaps also the string world sheets? The Kähler-Dirac equation certainly does not allow this for entire space-time surface.

2. Strong form of holography suggest that the light-like momenta for the Dirac equation at the ends of light-like string boundaries has interpretation as 8-D light-like momentum has M^4 projection equal to that of H spinor-harmonic. The mass squared of M^4 momentum would be same as the CP_2 momentum squared in both senses. Unless the gravitational four-momentum assignable to the induced Dirac operato r is conserved one cannot pose stronger condition.

3. If the induced spinor mode equals to imbedding space-spinor mode also at fermion line, the light like momentum is conserved. The fermion line would be also light-like geodesic of the imbedding space so that twistor polygons would have very concrete interpretation. This condition would be clearly analogous to the conditions in Witten's twistor string theory. A stronger condition would be that the mode of the imbedding space spinor field co-incides with induced spinor field at the string world sheet.

4. p-Adic mass calculations require that the massive excitations of imbedding space spinor fields with CP_2 mass scale are involved. The thermodynamics could be for fermion line, wormhole throat carrying possible several fermions, or wormhole contact carrying fermion at both throats. In the case of fermions physical intuition suggests that p-adic thermodynamics must be associated with single fermionic line. The massive excitations would correspond to light-like geodesics of the imbedding space.

To minimize confusion I must confess that until recently I have considered a different options for the momenta associated with fermionic lines.

1. The action of the Kähler-Dirac operator on the induced spinor field at the fermionic line equals to that of 4-D Dirac operator $p^k\gamma_k$ for a massless momentum identified as M^4 momentum [K14].

   Now the action reduces to that of 8-D massless algebraic Dirac operator for light-like string boundaries in the case of induced gamma matrices. Explicit calculation shows that in case of K-D gamma matrices and for light-like string boundaries it can happen that the 8-momentum of the mode can be tachyonic. Intriguingly, p-adic mass calculations require a tachyonic ground state?

2. For this option the helicities for virtual fermions were assumed to be non-physical in order to get non-vanishing fermion lines by residue integration: momentum integration for Dirac operator would replace Dirac propagators with Dirac operators. This would be the counterpart for the disappearance of bosonic propagators in residue integration.

3. This option has problems: quantum classical correspondence is not realized satisfactorily and the interpretation of p-adic thermodynamics is problematic.

12.4.5 About The Twistorial Description Of Light-Likeness In 8-D Sense Using Octonionic Spinors

The twistor approach to TGD [K76] require that the expression of light-likeness of M^4 momenta in terms of twistors generalizes to 8-D case. The light-likeness condition for twistors states that the 2×2 matrix representing M^4 momentum annihilates a 2-spinor defining the second half of the twistor. The determinant of the matrix reduces to momentum squared and its vanishing implies the light-likeness. This should be generalized to a situation in one has M^4 and CP_2 twistor which are not light-like separately but light-likeness in 8-D sense holds true.

The case of $M^8 = M^4 \times E^4$

$M^8 - H$ duality [K74] suggests that it might be useful to consider first the twistorialiation of 8-D light-likeness first the simpler case of M^8 for which CP_2 corresponds to E^4. It turns out that octonionic representation of gamma matrices provide the most promising formulation.

In order to obtain quadratic dispersion relation, one must have 2×2 matrix unless the determinant for the 4×4 matrix reduces to the square of the generalized light-likeness condition.

1. The first approach relies on the observation that the 2×2 matrices characterizing four-momenta can be regarded as hyper-quaternions with imaginary units multiplied by a commuting imaginary unit. Why not identify space-like sigma matrices with hyper-octonion units?

2. The square of hyper-octonionic norm would be defined as the determinant of 4×4 matrix and reduce to the square of hyper-octonionic momentum. The light-likeness for pairs formed by M^4 and E^4 momenta would make sense.

3. One can generalize the sigma matrices representing hyper-quaternion units so that they become the 8 hyper-octonion units. Hyper-octonionic representation of gamma matrices exists ($\gamma_0 = \sigma_z \times 1$, $\gamma_k = \sigma_y \times I_k$) but the octonionic sigma matrices represented by octonions span the Lie algebra of G_2 rather than that of $SO(1,7)$. This dramatically modifies the physical picture and brings in also an additional source of non-associativity. Fortunately, the flatness of M^8 saves the situation.

4. One obtains the square of $p^2 = 0$ condition from the massless octonionic Dirac equation as vanishing of the determinant much like in the 4-D case. Since the spinor connection is flat for M^8 the hyper-octonionic generalization indeed works.

This is not the only possibility that I have by-passingly considered [K14].

1. Is it enough to allow the four-momentum to be complex? One would still have 2×2 matrix and vanishing of complex momentum squared meaning that the squares of real and imaginary parts are same (light-likeness in 8-D sense) and that real and imaginary parts are orthogonal to each other. Could E^4 momentum correspond to the imaginary part of four-momentum?

2. The signature causes the first problem: M^8 must be replaced with complexified Minkowski space M^4_c for to make sense but this is not an attractive idea although M^4_c appears as subspace of complexified octonions.

3. For the extremals of Kähler action Euclidian regions (wormhole contacts identifiable as deformations of CP_2 type vacuum extremals) give imaginary contribution to the four-momentum. Massless complex momenta and also color quantum numbers appear also in the standard twistor approach. Also this suggest that complexification occurs also in 8-D situation and is not the solution of the problem.

The case of $M^8 = M^4 \times CP_2$

What about twistorialization in the case of $M^4 \times CP_2$? The introduction of wave functions in the twistor space of CP_2 seems to be enough to generalize Witten's construction to TGD framework and that algebraic variant of twistors might be needed only to realize quantum classical correspondence. It should correspond to tangent space counterpart of the induced twistor structure of space-time surface, which should reduce effectively to 4-D one by quaternionicity of the space-time surface.

1. For $H = M^4 \times CP_2$ the spinor connection of CP_2 is not trivial and the G_2 sigma matrices are proportional to M^4 sigma matrices and act in the normal space of CP_2 and to M^4 parts of octonionic imbedding space spinors, which brings in mind co-associativity. The octonionic charge matrices are also an additional potential source of non-associativity even when one has associativity for gamma matrices.

   Therefore the octonionic representation of gamma matrices in entire H cannot be physical. It is however equivalent with ordinary one at the boundaries of string world sheets, where induced gauge fields vanish. Gauge potentials are in general non-vanishing but can be gauge transformed away. Here one must be of course cautious since it can happen that gauge fields vanish but gauge potentials cannot be gauge transformed to zero globally: topological quantum field theories represent basic example of this.

2. Clearly, the vanishing of the induced gauge fields is needed to obtain equivalence with ordinary induced Dirac equation. Therefore also string world sheets in Minkowskian regions should have 1-D CP_2 projection rather than only having vanishing W fields if one requires that octonionic representation is equivalent with the ordinary one. For CP_2 type vacuum extremals electroweak charge matrices correspond to quaternions, and one might hope that one can avoid problems due to non-associativity in the octonionic Dirac equation. Unless this is the case, one must assume that string world sheets are restricted to Minkowskian regions. Also imbedding space spinors can be regarded as octonionic (possibly quaternionic or co-quaternionic at space-time surfaces): this might force vanishing 1-D CP_2 projection.

   (a) Induced spinor fields would be localized at 2-surfaces at which they have no interaction with weak gauge fields: of course, also this is an interaction albeit very implicit one! This would not prevent the construction of non-trivial electroweak scattering amplitudes since boson emission vertices are essentially due to re-groupings of fermions and based on topology change.

   (b) One could even consider the possibility that the projection of string world sheet to CP_2corresponds to CP_2 geodesic circle so that one could assign light-like 8-momentum to entire string world sheet, which would be minimal surface in $M^4 \times S^1$. This would mean enormous technical simplification in the structure of the theory. Whether the spinor harmonics of imbedding space with well-defined M^4 and color quantum numbers can co-incide with the solutions of the induced Dirac operator at string world sheets defined by minimal surfaces remains an open problem.

   (c) String world sheets cannot be present inside wormhole contacts which have 4-D CP_2 projection so that string world sheets cannot carry vanishing induced gauge fields.

12.4.6 How To Generalize Witten's Twistor String Theory To TGD Framework?

The challenge is to lift the geometric description of particle like aspects of twistorial amplitudes involving only algebraic curves (2-surfaces) in twistor space to TGD framework.

1. External particles correspond to the lifts of H-spinor harmonics to spinor harmonics in the twistor space and are labeled by four-momentum, helicity, color, and weak helicity (isospin) so that there should be no need to included fermions explicitly. The twistorial wave functions would be superpositions of the eigenstates of helicity operator which would become a non-local property in twistor space. Light-likeness would hold true in 8-D sense for spinor harmonics as well as for the corresponding twistorial harmonics.

2. The surfaces X^2 in Witten's theory would be replaced with the lifts of partonic 2-surfaces X^2 to 4-D surfaces with bundle structure with X^2 as base and S^2 as fiber. S^2 would be non-dynamical. Whether X^2 or its lift to 4-surface allows identification as algebraic surface is not quite clear but it seems that X^2 could be the relevant object identifiable as surface of the base space of $T(X^4)$. If X^2 is the basic object one would have the additional constraint (not present in Witten's theory) that it belongs to the base space X^4. The genus of the lift of X^2 would be that of its base space X^2. One obtains a union of partonic 2-surfaces rather than single surface and lines connecting them as boundaries of string world sheets.

   The n points of given X^2 would correspond to the ends of boundaries of string world sheets at the partonic 2-surface X^2 carrying fermion number so that the helicities of twistorial spinor modes would be highly fixed unless M^4 chiralities mix making fermions massive in M^4 sense. This picture is in accordance with the fact that the lines of fundamental fermions should correspond to QFT limit of TGD.

3. In TGD genus g of the partonic 2-surface labels various fermion families and $g < 3$ holds true for physical fermions. The explanation could be that Z^2 acts as global conformal symmetry (hyper-ellipticity) for $g < 3$ surfaces irrespective of their conformal moduli but for $g > 3$ only in for special moduli. Physically for $g > 2$ the additional handles would make the partonic 2-surface to behave like many-particle state of free particles defined by the handles.

   This assumption suggests that assigns to the partonic surface what I have called modular invariant elementary particle vacuum functional (EVPF) in modular degrees of freedom such that for a particle characterized by genus g one has $l \geq g$ and $l > g$ amplitudes are possible because the EPVF leaks partially to higher genera [K12]. This would also induce a mixing of boundary topologies explaining CKM mixing and its leptonic counterpart. In this framework it would be perhaps more appropriate to define the number of loops as $l_1 = l - g$.

   A more precise picture is as follows. Elementary particles have actually four wormhole throats corresponding to the two wormhole contacts. In the case of fermions the wormhole throat carrying the electroweak quantum numbers would have minimum value g of genus characterized by the fermion family. Furthermore, the universality of the standard model physics requires that the couplings of elementary fermions to gauge bosons do not depend on genus. This is the case if one has quantum superposition of the wormhole contacts carrying the quantum numbers of observed gauge bosons at their opposite throats over the three lowest genera $g = 0, 1, 2$ with identical coefficients. This meas SU(3) singlets for the dynamical SU(3) associated with genus degeneracy. Also their exotic variants - say octets - are in principle possible.

4. This description is not complete although already twistor string description involves integration over the conformal moduli of the partonic 2-surface. One must integrate over the "world of classical worlds" (WCW) -roughly over the generalized Feynman diagrams and their complements consisting of Minkowskian and Euclidian regions. TGD as almost topological QFT reduces this integration to that of the boundaries of space-time regions.

   By quaternion conformal invariance [K76] this functional integral could reduce to ordinary integration over the quaternionic-conformal moduli space of space-time surfaces for which the moduli space of partonic 2-surfaces should be contained (note that strong form of holography suggests that only the modular invariants associated with the tangent space data should enter the description). One might hope that twistor space approach allows an elegant description of the moduli assignable to the tangent space data.

12.4.7 Yangian Symmetry

One of the victories of the twistor Grassmannian approach is the discovery of Yangian symmetry [A27], [B31, B39], [K76], whose variant associated with extended super-conformal symmetries is expected to be in key role in TGD.

1. The very nature of the residue integral implies that the complex surface serving as the locus of the integrand of the twistor amplitude is highly non-unique. Indeed, the Yangian symmetry [K76] acting as multi-local symmetry and implying dual of ordinary conformal invariance acting on momentum twistors, has been found to reduce to diffeomorphisms of $G(k,n)$ respecting positivity property of the decomposition of $G(k,n)$ to polyhedrons. It is quite possible that this symmetry corresponds to quaternion conformal symmetries in TGD framework.

2. Positivity of a given regions means parameterization by non-negative coordinates in TGD framework a possible interpretation is based on the observation that canonical identification mapping reals to p-adic number and vice versa is well-defined only for non-negative real numbers. Number theoretical Universality of spinor amplitudes so that they make sense in all number fields, would therefore be implied.

3. Could the crucial Yangian invariance generalizing the extended conformal invariance of TGD could reduce to the diffeomorphisms of the extended twistor space $T(H)$ respecting positivity. In the case of CP_2 all coordinates could be regarded as angle coordinates and be replaced by phase factors coding for the angles which do not make sense p-adically. The number theoretical existence of phase factors in p-adic case is guaranteed if they belong to some algebraic extension of rationals and thus also p-adics containing these phases as roots of unity. This implies discretization of CP_2.

   ZEO allows to reduce the consideration to causal diamond CD defined as an intersection of future and past directed light-cones and having two light-like boundaries. CD is indeed a natural counterpart for S^4. One could use as coordinates light-cone proper time a invariant under Lorentz transformations of either boundary of CD, hyperbolic angle η and two spherical angles (θ,ϕ). The angle variables allow representation in terms of finite algebraic extension. The coordinate a is naturally non-negative and would correspond to positivity. The diffeomorphisms perhaps realizing Yangian symmetry would respect causality in the sense that they do not lead outside CD.

   Quaternionic conformal symmetries the boundaries of $CD \times CP_2$ continued to the interior by translation of the light-cones serve as a good candidates for the diffeomorphisms in question since they do not change the value of the Minkowski time coordinate associated with the line connecting the tips of CD.

12.4.8 Does BCFW Recursion Have Counterpart In TGD?

Could BCFW recursion for tree diagrams and its generalization to diagrams with loops have a generalization in TGD framework? Could the possible TGD counterpart of BCFW recursion have a representation at the level of the TGD twistor space allowing interpretation in terms of geometry of partonic 2-surfaces and associated string world sheets? Supersymmetry is essential ingredient in obtaining this formula and the proposed SUSY realized at the level of amplitudes and broken at the level of states gives hopes for it. One could however worry about the fact that spinors are Dirac spinors in TGD framework and that Majorana property might be essential element.

How to produce Yangian invariants

Nima Arkani-Hamed et al [B39] (`http://tinyurl.com/y97rlzqb`) describe in detail various manners to form Yangian invariants defining the singular parts of the integrands of the amplitudes allowing to construct the full amplitudes. The following is only a rough sketch about what is involved using particle picture and I cannot claim of having understood the details.

1. One can *add* particle ($(k,n) \rightarrow (k+1,n+1)$) to the amplitude by deforming the momentum twistors of two neighboring particles in a manner depending on the momentum twistor of the added particle. One inserts the new particle between n-1:th and 1st particle, modifies their momentum twistors without changing their four-momenta, and multiplying the resulting amplitude by a twistor invariant known as $[n-2,n-1,n,1,2]$ so that there is dependence on the added n:th momentum twistor.

2. One can *remove* particle ($(k,n) \rightarrow (k-1,n-1)$) by contour integrating over the momentum twistor variable of one particle.

3. One can *fuse* invariants simply by multiplying them.

4. One can *merge* invariants by identifying momentum twistors appearing in the two invariants. The integration over the common twistor leads to an elimination of particle.

5. One can form a *BCFW bridge* between n_1+1-particle invariant and n_2+1-particle invariant to get $n = n_1 + n_2$-particle invariant using the operations listed. One starts with the *fusion* giving the product $I_1(1,...,n_1,I)I_2(n_1+1,..n,I)$ of Yangian invariants followed by *addition* of n_1+1 to I_1 between n_1 and I and 1 to I_2 between I and n_1+1 (see the first item for details). After that follows the *merging* of lines labelled by I next to n_1 in I_1 and the predecessor of n_1+1 in I_2 reducing particle number by one unit and followed by residue integration over Z_I reducing particle number further by one unit so that the resulting amplitude is n-particle amplitude.

6. One can perform *entangled removal* of two particles. One could remove them one-by-one by independent contour integrations but one can also perform the contour integrations in such a manner that one first integrates over two twistors at the same complex line and then over the lines: this operation adds to n-particle amplitude loop.

BCFW recursion formula

BCFW recursion formula allows to express n-particle amplitudes with l loops in terms of amplitudes with amplitudes having at most $l-1$ loops. The basic philosophy is that singularities serve as data allowing to deduce the full integrands of the amplitudes by generalized unitarity and other kinds of arguments.

Consider first the arguments behind the BCFW formula.

1. BCFW formula is derived by performing the canonical momentum twistor deformation $Z_n \rightarrow z_n + zZ_{n-1}$, multiplying by $1/z$ and performing integration along small curve around origin so that one obtains original amplitude from the residue inside the curve. One obtains also and alternative of the residue integral expression as sum of residues from its complement. The singularities emerge by residue integral from poles of scattering amplitudes and eliminate two lines so that the recursion formula for n-particle amplitude can involve at most $n+2$-particle amplitudes.

   It seems that one must combine all n-particle amplitudes to form a single entity defining the full amplitude. I do not quite understand what how this is done. In ZEO zero energy state involving different particle numbers for the final state and expressible in terms of S-matrix (actually its generalization to what I call M-matrix) might allow to understand this.

2. In the general formula for the BCFW bridge of the "left" and "right" amplitudes one has $n_L + n_R = n+2$, $k_L + k_R = k-1$, and $l_L + l_R = l$.

3. The singularities are easy to understand in the case of tree amplitudes: they emerge from the poles of the conformally invariant quantities in the denominators of amplitudes. Physically this means that the sum of the momenta for a subset of particles corresponds to a complex pole (BCFW deformation makes two neighboring momenta complex). Hence one obtains sum over products of $j+1$-particle amplitudes BCFW bridged with $n-j$-particle amplitude to give n-particle amplitude by the merging process.

4. This is not all that is needed since the diagrams could be reduced to products of 1 loop 3-particle amplitudes which vanish by the triviality of coupling constant evolution in $\mathcal{N}=4$ SUSY. Loop amplitudes serving as a kind of source in the recursion relation save the situation. There is indeed also a second set of poles coming from loop amplitudes.

   One-loop case is the simplest one. One begins from $n+2$ particle amplitude with $l-1$ loops. At momentum space level the momenta the neighboring particles have opposite light-like momenta: one of the particles is not scattered at all. This is called forward limit. This limit suffers from collinear divergences in a generic gauge theory but in supersymmetric theories the limit is well-defined. This forward limit defines also a Yangian invariant at the level of twistor space. It can be regarded as being obtained by entangled removal of two particles combined with merge operation of two additional particles. This operation leads from $(n+2, l-1)$ amplitude to (n, l) amplitude.

Does BCFW formula make sense in TGD framework?

In TGD framework the four-fermion amplitude but restricted so that two outgoing particles have (in general) complex massless 8-momenta is the basic building brick. This changes the character of BCFW recursion relations although the four-fermion vertex effectively reduces to $F\overline{F}B$ vertex with boson identified as wormhole contact carrying fermion and antifermion at its throats.

The fundamental 4-fermion vertices assignable to wormhole contact could be formally expressed in terms of the product of two $F\overline{F}B_v$ vertices (MHV expression), where B_v is purely formal gauge boson, using the analog of MHV expression and taking into account that the second $F\overline{F}$ pair is associated with wormhole contact analogous to exchanged gauge boson.

If the fermions at fermion lines of the same partonic 2-surface can be assumed to be collinear and thus to form single coherent particle like unit, the description as superspace amplitude seems appropriate. Consequently, the effective $F\overline{F}B_v$ vertices could be assumed to have supersymmetry defined by the fermionic oscillator operator algebra at the partonic 2-surface (Clifford algebra). A good approximation is to restrict this algebra to that generating various spinor components of imbedding space spinors so that $\mathcal{N}=4$ SUSY is obtained in leptonic and quark sector. Together these give rise to $\mathcal{N}=8$ SUSY at the level of vertices broken however at the level of states.

Side note: The number of external fermions is always even suggesting that the superconformal anomalies plaguing the SUSY amplitudes with odd n (`http://tinyurl.com/yb85tnvc`) [B64] are absent in TGD: this would be basically due to the decomposition of gauge bosons to fermion pairs.

The leading singularities of scattering amplitudes would naturally correspond to the boundaries of the moduli space for the unions of partonic 2-surfaces and string world sheets.

1. The tree contribution to the gauge boson scattering amplitudes with $k=0,1$ vanish as found by Parke and Taylor who also found the simple twistorial form for the $k=2$ case (`http://tinyurl.com/y7nas26b`). In TGD framework, where lowest amplitude is 4-fermion amplitude, this situation is not encountered. According to Wikipedia article the so called CSW rules inspired by Witten's twistor theory have a problem due to the vanishing of $++-$ vertex which is not MHV form unless one changes the definition of what it is to be "wrong helicity". $++-$ is needed to construct $++++$ amplitude at one loop which does not vanish in YM theory. In SUSY it however vanishes.

   In TGD framework one does not encounter these problems since 4-fermion amplitudes are the basic building bricks. Fermion number conservation and the assumption that helicities do not mix (light-likeness in M^4 rather than only M^8-sense) implies $k=2(n(F)-n(\overline{F})$.

   In the general formula for the BCFW bridge of the "left" and "right" amplitudes one has $n_L+n_R=n+2$, $k_L+k_R=k-1$. If the TGD counterpart of the bridge eliminates two antifermions with the same "wrong" helicity -1/2, and one indeed has $k_L+k_R=k-1$ if fermions have well-defined M^4 helicity rather than being in superposition in completely correlated M^4 and CP_2 helicities.

2. In string theory loops correspond to handles of a string world sheet. Now one has partonic 2-surfaces and string world sheets and both can in principle have handles. The condition

$l \geq g$ of Witten's theory suggests that $l - g$ defines the handle number for string world sheet so that l is the total number of handles.

The identification of loop number as the genus of partonic 2-surface is second alternative: one would have $l = g$ and string world sheets would not contain handles. This might be forced by the Minkowskian signature of the induced metric at string world sheet. The signature of the induced metric would be presumably Euclidian in some region of string world sheet since the M^4 projection of either homology generator assignable with the handle would presumably define time loop in M^4 since the derivative of M^4 time coordinate with respect to string world sheet time should vanish at the turning points for M^4 time. Minimal surface property might eliminate Euclidian regions of the string world sheet. In any case, the area of string world sheet would become complex.

3. In the moduli space of partonic 2-surfaces first kind of leading singularities could correspond to pinches formed as n partonic 2-surfaces decomposes to two 2-surfaces having at least single common point so that moduli space factors into a Cartesian product. This kind of singularities could serve as counterparts for the merge singularities appearing in the BCFW bridging of amplitudes. The numbers of loops must be additive and this is consistent with both interpretations for l.

4. What about forward limit? One particle should go through without scattering and is eliminated by entangled removal. In ZEO one can ask whether there is also quantum entanglement between the positive and negative energy parts of this single particle state and state function reduction does not occur. The addition of particle and merging it with another one could correspond to a situation in which two points of partonic 2-surface touch. This means addition of one handle so that loop number l increases.

   It seems that analytically the loop is added by the entangled removal but at the level of partonic surface it is added by the merging. Also now both $l > g$ and $l = g$ options make sense.

12.4.9 Possible Connections Of TGD Approach With The Twistor Grassmannian Approach

For a non-specialist lacking the technical skills, the work related to twistors is a garden of mysteries and there are a lot of questions to be answered: most of them of course trivial for the specialist. The basic questions are following.

How the twistor string approach of Witten and its possible TGD generalization relate to the approach involving residue integration over projective sub-manifolds of Grassmannians $G(k,n)$?

1. In [B40] Nima et al argue that one can transform Grassmannian representation to twistor string representation for tree amplitudes. The integration over $G(k,n)$ translates to integration over the moduli space of complex curves of degree $d = k - 1 + l$, $l \geq g$ is the number of loops. The moduli correspond to complex coefficients of the polynomial of degree d and they form naturally a projective space since an overall scaling of coefficients does not change the surfaces. One can expect also in the general case that moduli space of the partonic 2-surfaces can be represented as a projective sub-manifold of some projective space. Loop corrections would correspond to the inclusion of higher degree surfaces.

2. This connection gives hopes for understanding the integration contours in $G(k,n)$ at deeper level in terms of the moduli spaces of partonic 2-surfaces possibly restricted by conformal gauge conditions.

Below I try to understand and relate the work of Nima Arkani Hamed et al with twistor Grassmannian approach to TGD.

The notion of positive Grassmannian

The notion of positive Grassmannian is one of the central notions introduced by Nima et al.

1. The claim is that the sub-spaces of the real Grassmannian $G(k,n)$ contributing to the amplitudes for $++--$ signature are such that the determinants of the $k\times k$ minors associated with ordered columns of the $k\times n$ matrix C representing point of $G(k,n)$ are positive. To be precise, the signs of all minors are positive or negative simultaneously: only the ratios of the determinants defining projective invariants are positive.

2. At the boundaries of positive regions some of the determinants vanish. Some k-volumes degenerate to a lower-dimensional volume. Boundaries are responsible for the leading singularities of the scattering amplitudes and the integration measure associated with $G(k,n)$ has a logarithmic singularity at the boundaries. These boundaries would naturally correspond to the boundaries of the moduli space for the partonic 2-surfaces. Here also string world sheets could contribute to singularities.

3. This condition has a partial generalization to the complex case: the determinants whose ratios serve as projectively invariant coordinates are non-vanishing. A possible further manner to generalize this condition would be that the determinants have positive real part so that apart from rotation by $\pi/2$ they would reside in the upper half plane of complex plane. Upper half plane is the hyperbolic space playing key role in complex analysis and in the theory of hyperbolic 2-manifolds for which it serves as universal covering space by a finite discrete subgroup of Lorentz group $SL(2,C)$. The upper half-plane having a deep meaning in the theory of Riemann surfaces might play also a key role in the moduli spaces of partonic 2-surfaces. The projective space would be based - not on projectivization of C^n but that of H^n, H the upper half plane.

Could positivity have some even deeper meaning?

1. In TGD framework the number theoretical universality of amplitudes suggests this. Canonical identification maps $\sum x_n p^n \rightarrow \sum x_n p^{-n}$ p-adic number to non-negative reals. p-Adicization is possible for angle variables by replacing them by discrete phases, which are roots of unity. For non-angle like variables, which are non-negative one uses some variant of canonical identification involving cutoffs [K104]. The positivity should hold true for all structures involved, the $G(2,n)$ points defined by the twistors characterizing momenta and helicities of particles (actually pairs of orthogonal planes defined by twistors and their conjugates), the moduli space of partonic 2-surfaces, etc...

2. p-Adicization requires discretization of phases replacing angles so that they come as roots of unity associated with the algebraic extension used. The p-adic valued counterpart of Riemann or Lebesque integral does not make sense p-adically. Residue integrals can however allow to define p-adic integrals by analytic continuation of the integral and discretization of the phase factor along the integration contour does not matter (not however the p-adically troublesome factor 2π!).

3. TGD suggests that the generalization of positive real projectively invariant coordinates to complex coordinates of the hyperbolic space representable as upper half plane, or equivalently as unit disk obtained from the upper half plane by exponential mapping $w = exp(iz)$: positive coordinate α would correspond to the radial coordinate for the unit disk (Poincare hyperbolic disk appearing in Escher's paintings). The real measure $d\alpha/\alpha$ would correspond to $dz = dw/w$ restricted to a radial line from origin to the boundary of the unit disk. This integral should correspond to integral over a closed contour in complex case. This is the case if the integrand is discontinuity over a radial cut and equivalent with an integral over curve including also the boundary of the unit disk. This integral would reduce to the sum of the residues of poles inside the unit disk.

The notion of amplituhedron

The notion of amplituhedron is the latest step of progress in the twistor Grassmann approach [B20, B19]. What is so remarkable, is the simplicity of the expressions for all-loop amplitudes and the fact that positivity implies locality and unitarity for $\mathcal{N}=4$ SUSY.

Consider first tree amplitudes with general value of k.

1. The notion of amplituhedron relies on the mapping of $G(k,n)$ to $G_+(k,k+m)$ $n \geq k+m$. $G_+(k,k+m)$ is positive Grassmannian characterized by the condition that all $k\times k$- minors $k\times(k+m)$ matrix representing the point of $G_+(k,k+m)$ are non-negative and vanish at the boundaries $G_+(k,k+m)$. The value of m is $m=4$ and follows from the conditions that amplitudes come out correctly. The constraint $Y = C\cdot Z$, where Y corresponds to point of $G_+(k,k+4)$ and Z to the point of $G(k,n)$ performs this mapping, which is clearly many-to one. One can decompose $G_+(k,k+4)$ to positive regions intersecting only along their common boundary portions. The decomposition of a convex polygon in plane represent the basic example of this kind of decomposition.

2. Each decomposition defines a sum of contributions to the scattering amplitudes involving integration of a projectively invariant volume form over the positive region in question. The form has a logarithmic singularity at the boundaries of the integration region but spurious singularities cancel so that only the contribution of the genuine boundary of $G_+(k,k+4)$ remains. There are additional delta function constraints fixing the integral completely in real case.

3. In complex case one has residue integral. The proposed generalization to the complex case is by analytic continuation. TGD inspired proposal is that the positivity condition in the real case is generalized to the condition that the positive coordinates are replaced by complex coordinates of hyperbolic space representable as upper half plane or equivalently as the unit disk obtained from upper half plane by exponential mapping $w = exp(iz)$. The measure $d\alpha/\alpha$ would correspond to $dz = dw/w$. If taken over boundary circle labelled by discrete phase factors $exp(i\phi)$ given by roots of unity the integral would be numerically a discrete Riemann sum making no sense p-adically but residue theorem could allow to avoid the discretizaton and to define the p-adic variant of the integral by analytic continuation. These conditions would be completely general conditions on various projectively invariant moduli involved.

4. One must extend the bosonic twistors Z_a of external particles by adding k coordinates. Somewhat surprisingly, these coordinates are anticommutative super-coordinates expressible as linear combinations of fermionic parts of super-twistor using coefficients, which are also Grassmann numbers. Integrating over these one ends up with the standard expression of the amplitude using canonical integration measure for the regions in the decomposition of amplituhedron.

   What looks to me intriguing is that there is only super-integration involved over the additional k degrees of freedom. In Witten's approach $k-1$ corresponds to the minimum degree of the polynomial defining the string world sheet representing tree diagram. In TGD framework $k+1$ (rather than $k-1$) could correspond to the minimum degree of partonic 2-surface. In TGD approximate SUSY would correspond to Grassmann algebra of fermionic oscillator operators defined by the spinor basis for imbedding space spinors. The interpretation could be that each fermion whose helicity differs from that allowed by light-likeness in M^4 sense (this requires non-vanishing M^4 mass), contributes $\Delta k = 1$ to the degree of corresponding partonic 2-surface. Since the partonic 2-surface is common for all particles, one must have $d = k+1$ at least. The k-fold super integration would be basically integral over the moduli characterizing the polynomials of degree k realizing quantum classical correspondence in fermionic degrees of freedom.

BFCW recursion formula involves also loop amplitudes for which amplituhedron provides also a very nice representation.

1. The basic operation is the addition of a loop to get (n,k,l) amplitude from $(n+2,k,l-1)$ amplitude. That 2 particles must be removed for each loop is not obvious in $\mathcal{N}=4$ SUSY but follows from the condition that positivity of the integration domain is preserved. This procedure removes from $(n+2,k,l-1)$-amplitude 2 particles with opposite four-momenta so that (n,k,l) amplitude is obtained. In the case of L-loops one extends $G(k,n)$ by adding its "complement" as a Cartesian factor $G(n-k,n)$ and imbeds to $G(n-k,n)$ 2-plane for each loop. Positivity conditions can be generalized so that they apply to $(k+2l)\times(k+2l)$-minors associated with matrices having as rows $0 \leq l \leq L$ ordered D_{i_k}:s and of C. The general

expressions of the loop contributions are of the same form as for tree contributions: only the number of integration variables is $4 \times (k + L)$.

2. As already explained, in TGD framework the addition of loop would correspond to the formation of a handle to the partonic surface by fusing two points of partonic 2-surface and thus creating a surface intermediate between topologies with g and g+1 handles. g would correspond to the genus characterizing fermion family and one would have $L \geq g$. In elementary particle wave functionals loop [K12] contributions would correspond to higher genus contributions $l_1 = l - g > 0$ with basic contribution coming from genus g. For scattering amplitudes loop contributions would involve the change of the genus of the incoming wormhole throat so that they correspond to singular surfaces at the boundaries of their moduli space identifiable as loop corrections. $l_1 = l - g > 0$ would represent the number of pinches of the partonic 2-surface.

What about non-planar amplitudes?

Non-planar Feynman diagrams have remained a challenge for the twistor approach. The problem is simple: there is no canonical ordering of the extrenal particles and the loop integrand involving tricky shifts in integrations to get finite outcome is not unique and well-defined so that twistor Grasmann approach encounters difficulties.

Recently Nima Arkani-Hamed et al have considered also non-planar MHV diagrams [B41] (having minimal number of "wrong" helicities) of N=4 SUSY, and shown that they can be reduced to non-planar diagrams for different permutations of vertices of planar diagrams ordered naturally. There are several integration regions identified as positive Grassmannians corresponding to different orderings of the external lines inducing non-planarity. This does not however hold true generally.

At the QFT limit the crossings of lines emerges purely combinatorially since Feynman diagrams are purely combinatorial objects with the ordering of vertices determining the topological properties of the diagram. Non-planar diagrams correspond to diagrams, which do not allow crossing-free imbedding to plane but require higher genus surface to get rid of crossings.

1. The number of the vertices of the diagram and identification of lines connecting them determines the diagram as a graph. This defines also in TGD framework Feynman diagram like structure as a graph for the fermion lines and should be behind non-planarity in QFT sense.

2. Could 2-D Feynman graphs exists also at geometric rather than only combinatorial level? Octonionization at imbedding space level requires identification of preferred $M^2 \subset M^4$ defining a preferred hyper-complex sub-space. Could the projection of the Fermion lines defined concrete geometric representation of Feynman diagrams?

3. Despite their purely combinatorial character Feynman diagrams are analogous to knots and braids. For years ago [K35] I proposed the generalization of the construction of knot invariants in which one gradually eliminates the crossings of the knot projection to end up with a trivial knot is highly suggestive as a procedure for constructing the amplitudes associated with the non-planar diagrams. The outcome should be a collection of planar diagrams calculable using twistor Grassmannian methods. Scattering amplitudes could be seen as analogs of knot invariants. The reduction of MHV diagrams to planar diagrams could be an example of this procedure.

One can imagine also analogs of non-planarity, which are geometric and topological rather than combinatorial and not visible at the QFT limit of TGD.

1. The fermion lines representing boundaries of string world sheets at the light-like orbits of partonic 2-surfaces can get braided. The same can happen also for the string boundaries at space-like 3-surfaces at the ends of the space-time surface. The projections of these braids to partonic 2-surfaces are analogs of non-planar diagrams. If the fermion lines at single wormhole throat are regarded effectively as a line representing one member of super-multiplet, this kind of braiding remains below the resolution used and cannot correspond to the braiding at QFT limit.

2. 2-knotting and 2-braiding are possible for partonic 2-surfaces and string world sheets as 2-surfaces in 4-D space-time surfaces and have no counterpart at QFT limit.

12.4.10 Permutations, Braidings, And Amplitudes

In [B37] Nima Arkani-Hamed demonstrates that various twistorially represented on-mass-shell amplitudes (allowing light-like complex momenta) constructible by taking products of the 3-particle amplitude and its conjugate can be assigned with unique permutations of the incoming lines. The article describes the graphical representation of the amplitudes and its generalization. For 3-particle amplitudes, which correspond to $++-$ and $+--$ twistor amplitudes, the corresponding permutations are cyclic permutations, which are inverses of each other. One actually introduces double cover for the labels of the particles and speaks of decorated permutations meaning that permutation is always a right shift in the integer and in the range $[1, 2\times n]$.

Amplitudes as representation of permutations

It is shown that for on mass shell twistor amplitudes the definition using on-mass-shell 3-vertices as building bricks is highly reducible: there are two moves for squares defining 4-particle sub-amplitudes allowing to reduce the graph to a simpler one. The first ove is topologically like the s-t duality of the old-fashioned string models and second one corresponds to the transformation black $\leftrightarrow$ white for a square sub-diagram with lines of same color at the ends of the two diagonals and built from 3-vertices.

One can define the permutation characterizing the general on mass shell amplitude by a simple rule. Start from an external particle a and go through the graph turning in in white (black) vertex to left (right). Eventually this leads to a vertex containing an external particle and identified as the image $P(a)$ of the a in the permutation. If permutations are taken as right shifts, one ends up with double covering of permutation group with $2\times n!$ elements - decorated permutations. In this manner one can assign to any any line of the diagram two lines. This brings in mind 2-D integrable theories where scattering reduces to braiding and also topological QFTs where braiding defines the unitary S-matrix. In TGD parton lines involve braidings of the fermion lines so that an assignment of permutation also to vertex would be rather nice.

BCFW bridge has an interpretation as a transposition of two neighboring vertices affecting the lines of the permutation defining the diagram. One can construct all permutations as products of transpositions and therefore by building BCFW bridges. BCFW bridge can be constructed also between disjoint diagrams as done in the BCFW recursion formula.

Can one generalize this picture in TGD framework? There are several questions to be answered.

(a) What should one assume about the states at the light-like boundaries of string world sheets? What is the precise meaning of the supersymmetry: is it dynamical or gauge symmetry or both?

(b) What does one mean with particle: partonic 2-surface or boundary line of string world sheet? How the fundamental vertices are identified: 4 incoming boundaries of string world sheets or 3 incoming partonic orbits or are both aspects involved?

(c) How the 8-D generalization of twistors bringing in second helicity and doubling the M^4 helicity states assignable to fermions does affect the situation?

(d) Does the crucial right-left rule relying heavily on the possibility of only 2 3-particle vertices generalize? Does M^4 massivation imply more than 2 3-particle vertices implying many-to-one correspondence between on-mass-shell diagrams and permutations? Or should one generalize the right-left rule in TGD framework?

Fermion lines for fermions massless in 8-D sense

What does one mean with particle line at the level of fermions?

(a) How the addition of CP_2 helicity and complete correlation between M^4 and CP_2 chiralities does affect the rules of $\mathcal{N}=4$ SUSY? Chiral invariance in 8-D sense guarantees fermion number conservation for quarks and leptons separately and means conservation of the product of M^4 and CP_2 chiralities for 2-fermion vertices. Hence only M^4 chirality need to be considered. M^4 massivation allows more 4-fermion vertices than $\mathcal{N}=4$ SUSY.

(b) One can assign to a given partonic orbit several lines as boundaries of string world sheets connecting the orbit to other partonic orbits. Supersymmetry could be understoond in two manners.

   i. The fermions generating the state of super-multiplet correspond to boundaries of different string world sheets which need not connect the string world sheet to same partonic orbit. This SUSY is dynamical and broken. The breaking is mildest breaking for line groups connected by string world sheets to same partonic orbit. Right handed neutrinos generated the least broken $\mathcal{N}=2$ SUSY.

   ii. Also single line carrying several fermions would provide realization of generalized SUSY since the multi-fermion state would be characterized by single 8-momentum and helicity. One would have $\mathcal{N}=4$ SUSY for quarks and leptons separately and $\mathcal{N}=8$ if both quarks and leptons are allowed. Conserved total for quark and antiquarks and leptons and antileptons characterize the lines as well.
   What would be the propagator associated with many-fermion line? The first guess is that it is just a tensor power of single fermion propagator applied to the tensor power of single fermion states at the end of the line. This gives power of $1/p^{2n}$ to the denominator, which suggests that residue integral in momentum space gives zero unless one as just single fermion state unless the vertices give compensating powers of p. The reduction of fermion number to 0 or 1 would simplify the diagrammatics enormously and one would have only 0 or 1 fermions per given string boundary line. Multi-fermion lines would represent gauge degrees of freedom and SUSY would be realized as gauge invariance. This view about SUSY clearly gives the simplest picture, which is also consistent with the earlier one, and will be assumed in the sequel

(c) The multiline containing n fermion oscillator operators can transform by chirality mixing in 2^n manners at 4-fermion vertex so that there is quite a large number of options for incoming lines with n_i fermions.

(d) In 4-D Dirac equation light-likeness implies a complete correlation between fermion number and chirality. In 8-D case light-likeness should imply the same: now chirality correspond to fermion number. Does this mean that one must assume just superposition of different M^4 chiralities at the fermion lines as 8-D Dirac equation requires. Or should one assume that virtual fermions at the end of the line have wrong chirality so that massless Dirac operator does not annihilate them?

Fundamental vertices

One can consider two candidates for fundamental vertices depending on whether one identifies the lines of Feynman diagram as fermion lines or as light-like orbits of partonic 2-surfaces. The latter vertices reduces microscopically to the fermionic 4-vertices.

(a) If many-fermion lines are identified as fundamental lines, 4-fermion vertex is the fundamental vertex assignable to single wormhole contact in the topological vertex defined by common partonic 2-surface at the ends of incoming light-like 3-surfaces. The discontinuity is what makes the vertex non-trivial.

(b) In the vertices generalization of OZI rule applies for many-fermion lines since there are no higher vertices at this level and interactions are mediated by classical induced gauge fields and chirality mixing. Classical induced gauge fields vanish if CP_2 projection is 1-dimensional for string world sheets and even gauge potentials vanish if the projection is to geodesic circle. Hence only the chirality mixing due to the mixing of M^4 and

CP_2 gamma matrices is possible and changes the fermionic M^4 chiralities. This would dictate what vertices are possible.

(c) The possibility of two helicity states for fermions suggests that the number of amplitudes is considerably larger than in $\mathcal{N} = 4$ SUSY. One would have 5 independent fermion amplitudes and at each 4-fermion vertex one should be able to choose between 3 options if the right-left rule generalizes. Hence the number of amplitudes is larger than the number of permutations possibly obtained using a generalization of right-left rule to right-middle-left rule.

(d) Note however that for massless particles in M^4 sense the reduction of helicity combinations for the fermion and antifermion making virtual gauge boson happens. The fermion and antifermion at the opposite wormhole throats have parallel four-momenta in good approximation. In M^4 they would have opposite chiralities and opposite helicities so that the boson would be M^4 scalar. No vector bosons would be obtained in this manner.

In 8-D context it is possible to have also vector bosons since the M^4 chiralities can be same for fermion and anti-fermion. The bosons are however massive, and even photon is predicted to have small mass given by p-adic thermodynamics [K39]. Massivation brings in also the M^4 helicity 0 state. Only if zero helicity state is absent, the fundamental four-fermion vertex vanishes for $++++$ and $----$ combinations and one extend the right-left rule to right-middle-left rule. There is however no good reason for he reduction in the number of 4-fermion amplitudes to take place.

Partonic surfaces as 3-vertices

At space-time level one could identify vertices as partonic 2-surfaces.

(a) At space-time level the fundamental vertices are 3-particle vertices with particle identified as wormhole contact carrying many-fermion states at both wormhole throats. Each line of BCFW diagram would be doubled. This brings in mind the representation of permutations and leads to ask whether this representation could be re-interpreted in TGD framework. For this option the generalization of the decomposition of diagram to 3-particle vertices is very natural. If the states at throats consist of bound states of fermions as SUSY suggests, one could characterize them by total 8-momentum and helicity in good approximation. Both helicities would be however possible also for fermions by chirality mixing.

(b) A genuine decomposition to 3-vertices and lines connecting them takes place if two of the fermions reside at opposite throats of wormhole contact identified as fundamental gauge boson (physical elementary particles involve two wormhole contacts).

The 3-vertex can be seen as fundamental and 4-fermion vertex becomes its microscopic representation. Since the 3-vertices are at fermion level 4-vertices their number is greater than two and there is no hope about the generalization of right-left rule.

OZI rule implies correspondence between permutations and amplitudes

The realization of the permutation in the same manner as for $\mathcal{N} = 4$ amplitudes does not work in TGD. OZI rule following from the absence of 4-fermion vertices however implies much simpler and physically quite a concrete manner to define the permutation for external fermion lines and also generalizes it to include braidings along partonic orbits.

(a) Already $\mathcal{N} = 4$ approach assumes decorated permutations meaning that each external fermion has effectively two states corresponding to labels k and $k + n$ (permutations are shifts to the right). For decorated permutations the number of external states is effectively 2^n and the number of decorated permutations is $2 \times n!$. The number of different helicity configurations in TGD framework is 2^n for incoming fermions at the vertex defined by the partonic 2-surface. By looking the values of these numbers for lowest integers one finds $2n \geq 2^n$: for $n = 2$ the equation is saturated. The inequality $log(n!) > nlog(n)/e) + 1$ (see `http://tinyurl.com/2bjk5h`). gives

$$\frac{log(2n!)}{log(2^n)} \geq \frac{log(2)+1+nlog(n/e)}{nlog(2)} = log(n/e)/log(2)+O(1/n)$$

so that the desired inequality holds for all interesting values of n.

(b) If OZI rule holds true, the permutation has very natural physical definition. One just follows the fermion line which must eventually end up to some external fermion since the only fermion vertex is 2-fermion vertex. The helicity flip would map $k \rightarrow k+n$ or vice versa.

(c) The labelling of diagrams by permutations generalizes to the case of diagrams involving partonic surfaces at the boundaries of causal diamond containing the external fermions and the partonic 2-surfaces in the interior of CD identified as vertices. Permutations generalize to braidings since also the braidings along the light-like partonic 2-surfaces are allowed. A quite concrete generalization of the analogs of braid diagrams in integrable 2-D theories emerges.

(d) BCFW bridge would be completely analogous to the fundamental braiding operation permuting two neighboring braid strands. The almost reduction to braid theory - apart from the presence of vertices conforms with the vision about reduction of TGD to almost topological QFT.

To sum up, the simplest option assumes SUSY as both gauge symmetry and broken dynamical symmetry. The gauge symmetry relates string boundaries with different fermion numbers and only fermion number 0 or 1 gives rise to a non-vanishing outcome in the residue integration and one obtains the picture used hitherto. If OZI rule applies, the decorated permutation symmetry generalizes to include braidings at the parton orbits and $k \rightarrow k \pm n$ corresponds to a helicity flip for a fermion going through the 4-vertex. OZI rules follows from the absence of non-linearities in Dirac action and means that 4-fermion vertices in the usual sense making theory non-renormalizable are absent. Theory is essentially free field theory in fermionic degrees of freedom and interactions in the sense of QFT are transformed to non-trivial topology of space-time surfaces.

3. If one can approximate space-time sheets by maps from M^4 to CP_2, one expects General Relativity and QFT description to be good approximations. GRT space-time is obtained by replacing space-time sheets with single sheet - a piece of slightly deformed Minkowski space but without assupmtion about imbedding to H. Induced classical gravitational field and gauge fields are sums of those associated with the sheets. The generalized Feynman diagrams with lines at various sheets and going also between sheets are projected to single piece of M^4. Many-sheetedness makes 1-homology non-trivial and implies analog of braiding, which should be however invisible at QFT limit.

A concrete manner to eliminate line crossing in non-planar amplitude to get nearer to non-planar amplitude could proceed roughly as follows. This is of course a pure guess motivated only by topological considerations. Professional might kill it in few seconds.

1. If the lines carry no quantum numbers, reconnection allows to eliminate the crossings. Consider the crossing line pair connecting AB in the initial state to CD in final state. The crossing lines are AD and BC. Reconnection can take place in two manners: AD and BC transform either to AB and CD or to AC and BD: neither line pair has crossing. The final state of the braid would be quantum superposition of the resulting more planar braids.

2. The crossed lines however carry different quantum numbers in the generic situation: for instance, they can be fermionic and bosonic. In this particular case the reconnection does not make sense since a line carrying fermion number would transform to a line carrying boson.

   In TGD framework all lines are fermion lines at fundamental level but the constraint due to different quantum numbers still remains and it is easy to see that mere reconnection is not enough. Fermion number conservation allows only one of the two alternatives to be realized. Conservation of quantum numbers forces to restrict gives an additional constraint which for simplest non-planar diagram with two crossed fermion lines forces the quantum numbers of fermions to be identical.

It seems also more natural to consider pairs of wormhole contacts defining elementary particles as "lines" in turn consisting of fermion lines. Yangian symmetry allows to develop a more detailed view about what this decomposition could mean.

Quantum number conservation demands that reconnection is followed by a formation of an additional internal line connecting the non-crossing lines obtained by reconnection. The additional line representing a quantum number exchange between the resulting non-crossing lines would guarantee the conservation of quantum numbers. This would bring in two additional vertices and one additional internal line. This would be enough to reduce planarity. The repeated application of this transformation should produced a sum of non-planar diagrams.

3. What could go wrong with this proposal? In the case of gauge theory the order of diagram increases by g^2 since two new vertices are generated. Should a multiplication by $1/g^2$ accompany this process? Or is this observation enough to kill the hypothesis in gauge theory framework? In TGD framework the situation is not understood well enough to say anything. Certainly the critical value of α_K implies that one cannot regard it as a free parameter and cannot treat the contributions from various orders as independent ones.

12.5 Could The Universe Be Doing Yangian Arithmetics?

One of the old TGD inspired really crazy ideas about scattering amplitudes is that Universe is doing some sort of arithmetics so that scattering amplitude are representations for computational sequences of minimum length. The idea is so crazy that I have even given up its original form, which led to an attempt to assimilate the basic ideas about bi-algebras, quantum groups [K6], Yangians [K76], and related exotic things. The work with twistor Grassmannian approach inspired a reconsideration of the original idea seriously with the idea that super-symplectic Yangian could define the arithmetics. I try to describe the background, motivation, and the ensuing reckless speculations in the following.

12.5.1 Do Scattering Amplitudes Represent Quantal Algebraic Manipulations?

I seems that tensor product $\otimes$ and direct sum $\oplus$ - very much analogous to product and sum but defined between Hilbert spaces rather than numbers - are naturally associated with the basic vertices of TGD. I have written about this a highly speculative chapter - both mathematically and physically [K92]. The chapter [K6] is a remnant of earlier similar speculations.

1. In $\otimes$ vertex 3-surface splits to two 3-surfaces meaning that the 2 "incoming" 4-surfaces meet at single common 3-surface and become the outgoing 3-surface: 3 lines of Feynman diagram meeting at their ends. This has a lower-dimensional shadow realized for partonic 2-surfaces. This topological 3-particle vertex would be higher-D variant of 3-vertex for Feynman diagrams.

2. The second vertex is trouser vertex for strings generalized so that it applies to 3-surfaces. It does not represent particle decay as in string models but the branching of the particle wave function so that particle can be said to propagate along two different paths simultaneously. In double slit experiment this would occur for the photon space-time sheets.

3. The idea is that Universe is doing arithmetics of some kind in the sense that particle 3-vertex in the above topological sense represents either multiplication or its time-reversal co-multiplication.

The product, call it $\circ$, can be something very general, say algebraic operation assignable to some algebraic structure. The algebraic structure could be almost anything: a random list of structures popping into mind consists of group, Lie-algebra, super-conformal algebra quantum algebra, Yangian, etc.... The algebraic operation $\circ$ can be group multiplication, Lie-bracket, its generalization to super-algebra level, etc...). Tensor product and thus linear (Hilbert) spaces are involved always, and in product operation tensor product $\otimes$ is replaced with $\circ$.

1. The product $A_k \otimes A_l \rightarrow C = A_k \circ A_l$ is analogous to a particle reaction in which particles A_k and A_l fuse to particle $A_k \otimes A_l \rightarrow C = A_k \circ A_l$. One can say that $\otimes$ between reactants is transformed to $\circ$ in the particle reaction: kind of bound state is formed.

2. There are very many pairs A_k, A_l giving the same product C just as given integer can be divided in many manners to a product of two integers if it is not prime. This of course suggests that elementary particles are primes of the algebra if this notion is defined for it! One can use some basis for the algebra and in this basis one has $C = A_k \circ A_l = f_{klm} A_m$, f_{klm} are the structure constants of the algebra and satisfy constraints. For instance, associativity $A(BC) = (AB)C$ is a constraint making the life of algebraist more tolerable and is almost routinely assumed.

   For instance, in the number theoretic approach to TGD associativity is proposed to serve as fundamental law of physics and allows to identify space-time surfaces as 4-surfaces with associative (quaternionic) tangent space or normal space at each point of octonionic imbedding space $M^4 \times CP_2$. Lie algebras are not associative but Jacobi-identities following from the associativity of Lie group product replace associativity.

3. Co-product can be said to be time reversal of the algebraic operation $\circ$. Co-product can be defined as $C = A_k \rightarrow \sum_{lm} f_k^{lm} A_l \otimes A_m$, where f_k^{lm} are the structure constants of the algebra. The outcome is quantum superposition of final states, which can fuse to C (the "reaction" $A_k \otimes A_l \rightarrow C = A_k \circ A_l$ is possible). One can say that $\circ$ is replaced with $\otimes$: bound state decays to a superposition of all pairs, which can form the bound states by product vertex.

There are motivations for representing scattering amplitudes as sequences of algebraic operations performed for the incoming set of particles leading to an outgoing set of particles with particles identified as algebraic objects acting on vacuum state. The outcome would be analogous to Feynman diagrams but only the diagram with minimal length to which a preferred extremal can be assigned is needed. Larger ones must be equivalent with it.

The question is whether it could be indeed possible to characterize particle reactions as computations involving transformation of tensor products to products in vertices and co-products to tensor products in co-vertices (time reversals of the vertices). A couple of examples gives some idea about what is involved.

1. The simplest operations would preserve particle number and to just permute the particles: the permutation generalizes to a braiding and the scattering matrix would be basically unitary braiding matrix utilized in topological quantum computation.

2. A more complex situation occurs, when the number of particles is preserved but quantum numbers for the final state are not same as for the initial state so that particles must interact. This requires both product and co-product vertices. For instance, $A_k \otimes A_l \rightarrow f_{kl}^m A_m$ followed by $A_m \rightarrow f_m^{rs} A_r \otimes A_s$ giving $A_k \rightarrow f_{kl}^m f_m^{rs} A_r \otimes A_s$ representing 2-particle scattering. State function reduction in the final state can select any pair $A_r \otimes A_s$ in the final state. This reaction is characterized by the ordinary tree diagram in which two lines fuse to single line and defuse back to two lines. Note also that there is a non-deterministic element involved. A given final state can be achieved from a given initial state after large enough number of trials. The analogy with problem solving and mathematical theorem proving is obvious. If the interpretation is correct, Universe would be problem solver and theorem prover!

3. More complex reactions affect also the particle number. 3-vertex and its co-vertex are the simplest examples and generate more complex particle number changing vertices. For instance, on twistor Grassmann approach on can construct all diagrams using two 3-vertices. This encourages the restriction to 3-vertice (recall that fermions have only 2-vertices)

4. Intuitively it is clear that the final collection of algebraic objects can be reached by a large - maybe infinite - number of ways. It seems also clear that there is the shortest manner to end up to the final state from a given initial state. Of course, it can happen that there is no

way to achieve it! For instance, if $\circ$ corresponds to group multiplication the co-vertex can lead only to a pair of particles for which the product of final state group elements equals to the initial state group element.

5. Quantum theorists of course worry about unitarity. How can avoid the situation in which the product gives zero if the outcome is element of linear space. Somehow the product should be such that this can be avoided. For instance, if product is Lie-algebra commutator, Cartan algebra would give zero as outcome.

12.5.2 Generalized Feynman Diagram As Shortest Possible Algebraic Manipulation Connecting Initial And Final Algebraic Objects

There is a strong motivation for the interpretation of generalized Feynman diagrams as shortest possible algebraic operations connecting initial and final states. The reason is that in TGD one does not have path integral over all possible space-time surfaces connecting the 3-surfaces at the ends of CD. Rather, one has in the optimal situation a space-time surface unique apart from conformal gauge degeneracy connecting the 3-surfaces at the ends of CD (they can have disjoint components).

Path integral is replaced with integral over 3-surfaces. There is therefore only single minimal generalized Feynman diagram (or twistor diagram, or whatever is the appropriate term). It would be nice if this diagram had interpretation as the shortest possible computation leading from the initial state to the final state specified by 3-surfaces and basically fermionic states at them. This would of course simplify enormously the theory and the connection to the twistor Grassmann approach is very suggestive. A further motivation comes from the observation that the state basis created by the fermionic Clifford algebra has an interpretation in terms of Boolean quantum logic and that in ZEO the fermionic states would have interpretation as analogs of Boolean statements $A \rightarrow B$.

To see whether and how this idea could be realized in TGD framework, let us try to find counterparts for the basic operations $\otimes$ and $\circ$ and identify the algebra involved. Consider first the basic geometric objects.

1. Tensor product could correspond geometrically to two disjoint 3-surfaces representing 3-particles. Partonic 2-surfaces associated with a given 3-surface represent second possibility. The splitting of a partonic 2-surface to two could be the geometric counterpart for co-product.

2. Partonic 2-surfaces are however connected to each other and possibly even to themselves by strings. It seems that partonic 2-surface cannot be the basic unit. Indeed, elementary particles are identified as pairs of wormhole throats (partonic 2-surfaces) with magnetic monopole flux flowing from throat to another at first space-time sheet, then through throat to another sheet, then back along second sheet to the lower throat of the first contact and then back to the thirst throat. This unit seems to be the natural basic object to consider. The flux tubes at both sheets are accompanied by fermionic strings. Whether also wormhole throats contain strings so that one would have single closed string rather than two open ones, is an open question.

3. The connecting strings give rise to the formation of gravitationally bound states and the hierarchy of Planck constants is crucially involved. For elementary particle there are just two wormhole contacts each involving two wormhole throats connected by wormhole contact. Wormhole throats are connected by one or more strings, which define space-like boundaries of corresponding string world sheets at the boundaries of CD. These strings are responsible for the formation of bound states, even macroscopic gravitational bound states.

12.5.3 Does Super-Symplectic Yangian Define The Arithmetics?

Super-symplectic Yangian would be a reasonable guess for the algebra involved.

1. The 2-local generators of Yangian would be of form $T_1^A = f^A_{BC} T^B \otimes T^C$, where f^A_{BC} are the structure constants of the super-symplectic algebra. n-local generators would be obtained

by iterating this rule. Note that the generator T_1^A creates an entangled state of T^B and T^C with f^A_{BC} the entanglement coefficients. T_n^A is entangled state of T^B and T^C_{n-1} with the same coefficients. A kind replication of T^A_{n-1} is clearly involved, and the fundamental replication is that of T^A. Note that one can start from any irreducible representation with well defined symplectic quantum numbers and form similar hierarchy by using T^A and the representation as a starting point.

That the hierarchy T_n^A and hierarchies irreducible representations would define a hierarchy of states associated with the partonic 2-surface is a highly non-trivial and powerful hypothesis about the formation of many-fermion bound states inside partonic 2-surfaces.

2. The charges T^A correspond to fermionic and bosonic super-symplectic generators. The geometric counterpart for the replication at the lowest level could correspond to a fermionic/bosonic string carrying super-symplectic generator splitting to fermionic/bosonic string and a string carrying bosonic symplectic generator T^A. This splitting of string brings in mind the basic gauge boson-gauge boson or gauge boson-fermion vertex.

   The vision about emission of virtual particle suggests that the entire wormhole contact pair replicates. Second wormhole throat would carry the string corresponding to T^A assignable to gauge boson naturally. T^A should involve pairs of fermionic creation and annihilation operators as well as fermionic and anti-fermionic creation operator (and annihilation operators) as in quantum field theory.

3. Bosonic emergence suggests that bosonic generators are constructed from fermion pairs with fermion and anti-fermion at opposite wormhole throats: this would allow to avoid the problems with the singular character of purely local fermion current. Fermionic and anti-fermionic string would reside at opposite space-time sheets and the whole structure would correspond to a closed magnetic tube carrying monopole flux. Fermions would correspond to superpositions of states in which string is located at either half of the closed flux tube.

4. The basic arithmetic operation in co-vertex would be co-multiplication transforming T_n^A to $T^A_{n+1} = f^A_{BC} T^B_n \otimes T^C$. In vertex the transformation of T^A_{n+1} to T^A_n would take place. The interpretations would be as emission/absorption of gauge boson. One must include also emission of fermion and this means replacement of T^A with corresponding fermionic generators F^A, so that the fermion number of the second part of the state is reduced by one unit. Particle reactions would be more than mere braidings and re-grouping of fermions and anti-fermions inside partonic 2-surfaces, which can split.

5. Inside the light-like orbits of the partonic 2-surfaces there is also a braiding affecting the M-matrix. The arithmetics involved would be therefore essentially that of measuring and "co-measuring" symplectic charges.

   Generalized Feynman diagrams (preferred extremals) connecting given 3-surfaces and many-fermion states (bosons are counted as fermion-anti-fermion states) would have a minimum number of vertices and co-vertices. The splitting of string lines implies creation of pairs of fermion lines. Whether regroupings are part of the story is not quite clear. In any case, without the replication of 3-surfaces it would not be possible to understand processes like e-e scattering by photon exchange in the proposed picture.

It is easy to hear the comments of the skeptic listener in the back row.

1. The attribute "minimal" - , which could translate to minimal value of Kähler function - is dangerous. It might be very difficult to determine what the minimal diagram is - consider only travelling salesman problem or the task of finding the shortest proof of theorem. It would be much nicer to have simple calculational rules.

   The original proposal might help here. The generalization of string model duality was in question. It stated that that it is possible to move the positions of the vertices of the diagrams just as one does to transform s-channel resonances to t-channel exchange. All loops of generalized diagrams could be be eliminated by transforming the to tadpoles and snipped away so that only tree diagrams would be left. The variants of the diagram were identified as different continuation paths between different paths connecting sectors of WCW corresponding to different 3-topologies. Each step in the continuation procedure would involve product or co-product defining what continuation between two sectors means for WCW spinors. The continuations between two states require some minimal number of steps. If this is true, all computations connecting identical states are also physically equivalent. The value of the vacuum functional be same for all of them. This looks very natural.

That the Kähler action should be same for all computational sequences connecting the same initial and final states looks strange but might be understood in terms of the vacuum degeneracy of Kähler action.

2. QFT perturbation theory requires that should have superposition of computations/continuations. What could the superposition of QFT diagrams correspond to in TGD framework?

   Could it correspond to a superposition of generators of the Yangian creating the physical state? After all, already quantum computer perform superpositions of computations. The fermionic state would not be the simplest one that one can imagine. Could AdS/CFT analogy allow to identify the vacuum state as a superposition of multi-string states so that single super-symplectic generator would be replaced with a superposition of its Yangian counterparts with same total quantum numbers but with a varying number of strings? The weight of a given superposition would be given by the total effective string world sheet area. The sum of diagrams would emerge from this superposition and would basically correspond to functional integration in WCW using exponent of Kähler action as weight. The stringy functional integral ("functional" if also wormhole contacts contain string portion, otherwise path integral) would give the perturbation theory around given string world sheet. One would have effective reduction of string theory.

12.5.4 How Does This Relate To The Ordinary Perturbation Theory?

One can of course worry about how to understand the basic results of the usual perturbation theory in this picture. How does one obtain a perturbation theory in powers of coupling constant, what does running coupling constant mean, etc...? I have already discussed how the superposition of diagrams could be understood in the new picture.

1. The QFT picture with running coupling constant is expected at QFT limit, when many-sheeted space-time is replaced with a slightly curved region of M^4 and gravitational field and gauge potentials are identified as sums of the deviations of induced metric from M^4 metric and classical induced gauge potentials associated with the sheets of the many-sheeted space-time. The running coupling constant would be due to the dependence of the size scale of CD, and p-adic coupling constant evolution would be behind the continuous one.

2. The notion of running coupling constant is very physical concept and should have a description also at the fundamental level and be due to a finite computational resolution, which indeed has very concrete description in terms of Noether charges of super-symplectic Yangian creating the states at the ends of space-time surface at the boundaries of CD. The space-time surface and the diagram associated with a given pair of 3-surfaces and stringy Noether charges associated with them can be characterized by a complexity measured in terms of the number of vertices (3-surface at which three 3-surfaces meet).

   For instance, 3-particle scattering can be possible only by using the simplest 3-vertex defined by product or co-product for pairs of 3-surfaces. In the generic case one has more complex diagram and what looks first 3-particle vertex has complex substructure rather than being simple product or co-product.

3. Complexity seems to have two separate aspects: the complexities of the positive and negative parts of zero energy state as many-fermion states and the complexity of associated 3-surfaces. The generalization of AdS/CFT however suggests that once the string world sheets and partonic 2-surfaces appearing in the diagram have been fixed, the space-time surface itself is fixed. The principle also suggests that the fixing partonic 2-surface and the strings connecting them at the boundaries of CD fixes the 3-surface apart from the action of sub-algebra of

Yangian acting as gauge algebra (vanishing classical Noether charges). If one can determine the minimal sequence of allowed algebraic operation of Yangian connecting initial and final fermion states, one knows the minimum number of vertices and therefore the topological structure of the connecting minimal space-time surface.

4. In QFT spirit one could describe the finite measurement resolution by introducing effective 3-point vertex, which is need not be product/co-produce anymore. 3-point scattering amplitudes in general involve microscopic algebraic structure involving several vertices. One can however give up the nice algebraic interpretation and just talk about effective 3-vertex for practical purposes. Just as the QFT vertex described by running coupling constant decomposes to sum of diagrams, product/co-product in TGD could be replaced with effective product/co-product expressible as a longer computation. This would imply coupling constant evolution.

   Fermion lines could however remain as such since they are massless in 8-D sense and mass renormalization does not make sense.

   Similar practical simplification could be done the initial and final states to get rid of superposition of the Yangian generators with different numbers of strings ("cloud of virtual particles"). This would correspond to wave function renormalization.

5. The number of vertices and wormhole contact orbits serves as a measure for the complexity of the diagram. Since fermion lines are associated with wormhole throats assignable with wormhole contacts identifiable as deformations CP_2 type vacuum extremals, one expects that the exponent of the Kähler function defining vacuum functional is in the first approximation the total CP_2 volume of wormhole contacts giving a measure for the importance of the contribution in functional integral. If it converges very rapidly only Gaussian approximation around maximum is needed.

6. Convergence depends on how large the fraction of volume of CP_2 is associated with a given wormhole contact. The volume is proportional to the length of the wormhole contact orbit. One expects exponential convergence with the number of fermion lines and their lengths for long lines. For short distances the exponential damping is small so that diagrams with microscopic structure of diagrams are needed and are possible. This looks like adding small scale details to the algebraic manipulations.

7. One must be of course be very cautious in making conclusions. The presence of $1/\alpha_K \propto h_{eff}$ in the exponent of Kähler function would suggest that for large values of h_{eff} only the 3-surfaces with smallest possible number of wormhole contact orbits contribute. On the other hand, the generalization of AdS/CFT duality suggests that Kähler action reducible to area of string world sheet in the effective metric defined by canonical momentum currents of Kähler action behaves as $\alpha_K^2 \propto 1/h_{eff}^2$. What does this mean?

To sum up, the identification of vertex as a product or co-product in Yangian looks highly promising approach. The Nother charges of the super-symplectic Yangian are associated with strings and are either linear or bilinear in the fermion field. The fermion fields associated with the partonic 2-surface defining the vertex are contracted with fermion fields associated with other partonic 2-surfaces using the same rule as in Wick expansion in quantum field theories. The contraction gives fermion propagator for each leg pair associated with two vertices. Vertex factor is proportional to the contraction of spinor modes with the operators defining the Noether charge or super charge and essentially Kähler-Dirac gamma matrix and the representation of the action of the symplectic generator on fermion realizable in terms of sigma matrices. This is very much like the corresponding expression in gauge theories but with gauge algebra replaced with symplectic algebra. The possibility of contractions of creation and annihilation operator for fermion lines associated with opposite wormhole throats at the same partonic 2-surface (for Noether charge bilinear in fermion field) gives bosonic exchanges as lines in which the fermion lines turns in time direction: otherwise only regroupings of fermions would take place.

12.5.5 This Was Not The Whole Story Yet

The proposed amplitude represents only the value of WCW spinor field for single pair of 3-surfaces at the opposite boundaries of given CD. Hence Yangian construction does not tell the whole story.

1. Yangian algebra would give only the vertices of the scattering amplitudes. On basis of previous considerations, one expects that each fermion line carries propagator defined by 8-momentum. The structure would resemble that of super-symmetric YM theory. Fermionic propagators should emerge from summing over intermediate fermion states in various vertices and one would have integrations over virtual momenta which are carried as residue integrations in twistor Grassmann approach. 8-D counterpart of twistorialization would apply.

2. Super-symplectic Yangian would give the scattering amplitudes for single space-time surface and the purely group theoretical form of these amplitudes gives hopes about the independence of the scattering amplitude on the pair of 3-surfaces at the ends of CD near the maximum of Kähler function. This is perhaps too much to hope except approximately but if true, the integration over WCW would give only exponent of Kähler action since metric and poorly defined Gaussian and determinants would cancel by the basic properties of Kähler metric. Exponent would give a non-analytic dependence on α_K.

   The Yangian supercharges are proportional to $1/\alpha_K$ since covariant Kähler-Dirac gamma matrices are proportional to canonical momentum currents of Kähler action and thus to $1/\alpha_K$. Perturbation theory in powers of $\alpha_K = g_K^2/4\pi\hbar_{eff}$ is possible after factorizing out the exponent of vacuum functional at the maximum of Kähler function and the factors $1/\alpha_K$ multiplying super-symplectic charges.

   The additional complication is that the characteristics of preferred extremals contributing significantly to the scattering amplitudes are expected to depend on the value of α_K by quantum interference effects. Kähler action is proportional to $1/\alpha_K$. The analogy of AdS/CFT correspondence states the expressibility of Kähler function in terms of string area in the effective metric defined by the anti-commutators of K-D matrices. Interference effects eliminate string length for which the area action has a value considerably larger than one so that the string length and thus also the minimal size of CD containing it scales as h_{eff}. Quantum interference effects therefore give an additional dependence of Yangian super-charges on h_{eff} leading to a perturbative expansion in powers of α_K although the basic expression for scattering amplitude would not suggest this.

3. Non-planar diagrams of quantum field theories should have natural counterpart and linking and knotting for braids defines it naturally. This suggests that the amplitudes can be interpreted as generalizations of braid diagrams defining braid invariants: braid strands would appear as legs of 3-vertices representing product and co-product. Amplitudes could be constructed as generalized braid invariants transforming recursively braided tree diagram to an un-braided diagram using same operations as for braids. In [L17] I considered a possible breaking of associativity occurring in weak sense for conformal field theories and was led to the vision that there is a fractal hierarchy of braids such that braid strands themselves correspond to braids. This hierarchy would define an operad with subgroups of permutation group in key role. Hence it seems that various approaches to the construction of amplitudes converge.

12.6 Appendix: Some Mathematical Details About Grassmannian Formalism

In the following I try to summarize my amateurish understanding about the mathematical structure behind the Grassmann integral approach. The representation summarizes what I have gathered from the articles of Arkani-Hamed and collaborators [B38, B39]. These articles are rather sketchy and the article of Bullimore provides additional details [B60] related to soft factors. The article of Mason and Skinner provides excellent introduction to super-twistors [B31] and dual superconformal invariance. I apologize for unavoidable errors.

Before continuing a brief summary about the history leading to the articles of Arkani-Hamed and others is in order. This summary covers only those aspects which I am at least somewhat familiar with and leaves out many topics about existence which I am only half-conscious.

1. It is convenient to start by summarizing the basic facts about bi-spinors and their conjugates allowing to express massless momenta as $p^{aa'} = \lambda_a \tilde{\lambda}_{a'}$ with $\tilde{\lambda}$ defined as complex conjugate of λ and having opposite chirality. When λ is scaled by a complex number $\tilde{\lambda}$ suffers an opposite scaling. The bi-spinors allow the definition of various inner products

$$\begin{aligned}
\langle \lambda, \mu \rangle &= \epsilon_{ab} \lambda^a \mu^b \ , \\
\left[\tilde{\lambda}, \tilde{\mu}\right] &= \epsilon_{a'b'} \lambda^{a'} \mu^{b'} \ , \\
p \cdot q &= \langle \lambda, \mu \rangle \left[\tilde{\lambda}, \tilde{\mu}\right] \ , \ (q_{aa'} = \mu_a \tilde{\mu}_{a'}) \ .
\end{aligned} \tag{12.6.1}$$

   If the particle has spin one can assign it a positive or negative helicity $h = \pm 1$. Positive helicity can be represented by introducing artitrary negative (positive) helicity bispinor μ_a ($\mu_{a'}$) not parallel to λ_a ($\mu_{a'}$) so that one can write for the polarization vector

$$\begin{aligned}
\epsilon_{aa'} = \frac{\mu_a \tilde{\lambda}_{a'}}{\langle \mu, \lambda \rangle} \ , \ \text{positive helicity} \ , \\
\epsilon_{aa'} = \frac{\lambda_a \tilde{\mu}_{a'}}{\left[\tilde{\mu}, \tilde{\lambda}\right]} \ , \ \text{negative helicity} \ .
\end{aligned} \tag{12.6.2}$$

   In the case of momentum twistors the μ part is determined by different criterion to be discussed later.

2. Tree amplitudes are considered and it is convenient to drop the group theory factor $Tr(T_1 T_2 \cdots T_n)$. The starting point is the observation that tree amplitude for which more than $n-2$ gluons have the same helicity vanish. MHV amplitudes have exactly $n-2$ gluons of same helicity- taken by a convention to be negative- have extremely simple form in terms of the spinors and reads as

$$A_n = \frac{\langle \lambda_x, \lambda_y \rangle^4}{\prod_{i=1}^{n} \langle \lambda_i, \lambda_{i+1} \rangle} \tag{12.6.3}$$

   When the sign of the helicities is changed $\langle .. \rangle$ is replaced with $[..]$.

3. The article of Witten [B33] proposed that twistor approach could be formulated as a twistor string theory with string world sheets "living" in 6-dimensional CP_3 possessing Calabi-Yau structure and defining twistor space. In this article Witten introduced what is known as half Fourier transform allowing to transform momentum integrals over light-cone to twistor integrals. This operation makes sense only in space-time signature $(2,2)$. Witten also demonstrated that maximal helicity violating (MHV) twistor amplitudes (two gluons with negative helicity) with n particles with $k+2$ negative helicities and l loops correspond in this approach to holomorphic 2-surfaces defined by polynomials defined by polynomials of degree $D = k-1+l$, where the genus of the surface satisfies $g \leq l$. AdS/CFT duality provides a second stringy approach to $\mathcal{N} = 4$ theory allowing to understand the scattering amplitudes in terms of Wilson loops with light-like edges: about this I have nothing to say. In any case, the generalization of twistor string theory to TGD context is highly attractive idea and will be considered later.

4. In the article [B27] Cachazo, Svrcek, and Witten propose the analog of Feynman diagrammatics in which MHV amplitudes can be used as analogs of vertices and ordinary $1/P^2$ propagator as propagator to construct tree diagrams with arbitrary number of negative helicity gluons. This approach is not symmetric with respect to the change of the sign of helicities since the amplitudes with two positive helicities are constructed as tree diagrams. The construction is non-trivial because one must analytically continue the on mass shell tree amplitudes to off mass shell momenta. The problem is how to assign a twistor to these momenta. This is achieved by introducing an arbitrary twistor $\eta^{a'}$ and defining λ_a as $\lambda_a = p_{aa'}\eta^{a'}$. This works for both massless and massive case. It however leads to a loss of the manifest Lorentz invariance. The paper however argues and the later paper [B26, B26] shows rigorously that the loss is only apparent. In this paper also BCFW recursion formula is introduced allowing to construct tree amplitudes recursively by starting from vertices with 2 negative helicity gluons. Also the notion which has become known as BCFW bridge representing the massless exchange in these diagrams is introduced. The tree amplitudes are not tree amplitudes in gauge theory sense where correspond to leading singularities for which 4 or more lines of the loop are massless and therefore collinear. What is important that the very simple MHV amplitudes become the building blocks of more complex amplitudes.

5. The nex step in the progress was the attempt to understand how the loop corrections could be taken into account in the construction BCFW formula. The calculation of loop contributions to the tree amplitudes revealed the existence of dual super-conformal symmetry which was found to be possessed also by BCFW tree amplitudes besides conformal symmetry. Together these symmetries generate infinite-dimensional Yangian symmetry [B31].

6. The basic vision of Arkani-Hamed and collaborators is that the scattering amplitudes of $\mathcal{N}=4$ SYM are constructible in terms of leading order singularities of loop diagrams. These singularities are obtained by putting maximum number of momenta propagating in the lines of the loop on mass shell. The non-leading singularities would be induced by the leading singularities by putting smaller number of momenta on mass shell are dictated by these terms. A related idea serving as a starting point in [B38] is that one can define loop integrals as residue integrals in momentum space. If I have understood correctly, this means that one an imagine the possibility that the loop integral reduces to a lower dimensional integral for on mass shell particles in the loops: this would resemble the approach to loop integrals based on unitarity and analyticity. In twistor approach these momentum integrals defined as residue integrals transform to residue integrals in twistor space with twistors representing massless particles. The basic discovery is that one can construct leading order singularities for n particle scattering amplitude with $k+2$ negative helicities as Yangian invariants $Y_{n,k}$ for momentum twistors and invariants constructed from them by canonical operations changing n and k. The correspondence $k=l$ does not hold true for the more general amplitudes anymore.

12.6.1 Yangian Algebra And Its Super Counterpart

The article of Witten [B30] gives a nice discussion of the Yangian algebra and its super counterpart. Here only basic formulas can be listed and the formulas relevant to the super-conformal case are given.

Yangian algebra

Yangian algebra $Y(G)$ is associative Hopf algebra. The elements of Yangian algebra are labelled by non-negative integers so that there is a close analogy with the algebra spanned by the generators of Virasoro algebra with non-negative conformal weight. The Yangian symmetry algebra is defined by the following relations for the generators labeled by integers $n=0$ and $n=1$. The first half of these relations discussed in very clear manner in [B30] follows uniquely from the fact that adjoint representation of the Lie algebra is in question

$$\left[J^A, J^B\right] = f_C^{AB} J^C \quad , \quad \left[J^A, J^{(1)B}\right] = f_C^{AB} J^{(1)C} \quad . \tag{12.6.4}$$

Besides this Serre relations are satisfied. These have more complex and read as

$$\begin{aligned}
&\left[J^{(1)A},\left[J^{(1)B},J^{C}\right]\right]+\left[J^{(1)B},\left[J^{(1)C},J^{A}\right]\right]+\left[J^{(1)C},\left[J^{(1)A},J^{B}\right]\right]\\
&\qquad=\frac{1}{24}f^{ADK}f^{BEL}f^{CFM}f_{KLM}\{J_D,J_E,J_F\}\ ,\\
&\left[\left[J^{(1)A},J^{(1)B}\right],\left[J^{C},J^{(1)D}\right]\right]+\left[\left[J^{(1)C},J^{(1)D}\right],\left[J^{A},J^{(1)B}\right]\right]\\
&\qquad=\frac{1}{24}(f^{AGL}f^{BEM}f_K^{CD}\\
&\qquad+f^{CGL}f^{DEM}f_K^{AB})f^{KFN}f_{LMN}\{J_G,J_E,J_F\}\ .
\end{aligned} \tag{12.6.5}$$

The indices of the Lie algebra generators are raised by invariant, non-degenerate metric tensor g_{AB} or g^{AB}. $\{A,B,C\}$ denotes the symmetrized product of three generators.

Repeated commutators allow to generate the entire algebra whose elements are labeled by non-negative integer n. The generators obtain in this manner are n-local operators arising in $(n-1)$-commutator of $J^{(1)}$: s. For $SU(2)$ the Serre relations are trivial. For other cases the first Serre relation implies the second one so the relations are redundant. Why Witten includes it is for the purposed of demonstrating the conditions for the existence of Yangians associated with discrete one-dimensional lattices (Yangians exists also for continuum one-dimensional index).

Discrete one-dimensional lattice provides under certain consistency conditions a representation for the Yangian algebra. One assumes that each lattice point allows a representation R of J^A so that one has $J^A=\sum_i J_i^A$ acting on the infinite tensor power of the representation considered. The expressions for the generators J^{1A} are given as

$$J^{(1)A} = f^A_{BC}\sum_{i<j}J_i^BJ_j^C\ . \tag{12.6.6}$$

This formula gives the generators in the case of conformal algebra. This representation exists if the adjoint representation of G appears only one in the decomposition of $R\otimes R$. This is the case for SU(N) if R is the fundamental representation or is the representation of by k^{th} rank completely antisymmetric tensors.

This discussion does not apply as such to $\mathcal{N}=4$ case the number of lattice points is finite and corresponds to the number of external particles so that cyclic boundary conditions are needed guarantee that the number of lattice points reduces effectively to a finite number. Note that the Yangian in color degrees of freedom does not exist for SU(N) SYM.

As noticed, Yangian algebra is a Hopf algebra and therefore allows co-product. The co-product Δ is given by

$$\begin{aligned}
\Delta(J^A) &= J^A\otimes 1+1\otimes J^A\ ,\\
\Delta(J^{(1)A}) &= J^{(1)A}\otimes 1+1\otimes J^{(1)A}+f^A_{BC}J^B\otimes J^C
\end{aligned} \tag{12.6.7}$$

Δ allows to imbed Lie algebra to the tensor product in non-trivial manner and the non-triviality comes from the addition of the dual generator to the trivial co-product. In the case that the single spin representation of $J^{(1)A}$ is trivial, the co-product gives just the expression of the dual generator using the ordinary generators as a non-local generator. This is assumed in the recent case and also for the generators of the conformal Yangian.

Super-Yangian

Also the Yangian extensions of Lie super-algebras make sense. From the point of physics especially interesting Lie super-algebras are $SU(m|m)$ and $U(m|m)$. The reason is that $PSU(2,2|4)$ (P refers to "projective") acting as super-conformal symmetries of $\mathcal{N}=4$ SYM and this super group is a real form of $PSU(4|4)$. The main point of interest is whether this algebra allows Yangian representation and Witten demonstrated that this is indeed the case [B30].

These algebras are Z_2 graded and decompose to bosonic and fermionic parts which in general correspond to n- and m-dimensional representations of $U(n)$. The representation associated with the fermionic part dictates the commutation relations between bosonic and fermionic generators. The anti-commutator of fermionic generators can contain besides identity also bosonic generators if the symmetrized tensor product in question contains adjoint representation. This is the case if fermions are in the fundamental representation and its conjugate. For $SU(3)$ the symmetrize tensor product of adjoint representations contains adjoint (the completely symmetric structure constants d_{abc}) and this might have some relevance for the super $SU(3)$ symmetry.

The elements of these algebras in the matrix representation (no Grassmann parameters involved) can be written in the form

$$x = \left(\begin{array}{cc} a & b \\ c & d \end{array} \right) \quad .$$

a and d representing the bosonic part of the algebra are $n \times n$ matrices and $m \times m$ matrices corresponding to the dimensions of bosonic and fermionic representations. b and c are fermionic matrices are $n \times m$ and $m \times n$ matrices, whose anti-commutator is the direct sum of $n \times n$ and $n \times n$ matrices. For $n = m$ bosonic generators transform like Lie algebra generators of $SU(n) \times SU(n)$ whereas fermionic generators transform like $n \otimes \overline{n} \oplus \overline{n} \otimes n$ under $SU(n) \times SU(n)$. Supertrace is defined as $Str(x) = Tr(a) - Tr(b)$. The vanishing of Str defines $SU(n|m)$. For $n \neq m$ the super trace condition removes identity matrix and $PU(n|m)$ and $SU(n|m)$ are same. That this does not happen for $n = m$ is an important delicacy since this case corresponds to $\mathcal{N} = 4$ SYM. If any two matrices differing by an additive scalar are identified (projective scaling as now physical effect) one obtains $PSU(n|n)$ and this is what one is interested in.

Witten shows that the condition that adjoint is contained only once in the tensor product $R \otimes \overline{R}$ holds true for the physically interesting representations of $PSU(2,2|4)$ so that the generalization of the bilinear formula can be used to define the generators of $J^{(1)A}$ of super Yangian of $PU(2,2|4)$. The defining formula for the generators of the Super Yangian reads as

$$\begin{aligned} J_C^{(1)} &= g_{CC'} J^{(1)C'} = g_{CC'} f^{C'}_{AB} \sum_{i<j} J_i^A J_j^B \\ &= g_{CC'} f^{C'}_{AB} g^{AA'} g^{BB'} \sum_{i<j} J^i_{A'} J^j_{B'} \quad . \end{aligned} \tag{12.6.8}$$

Here $g_{AB} = Str(J_A J_B)$ is the metric defined by super trace and distinguishes between $PSU(4|4)$ and $PSU(2,2|4)$. In this formula both generators and super generators appear.

Generators of super-conformal Yangian symmetries

The explicit formula for the generators of super-conformal Yangian symmetries in terms of ordinary twistors is given by

$$\begin{aligned} j_B^A &= \sum_{i=1}^n Z_i^A \partial_{Z_i^B} \quad , \\ j_B^{(1)A} &= \sum_{i<j} (-1)^C \left[Z_i^A \partial_{Z_j^C} Z_j^C \partial_{Z_j^B} \right] \quad . \end{aligned} \tag{12.6.9}$$

This formula follows from completely general formulas for the Yangian algebra discussed above and allowing to express the dual generators $j_N^{(1)}$ as quadratic expression of j_N involving structures constants. In this rather sketchy formula twistors are ordinary twistors. Note however that in the recent case the lattice is replaced with its finite cutoff corresponding to the external particles of the scattering amplitude. This probably corresponds to the assumption that for the representations considered only finite number of lattice points correspond to non-trivial quantum numbers or to cyclic symmetry of the representations.

In the expression for the amplitudes the action of transformations is on the delta functions and by partial integration one finds that a total divergence results. This is easy to see for the linear generators but not so for the quadratic generators of the dual super-conformal symmetries. A similar formula but with j_B^A and $j_B^{(1)A}$ interchanged applies in the representation of the amplitudes as Grassmann integrals using ordinary twistors. The verification of the generalization of Serre formula is also straightforward.

12.6.2 Twistors And Momentum Twistors And Super-Symmetrization

In [B31] the basics of twistor geometry are summarized. Despite this it is perhaps good to collect the basic formulas here.

Conformally compactified Minkowski space

Conformally compactified Minkowski space can be described as $SO(2,4)$ invariant (Klein) quadric

$$T^2+V^2-W^2-X^2-Y^2-Z^2=0 \ . \tag{12.6.10}$$

The coordinates (T,V,W,X,Y,Z) define homogenous coordinates for the real projective space RP^5. One can introduce the projective coordinates $X_{\alpha\beta}=-X_{\beta\alpha}$ through the formulas

$$\begin{array}{lll} X_{01}=W-V \ , & X_{02}=Y+iX \ , & X_{03}=\frac{i}{\sqrt{2}}T-Z \ , \\ X_{12}=-\frac{i}{\sqrt{2}}(T+Z) \ , & X_{13}=Y-iX \ , & X_{23}=\frac{1}{2}(V+W) \ . \end{array} \tag{12.6.11}$$

The motivation is that the equations for the quadric defining the conformally compactified Minkowski space can be written in a form which is manifestly conformally invariant:

$$\epsilon^{\alpha\beta\gamma\delta}X_{\alpha\beta}X_{\gamma\delta} \quad = \quad 0 \ per. \tag{12.6.12}$$

The points of the conformally compactified Minkowski space are null separated if and only if the condition

$$\epsilon^{\alpha\beta\gamma\delta}X_{\alpha\beta}Y_{\gamma\delta}=0 \tag{12.6.13}$$

holds true.

Correspondence with twistors and infinity twistor

One ends up with the correspondence with twistors by noticing that the condition is equivalent with the possibility to expression $X_{\alpha\beta}$ as

$$X_{\alpha\beta}=A_{[\alpha}B_{\beta]} \ , \tag{12.6.14}$$

where brackets refer to antisymmetrization. The complex vectors A and B define a point in twistor space and are defined only modulo scaling and therefore define a point of twistor space CP_3 defining a covering of 6-D Minkowski space with metric signature $(2,4)$. This corresponds to the fact that the Lie algebras of $SO(2,4)$ and $SU(2,2)$ are identical. Therefore the points of conformally compactified Minkowski space correspond to lines of the twistor space defining spheres CP_1 in CP_3.

One can introduce a preferred scale for the projective coordinates by introducing what is called infinity twistor (actually a pair of twistors is in question) defined by

$$I_{\alpha\beta} = \begin{pmatrix} \epsilon^{A'B'} & 0 \\ 0 & 0 \end{pmatrix} \quad . \tag{12.6.15}$$

Infinity twistor represents the projective line for which only the coordinate X_{01} is non-vanishing and chosen to have value $X_{01} = 1$.

One can define the contravariant form of the infinite twistor as

$$I^{\alpha\beta} = \epsilon^{\alpha\beta\gamma\delta} I_{\gamma\delta} = \begin{pmatrix} 0 & 0 \\ 0 & \epsilon^{AB} \end{pmatrix} \quad . \tag{12.6.16}$$

Infinity twistor defines a representative for the conformal equivalence class of metrics at the Klein quadric and one can express Minkowski distance as

$$(x-y)^2 = \frac{X^{\alpha\beta} Y_{\alpha\beta}}{I_{\alpha\beta} X^{\alpha\beta} I_{\mu\nu} Y^{\mu\nu}} \quad . \tag{12.6.17}$$

Note that the metric is necessary only in the denominator. In twistor notation the distance can be expressed as

$$(x-y)^2 = \frac{\epsilon(A,B,C,D)}{\langle AB\rangle\langle CD\rangle} \quad . \tag{12.6.18}$$

Infinite twistor $I_{\alpha\beta}$ and its contravariant counterpart project the twistor to its primed and unprimed parts usually denoted by $\mu^{A'}$ and λ^A and defined spinors with opposite chiralities.

Relationship between points of M^4 and twistors

In the coordinates obtained by putting $X_{01} = 1$ the relationship between space-time coordinates $x^{AA'}$ and $X^{\alpha\beta}$ is

$$X_{\alpha\beta} = \begin{pmatrix} -\frac{1}{2}\epsilon^{A'B'}x^2 & -ix^{A'}_{B} \\ ix_{A}^{B'} & \epsilon_{A,B} \end{pmatrix} \quad , \quad X^{\alpha\beta} = \begin{pmatrix} \epsilon_{A'B}{}'x^2 & -ix_{A'}^{B} \\ ix^{A}_{B'} & -\frac{1}{2}\epsilon^{AB}x^2 \end{pmatrix} \quad , \tag{12.6.19}$$

If the point of Minkowski space represents a line defined by twistors (μ_U, λ_U) and (μ_V, λ_V), one has

$$x^{AC'} = i\frac{(\mu_V\lambda_U - \mu_U\lambda_V)^{AC'}}{\langle UV\rangle} \tag{12.6.20}$$

The twistor μ for a given point of Minkowski space in turn is obtained from λ by the twistor formula by

$$\mu^{A'} = -ix^{AA'}\lambda_A \quad . \tag{12.6.21}$$

Generalization to the super-symmetric case

This formalism has a straightforward generalization to the super-symmetric case. CP_3 is replaced with $CP_{3|4}$ so that Grassmann parameters have four components. At the level of coordinates this means the replacement $[W_I] = [W_\alpha, \chi_\alpha]$. Twistor formula generalizes to

$$\mu^{A'} = -ix^{AA'}\lambda_A \quad , \quad \chi_\alpha = \theta^A_\alpha\lambda_A \quad . \tag{12.6.22}$$

The relationship between the coordinates of chiral super-space and super-twistors generalizes to

$$(x,\theta) = \left(i\frac{(\mu_V\lambda_U - \mu_U\lambda_V)}{\langle UV\rangle}, \frac{(\chi_V\lambda_U - \chi_U\lambda_V)}{\langle UV\rangle}\right) \quad (12.6.23)$$

The above formulas can be applied to super-symmetric variants of momentum twistors to deduce the relationship between region momenta x assigned with edges of polygons and twistors assigned with the ends of the light-like edges. The explicit formulas are represented in [B31]. The geometric picture is following. The twistors at the ends of the edge define the twistor pair representing the region momentum as a line in twistor space and the intersection of the twistor lines assigned with the region momenta define twistor representing the external momenta of the graph in the intersection of the edges.

Basic kinematics for momentum twistors

The super-symmetrization involves replacement of multiplets with super-multiplets

$$\Phi(\lambda,\tilde{\lambda},\eta) = G^{+}(\lambda,\tilde{\lambda}) + \eta_i\Gamma^a\lambda,\tilde{\lambda}) + \cdots + \epsilon_{abcd}\eta^a\eta^b\eta^c\eta^d G^{-}(\lambda,\tilde{\lambda}) \ . \quad (12.6.24)$$

Momentum twistors are dual to ordinary twistors and were introduced by Hodges. The light-like momentum of external particle a is expressed in terms of the vertices of the closed polygon defining the twistor diagram as

$$p_i^{\mu} = x_i^{\mu} - x_{i+1}^{\mu} = \lambda_i\tilde{\lambda}_i \ , \ \ \theta_i - \theta_{i+1} = \lambda_i\eta_i \ . \quad (12.6.25)$$

One can say that massless momenta have a conserved super-part given by $\lambda_i\eta_i$. The dual of the super-conformal group acts on the region momenta exactly as the ordinary conformal group acts on space-time and one can construct twistor space for dua region momenta.

Super-momentum conservation gives the constraints

$$\sum p_i = 0 \ , \ \ \sum \lambda_i\eta_i = 0 \ . \quad (12.6.26)$$

The twistor diagrams correspond to polygons with edges with lines carrying region momenta and external massless momenta emitted at the vertices.

This formula is invariant under overall shift of the region momenta x_a^{μ}. A natural interpretation for x_a^{μ} is as the momentum entering to the vertex where p_a is emitted. Overall shift would have interpretation as a shift in the loop momentum. x_a^{μ} in the dual coordinate space is associated with the line $Z_{a-1}Z_a$ in the momentum twistor space. The lines $Z_{a-1}Z_a$ and Z_aZ_{a+1} intersect at Z_a representing a light-like momentum vector p_a^{μ}.

The brackets $\langle abcd\rangle \equiv \epsilon_{IJKL}Z_a^IZ_b^JZ_c^KZ_d^L$ define fundamental bosonic conformal invariants appearing in the tree amplitudes as basic building blocks. Note that Z_a define points of 4-D complex twistor space to be distinguished from the projective twistor space CP_3. Z_a define projective coordinates for CP_3 and one of the four complex components of Z_a is redundant and one can take $Z_a^0 = 1$ without a loss of generality.

12.6.3 Brief Summary Of The Work Of Arkani-Hamed And Collaborators

The following comments are an attempt to summarize my far from complete understanding about what is involved with the representation as contour integrals. After that I shall describe in more detail my impressions about what has been done.

Limitations of the approach

Consider first the limitations of the approach.

1. The basis idea is that the representation for tree amplitudes generalizes to loop amplitudes. On other words, the amplitude defined as a sum of Yangian invariants expressed in terms of Grassmann integrals represents the sum of loops up to some maximum loop number. The problem is here that shifts of the loop momenta are essential in the UV regularization procedure. Fixing the coordinates $x_1, \cdots, x_n$ having interpretation as momenta associated with lines in the dual coordinate space allows to eliminate the non-uniqueness due to the common shift of these coordinates.

2. It is not however not possible to identify loop momentum as a loop momentum common to different loop integrals unless one restricts to planar loops. Non-planar diagrams are obtained from a planar diagram by permuting the coordinates x_i but this means that the unique coordinate assignment is lost. Therefore the representation of loop integrands as Grassmann integrals makes sense only for planar diagrams. From TGD point of view one could argue that this is one good reason for restricting the loops so that they are for on mass shell particles with non-parallel on mass shell four-momenta and possibly different sign of energies for given wormhole contact representing virtual particle.

3. IR regularization is needed even in $\mathcal{N} = 4$ for SYM given by "moving out on the Coulomb branch theory" so that IR singularities remain the problem of the theory.

What has been done?

The article proposes a generalization of the BCFW recursion relation for tree diagrams of $\mathcal{N} = 4$ for SYM so that it applies to planar diagrams with a summation over an arbitrary number of loops.

1. The basic goal of the article is to generalize the recursion relations of tree amplitudes so that they would apply to loop amplitudes. The key idea is following. One can formally represent loop integrand as a contour integral in complex plane whose coordinate parameterizes the deformations $Z_n \rightarrow Z_n + \epsilon Z_{n-1}$ and re-interpret the integral as a contour integral with oppositely oriented contour surrounding the rest of the complex plane which can be imagined also as being mapped to Riemann sphere. What happens only the poles which correspond to lower number of loops contribute this integral. One obtains a recursion relation with respect to loop number. This recursion seems to be the counterpart for the recursive construction of the loops corrections in terms of absorptive parts of amplitudes with smaller number of loop using unitarity and analyticity.

2. The basic challenge is to deduce the Grassmann integrands as Yangian invariants. From these one can deduce loop integrals by integration over the four momenta associated with the lines of the polygonal graph identifiable as the dual coordinate variables x_a. The integration over loop momenta can induce infrared divergences breaking Yangian symmetry. The big idea here is that the operations described above allow to construct loop amplitudes from the Yangian invariants defining tree amplitudes for a larger number of particles by removing external particles by fusing them to form propagator lines and by using the BCFW bridge to fuse lower-dimensional invariants. Hence the usual iterative procedure (bottom-up) used to construct scattering amplitudes is replaced with a recursive procedure (top-down). Of course, once lower amplitudes has been constructed they can be used to construct amplitudes with higher particle number.

3. The first guess is that the recursion formula involves the same lower order contributions as in the case of tree amplitudes. These contributions have interpretation as factorization of channels involving single particle intermediate states. This would however allow to reduce loop amplitudes to 3-particle loop amplitudes which vanish in $\mathcal{N} = 4$ SYM by the vanishing of coupling constant renormalization. The additional contribution is necessary and corresponds to a source term identifiable as a "forward limit" of lower loop integrand. These terms are obtained by taking an amplitude with two additional particles with opposite four-momenta

and forming a state in which these particles are entangled with respect to momentum and other quantum numbers. Entanglement means integral over the massless momenta on one hand. The insertion brings in two momenta x_a and x_b and one can imagine that the loop is represented by a branching of propagator line. The line representing the entanglement of the massless states with massless momentum define the second branch of the loop. One can of course ask whether only massless momentum in the second branch. A possible interpretation is that this state is expressible by unitarity in terms of the integral over light-like momentum.

4. The recursion formula for the loop amplitude $M_{n,k,l}$ involves two terms when one neglects the possibility that particles can also suffer trivial scattering (cluster decomposition). This term basically corresponds to the Yangian invariance of n arguments identified as Yangian invariant of $n-1$ arguments with the same value of k.

   (a) The first term corresponds to single particle exchange between particle groups obtained by splitting the polygon at two vertices and corresponds to the so called BCFW bridge for tree diagrams. There is a summation over different splittings as well as a sum over loop numbers and dimensions k for the Grassmann planes. The helicities in the two groups are opposite.

   (b) Second term is obtained from an amplitude obtained by adding of two massless particles with opposite momenta and corresponds to $n+2, k+1, l-1$. The integration over the light-like momentum together with other operations implies the reduction $n+2 \rightarrow n$. Note that the recursion indeed converges. Certainly the allowance of added zero energy states with a finite number of particles is necessary for the convergence of the procedure.

12.6.4 The General Form Of Grassmannian Integrals

If the recursion formula proposed in [B39] is correct, the calculations reduce to the construction of N^kMHV (super) amplitudes. MHV refers to maximal helicity violating amplitudes with 2 negative helicity gluons. For N^kMHV amplitude the number of negative helicities is by definition $k+2$ [B38]. Note that the total right handed R-charge assignable to 4 super-coordinates η_i of negative helicity gluons can be identified as $R = 4k$. BCFW recursion formula [B26, B26] allows to construct from MHV amplitudes with arbitrary number of negative helicities.

The basic object of study are the leading singularities of color-stripped n-particle N^kMHV amplitudes. The discovery is that these singularities are expressible in terms Yangian invariants $Y_{n,k}(Z_1, \cdots, Z_n)$, where Z_i are momentum super-twistors. These invariants are defined by residue integrals over the compact $nk-1$-dimensional complex space $G(n,k) = U(n)/U(k) \times U(n-k)$ of k-planes of complex n-dimensional space. n is the number of external massless particles, k is the number negative helicity gluons in the case of N^kMHV amplitudes, and Z_a, $i = 1, \cdots, n$ denotes the projective 4-coordinate of the super-variant $CP^{3|4}$ of the momentum twistor space CP_3 assigned to the massless external particles is following. $Gl(n)$ acts as linear transformations in the n-fold Cartesian power of twistor space. Yangian invariant $Y_{n,k}$ is a function of twistor variables Z^a having values in super-variant $CP_{3|3}$ of momentum twistor space CP_3 assigned to the massless external particles being simple algebraic functions of the external momenta.

It is also possible to define N^kMHV amplitudes in terms of Yangian invariants $L_{n,k+2}(W_1, \cdots, W_n)$ by using ordinary twistors W_a and identical defining formula. The two invariants are related by the formula $L_{n,k+2}(W_1, \cdots, W_n) = M^{tree}_{MHV} \times Y_{n,k}(Z_1, \cdots, Z_n)$. Here M^{tree}_{MHV} is the tree contribution to the maximally helicity violating amplitude for the scattering of n particles: recall that these amplitudes contain two negative helicity gluons whereas the amplitudes containing a smaller number of them vanish [B27]. One can speak of a factorization to a product of n-particle amplitudes with $k-2$ and 2 negative helicities as the origin of the duality. The equivalence between the descriptions based on ordinary and momentum twistors states the dual conformal invariance of the amplitudes implying Yangian symmetry. It has been conjectured that Grassmannian integrals generate all Yangian invariants.

The formulas for the Grassmann integrals for twistors and momentum twistors appearing in the expressions of N^kMHV amplitudes are given by following expressions.

1. The integrals $L_{n,k}(W_1, \cdots, W_n)$ associated with $N^{k-2}MHV$ amplitudes in the description based on ordinary twistors correspond to k negative helicities and are given by

$$
\begin{aligned}
L_{n,k}(W_1,\cdots,W_n) &= \frac{1}{Vol(GL(2))}\int \frac{d^{k\times n}C_{\alpha a}}{(1\cdots k)(2\cdots k+1)\cdots(n1\cdots k-1)}\times \\
&\times \prod_{\alpha=1}^{k} d^{4|4}Y_\alpha \prod_{i=1}^{n}\delta^{4|4}(W_i-C_{\alpha i}Y_\alpha) \ .
\end{aligned}
\tag{12.6.27}
$$

Here $C_{\alpha a}$ denote the $n\times k$ coordinates used to parametrize the points of $G_{k,n}$.

2. The integrals $Y_{n,k}(W_1,\cdots,W_n)$ associated with N^kMHV amplitudes in the description based on momentum twistors are defined as

$$
Y_{n,k}(Z_1,\cdots,Z_n) = \frac{1}{Vol(GL(k))}\times\int \frac{d^{k\times n}C_{\alpha a}}{(1\cdots k)(2\cdots k+1)\cdots(n1\cdots k-1)}\times\prod_{\alpha=1}^{k}\delta^{4|4}(C_{\alpha a}Z_a) \ .
\tag{12.6.28}
$$

The possibility to select $Z_a^0=1$ implies $\sum_k C_{\alpha k}=0$ allowing to eliminate $C_{\alpha n}$ so that the actual number of coordinates Grassman coordinates is $nk-1$. As already noticed, $L_{n,k+2}(W_1,\cdots,W_n) = M_{MHV}^{tree}\times Y_{n,k}(Z_1,\cdots,Z_n)$. Momentum twistors are obviously calculationally easier since the value of k is smaller by two units.

The $4k$ delta functions reduce the number of integration variables of contour integrals from nk to $(n-4)k$ in the bosonic sector (the definition of delta functions involves some delicacies not discussed here). The n quantities $(m,\cdots m+k)$ are $k\times k$-determinants defined by subsequent columns from m to $m+k-1$ of the $k\times n$ matrix defined by the coordinates $C_{\alpha a}$ and correspond geometrically to the k-volumes of the k-dimensional parallel-pipeds defined by these column vectors. The fact that the scalings of twistor space coordinates Z_a can be compensated by scalings of $C_{\alpha a}$ deforming integration contour but leaving the residue integral invariant so that the integral depends on projective twistor coordinates only.

Since the integrand is a rational function, a multi-dimensional residue calculus allows to deduce the values of these integrals as residues associated with the poles of the integrand in a recursive manner. The poles correspond to the zeros of the $k\times k$ determinants appearing in the integrand or equivalently to singular lower-dimensional parallel-pipeds. It can be shown that local residues are determined by $(k-2)(n-k-2)$ conditions on the determinants in both cases. The value of the integral depends on the explicit choice of the integration contour for each variable $C_{\alpha a}$ left when delta functions are taken into account. The condition that a correct form of tree amplitudes is obtained fixes the choice of the integration contours.

For the ordinary twistors W the residues correspond to projective configurations in CP_{k-1}, or more precisely in the space $CP_{k-1}^n/Gl(k)$, which is $(k-1)n-k^2$-dimensional space defining the support for the residues integral. $Gl(k)$ relates to each other different complex coordinate frames for k-plane and since the choice of frame does not affect the plane itself, one has $Gl(k)$ gauge symmetry as well as the dual $Gl(n-k)$ gauge symmetry.

CP_{k-1} comes from the fact that C_{α_k} are projective coordinates: the amplitudes are indeed invariant under the scalings $W_i\rightarrow t_iW_i$, $C_{\alpha i}\rightarrow tC_{\alpha i}$. The coset space structure comes from the fact that $Gl(k)$ is a symmetry of the integrand acting as $C_{\alpha i}\rightarrow \Lambda_\alpha^{\ \beta}C_{\beta_i}$. This analog of gauge symmetry allows to fix k arbitarily chosen frame vectors $C_{\alpha i}$ to orthogonal unit vectors. For instance, one can have $C_{\alpha i}=\delta_{\alpha_i}$ for $\alpha=i\in 1,\cdots,k$. This choice is discussed in detail in [B38]. The reduction to CP_{k-1} implies the reduction of the support of the integral to line in the case of MHV amplitudes and to plane in the case of NMHV as one sees from the expression $d\mu=\prod_\alpha d^{4|4}Y_\alpha\prod_{i=1}^n\delta^{4|4}(W_i-C_{\alpha i}Y_\alpha)$. For $(i_1,\cdots,i_k)=0$ the vectors $i_1,..i_k$ belong to $k-2$-dimensional plane of CP_{k-1}. In the case of $NMHV$ (N^2MHV) amplitudes this translates at the level of twistors to the condition that the corresponding twistors $\{i_1,i_2,i_3\}$ ($\{i_1,i_2,i_3,i_4\}$) are collinear (in the same plane) in twistor space. This can be understood from the fact that the delta functions in $d\mu$ allow to express W_i in terms of $k-1$ Y_α: s in this case.

The action of conformal transformations in twistor space reduces to the linear action of $SU(2,2)$ leaving invariant Hermitian sesquilinear form of signature $(2,2)$. Therefore the conformal invariance of the Grassmannian integral and its dual variant follows from the possibility to perform a compensating coordinate change for $C_{\alpha a}$ and from the fact that residue integral is invariant under small deformations of the integration contour. The above described relationship between representations based on twistors and momentum twistors implies the full Yangian invariance.

12.6.5 Canonical Operations For Yangian Invariants

General l-loop amplitudes can be constructed from the basic Yangian invariants defined by N^kMHV amplitudes by various operations respecting Yangian invariance apart from possible IR anomalies. There are several operations that one can perform for Yangian invariants $Y_{n,k}$ and all these operations appear in the recursion formula for planar all loop amplitudes. These operations are described in [B39] much better than I could do it so that I will not go to any details. It is possible to add and remove particles, to fuse two Yangian invariants, to merge particles, and to construct from two Yangian invariants a higher invariant containing so called BCFW bridge representing single particle exchange using only twistorial methods.

Inverse soft factors

Inverse soft factors add to the diagram a massless collinear particles between particles a and b and by definition one has

$$O_{n+1}(a,c,b,\cdots) = \frac{\langle ab\rangle}{\langle ac\rangle\langle cb\rangle}O_n(a'b') \ . \qquad (12.6.29)$$

At the limit when the momentum of the added particle vanishes both sides approach the original amplitude. The right-handed spinors and Grassmann parameters are shifted

$$\begin{array}{ll} \tilde{\lambda}'_a = \tilde{\lambda}_a + \frac{\langle cb\rangle}{\langle ab\rangle}\tilde{\lambda}_c \ , & \tilde{\lambda}'_b = \tilde{\lambda}_b + \frac{\langle ca\rangle}{\langle ba\rangle}\tilde{\lambda}_c \ , \\ \eta'_a = \eta_a + \frac{\langle cb\rangle}{\langle ab\rangle}\eta_c \ , & \eta'_b = \eta_b + \frac{\langle ca\rangle}{\langle ba\rangle}\eta_c \ . \end{array} \qquad (12.6.30)$$

There are two kinds of inverse soft factors.

1. The addition of particle leaving the value k of negative helicity gluons unchanged means just the re-interpretation

$$Y'_{n,k}(Z_1,\cdots,Z_{n-1},Z_n) = Y_{n-1,k}(Z_1,\cdots,Z_{n-1}) \qquad (12.6.31)$$

without actual dependence on Z_n. There is however a dependence on the momentum of the added particle since the relationship between momenta and momentum twistors is modified by the addition obtained by applying the basic rules relating region super momenta and momentum twistors (light-like momentum determines λ_i and twistor equations for x_i and λ_i,η_i determine (μ_i,χ_i)) is expressible assigned to the external particles [B60]. Modifications are needed only for the new vertex and its neighbors.

2. The addition of a particle increasing k with single unit is a more complex operation which can be understood in terms of a residue of $Y_{n,k}$ proportional to $Y_{n-1,k-1}$ and Yangian invariant $[z_1\cdots z_5]$ with five arguments constructed from basic Yangian invariants with four arguments. The relationship between the amplitudes is now

$$Y'_{n,k}(..,Z_{n-1}Z_n,Z_1\cdots) = [n-2\ n-1\ n\ 1\ 2]\times Y_{n-1,k-1}(\cdots\hat{Z}_{n-1},\hat{Z}_1,\cdots) (12.6.32)$$

Here

$$[abcde] = \frac{\delta^{0|4}(\eta_a\langle bcde\rangle + cyclic)}{\langle abcd\rangle\langle bcde\rangle\langle cdea\rangle\langle deab\rangle\langle eabc\rangle} . \quad (12.6.33)$$

denoted also by $R(a,b,c,d,e)$ is the fundamental R-invariant appearing in one loop corrections of MHV amplitudes and will appears also in the recursion formulas. $\langle abcd\rangle$ is the fundamental super-conformal invariant associated with four super twistors defined in terms of the permutation symbol.

$\hat{Z}_{n-1}, \hat{Z}_1$ are deformed momentum twistor variables. The deformation is determined from the relationship between external momenta, region momenta and momentum twistor variables. $\hat{Z}^1$ is the intersection $\hat{Z}^1 = (n-2\ n-1\ 2) \cap (12)$ of the line (12) with the plane $(n-2\ n-1\ 2)$ and $\hat{Z}^{n-1}$ the intersection $\hat{Z}^1 = (12n) \cap (n-2\ n-1)$ of the line $(n-2\ n-1)$ with the plane $(12n)$. The interpretation for the intersections at the level of ordinary Feynman diagrams is in terms of the collinearity of the four-momenta involved with the underlying box diagram with parallel on mass shell particles. These result from unitarity conditions obtained by putting maximal number of loop momenta on mass shell to give the leading singularities.

The explicit expressions for the momenta are

$$\begin{aligned}
\hat{Z}^1 &\equiv (n-2\ n-1\ 2) \cap (12)Z_1 = \langle 2\ n-2\ n-1\ n\rangle + Z_2\langle n-2\ n-1\ n\ 1\rangle\ , \\
\hat{Z}^{n-1} &\equiv (12n) \cap (n-2\ n-1) = Z_{n-2}\langle n-2\ n-1\ n\ 2\rangle + Z_{n-1}\langle n\ 1\ 2\ n-2\rangle\ .
\end{aligned} \quad (12.6.34)$$

These intersections also appear in the expressions defining the recursion formula.

Removal of particles and merge operation

Particles can be also removed. The first manner to remove particle is by integrating over the twistor variable characterizing the particle. This reduces k by one unit. Merge operation preserves the number of loops but removes a particle particle by identifying the twistor variables of neighboring particles. This operation corresponds to an integral over on mass shell loop momentum at the level of tree diagrams and by Witten's half Fourier transform can be transformed to twistor integral.

The product

$$Y'(Z_1, \cdots Z_n) = Y_1(Z_1, \cdots Z_m) \times Y_2(Z_{m+1}, \cdots Z_n) \quad (12.6.35)$$

of two Yangian invariants is again a Yangian invariant. This is not quite trivial since the dependence of region momenta and momentum twistors on the momenta of external particles makes the operation non-trivial.

Merge operation allows to construct more interesting invariants from the products of Yangian invariants. One begins from a product of Yangian invariants (Yangian invariant trivially) represented cyclically as points of circle and identifies the last twistor argument of given invariant with the first twistor argument of the next invariant and performs integrals over the momentum twistor variables appearing twice. The soft k-increasing and preserving operations can be described also in terms of this operation for Yangian invariants such that the second invariant corresponds to 3-vertex. The cyclic merge operation applied to four MHV amplitudes gives NMHV amplitudes associated with on mass shell momenta in box diagrams. By applying similar operation to NMHV amplitudes and MHV amplitudes one obtains 2-loop amplitudes. In [B39] examples about these operations are described.

BCFW bridge

BCFW bridge allows to build general tree diagrams from MHV tree diagrams [B26, B26] and recursion formula of [B39] generalizes this to arbitrary diagrams. At the level of Feynman diagrams it corresponds to a box diagram containing general diagrams labeled by L and R and MHV and $\overline{MHV}$ 3-vertices ($\overline{MHV}$ 3-vertex allows expression in terms of MHV diagrams) with the lines of the box on mass shell so that the three momenta emanating from the vertices are parallel and give rise to a one-loop leading singularity.

At the level of Feynman diagrams BCFW bridge corresponds to so called "two-mass hard" leading singularities associated with box diagrams with light-like momenta at the four lines of the diagram [B38]. The motivation for the study of these diagrams comes from the hypothesis the leading order singularities obtained by putting as many particles as possible on mass shell contain the data needed to construct scattering amplitudes of $\mathcal{N}=4$ SYM completely. This representation of the leading singularities generalizes to arbitrary loops. The recent article is a continuation of this program to planar amplitudes.

Also BCFW bridge allows an interpretation as a particular kind fusion for Yang invariants and involves all the basic operations. One starts from the amplitudes $Y^L_{n_L,k_L}$ and $Y^R_{n_R,k_R}$ and constructs an amplitude $Y'_{n_L+n_R,k_L+k_R+1}$ representing the amplitude which would correspond to a generalization of the MHV diagrams with the two tree diagrams connected by the MHV propagator (BCFW bridge) replaced with arbitrary loop diagrams. Particle "1" *resp.* "j+1" is added by the soft k-increasing factor to Y_{n_L+1,k_L+1} *resp.* Y_{n_R+1,k_R+1} giving amplitude with $n+2$ particles and with k-charge equal to k_L+k_R+2. The subsequent operations must reduce k-charge by one unit. First repeated "1" and "j+1" are identified with their copies by k conserving merge operation, and after that one performs an integral over the twistor variable Z^I associated with the internal line obtained and reducing k by one unit. The soft k-increasing factors bring in the invariants $[n-1\ n\ 1\ I\ j+2]$ associated with Y_L and $[1\ I\ j+1\ j\ j-1]$ associated with Y_R. The integration contour is chosen so that it selects the pole defined by $\angle n-1\ n\ 1\ I\rangle$ in the denominator of $[n-1\ n\ 1\ I\ j+2]$ and the pole defined by $\langle 1\ I\ j+1\ j\rangle$ in the denominator of $[1\ I\ j+1\ j\ j-1]$.

The explicit expression for the BCFW bridge is very simple:

$$\begin{aligned} (Y_L\otimes_{BCFW}Y_R)(1,\cdots,n) &= [n-1\ n\ 1\ j\ j+1]\times Y_R(1,\cdots,j,I)Y_L(I,j+1,\cdots,n-1,\hat{n})\ , \\ \hat{n} &= (n-1\ n)\cap(j\ j+1\ 1)\ ,\ \ I=(j\ j+1)\cap(n-1\ n\ 1) \end{aligned} \tag{12.6.36}$$

Single cuts and forward limit

Forward limit operation is used to increase the number of loops by one unit. The physical picture is that one starts from say 1-loop amplitude and cuts one line by assigning to the pieces of the line opposite light-like momenta having interpretation as incoming and outgoing particles. The resulting amplitude is called forward limit. The only reasonable interpretation seems to be that the loop integration is expressed by unitarity as forward limit meaning cutting of the line carrying the loop momentum. This operation can be expressed in a manifestly Yangian invariant way as entangled removal of two particles with the merge operation meaning the replacement $Z_n\rightarrow Z_{n-1}$. Particle $n+1$ is added adjacent to A,B as a k-increasing inverse soft factor and then A and B are removed by entangled integration, and after this merge operation identifies $n+1$ and 1.

Forward limit is crucial for the existence of loops and for Yangian invariants it corresponds to the poles arising from $\langle (AB)_q Z_n(z)Z_1)\rangle$ the integration contour Z_n+zZ_{n-1} around Z_b in the basic formula $M=\oint(dz/z)M_n$ leading to the recursion formula. A and B denote the momentum twistors associated with opposite light-like momenta. In the generalized unitarity conditions the singularity corresponds to the cutting of line between particles n and 1 with momenta q and $-q$, summing over the multiplet of stats running around the loop. Between particles n_2 and 1 one has particles $n-1$, n with momenta $q,-q$. $q=x_1-x_n=-x_n+x_{n-1}$ giving $x_1=x_{n-1}$. Light-likeness of q means that the lines $(71)=(76)$ and (15) intersect. At the forward limit giving rise to the pole Z_6 and Z_7 approach to the intersection point $(76)\cap(15)$. In a generic gauge theories the forward limits are ill-defined but in super-symmetric gauge theories situation changes.

The corresponding Yangian operation removes two external particles with opposite four-momenta and involves integration over two twistor variables Z_a and Z_b and gives rise to the following expression

$$\int_{GL(2)} Y(\cdots, Z_n, Z_A, Z_B, Z_1, \cdots) \ . \tag{12.6.37}$$

The integration over $GL(2)$ corresponds to integration over twistor variables associated Z_A and Z_B. This operation allows addition of a loop to a given amplitude. The line Z_aZ_b represents loop momentum on one hand and the dual x-coordinate identified as momentum propagating along the line on the other hand.

The integration over these variables is equivalent to an integration over loop momentum as the explicit calculation of [B39] (see pages 12-13) demonstrates. If the integration contours are products in the product of twistor spaces associated with a and b the and gives lower order Yangian invariant as answer. It is however also possible to choose the integration contour to be entangled in the sense that it cannot be reduced to a product of integration contours in the Cartesian product of twistor spaces. In this case the integration gives a loop integral. In the removal operation Yangian invariance can be broken by IR singularities associated with the integration contour and the procedure does not produce genuine Yangian invariant always.

What is highly interesting from TGD point of view is that this integral can be expressed as a contour integral over $CP_1 \times CP_1$ combined with integral over loop momentum. If TGD vision about generalized Feynman graps in zero energy ontology is correct, the loop momentum integral is discretized to an an integral over discrete mass shells and perhaps also to a sum over discretized momenta and one can therefore avoid IR singularities.

12.6.6 Explicit Formula For The Recursion Relation

Recall that the recursion formula is obtained by considering super-symmetric momentum-twistor deformation $Z_n \rightarrow Z_n + zZ_{n-1}$ and by integrating over z to get the identity

$$M_{n,k,l} \ = \ \oint \frac{dz}{z}\hat{M}_{n,k,l}(z) \ . \tag{12.6.38}$$

This integral equals to integral with reversed integration contour enclosing the exterior of the contour. The challenge is to deduce the residues contributing to the residue integral and the claim of [B39] is that these residues reduce to simple basic types.

1. The first residue corresponds to a pole at infinity and reduces the particle number by one giving a contribution $M_{n-1,k,l}(1,\cdots,n-1)$ to $M_{n,k,l}(1,\cdots,n-1,n)$. This is not totally trivial since the twistor variables are related to momenta in different manner for the two amplitudes. This gives the first contribution to the right hand side of the formula below.

2. Second pole corresponds to the vanishing of $\langle Z_n(z)Z_1Z_jZ_{j+1}\rangle$ and corresponds to the factorization of channels. This gives the second BCFW contribution to the right hand side of the formula below. These terms are however not enough since the recursion formula would imply the reduction to expressions involving only loop corrections to 3-loop vertex which vanish in $\mathcal{N} = 4$ SYM.

3. The third kind of pole results when $\langle (AB)_q Z_n(z)Z_1\rangle$ vanishes in momentum twistor space. $(AB)_q$ denotes the line in momentum twistor space associated with q: th loop variable.

The explicit formula for the recursion relation yielding planar all loop amplitudes is obtained by putting all these pieces together and reads as

$$\begin{aligned} M_{n,k,l}(1,\cdots,n) \ &= \ M_{n-1,k,l}(1,\cdots,n-1) \\ &+ \ \sum_{n_L,k_L,l_L;j} [j\ j+1\ n-1\ n\ 1]\, M^R_{n_R,k_R,l_R}(1,\cdots,j,I_j) \times M^L_{n_L,k_L,l_L}(I_j,j+1,\cdots,\hat{n}_j) \\ &+ \ \int_{GL(2)} [AB\ n-1\ n\ 1]\, M_{n+2,k+1,n,k-1}(1,\cdots,\hat{n}_{AB},\hat{A},B) \ , \\ n_L \ &+ \ n_R = n+2 \ , \ k_L + k_R = k-1 \ , \ l_R + l_L = l \ . \end{aligned} \tag{12.6.39}$$

The momentum super-twistors are given by

$$\begin{array}{ll} \hat{n}_j = (n-1\ n)\cap(j\ j+1\ 1)\ , & I_j = (j\ j+1\ 1)\cap(n-1\ n\ 1)\ , \\ \hat{n}_{AB} = (n-1\ n)\cap(AB\ 1)\ , & \hat{A} = (AB)\cap(n-1\ n\ 1)\ . \end{array} \tag{12.6.40}$$

The index l labels loops in $n+2$-particle amplitude and the expression is fully symmetrized with equal weight for all loop integration variables $(AB)_l$. A and B are removed by entangled integration meaning that $GL(2)$ contour is chosen to encircle points where both points A, B on the line (AB) are located at the intersection of the line (AB) with the plane $(n-1\ n\ 1)$. GL(2) integral can be done purely algebraically in terms of residues.

In [B39] and [B60] explicit calculations for N^kMHV amplitudes are carried out to make the formulas more concrete. For N^1MHV amplitudes second line of the formula vanishes and the integrals are rather simple since the determinants are 1×1 determinants.

Chapter 13

From Principles to Diagrams

13.1 Introduction

The generalization of twistor diagrams to TGD framework has been very inspiring (and also frightening) mission impossible and allowed to gain deep insights about what TGD diagrams could be mathematically. I of course cannot provide explicit formulas but the general structure for the construction of twistorial amplitudes in $\mathcal{N} = 4$ SUSY suggests an analogous construction in TGD thanks to huge symmetries of TGD and unique twistorial properties of $M^4 \times CP_2$. The twistor program in TGD framework has been summarized in [K76].

Contrary to the original expectations, the twistorial approach is not a mere reformulation but leads to a first principle identification of cosmological constant and perhaps also of gravitational constant and to a modification of the dynamics of Kähler action however preserving the known extremals and basic properties of Kähler action and allowing to interpret induced Kähler form in terms of preferred imaginary unit defining twistor structure.

There are some new results forcing a profound modification of the recent view about TGD but consistent with the general picture. A more explicit realization of twistorialization as lifting of the preferred extremal X^4 of Kähler action to corresponding 6-D twistor space X^6 identified as surface in the 12-D product of twistor spaces of M^4 and CP_2 allowing Kähler structure suggests itself. The fiber F of Minkowskian twistor space must be identified with sphere S^2 with signature $(-1,-1)$ and would be a variant of the complex space with complex coordinates associated with S^2 and transversal space E^2 in the decomposition $M^4 = M^2 \times E^2$ and one hyper-complex coordinate associated with M^2.

The action principle in 6-D context is also Kähler action, which dimensionally reduces to Kähler action plus cosmological term. This brings in the radii of spheres $S^2(M^4)$ and $S^2(CP_2)$ associated with the twistors space of M^4 and CP_2. For $S(CP_2)$ the radius is of order CP_2 radius R. $R(S^2(M^4))$ could be of the order of Planck length l_P, which would thus become purely classical parameter contrary the expectations. An alternative option is $R(S^2(M^4)) = R$ The radius of S^2 associated with space-time surface is determined by the induced metric and is emergent length scale. The normalization of 6-D Kähler action by a scale factor $1/L^2$ with dimension, which is inverse length squared brings in a further length scale closely related to cosmological constant which is also dynamical and has correct sign to explain accelerated expansion of the Universe. The order of magnitude for L must be radius of the $S^2(X^4)$ and therefore small. This could mean a gigantic cosmological constant. Just as in GRT based cosmology!

This issue can be solved by using the observation that thanks to the decomposition $H = M^4 \times CP_2$ 6-D Kähler action is a sum of two independent terms. The first term corresponds to the 6-D lift of the ordinary Kähler action and for it the contribution from $S^2(CP_2)$ fiber is assumed to be absent: this could be due to the imbedding of $S^2(X^4)$ reducing to identification $S^2(M^4)$ and is not true generally. Second term in action is assumed to come from the $S^2(M^4)$ fiber of twistor space $T(M^4)$. The independency implies that couplings strengths are independent for them.

The analog for Kähler coupling strength (analogous to critical temperature) associated with $S^2(M^4)$ must be extremely large - so large that one has $\alpha_K(M^4) \times R(M^4)^2 \sim L^2$, L size scale of the recent Universe. This makes possible the small value of cosmological constant assignable to the volume term given by this part of the dimensionally reduced action. Both Kähler coupling strengths are assumed to have a spectrum determined by quantum criticality and the spectrum of $\alpha_K(M^4)$ comes essentially as p-adic primes satisfying p-adic length scale hypothesis $p \simeq 2^k$, k prime. In fact, it turns that one can assumed that the entire 6-D Kähler action contributes if one assumes that the winding numbers (w_1, w_2) for the map $S^2(X^4) \to S^2(M^4) \times S^2(CP_2)$ satisfy $(w_1, w_2) = (n, 0)$ in cosmological scales. The identification of w_1 as $h_{eff}/h = n$ is highly suggestive.

The dimensionally reduced dynamics is a highly non-trivial modification of the dynamics of Kähler action however preserving the known extremals and basic properties of Kähler action and allowing to interpret induced Kähler form in terms of preferred imaginary unit defining twistor structure. Strong constraints come also from the condition that induced spinor structure coming from that for twistor space $T(H)$ is essentially that coming from that of H.

Second new element is the fusion of the twistorial approach with the vision that diagrams are representations for computations. This as also quantum criticality demands that the diagrams should allow huge symmetries allowing to transform them to braided generalizations of tree-diagrams. Several guiding principles are involved and what is new is the observation that they indeed seem to form a coherent whole.

In the sequel I will discuss the recent understanding of twistorizalization, which is considerably improved from that in the earlier formulation. I formulate the dimensional reduction of 6-D Kähler action and consider the physical interpretation. There are considerable uncertainties at the level of details I dare believe that basically the situation is understood. After that I proceed to discuss the basic principles behind the recent view about scattering amplitudes as generalized Feynman diagrams.

13.2 twistor lift of Kähler action

First I will try to clarify the mathematical details related to the twistor spaces and how they emerge in the recent context. I do not regard myself as a mathematician in technical sense and I can only hope that the representation based on physical intuition does not contain serious mistakes.

13.2.1 Imbedding space is twistorially unique

It took roughly 36 years to learn that M^4 and CP_2 are twistorially unique. Space-times are surfaces in $H = M^4 \times CP_2$. M^4 and CP_2 are unique 4-manifolds in the sense that both allow twistor space with Kähler structure: Kähler structure is the crucial concept. Strictly speaking, it is E^4 and S^4 allow twistor space with Kähler structure [A63] : in the case of M^4 signature could cause problems. The standard identification for the twistor space of M^4 would be Minkowskian variant $PT = P_3 = SU(2,2)/SU(2,1) \times U(1)$ of 6-D twistor space $PT = CP_3 = SU(4)/SU(3) \times U(1)$ of E^4. The twistor space of CP_2 is 6-D $T(CP_2) = SU(3)/U(1) \times U(1)$, the space for the choices of quantization axes of color hypercharge and isospin.

The case of M^4 is however problematic. It is often stated that the twistor space is $PT = CP_3 = SU(4)/SU(3) \times U(1)$. The metric of twistor space does not appear in the construction of twistor amplitudes. Already the basic structure of PT suggests that this identification cannot be correct.

As if the situation were not complicated enough, there are two notions of twistor space: the twistor space identified as P_3 and as a trivial sphere bundle $M^4 \times CP_1$ having Kähler structure - what Kähler structure actually means in case of M^4 is hower not quite clear.

These considerations lead to a proposal - just a proposal - for the formulation of TGD in which space-time surfaces X^4 in H are lifted to twistor spaces X^6, which are sphere bundles over X^4 and such that they are surfaces in 12-D product space $T(M^4) \times T(CP_2)$ such the twistor structure of X^4 are in some sense induced from that of $T(M^4) \times T(CP_2)$. In the following $T(M^4)$ therefore denotes the trivial sphere bundle $M^4 \times CP_1$ over M^4 and twistorialization of scattering amplitudes would involve the projection from $T(M^4)$ to P_3. What is nice in this formulation is that one could use all the machinery of algebraic geometry so powerful in superstring theory (Calabi-Yau manifolds).

13.2.2 Some basic definitions

What twistor structure in Minkowskian signature does really mean geometrically has remained a confusing question for me. The problems associated with the Minkowskian signature of the metric are encountered also in twistor Grassmann approach to the scattering amplitudes but are circumvented by performing Wick rotation that is using E^4 or S^4 instead of M^4 and applying algebraic continuation. Also complexification of Minkowksi space for momenta is used. These tricks do not apply now.

To make this more concrete, let us sum up the basic definitions.

1. Bi-spinors in representations (1/2,0) and (0,1/2) of Lorentz group are the building bricks of twistors. Bi-spinors v^a and their conjugates $v^{a'}$ have the following inner products:

$$\begin{aligned} &\langle vw\rangle = \epsilon_{ab}v^a w^b \ , \qquad [vw] = \epsilon_{a'b'}v^{a'}w^{b'} \ , \\ &\epsilon_{ab} = (0,1;-1,0) \ , \quad \epsilon_{a'b'} = (0,1;-1,0) \ . \end{aligned} \tag{13.2.1}$$

   Unprimed spinor and its primed variant of the spinor are related by complex conjugation. Index raising is by the inverse ϵ^{ab} of ϵ_{ab}.

2. Twistors are identified as pairs of 2-spinor and its conjugate

$$Z^\alpha = (\lambda_a, \mu^{a'}) \ , \quad \overline{Z}_\alpha = (\overline{\mu}^a, \lambda_{a'}) \tag{13.2.2}$$

   The norm for Z^α is defined as

$$Z^\alpha \overline{Z}^\alpha = \langle \lambda\overline{\mu}\rangle + \left[\overline{\lambda}\mu\right] \ . \tag{13.2.3}$$

   One can write the metric explicitly as direct sum of terms of form $dudv$ (metric of M^2) and each of the can be taken to diagonal form (1,-1). Hence the metric can be written as $diag(1,1,1,1,-1,-1,-1,-1)$.

3. This norm allows to decompose PT to 3 parts PT_+,PT_- and PN in a projectively invariant manner depending on whether the sign of the norm is negative, positive, or whether it vanishes. PT_+ and PT_- serve as loci for the twistor lifts of positive and negative energy modes of massless fields. PN corresponds to the 5-D boundary of the lightcone of $M(2,4)$. By projective identification along light-like radial coordinate it reduces to what is known as conformal compactification of M^4, whose metric is defined only apart from a conformal factor. The natural metric of $PT = P_3$ does not seem to play any role in the construction of the amplitudes relying on projective invariants. The signature of M^4 metric however makes itself visible in the structure of PT: for the Euclidian variant of twistor space one would not have this decomposition to three parts.

Another definition of twistor space - to be used in the geometrization of twistor approach to be proposed - is as a trivial S^2 bundle $M^4 \times CP_1$ over M^4. Since the twistor spheres associated with the points of M^4 with light-like separation intersect, these two definitions cannot be equivalent. In fact, the proper definition of twistor space relies on double fibration involving both views about twistor space discussed in [B72] (see `http://tinyurl.com/yb4bt74l`).

1. The twistor bundle denoted as PS is the product $M^4 \times CP_1$ with CP_1 realized as projective space and having coordinates $(x^{aa'}, \lambda_a)$, $\{x^{aa'}\} \leftrightarrow x^\mu\sigma_\mu$, where the spinor λ_a is projective 2-spinor in $(1/2, 0)$ representation.

2. The twistors defined in this manner have a trivial projection q to M^4 and non-trivial projection p to P_3 with local projective coordinates $(\lambda_a, \mu^{a'})$. The projection p is defined by the projectively invariant incidence relation

$$\mu^{a'} = ix^{aa'}\lambda_a$$

If $y^{aa'}$ and $a^{aa'}$ differ by light-like vector there exists spinor λ annihilated by the difference vector and there exists twistor $(\lambda_a, \mu^{a\prime})$ to which both (x, λ) and (y, λ) are mapped by the incidence relation. Thus the images of twistor spheres associated for points with light-like separation intersect so that one does not have a proper CP_1 bundle structure.

3. The trivial twistor bundle $T(M^4) = M^4 \times CP_1$ would define the twistor space of M^4 in geometric sense. For this space the metric matters and the radius of CP_1 turns out to allow identification in terms of Planck length. Gravitational interaction would bring in Planck length as a basic scale in this manner. PT in turn would define the twistor space in which the twistor lifts of imbedding space-spinor fields are defined. For this space the metric, which is degenerate and seems to be only projectively defined should not be relevant as the construction of twistorial amplitudes suggests. Note however that the identification as the Minkowskian variant of P_3 allows also the introduction of metric.

This picture has an important immediate implication for the construction of quantum TGD. Positive and negative energy parts of zero energy states are defined at light-like boundaries of $CD \times CP_2$, where CD is the intersection of future and past directed light-cones. The twistor lifts of the amplitudes from $\delta CD \times CP_2$ must be single valued. The strongest condition guaranteing this is that they do not depend on the radial light-like coordinate at δCD. Super-symplectic symmetry implying the analog of conformal gauge symmetry for the radial light-like coordinate could guarantee this. There is however a hierarchy of conformal gauge symmetry breakings corresponding to the inclusion hierarchy of isomorphic sub-algebras so that this condition is too strong. A weaker condition is that the amplitude $F(m, \lambda)$ in $T(M^4)$ is constant along the light-like ray for the λ associated with the m along this ray. An even stronger condition is that $F(m, \lambda)$ vanishes along the ray. Particle would not propagate along δCD and would avoid remaining at the boundary of CD, a condition which is perfectly sensible physically.

13.2.3 What does twistor structure in Minkowskian signature really mean?

The following considerations relate to $T(M^4)$ identified as trivial bundle $M^4 \times CP_1$ with natural coordinates $(m^{aa'}, \lambda_a)$, where λ_a is projective spinor. The challenge is to generalize the complex structure of twistor space of E^4 to that for M^4. It turns out that the assumption that twistor space has ordinary complex structure fails. The first guess was that the fiber of twistor space is hyperbolic sphere with metric signature $(1, -1)$ having infinite area so that the 6-D Kähler action would be infinite. This makes no sense. The only alternative, which comes in mind is a hypercomplex generalization of the Kähler structure for M^4 lifted to twistor space, which locally means only adding of S^2 fiber with metric signature $(-1, -1)$.

1. To proceed one must make an explicit the definition of twistor space. The 2-D fiber S^2 consists of antisymmetric tensors of X^4 which can be taken to be self-dual or anti-self-dual by taking any antisymmetric form and by adding to its plus/minus its dual. Each tensor of this kind defines a direction - point of S^2. These points can be also regarded as quaternionic imaginary units. One has a natural metric in S^2 defined by the X^4 inner product for antisymmetric tensors: this inner product depends on space-time metric. Kähler action density is example of a norm defined by this inner product in the special case that the antisymmetric tensor is induced Kähler form. Induced Kähler form defines a preferred imaginary unit and is needed to define the imaginary part $\omega(X, Y) = ig(X, -JY)$ of hermitian form $h = h + i\omega$.

2. To define the analog of Kähler structure for M^4, one must start from a decomposition of $M^4 = M^2 \times E^2$ (M^2 is generated by light-like vector and its dual) and E^2 is orthogonal to it. M^2 allows hypercomplex structure, which light-like coordinates $(u = t - z, v = t + z)$ and E^2

complex structure and the metric has form $ds^2 = dudv + dzd\overline{z}$. Hypercomplex numbers can be represented as $h = t + iez$, $i^2 = -1, e^2 = -1$ $i^2 = -1, e^2 = -1$. Hyper-complex numbers do not define number field since for light-like hypercomplex numbers $t + iez$, $t = \pm z$ do not have finite inverse. Hypercomplex numbers allow a generalization of analytic functions used routinely in physics. Kähler form representing hypercomplex imaginary unit would be replaced with eJ. One would consider sub-spaces of complexified quaternions spanned by real unit and units eI_k, $k = 1, 2, 3$ as representation of the tangent space of space-time surfaces in Minkowskian regions. This is familiar already from M^8 duality [K111].

$M^4 = M^2 \times E^2$ decomposition can depend on point of M^4 (polarization plane and light-like momentum direction depend on point of M^4. The condition that this structure allows global coordinates analogous to $(u, v, z, \overline{z})$ requires that the distributions for M^2 and E^2 are integrable and thus define 2-D surfaces. I have christened this structure Hamilton-Jacobi structure. It emerges naturally in the construction of extremals of Kähler action that I have christened massless extremals (MEs, [K7]) and also in the proposal for the generalization of complex structure to Minkowskian signature [K103].

One can define the analog of Kähler form by taking sum of induced Kähler form J and its dual $*J$ defined in terms of permutation tensor. The normalization condition is that this form integrates to the negative of metric $(J \pm *J)^2 = -g$. This condition is possible to satisfy.

3. How to lift the Hamilton Jacobi structure of M^4 to Kähler structure of its twistor space? The basic definition of twistors assumes that their exists a field of time-like directions, and that one considers projections of 4-D antisymmetric tensors to the 3-space orthogonal to the time-like direction at given point. One can say that the projection yields magnetic part of the antisymmetric tensor (say induced Kähler form J) with positive norm with respect to natural metric induced to the twistor fiber from the inner product between two-forms. This unique time direction would be defined the light-like vector defining M^2 and its dual. Therefore the signature of the metric of S^2 would be $(-1, -1)$. In quaternionic picture this direction corresponds to real quaternionic unit.

4. To sum up, the metric of the Minkowskian twistor space has signature $(-1, -1, 1, -1, -1, -1)$. The Minkowskian variant of the twistor space would give 2 complex coordinates and one hyper-complex coordinate. Cosmological term would be finite and the sign of the cosmological term in the dimensionally reduced action would be positive as required. Also metric determinant would be imaginary as required. At this moment I cannot invent any killer objection against this option.

It must be made clear that the proposed definition of twistor space of M^4 does not seem to be equivalent with the twistor space assignable to conformally compactified M^4. One has trivial S^2 bundle and Hamilton-Jacobi structure, which is hybrid of complex and hyper-complex structure.

13.2.4 What does the induction of the twistor structure to space-time surface really mean?

Consider now what the induction of the twistor structure to space-time surface X^4 could mean.

1. The induction procedure for Kähler structure of 12-D twistor space T requires that the induced metric and Kähler form of the base space X^4 of X^6 obtained from T is the same as that obtained by inducing from $H = M^4 \times CP_2$. Since the Kähler structure and metric of T is lift from H this seems obvious. Projection would compensate the lift.

2. This is not yet enough. The Kähler structure and metric of S^2 projected from T must be same as those lifted from X^4. The connection between metric and ω implies that this condition for Kähler form is enough. The antisymmetric Kähler forms in fiber obtained in these two manners co-incide. Since Kähler form has only one component in 2-D case, one obtains single constraint condition giving a commutative diagram stating that the direct projection to S^2 equals with the projection to the base followed by a lift to fiber. The resulting induced Kähler form is not covariantly constant but in fiber S^2 one has $J^2 = -g$.

As a matter of fact, this condition might be trivially satisfied as a consequence of the bundle structure of twistor space. The Kähler form from $S^2 \times S^2$ can be projected to S^2 associated with X^4 and by bundle projection to a two-form in X^4. The intuitive guess - which might be of course wrong - is that this 2-form must be same as that obtained by projecting the Kähler form of CP_2 to X^4. If so then the bundle structure would be essential but what does it really mean?

3. Intuitively it seems clear that X^6 must decompose locally to a product $X^4 \times S^2$ in some sense. This is true if the metric and Kähler form reduce to direct sums of contributions from the tangent spaces of X^4 and S^2. This guarantees that 6-D Kähler action decomposes to a sum of 4-D Kähler action and Kähler action for S^2.

   This could be however too strong a condition. Dimensional reduction occurs in Kaluza-Klein theories and in this case the metric can have also components between tangent spaces of the fiber and base being interpreted as gauge potentials. This suggests that one should formulate the condition in terms of the matrix $T \leftrightarrow g^{\alpha\mu}g^{\beta\nu} - g^{\alpha\nu}g^{\beta\mu}$ defining the norm of the induced Kähler form giving rise to Kähler action. T maps Kähler form $J \leftrightarrow J_{\alpha\beta}$ to a contravariant tensor $J_c \leftrightarrow J^{\alpha\beta}$ and should have the property that $J_c(X^4)$ ($J_c(S^2)$) does not depend on $J(S^2)$ ($J(X^4)$).

   One should take into account also the self-duality of the form defining the imaginary unit. In X^4 the form $S = J \pm *J$ is self-dual/anti-self dual and would define twistorial imaginary unit since its square equals to $-g$ representing the negative of the real unit. This would suggest that 4-D Kähler action is effectively replaced with $(J \pm *J) \wedge (J \pm *J) = J^* J \pm J \wedge J$, where $*J$ is the Hodge dual defined in terms of 4-D permutation tensor ϵ. The second term is topological term (Abelian instanton term) and does not contribute to field equations. This in turn would mean that it is the tensor $T \pm \epsilon$ for which one can demand that $S_c(X^4)$ ($S_c(S^2)$) does not depend on $S(S^2)$ ($S(X^4)$).

4. The preferred quaternionic imaginary unit should be represented as a projection of Kähler form of 12-D twistor space $T(H)$. The preferred imaginary unit defining twistor structure as sum of projections of both $T(CP_2)$ and $T(M^4)$ Kähler forms would guarantee that vacuum extremals like canonically imbedded M^4 for which $T(CP_2)$ Kähler form contributes nothing have well-defined twistor structure. $T(M^4)$ or $T(CP_2)$ are treated completely symmetrically but the maps of $S^2(X^4)$ to $S^2(M^4)$ and $S^2(CP_2)$ characterized by winding numbers induce symmetry breaking.

   For Kähler action $M^4 - CP_2$ symmetry does not make sense. 4-D Kähler action to which 6-D Kähler action dimensionally reduces can depend on CP_2 Kähler form only. I have also considered the possibility of covariantly constant self-dual M^4 term in Kähler action but given it up because of problems with Lorentz invariance. One should couple the gauge potential of M^4 Kähler form to induced spinors. This would mean the existence of vacuum gauge fields coupling to sigma matrices of M^4 so that the gauge grop would be non-compact $SO(3,1)$ leading to a breakdown of unitarity.

There is still one difficulty to be solved.

1. The normalization of 6-D Kähler action by a scale factor $1/L^2$ with dimension, which is inverse length squared, brings in a further length scale. The first guess is that $1/L^2$ is closely related to cosmological constant, which is also dynamical and $1/L^2$has indeed correct sign to explain accelerated expansion of the Universe. Unfortunately, if $1/L^2$ is of order cosmological constant, the value of the ordinary Kähler coupling strength α_K would be enormous. As a matter of fact, the order of magnitude for L^2 must be equal to the area of $S^2(X^4)$ and in good approximation equal to $L^2 = 4\pi R^2(S^2(M^4))$ and therefore in the same range as Planck length l_P and CP_2 radius R. This would imply a gigantic value of cosmological constant. Just as in GRT based cosmology!

2. This issue can be solved by using the observation that thanks to the decomposition $H = M^4 \times CP_2$, 6-D Kähler action is sum of two independent terms. The first term corresponds to the 6-D lift of the ordinary Kähler action. For it the contribution from $S^2(CP_2)$ fiber

is absent if the imbedding of $S^2(X^4)$ to $S^2(M^4) \times S^2(CP_2)$ reduces to identification with $S^2(M^4)$ so that $S^2(CP_2)$ is effectively absent: this is not true generally. Second term in the action is assumed to come from the $S^2(M^4)$ fiber of twistor space $T(M^4)$, which can indeed contribute without breaking of Lorentz symmetry. In fact, one can assume that also the Kähler form of M^4 contributes as will be found.

3. The independency implies that Kähler couplings strengths are independent for them. If one wants that cosmological constant has a reasonable order of magnitude, $L \sim R(S^2(M^4))$ must hold true and the analog $\alpha_K(S^2(M^4))$ of the ordinary Kähler coupling strength (analogous to critical temperature) must be extremely large - so large that one has

$$\alpha_K(M^4) \times 4\pi R(M^4)^2 \sim L^2 \ ,$$

where L is the size scale of the recent Universe.

This makes possible the small value of cosmological constant assignable to the volume term given by this part of dimensionally reduced action. Both Kähler coupling strengths are assumed to have a spectrum determined by quantum criticality and the spectrum of $\alpha_K(M^4)$ would be essentially as p-adic primes satisfying p-adic length scale hypothesis $p \simeq 2^k$, k prime. One can criticize this identification of 6-D Kähler action as artificial but it seems to be the only option that works. Interestingly also the contribution from M^4 Kähler form can be allowed since it is also extremely small. For canonically imbedded M^4 this contribution vanishes by self-duality of M^4 Kähler form and is extremely small for the vacuum extremals of Kähler action.

4. For general winding numbers of the map $S^2(X^4) \to S^2(M^4) \times S^2(CP_2)$ also $S^2(CP_2)$ Kähler form contributes and cosmological constant is gigantic. It would seem that only the winding numbers $(w_1, w_2) = (n, 0)$ are consistent with the observed value of cosmological constant. Hence it seems that there is no need to pose any additional conditions to the Kähler action if one uses the fact that $T(M^4)$ and $T(CP_2)$ parts are independent!

It is good to list the possible open issues related to the precise definition of the twistor structure and of M^4 Kähler action.

1. The proposed definition of M^4 twistor space a Cartesian product of M^4 and $S^2(M^4)$ parts involving Hamilton-Jacobi structure does not seem to be equivalent with the twistor identification as $SU(2,2)/SU(2,1) \times U(1)$ having conformally compactified M^4 as base space. There exists an entire moduli space of Hamilton-Jacobi structures. If the M^4 part of Kähler form participates in dynamics, one must include the specification of the Hamilton-Jacobi structure to the definition of CD and integrate over Hamilton Jacobi-structures as part of integral over WCW in order to gain Lorentz invariance. Note that Hamilton-Jacobi structure enters to dynamics also through the construction of massless extremals [K7].

2. The presence of M^4 part of Kähler form in action implies breaking of Lorentz invariance for extremals of lifted Kähler action. The same happens at the level of induced spinors if this Kähler form couples to imbedding space spinors. If $T(M^4)$ is trivial bundle, one can include only the $T(S^2(M^4))$ part of Kähler form to Kähler action and couple only this to the spinors of $T(H)$. The integration over Hamilton-Jacobi structures becomes un-necessary.

3. If one includes M^4 part of Kähler form to 6-D Kähler action, one has several options. One can have sum of the Kähler actions for $T(M^4)$ and $T(CP_2)$ or Kähler action defined by the sum $J(T(M^4)/g_K$ and $J(T(CP_2)/\alpha_K$ with $\alpha_K(M^4) = g_K^2(M^4)/4\pi\hbar$ and $\alpha_K = g_K^2/4\pi\hbar$ with a proper normalization to guarantee that the squares of induced Kähler forms give sum of Kähler actions as in the first option. In this case one obtains interference term proportional to $Tr(J(M^4)J(CP_2)$. For the proposed value of α_K also the interference term is extremely small as compared to Kähler action in recent cosmology.

13.2.5 Could M^4 Kähler form introduce new gravitational physics?

The introduction of M^4 Kähler form could bring in new gravitational physics.

1. As found, the twistorial formulation of TGD assigns to M^4 a self dual Kähler form whose square gives Minkowski metric. It can (but need not if M^4 twistor space is trivial as bundle) contribute to the 6-D twistor counterpart of Kähler action inducing M^4 term to 4-D Kähler action vanishing for canonically imbedded M^4.

2. Self-dual Kähler form in empty Minkowski space satisfies automatically Maxwell equations and has by Minkowskian signature and self-duality a vanishing action density. Energy momentum tensor is proportional to the metric so that Einstein Maxwell equations are satisfied for a non-vanishing cosmological constant! M^4 indeed allows a large number of self dual Kähler fields (I have christened them as Hamilton-Jacobi structures). These are probably the simplest solutions of Einstein-Maxwell equations that one can imagine!

3. There however exist quite a many Hamilton-Jacobi structures. However, if this structure is to be assigned with a causal diamond (CD) it must satisfy additional conditions, say $SO(3)$ symmetry and invariance under time translations assignable to CD. Alternatively, covariant constancy and $SO(2) \subset SO(3)$ symmetry might be required.

This raises several questions. Could M^4 Kähler form replace CP_2 Kähler form in the picture for how gravitational interaction is mediated at quantal level? Could one speak of flux tubes of the magnetic part of this Kähler form? Or should one consider the Kähler field as a sum of the two Kähler forms weighted by the inverses $1/g_K$ of corresponding Kähler couplings. If so then M^4 contribution would be negligible except for canonically imbedded M^4 in the recent cosmology. Note that α_K and $\alpha_K(M^4)$ have interpretation as analogs of quantum critical temperatures but can depend on the p-adic lengths scale defining the cosmology.

1. The natural expectation is that Kähler form characterizes CD having preferred time direction suggested strongly by number theoretical considerations involving quaternionic structure with preferred direction of time axis assignable to real unit quaternion.

   Self-duality gives rise to Kähler magnetic and electric fields in the same spatial direction identifiable as a local quantization axis for spin assignable to CD assignable to observer. CD indeed serves as a correlate for conscious entity in TGD inspired theory of consciousness. Flux tube would connect mass M to mass m assignable to observer and flux tube direction would define spin quantization axes for the CD of the observer. Spin quantization axis would be naturally in the direction of magnetic field, which is direction of the flux tube.

2. The self-dual Kähler form could be spherically symmetric for CDs and represent self dual magnetic monopole field (dyon) with monopole charge at the line connecting the tips of CD and have non-vanishing components $J^{tr} = \epsilon^{tr\theta\phi} J_{\theta\phi}$, $J_{\theta\phi} = sin(\theta)$. One would have genuine monopole, which is somewhat questionable feature. Only the entire radial flux would be quantized. CD could be associated with the mass M of the central object. The gauge potential associated with J could be chosen to be $A_\mu \leftrightarrow (1/r, 0, 0, cos(\theta)$. I have considered this kind of possibility earlier in context of TGD inspired model of anyons but gave up the idea.

   The moduli space for CDs with second tip fixed would be hyperbolic space $H^3 = SO(3,1)/SO(3)$ or a space obtained by identifying points at the orbits of some discrete subgroup of $SO(3,1)$ as suggested by number theoretic considerations. This induced Kähler field could make the blackholes with center at this line to behave like M^4 magnetic monopoles if the M^4 part of Kähler form is induced into the 6-D lift of Kähler action with extremely small coefficients of order of magnitude of cosmological constant. Cosmological constant and the possibility of CD monopoles would thus relate to each other.

3. The self-dual M^4 Kähler form could be also covariantly constant ($J_{tz} = J_{xy} = 1$) and represent electric and magnetic fluxes in a fixed direction identifiable as a quantization axes for spin and characterizing CD. In this case the CD would be associated with the mass m of observer. The moduli space of CDs would be now $SO(3,1)/SO(1,1) \times SO(2)$ which is completely analogous to the twistor space $SU(3)/U(1) \times U(1)$.

4. Boundary conditions (allowing no boundaries!) demand that the flux tubes have closed cross section - say sphere S^2 - rather than disk: stability is guaranteed if the S^2 cross section is mapped to homologically non-trivial surface of CP_2 or is projection of it. This would give monopole flux also for CP_2 Kähler form so that the original hypothesis would be correct.

5. Radial flux tubes are possible both spherically symmetric and covariantly constant Kähler form possibly mediating gravitational interaction but the flux is not quantized unless preferred extremal property implies this: in any case M^4 flux would be very small unless one has large value of gravitational Planck constant implying n-sheeted covering of M^4 and flux is scale up by n since every sheet gives a contribution. For spherically symmetric M^4 Kähler form the flux tubes would have naturally conical structure spanning a constant solid angle. For covariantly constant Kähler form the flux tubes would be cylindrical.

There are further interpretational problems.

1. The classical coupling of M^4 Kähler gauge potential to induced spinors is not small. Can one really tolerate this kind of coupling equivalent to a coupling to a self dual monopole field carrying electric and magnetic charges? One could of course consider the condition that the string world sheets carrying spinor modes are such that the induced M^4 Kähler form vanishes and gauge potential become pure gauge. M^4 projection would be 2-D Lagrange manifold whereas CP_2 projection would carry vanishing induce $W$and possibly also Z^0 field in order that em charge is well defined for the modes. These conditions would fix the string world sheets to a very high degree in terms of maps between this kind of 2-D sub-manifolds of M^4 and CP_2. Spinor dynamics would be determined by the avoidance of interaction!

   Recall that one could interpret the localization of spinor modes to 2-surfaces in the sense of strong form of holography: one can continued induced spinor fields to the space-time interior as indeed assumed but the continuation is completely determined by the data at 2-D string world sheets.

   It must be emphasized that the imbedding space spinor modes characterizing the ground states of super-symplectic representations would not couple to the monopole field so that at this level Poincare invariance is not broken. The coupling would be only at the space-time level and force spinor modes to Lagrangian sub-manifolds.

2. At the static limit of GRT and for $g_{ij} \simeq \delta_{ij}$ implying $SO(3)$ symmetry there is very close analogy with Maxwell's equations and one can speak of gravi-electricity and gravi-magnetism with 4-D vector potential given by the components of $g_{t\alpha}$. The genuine $U(1)$ gauge potential does not however relate to the gravimagnetism in GRT sense. Situation would be analogous to that for CP_2, where one must add to the spinor connection $U(1)$ term to obtain respectable spinor structure. Now the $U(1)$ term would be added to trivial spinor connection of flat M^4: its presence would be justified by twistor space Kähler structure. If the induced M^4 Kähler form is present as a classical physical field it means genuinely new contribution to U(1) electroweak of standard model. If string world sheets carry vanishing M^4 Kähler form, this contribution vanishes classically.

13.2.6 A connection with the hierarchy of Planck constants?

A connection with the hierarchy of Planck constants is highly suggestive. Since also a connection with the p-adic length scale hierarchy suggests itself for the hierarchy of p-adic length scales it seems that both length scale hierarchies might find first principle explanation in terms of twistor lift of Kähler action.

1. Cosmological considerations encourage to think that $R_1 \simeq l_P$ and $R_2 \simeq R$ hold true. One would have in early cosmology $(w_1, w_2) = (1, 0)$ and later $(w_1, w_2) = (0, 1)$ guaranteeing $R_D$grows from l_P to R during cosmological evolution. These situations would correspond the solutions $(w_1 = n, 0)$ and $(0, w_2 = n)$ one has $A = n4\pi R_1^2$ and $A = n \times 4\pi R_2^2$ and both Kähler coupling strengths are scaled down to α_K/n. For $\hbar_{eff}/h = n$ exactly the same thing happens!

There are further intriguing similarities. $h_{eff}/h = n$ is assumed to correspond *multi-sheeted* (to be distinguished from *many-sheeted*!) covering space structure for space-time surface. Now one has covering space defined by the lift $S^2(X^4) \rightarrow S^2(M4) \times S^2(CP_2)$. These lifts define also lifts of space-time surfaces.

Could the hierarchy of Planck constants correspond to the twistorial surfaces for which $S^2(M^4)$ is n-fold covering of $S^2(X^4)$? The assumption has been that the n-fold multi-sheeted coverings of space-time surface for $h_{eff}/h = n$ are singular at the ends of space-time surfaces at upper and lower boundaries if causal diamond (CD). Could one consider a more precise definition of twistor space in such a manner that CD replaces M^4 and the covering becomes singular at the light-like boundaries of CD - the branches of space-time surface would collapse to single one.

Does this collapse have a clear geometric meaning? Are the projections of various branches of the S^2 lift automatically identical so that one would have the original picture in which one has n identical copies of the same space-time surface? Or can one require identical projections only at the light-like boundaries of CD?

2. $w_1 = w_2 = w$ is essentially the first proposal for conditions associated with the lifting of twistor space structure. $w_1 = w_2 = n$ gives $ds^2 = (R_1^2 + R_2^2)(d\theta^2 + w^2 d\phi^2)$ and $A = n \times 4\pi(R_1^2 + R_2^2)$. Also now Kähler coupling strength is scaled down to α/n. Again a connection with the hierarchy of Planck constants suggests itself.

3. One can consider also the option $R_1 = R_2$ option giving $ds^2 = R_1^2(2d\theta^2 + (w_1^2 + w_2^2)d\phi^2$. If the integers w_i define Pythagorean square one has $w_1^2 + w_2^2 = n^2$ and one has $R_1 = R_2$ option that one has $A = n \times 4\pi R^2$. Also now the connection with the hierarchy of Planck constants might make sense.

13.2.7 Twistorial variant for the imbedding space spinor structure

The induction of the spinor structure of imbedding space is in key role in quantum TGD. The question arises whether one should lift also spinor structure to the level of twistor space. If so one must understand how spinors for $T(M^4)$ and $T(CP_2)$ are defined and how the induced spinor structure is induced.

1. In the case of CP_2 the definition of spinor structure is rather delicate and one must add to the ordinary spinor connection U(1) part, which corresponds physically to the addition of classical U(1) gauge potential and indeed produces correct electroweak couplings to quarks and leptons. It is assumed that the situation does not change in any essential manner: that is the projections of gauge potentials of spinor connection to the space-time surface give those induced from $M^4 \times CP_2$ spinor connection plus possible other parts coming as a projection from the fiber $S^2(M^2) \times S^2(CP_2)$. As a matter of fact, these other parts should vanish if dimensional reduction is what it is meant to be.

2. The key question is whether the complications due to the fact that the geometries of twistor spaces $T(M^4)$ and $T(CP_2)$ are not quite Cartesian products (in the sense that metric could be reduced to a direct sum of metrics for the base and fiber) can be neglected so that one can treat the sphere bundles approximately as Cartesian products $M^4 \times S^2$ and $CP_2 \times S^2$. This will be assumed in the following but should be carefully proven.

3. Locally the spinors of the twistorspace $T(H)$ are tensor products of imbedding spinors and those for of $S^2(M^4) \times S^2(CP_2)$ expressible also as tensor products of spinors for $S^2(M^4)$ and $S^2(CP_2)$. Obviously, the number of spinor components increases by factor $2 \times 2 = 4$ unless one poses some additional conditions taking care that one has dimensional reduction without the emergence of any new spin like degrees of freedom for which there is no physical evidence. The only possible manner to achieve this is to pose covariant constancy conditions already at the level of twistor spaces $T(M^4)$ and $T(CP_2)$ leaving only single spin state in these degrees of freedom.

4. In CP_2 covariant constancy is possible for right-handed neutrino so that CP_2 spinor structure can be taken as a model. In the case of CP_2 spinors covariant constancy is possible for right-handed neutrino and is essentially due to the presence of U(1) part in spinor connection forced by the fact that the spinor structure does not exist otherwise. Ordinary S^2spinor connection defined by vielbein exists always. One can however add a coupling to a suitable multiple of Kähler potential satisfying the quantization of magnetic charge (the magnetic flux defined by $U(1)$ connection is multiple of 2π so that its imaginary exponential is unity).

   S^2 spinor connections must must have besides ordinary vielbein part determined by S^2 metric also $U(1)$ part defined by Kähler form coupled with correct coupling so that the curvature form annihilates the second spin state for both $S^2(M^4)$ and $S^2(CP_2)$. $U(1)$ part of the spinor curvature is proportional to Kähler form $J \propto sin(theta)d\theta d\phi$ so that this is possible. The vielbein and $U(1)$ parts of the spinor curvature ear proportional Pauli spin matrix $\sigma_z = (1,0;0,-1)/2$ and unit matrix $(1,0;0,1)$ respectively so that the covariant constancy is possible to satisfy and fixes the spin state uniquely.

5. The covariant derivative for the induced spinors is defined by the sum of projections of spinor gauge potentials for $T(M^4)$ and $T(CP_2)$. With above assumptions the contributions gauge potentials from $T(M^4)$ and $T(CP_2)$ separately annihilate single spinor component. As a consequence there are no constraints on the winding numbers w_i, $i = 1,2$ of the maps $S^2(X^4) \rightarrow S^2(M^4)$ and $S^2(X^4) \rightarrow S^2(CP_2)$. Winding number w_i corresponds to the imbedding map $(\Theta_i = \theta, \Phi_i = w_i\phi)$.

6. If the square of the Kähler form in fiber degrees of freedom gives metric to that its square is metric, one obtains just the area of S^2 from the fiber part of action. This is given by the area $A = 4\pi\sqrt{2(w_1^2R_1^2 + w_2^2R_2^2)}$ since the induced metric is given by $ds^2 = (R_1^2 + R_2^2)d\theta^2 + (w_1^2R_1^2 + w_2^2R_2^2)d\phi^2$ for $(\Theta_1 = \theta, \Phi = n_1\phi, \Phi_2 = n_2\phi)$.

13.2.8 Twistor googly problem transforms from a curse to blessing in TGD framework

There was a nice story with title "Michael Atiyahs Imaginative State of Mind" about mathematician Michael Atyiah in Quanta Magazine (see `http://tinyurl.com/jta2va8`). The works of Atyiah have affected profoundly the development of theoretical physics. What was pleasant to hear that Atyiah belongs to those scientists who do not care what others think. As he tells, he can afford this since he has got all possible prices. This is consoling and encouraging even for those who have not cared what others think and for this reason have not earned any prizes. Nor even a single coin from what they have been busily doing their whole lifetime!

In the beginning of the story "twistor googly problem" was mentioned. I had to refresh my understanding about googly problem. In twistorial description the modes of massless fields (rather than entire massless fields) in space-time are lifted to the modes in its 6-D twistor-space and dynamics reduces to holomorphy. The analog of this takes place also in string models by conformal invariance and in TGD by its extension.

One however encounters what is known as googly problem: one can have twistorial description for circular polarizations with well-defined helicity +1/-1 but not for general polarization states - say linear polarizations, which are superposition of circular polarizations. This reflects itself in the construction of twistorial amplitudes in twistor Grassmann program for gauge fields but rather implicitly: the amplitudes are constructed only for fixed helicity states of scattered particles. For gravitons the situation gets really bad because of non-linearity.

Mathematically the most elegant solution would be to have only +1 or -1 helicity but not their superpositions implying very strong parity breaking and chirality selection. Parity parity breaking occurs in physics but is very small and linear polarizations are certainly possible! The discusion of Penrose with Atyiah has inspired a possible solution to the problem known as "palatial twistor theory" (see `http://tinyurl.com/hr7hmh2`). Unfortunately, the article is behind paywall too high for me so that I cannot say anything about it.

What happens to the googly problem in TGD framework? There is twistorialization at space-time level and imbedding space level.

1. One replaces space-time with 4-surface in $H = M^4 \times CP_2$ and lifts this 4-surface to its 6-D twistor space represented as a 6-surface in 12-D twistor space $T(H) = T(M^4) \times T(CP_2)$. The twistor space has Kähler structure only for M^4 and CP_2 so that TGD is unique. This Kähler structure is needed to lift the dynamics of Kähler action to twistor context and the lift leads to the a dramatic increase in the understanding of TGD: in particular, Planck length and cosmological constant with correct sign emerge automatically as dimensional constants besides CP_2 size.

2. Twistorialization at imbedding space level means that spinor modes in H representing ground states of super-symplectic representations are lifted to spinor modes in T(H). M^4 chirality is in TGD framework replaced with H-chirality, and the two chiralities correspond to quarks and leptons. But one cannot superpose quarks and leptons! "Googly problem" is just what the superselection rule preventing superposition of quarks and leptons requires in TGD!

One can look this in more detail.

1. Chiral invariance makes possible for the modes of massless fields to have definite chirality: these modes correspond to holomorphic or antiholomorphic amplitudes in twistor space and holomorphy (antiholomorphy is holomorphy with respect to conjugates of complex coordinates) does not allow their superposition so that massless bosons should have well-defined helicities in conflict with experimental facts. Second basic problem of conformally invariant field theories and of twistor approach relates to the fact that physical particles are massive in 4-D sense. Masslessness in 4-D sense also implies infrared divergences for the scattering amplitudes. Physically natural cutoff is required but would break conformal symmetry.

2. The solution of problems is masslessness in 8-D sense allowing particles to be massive in 4-D sense. Fermions have a well-defined 8-D chirality - they are either quarks or leptons depending on the sign of chirality. 8-D spinors are constructible as superpositions of tensor products of M^4 spinors and of CP_2 spinors with both having well-defined chirality so that tensor product has chiralities (ϵ_1, ϵ_2), $\epsilon_i = \pm 1$, $i = 1, 2$. H-chirality equals to $\epsilon = \epsilon_1\epsilon_2$. For quarks one has $\epsilon = 1$ (a convention) and for leptons $\epsilon = -1$. For quark states massless in M^4 sense one has either $(\epsilon_1, \epsilon_2) = (1, 1)$ or $(\epsilon_1, \epsilon_2) = (-1, -1)$ and for massive states superposition of these. For leptons one has either $(\epsilon_1, \epsilon_2) = (1, -1)$ or $(\epsilon_1, \epsilon_2) = (-1, 1)$ in massless case and superposition of these in massive case.

3. The twistor lift to $T(M^4) \times T(CP_2)$ of the ground states of super-symplectic representations represented in terms of tensor products formed from H-spinor modes involves only quark and lepton type spinor modes with well-defined H-chirality. Superpositions of amplitudes in which different M^4 helicities appear but M^4 chirality is always paired with completely correlating CP_2 chirality to give either $\epsilon = 1$ or $\epsilon = -1$. One has never a superposition of of different chiralities in either M^4 or CP_2 tensor factor. I see no reason forbidding this kind of mixing of holomorphicities and this is enough to avoid googly problem. Linear polarizations and massive states represent states with entanglement between M^4 and CP_2 degrees of freedom. For massless and circularly polarized states the entanglement is absent.

4. This has interesting implications for the massivation. Higgs field cannot be scalar in 8-D sense since this would make particles massive in 8-D sense and separate conservation of B and L would be lost. Theory would also contain a dimensional coupling. TGD counterpart of Higgs boson is actually CP_2 vector, and one can say that gauge bosons and Higgs combine to form 8-D vector. This correctly predicts the quantum numbers of Higgs. Ordinary massivation by constant vacuum expectation value of vector Higgs is not an attractive idea since no covariantly constant CP_2 vector field exists so that Higgsy massivation is not promising except at QFT limit of TGD formulated in M^4. p-Adic thermodynamics gives rise to 4-D massivation but keeps particles massless in 8-D sense. It also leads to powerful and correct predictions in terms of p-adic length scale hypothesis.

Anonymous reader gave me a link to the paper of Penrose and this inspired further more detailed considerations of googly problem.

1. After the first reading I must say that I could not understand how the proposed elimination of conjugate twistor by quantization of twistors solves the googly problem, which means that both helicities are present (twistor Z and its conjugate) in linearly polarized classical modes so that holomorphy is broken classically.

2. I am also very skeptic about quantizing of either space-time coordinates or twistor space coordinates. To me quantization is natural only for linear objects like spinors. For bosonic objects one must go to higher abstraction level and replace superpositions in space-time with superpositions in field space. Construction of "World of Classical Worlds" (WCW) in TGD means just this.

3. One could however think that circular polarizations are fundamental and quantal linear combination of the states carrying circularly polarized modes give rise to linear and elliptic polarizations. Linear combination would be possible only at the level of field space (WCW in TGD), not for classical fields in space-time. If so, then the elimination of conjugate of Z by quantization suggested by Penrose would work.

4. Unfortunately, Maxwell's equations allow classically linear polarisations! In order to achieve classical-quantum consistency, one should modify classical Maxwell's equations somehow so that linear polarizations are not possible. Googly problem is still there!

What about TGD?

1. Massless extremals representing massless modes are very "quantal": they cannot be superposed classically unless both momentum and polarisation directions for them (they can depend space-time point) are exactly parallel. Optimist would guess that the classical local classical polarisations are circular. No, they are linear! Superposition of classical linear polarizations at the level of WCW can give rise to local linear but not local circular polarization! Something more is needed.

2. The only sensible conclusion is that only gauge boson quanta (not classical modes) represented as pairs of fundamental fermion and antifermion in TGD framework can have circular polarization! And indeed, massless bosons - in fact, all elementary particles- are constructed from fundamental fermions and they allow only two M^4, CP_2 and $M^4 \times CP_2$ helicities/-chiralities analogous to circular polarisations. B and L conservation would transform googly problem to a superselection rule as already described.

To sum up, both the extreme non-linearity of Kähler action, the representability of all elementary particles in terms of fundamental fermions and antifermions, and the generalization of conserved M^4 chirality to conservation of H-chirality would be essential for solving the googly problem in TGD framework.

13.3 Surprise: Twistorial Dynamics Does Not Reduce to a Trivial Reformulation of the Dynamics of Kähler Action

I have thought that twistorialization classically means only an alternative formulation of TGD. This is definitely not the case as the explicit study demonstrated. Twistor formulation of TGD is in terms of of 6-D twistor spaces $T(X^4)$ of space-time surfaces $X^4 \subset M^4 \times CP_2$ in 12-dimensional product $T = T(M^4) \times T(CP_2)$ of 6-D twistor spaces of $T(M^4)$ of M^4 and $T(CP_2)$ of CP_2. The induced Kähler form in X^4 defines the quaternionic imaginary unit defining twistor structure: how stupid that I realized it only now! I experienced during single night many other "How stupid I have been" experiences.

Classical dynamics is determined by 6-D variant of Kähler action with coefficient $1/L^2$ having dimensions of inverse length squared. Since twistor space is bundle, a dimensional reduction of 6-D Kähler action to 4-D Kähler action plus a term analogous to cosmological term - space-time volume - takes place so that dynamics reduces to 4-D dynamics also now. Here one must be careful:

this happens provided the radius of $S2$ associated with X^4 does not depend on point of X^4. The emergence of cosmological term was however completely unexpected: again "How stupid I have been" experience. The scales of the spheres and the condition that the 6-D action is dimensionless bring in 3 fundamental length scales!

13.3.1 New scales emerge

The twistorial dynamics gives to several new scales with rather obvious interpretation. The new fundamental constants that emerge are the radii of the spheres associated with $T(M^4)$ and $T(CP_2)$. The radius of the sphere associated with X^4 is not a fundamental constant but determined by the induced metric. By above argument the fiber is sphere for both Euclidian signature and Minkowskian signatures.

1. For CP_2 twistor space the radius of $S^2(CP_2)$ must be apart from numerical constant equal to CP_2 radius R. For $S^2(M^4)$ one an consider two options. The first option is that also now the radius for $S^2(M^4)$ equals to $R(M^4) = R$ so that Planck length would not emerge from fundamental theory classically as assumed hitherto. Second imaginable option is that it does and one has $R(M^4) = l_P$.

2. If the signature of $S^2(M^4)$ is $(-1,-1)$ both Minkowskian and Euclidian regions have $S^2(X^4)$ with the same signature $(-1,-1)$. The radius R_D of $S^2(X^4)$ is dynamically determined.

Recall first how the cosmological constant emerges from TGD framework. The key point is that the 6-D Kähler action contains two terms.

1. The first term is essentially the ordinary Kähler action multiplied by the area of $S^2(X^4)$ which is compensated by the length scale, which can be taken to be the area $4\pi R^2(M^4)$ of $S^2(M^4)$. This makes sense for winding numbers $(w_1, w_2) = (1, 0)$ meaning that $S^2(CP_2)$ is effectively absent but $S^2(M^4)$ is present.

2. Second term is the analog of Kähler action assignable assignable to the projection of $S^2(M^4)$ Kähler form. The corresponding Kähler coupling strength $\alpha_K(M^4)$ is huge - so huge that one has $\alpha_K(M^4)4\pi R^2(M^4) \equiv L^2$, where $1/L^2$ is of the order of cosmological constant and thus of the order of the size of the recent Universe. $\alpha_K(M^4)$ is also analogous to critical temperature and the earlier hypothesis that the values of L correspond to p-adic length scales implies that the values of come as $\alpha_K(M^4) \propto p \simeq 2^k$, p prime, k prime.

   The assignment of different value of α_K to M^4 and CP_2 degrees of freedom can be criticized as ad hoc assumption. In [L38] a scenario in which the value of α_K is universal. This option has very nice properties and one can overcome the problem associated with cosmological constant by assuming that it the *entire* 4-D action corresponds to the effective cosmological constant. The cancellation between Kähler action and volume term would give rise to very small cosmological constant and also its p-adic evolution could be understood.

3. One can get an estimate for the relative magnitude of the Kähler action $S(CP_2) = \pi/8\alpha_K$ assignable to CP_2 type vacuum extremal and the corresponding cosmological term. The magnitude of the volume term is of order $1/4\pi\alpha_K(M^4)$ with $\alpha_K(M^4)$ given by $\alpha_K(M^4) = L^2/4\pi R^2(M^4)$. The sequel the magnitude of L is estimated to be $L = (2^{3/2}\pi l_P/R_D) \times R_U$, where R_U is the recent size of the Universe. This estimate follows from the identification of the volume term as cosmological constant term.

   For $R_D = R_M = l_P$ this gives $\alpha_K(M^4) = 2\pi(R_U/l_P)^2 \sim 2 \times 10^{18}$. For $\alpha_K \simeq 1/137$ the ratio of the two terms is of order 10^{-20}. The cosmological terms is completely negligible in elementary particle scales. For vacuum extremals the situation changes and the overall effect is presumably the transformation of 4-D spin glass degeneracy so that the potentials wells in the analog spin glass energy landscape do not correspond to vacuum extremal anymore and perturbation theory around them is in principle possible. The huge value of $\alpha_K(M^4)$ implies that the system corresponds mathematically to an extremely strongly interacting system so that perturbation theory fails to converge. The geometry of "world of classical worlds" (WCW) provides the needed non-perturbative approach and leads to to strong form of holography.

4. One could argue that the Kähler form assignable to M^4 cannot contribute to the action since it does not contribute to spinor connection of M^4 - an assumption that can be challenged. For canonically imbedded M^4 self-duality implies that this contribution to action vanishes. For vacuum extremals of ordinary Kähler action the contribution to the action density is proportional to the CP_2 part of induced metric and to $1/\alpha_K(M^4)$, and therefore extremely small.

   The breaking of Lorentz invariance can be seen as a possible problem for the induced spinor fields coupling to the self-dual Kähler potential. This corresponds to coupling to constant magnetic field and constant electric field, which are duals of each other. This would give rise to the analogs of cyclotron energy states in transversal directions and to the analogs of states in constant electric field in longitudinal directions. Could this extremely small effect serve as a seed for the generation of Kähler magnetic flux tubes carrying longitudinal electric fields in various scales? Note also that the value of $\alpha_K(M^4)$ is predicted to decrease as p-adic length scale so that the effect would be larger in early cosmology and in short length scales.

Hence one can consider the possibility that the action is just the sum of full 6-D Kähler actions assignable to $T(M^4)$ and $T(CP_2)$ but with different values of α_K if one has $(w_1, w_2) = (n, 0)$. Also other $w_2 \neq 0$ is possible but corresponds to gigantic cosmological constant.

Given the parameter L^2 as it is defined above, one can deduce an expression for cosmological constant Λ and show that it is positive.

1. 6-D Kähler action has dimensions of length squared and one must scale it by a dimensional constant: call it $1/L^2$. L is a fundamental scale and in dimensional reduction it gives rise to cosmological constant. Cosmological constant Λ is defined in terms of vacuum energy density as $\Lambda = 8\pi G\rho_{vac}$ can have two interpretations. Λ can correspond to a modification of Einstein-Hilbert action or - as now - to an additional term in the action for matter. In the latter case positive Λ means negative pressure explaining the observed accelerating expansion. It is actually easy to deduce the sign of Λ.

   $1/L^2$ multiplies both Kähler action - $F^{ij}F_{ij}$ ($\propto E^2 - B^2$ in Minkowskian signature). The energy density is positive. For Kähler action the sign of the multiplier must be positive so that $1/L^2$ is positive. The volume term is fiber space part of action having same form as Kähler action. It gives a positive contribution to the energy density and negative contribution to the pressure.

   In $\Lambda = 8\pi G\rho_{vac}$ one would have $\rho_{vac} = \pi/L^2R_D^2$ as integral of the $-F^{ij}F_{ij}$ over S^2 given the π/R_D^2 (no guarantee about correctness of numerical constants). This gives $\Lambda = 8\pi^2 G/L^2R_D^2$. Λ is positive and the sign is same as as required by accelerated cosmic expansion. Note that super string models predict wrong sign for Λ. Λ is also dynamical since it depends on R_D, which is dynamical. One has $1/L^2 = k\Lambda$, $k = 8\pi^2 G/R_D^2$ apart from numerical factors.

   The value of L of deduced from Euclidian and Minkowskian regions in this formal manner need not be same. Since the GRT limit of TGD describes space-time sheets with Minkowskian signature, the formula seems to be applicable only in Minkowskian regions. Again one can argue that one cannot exclude Euclidian space-time sheets of even macroscopic size and blackholes and even ordinary concept matter would represent this kind of structures.

2. L is not size scale of any fundamental geometric object. This suggests that L is analogous to α_K and has value spectrum dictated by p-adic length scale hypothesis. In fact, one can introduce the ratio of $\epsilon = R^2/L^2$ as a dimensionless parameter analogous to coupling strength what it indeed is in field equations. If so, L could have different values in Minkowskian and Euclidian regions.

3. I have earlier proposed that $R_U \equiv 1/\sqrt{1/\Lambda}$ is essentially the p-adic length scale $L_p \propto \sqrt{p} = 2^{k/2}$, $p \simeq 2^k$, $k$prime, characterizing the cosmology at given time and satisfies $R_U \propto a$ meaning that vacuum energy density is piecewise constant but on the average decreases as $1/a^2$, a cosmic time defined by light-cone proper time. A more natural hypothesis is that L satisfies this condition and in turn implies similar behavior or R_U. p-Adic length scales would be the critical values of L so that also p-adic length scale hypothesis would emerge from

quantum critical dynamics! This conforms with the hypothesis about the value spectrum of α_K labelled in the same manner [L16].

4. At GRT limit the magnetic energy of the flux tubes gives rise to an average contribution to energy momentum tensor, which effectively corresponds to negative pressure for which the expansion of the Universe accelerates. It would seem that both contributions could explain accelerating expansion. If the dynamics for Kähler action and volume term are coupled, one would expect same orders of magnitude for negative pressure and energy density - kind of equipartition of energy.

Consider first the basic scales emerging also from GRT picture. $R_U \sim \sqrt{1/\Lambda} \sim 10^{26}$ m = 10 Gly is not far from the recent size of the Universe defined as $c \times t \sim 13.8$ Gly. The derived size scale $L_1 \equiv (R_U \times l_P)^{1/2}$ is of the order of $L_1 = .5 \times 10^{-4}$ meters, the size of neuron. Perhaps this is not an accident. To make life of the reader easier I have collected the basic numbers to the following table.

$$\begin{array}{lll} m(CP_2) \simeq 5.7 \times 10^{14}\ \ GeV\ \ , & m_P = 2.435 \times 10^{18}\ \ GeV\ \ , & \frac{R(CP_2)}{l_P} \simeq 4.1 \times 10^3\ \ , \\ R_U = 10\ \ Gy\ \ , & t = 13.8\ \ Gy\ \ , & L_1 = \sqrt{l_P R_U} = .5 \times 10^{-4}\ \ m\ \ . \end{array} \tag{13.3.1}$$

Let us consider now some quantitative estimates. $R(X^4)$ depends on homotopy equivalence classes of the maps from $S^2(X^4) \rightarrow S^2(M^4)$ and $S^2(X^4) \rightarrow S^2(CP_2)$ - that is winding numbers w_i, $i = 1, 2$ for these maps. The simplest situations correspond to the winding numbers $(w_1, w_2) = (1, 0)$ and $(w_1, w_2) = (0, 1)$. For $(w_1, w_2) = (1, 0)$ M^4 contribution to the metric of $S^2(X^4)$ dominates and one has $R(X^4) \simeq R(M^4)$. For $R(M^4) = l_P$ so Planck length would define a fundamental length and Planck mass and Newton's constant would be quantal parameters. For $(w_1, w_2) = (0, 1)$ the radius of sphere would satisfy $R_D \simeq R$ (CP_2 size): now also Planck length would be quantal parameter.

Consider next additional scales emerging from TGD picture.

1. One has $L = (2^{3/2}\pi l_P/R_D) \times R_U$. In Minkowskian regions with $R_D = l_P$ this would give $L = 8.9 \times R_U$: there is no obvious interpretation for this number in recent cosmology. For $(R_D = R)$ one obtains the estimate $L = 29$ Mly. The size scale of large voids varies from about 36 Mly to 450 Mly (see `http://tinyurl.com/jyqcjhl`).

2. Consider next the derived size scale $L_2 = (L \times l_P)^{1/2} = \sqrt{L/R_U} \times L_1 = \sqrt{2^{3/2}\pi l_P/R_D} \times L_1$. For $R_D = l_P$ one has $L_2 \simeq 3L_1$. For $R_D = R$ making sense in Euclidian regions, this is of the order of size of neutrino Compton length: 3 μm, the size of cellular nucleus and rather near to the p-adic length scale $L(167) = 2.6$ m, corresponds to the largest miracle Gaussian Mersennes associated with $k = 151, 157, 163, 167$ defining length scales in the range between cell membrane thickness and the size of cellular nucleus. Perhaps these are co-incidences are not accidental. Biology is something so fundamental that fundamental length scale of biology should appear in the fundamental physics.

The formulas and predictions for different options are summarized by the following table.

$$\begin{array}{lll} Option & L = \frac{2^{3/2}\pi l_P}{R_D} \times R_U & L_2 = \sqrt{L l_P} = \sqrt{\frac{2^{3/2}\pi l_P}{R_D}} \times L_1 \\ R_D = R\ \ , & 29\ \ Mly\ \ , & \simeq 3\ \ \mu m\ \ , \\ R_D = l_P\ \ , & 8.9 R_U\ \ , & \simeq 3L_1 = 1.5 \times 10^{-4}\ \ m\ \ , \end{array} \tag{13.3.2}$$

In the case of M^4 the radius of S^2 cannot be fixed it remains unclear whether Planck length scale is fundamental constant or whether it emerges.

13.3.2 Estimate for the cosmic evolution of R_D

One can actually get estimate for the evolution of R_D as function of cosmic time if one accepts Friedman cosmology as an approximation of TGD cosmology.

1. Assume critical mass density so that one has

$$\rho_{cr} = \frac{3H^2}{8\pi G} \ .$$

2. Assume that the contribution of cosmological constant term to the mass mass density dominates. This gives $\rho \simeq \rho_{vac} = \Lambda/8\pi G$. From $\rho_{cr} = \rho_{vac}$ one obtains

$$\Lambda = 3H^2 \ .$$

3. From Friedman equations one has $H^2 = ((da/dt)/a)^2$, where a corresponds to light-cone proper time and t to cosmic time defined as proper time along geodesic lines of space-time surface approximated as Friedmann cosmology. One has

$$\Lambda = \frac{3}{g_{aa}a^2}$$

in Robertson-Walker cosmology with $ds^2 = g_{aa}da^2 - a^2 d\sigma_3^2$.

4. Combining this equations with the TGD based equation

$$\Lambda = \frac{8\pi^2 G}{L^2 R_D^2}$$

one obtains

$$\frac{8\pi^2 G}{L^2 R_D^2} = \frac{3}{g_{aa}a^2} \ . \tag{13.3.3}$$

5. Assume that quantum criticality applies so that L has spectrum given by p-adic length scale hypothesis so that one discrete p-adic length scale evolution for the values of L. There are two options to consider depending on whether p-adic length scales are assigned with light-cone proper time a or with cosmic time t

$$T = a \ \ (Option \ \ I) \ \ , \ \ \ T = t \ \ (Option \ \ II) \tag{13.3.4}$$

Both options give the same general formula for the p-adic evolution of $L(k)$ but with different interpretation of $T(k)$.

$$\frac{L(k)}{L_{now}} = \frac{T(k)}{T_{now}} \ \ , \ \ \ T(k) = L(k) = 2^{(k-151)/2} \times L(151) \ \ , \ \ \ L(151) \simeq 10 \ \ nm \ \ . \tag{13.3.5}$$

Here $T(k)$ is assumed to correspond to primary p-adic length scale. An alternative - less plausible - option is that $T(k)$ corresponds to secondary p-adic length scale $L_2(k) = 2^{k/2}L(k)$ so that $T(k)$ would correspond to the size scale of causal diamond. In any case one has $L \propto L(k)$. One has a discretized version of smooth evolution

$$L(a) = L_{now} \times \frac{T}{T_{now}} \ . \tag{13.3.6}$$

6. Feeding into this to Eq. 13.3.3 one obtains an expression for $R_D(a)$

$$\frac{R_D}{l_P} = (\frac{8}{3})^{1/2}\pi \times \frac{a}{L(a)} \times g_{aa}^{1/2} \quad . \tag{13.3.7}$$

Unless the dependences on cosmic time compensate each other, R_D is dynamical and becomes very small at very early times since g_{aa} becomes very small. $R(M^4) = l_P$ however poses a lower boundary since either of the maps $S^2(X^4) \to S^2(M^4)$ and $S^2(X^4) \to S^2(CP_2)$ must be homotopically non-trivial. For $R(M^4) = l_P$ one would obtain $R_D/l_P = 1$ at this limit giving also lower bound for g_{aa}. For $T = t$ option $a/L(a)$ becomes large and g_{aa} small.

As a matter of fact, in very early cosmic string dominated cosmology g_{aa} would be extremely small constant [K67]. In late cosmology $g_{aa} \to 1$ holds true and one obtains at this limit

$$\frac{R_D(now)}{l_P} = (\frac{8}{3})^{1/2}\pi \times \frac{a_{now}}{L_{now}} \times l_P \simeq 4.4\frac{a_{now}}{L_{now}} \quad . \tag{13.3.8}$$

7. For $T = t$ option R_D/l_P remains constant during both matter dominated cosmology, radiation dominated cosmology, and string dominated cosmology since one has $a \propto t^n$ with $n = 1/2$ during radiation dominated era, $n = 2/3$ during matter dominated era, and $n = 1$ during string dominated era [K67]. This gives

$$\frac{R_D}{l_P} = (\frac{8}{3})^{1/2}\pi \times \frac{a}{t}\sqrt{g_{aa}}\frac{t(end)}{L(end)} = (\frac{8}{3})^{1/2}\frac{\pi}{n}\frac{t(end)}{L(end)} \quad .$$

Here "end" refers the end of the string or radiation dominated period or to the recent time in the case of matter dominated era. The value of n would have evolved as $R_D/l_P \propto (1/n)(t_{end}/L_{end})$, $n \in \{1, 3/2, 2\}$. During radiation dominated cosmology $R_D \propto a^{1/2}$ holds true. The value of R_D would be very nearly equal to $R(M^4)$ and $R(M^4)$ would be of the same order of magnitude as Planck length. In matter dominated cosmology would would have $R_D \simeq 2.2(t(now)/L(now)) \times l_P$.

8. For $R_D(now) = l_P$ one would have

$$\frac{L_{now}}{a_{now}} = (\frac{8}{3})^{1/2}\pi \simeq 4.4 \quad .$$

In matter dominated cosmology $g_{aa} = 1$ gives $t_{now} = (2/3) \times a_{now}$ so that predictions differ only by this factor for options I and II. The winding number for the map $S^2(X^4) \to S^2(CP_2)$ must clearly vanish since otherwise the radius would be of order R.

9. For $R_D(now) = R$ one would obtain

$$\frac{a_{now}}{L_{now}} = (\frac{8}{3})^{1/2} \times \frac{R}{l_P} \simeq 2.1 \times 10^4 \quad .$$

One has $L_{now} = 10^6$ ly: this is roughly the average distance scale between galaxies. The size of Milky Way is in the range $1 - 1.8 \times 10^5$ ly and of an order of magnitude smaller.

10. An interesting possibility is that $R_D(a)$ evolves from $R_D \sim R(M^4) \sim l_P$ to $R_D \sim R$. This could happen if the winding number pair $(w_1, w_2) = (1, 0)$ transforms to $(w_1, w_2) = (0, 1)$ during transition to from radiation (string) dominance to matter (radiation) dominance. R_D/l_P radiation dominated cosmology would be related by a factor

$$\frac{R_D(rad)}{R_D(mat)} = (3/4)\frac{t(rad, end)}{L(rad, end)} \times \frac{L(now)}{t(now)}$$

to that in matter dominated cosmology. Similar factor would relate the values of R_D/l_P in string dominated and radiation dominated cosmologies. The condition $R_D(rad)/R_D(mat) = l_P/R$ expressing the transformation of winding numbers would give

$$\frac{L(now)}{L(rad,end)} = \frac{4}{3}\frac{l_P}{R}\frac{t(now)}{t(rad,end)} \quad .$$

One has $t(now)/t(rad,end) \simeq .5 \times 10^6$ and $l_P/R = 2.5 \times 10^{-4}$ giving $L(now)/L(rad,end) \simeq 125$, which happens to be near fine structure constant.

11. For the twistor lifts of space-time surfaces for which cosmological constant has a reasonable value , the winding numbers are equal to $(w_1, w_2) = (n, 0)$ so that $R_D = \sqrt{n}R(S^2(M^4))$ holds true in good approximation. This conforms with the observed constancy of R_D during various cosmological eras, and would suggest that the ratio $\frac{t(end)}{L(end)}$ characterizing these periods is same for all periods. This determines the evolution for the values of $\alpha_K(M^4)$.

$R(M^4) \sim l_P$ seems rather plausible option so that Planck length would be fundamental classical length scale emerging naturally in twistor approach. Cosmological constant would be coupling constant like parameter with a spectrum of critical values given by p-adic length scales.

13.3.3 What about the extremals of the dimensionally reduced 6-D Kähler action?

It seems that the basic wisdom about extremals of Kähler action remains unaffected and the motivations for WCW are not lost in the case that M^4 Kähler form does not contribute to 6-D Kähler action (the case to be considered below): otherwise the predicted effects are extremely small in the recent Universe. What is new is that the removal of vacuum degeneracy is forced by twistorial action.

1. All extremals, which are minimal surfaces remain extremals. In fact, all the known extremals except vacuum extremals. For minimal surfaces the dynamics of the volume term and 4-D Kähler action separate and field equations for them are separately satisfied. The vacuum degeneracy motivating the introduction of WCW is preserved. The induced Kähler form vanishes for vacuum extremals and the imaginary unit of twistor space is ill-defined. Hence vacuum extremals cannot belong to WCW. This correspond to the vanishing of WCW metric for vacuum extremals.

2. For non-minimal surfaces Kähler coupling strength does not disappear from the field equations and appears as a genuine coupling very much like in classical field theories. Minimal surface equations are a generalization of wave equation and Kähler action would define analogs of source terms. Field equations would state that the total isometry currents are conserved. It is not clear whether other than minimal surfaces are possible, I have even conjectured that all preferred extremals are always minimal surfaces having the property that being holomorphic they are almost universal extremals for general coordinate invariant actions.

3. Thermodynamical analogy might help in the attempts to interpret. Quantum TGD in zero energy ontology (ZEO) corresponds formally to a complex square root of thermodynamics. Kähler action can be identified as a complexified analog of free energy. Complexification follows both from the fact that $\sqrt{g}$ is real/imaginary in Euclidian/Minkowskian space-time regions. Complex values are also implied by the proposed identification of the values of Kähler coupling strength in terms of zeros and pole of Riemann zeta in turn identifiable as poles of the so called fermionic zeta defining number theoretic partition function for fermions [K111] [L16, L18]. The thermodynamical for Kähler action with volume term is Gibbs free energy $G = F - TS = E - TS + PV$ playing key role in chemistry.

4. The boundary conditions at the ends of space-time surfaces at boundaries of CD generalize appropriately and symmetries of WCW remain as such. At light-like boundaries between

Minkowskian and Euclidian regions boundary conditions must be generalized. In Minkowkian regions volume can be very large but only the Euclidian regions contribute to Kähler function so that vacuum functional can be non-vanishing for arbitrarily large space-time surfaces since exponent of Minkowskian Kähler action is a phase factor.

5. One can worry about almost topological QFT property. Although Kähler action from Minkowskian regions at least would reduce to Chern-Simons terms with rather general assumptions about preferred extremals, the extremely small cosmological term does not. Could one say that cosmological constant term is responsible for "almost"?

   It is interesting that the volume of manifold serves in algebraic geometry as topological invariant for hyperbolic manifolds, which look locally like hyperbolic spaces $H_n = SO(n,1)/SO(n)$ [A31] [K90]. See also the article "Volumes of hyperbolic manifolds and mixed Tate motives" (see `http://tinyurl.com/yargy3uw`). Now one would have $n = 4$. It is probably too much to hope that space-time surfaces would be hyperbolic manifolds. In any case, by the extreme uniqueness of the preferred extremal property expressed by strong form of holography the volume of space-time surface could also now serve as topological invariant in some sense as I have earlier proposed. What is intriguing is that AdS_n appearing in AdS/CFT correspondence is Lorentzian analogue H_n.

6. $\alpha(M^4)$ is extremely large so that there is no hope of quantum perturbation theory around canonically imbedded M^4 although the propagator for CP_2 coordinate exists. In the new framework WCW can be seen as a solution to how to construct non-perturbative quantum TGD.

To sum up, I have the feeling that the final formulation of TGD has now emerged and it is clear that TGD is indeed a quantum theory of gravitation allowing to understand standard model symmetries. The existence of twistorial formulation is all that is needed to fix the theory completely. It makes possible gravitation and predicts standard model symmetries. This cannot be said about any competitor of TGD.

13.4 Basic Principles Behind Construction of Amplitudes

Basic principles of the construction summarized in this section could be seen as axioms trying to abstract the essentials. The explicit construction of amplitudes is too heavy challenge at this stage and at least for me.

13.4.1 Imbedding space is twistorially unique

It took roughly 36 years to learn that M^4 and CP_2 are twistorially unique.

1. As already explained, M^4 and CP_2 are unique 4-manifolds in the sense that both allow twistor space with Kähler structure: Kähler structure is the crucial concept as one might guess from the fact that the projection of Kähler form naturally defines the preferred quaternionic imaginary unit defining the twistor structure for space-time surface. Both M^4 and its Euclidian variant E^4 allow twistor space. The first guess is that the twistor space of M^4 is Minkowskian variant $T(M^4) = SU(2,2)/SU(2,1)\times U(1)$ of 6-D twistor space $CP_3 = SU(4)/SU(3)\times U(1)$ of E^4. This is sensible assumption at the level of momentum space but the second candidate, which is simply $T(M^4) = M^4 \times CP_1$, is the only sensible option at space-time level. The twistor space of CP_2 is 6-D $T(CP_2) = SU(3)/U(1) \times U(1)$, the space for the choices of quantization axes of color hypercharge and isospin.

2. This leads to a proposal for the formulation of TGD in which space-time surfaces X^4 in H are lifted to twistor spaces X^6, which are sphere bundles over X^4 and such that they are surfaces in 12-D product space $T(M^4)\times T(CP_2)$ such the twistor structure of X^4 are in some sense induced from that of $T(M^4) \times T(CP_2)$.

   What is nice in this formulation is that one might be able to use all the machinery of algebraic geometry so powerful in superstring theory (Calabi-Yau manifolds) provided one can generalize the notion of Kähler structure from Euclidian to Minkowskian signature. It has been already described how this approach leads to a profound understanding of the relationship between TGD and GRT. Planck length emerges whereas fundamental constant as also cosmological constant emerges dynamically from the length scale parameter appearing in 6-D Kähler action. One can say, that twistor extension is absolutely essential for really understanding the gravitational interactions although the modification of Kähler action is extremely small due to the huge value of length scale defined by cosmological constant.

3. Masslessness (masslessness in complex sense for virtual particles in twistorialization) is essential condition for twistorialization. In TGD massless is masslessness in 8-D sense for the representations of superconformal algebras. This suggests that 8-D variant of twistors makes sense. 8-dimensionality indeed allows octonionic structure in the tangent space of imbedding space. One can also define octonionic gamma matrices and this allows a possible generalization of 4-D twistors to 8-D ones using generalization of sigma matrices representing quaternionic units to to octonionic sigma "matrices" essential for the notion of twistors. These octonion units do not of course allow matrix representation unless one restricts to units in some quaternionic subspace of octonions. Space-time surfaces would be associative and thus have quaternionic tangent space at each point satisfying some additional conditions.

13.4.2 Strong form of holography

Strong form of holography (SH) following from general coordinate invariance (GCI) for space-times as surfaces states that the data assignable to string world sheets and partonic 2-surfaces allows to code for scattering amplitudes. The boundaries of string world sheets at the space-like 3-surfaces defining the ends of space-time surfaces at boundaries of causal diamonds (CDs) and the fermionic lines along light-like orbits of partonic 2-surfaces representing lines of generalized Feynman diagrams become the basic elements in the generalization of twistor diagrams (I will not use the attribute "Feynman" in precise sense, one could replace it with "twistor" or even drop away). One can assign fermionic lines massless in 8-D sense to flux tubes, which can also be braided. One obtains a fractal hierarchy of braids with strands, which are braids themselves. At the lowest level one has braids for which fermionic lines are braided. This fractal hierarchy is unavoidable and means generalization of the ordinary Feynman diagram. I have considered some implications of this hierarchy in [L17].

The precise formulation of strong form of holography (SH) is one of the technical problems in TGD. A comment in FB page of Gareth Lee Meredith led to the observation that besides the purely number theoretical formulation based on commutativity also a symplectic formulation in the spirit of non-commutativity of imbedding space coordinates can be considered. One can however use only the notion of Lagrangian manifold and avoids making coordinates operators leading to a loss of General Coordinate Invariance (GCI).

13.4.3 The existence of WCW demands maximal symmetries

Quantum TGD reduces to the construction of Kähler geometry of infinite-D "world of classical worlds" (WCW), of associated spinor structure, and of modes of WCW spinor fields which are purely classical entities and quantum jump remains the only genuinely quantal element of quantum TGD. Quantization without quantization, would Wheeler say.

By its infinite-dimensionality, the mere mathematical existence of the Kähler geometry of WCW requires maximal isometries. Physics is completely fixed by the mere condition that its mathematical description exists. Super-symplectic and other symmetries of "world of classical worlds" (WCW) are in decisive role. These symmetry algebras have conformal structure and generalize and extend the conformal symmetries of string models (Kac-Moody algebras in particular). These symmetries give also rise to the hierarchy of Planck constants. The super-symplectic symmetries extend to a Yangian algebra, whose generators are polylocal in the sense that they involve products of generators associated with different partonic surfaces. These symmetries leave scattering amplitudes invariant. This is an immensely powerful constraint, which remains to be understood.

13.4.4 Quantum criticality

Quantum criticality (QC) of TGD Universe is a further principle. QC implies that Kähler coupling strength is mathematically analogous to critical temperature and has a discrete spectrum. Coupling constant evolution is replaced with a discrete evolution as function of p-adic length scale: sequence of jumps from criticality to a more refined criticality or vice versa (in spin glass energy landscape you at bottom of well containing smaller wells and you go to the bottom of smaller well). This implies that either all radiative corrections (loops) sum up to zero (QFT limit) or that diagrams containing loops correspond to the same scattering amplitude as tree diagrams so that loops can eliminated by transforming them to arbitrary small ones and snipping away moving the end points of internal lines along the lines of diagram (fundamental description).

Quantum criticality at the level of super-conformal symmetries leads to the hierarchy of Planck constants $h_{eff} = n \times h$ labelling a hierarchy of sub-algebras of super-symplectic and other conformal algebras isomorphic to the full algebra. Physical interpretation is in terms of dark matter hierarchy. One has conformal symmetry breaking without conformal symmetry breaking as Wheeler would put it.

13.4.5 Physics as generalized number theory, number theoretical universality

Physics as generalized number theory vision has important implications. Adelic physics is one of them. Adelic physics implied by number theoretic universality (NTU) requires that physics in real and various p-adic numbers fields and their extensions can be obtained from the physics in their intersection corresponding to an extension of rationals. This is also enormously powerful condition and the success of p-adic length scale hypothesis and p-adic mass calculations can be understood in the adelic context.

In TGD inspired theory of consciousness various p-adic physics serve as correlates of cognition and p-adic space-time sheets can be seen as cognitive representations, “thought bubbles”. NTU is closely related to SH. String world sheets and partonic 2-surfaces with parameters (WCW coordinates) characterizing them in the intersection of rationals can be continued to space-time surfaces by preferred extremal property but not always. In p-adic context the fact that p-adic integration constants depend on finite number of pinary digits makes the continuation easy but in real context this need not be possible always. It is always possible to imagine something but not always actualize it!

13.4.6 Scattering diagrams as computations

Quantum criticality as possibility to eliminate loops has a number theoretic interpretation. Generalized Feynman diagram can be interpreted as a representation of a computation connecting given set X of algebraic objects to second set Y of them (initial and final states in scattering) (trivial example: $X = \{3, 4\} \to 3 \times 4 = 12 \to 2 \times 6 \to \{2, 6\} = Y$. The 3-vertices ($a \times b = c$) and their time-reversals represent algebraic product and co-product.

There is a huge symmetry: all diagrams representing computation connecting given X and Y must produce the same amplitude and there must exist minimal computation. This generalization of string model duality implies an infinite number of dualities unless the finite size of CD allows only a finite number of equivalent computations. These dualities are analogous to the dualities of super-string model, in particular mirror symmetry stating that same quantum physical situation does not correspond to a unique space-time geometry and topology (Calabi-Yau and its mirror represent the same situation). The task of finding this computation is like finding the simplest representation for the formula X=Y and the noble purpose of math teachers is that we should learn to find it during our school days. This generalizes the duality symmetry of old fashioned string models: one can transform any diagram to a tree diagram without loops. This corresponds to quantum criticality in TGD: coupling constants do not evolve. The evolution is actually there but discrete and corresponds to infinite number critical values for Kahler coupling strength analogous to temperature.

13.4.7 Reduction of diagrams with loops to braided tree-diagrams

1. In TGD pointlike particles are replaced with 3-surfaces and by SH by partonic 2-surfaces. The important implication of 3-dimensionality is braiding. The fermionic lines inside light-like orbits of partonic 2-surfaces can be knotted and linked - that is braided (this is dynamical braiding analogous to dance). Also the fermionic strings connecting partonic 2-surfaces at space-like 3-surfaces at boundaries of causal diamonds (CDs) are braided (space-like braiding).

   Therefore ordinary Feynman diagrams are not enough and one must allow braiding for tree diagrams. One can also imagine of starting from braids and allowing 3-vertices for their strands (product and co-product above). It is difficult to imagine what this braiding could mean. It is better to imagine braid and allow the strands to fuse and split (annihilation and pair creation vertices).

2. This braiding gives rise in the planar projection representation of braids to a generalization of non-planar Feynman diagrams. Non-planar diagrams are the basic unsolved problem of twistor approach and have prevented its development to a full theory allowing to construct exact expressions for the full scattering amplitudes (I remember however that Nima Arkani-Hamed et al have conjectured that non-planar amplitudes could be constructed by some procedure: they notice the role of permutation group and talk also about braidings (describable using covering groups of permutation groups)). In TGD framework the non-planar Feynman diagrams correspond to non-trivial braids for which the projection of braid to plane has crossing lines, say a and b, and one must decide whether the line a goes over b or vice versa.

3. An interesting open question is whether one must sum over all braidings or whether one can choose only single braiding. Choice of single braiding might be possible and reflect the failure of string determinism for Kähler action and it would be favored by TGD as almost topological quantum field theory (TQFT) vision in which Kähler action for preferred extremal is topological invariant.

13.4.8 Scattering amplitudes as generalized braid invariants

The last big idea is the reduction of quantum TGD to generalized knot/braid theory (I have talked also about TGD as almost TQFT). The scattering amplitude can be identified as a generalized braid invariant and could be constructed by the generalization of the recursive procedure transforming in a step-by-step manner given braided tree diagram to a non-braided tree diagram: essentially what Alexander the Great did for Gordian knot but tying the pieces together after cutting. At each step one must express amplitude as superposition of amplitudes associated with the different outcomes of splitting followed by reconnection. This procedure transforms braided tree diagram to a non-braided tree diagrams and the outcome is the scattering amplitude!

13.5 Tensor Networks and S-matrices

The concrete construction of scattering amplitudes has been the toughest challenge of TGD and the slow progress has occurred by identification of general principles with many side tracks. One of the key problems has been unitarity. The intuitive expectation is that unitarity should reduce to a local notion somewhat like classical field equations reduce the time evolution to a local variational principle. The presence of propagators have been however the obstacle for locally realized unitarity in which each vertex would correspond to unitary map in some sense.

TGD suggests two approaches to the construction of S-matrix.

1. The first approach is generalization of twistor program [K76]. What is new is that one does not sum over diagrams but there is a large number of equivalent diagrams giving the same outcome. The complexity of the scattering amplitude is characterized by the minimal diagram. Diagrams correspond to space-time surfaces so that several space-time surfaces give rise to the same scattering amplitude. This would correspond to the fact that the dynamics

breaks classical determinism. Also quantum criticality is expected to be accompanied by quantum critical fluctuations breaking classical determinism. The strong form of holography would not be unique: there would be several space-time surfaces assignable as preferred extremals to given string world sheets and partonic 2-surfaces defining "space-time genes".

2. Second approach relies on the number theoretic vision and interprets scattering amplitudes as representations for computations with each 3-vertex identifiable as a basic algebraic operation [K76]. There is an infinite number of equivalent computations connecting the set of initial algebraic objects to the set of final algebraic objects. There is a huge symmetry involved: one can eliminate all loops moving the end of line so that it transforms to a vacuum tadpole and can be snipped away. A braided tree diagram is left with braiding meaning that the fermion lines inside the line defined by light-like orbit are braided. This kind of braiding can occur also for space-like fermion lines inside magnetic flux tubes and defining correlate for entanglement. Braiding is the TGD counterpart for the problematic non-planarity in twistor approach.

Third approach involving local unitary as an additional key element is suggested by tensor networks relying on the notion of perfect entanglement discussed by Preskill et al [B43].

1. Tensor networks provide an elegant representation of holography mapping interior states isometrically (in Hilbert space sense) to boundary states or vice versa for selected subsets of states defining the code subspace for holographic quantum error correcting code. Again the tensor net is highly non-unique but there is some minimal tensor net characterizing the complexity of the entangled boundary state.

2. Tensor networks have two key properties, which might be abstracted and applied to the construction of S-matrix in zero energy ontology (ZEO): perfect tensors define isometry for any subspace defined by the index subset of perfect tensor to its complement and the non-unique graph representing the network. As far as the construction of Hilbert space isometry between local interior states and highly non-local entangled boundary states is considered, these properties are enough.

One cannot avoid the question whether these three constructions could be different aspects of one and same construction and that tensor net construction with perfect tensors representing vertices could provide and additional strong constraint to the long sought for explicit recipe for the construction of scattering amplitudes.

13.5.1 Objections

It is certainly clear from the beginning that the possibly existing description of S-matrix in terms of tensor networks cannot correspond to the perturbative QFT description in terms of Feynman diagrams.

1. Tensor network description relates interior and boundary degrees in holography by a isometry. Now however unitary matrix has quite different role. It could correspond to U-matrix relating zero energy states to each other or to the S-matrix relating to each other the states at boundary of CD and at the shifted boundary obtained by scaling. These scalings shifting the second boundary of CD and increasing the distance between the tips of CD define the analog of unitary time evolution in ZEO. The U-matrix for transitions associated with the state function reductions at fixed boundary of CD effectively reduces to S-matrix since the other boundary of CD is not affected.

   The only manner one could see this as holography type description would be in terms of ZEO in which zero energy states are at boundaries of CD and U-matrix is a representation for them in terms of holography involving the interior states representing scattering diagram in generalized sense.

2. The appearance of small gauge coupling constant tells that the entanglement between "states" in state spaces whose coordinates formally correspond to quantum fields is weak and just

opposite to that defined by a perfect tensor. Quite generally, coupling constant might be the fatal aspect of the vertices preventing the formulation in terms of perfect entanglement.

One should understand how coupling constant emerges from this kind of description - or disappears from standard QFT description. One can think of including the coupling constant to the definition of gauge potentails: in TGD framework this is indeed true for induced gauge fields. There is no sensical manner to bring in the classical coupling constants in the classical framework and the inverse of Kähler coupling strength appears only as multiplier of the Kähler action analogous to critical temperature.

More concretely, there are WCW spin degrees of freedom (fermionic degrees of freedom) and WCW orbital degrees of freedom involving functional integral over WCW. Fermionic contribution would not involve coupling constants whereas the functional integral over WCW involving exponential of vacuum functional could give rise to the coupling constants assignable to the vertices in the minimal tree diagram.

3. The decomposition $S = 1 + iT$ of unitary S-matrix giving unitarity as the condition $-i(T - T^\dagger) + T^\dagger T = 0$ reflects the perturbative thinking. If one has only isometry instead of unitary transformation, this decomposition becomes problematic since T and $T^\dagger$ whose some appears in the formula act in different spaces. One should have the generalization of Id as a "trivial" isometry. Alternatively, one should be able to extend the state space H_{in} by adding a tensor factor mapped trivially in isometry.

4. There are 3- and 4-vertices rather than only -say, 3-vertices as in tensor networks. For non-Abelian Chern-Simons term for simple Lie group one would have besides kinetic term only 3-vertex $Tr(A \wedge A \wedge A)$ defining the analog of perfect tensor entanglement when interpreted as co-product involving 3-D permutation symbol and structure constants of Lie algebra. Note also that for twistor Grassmannian approach the fundamental vertices are 3-vertices. It must be however emphasized that QFT description emerges from TGD only at the limit when one identifies gauge potentials as sums of induced gauge potentials assignable to the space-time sheets, which are replaced with single piece of Minkowski space.

5. Tensor network description does not contain propagators since the contractions are between perfect tensors. It is to make sense propagators must be eliminated. The twistorial factorization of massless fermion propagator suggest that this might be possible by absorbing the twistors to the vertices.

These reasons make it clear that the proposed idea is just a speculative question. Perhaps the best strategy is to look this crazy idea from different view points: the overly optimistic view developing big picture and the approach trying to debunk the idea.

13.5.2 The overly optimistic vision

With these prerequisites on one can follow the optimistic strategy and ask how tensor networks could the allow to generalize the notion of unitary S-matrix in TGD framework.

1. Tensor networks suggests the replacement of unitary correspondence with the more general notion of Hilbert space isometry. This generalization is very natural in TGD since one must allow phase transitions increasing the state space and it is quite possible that S-matrix represents only isometry: this would mean that $S^\dagger S = Id_{in}$ holds true but $SS^\dagger = Id_{out}$ does not even make sense. This conforms with the idea that state function reduction sequences at fixed boundary of causal diamonds defining conscious entities give rise evolution implying that the size of the state space increases gradually as the system becomes more complex. Note that this gives rise to irreversibility understandandable in terms of NMP [K41]. It might be even impossible to formally restore unitary by introducing formal additional tensor factor to the space of incoming states if the isometric map of the incoming state space to outgoing state space is inclusion of hyperfinite factors.

2. If the huge generalization of the duality of old fashioned string models makes sense, the minimal diagram representing scattering is expected to be a tree diagram with braiding

and should allow a representation as a tensor network. The generalization of the tensor network concept to include braiding is trivial in principle: assign to the legs connecting the nodes defined by perfect tensors unitary matrices representing the braiding - here topological QFT allows realization of the unitary matrix. Besides fermionic degrees of freedom having interpretation as spin degrees of freedom at the level of "World of Classical Worlds" (WCW) there are also WCW orbital degrees of freedom. These two degrees of freedom factorize in the generalized unitarity conditions and the description seems much simpler in WCW orbital degrees of freedom than in WCW spin degrees of freedom.

3. Concerning the concrete construction there are two levels involved, which are analogous to descriptions in terms of boundary and interior degrees of freedom in holography. The level of fundamental fermions assignable to string world sheets and their boundaries and the level of physical particles with particles assigned to sets of partonic 2-surface connected by magnetic flux tubes and associated fermionic strings. One could also see the ends of causal diamonds as analogous to boundary degrees of freedom and the space-time surface as interior degrees of freedom.

The description at the level of fundamental fermions corresponds to conformal field theory at string world sheets.

1. The construction of the analogs of boundary states reduces to the construction of N-point functions for fundamental fermions assignable to the boundaries of string world sheets. These boundaries reside at 3-surfaces at the space-like space-time ends at CDs and at light-like 3-surfaces at which the signature of the induced space-time metric changes.

2. In accordance with holography, the fermionic N-point functions with points at partonic 2-surfaces at the ends of CD are those assignable to a conformal field theory associated with the union of string world sheets involved. The perfect tensor is assignable to the fundamental 4-fermion scattering which defines the microscopy for the geometric 3-particle vertices having twistorial interpretation and also interpretation as algebraic operation.

   What is important is that fundamental fermion modes at string world sheets are labelled by conformal weights and standard model quantum numbers. No four-momenta nor color quantum numbers are involved at this level. Instead of propagator one has just unitary matrix describing the braiding.

3. Note that four-momenta emerging in somewhat mysterious manner to stringy scattering amplitudes and mean the possibility to interpret the amplitudes at the particle level.

Twistorial and number theoretic constructions should correspond to particle level construction and also now tensor network description might work.

1. The 3-surfaces are labelled by four-momenta besides other standard model quantum numbers but the possibility of reducing diagram to that involving only 3-vertices means that momentum degrees of freedom effectively disappear. In ordinary twistor approach this would mean allowance of only forward scattering unless one allows massless but complex virtual momenta in twistor diagrams. Also vertices with larger number of legs are possible by organizing large blocks of vertices to single effective vertex and would allow descriptions analogous to effective QFTs.

2. It is highly non-trivial that the crucial factorization to perfect tensors at 3-vertices with unitary braiding matrices associated with legs connecting them occurs also now. It allows to split the inverses of fermion propagators into sum of products of two parts and absorb the halves to the perfect tensors at the ends of the line. The reason is that the inverse of massless fermion propagator (also when masslessness is understood in 8-D sense allowing M^4 mass to be non-vanishing) to be express as bilinear of the bi-spinors defining the twistor representing the four-momentum. It seems that this is absolutely crucial property and fails for massive (in 8-D sense) fermions.

13.5.3 Twistorial and number theoretic visions

Both twistorial and number theoretical ideas have given a strong boost to the development of ideas.

1. With experience coming from twistor Grassmannian approach, twistor approach is conjectured to allow an extension of super-symplectic and other superconformal symmetry algebras to Yangian algebras by adding a hierarchy of multilocal generators [K76]. The twistorial diagrams for $\mathcal{N} = 4$ SUSY can be reduced to a finite number and there is large number of equivalent diagrams. One expects that this is true also in TGD framework.

   Twistorial approach is extremely general and quite too demanding to my technical skills but its is a useful guideline. An important outcome of twistor approach is that the intermediate states are massless on-mass-shell states but with complex momenta. Does this generalize and could each vertex define unitary scattering event with complex four-momenta in possibly complexified Minkowski space? Or could even real momenta be possible for massive particles, which would be massless in 8-D sense thanks to the existence of octonionic tangent space structure of 8-D imbedding space? And what is the role of the unique twistorial properties of M^4 and CP_2?

2. Number theoretical vision suggests that the scattering amplitudes correspond to sequences of algebraic operations taking inputs and producing outputs, which in turn serve as inputs for a neighboring node [K76]. The vertices form a diagram defining a network like structure defining kind of distributed computations leading from given inputs to given outputs. A computation leading from given inputs to given outputs is suggestive. There exists an infinite number of this kind of computations and there must be the minimal one which defines the complexity of the scattering. The maximally simplifying guess is that this diagram would correspond to a braided tree diagram. At space-time level these diagrams would correspond to different space-time surfaces defining same physics: this is because of holography meaning that only the ends of space-time surfaces at boundaries of CD matter.

   This vision generalizes of the old-fashioned stringy duality. It states that all diagrams can be reduced to minimal diagrams. This is achieved by by moving the ends of internal lines so that loops becomes vacuum tadpoles and can be snipped off. Tree diagrams must be however allowed to braid and outside the vertices the diagrams look like braids. Braids for which threads can split and glue together is the proper description for what the diagrams could be. Braiding would provide the counterpart for the non-planar twistor diagrams.

   The fermion lines inside the light-like 3-surfaces can get braided. Smaller partonic 2-surfaces can topologically condense at given bigger partonic 2-surface (electronic parton surface can topologically condense to nano-scopic parton surface) and the orbits of the condensed partonic 2-surfaces at the light-like orbit of the parton surface can get braided. This gives rise to a hierarchy of braids with braids.

13.5.4 Generalization of the notion of unitarity

The understanding of unitarity has been the most difficult issue in my attempts to understand S-matrix in TGD framework. When something turns out to be very difficult to understand, it might make sense to ask whether the definition of this something involves un-necessary assumptions. Could unitarity be this kind of notion?

The notion of tensor network suggests that unitarity can generalized and that this generalization allows the realization of unitarity in extremely simple manner using perfect tensors as building bricks of diagrams.

1. Both twistorial and number theoretical approaches define M-matrix and associated S-matrix as a map between the state spaces H_{in} and H_{out} assignable to the opposite boundaries of CD - say positive and negative energy parts of zero energy state. In QFT one has $H_{in} = H_{out}$ and the map would be Hilbert space unitary transformation satisfying $SS^\dagger = S^\dagger S = Id$.

2. The basic structure of TGD (NMP favoring generation of negentropic entanglement, the hierarchy of Planck constants, length scale hierarchies, and hierarchy of space-time sheets)

suggests that the time evolution leads to an increasingly complex systems with higher-dimensional Hilbert space so that $H_{in} = H_{out}$ need not hold true but is replaced with $H_{in} \subset H_{out}$. This view is very natural since one must allow quantum phase transitions increasing the value of h_{eff} and the value of p-adic prime defining p-adic length scale.

S-matrix would thus define isometric map $H_{in} \subset H_{out}$. Isometry property requires $U^{\dagger}U = Id_{in}$. If the inclusion of H_{in} to H_{out} is a genuine subspace of H_{out}, the condition $UU^{\dagger} = Id_{out}$ does not make sense anymore. This means breaking of reversibility and is indeed implied by the quantum measurement theory based on ZEO.

3. It would be at least formally possible to fuse all state spaces to single very large state space by replacing isometry $H_{in} \subset H_{out}$ with unitary map $H_{out} \rightarrow H_{out}$ by adding a tensor factor in which the map acts as identity transformation. This is not practical since huge amounts of redundant information would be introduced. Also the information about hierarchical structure essential for the idea of evolution would be lost. This hierarchical of inclusions should also be crucial for understanding the construction of S-matrix or rather, the hierarchy of S-matrices of isometric inclusions including as a special case unitary S-matrices.

4. There is also a further intricacy, which might prevent the formal unitarization by the addition of an inert tensor factor. I have talked a lot about HFFs referring to hyper-finite factors of type II_1 (possibly also of type III_1) and their inclusions [K87]. The reason is that WCW spinors form a canonical representation for these von Neumann algebras.

   Could the isometries replacing unitary S-matrix correspond to inclusions of HFFs? In the recent interpretation the included factor (now H_{in}) corresponds to the degrees of freedom below measurement resolution. Certainly this does not make sense now. The interpretation in terms of finite measurement resolution need not however be the only possible interpretation and the interpretation in terms of measurement resolution might of course be wrong. Therefore one can ask whether the relation between H_{in} and H_{out} could be more complex than just $H_{out} = H_{in} \otimes H_1$ so that formal unitarization would fail.

13.5.5 Scattering diagrams as tensor networks constructed from perfect tensors

Preskill's tensor network construction [B43] realizes isometric maps as representations of holography and as models for quantum error correcting codes. These tensor networks have remarkable similarities with twistorial and number theoretical visions, which suggests that it could be used to construct scattering amplitudes. A further idea inspired by holography is that the description of scattering amplitudes in terms of fundamental fermions and physical particles are dual to each other.

1. In the construction of quantum error codes tensor network defines an isometric imbedding of local states in the interior to strongly entangled non-local states at boundary. Their vertices correspond to tensors, which in the proposal of Preskill et al [B43] are perfect tensors such that one can take any m legs of the vertex and the tensor defines isometry from the state space of m legs to that of $n - m$ legs. When the number of indices is $2n$, the entanglement defined by perfect tensor between any n-dimensional subspace and its complement is maximal

   TGD framework maximal entanglement corresponds to negentropic entanglement with density matrix proportional to identity matrix. What is important that the isometry is constructed by composing local isometries associated with a network. Given isometry can be constructed in very many manners but there is some minimal realization.

2. The tensor networks considered in [B43] are very special since they are determined by tesselations of hyperbolic space H_2. This kind of tesselations of H_3 could be crucial for understanding the analog of condensed matter physics for dark matter and could appear in biology [L23]. What is crucial is that only the graph property and perfect tensor property matter as far as isometricity is considered so that it is possible to construct very general isometries by using tensor networks.

13.5.6 Eigenstates of Yangian co-algebra generators as a manner to generate maximal entanglement?

Negentropically entangled objects are key entities in TGD inspired theory of consciousness and also of tensor networks, and the challenge is to understand how these could be constructed and what their properties could be. These states are diametrically opposite to unentangled eigenstates of single particle operators, usually elements of Cartan algebra of symmetry group. The entangled states should result as eigenstates of poly-local operators. Yangian algebras involve a hierarchy of poly-local operators, and twistorial considerations inspire the conjecture that Yangian counterparts of super-symplectic and other algebras made poly-local with respect to partonic 2-surfaces or end-points of boundaries of string world sheet at them are symmetries of quantum TGD [L22]. Could Yangians allow to understand maximal entanglement in terms of symmetries?

1. In this respect the construction of maximally entangled states using bi-local operator $Q^z = J_x \otimes J_y - J_x \otimes J_y$ is highly interesting since entangled states would result by state function. Single particle operator like J_z would generate un-entangled states. The states obtained as eigenstates of this operator have permutation symmetries. The operator can be expressed as $Q^z = f^z_{ij} J^i \otimes J^j$, where f^A_{BC} are structure constants of SU(2) and could be interpreted as co-product associated with the Lie algebra generator J^z. Thus it would seem that unentangled states correspond to eigenstates of J^z and the maximally entangled state to eigenstates of co-generator Q^z. Kind of duality would be in question.

2. Could one generalize this construction to n-fold tensor products? What about other representations of SU(2)? Could one generalize from SU(2) to arbitrary Lie algebra by replacing Cartan generators with suitably defined co-generators and spin 1/2 representation with fundamental representation? The optimistic guess would be that the resulting states are maximally entangled and excellent candidates for states for which negentropic entanglement is maximized by NMP [K41].

3. Co-product is needed and there exists a rich spectrum of algebras with co-product (quantum groups, bialgebras, Hopf algebras, Yangian algebras). In particular, Yangians of Lie algebras are generated by ordinary Lie algebra generators and their co-generators subject to constraints. The outcome is an infinite-dimensional algebra analogous to one half of Kac-Moody algebra with the analog of conformal weight N counting the number of tensor factors. Witten gives a nice concrete explanation of Yangian [B30] for which co-generators of T^A are given as $Q^A = \sum_{i<j} f^A_{BC} T^B_i \otimes T^C_j$, where the summation is over discrete ordered points, which could now label partonic 2-surfaces or points of them or points of string like object (see `http://tinyurl.com/y727n8ua`). For a practically totally incomprehensible description of Yangian one can look at the Wikipedia article (see `http://tinyurl.com/y7heufjh`).

4. This would suggest that the eigenstates of Cartan algebra co-generators of Yangian could define an eigen basis of Yangian algebra dual to the basis defined by the totally unentangled eigenstates of generators and that the quantum measurement of poly-local observables defined by co-generators creates entangled and perhaps even maximally entangled states. A duality between totally unentangled and completely entangled situations is suggestive and analogous to that encountered in twistor Grassmann approach where conformal symmetry and its dual are involved. A beautiful connection between generalization of Lie algebras, quantum measurement theory and quantum information theory would emerge.

13.5.7 Two different tensor network descriptions

The obvious question is whether also unitary S-matrix of TGD could be constructed using tensor network built from perfect tensors. In ZEO the role of boundary would be taken by the ends of the space-time at upper and lower light-like boundaries of CD carrying the particles characterized by standard model quantum numbers. Strong form of holography would suggest that partonic surfaces and strings at the ends of CD provide information for the description of zero energy states and therefore of scattering amplitudes. The role of interior would be taken by the space-time surface - in particular the light-like orbits of partonic surfaces carrying the fermion lines identified as boundaries of string world sheets. Conformal field theory description would apply to fermions residing at string world sheets with boundaries at light-like orbits of partonic 2-surfaces.

In QFT Feynman diagrammatics one obtains a sum over diagrams with arbitrary numbers of loops. In both twistorial and number theoretic approach however only a finite number of diagrams with possibly complex on mass shell massless momenta are needed. If the vertices are however such that particles remain on-mass-shell but are allowed to have complex four-momenta then the integration over internal momenta (loops) is not present and tensor network description could make sense. This encourages the conjecture that tensor networks could be used to construct the scattering amplitudes in TGD framework.

What could perfect tensor property mean for the vertices identified as nodes of a tensor network? There are two levels to be considered: the geometric level identifying particles as 3-surfaces with net quantum numbers and the fermion level identifying particles as fundamental fermions at the boundaries of string world sheets.

1. At the geometric level vertices corresponds to light-like orbits of partonic 2-surfaces meeting at common end which is partonic 2-surface. This is 3-D generalization of Feynman diagram as a geometric entity. At the level of fermion lines associated with the light-like 3-surfaces one the basic interaction corresponds to the scattering of 2-fermions leading to re-sharing of fermion lines between outgoing light-like 3-surfaces, which include also representations for virtual particles. One has 4-fermion vertex but not in the sense that it appears in the interaction of weak interactions at low energies.

   Geometrically the basic vertex could be 3-vertex: $n > 3$-vertices are unstable against deformation to lower vertices. For 3-vertex perfect tensor property means that the tensor defining the vertex maps any 1-particle subspaces to 2-particle subspace isometrically. The geometric vertices define a network consisting of 3-D "lines" and 2-D vertices but one cannot tell what is within the 3-D lines and what happens in the 2-D nodes. The lines would consist of braided fundamental fermion lines and in nodes the basic process would be 2+2 scattering for fermions. In the case of 3-vertex momentum conservation would effectively eliminate the four-momentum and the state spaces associated with vertex would be effectively discrete. This is p-adically of utmost importance.

2. At the level of fundamental fermion lines in the interior of particle lines one would have 4-vertices and if a perfect tensor describes it, it gives rise to a unitary map of any 2-fermion subspace to its complement plus isometric maps of 1-fermion subspaces to 3-fermion subspaces. In this case momenta cannot act as labels of fermion lines for rather obvious reasons: the solution of the problem is that conformal weights label fundamental fermion lines

   The conservation of discrete quark and lepton numbers allows only vertices of type $qL \rightarrow qL$ and its variants obtained by crossing. In this case the isometries might allow realization. The isometries must be defined to take into account quark and lepton number conservation by crossing replacing fermion with antifermion. By allowing the states of Hilbert space in node to be both quarks and leptons, difficulties can be avoided.

Tensor network description in terms of fundamental fermions and CFT

Consider first fundamental fermions. What are the labels characterizing the states of fundamental fermions fermions propagating along the lines? There are two options: the labels are either conformal weights or four-momenta.

1. Since fermions corresponds to strings defining the boundaries of string world sheets and since strong form of holography implies effective 2-dimensionality also in fermion sector, the natural guess is that the conformal weights plus some discrete quantum numbers - standard model quantum numbers at least - are in question. The situation would be well-defined also p-adically for this option. In this case one can hope that conformal field theory at partonic 2-surface could define the fermionic 4-vertex more or less completely. There would be no need to assign propagators between different four-fermion vertices. The scattering diagram would define a composite formed from light-like 3-surfaces and one would have single isometry build from 4-fermion perfect tensors. There would be no integrations over internal momenta.

2. Second option is that fundamental fermions are labelled by four-momenta. The outgoing four-momenta in 4-vertices would not be completely fixed by the values of the incoming momenta and this extends the state space. Concerning p-adicization this integral is not desirable and this forces to consider seriously discrete labelling. The unitarity condition for 2+2 scattering would involve integral over 2-sphere. Four-fermion scattering must be unitary process in QFT so that this condition might be possible to satisfy. The problem would be how to fix this fundamental scattering matrix uniquely. This option does not look attractive number theoretically.

The most plausible option is that holography means that conformal field theory describes the scattering of fundamental fermions and QFT type description analogous to twistorial approach describes the scattering of physical fermions. If only 3-vertices are allowed, and if masslessness corresponds to masslessness in 8-D sense, one obtains non-trivial scattering vertices (for ordinary twistor approach all massless momenta would be collinear if real).

Tensor network description for physical particles

Could the twistorial description expected to correspond to the description in terms of particles allow tensor network description?

1. Certainly one must assign four-momenta to incoming *physical* particles - also fermions - but they correspond to pairs of wormhole contacts rather than fundamental fermions at the boundaries of string world sheets. It would be natural to assign four-momenta also to the virtual *physical* fermions appearing in the diagram and the geometric view about scattering would allow only 3-vertices so that momentum conservation would eliminate momentum degrees of freedom effectively. This would be a p-adically good news.

2. At the level of fundamental fermions entanglement is described as a tensor contraction of the CFT vertices. This locality is natural since the vertices are at null distance from each other. At QFT limit the entanglement between the ends of the line is characterized the propagator.

   One must get rid of propagators in order to have tensor network description. The inclusion of propagators to the fundamental tensor diagrams would break the symmetry between the legs of vertex since the propagator cannot be included to its both ends. Situation changes if one can represent the propagator as a bilinear of something more primitive and include the halves to the opposite ends of the line. Twistor representation of four-momentum indeed defines this kind of representation as a bilinear $p^{a\tilde{b}} = \lambda\tilde{\mu}^{\tilde{b}}$ of twistors λ and $\tilde{\mu}$. There is problem due to the diverging $1/p^2$ factor but residue integral eliminates this factor and one can write directly the fermionic propagator factors as $p^{a\tilde{b}}$.

3. In QFT description the perturbative expansion is in powers of coupling constant. If the reduction to braided tree diagrams analogous to twistor diagrams occurs, power g^{N-2} of coupling constant is expected to factorize as a multiplier of a tree diagram with N external legs. One should understand this aspect in the tensor net-work picture.

   For $\mathcal{N} = 4$ SUSY there is coupling constant renormalization. Similar prediction is expected from TGD. Coupling constant evolution is expected to be discrete and induced by the discrete evolution of Kähler coupling strength defined by the spectrum of its critical values. The conjecture is that critical values are naturally labelled by p-adic primes $p \simeq 2^k$, k prime, labelling p-adic length scales. Therefore one might hope that problems could be avoided.

These observations encourage the expectation that twistorial approach involving only 3-vertices allows to realize tensor network idea also at the level of physical particles. It might be essential that twistors can be generalized to 8-D twistors. Octonionic representation of gamma matrices might make this possible. Also the fact twistorial uniqueness of M^4 and CP_2 might be crucial.

Gauge theory follows as QFT limit of TGD so that one cannot in principle require that gauge theory vertices satisfy the isometricity conditions. Nothing however prevents from checking whether gauge theory limit might inherit this property.

1. For instance, could 3-vertices of Yang-Mills theory define isometric imbedding of 1-particle states to 2 particle states? For a given gauge boson there should exist always a pair of gauge bosons, which can fuse to it. Consider a basis for Lie-algebra generators of the gauge group. If the generator T is such that there exists no pair $[A, B]$ with the property $[A, B] = T$, Jacobi identity implies that T must commute with all generators and one has direct sum of Lie algebras generated by T and remaining generators.

2. In the case of weak algebra $SU(2) \times U(1)$ the weak mixing of Y and I_3 might allow the isometric imbeddings of type $1 \rightarrow 2$. Does this mean that Weinberg angle must be non-vanishing in order to have consistent theory? A realistic manner to get rid of the problem is to allow at QFT limit the lines to be also fermions so that also U(1) gauge boson can be constructed as fermion pair.

How the two tensor network descriptions would be related?

There are two descriptions for the zero energy states providing representation of scattering amplitudes: the CFT description in terms of fundamental fermions at the boundaries of string world sheets, and the description in terms of physical particles to which one can assign light-like 3-surfaces as virtual lines and total quantum numbers.

1. CFT description in terms of fundamental fermions in some aspects very simple because of its 2-dimensionality and conformal invariance. The description is in terms of physical particles having light-like 3-surfaces carrying some total quantum numbers as correlates and is simpler in different sense. These descriptions should be related by an Hilbert space isometry.

2. The perfect tensor property for 4-fermion vertices makes fundamental fermion states analogous to physical states realizing logical qubits as highly entangled structures. Geometric description in terms of 3-surfaces is in turn analogous to the description in terms of logical qubits.

3. Holography-like correspondence between these descriptions of zero energy states (scattering diagrams) should exist. Physical particles should correspond to the level, at which resolution is smaller and which should be isometrically mapped to the strongly entangled level defined by fundamental fermions and analogous to boundary degrees of freedom (fundamental fermions *are* at the boundaries of string world sheets!).

   The map relating the two descriptions seems to exist. One can assign four-momenta to the legs of conformal four-point function as parameters so that one obtains a mapping from the states labelled by conformal weights to the states labelled by four-momenta! The appearance of 4-momenta from conformal theory is somewhat mysterious looking phenomenon but this duality makes it rather natural.

13.5.8 Taking into account braiding and WCW degrees of freedom

One must also take intro account braiding and orbital degrees of freedom of WCW. The generalization of tensor network to braided tensor network is trivial. Thanks to the properties of tensor network orbital and spinor degrees of freedom factorize so that also the treatment of WCW degrees of freedom seems to be possible.

What about braiding?

The scattering diagrams would be tree diagrams with braiding of fermionic lines along light-like 3-surfaces - dance of fundamental quarks and leptons at parquette defined by the partonic 2-surface one might say. Also space-like braiding at magnetic flux tubes at the ends of CD is possible and its time evolution between the ends of space-time surfaces defines 2-braiding which is generalization of the ordinary braiding but will not be discussed here. This gives rise to a hierarchy of braidings. One can talk about flux tubes within flux tubes and about light-like 3-surface within light-like 3-surfaces. The smaller light-like 3-surface would be glued by a wormhole contact to the larger one and contact could have Euclidian signature of induced metric.

How can one treat the braiding in the tensor network picture? The answer is simple. Braiding corresponds to an element of braid group and one can represent it by a unitary matrix as one does in topological QFT as one constructs knot invariants. In particular, the trace of this unitary matrix defines a knot invariant. The generalization of the tensor network is simple. One attaches to the links connecting two nodes unitary transformation defining a representation of the braid involved. Local variant of unitarity would mean isometricity at nodes and unitarity at links.

What about WCW degrees of freedom?

The above considerations are about fermions that its WCW spinor degrees of freedom and the space-time surface itself has been regarded as a fixed background. How can one take into account WCW degrees of freedom?

The scattering amplitude involves a functional integral over the 3-surfaces at the ends of CD. The functional integration over WCW degrees of freedom gives an expression depending on Kähler coupling strength α_K and determines the dependence on various gauge coupling strengths expressible in terms of α_K. This makes it possible to have the tensor network description in fermionic degrees of freedom without losing completely the dependence of the scattering amplitudes on gauge couplings. By strong form of holography the functional integral should reduce to that over partonic 2-surfaces and strings connecting them. Number theoretic discretization with a cutoff determined by measurement resolution forces the parameters characterizing the 2-surfaces to belong to an algebraic extension of rationals and is expected to reduce functional integral to a sum over discretized WCW so that it makes sense also in p-adic sectors [K110, K111].

A brief summary of quantum measurement theory in ZEO is necessary. The repeated state function reduction shifts active boundary A of CD and affects the states at it. The passive boundary of CD- call it P - and the states at it - remain unaffected. The repeated state function reductions leaving P unaffected and giving usually rise to Zeno effect, correspond now to the TGD counterpart of unitary time evolution by shifts between subsequent state function reductions. Call A and its shifted version A_{in} and A_{out} and the corresponding state spaces H_{in} and H_{out}. The unitary (or more generally isometric) Smatrix represents this shift. This is the TGD counterpart of a unitary evolution of QFTs. S forms a building brick of a more general unitary matrix U acting in the space of zero energy states but U is not considered now.

Consider now the isometricity conditions.

1. Unitarity conditions generalized to isometricity conditions apply to S. Isometricity conditions $S^\dagger S = Id_{in}$ can be applied at A_{in}. The states appearing in the isometry conditions as initial and final states correspond to A_{in} and A_{out}. There is a trace over WCW spin indices (labels for many-fermion states) of H_{out} in the conditions $S^\dagger S = Id_{in}$. Isometricity conditions involve also an integral over WCW orbital degrees of freedom at both ends: these degrees of freedom are strongly correlated and for a strict classical determinism the correlation between the ends is complete. If the tensor network idea works, the summation over spinor degrees of freedom at A_{out} gives just a unit matrix in the spinor indices at A_{in} and leaves only the WCW orbital degrees of freedom in consideration. This factorization of spinor and orbital WCW degrees of freedom simplifies the situation dramatically.

2. One can express isometricity conditions for modes with $\Psi_{in,M}$ and $\Psi_{out,N}$ at A_{in} and A_{out}: this requires functional integration over 3-surfaces WCW at A_{in} and A_{out}. The conditions are formulated in terms of the labels - call them M_{in}, N_{in} - of WCW spinor modes at A_{in} including standard model quantum numbers and labels characterizing the states of supersymplectic and super-conformal representations. The trace is over the corresponding indices R_{out} at A_{out}. The WCW functional integrals in the generalized unitarity conditions are therefore over A_{in} and A_{out} and should give Kronecker delta $\sum_{R_{out}} S^\dagger_{M_{in}R_{out}} S_{R_{out}N_{in}} = \delta_{M_{in},N_{in}}$.

3. The simplest view would be that Kähler action with boundary conditions implies completely deterministic dynamics. The conditions expressing strong form of holography state that sub-algebras of super-symplectic algebra and related conformal algebras isomorphic to the entire algebra give rise to vanishing Noether charges. Suppose that these conditions posed at the ends of CD are so strong that they fix the time evolution of the space-time surface as preferred extremal completely when posed at either boundary. In this case the isometricity conditions

would be so strong that the double functional integration appearing in the matrix product reduces to that at A_{in} and the isometricity conditions would state just the orthonormality of the basis of WCW spinor modes at A_{in}.

4. Quantum criticality and in particular, the hierarchy of Planck constants providing a geometric description for non-deterministic long range fluctuations, does not support this view. Also the fact that string world sheets connect the boundaries of CD suggests that determinism must be broken. The inner product defining the completeness of the WCW state basis in orbital degrees of freedom can be however generalized to a bi-local inner product involving functional integration over 3-surfaces at both A_{in} and A_{out}. There is however a very strong correlation so that integration volume at A_{out} is expected to be small. This also suggests that one can have only isometricity conditions.

13.5.9 How do the gauge couplings appear in the vertices?

Reader is probably still confused and wondering how the gauge couplings appear in the vertices from the functional integral over WCW degrees of freedom. In twistorial approach, the vanishing of loops in $\mathcal{N}=4$ SYM theory gives just g^N, N the number of 3-vertices. Each vertex should give gauge coupling. Or equivalently, each propagator line connecting vertices should give α_K. The functional integral should give this factor for each propagator line. Generalization of conformal invariance is expected to give this picture.

To proceed some basic facts about N-point functions of CFTs are needed.

1. In conformal field theory the functional form of two-point function is completely fixed by conformal symmetry:

$$\begin{aligned} G^{(2)}(z_i,\overline{z}_i) &= \frac{C_{12}}{z_{12}^{2h}\overline{z}_{12}^{2\overline{h}}} \ , \\ z_{ij} &= z_i - z_j \ , \ \ \overline{z}_{ij} = \overline{z}_i - \overline{z}_j \ , \\ h_1 = h_2 = h &= h_a + ih_b \ , \ \ \overline{h} = \overline{h}_a + i\overline{h}_b \ . \end{aligned} \tag{13.5.1}$$

$h_1 = h_2 \equiv h$ and its conjugate $\overline{h}$ are conformal weights of conformal field and its conjugate. Note that the conformal weights of conformal fields Φ_1 and Φ_2 must be same. In TGD context C_{12} is expected to be proportional to α_K and this would give to each vertex g_K when couplings are absorbed into vertices.

2. The 3-point function for 3 conformal fields Φ_i, $i=1,2,3$ is dictated by conformal symmetries apart from constant C_{123}:

$$G^{(3)}(z_i,\overline{z}_i) = C_{123}\times\frac{1}{z_{12}^{h_1+h_2-h_3}z_{23}^{h_2+h_3-h_1}z_{31}^{h_3+h_1-h_2}}\times\frac{1}{\overline{z}_{12}^{\overline{h}_1+\overline{h}_2-\overline{h}_3}\overline{z}_{23}^{\overline{h}_2+\overline{h}_3-\overline{h}_1}\overline{z}_{31}^{\overline{h}_3+\overline{h}_1-\overline{h}_2}} \ . \tag{13.5.2}$$

Here C_{123} should bef fixed by super-symplectic and related symmetries and determined the numerical coefficients various couplings when expressed in terms of g_K.

3. 4-point functions have analogous form

$$\begin{aligned} G^{(4)}(z_i,\overline{z}_i) &= f_{1234}(x,\overline{x})\prod_{i<j} z_{ij}^{-(h_i+h_j)+h/3}\prod_{i<j}\overline{z}_{ij}^{-(\overline{h}_i+\overline{h}_j)+\overline{h}/3} \ , \\ h &= \sum_i h_i \ , \end{aligned} \tag{13.5.3}$$

but are proportional to an arbitrary function f_{1234} of conformal invariant $x = z_{12}z_{34}/z_{13}z_{24}$ and its conjugate.

If only 3-vertices appear/are needed for physical particles - as both twistorial and number theoretic approaches strongly suggest - the conformal propagators and vertices are fixed apart from constants C_{ijk}, which in turn should be fixed by the huge generalization of conformal symmetries. α_K emerges in the expected manner.

This picture seems to follow from first principles.

1. One can fix the partonic 2-surfaces at the boundaries of CD but there is a functional integral over partonic 2-surfaces defining the vertices: their deformations induce deformations of the legs. One can expand the exponent of Kähler action and in the lowest order the perturbation term is trilinear and non-local in the perturbations. This gives rise to 3-point function of CFT nonlocal in z_i. The functional integral over perturbations gives the propagators in legs proportional to α_K in terms of two point function of CFT. Note that the external propagator legs can be eliminated in S-matrix.

2. The cancellation of higher order perturbative corrections in WCW functional integral is required by the quantum criticality and means trivial coupling constant evolution for α_K and other coupling constants. Coupling constant evolution is discretized with values of α_K analogous to critical temperatures and should correspond to p-adic coupling constant evolution [L16].

3. This picture leaves a lot of details open. An integration over the values of z_i is needed and means a kind of Fourier analysis leading from complex domain. The analog of Fourier analysis would be for deformations of partonic 2-surface labelled by some natural labels. Conformal weights could be natural labels of this kind.

It is easy to get confused since there are several diagrammatics involved: the topological diagrammatics of 3-surfacse assignable to the physical particles with partonic 2-surfaces as vertices, the diagrammatics associated with the perturbative functional integral for the Kähler action, and the fermionic diagrammatics suggested to reduce to tensor network. The conjectures are as follows.

1. The "primary" vertices $G^{(n)}$, $n > 3$ assignable to single partonic 2-surface and coming from a functional integral for Kähler action vanishes. This corresponds to quantum criticality and trivial RG evolution.

2. $G^{(n)}$, $n > 3$ in the sense of topological diagrammatics without loops and involving n partonic 2-surfaces do not vanish. One can construct the analog of $G^{(4)}$ from two $G^{(3)}$:s at different partonic 2-surfaces and propagator defined by 2-point function connecting them as string diagram.

   Also topological variant of $G^{(4)}$ assignable to single partonic 2-surface can be constructed by allowing the 3-D propagator "line" to return back to the partonic 2-surface. This would correspond to an analog of loop. Similar construction applies to "primary" $G^{(n)}$,$n > 4$. In number theoretic vision these loops are eliminated as redundant representations so that one has only braided tree diagrams. Also twistor Grassmann approach supports this view.

To sum up, the tensor network description would apply to fermionic degrees of freedom. In bosonic degrees of freedom functional integral would give CFT picture with 3-vertex as the only "primary" vertex and from this twistorial and number theoretic visions follow via the super-symplectic symmetries of the vertex coefficients C_{ijk} extended to Yangian symmetries.

Chapter 14

About Twistor Lift of TGD

14.1 Introduction

The twistor lift of classical TGD [L22] is attractive physically but it is still unclear whether it satisfies all constraints. The basic implication of twistor lift would be the understanding of gravitational and cosmological constants. Volume term in action removes the infinite vacuum degeneracy of Kähler action but because of the extreme smallness of cosmological constant Λ playing the role of inverse of gauge coupling strength, the situation for nearly vacuum extremals of Kähler action in the recent cosmology is non-perturbative.

What is remarkable that twistor lift is possible only in zero energy ontology (ZEO) since the volume term would be infinite by infinite volume of space-time surface in ordinary ontology: by the finite size of causal diamond (CD) the space-time volume is however finite in ZEO. Furthermore, the condition that the destructive interference does not cancel vacuum functional implies Bohr quantization for the action in ZEO. The scale of CD corresponds naturally to the length scale $L_\Lambda = \sqrt{8\pi/\Lambda}$ defined by the cosmological constant.

One motivation for introducing the hierarchy of Planck constants [K22, K106] was that the phase transition increasing Planck constant makes possible perturbation theory in strongly interacting system. Nature itself would take care about the converge of the perturbation theory by scaling Kähler coupling strength α_K to α_K/n, $n = h_{eff}/h$. This hierarchy might allow to construct gravitational perturbation theory as has been proposed already earlier. This would for gravitation to be quantum coherent in astrophysical and even cosmological scales.

In this chapter two options for the twistor lift are studied in detail.

1. Option I (the original option): The values of $\alpha_K(M^4)$ and $\alpha_K(CP_2)$ are widely different with $\alpha_K(M^4)$ being extremely large so that M^4 part of the 6-D Kähler action gives in dimensional reduction extremely small cosmological term. Allowing Kähler coupling strength $\alpha_K(CP_2)$ to correspond to zeros of zeta implies that for complex zeros the preferred extremals for $\alpha_K(M^4)$ having different phase are mimimal surface extremals of Kähler action so that the values of coupling constants do not matter and extremals depend on couplings only through the boundary conditions stating the vanishing of certain super-symplectic conserved charges.

   It has turned out that this option has several shortcomings. First of all, $\alpha_K(M^4) \neq \alpha_K(CP_2)$ looks like ad hoc assumption tailored to make cosmological constant small. Secondly, the decoupling between Kähler action and volume term implies separately conserved Noether charges which looks strange. Thirdly, for $\sqrt{g_4}$ instead of $\sqrt{|g_4|}$ in the volume element assumed hitherto, there is no charge transfer between Minkowski and Euclidian regions.

2. Option II: $\alpha_K(M^4) = \alpha_K(CP_2)$ is satisfied. Now entire action is identified as the cosmological term. A small effective value of cosmological constant is obtained if the Kähler action and volume term tend to cancel each other. Minimal surface extremals of Kähler action correspond naturally to asymptotic dynamics near the boundaries of CDs, where the analog of free geodesic motion as minimal surfaces is expected. For $\sqrt{|g_4|}$ option there is charge transfer between Minkowski and Euclidian regions.

The two options provide different generalizations of Chladni mechanism [K113] [L27, L28] (see "An Amazing Resonance Experiment" at `http://tinyurl.com/kcbmrzz`)to a "dynamics of avoidance". Both options have profound implications for the views about what happens in particle physics experiment and in quantum measurement, and for consciousness theory and for quantum biology. It is however clear that Option II is the favored one.

The need to understand the twistor lift leads to a critics of the formulation of the basic action principle and the outcome is a more elegant formulation with non-trivial physical consequences.

1. Dimensionless gauge field is obtained from dimension 2 induced Kähler form by division with constant R_1^2 with dimension two. This parameter defines a hidden coupling parameter in the action and the identification in terms of CP_2 radius made hitherto rather implicitly is probably reasonable but ad hoc. The simple idea is to use the induced Kähler form as basic object and formulate the action principle accordingly. This brings in the dimensional parameter $1/R_1^4$ compensating for the dimension of $\sqrt{g_4}$ in the action.

2. One ends up to a general formulation of both bosonic and fermionic action principles showing that the overall scaling factor of fermionic and bosonic actions - call it X, disappears from classical dynamics so that extremals have no explicit independence on X. This is crucial for number theoretical universality.

   Quantum Classical Correspondence (QCC) realized as the condition that classical Noether charges in Cartan algebra correspond to eigenvalues of quantal fermionic charges however breaks the invariance with respect to scalings of action via fermionic anticommutation relations which depend on the scaling factor. The new formulation leads to a unique guesses for the 6-D actions, their 4-D dimensionally reduced variants, and 2-D effective actions.

3. The formulation helps to realize that Number Theoretical Universality (NTU) requires that $\sqrt{|g_4|}$ option is the only possible one. Physically the need to have charge transfer between Euclidian and Minkowskian space-time regions implies the same result.

This leads to two different views about cosmological constant.

1. For Option I the explanation for dark energy is in terms of volume term of the action and small value of cosmological constant obeying p-adic coupling constant evolution as function of p-adic length scale. For Option II the cancellation of Kähler action and volume term would give rise to a small value of cosmological constant and its p-adic evolution.

2. Either $L_\lambda = \sqrt{8\pi/\Lambda}$ or the length L characterizing vacuum energy density as $\rho_{vac} = \hbar/L^4$ or both can obey p-adic length scale hypothesis as analogs of coupling constant parameters. The third option makes sense if the ratio R/l_P of CP_2 radius and Planck length is power of two: it can be indeed chosen to be $R/l_P = 2^{12}$ within measurement uncertainties. $L(now)$ corresponds to the p-adic length scale $L(k) \propto 2^{k/2}$ for $k = 175$, size scale of neuron and axon.

3. A microscopic explanation for the vacuum energy realizing strong form of holography (SH) is in terms of vacuum energy for radial flux tubes emanating from the source of gravitational field. The independence of energy from the value of $h_{eff}/h = n$ implies analog of Uncertainty Principle: the product Nn for the number N of flux tubes and the value of n defining the number of sheets of the covering associated with $h_{eff} = n \times h$ is constant. This picture suggests that holography is realized in biology in terms of pixels whose size scale is characterized by L rather than Planck length.

4. A interesting observation is that a fundamental length scale of biology - size scale of neuron and axon - would correspond to the p-adic length scale assignable to vacuum energy density characterized by cosmological constant and be therefore a fundamental physics length scale. An especially interesting result is that in the recent cosmology the size scale of a large neuron would be fundamental physical length scale determined by cosmological constant. This gives additional boost to the idea that biology and fundamental physics could relate closely to each other: the size scale of neuron would not be an accident but "determined in stars" and even beyond them!

14.2 More about twistor lift of Kähler action

The following piece of text was motivated by some observations relating to the twistor lift of Kähler action forcing a criticism of the earlier view about twistor lift.

The first observation was that the correct formulation of 6-D Kähler action in the framework of adelic physics implies that the *classical* physics of TGD does not depend on the overall scaling of Kähler action. This implies that the preferred extremals need not be minimal surface extremals of Kähler action. It is enough that they are so asymptotically - near the boundaries of CDs where they behave like free particles. This also nicely conforms with the physical idea that they are 4-D generalizations for orbits of particles in induced Kähler field.

The independence of the classical physics on the scale of the action inspires a detailed discussion of the number theoretic vision. Quantum Classical Correspondence (QCC) breaks the invariance with respect to the scalings via fermionic anti-commutation relations and Number Theoretical Universality (NTU) can fix the spectrum of values of the over-all scaling parameter of the action. One ends up to a condition guaranteeing NTU of the action exponential and finds an answer to the nagging question whether one should use $\sqrt{g_4}$ (imaginary in Minkowskian regions) or $\sqrt{|g_4|}$ in the action. Complex α_K allows $\sqrt{|g_4|}$ and NTU assuming that $1/\alpha_K = s$, $s = 1/2 + iy$ zero of Riemann zeta, implies $y = q\pi$, q rational as proposed also in [L16].

Second observation relates to cosmological constant. The proposed vision for the p-adic evolution of cosmological constant assumes that $\alpha_K(M^4)$ and $\alpha_K(CP_2)$ are different for the twistor lift. One however finds that single value of α_K is the natural choice. This destroys the original proposal for the p-adic length scale evolution of cosmological constant explaining why it is so small in cosmological scale.

The solution to the problem of the cosmological constant would be that the *entire* 6-D action decomposing to 4-D Kähler action and volume term is identified in terms of cosmological constant. The cancellation of Kähler electric contribution and remaining contributions would explain why the cosmological constant is so small in cosmological scales and also allows to understand p-adic coupling constant evolution of cosmological constant. One must however remain cautious: also the original proposal can be defended.

14.2.1 Kähler action contains overall scale as a hidden coupling parameter

The first observation leads to a more precise understanding of 6-D Kähler action relates to the induction procedure.

1. Kähler form has dimension two since its square gives metric: $J^2 = -g$. Gauge fields are however 2-forms, which are usually taken to be dimensionless (this requires that coupling constant g is included as multiplicative factor to gauge potential). Accordingly, I have assumed that induced Kähler form is obtained by diving Kähler form by $1/R^2$, R the radius of CP_2 identified as the radius of its geodesic sphere. One can however argue that the identification of the scaling factor is ad hoc since its value does not affect classical field equations.

2. What would happen if one induces the dimensional Kähler form as such? Kähler action density $L_K\sqrt{g_4}$ would have dimension of volume so that $1/\alpha_K$ must be replaced with $1/8\pi\alpha_K R_1^4$, where R_1 a fundamental coupling constant with dimension of length. This coupling however disappears from the classical field equations and in the recent adelic formulation also from quantum theory [L38].

3. For the 6-D twistor lift of Kähler action one must introduce an additional dimensional factor to get a dimensionless action. One has $R_1^4 \to R_1^4 R_0^2$, where R_0^2 has dimensions of area. The 4-D action density obtained from dimensional reduction for twistor sphere $S^2(X^4)$ assuming that the induced Kähler form for the sphere satisfies $J^4 = -g$ for $S^2(X^4)$ is proportional to

$$L = X \times (J \cdot J - 2)\sqrt{g_4} \ , \quad X = \frac{1}{2\alpha_K}\frac{Area(S^2(X^4))}{S_0}\frac{1}{R_1^4} \ , \quad S_0 = 4\pi R_0^2 \ . \qquad (14.2.1)$$

The shift of Kähler action density by -2 comes from $S^2(X^4)$ part of 6-D Kähler action.

4. From this form one can immediately see that the factor X in Eq. 14.2.1 disappears from field equations, and the functional form of preferred extremals has no dependence on coupling parameters! The quantum classical correspondence (QCC) stating that fermionic Noether charges in Cartan algebra have eigenvalues equals to their classical counterparts however implies this dependence.

Modified Dirac action and string world sheet action in the new formalism

What about the modified Dirac action related super-symmetrically to Kähler action in the new formalism? The 6-D formalism for the induced spinors doubles the number of spinor components and dimensional reduction must eliminate half of them to give something equivalent with the ordinary induced spinor structure. Chirality condition is the most plausible manner to achieve this. This answers the old question whether one could assume only leptonic spinors as fundamental spinors and construct quarks as some of anyonic leptons. This would require two chirality conditions and this is very probably not possible. The 6-D modified Dirac action can be written using the same rules as applied in 4-D case. The possible delicacies of the fermionic dimensional reduction require a separate discussion.

The 4-D dimensionally reduced part of 6-D modified Dirac action must reduce to the 4-D modified Dirac action associated with the full bosonic action. The modified gamma matrices Γ^α are expressible as contractions of the canonical momentum currents with imbedding space gamma matrices (this applies also in $D = 6$). Therefore they are proportional to the dimensionless quantity $X\sqrt{g_4}$. Γ^α has dimension $1/L$ so that induced spinors must have dimension $L^{1/2}$. In the usual approach the dimension would be $1/L^{3/2}$.

With these conventions X apparently drop from the equations stating QCC as identity of eigenvalues of fermionic Noether charges and corresponding classical Noether charges in Cartan algebra. This not true. The anti-commutations for Ψ and time component J^0 of the canonical momentum density $J^\alpha = \partial L/\partial(\partial_\alpha\Psi) = \overline{\Psi}\Gamma^\alpha$ involve X and affect the scale of anti-commutation relations and therefore QCC. That the anti-commutations can be indeed realized under these dimensional constraints, requires a proof.

What about the spinors restricted to 2-D string world sheets and corresponding space-time action? Perhaps the most plausible option is that they do not appear at the fundamental level and appear only as the effective action suggested by SH. If this is the case, it is rather easy to guess the form of the bosonic and fermion 2-D effective actions. Their forms could be exactly the same as the form of 4-D actions. The only modification would be in the bosonic case the replacement of $1/R_1^4$ with $1/R_1^2$ to get the dimensions correctly! The bosonic action would dictate the fermionic action by above rules.

The bosonic string world sheet action would differ from the area action. The action density would be $XR_1^2(J \cdot J - 2)\sqrt{g}$ in complete analogy with the 4-D case. Two special cases deserve to be mentioned.

1. This action vanishes for string world sheets with $J \cdot J = 2$. This is the case if one has $J = M(M^4)$ and J is self-dual. This is true if string world sheet is the preferred plane M^2 defining the symplectic structure of M^4 (there is moduli space form them in order to gain Lorentz invariance and giving rise to sectors of WCW).

   Small deformations of this plane would give rise to strings with small string tension and be naturally relating to the small value of the cosmological constant. These strings should accompany long strings mediating gravitational interaction in long length scales. The small action would require large value of $h_{eff}/h = n = h_{gr}$ for the perturbation theory to work.

2. Second special case corresponds to Lagrangian surfaces for which $J(M^4) + J(CP_2)$ induced to string world sheet vanishes. One would have ordinary strings with area action. String tension would be determined by CP_2 size scale. The appearance of also light strings would distinguish between TGD and super string models.

Kähler action can contain also a topological instanton term affecting the field equations only via boundary condition. This term could induce to the string world sheet action a magnetic flux term reducing to a boundary term at the boundaries of string world sheets adding an interaction term to the usual action defined by word-line length. The outcome would be equation of motion for a point-like particle experiencing Kähler force. These topological terms give additional terms to corresponding modified Dirac equations.

It would seem that the new approach to action principle allows a more unified approach to the details of the variational principle in dimension $D = 4$ and allows also to deduce the general form of 6-D and 2-D effective action. It must be however made clear that one could have brane like hierarchy of structures already at fundamental level. Also in this case the new approach applies.

Action principle, quantum classical correspondence, and number theoretical universality

The above observations force to reconsider the interpretation of the action principle. Here the adelic physics based vision can be used as a guideline.

1. It is good to list the geometric parameters and coupling constant like parameters of TGD. CP_2 scale $R(CP_2)$ certainly appears in the theory. The radius of $S^2(M^4)$ makes l_P^2 a natural scale factor of M^4 metric. One can re-scale $J(M^4)$ and the M^4 part of the metric of $T(M^4)$ but not the entire metric.

2. $r = R_1/R(CP_2)$ can be seen as a dimensionless coupling constant like parameter and in principle quantum criticality allows it to have a spectrum values determined by the extension of rationals defining adeles. The QCC condition stating the quantized values of the fermionic Noether charges are equal to their classical counterparts having non-local expressions forces to consider the possibility that the value of R_1 can indeed vary and has value guaranteeing that QCC holds true. Also α_K has spectrum of values: one possible spectrum corresponds to the zeros of Riemann zeta [L16]. Even the number theoretically problematic exponent of action could belong to the extension with a suitable choice of R_1.

   This would allow to speak about the exponent of action and of Kähler function making sense also p-adically in the intersection of real and p-adic WCWs. Both action and its exponent should exist in the extension. This is true if the action is of form $q_1 + q_2\pi$, q_i rational numbers. One might hope that a suitable choice of R_1 could make possible to realize QCC and this condition.

QCC and the value spectrum of R_1

Classical field equations do not depend at all on the value on the overall coefficient X of the action in Eq. 14.2.1. Also boundary conditions are independent of the scaling of X. Does this mean that one has projective invariance in the sense that the value of R_1 does not matter at all? No!

1. QCC for the Cartan algebra of fermionic and classical Noether charges gives meaning for the scale R_1. QCC states that the eigenvalues of the Cartan algebra charges are equal to the corresponding values of classical Noether charges. Since the normalization of quantal charges is fixed by the value of $\hbar$, this fixes the normalization of classical charges and thus the parameter R_1. If Ψ is taken dimensionless, the modified Dirac action can be taken to be proportional to factor $1/R_1^3$. Therefore R_1 has physical meaning. The above argument suggests that R_1 is fixed by quantum criticality and characterizes the extension of rationals.

2. Could one require that the values of classical charges belong to the extension of rationals defining the adeles in question? This condition involves in real context integral over 3-surface and is thus a non-local operation. How can one know, which 3-surfaces satisfy the condition? Is the choice of R_1 dictated by this condition so that it depends on the extension of rationals involved and obeys number theoretic coupling constant evolution?

   Note that classical Noether charges serve as WCW coordinates, and the interpretation would be the same as at space-time level: these special 3-surfaces would form a kind of cognitive representation analogous to that formed by the points of space-time surface with coordinates in extension. The quantization of these WCW coordinates would give a cognitive representation!

3. The action would be same for the symmetry related 3-surfaces and one could have WCW wave functions at the orbits of symmetries with coordinates which are conjugate variables for the quantized Noether charges. For the orbits of symmetry groups the allowed points in WCW would correspond to values of group parameters in the extension. Besides isometries and corresponding Kac-Moody algebras supersymplectic symmetry gives rise to this kind of wave functions. In case of four-momentum, the basic number theoretic conditions would be for rest masses.

   Strong form of holography (SH) could be realized by the reduction of both bosonic and fermionic action to an effective action restricted to string world sheets and partonic 2-surfaces. This option looks more attractive from the point of view of SH than fundamental action containing terms located at lower-dimensional surfaces.

Number theoretical universality and action exponential

In adelic physics number theoretical universality plays a key role.

1. Adelic physics leads to the proposal that the action exponentials appearing in the scattering amplitudes disappear. The normalization factor defined by functional integral of action exponential to which also the scattering amplitude is proportional would cancel them as in QFTs [L38].

   This would require that each maximum of Kähler function with respect to variations of 3-surface and having fixed topological scattering diagram defined by light-like partonic orbits and same action defines its own zero energy state as functional integral and these states can be freely superposed. One would not functionally integrate over different topological scattering diagrams: this would allow to interpret topological scattering diagram as a representation of computation.

2. At the level of scattering amplitudes - but not at the level of WCW geometry - the absence of exponents would allow to get rid of the grave difficulty posed by the fact that the exponent of Kähler action belongs to an extension of rationals only when powerful additional conditions are satisfied. The cancellation of exponents of action from scattering amplitudes looks compelling if one requires number theoretical universality since there are no practical means for checking that the exponent of action is in the extension of rationals for an arbitrary preferred extremal. Also the definition of the action as integral is problematic in p-adic context and the only possible means to define it seems to be in terms of algebraic continuations from the real sector.

   One can however argue against number theoretical extremism. Action exponentials are needed for the interpretation of the theory. Maxima of Kähler function, which also correspond to stationary phase correspond to the most probable 3-surfaces. Hence one can argue that the exponents should appear in the scattering amplitudes. Number theoretical cognition theorist could however argue that the points of WCW, which correspond to maxima have WCW coordinates in an extension of rationals and thus define cognitive representation at the level of WCW. Furthermore, one can argue that scattering amplitudes are not the entire physics. Kähler action and its exponent have real meaning independent of scattering amplitudes.

3. On the other hand, if the value of R_1 adjusts to guarantee that the action is of form

$$S = q_1 + iq_2\pi \quad . \tag{14.2.2}$$

   exponents can appear in the amplitudes and the standard approach allowing functional integral giving sum of several exponents makes sense. In this case the scattering amplitudes are proportional to X_i/X, $X = \sum_i X_i$, where X_i denotes action exponent for a particular maximum of action as function of WCW coordinates. Note however that the action itself is not number theoretically universal: only its exponent. This is completely analogous with the fact that angles do not make sense p-adically and one can speak about corresponding phases identified as roots of unity.

Number theoretical universality (NTU) allows two options to consider depending on whether the action exponentials can appear in the scattering amplitudes or not. In WCW geometry action and also its exponent certainly appear.

1. The elimination of exponents of 6-D action from the scattering amplitudes would be a huge simplification and make practical calculations possible. This kind of assumption is in practice made also in standard path integral approach as approximation. ZEO allows this and the interpretation is in terms of the notion of quantum phase of matter: different topologies for partonic 2-surfaces correspond to different phases and the localization to single phase for zero energy states is possible: space-time would be much more classical object than without localization. One must however remain critical: the value of R_1 depending on extension of rationals could allow to achieve QCC conditions.

2. If something is gained, something is also lost. The earlier arguments involving exponent of Kähler function are lost if the exponentials do not appear in scattering amplitudes. In particular, the estimate for the value of gravitational coupling strength in terms of exponent of Kähler function and α_K (see the last section of [K3]) is lost if exponents do not appear anywhere. One can argue that this argument was actually lost already when the twistor lift was introduced and Planck length was transformed to a fundamental parameter appearing as scaling factor of M^4 Kähler form and metric.

There is a further challenge for the adelic physics. What could fix the value of the fundamental parameter $l_P^2/R^2(CP_2)$ (of order 10−7)? It seems that quantum criticality cannot help here. Both l_P^2 and R^2 appear in the induced metric of space-time surface and number theoretical universality for field equations demands that $l_P^2/R^2(CP_2)$ is a rational number. The p-adic evolution scenario of cosmological constant and empirical input for the cosmological constant gives $l_P^2/R^2(CP_2) = 2^{-12}$ [L24]. Why power of 2 which having unit p-adic norm for all odd primes and why just this power?

To sum up, a more precise adelic formulation of the classical action has allowed to detect a hitherto hidden scaling parameter in the action appearing as an additional coupling parameter depending on the extension of rationals, to understand better the number theoretical role of QCC, and allowed to answer a nagging question about whether to use metric determinant or its absolute value in the action assuming NTU for the exponential of action, and deduce the earlier conjecture for the zeros of zeta.

Answer to an old nagging question

Eq. 15.4.1 can be applied to the situation in which the extremal is known. For CP_2 type extremals volume and Kähler action (-4 times volume) are indeed known. Quite surprisingly, this suggest a solution to an old problem whether one should use $\sqrt{g_4}$ giving imaginary volume element in Minkowskian space-time regions or $\sqrt{|g_4|}$ used usually.

1. The action exponent

$$e^{\frac{x}{2\alpha_K}} \quad , \quad x = \frac{6Vol(CP_2)}{R_1^4}$$

is a number in an extension of rationals guaranteed if one has

$$(1/2)Re(\frac{1}{\alpha_K}) \times x = q_1 \quad , \quad (1/2)Im(\frac{1}{\alpha_K}) \times x = q_2\pi \quad .$$

.

2. Suppose that the volume integral uses volume element $\sqrt{g_4}$, which is imaginary in Minkowskian space-time regions and real in Euclidian regions. The motivation is that for real α_K the action exponential from Minkowskian space-time regions is phase as QFT picture demands.

For $1/\alpha_K = is = i/2 + y$, s a complex zero of zeta, the phase of the action exponential coming from Minkowskian regions is proportional to iy and in a good approximation equal to $1/Re(\alpha_K)$. The conditions give $Vol(CP_2)/R_1^4 \propto \pi$ and $y = q$. Note that $Vol(CP_2)$ is proportional to π^2 so that the normalization volume R_1^4 would be proportional to π. Since $R_1^4 = q \times Vol(CP_2)$ is natural normalization factor one would have expected x to be rational. This does not look promising.

That the zeros of zeta should be complex rationals is totally unexpected but would conform with the number theoretical universality. This would be of course very nice from TGD point of view strongly suggesting that zeros belong to some extension of rationals. I have proposed that the zeros of zeta appear as conformal weights in TGD framework [L16].

3. Suppose that the volume element is given by $\sqrt{|g_4|}$ as was done originally. If α_K is complex, the phase factor is obtained in any case. This option favours $1/\alpha_K = s$, s a complex zero of zeta. Eq. 15.4.1 would predict $Vol(CP_2)/R_1^4 = q$ and $y = q\pi$. These predictions conform with the physical intuition. I have proposed earlier [L16] that the exponents of imaginary parts for the zeros of zeta could correspond to roots of unity. Only the exponents of zeros of zeta would be number theoretically universal and continuable to the p-adic sectors.

To sum up, a more precise adelic formulation of the classical action has allowed to detect a hitherto hidden scaling parameter in the action appearing as an additional coupling parameter depending on the extension of rationals, to understand better the number theoretical role of QCC, and allowed to answer a nagging question about whether to use metric determinant or its absolute value in the action assuming NTU for the exponential of action, and deduce the earlier conjecture for the zeros of zeta.

There is however a further challenge for the adelic physics. What could fix the value of the fundamental parameter $l_P^2/R^2(CP_2)$ (of order 10−7)? It seems that quantum criticality cannot help here. Both l_P^2 and R^2 appear in the induced metric of space-time surface and number theoretical universality for field equations demands that $l_P^2/R^2(CP_2)$ is a rational number. The p-adic evolution scenario of cosmological constant and empirical input for the cosmological constant gives $l_P^2/R^2(CP_2) = 2^{-12}$ [L24]. Why power of 2 which having unit p-adic norm for all odd primes and why just this power?

14.2.2 The problem with cosmological constant

Second (unpleasant) observation was that the previous proposal for the twistor lift of Kähler action has an ad hoc feature.

Can the original proposal for the twistor lift of Kähler action be correct?

Consider first the unpleasant observation about cosmological constant.

1. α_K is also assumed to be complex and the conjecture [L16] has been that its values correspond to zeros of Riemann zeta. In the earlier proposal for twistor lift cosmological constant and α_K are assumed to obey independent p-adic evolutions, and cosmological constant was assumed to be real and to behave like $1/p$ as function of p-adic prime in p-adic length scale evolution so that its extreme smallness in cosmological scales could be understood [L22, L24].

   The motivation for the proposal was the decomposition $T(H) = T(M^4) \times T(CP_2)$ of the twistor space of H. It was argued that this allows to decompose the Kähler action of $T(H)$ to a sum of two parts with *different* values of α_K. For M^4 part the value of α_K, call it $\alpha_K(M^4)$, would be enormous and the resulting volume term in the dimensionally reduced 6-D Kähler action would have cosmological constant $\hbar/l_D^4$ as its coefficient: l_D would be of the order of the size about 10^{-4} meters of a large neuron in cosmological length scales.

2. If the value of $\alpha_K(M^4)$ is real or has different phase than $1/\alpha_K$, whose spectrum is proposed to correspond to zeros of zeta [L16], the action is complex, and one has separate field equations for real and imaginary part of action. The extremals would be minimal surface extremals of Kähler action. That all known extremals of Kähler action have this property was seen as a support for the hypothesis.

   The physically problematic aspect is that Kähler action and volume term effectively decouple. This would make sense asymptotically but looks strange as a general property [**?**] On the other hand, the independence of the extremals on coupling constants is a highly desirable outcome from the point of view of number theoretical universality.

3. The assumption about different Kähler coupling strengths admittedly looks somewhat ad hoc. If one assumes that also M^4 possesses Kähler form $J(M^4)$ [L40], and induced Kähler form corresponds to the sum $J(M^4) + J(M^2)$, universal value of α_K is the natural option. This assumption however allowed to understand the smallness of cosmological term in 4-D action and also the p-adic coupling constant evolution for the cosmological constant.

4. Also boundary conditions are problematic for this option. It would be highly desirable to have flow of classical Noether charges between Euclidian and Minkowskian space-time regions as a correlate for classical interactions between physical objects having Euclidian regions as space-time correlates (analogous to lines of scattering diagrams). The conditions stating the conservation of sums of complex Kähler and volume charges from Minkowskian and Euclidian regions however give 2+2 conditions if the phases of Kähler action and volume term are different and the metric determinant $\sqrt{g_4}$ is imaginary for Minkowskian regions. It is easy to see that Kähler and volume charges are conserved separately and that there is no charge transfer between Euclidian and Minkowskian regions. The alternative $\sqrt{|g_4|}$ allows the flow of real and imaginary charges between the two regions. One can however insist that the existence of two separate conserved energies should have been discovered long time ago.

What if one gives up the assumption $\alpha_K(M^4) \neq \alpha_K(CP_2)$?

1. The volume term would be also proportional to $1/\alpha_K$ so that the phases of both Kähler action and volume term would be identical. The pleasant surprise is that coupling constants disappear from the field equations altogether! It is not necessary to postulate minimal surface property of the preferred extremals anymore to guarantee number theoretical universality.

   Minimal surface property could be however asymptotic so that there would be no exchange of conserved quantities between these degrees of freedom. This would conform with the idea that incoming and outgoing particles are free and thus minimal surfaces as 4-D generalization of a geodesic line resulting when 4-D generalization of Abelian Maxwell force vanishes. Causal diamond (CD) would represent a region with the property that the extremals approach minimal surfaces at its boundary. One can loosely say that interactions are coupled on and off near the opposite boundaries of CD: CD corresponds to scattering volume.

   The vertices of topological diagrams defined by as 2-D intersections of the ends of orbits of partonic 2-surfaces - analogous to vertices of Feynman diagrams - would be also accompanied by transient regions, where there the motion of 3-surface is not geodesic. The results are extremely nice from the point of view of number theoretical universality.

2. Also in this case the charge transfer between Euclidian and Minkowskian regions is impossible if $\sqrt{g_4}$ defines volume element (imaginary in Minkowskian regions). $\sqrt{|g_4|}$ this is not the case. As found, also NTU favors this option.

3. The above result is extremely nice. What makes the shower cold is that one ends up with problems with cosmological constant since Kähler and volume terms in the action are of same order of magnitude. Also the proposed p-adic evolution scenario for the cosmological constant is lost. The only cure that I can imagine is that the entire 4-D action has interpretation as a cosmological term, and that a cancellation between Kähler action and volume term take place giving rise to a very small effective value of cosmological constant.

Can one understand the p-adic evolution of cosmological constant?

The above findings lead to a problem with cosmological constant.

1. If the cosmological constant corresponds to the volume term in the dimensionally reduced 6-D Kähler action with scaling factor $X = 1/2\alpha_K R_1^2 S_0$, one has from Eq. 14.2.1

$$\rho_{vac} = \frac{1}{l_D^4} = \frac{2}{\alpha_K R_1^4} \frac{Area(S^2(X^4))}{S_0} = \frac{\Lambda}{8\pi l_P^2} \quad . \tag{14.2.3}$$

Here l_D corresponds to a length scale which is roughly the size 10^{-4} meters of large neuron for cosmological constant in cosmic scales. Also Kähler action would be extremely small. It would however seem that the ratio of these actions should be extremely small. The simplest solution corresponds to $\frac{Area(S^2(X^4))}{S_0} = 1$.

2. The Kähler action for CP_2 type extremal with light-like geodesic as M^4 projection the action would be

$$S = -3\frac{Vol(CP_2)}{l_D^4} \quad .$$

The action has totally different order of magnitude than assumed earlier if R_1 corresponds to the value of cosmological constant. If one assumes $R_1 = R(CP_2)$, cosmological constant is enormous. Something seems to go wrong.

How could one overcome this problem?

1. Could l_D be small and imply large cosmological constant? Could the parameter $X = \frac{Area(S^2(X^4)}{S_0}$ be small and increase the effective size of l_D? Could the time-like signature for $S^2(M^4)$ allow this by reducing the value of $Area(S^2(X^4)$?

   One can study the imbedding of $S^2(X^4)$ to $S^2(M^4)$ and $S^2(CP_2)$ characterized by winding numbers n_1 and n_2. One can choose S_0 to be the area for the imbedding with $n_1 = n_2 = 1$. This gives $\frac{Area(S^2(X^4)}{S_0} = (n_1X^2 - n_2)(X^2 - 1)$, $X^2 = (R^2(CP_2)/l_P^2)$ for time-like signature for $S^2(M^4)$. The condition $\frac{Area(S^2(X^4)}{S_0} = 1/p$ would give p-adic length scales but could be satisfied for finite number of primes p only. Second problem is that this would *not* affect the ratio of Kähler and volume contributions to the action.

2. Could effective cosmological constant correspond to the entire action so that Kähler would cancel the real cosmological term in cosmological scales?

   Could $J \cdot J - 2$ should become small in Minkowskian regions and be necessarily large in Euclidian regions? The positive Kähler electric contribution to the action should sum up to almost zero with the negative magnetic contribution and cosmological term. This cancellation should take place in cosmic scales at least and require long range induced Kähler electric fields. They are assumed to be present in the model for large voids. If M^4 Kähler form is present as CP breaking and some other arguments suggest [L40] [L24], it could give a large Kähler electric contribution in long scales if CP_2 contribution becomes small as one might expect.

   The values of 6-D Kähler action should have tendency to concentrate around values inversely proportional to prime p near power of 2 (also other small primes can be considered). The values of Kähler action for the maxima of Kähler function could have this property. This conjecture was made earlier in an attempt to understand gravitational constant in terms of p-adic length scale hypothesis and the exponent of Kähler action for CP_2 type extremals (see the last section of [K3]).

3. This interpretation would mean that for strings like objects having both vanishing induced M^4 and CP_2 parts of induced Kähler fields the action would be large and coming from cosmological constant in CP_2 scale, and one could at least formally say that the situation is perturbative. Strings could however carry non-vanishing and large M^4 parts of Kähler electric fields and the action could be small in this case.

4. I must be added that the interpretation of cosmological constant has varied during years. For the 4-D Kähler action the proposal was that cosmological constant corresponds to the magnetic part of Kähler action with magnetic tension responsible for the negative pressure. The twistor lift in turn led to ask whether Kähler action and volume term could provide alternative, dual manners to understand cosmological constant. For the recent option the small effective cosmological constant results from the cancellation of Kähler action and volume term.

The cautious conclusion would be following. If the 6-D Kähler action contains only single α_K, the cosmological constant is very large at short scales and for Euclidian space-time regions. The cancellation of Minkowskian Kähler electric contribution and Kähler magnetic action in 6-D sense however makes the effective value of cosmological very small. The solution of the problem of cosmological constant would be dynamical. The previous option for which Kähler action decomposes to M^4 and CP_2 parts with different values of $\alpha_K(M^4)$ and $\alpha_K(CP_2) \leq\leq \alpha_K(M^4)$ cannot be however excluded.

14.3 Twistor lift of TGD, hierarchy of Planck constant, quantum criticality, and p-adic length scale hypothesis

Kähler action is characterized by enormous vacuum degeneracy: any four-surface, whose CP_2 projection is Lagrangian sub-manifold of CP_2 having therefore vanishing induced Kähker form, defines a vacuum extremal. The perturbation theory around canonically imbedded M^4 in $M^4 \times CP_2$ defined in terms of path integral fails completely as also canonical quantization. This led to the construction of quantum theory in "world of classical worlds" (WCW) and to identification of quantum theory as classical physics for the spinor fields of WCW: WCW spinors correspond to fermionic Fock states. The outcome is 4-D spin glass degeneracy realizing non-determinism at classical space-time level [K34, K15, K88, K110].

The twistor lift of TGD is based on unique properties of the twistor spaces of M^4 and CP_2. Note that M^4 allows two notions of twistor space. The first one involves conformal compactification allowing only conformal equivalence class of metrics. Second one is equal to Cartesian product $M^4 \times S^2$ [B72] (see `http://tinyurl.com/yb4bt74l`). CP_2 has flag manifold $SU(3)/U(1) \times U(1)$ as twistor space having interpretation as the space for the choices for quantization axis of color hypercharge and isospin. Both these spaces Kähler structure (strictly speaking E^4 and S^4 allow it but the notion generalizes to M^4) and there are no others. Therefore TGD is unique both from standard model symmetries and twistorial considerations.

The existence of Kähler structure is a unique hint for how to proceed in the twistorial formulation of classical TGD. One must lift Kähler action to that in the twistor space of space-time surface having also S^2 as a fiber and identify the preferred extremals of this 6-D Kähler action as those of dimensionally reduced Kähler action, which is 4-D Kähler action plus volume term identifiable in terms of cosmological constant. As found, there are two options to consider.

1. Option I: The values of $\alpha_K(M^4)$ and $\alpha_K(CP_2)$ are widely different with $\alpha_K(M^4)$ being extremely large so that M^4 part of the 6-D Kähler action gives in dimensional reduction extremely small cosmological term. Allowing Kähler coupling strength $\alpha_K(CP_2)$ to correspond to zeros of zeta implies that for complex zeros the preferred extremals for $\alpha_K(M^4)$ having different phase are mimimal surface extremals of Kähler action so that the values of coupling constants do not matter and extremals depend on couplings only through the boundary conditions stating the vanishing of certain super-symplectic conserved charges. In this case the cosmological constant would correspond to running $\alpha_K(M^4)$ and would behave like $1/p$, p p-adic prime. This was the original proposal.

2. Option II: $\alpha_K(M^4) = \alpha_K(CP_2)$ is satisfied. A small effective value of cosmological constant is obtained if the Kähler action and volume term tend to cancel each other. In this case minimal surface extremals of Kähler action correspond naturally to asymptotic dynamics near the boundaries of CDs, where the analog of free geodesic motion as minimal surfaces is expected. In this case effective cosmological constant would correspond to the *entire action*: volume term and Kähler action receiving also M^4 contribution would cancel almost completely in cosmic scales.

   One can in fact argue that one cannot distinguish between Kähler and volume contributions to the action so that Option II remains the only possible one. Option I also breaks the symmetry between Kähler forms of M^4 and CP_2. It is natural that the induced Kähler form is the sum of both and appears in the Kähler action: hence $\alpha_K(M^4) = \alpha_K(CP_2)$.

Option I might be argued to be adhoc but at this moment it is not yet wise to select between these two options. The most conservative assumption is that the twistorial approach is only an alternative for the space-time formulation: in this formulation preferred extremal property might reduce to twistor space property.

Kähler action gives as fundamental constants the radius $R \simeq 2^{12} l_P$ of CP_2 serving as the TGD counterpart of the unification scale of GUTs and Kähler coupling strength α_K in terms of which gauge coupling strengths can be expressed. Twistor lift gives 2 additional dimensional constants. The radius of S^2 fiber of M^4 twistor space $M^4 \times S^2$ is essentially Planck length $l_P = \sqrt{G/\hbar}$, and the cosmological constant $\Lambda = 8\pi G\rho_{vac}$ defining vacuum energy density is dynamical in the sense that it allows p-adic coupling constant evolution as does also α_K.

For both Option I and II one can imagine two options for the p-adic coupling constant evolution of cosmological constant.

1. $\rho_{vac} = k_1 \times \hbar/L_p^4$, where $p \simeq 2^k$ characterizes a given level in the p-adic length scale hierarchy for space-time sheets. Here one can in principle allow $k_1 \neq 1$.

2. $\Lambda/8\pi = k_2/L_\Lambda^2 \propto \frac{1}{p_\Lambda^2}$. Also k_2 could differ from unity. Number theoretical universality suggests $k_1 = k_2 = 1$. The that here secondary p-adic length scale is assumed.

The first option seems more natural physically. During very early cosmology $\Lambda R^2/8\pi$ approaches l_P^2/R^2 for the first option, where $R \simeq 2^{12} l_P$ is the size scale of CP_2 so that one has $\Lambda R^2/8\pi \simeq 2^{-24} \simeq 6 \times 10^{-8}$ at this limit. Therefore perturbation theory would fail for Option I also in early cosmology near vacuum extremals. In the recent cosmology Λ is extremely small. Note that vacuum energy density would be always smaller than $\hbar/R^4$ and thus by a factor $(l_P/R)^4 \simeq 2^{-48} \simeq 3.6 \times 10^{-15}$ lower than in GRT based cosmology.

It it is good go recall that the earlier identification of the cosmological constant was in terms of the effective description for the magnetic energy density of the magnetic flux tubes. Magnetic tension would give rise to effective negative pressure. For Option II the cosmological constant would correspond to the entire action with magnetic and volume contributions slightly larger than Kähler electric contribution. For Option I it wold correspond to the volume term.

14.3.1 Twistor lift brings volume term back

Concerning volume term the situation changed as I introduced twistor lift of TGD. One could say that twistor lift forces cosmological constant. As already described, there are two options: Option I and Option II. The following arguments developed for Option I apply with small modifications also to Option II. The only difference is that the volume term has complex phase for complex α_K [L16] and effective cosmological constant follows from the compensation of Kähler and volume contributions.

1. The twistor lift of Kähler action is 6-D Kähler action for the twistor space $T(X^4)$ of space-time surface X^4. The analog of twistor structure would be induced from the product $T(M^4) \times T(CP_2)$, of twistor spaces $T(M^4) = M^4 \times S^2$ of M^4 [B72] and $T(CP_2) = SU(3)/U(1) \times U(1)$ of CP_2 having Kähler structure so that the induction of Kähler structure to $T(X^4)$ makes sense. Besides M^4 and CP_2 only the spaces E^4 and the S^4, which are variants of M^4 have twistor space with Kähler structure or analog of it. The induction conditions would imply dimensional reduction so that the 6-D Kähler action for the twistor lift would reduce to 4-D Kähler action plus volume term identifiable in terms of cosmological constant Λ.

2. 4-D Kähler action has Kähler coupling strength α_K as coupling parameter and volume term has coefficient $1/L^4$ identifiable in terms of cosmological constant

$$\frac{1}{L^4} \equiv \frac{\Lambda}{8\pi l_P^2} \quad .$$

$l_P = \sqrt{G/\hbar}$ would correspond to the radius of twistor sphere for M^4 and thus becomes fundamental length scale of twistorially lifted TGD besides radius of CP_2. Note that the radius of twistor sphere of CP_2 is naturally CP_2 radius.

L is in the role of coupling constant and expected to obey discrete p-adic coupling constant evolution $L \propto \sqrt{p}$, prime or prime near power of two if p-adic length scale hypothesis is accepted. In the recent cosmology L could correspond to the p-adic length scale $L(175) \simeq 40$ μm, the size of large neuron.

L $\simeq 40\mu$m corresponds to the energy scale $E = 1/L \simeq .031$ eV, which is thermal energy at temperature of 310 K (40 C) - the physiological temperature. A deep connection with quantum biology is suggestive. Also the energy scale defined by cell membrane potential is in this energy scale. This energy scale about 10 times smaller than the mass scale of neutrinos.

Also $L_\Lambda = \sqrt{8\pi/\Lambda}$ would satisfy p-adic coupling constant evolution as already discussed. Now the p-adic length scale would be secondary p-adic length scale $L_\Lambda = L(2,p) = \sqrt{p} \times (R/l_P)$, l_P Planck length. p-Adic length scale hypothesis demands that R/l_P - the ratio for the radii of CP_2 and twistor sphere is power of 2. p-Adic mass calculations indeed allow this ratio can be indeed chosen to be equal to $R/l_P = 2^{12}$.

14.3.2 ZEO and twistor lift

The volume term, which I gave up 38 years ago, has creeped back to the theory! The infinite value of volume for space-time surfaces of infinite duration? This would not make the notion of vacuum functional poorly defined. Should one forget twistor lift because of this? No! ZEO saves the situation.

In ZEO given CD defines a sub-WCW consisting of space-time surfaces inside CD. This implies that the volumes for the M^4 projections of allowed space-time surfaces are smaller than CD volume having the order of magnitude $L^4(CD)$, $L(CD)$ is the temporal distance between the tips of CD (one has $c = 1$). I have also proposed that $L(CD)$ is quantized in multiples of integers, primes or primes near power of two so that the identification might make sense. $L(CD) = L$ is not possible due to the small value 40 μm of L but $L(CD) = L_\Lambda$ could make sense.

Stationary phase condition and ZEO

The preferred extremal property realizing SH poses extremely strong constraints on the value of total action and it should force the phase defined by action to be stationary so that interference effects would be practically absent. This argument assumes that the action exponentials indeed appear in the scattering amplitudes defined by the WCW spinor fields in ZEO. NTU however forces to challenge this assumption unless one assumes that action is quantized as $q_1 + iq_2\pi$: this might be achieved by the quantization of the overall scale factor X of the action. The construction of twistor scattering amplitude suggests that the cancellation of action exponentials might be indeed achieved. If the exponents are present, the question is how the stationarity of phase could be achieved.

1. The most general possibility is that the phase of the vacuum functional can be large but is localized around very narrow range of values. The imaginary part of the action S_{Im} for preferred extremals should be around values $S_{Im} = A_0 + n2\pi$. Standard Bohr orbitology indeed assumes the quantization of action in this manner. One could also argue that just the absence of destructive interference demands Bohr quantization of the action in the vacuum functional. Whether preferred extremal property indeed gives rise to this kind of Bohr quantization, is an open problem. The real exponent of the vacuum functional should in turn be large enough and positive values are favored. They are however bounded in ZEO because of the finite size of CDs.

2. To proceed further one must say something about the value spectrum of α_K. In the most general situation α_K is complex number: the proposal of [L16] is that the discrete p-adic coupling constant evolution for $1/\alpha_K$ corresponds to a complex zero $s = 1/2 + iy$ of Riemann zeta: also the trivial real zeros can be considered. For large values of y the imaginary part of y would determine $1/\alpha_K$ and $Re(s) = 1/2$ would be responsible for complex value of α_K. This makes sense since quantum TGD can be regarded formally as a complex square root of thermodynamics.

3. Denote by $S = S_{Re} + iS_{Im}$ the exponent of vacuum functional. For complex values of $1/\alpha_K$ S_{Im} and S_{Re} receive a contribution from both Euclidian and Minkowskian regions and a

contribution also from the Minkowskian regions. For S_{Im} the contributions should obey the condition

$$S_{Im} = S_{Im}(M) + S_{Im}(E) \simeq A_0 + n2\pi \quad (14.3.1)$$

to achieve constructive interference.

For real parts the condition $S_{Re} = S_{Re}(M)+S_{Re}(E)$ must be small if negative. Large positive values of S_{Re} are favored. S_{Re} automatically selects the configurations, which contribute most and among these configurations the phase $exp(iS_{Im})$ must be stationary. The conditions for S_{Im} relate the values of action in the Euclidian and Minkowskian regions. If α_K is real, one has $S_{Im}(M) \simeq A_0 + n2\pi$ and $S_{Re}(E)$ small if negative and Euclidian and Minkowskian regions effectively decouple in the conditions. It seems that complex values of α_K are indeed needed.

4. $S_{Re}(E) = S_{Re}(M)+S_{Re}(E)$ receives a positive contribution from Euclidian regions. Minkowskian regions a contributions for complex value α_K. Both positive and negative contributions are present and the character of these contributions depends on sign of the imaginary part of α_K. Depending on the sign factor ± 1 of $Im(1/\alpha_K)$ Minkowskian regions give negative (positive) contribution from the space-time regions dominated by Kähler electric fields and positive (negative) contribution from the volume term and the regions dominated by Kähler magnetic field.

   The option "+" for which Kähler magnetic action and volume term give positive contribution to $S_{Re}(M)$ looks physically attractive. "+" option would have no problems in ZEO since the contribution to S_{Re} would be automatically positive but bounded by the finite size of CD: this would give a deep reason for the notion of CD (also the realization of super-symplectic symmetries gives it). For "-" option Minkowskian regions containing Kähler electric fields would be essential in order to obtain $S_{Re} > 0$: Kähler magnetic fields would not be favored and the unavoidable volume term would give wrong sign contribution to $S_{Re} > 0$.

The condition $S_{Im} \leq \pi/2$ is not realistic

One can look what the mere volume term contributes to S_{Im} assuming $S_{Im} \leq \pi/2$. Volume term dominates for near to vacuum extremals with a small Kähler action: in particular, for string like objects $X^2 \times S^2$, S^2 a homologically trivial geodesic sphere with vanishing induced Kähler form. It turns out that these conditions are not physically plausible and that $S_{Im} \simeq A_0 + n2\pi$ is the only realistic option.

1. Cosmological constant (parametrizable using the scale L) together with the finite size of CD gives a very stringent upper bound for the volume term of the action: $A = vol(X^4)/L^4$. The rough estimate is that for the largest CDs involved the volume action is not much larger than $L^4\pi/2$ in the recent cosmology. In the recent cosmology L would be only about 40 μm so that the bound is extremely strong! and suggests that $S_{Im} < \pi/2$ is not a realistic condition.

2. $L(CD) = L$ is certainly excluded. Can one have $L(CD) = L_\Lambda$? How can one achieve space-time volume not much larger that L^4 for space-time surfaces with duration $L(CD)$? Could magnetic flux tubes help! For the simplest string like objects $X^2 \times Y^2$, where $X^2 \subset M^4$ is minimal surface and Y^2 a 2-D surface (complex sub-manifold of CP_2) the volume action is essentially

$$Action = \frac{V}{l_P^2 L_\Lambda^2} = \frac{Area(X^2)}{L_\Lambda^2} \times \frac{Area(Y^2)}{l_P^2} \quad . \quad (14.3.2)$$

   The conservative condition for the absence of destructive interference is roughly $Action < \pi/2$.

3. To get a more concrete idea about the situation one can use the parameterization

$$Area(string) = L(CD) \times L(string) \ \ , \ \ Area(Y^2) = x \times 4\pi R^2 \ \ . \tag{14.3.3}$$

x is a numerical parameter, which can be quite large for deformations of cosmic strings with thick transversal M^4 projection. The condition for the absence of destructive interference is roughly

$$\frac{L(CD)\times L(string))}{L_\Lambda^2} \times x \times \frac{4\pi R^2}{l_P^2} < \frac{\pi}{2} \ \ . \tag{14.3.4}$$

For $L(string) \ll L(CD)$ one can have space-time surfaces of temporal duration $L(CD) = L_\Lambda$. For these the condition reduces to

$$\begin{aligned} & y \times x < \pi \frac{l_P^2}{4\pi R^2} = 2^{-13}\pi \ \ , \\ & y \equiv \frac{L(string)}{L_\Lambda} \ \ . \end{aligned} \tag{14.3.5}$$

For deformations the transversal area of string like object can be also chosen to be considerably larger than the area of geodesic sphere. For flux tubes of length of order 1 AU the one have $y \sim 10^{-16}$. This would require $x \leq 10^{13}$. This would correspond to a radius $L(Y^2)$ about $10^6 R$ much smaller than required.

For $L(string) \sim L$ this would give $y \sim 10^{-31}$ giving $x \leq 10^{28}$ $L(Y^2) \leq 10^{14}R$, which corresponds to elementary particle scale. Still this fails to fit with intuitive expectations, which are of course inspired by the standard positive energy ontology.

4. One could try to invent mechanisms making volume term small. The required reduction would be enormous. This does look sensible. One can have vacuum extremals of Kähler action for which CP_2 projection is a geodesic line: $\Phi = \omega t$. The time component $g_{tt} = 1 - R^2\omega^2$ of the flat metric can be arbitrarily small so that the volume proportional to $\sqrt{gtt}$ can be arbitrarily small. One expects that this happens in early cosmology but as a general mechanism this is not plausible. Also very rapidly rotating string like objects with small area of string world sheet are in principle possible but do not represent a realistic option.

The cautious conclusion is that Bohr quantization $S_{Im} \simeq A_0 + n2\pi$ is the only sensible option. The hypothesis that the coupling constant evolution for $1/\alpha_K$ is given in terms of zeros of Riemann zeta seems to be consistent with this picture and correlates the values of actions in Minkowskian and Euclidian regions.

14.3.3 Hierarchy of Planck constants

One motivation (besides motivations from bio-electromagnetism and Nottale's work [E1]) for the hierarchy of Planck constants $h_{eff} = n \times h$ identified as gravitational Planck constants $\hbar_{gr} = GMm/v_0$ at the magnetic flux tubes mediating the gravitational interaction was that it effectively replaces the large coupling parameter GMm with dimensionless coupling $v_0/c < 1$. This assumes quantum coherence in even astrophysical length and time scales. For gauge interaction corresponding to gauge coupling g one $\hbar_g = Q_1Q_2\alpha/v_0$. Also Kähler coupling strength α_K to α_K/n and makes perturbation theory converging for large enough value of n.

The geometric interpretation for $h_{eff} = n \times h$ emerges if one asks how to make the action large for very large value of coupling parameter to guarantee convergence of functional integral.

1. The answer is simple: space-time surfaces are replaced with n-fold coverings of a space-space giving n-fold action and effectively scaling h to $h_{eff} = n \times h$ so that coupling strength scale down by $1/n$. The coverings would be singular in the sense that at the 3-D ends of space-time surface at the boundaries of causal diamond (CD) the sheets co-incide.

2. The branches of the space-time surface would be related by discrete symmetries. The symmetry group could be Galois group in number theoretic vision about finite measurement resolution realized in terms of what I call monadic or adelic geometries [L26] [K117].

   On the other hand, the twistor lift suggests that covering could be induced by the covering of the fiber $S^2(X^6)$ by the spheres $S^2(M^4 \times S^2)$ and the twistor space $S^2(SU(3)/U(1) \times U(1))$ defining fibers of twistor spaces of M^4 and CP_2. There would be gauge transformations transforming the light-like parton orbits to each other and the discrete set would consists of gauge equivalence classes. These two identifications for the symmetries could be equivalent.

$h_{eff} = h_{gr} = n \times h$ would make perturbation theory possible for the space-time surfaces near vacuum extremals. For far from vacuum extremals Kähler action dominates and one would have $h_{eff} = h_{g_K} = n \times h$. This picture would conform with the idea that gravitational interactions are mediated by massless extremals (MEs) topologically condensed at magnetic flux tubes obtained as deformations of string like objects $X^2 \times S^2_I$, S^2_I a homologically trivial geodesic sphere of CP_2. The other interactions could be mediated in the similar manner. The flux tubes would be deformations of $X^2 \times S^2_{II}$, S^2_{II} a homologically non-trivial sphere so that the flux tubes would carry monopole flux.

The enormously small value of cosmological constant would require large value of $h_{eff}/h = n$ explaining the huge value of h_{gr} whereas for other interactions the value of n would be much smaller. Since only the size of the action matters, this is true for both Option I and Option II. One can consider also variants of this working hypothesis. For instance, all long range interactions mediated by massless quanta could correspond to extremals for which cosmological constant is small.

What smallness requires depends on option. For Option I the reason is that very long homologically non-trivial magnetic flux tubes tend to have large energy (the energy goes as $1/S$) so that homologically trivial flux tubes having only vacuum energy are favored. For Option II the cancellation of Kähler action and volume term is necessary. The compensating Kähler electric action could come from the M^4 Kähler from $J(M^4)$. These flux tubes could be also homologically non-trivial

Quantum criticality would suggest that both homologically trivial and non-trivial phases are important. In TGD inspired quantum biology [K37] I have considered the possibility that structures with size scaled by $h_{eff}/h = n$ can transform to structures with $n = 1$ but p-adic length scale scaled up by n. Here n would be power of two by p-adic length scale hypothesis.

This would have interpretation in terms of quantum criticality. Homologically non-trivial string like objects with given string tension determined by Kähler action would be transformed to homologically trivial string like objects with the same string tension but determined by the cosmological constant term. This would give a condition on the value of the cosmological constant and thickness of flux tubes to be discussed later.

14.3.4 Magnetic flux tubes as mediators of interactions

The gravitational Planck constant $\hbar_{gr} = GMm/v_0$ [K66, K53, K109, K106] introduced originally by Nottale [E1] depends on the large central mass M and small mass m. This makes sense only if h_{gr} characterizes a magnetic flux tube connecting the two masses. Similar conclusion holds true for h_g. This leads to a picture in which mass M has involves a collection of radial flux tubes emanating radially from it. This assumption makes sense in many-sheeted space-time since the fluxes can go to the another space-time sheets through wormhole contacts associated also with elementary particles. For single-sheeted space-time one should have genuine magnetic charges.

This picture encourages a strongly simplified vision about how holography is realized. From center mass flux tubes emanate and in given size scale of the space-time sheet from by the flux tubes having say spherical boundary, the boundary is decomposed of pixels representing finite number of qubits. Each pixel receives one flux tube.

Vacuum energy for Options I and II

For Option I and magnetic flux tubes with vanishing Kähler form carry mere vacuum energy and are candidates for the mediators of long range interactions including gravitation. The homologically trivial flux tubes carry vacuum energy, which by flux conservation is proportional to $1/S$, where S is surface area. Long flux tubes are necessarily thick.

For Option II the thin magnetic flux tubes with vanishing induced Kähler form have very large tension and could be perturbative so that there would be no need for large values of $h_{eff}/h = n$. These flux tubes are expected to be short. The string world sheets mediating gravitational interaction should be long and have small string tension. They would naturally carry non-vanishing Kähler electric field in the direction of string (and flux tube).

1. Gravitational action (interaction energy from $J(M^4)$) and volume action (energy) would compensate to give a small cosmological constant forcing $h_{eff}/h = n$ hierarchy describing dark matter. Thus $J(M^4)$ crucial for understanding CP breaking and matter antimatter asymetry would be also crucial for the smallness of cosmological constant. This option looks physically rather attractive.

2. For flux tubes with vanishing induced $J(CP_2)$ the condition for cancellation would be $J \cdot J - 2 \simeq 0$. The compensating Kähler field would be electric and would naturally due to $J(M^4)$ and also responsible for the gravitational field along flux tube at QFT limit. Compensation of actions giving a small and scale dependent cosmological constant requiring large $h_{eff}/h = n = h_{gr}/h$ is possible.

3. For flux tubes with Kähler magnetic tube carrying magnetic monopole flux the cancellation condition would $J(M^4) \cdot J(M^4) - 2 - J(CP_2) \cdot J(CP_2) \simeq 0$. The thickening of flux tubes weakening the value of $J(CP_2)$ behaving from flux conservation like $J(CP_2) \propto 1/S$, S the cross sectional area of the flux tube, should make approximate cancellation possible. Elementary particles would represent an example of structures formed by closed monopole flux tubes assignable with a pair of space-time sheets. Homologically non-trivial magnetic flux tubes with small string tension could explain the mysterious cosmic magnetic fields: homological non-triviality implies that no current is needed to create the fields.

Magnetic flux tubes as carriers of magnetic energy

The holographic picture leads to a picture about vacuum energy. The following arguments developed originally for Option I should apply to both options since it is enough that magnetic flux tubes have only low vacuum energy density. Possible delicacies relate to the fact that small Kähler action $(E^2 - B^2)$ does not necessarily mean small Kähler energy. For Option II this situation is however not encountered.

1. Vacuum energy can be expressed as a sum of energies assignable to the flux tubes. Same applies to Kähler interaction energy. The contribution of individual flux tube is proportional to its length given by radius r of the large sphere considered. The total vacuum energy must be proportional to r^3 so that the number of flux tubes must be proportional to r^2. This implies that single flux tube correponds to constant area ΔS of the boundary sphere for given value of cosmological constant. The natural guess is that ΔS is of the same order of magnitude as the area defined by the length scale defined L by the vacuum energy density $\rho_{vac} = \Lambda/8\pi G$ allowing parameterization $\rho_{vac} = k_1 \hbar/L^4$.

2. In the recent cosmology one has $\hbar/L(now) \simeq .029$ eV, which equals roughly to $M/10$, where $M = \sum m(\nu_i) \simeq .032 \pm 0.081$ eV is the sum of the three neutrino masses. L is given as a geometric mean

$$L = \sqrt{L_\Lambda l_P} \simeq .42 \times 10^{-4}$$

meters of length scales $l_P = \sqrt{G/\hbar}$ and $L_\Lambda = (8\pi/\Lambda)^{1/2}$. $L(now)$ corresponds to the size scale of large neuron. This is perhaps not an accident.

The area of pixel must be of order $L^2(now)$ suggesting strongly a p-adic length scale assignable with neuron: maybe neuronal system wold realize holography. $L(151) = 10$ nm (cell length scale thickness) and $L(k) \propto \sqrt{p} \simeq 2^{k/2}$ gives the estimate $p \simeq 2^k$, $k = 175$: the p-adic length scale is 4 per cent smaller than $L(now)$.

3. The pixel area would be by a factor $L^2(now)/l_P^2$ larger than Planck length squared usually assumed to define the pixel size but would conform with the p-adic variant of Hawking-Bekenstein law in which p-adic length scale replaces Planck length [K49].

The value of the vacuum energy density for a given flux tube is proportional to the value of $h_{eff}/h = n$ by the multi-sheeted covering property. Vacuum energy cannot however depend on n. There are two manners to achieve this: local and global.

1. For the local option the energy of each flux tube would remain invariant under $h \rightarrow n \times h$ as would also the number N of flux tubes. This requires that the cross section S of the radial gravitational flux tube to which energy is proportional, scales down as S/n. This looks strange.

2. For the global option flux tubes are not changed but the number N of the radial flux tubes scales down as $N \propto 1/n$: one has $Nn = constant$. In the situation in which Kähler magnetic energy dominant local option demands $S \propto n$ and global option $N \propto 1/n$. Nn constant conditions brings in mind something analogous to Uncertainty Principle. The resolutions characterized by N and n are associated with complementary variables.

The global option applies to both homologically trivial and non-trivial options and is more promising.

Could the value of endogenous dark magnetic field relate to cosmological constant?

TGD development of inspired model for quantum biology was initiated by the observation [J2] that ELF em fields have non-trivial effects on the brain physiology and behavior of vertebrates [K94, K59]. Since the energies of ELF photons (with frequencies in EEG range) are many orders of magnitude below thermal energy, the proposal was that one has dark photons having $h_{eff}/h = n$ increasing the value of the energy $E = h_{eff}f$ of ELF photons above thermal energy, possibly even to the energies of bio-photons in visible and UV range identified as resulting in a phase transition reducing h_{eff} to its value for visible matter.

The effects appear at multiples of cyclotron frequencies of biologically important ions in endogenous ("dark") magnetic field of $B_{end} \simeq .2$ Gauss. This corresponds to magnetic length $1/\sqrt{eB}$ not far from the size of large neuron. Could this field strength correspond to the Kähler magnetic field assignable to the flux tubes carrying monopole magnetic field, whose strength is determined by the value of cosmological constant? This would give a direct connection between cosmology and biology!

1. In recent cosmology the value of B_K (more precisely, $g_K B_K$ using ordinary conventions) at criticality would be

$$B_K = \frac{\Phi_0}{4\pi}\frac{1}{L^2(175)} \ .$$

B_K corresponds to the U(1) magnetic field in standard model and is therefore as such not the ordinary magnetic field. For S^2_{II} Kähler magnetic field is non-vanishing. If Z^0 field vanishes, classical em field (with e included as normalization factor) equals to $\gamma = 3J$, where J is K"ahler induced Kähler form (see [L2]. One has

$$B_K = \frac{eB_{em}}{3} \ . \tag{14.3.6}$$

2. An interesting question is whether one could identify physically the ordinary magnetic field assignable to the critical Kähler magnetic field.

   Earth's magnetic field $B_E = .5$ Gauss corresponds to magnetic length $L_B = \sqrt{\hbar e B} = 5\mu$m. Endogenous magnetic field $B_{end} \simeq 2B_E/5$ explaining the findings of Blackman [J2] about the effects of ELF em fields on vertebrate brain in terms of cyclotron transitions corresponds to $L_B = 12.5$ μm to be compared with the p-adic length scale $L(175) = 40$ μm. Also these findings served as inspiration of $h_{eff} = n \times h$ hypothesis [K94, K93].

   I have assigned large Planck constant phases with the flux tubes of B_{end}, which have however remained somewhat mysterious entity. Could B_{end} correspond to quantum critical value of B_K and therefore relate directly to cosmology?

   One can check whether $B_K = eB_{end}/3$ holds true. The hypothesis would give

   $$eB_{end} = \frac{1}{L_B^2} = 3 \times \frac{\Phi_0}{4\pi\hbar} \frac{1}{L^2(175)} \quad .$$

   implying

   $$r = \frac{L^2(175)}{L_B^2} = \frac{3\Phi_0}{4\pi\hbar} \quad .$$

   The left hand side gives $r = 10.24$. For $\Phi_0 = 8\pi\hbar$ the right hand side gives $r = 6$. $B_E = .34$ Gauss left and right hand sides of the formula are identical.

3. One can wonder the proposed formulas might be exact for preferred extremals satisfying extremely powerful conditions to guarantee strong form of holography. This would require in both cases bundle structure with transversal cross section action as fiber. In the case of extremals of Kähler this would require that induce Kähler magnetic field is covariantly constant.

14.3.5 Two variants for p-adic length scale hypothesis for cosmological constant

There are two options for the dependence string tension Tandarea S of the cross section of the flux tube on p-adic length scale: either $L_\Lambda = \sqrt{8\pi/\Lambda}$ or $L = (\hbar/\rho_{vac})^{1/4}$ satisfies p-adic length scale hypothesis. The "boundary condition" is that the radius of flux tubes would be of the order of neutron size scale in recent cosmology.

1. $L(now) = L_p$ scaling gives

   $$S = S(now)\frac{p(now)}{p} \tag{14.3.7}$$

   with $p_{now} \simeq 2^{175}$ by p-adic length scale hypothesis. $L(175)$ is by about 4 per cent smaller than the Compton length assignable to $\hbar/L(now) = .029$ eV.

   If one wants $L(now) = L(175)$ exactly, one must increase R by 4 per cent, which is allowed by p-adic mass calculations fixing the value of R only with 10 per cent accuracy. Indeed, the second order contribution in p-adic mass calculations is uncertain and the ratio of maximal and minimal values of R is $R_{max}/R_{min} = \sqrt{6/5} \simeq 1.1$.

   As already noticed, $L(now)$ corresponds to neutron size scale, which conforms with p-adic mass calculations since the radius of flux tubes would correspond to p-adic length scale. This option looks more natural and suggest a profound connection with biology and fundamental physics.

2. $L_\lambda \equiv \sqrt{8\pi/\Lambda}$ could be proportional to secondary p-adic length scale $L(2,p_\Lambda) \equiv \sqrt{p_\Lambda}L_{p_\Lambda}$. The scaling law

$$L_\Lambda \propto \frac{p_\Lambda(now)}{p_\Lambda} \tag{14.3.8}$$

gives

$$L_\Lambda^2(now) = \frac{8\pi}{\Lambda(now)} = (\frac{p}{p(now)})^2 \times \frac{L^4(now)}{l_P^2} \quad . \tag{14.3.9}$$

$L_\Lambda(now) \sim 50$ Gly (roughly the age of the Universe) holds true. Note that one has $S \propto \sqrt{p_{now}/p}S(now)$ and $T = T_{now}\sqrt{p/p_{now}}$.

$1/p$-dependence for the string tension T looks more natural in light of p-adic mass calculations. One must however notice that the $L = L(175)$ is 4 per cent small than $L(now)$.

The density of dark energy is uncertain by few per cent at least and one can ask whether $L(now) = L(175)$ could fix it. The change induced to ρ_{vac} by that of $L(now)$ is

$$\frac{\Delta\rho_{vac}}{\rho_{vac}} = -4\frac{\Delta L(now)}{L(now)}$$

and the reduction L by 4 per cent would reduce vacuum density by 16 per cent, which looks rather large change. The value of R can be determined by 10 per cent accuracy and the increase of R by four per cent is another manner to achieve $L(now) = L(175)$.

One can of course ask, whether both variants of p-adic length scale hypothesis could be correct. The reader night protest that this leads to the murky waters of p-adic numerology.

1. Could L_Λ be proportional to the secondary p-adic length scale $L(p,2) = \sqrt{p}L_p = 2^{k/2} \times L(k)$ associated with p characterizing L such that the proportionality constant is power of $\sqrt{2}$. The application of the condition defining L in terms of $L_\Lambda^2 = 8\pi/\Lambda$ gives

$$L_\Lambda^2 = \frac{L^4}{l_P^2} \quad .$$

Using $L_\Lambda = \sqrt{p_\Lambda}R$ and taking square roots, this gives

$$\sqrt{p_\Lambda} = pk^2 \quad , \quad k = \frac{R_{CP_2}}{l_P} \quad . \tag{14.3.10}$$

This conforms with the p-adic length scales hypothesis in its simplest form if k is power of $\sqrt{2}$.

2. The estimate from p-adic mass calculations for $r \equiv R(CP_2)/l_P$ is $r = 4.167 \times 10^3$ and is 2 per cent larger than 2^{12}. Could the $R(CP_2)/l_P = 2^{12}$ for the radii of CP_2 and M^4 twistorial sphere be an exact formula between fundamental length scales? As noticed, the second order contribution in p-adic mass calculations is uncertain by 10 per cent. This would allow the reduction of $R(CP_2)$ by 2 percent.

This looks an attractive option. The bad news is that the *increase* of $R(CP_2)$ by about 4 per cent to achieve $L(now) = L(175)$ is in conflict with its *reduction* by 2 per cent to achieve $R(CP_2)/l_P = 2^{12}$: this would reduce $L(175)$ by 2 per cent and increase ρ_{vac} by about 8 per cent. ρ_{vac} is however an experimental parameter depending on theoretical assumption and it value could allow this tuning. Therefore

$$\begin{aligned} \frac{R_{CP_2}}{l_P} &= 2^{12} \ , \\ p_\Lambda &= 2^{48} \times p^2 \ . \end{aligned} \tag{14.3.10}$$

is an attractive option fixing completely the value of $R(CP_2)/l_P$ and predicting relation between cosmological scale L_Λ and a fundamental scale in recent biology, which could be assigned to magnetic flux tubes assignable to axons. Note that for $k_{now} = 175$ the value of $k_\Lambda = k_{now} + 48$ is $k_\Lambda = 175 + 48 = 223$ which corresponds to p-adic length scale of 64 m.

3. Needless to say that one must be take these estimates with a big grain of salt. Number theoretical universality suggests that one might apply number theoretical constraints to fundamental constants like R, l_P, and Λ but one should be very critical concerning the values of empirical parameters such as ρ_{vac} depending on theoretical assumptions. Furthermore, p-adic length scale hypothesis is applied at the level of imbedding space metric and one can ask whether it actually applies for the induced metric (Robertson-Walker metric now).

14.4 What happens for the extremals of Kähler action in twistor lift

As I started to work with TGD around 1977, I adopted path integral and canonical quantization as the first approaches. One of the first guesses for the action principle was 4-volume in induced metric giving minimal surfaces as preferred extremals. The field equations are a generalization of massless field equation and at least in the case of string models Hamiltonian formalism and second quantization is possible. The reason why for giving up this option was that for space-time surfaces of infinite duration the volume is infinite. This is not pleasant news concerning quantization since subtraction of exponent of infinite volume factor looked really ugly thing to do. At that time I did of course have no idea about ZEO and CDs.

For Kähler action there is however infinite vacuum degeneracy. All space-time surfaces with CP_2 projection, which is Lagrangian manifold (at most 2-dimensional) are vacuum extremals and canonical quantization fails completely. This implies classical non-determinism also for non-vacuum extremals obtained as small deformations of vacuum extremals. This feature seems to have nice implications such as 4-D spin glass degeneracy. It would however make WCW metric singular for nearly vacuum extremals.

The twistor lift brings volume term to the action. For option II there is also coupling between Kähler action and volume term but asymptotically one expects minimal surface extremals as analogs for free geodesic motion. The question is what happens to the known extremals of Kähler action, most of which are minimal surfaces.

14.4.1 The coupling between Kähler action and volume term

The addition of the volume term to Kähler action has very nice interpretation as a generalization of equations of motion for a world-line extended to a 4-D space-time surface. The field equations generalize in the same manner for 3-D light-like surfaces at which the signature of the induced metric changes from Minkowskian to Euclidian, for 2-D string world sheets, and for their 1-D boundaries defining world lines at the light-like 3-surfaces. For 3-D light-like surfaces the volume term is absent. Either light-like 3-surface is freely choosable in which case one would have Kac-Moody symmetry as gauge symmetry or that the extremal property for Chern-Simons term fixes the gauge.

The condition that the dynamics based on Kähler action and volume term is number theoretically universal demands that coupling constants do not appear in it. This leaves only Option I ($\alpha_K(M^4) \neq \alpha_K(CP_2)$ with different phases) and option II ($\alpha_K(M^4) = \alpha_K(CP_2)$ with the same phase). This condition is taken as granted in the following.

The dynamics of twistor lift as a generalization of the dynamics of point like particle coupling to Maxwell field

Almost all the known non-vacuum extremals are minimal surface extremals of Kähler action [K7, K112] and it might well be that the preferred extremal property realizing SH quite generally demands this. CP_2 type vacuum extremals are also minimal surfaces if one assumes that the M^4 projection is light-like geodesic rather than only geodesic line.

The addition of the volume term could however make Kähler coupling strength a manifest coupling parameter also classically when the phases of Λ and α_K are same. Therefore quantum criticality for Λ and α_K would have a precise local meaning also classically in the interior of space-time surface. The equations of motion for a world line of U(1) charged particle would generalize to field equations for a "world line" of 3-D extended particle.

This is an attractive idea consistent with standard wisdom but for Option I one can invent strong objections against it.

1. The conjecture is that α_K has zeros of zeta as its spectrum of critical values [L16]. If so then all preferred extremals are minimal surface extremals of Kähler action for a real value of cosmological constant Λ possible for Option I ($\alpha_K(M^2)$ would be real). Hence the two actions decouple: this does not look nice. For Option II the phase is same and there is interaction between these degrees of freedom. One could of course force also the phase for Option I to be same.

2. All known non-vacuum extremals of Kähler action are minimal surfaces and the minimal surface vacuum extremals of Kähler action become non-vacuum extremals. This allows to consider the possibility that preferred extremals are minimal surface extremals of Kähler action so that the two dynamics apparently decouple. For Option II this makes sense since the solutions do not depend at all on the common over-all scaling factor of Kähler action and volume term. Minimal surface extremals are analogs for geodesics in the case of point-like particles: one might say that one has only gravitational interaction. This conforms with SH stating that gauge interactions at boundaries (orbits of partonic 2-surfaces and 2-surfaces at the ends of CD) correspond classically to the gravitational dynamics in the space-time interior.

   Note that at the boundaries of the string world sheets at light-like 3-surfaces the situation is different: one has equations of motion for geodesic line coupled to induce Kähler gauge potential and gauge coupling indeed appears classically as one might expect! For string world sheets one has only the topological magnetic flux term and minimal surface equation in string world sheet. Magnetic flux term gives the Kähler coupling at the boundary.

3. For Option I decoupling implied by extremal property of both real and imaginary parts of action would allow to realize number theoretical universality [K111] since the field equations would not depend on coupling parameters at all. For Option II same is achieved even without decoupling.

4. One can argue that the decoupling for Option I makes it impossible to understand coupling constant evolution. This need not be the case. The point is that the classical charges assignable to super-symplectic algebra are sums over contributions from Kähler action and volume term and therefore depend on the coupling parameters. Their vanishing conditions for sub-algebra and its commutator with entire algebra give boundary conditions on preferred extremals so that coupling constant evolution creeps in classically!

   Quantum classical correspondence realized as the condition that the eigenvalues of fermionic charge operators are equal to the classical charges brings in the dependence of quantum charges on coupling parameters. Since the elements of scattering matrix are expected to involve as building bricks the matrix elements of super-symplectic algebra and Kac-Moody algebra of isometry charges, one expects that discrete coupling constant evolution creeps in also quantally via the boundary conditions for preferred extremals.

Options I and II and Chladni mechanism

One can compare Options I and II.

1. For Option I the coupling between the two dynamics could be induced just by the condition that the space-time surface becomes an analog of geodesic line by arranging its interior so that the U(1) force vanishes! This would generalize Chladni mechanism (see `http://tinyurl.com/j9rsyqd`)!

   The interaction would be present but be based on going to the nodal surfaces! Also the dynamics of string world sheets is similar: if the string sheets carry vanishing W boson classical fields, em charge is well-defined and conserved. One would also avoid the problems produced by large coupling constant between the two-dynamics present already at the classical level. At quantum level the fixed point property of quantum critical couplings would be the counterparts for decoupling. This option however seems to be missing the transient phase preceding the Chladni configuration.

2. For Option II the coupling would be present during transient periods leading to decoupling.. The alternative view is that the deviation from minimal surface and can act as a controller of the dynamics defined by the volume term providing a small push or pull now and then. Could this sensitivity relate to quantum criticality and to the view about morphogenesis relying on Chladni mechanism in which field patterns control the dynamics with charged flux tubes ending up to the nodal surfaces of (Kähler) electric field [L27]? Magnetic flux tubes containing dark matter would in turn control and serve as template for the dynamics of ordinary matter.

   Chladni mechanism would not be instantaneous but lead via transient phase to minimal surface extremals near either or both boundaries of CDs analogous to external particles in particle reaction. The space-time regions assignable to particle interaction vertices identified as 2-surfaces at which the ends of three 3-D light-like partonic orbits meet, would correspond to transient regions, where the coupling is present. This option looks clearly more realistic.

Admittedly Option II looks more attractive.

As an example one can consider a typical particle physics experiment. There are incoming and outgoing free particles moving along geodesics, these particles interact, and emanate as free particles from the interaction volume. This phenomenological picture does not follow from QFT but is put in by hand, in particular the idea about interaction couplings becoming non-zero is involved. Also the role of the observer remains poorly understood.

The motion of incoming and outgoing particles is analogous to free motion along geodesic lines with particles generalized to 3-D extended objects. For both options these would correspond to the preferred extremals in the complement of CD within larger CD representing observer or measurement instrument. Decoupling would take place. In interaction volume interactions are "coupled on" and particles interact inside the volume characterized by causal diamond (CD). What could be the TGD view translation of this picture?

1. For Option I one would still have decoupling and the interpretation would be in terms of twistor picture in which one always has also in the internal lines on mass shell particles but with complex four-momenta. In TGD framework the momenta would be always complex due to the contribution of Euclidian regions defining the lines of generalized scattering diagrams. Note however that the real and imaginary parts of the conserved charges are predicted to be proportional to each other. This result is obtained also in twistor approach from 8-D light-likeness and is crucial for twistorialization in TGD sense [L38]. As explained, coupling constant evolution can be understood also in this case and also classical dynamics depends on coupling parameters via the boundary conditions. There would be no counterpart for transitory period (interaction on) leading to the decoupled situation so that Option I is not attractive.

2. For Option II the transitory period would correspond to the coupling between the two classical dynamics in regions assignable to the vertices of topological scattering diagrams at which the ends of the parton orbits meet. Near the ends the dynamics would decouple and one would have the analog of free geodesic motion.

Second example comes from biology. The free geodesic line dynamics with vanishing $U(1)$ Kähler force indeed brings in mind the proposed generalization of Chladni mechanism generating nodal surfaces at which charged magnetic flux tubes are driven [K113] [L27, L28] . Chlandi mechanism could be seen as a basic mechanism behind morphogenesis.

1. For Option I the interiors of all space-time surfaces would be analogous to nodal surfaces and "big" state function reductions would correspond to transition periods between different nodal surfaces. The decoupling would be dynamics of avoidance and could highly analogous to Chladni mechanism.

2. For Option II transition period would correspond to a period during which nodal surfaces are formed.

It seems that Option II is favored by both SH, number theoretical universality, and generalization of Chladni mechanism to a dynamics of avoidance.

14.4.2 Twistor lift and the extremals of Kähler action

The addition of the volume term makes Kähler coupling strength a genuine coupling parameter also classically when the variation of Kähler action is non-vanishing. Therefore quantum criticality for Λ and α_K gets precise meaning also classically. The equations of motion for a worldline of U(1) charged particle generalize to field equations for a "world line" of 3-D extended particle.

The field equations generalize in the same manner for 3-D light-like surfaces at which the signature of the induced metric changes from Minkowskian to Euclidian, for 2-D string world sheets, and for their 1-D boundaries defining world lines at the light-like 3-surfaces. For 3-D light-like surfaces the volume term is absent. Either light-like 3-surface is freely choosable in which case one would have Kac-Moody symmetry as gauge symmetry or that the extremal property for Chern-Simons term fixes the gauge.

What happens to the extremals of Kähler action?

What happens to the extremals of Kähler action when volume term is introduced?

1. The known non-vacuum extremals [K7, K112] such as massless extremals (topological light rays) and cosmic strings are minimal surfaces.

2. For $J(M^4) = 0$ these extremals remain extremals for both Option I and II and only the classical Noether charges receive an additional volume term. In particular, string tension is modified by the volume term. Homologically non-trivial cosmic strings are of form $X^2 \times Y^2$, where $X^2 \subset M^4$ is minimal surface and $Y^2 \subset CP_2$ is complex 2-surface and therefore also minimal surface.

3. For $J(M^4) \neq 0$ essential for obtaining small cosmological constant for Option II, the situation changes and minimal surface property is possible only under additional conditions. For instance, one can have minimal surfaces of form $X^2 \times Y^2 \subset M^4 \times Y^2$, where Y^2 is minimal surface in CP_2. X^2 cane be $M^2 \subset N^2 \times E^2$ defining the $J(M^4)$ giving $J(M^4) \cdot J(M^4) - 2 = 0$. X^2 can be also minimal surface, which is an analog of Lagrangian manifold for $J(M^4)$.

4. Vacuum degeneracy is lifted for both options. For $J(M^4) = 0$ vacuum extremals, which are minimal surfaces survive as extremals for both options. For $J(M^4) \neq 0$ the situation is more complex.

Vacuum extremals

For CP_2 type vacuum extremals [K7, K112] the roles of M^4 and CP_2 are changed. M^4 projection is light-like curve, and can be expressed as $m^k = f^k(s)$ with light-likeness conditions reducing to Virasoro conditions. These surfaces are isometric to CP_2 and have same Kähler and symplectic structures as CP_2 itself. What is new as compared to GRT is that the induced metric has Euclidian signature. The interpretation is as lines of generalized scattering diagrams. The addition of the volume term forces the random light-like curve to be light-like geodesic and the action becomes the volume of CP_2 in the normalization provided by cosmological constant. What looks strange is that the volume of any CP_2 type vacuum extremals equals to CP_2 volume but only the extremal with light-like geodesic as M^4 projection is extremal of volume term. A little calculation shows that for CP_2 type extremals the contribution of the volume term to the action would be completely negligible as compared to the Kähler action.

Consider next vacuum extremals, which have vanishing induced Kähler form and are thus have CP_2 projection belonging to at most 2-D Lagrangian manifold of CP_2 [K7, K112].

1. Vacuum extremals with 2-D projections to CP_2 and M^4 are possible and are of form $X^2 \times Y^2$, X^2 arbitrary 2-surface and Y^2 a Lagrangian manifold. Volume term forces X^2 to be a minimal surface and Y^2 is Lagrangian minimal surface unless the minimal surface property destroys the Lagrangian character.

   If the Lagrangian sub-manifold is homologically trivial geodesic sphere, one obtains string like objects with string tension determined by the cosmological constant alone.

   Do more general 2-D Lagrangian minimal surfaces than geodesic sphere exist? For general Kähler manifold there are obstructions but for Kähler-Einstein manifolds such as CP_2, these obstructions vanish (see `http://tinyurl.com/gtkpya6`). The case of CP_2 is also discussed in the slides "On Lagrangian minimal surfaces on the complex projective plane" (see `http://tinyurl.com/jrhl6gy`). The discussion is very technical and demonstrates that Lagrangian minimal surfaces with all genera exist. In some cases these surfaces can be also lifted to twistor space of CP_2.

2. More general vacuum extremals have 4-D M^4 projection. Could the minimal surface condition for 4-D M^4 projection force a deformation spoiling the Lagrangian property? The physically motivated expectation is that string like objects give as deformations magnetic flux tubes for which string is thicknened so that it has a 2-D cross section. This would suggest that the deformations of string like objects $X^2 \times Y^2$, where Y^2 is Lagrangian minimal surface, give rise to homologically trivial magnetic flux tubes. In this case Kähler magnetic field would vanish but the spinor connection of CP_2 would give rise to induced magnetic field reducing to some U(1) subgroup of U(2). In particular, electromagnetic magnetic field could be present.

3. p-Adically Λ behaves like $1/p$ as also string tension. Could hadronic string tension be understood also in terms of cosmological constant in hadronic p-adic length scale for strings if one assumes that cosmological constant for given space-time sheet is determined by its p-adic length scale?

Maxwell phase

What might be called Maxwell phase which would correspond to small perturbations of M^4 is also possible for 4-D Kähler action. For the twistor lift the volume term makes this phase possible. Maxwell phase is highly interesting since it corresponds to the intuitive view about what QFT limit of TGD could be. The following arguments apply only for $J(M^4) = 0$.

1. The field equations are a generalization of massless field equations for fields identifiable as CP_2 coordinates and with a coupling to the deviation of the induced metric from M^4 metric. It representes very weak perturbation. Hence the linearized field equations are expected to be an excellent approximation. The general challenge would be however the construction of exact solutions. One should also understand the conditions defining preferred extremals and stating that most of symplectic Noether charges vanish at the ends of space-time surface about boundaries of CD.

2. Maxwell phase is the TGD analog for the perturbative phase of gauge theories. The smallness of the cosmological constant in cosmic length scales would make the perturbative approach useless in the path integral formulation. In TGD approach the path integral is replaced by functional integral involving also a phase but also now the small value of cosmological constant is a problem in long length scales. As proposed, the hierarchy of Planck constants would provide the solution to the problem.

3. The value of cosmological constant behaving like $\Lambda \propto 1/p$ as the function of p-adic prime could be in short p-adic length scales large enough to allow a converging perturbative expansion in Maxwellian phase. This would conform with the idea that Planck constant has its ordinary value in short p-adic length scales.

4. Does Maxwell phase allow extremals for which the CP_2 projection is 2-D Lagrangian manifold - say a perturbation of a minimal Lagrangian manifold? This perturbation could be seen also as an alternative view about thickened minimal Lagrangian string allowing also M^4 coordinates as local coordinates. If the projection is homologically trivial geodesic sphere this is the case. Note that solutions representable as maps $M^4 \rightarrow CP_2$ are also possible for homologically non-trivial geodesic sphere and involve now also the induced Kähler form.

5. The simplest deformations of canonically imbedded M^4 are of form $\Phi = k \cdot m$, where Φ is an angle coordinate of geodesic sphere. The induced metric in M^4 coordinates reads as $g_{kl} = m_{kl} - R^2 k_k k_l$ and is flat and in suitably scaled space-time coordinates reduces to Minkowski metric or its Euclidian counterpart. k_k is proportional to classical four-momentum assignable to the dark energy. The four-momentum is given by

$$P^k = A \times \hbar k^k \quad , \quad A = \frac{Vol(X^3)}{L_\Lambda^4} \times \frac{1+2x}{1+x} \quad , \quad x = R^2 k^2 \quad .$$

   Here k^k is dimensionless since the the coordinates m^k are regarded as dimensionless.

6. There are interesting questions related to the singularities forced by the compactness of CP_2. Eguchi-Hanson coordinates (r, θ, Φ, Ψ) [L2] (see http://tinyurl.com/z86o5qk) allow to get grasp about what could happen.

   For the cyclic coordinates Ψ and Φ periodicity conditions allow to get rid of singularities. One can however have n-fold coverings of M^4 also now.

   (r, θ) correspond to canonical momentum type canonical coordinates. Both of them correspond to angle variables ($r/\sqrt{1+r^2}$ is essentially sine function). It is convenient to express the solution in terms of trigonometric functions of these angle variables. The value of the trigonometric function can go out of its range $[-1, 1]$ at certain 3-surface so that the solution ceases to be well-defined. The intersections of these surfaces for r and θ are 2-D surfaces. Many-sheeted space-time suggests a possible manner to circumvent the problem by gluing two solutions along the 3-D surfaces at which the singularities for either variable appear. These surfaces could also correspond to the ends of the space-time surface at the boundaries of CD or to the light-like orbits of the partonic 2-surfaces.

   Could string world sheets and partonic 2-surfaces correspond to the singular 2-surfaces at which both angle variables go out of their allowed range. If so, 2-D singularities would code for data as assumed in strong form of holography (SH). SH brings strongly in mind analytic functions for which also singularities code for the data. Quaternionic analyticity which makes sense would indeed suggest that co-dimension 2 singularities code for the functions in absence of 3-D counterpart of cuts (light-like 3-surfaces?) [L22].

7. A more general picture might look like follows. Basic objects come in two classes. Surfaces $X^2 \times Y^2$, for which Y^2 is either homologically non-trivial complex minimal 2-surface of CP_2 of Lagrangian minimal surface. The perturbations of these two surfaces would also produce preferred extremals, which look locally like perturbations of M^4. Quaternionic analyticity might be shared by both solution types. Singularities force many-sheetedness and strong form of holography.

Astrophysical and cosmological solutions

Cosmological constant is expected to obey p-adic evolution and in very early cosmology the volume term becomes large. What are the implications for the vacuum extremals representing Robertson-Walker metrics having arbitrary 1-D CP_2 projection? [K7, K112, K67]. One can also ask what is the fate of spherically symmetric solutions of GRT providing a model of star.

Already the existing physical picture explaining $h_{gr}/hh_{eff}/h = n$ in terms of flux tubes mediating gravitational interactions suggests that Robertson-Walker metrics and spherically symmetric metrics are possible only at QFT limit. The presence of covariantly constant $J(M^4)$ breaking Lorentz symmetry and rotational symmetry makes this obvious. One could consider variants of $J(M^4)$ invariant under Lorentz group or some subgroup of Lorentz group but $J(M^4)$ would not be covariantly constant anymore. It is not clear when it makes sense to extend the moduli space for $J(M^4)$.

1. The TGD inspired cosmology involves primordial phase during a gas of cosmic strings in M^4 with 2-D M^4 projection dominates. The value of cosmological constant at that period could be fixed from the condition that homologically trivial and non-trivial cosmic strings have the same value of string tension. After this period follows the analog of inflationary period when cosmic strings condense are the emerging 4-D space-time surfaces with 4-D M^4 projection and the M^4 projections of cosmic strings are thickened. A fractal structure with cosmic strings topologically condensed at thicker cosmic strings suggests itself.

2. GRT cosmology is obtained as an approximation of the many-sheeted cosmology as the sheets of the many-sheeted space-time are replaced with region of M^4, whose metric is replaced with Minkowski metric plus the sum of deformations from Minkowski metric for the sheet. The vacuum extremals with 4-D M^4 projection and arbitrary 1-D projection could serve as an approximation for this GRT cosmology. Note however that this representability is not required by basic principles.

3. For cosmological solutions with 1-D CP_2 projection minimal surface property forces the CP_2 projection to belong to a geodesic circle S^1. Denote the angle coordinate of S^1 by Φ and its radius by R. For the future directed light-cone M^4_+ use the Robertson-Walker coordinates $(a = \sqrt{m_0^2 - r_M^2}, r = ar_M, \theta, \phi)$, where (m^0, r_M, θ, ϕ) are spherical Minkowski coordinates. The metric of M^4_+ is that of empty cosmology and given by $ds^2 = da^2 - a^2 d\Omega^2$, where Ω^2 denotes the line element of hyperbolic 3-space identifiable as the surface $a = constant$.

   One can can write the ansatz as a map from M^4_+ to S^1 given by $\Phi = f(a)$. One has $g_{aa} = 1 \rightarrow g_{aa} = 1 - R^2(df/da)^2$. The field equations are minimal surface equations and the only non-trivial equation is associated with Φ and reads $d^2f/da^2 = 0$ giving $\Phi = \omega a$, where ω is analogous to angular velocity. The metric corresponds to a cosmology for which mass density goes as $1/a^2$ and the gravitational mass of comoving volume (in GRT sense) behaves is proportional to a and vanishes at the limit of Big Bang smoothed to "Silent whisper amplified to rather big bang" for the critical cosmology for which the 3-curvature vanishes. This cosmology is proposed to results at the limit when the cosmic temperature approaches Hagedorn temperature [K67].

4. The TGD counterpart for inflationary cosmology corresponds to a cosmology for which CP_2 projection is homologically trivial geodesic sphere S^2 (presumably also more general Lagrangian (minimal) manifolds are allowed). This cosmology is vacuum extremal of Kähler action. The metric is unique apart from a parameter defining the duration of this period serving as the TGD counterpart for inflationary period during which the gas of string like objects condensed at space-time surfaces with 4-D M^4 projection. This cosmology could serve as an approximate representation for the corresponding GRT cosmology.

   The form of this solution is completely fixed from the condition that the induced metric of $a = constant$ section is transformed from hyperbolic metric to Euclidian metric. It should be easy to check whether this condition is consistent with the minimal surface property. It seems that one cannot satisfy minimal surface equations.

5. For $J(M^4) \neq 0$ the spherical and Lorentz symmetries are lost and the only cosmological solution are light-cones $M^4_\pm$. Also the existence of stationary spherically symmetric minimal surface extremals is impossible for $J(M^4) \neq 0$. Spherically symmetric metrics and Robertson-Walker metric would serve only as long length scale approximations providing a statistical description of the gravitational interaction described microscopically in terms of a flux tube network.

14.4.3 Are minimal surface extremals of Kähler action holomorphic surfaces in some sense?

If the spectrum for the critical value of Kähler coupling strength is complex - say given by the complex zeros of zeta [L16] - the preferred extremals of Kähler action are minimal surfaces for Option I. For Option II they correspond to asymptotic solutions.

I have considered several ansätze for the general solutions of the field equations for the preferred extremals. One proposal is that preferred extremals as 4-surfaces of imbedding space with octonionic tangent space structure have quaternionic tangent space or normal space (so called $M^8 - H$ duality [K74]). Second proposal is that preferred extremals can be seen as quaternion analytic [A94] surfaces [K110, K76] [L14]. Third proposal relies on a fusion of complex and hyper-complex structures to what I call Hamilton-Jacobi structure [K79, K112]. In Euclidian regions this would correspond to complex structure. Twistor approach [L22] suggests that the condition that the twistor lift of the space-time surface to a 6-D surface in the product of twistor spaces of M^4 and CP_2 equals to the twistor space of CP_2. This proposal is highly interesting since twistor lift works only fr $M^4 \times CP_2$. The intuitive picture is that the field equations are integrable and all these views might be consistent.

Preferred extremals of Kähler action as minimal surfaces would be a further proposal. Can one make conclusions about general form of solutions assuming that one has minimal surface extremals of Kähler action?

In $D = 2$ case minimal surfaces are holomorphic surfaces or they hyper-complex variants and the imbedding space coordinates can be expressed as complex-analytic functions of complex coordinate or a hypercomplex analog of this. Field equations stating the vanishing of the trace $g\alpha\beta H^k_{\alpha\beta}$ if the second fundamental form $H^k_{\alpha\beta} \equiv D_\alpha \partial_\beta h^k$ are satisfied because the metric is tensor of type $(1,1)$ and second fundamental form of type $(2,0) \oplus (2,0)$. Field equations reduce to an algebraic identity and functions involved are otherwise arbitrary functions. The constraint comes from the condition that metric is of form $(1,1)$ as holomorphic tensor.

This raises the question whether this finding generalizes to the level of 4-D space-time surfaces and perhaps allows to solve the field equations exactly in coordinates generalizing the hypercomplex coordinates for string world sheet and complex coordinates for the partonic 2-surface.

Almost all the known non-vacuum extremals are minimal surface extremals of Kähler action [K7, K112] and it might well be that the preferred extremal property realizing SH quite generally demands this. CP_2 type vacuum extremals are also minimal surfaces if one assumes that the M^4 projection is light-like geodesic rather than only geodesic line. The common feature suggested already earlier to be common for all preferred extremals is the existence of generalization of complex structure.

1. For Minkowskian regions this structure would correspond to what I have called Hamilton-Jacobi structure [K79, K112]. The tangent space of the space-time surface X^4 decomposes to local direct sum $T(X^4) = T(X^2) \oplus T(Y^2)$, where the 2-D tangent places $T(X^2)$ and $T(Y^2)$ define an integrable distribution integrating to a decomposition $X^4 = X^2 \times Y^2$. The complex structure is generalized to a direct some of hyper-complex structure in X^2 meaning that there is a local light-like direction defining light-like coordinate u and its dual v. $Y2$ has complex complex coordinate $(w, \overline{w})$. Minkowski space M^4 has similar structure. It is still an open question whether metric decomposes to a direct sum of orthogonal metrics assignable to X^2 and Y^2 or is the most general analog of complex metric in question. g_{uv} and $g_{w\overline{w}}$ are certainly non-vanishing components of the induced metric. Metric could allow as non-vanishing components also g_{uw} and $g_{v\overline{w}}$. This slicing by pairs of surfaces would correspond to decomposition to a product of string world sheet and partonic 2-surface everywhere.

   In Euclidian regions ne would have 4-D complex structure with two complex coordinates (z, w) and their conjugates and completely analogous decompositions. In CP_2 one has similar complex structure and actually Kähler structure extending to quaternionic structure. I have actually proposed that quaternion analyticity could provide the general solution of field equations.

2. Assuming minimal surface property the field equations for Kähler action reduce to the vanishing of a sum of two terms. The first term comes from the variation with respect to the induced metric and is proportional to the contraction

$$A = J^{\alpha}_{\ \gamma} J^{\gamma\beta} H^k_{\alpha\beta} \quad . \tag{14.4.1}$$

Second term comes from the variation with respect to induced Kähler form and is proportional to

$$B = j^{\alpha} P^k_{\ s} J^s_{\ l} \partial_{\alpha} h^l \quad . \tag{14.4.2}$$

Here P^k_l is projector to the normal space of space-time surface and $j^{\alpha} = D_{\beta} J^{\alpha\beta}$ is the conserved Kähler current.

For the known extremals j vanishes or is light-like (for massless extremals) in which case A and B vanish separately.

3. An attractive manner to satisfy field equations would be by assuming that the situation for 2-D minimal surface generalizes so that minimal surface equations are identically satisfied. Extremal property for Kähler action could be achieved by requiring that energy momentum tensor also for Kähler action is of type $(1,1)$ so that one would have $A = 0$. This implies $j^{\alpha}\partial_{\alpha} s^k = 0$. This is true if j vanishes or is light-like as it is for the known extremals. In Euclidian regions one would have $j = 0$.

4. The proposed generalization is especially interesting in the case of cosmic string extremals of form $X^2 \times Y^2$, where $X^2 \subset M^4$ is minimal surface (string world sheet) and Y^2 is complex homologically non-trivial sub-manifold of CP_2 carrying Kähler magnetic charge. The generalization would be that the two transversal coordinates $(w, \overline{w})$ in the plane orthogonal to the string world sheet defining polarization plane depend holomorphically on the complex coordinates of complex surface of CP_2. This would transform cosmic string to flux tube.

5. There are also solutions of form $X^2 \times Y^2$, where Y^2 is Lagrangian sub-manifold of CP_2 with vanishing Kähler magnetic charge and their deformations with $(w, \overline{w})$ depending on the complex coordinates of Y^2 (see the slides "On Lagrangian minimal surfaces on the complex projective plane" at `http://tinyurl.com/jrhl6gy`). In this case Y^2 is not complex sub-manifold of CP_2 with arbitrary genus and induced Kähler form vanishes. The simplest choice for Y^2 would be as homologically trivial geodesic sphere. Because of its 2-dimensionality Y^2 has a complex structure defined by its induced metric so that solution ansatz makes sense also now.

14.5 About string like objects

String like objects and partonic 2-surfaces carry the information about quantum states and about space-time surfaces as preferred extremals if strong form of holography (SH) holds true. SH has of course some variants. The weakest variant states that fundamental information carrying objects are metrically 2-D. The light-like 3-surfaces separating space-time regions with Minkowskian and Euclidian signature of the induced metric are indeed metrically 2-D, and could thus carry information about quantum state.

The original observation was that string world sheets should carry vanishing W boson fields in order that the em charge for the modes of the induced spinor field is well-defined. This condition can be satisfied in certain situations also for the entire space-time surface. This raises several questions. What is the fundamental condition forcing the restriction of the spinor modes to string world sheets - or more generally, to a surface of given dimension?

Can one have an analog of brane hierarchy in which also higher-D objects can carry modes of induced spinor field [K95]. Or should one identify 2-surfaces in terms of effective action, which by SH allows to describe the dynamics in terms of 2-D data? Both options have their nice features.

14.5.1 Two options for fundamental variational principle

String world sheets and partonic 2-surfaces seems to be fundamental for TGD - especially so in the fermionic sector - but also the 4-D action seems to necessary and supersymmetry forces 4-D modified Dirac action too. The interpretation of the situation is far from obvious. One ends up to two options for the fundamental variational principle.

Option A: The *fundamental* action principle for space-time surfaces contains besides 4-D action also 2-D action assignable to string world sheets, whose topological part (magnetic flux) gives rise to a coupling term to Kähler gauge potentials assignable to the 1-D boundaries of string world sheets containing also geodesic length part. Super-symplectic symmetry demands that modified Dirac action has 1-, 2-, and 4-D parts: spinor modes would exist at both string boundaries, string world sheets, and space-time interior. A possible interpretation for the interior modes would be as generators of space-time super-symmetries [K95].

This option is not quite in the spirit of SH and string tension appears as an additional parameter. Also the conservation of em charge forces 2-D string world sheets carrying vanishing induced W fields and this is in conflict with the existence of 4-D spinor modes unless they satisfy the same condition. This looks strange.

Option B: Stringy action and its fermionic counterpart are effective actions only and justified by SH. In this case there are no problems of interpretation. SH requires only that the induced spinor fields at string world sheets determine them in the interior much like the values of analytic function at curve determine it in an open set of complex plane. At the level of quantum theory the scattering amplitudes should be determined by the data at string world sheets. If the induced W fields at string world sheets are vanishing, the mixing of different charge states in the interior of X^4 would not make itself visible at the level of scattering amplitudes!

If string world sheets are generalized Lagrangian sub-manifolds, only the induced em field would be non-vanishing and electroweak symmetry breaking would be a fundamental prediction. This however requires that M^4 has the analog of symplectic structure suggested also by twistorialization. This in turn provides a possible explanation of CP breaking and matter-antimatter asymmetry. In this case 4-D spinor modes do not define space-time super-symmetries.

The latter option conforms with number theoretically broken SH and would mean that the theory is amazingly simple. String world sheets together with number theoretical space-time discretization meaning small breaking of SH would provide the basic data determining classical and quantum dynamics. The Galois group of the extension of rationals defining the number-theoretic space-time discretization would act as a covering group of the covering defined by the discretization of the space-time surface, and the value of $h_{eff}/h = n$ would correspond to the dimension of the extension dividing the order of its Galois group. The phase transitions reducing $ord(G) \geq n$ would correspond to spontaneous symmetry breaking leading from Galois group to a subgroup H so that $ord(H)$ would divide $ord(G)$ and the new value of n would divide n.

The ramified primes of the extension would be preferred primes of given extension. The extensions for which the number of p-adic space-time surfaces representable also as a real algebraic continuation of string world sheets to preferred extrenal is especially large would be physically favored as also corresponding ramified primes. In other words, maximal number of p-adic imaginations would be realizable so that these extensions and corresponding ramified primes would be winners in the number-theoretic fight for survival. Whether this conforms with p-adic length scale hypothesis, remains an open question.

An attractive possibility is that this information is basically topological. For instance, the value of Planck constant $h_{eff} = n \times h$ would tell the number sheets of the singular covering defining this surface such that the sheets co-incide at partonic 2-surfaces at the ends of space-time surface at boundaries of CD. In the following some questions related to string world sheets are considered. The information could be also number theoretical. Galois group for the algebraic extension of rationals defining particular adelic physics would transform to each other the number theoretic discretizations of light-like 3-surfaces and give rise to covering space structure. The action is trivial at partonic 2-surfaces should be trivial if one wants singular covering: this would mean that discretizations of partonic 2-surfaces consist of rational points. $h_{eff}/h = n$ could in this case be a factor of the order of Galois group.

14.5.2 How to achieve low value of string tension?

String tension should be low for string world sheets in long scales. If string actions are effective actions (Option B), the same should be true for the string tensions of the magnetic flux tubes accompanying strings. Minimal surface property for string world sheets is natural. Let us consider only Option B in the following.

1. Could the analogs of Lagrangian sub-manifolds of $X^4 \subset M^4 \times CP_2$ satisfying $J(M^4) + J(CP_2) = 0$ define string world sheets and their variants with varying dimension? For Option I ($\alpha_K(M^4) \neq \alpha_K(CP_2)$) this could make sense if the flux tubes are homologically trivial. Homologically non-trivial (monopole) flux tubes should be thick enough to have small enough string tension, which is inversely proportional to the cross sectional area of the flux tube.

2. For Option II ($\alpha_K(M^4) = \alpha_K(CP_2)$) the action density is proportional to $J \cdot J - 2$ also for stringy action and this does not seem to make sense. Could the additional condition be $J(M^4) \cdot J(M^4) - 2 \sim 0$ holding true in 4-D sense for space-time regions with a small value of cosmological constant behaving like $1/p$, p preferred p-adic prime near power of 2. That low string tension and small cosmological constant would have the same origin, would be nice.

   The cancellation mechanism involving in an essential manner $J(M^4)$ would give rise to low mass strings and light hadron like particles and small cosmological constant instead of only high mass strings as in super string models. p-Adic thermodynamic for CP_2-mass excitations assignable to wormhole throats would determine elementary particle masses and long monopole flux tubes with small string tension connecting pairs of wormhole contacts would give stringy contribution to particle masses. In the case of hadrons this contribution from color magnetic flux tubes would dominate over quark masses. Clearly, Option II seems to conform with the existing picture about masses of elementary particles and hadrons.

14.5.3 How does the gravitational coupling emerge?

The appearance of $G = l_P^2$ has coupling constant remained for a long time actually somewhat of a mystery in TGD. l_P defines the radius of the twistor sphere of M^4 replaced with its geometric twistor space $M^4 \times S^2$ in twistor lift. G makes itself visible via the coefficients $\rho_{vac} = 8\pi\Lambda/G$ volume term but not directly and if preferred extremals are minimal surface extremals of Kähler action ρ_{vac} makes itself visible only via boundary conditions. How G appears as coupling constant?

Somehow the M^4 Kähler form should appear in field equations. $1/G$ could naturally appear in the string tension for string world sheets as string models suggest. p-Adic mass calculations identify the analog of string tension as something of order of magnitude of $1/R^2$ [K39]. This identification comes from the fact that the ground states of super-conformal representations correspond to imbedding space spinor modes, which are solutions of Dirac equation in $M^4 \times CP_2$. This argument is rather convincing and allows to expect that the p-adic mass scale is not determined by string tension.

The problem is that the length of string like objects would be given by Planck length or CP_2 length if either of these pictures is the whole truth. One expects long gravitational flux tubes mediating gravitational interactions. The hypothesis $\hbar_{eff} = n\hbar = \hbar_{gr} = GMm/v_0$, where $v_0 < c$ is a parameter with dimensions of velocity, suggests that the string tension assignable to the flux tubes mediating gravitational interaction between masses M and m is apart from a numerical factor equal to Λ_{gr}^{-2}, where gravitational Compton length is $\Lambda_{gr} = h_{gr}/m = GM/v_0$ so that the length of the flux tubes is of order Λ_{gr}.

The problem is that the length of string like objects would be given by Planck length or CP_2 length if either of these pictures is the whole truth. One would like to have long gravitational flux tubes mediating gravitational interactions. Strong form of holography (SH) indeed suggests that stringy action appears as effective action expressing 4-D space-time action and modified Dirac action as 2-D actions assignable to string world sheets [L34] (see `http://tinyurl.com/zylrd7w`). This view would allow to understand the localization of spinor modes to string world sheets carrying vanishing W fields in terms as an effective description implying well-defineness of classical em charge and conservation of em charge at the level of scattering amplitudes. In fact that the introduction of the Kähler form $J(M^4)$ would allow to understand string world sheets as analogs of Lagrangian sub-manifolds.

14.5.4 Non-commutative imbedding space and strong form of holography

Quantum group theorists have studied the idea that space-time coordinates are non-commutative and tried to construct quantum field theories with non-commutative space-time coordinates (see `http://tinyurl.com/z3m8sny`). My impression is that this approach has not been very successful. The non-commutativity is introduced by postulating the Minkowskian analog of symplectic form and $J(M^4)$ forced by Option II indeed is symplectic form. The loss of Lorentz invariance induced by $J(M^4)$ is the basic stumbling block. In TGD framework the moduli space for $J(M^4)$ emerges already when one introduces the moduli space for CDs. $J(M^4)$ would define quantization axis of energy (rest system) and quantization axis of spin. The nice features of $J(M^4)$ is that it could allow to understand CP breaking and matter antimatter asymmetry at fundamental level.

The analog of non-commutative space-time in TGD framework

In Minkowski space one introduces antisymmetry tensor J_{kl} and uncertainty relation in linear M^4 coordinates m^k would look something like $[m^k, m^l] = l_P^2 J^{kl}$, where l_P is Planck length. This would be a direct generalization of non-commutativity for momenta and coordinates expressed in terms of symplectic form J^{kl}.

1+1-D case serves as a simple example. The non-commutativity of p and q forces to use either p or q. Non-commutativity condition reads as $[p, q] = \hbar J^{pq}$ and is quantum counterpart for classical Poisson bracket. Non-commutativity forces the restriction of the wave function to be a function of p or of q but not both. More geometrically: one selects Lagrangian sub-manifold to which the projection of J_{pq} vanishes: coordinates become commutative in this sub-manifold. This condition can be formulated purely classically: wave function is defined in Lagrangian sub-manifolds to which the projection of J vanishes. Lagrangian manifolds are however not unique and this leads to problems in this kind of quantization. In TGD framework the notion of "World of Classical Worlds" (WCW) allows to circumvent this kind of problems and one can say that quantum theory is purely classical field theory for WCW spinor fields. "Quantization without quantization" would have Wheeler stated it.

General Coordinate Invariance (GCI) poses however a problem if one wants to generalize quantum group approach from M^4 to general space-time: linear M^4 coordinates assignable to Lie-algebra of translations as isometries do not generalize. In TGD space-time is surface in imbedding space $H = M^4 \times CP_2$: this changes the situation since one can use 4 imbedding space coordinates (preferred by isometries of H) also as space-time coordinates. The analog of symplectic structure J for M^4 makes sense and number theoretic vision involving octonions and quaternions leads to its introduction. Note that CP_2 has naturally symplectic form.

Could it be that the coordinates for space-time surface are in some sense analogous to symplectic coordinates (p_1, p_2, q_1, q_2) so that one must use either (p_1, p_2) or (q_1, q_2) providing coordinates for a Lagrangian sub-manifold. This would mean selecting a Lagrangian sub-manifold of space-time surface? Could one require that the sum $J_{\mu\nu}(M^4) + J_{\mu\nu}(CP_2)$ for the projections of symplectic forms vanishes and forces in the generic case localization to string world sheets and partonic 2-surfaces. In special case also higher-D surfaces - even 4-D surfaces as products of Lagrangian 2-manifolds for M^4 and CP_2 are possible: they would correspond to homologically trivial cosmic strings $X^2 \times Y^2 \subset M^4 \times CP_2$, which are not anymore vacuum extremals but minimal surfaces if the action contains besides Käction also volume term.

But why this kind of restriction? In TGD one has strong form of holography (SH): 2-D string world sheets and partonic 2-surfaces code for data determining classical and quantum evolution. Could this projection of $M^4 \times CP_2$ symplectic structure to space-time surface allow an elegant mathematical realization of SH and bring in the Planck length l_P defining the radius of twistor sphere associated with the twistor space of M^4 in twistor lift of TGD? Note that this can be done without introducing imbedding space coordinates as operators so that one avoids the problems with general coordinate invariance. Note also that the non-uniqueness would not be a problem as in quantization since it would correspond to the dynamics of 2-D surfaces.

The analog of brane hierarchy at fundamental level or from SH?

The analog of brane hierarchy for the localization of spinors - space-time surfaces; string world sheets and partonic 2-surfaces; boundaries of string world sheets - is suggestive (note however that SH does not favour it). Could this hierarchy correspond to a hierarchy of Lagrangian sub-manifolds of space-time in the sense that $J(M^4) + J(CP_2) = 0$ is true at them? Boundaries of string world sheets would be trivially Lagrangian manifolds. String world sheets allowing spinor modes should have $J(M^4)+J(CP_2) = 0$ at them. The vanishing of induced W boson fields is needed to guarantee well-defined em charge at string world sheets and that also this condition allow also 4-D solutions besides 2-D generic solutions. As already found, for the physically favoured Option II the more plausible option is $J(M^4) \cdot J(M^4) - 2 \sim 0$ for space-time regions with small cosmological constant. Despite this one can discuss this idea.

This condition is physically obvious but mathematically not well-understood: could the condition $J(M^4) + J(CP_2) = 0$ force the vanishing of induced W boson fields? Lagrangian cosmic string type minimal surfaces $X^2 \times Y^2$ would allow 4-D spinor modes. If the light-like 3-surface defining boundary between Minkowskian and Euclidian space-time regions is Lagrangian surface, the total induced Kähler form Chern-Simons term would vanish. The 4-D canonical momentum currents would however have non-vanishing normal component at these surfaces. I have considered the possibility that TGD counterparts of space-time super-symmetries could be interpreted as addition of higher-D right-handed neutrino modes to the 1-fermion states assigned with the boundaries of string world sheets [K95].

Induced spinor fields at string world sheets could obey the "dynamics of avoidance" in the sense that *both* the induced weak gauge fields W, Z^0 and induced Kähler form (to achieve this U(1) gauge potential must be sum of M^4 and CP_2 parts) would vanish for the regions carrying induced spinor fields. They would couple only to the *induced em field (!)* given by the R_{12} part of CP_2 spinor curvature [L2] for $D = 2, 4$. For $D = 1$ at boundaries of string world sheets the coupling to gauge potentials would be non-trivial since gauge potentials need *not* vanish there. Spinorial dynamics would be extremely simple and would conform with the vision about symmetry breaking of electro-weak group to electromagnetic gauge group.

It seems relatively easy to construct am infinite family of Lagrangian string world sheets satisfying $J(M^4) + J(CP_2) = 0$ using generalized symplectic transformations of M^4 and CP_2 as Hamiltonian flows to generate new ones from a given Lagrangian string world sheets. One must pose minimal surface property as a separate condition. Consider a piece of M^2 with coordinates (t, z) and homologically non-trivial geodesic sphere S^2 of CP_2 with coordinates $(u = cos(\Theta), \Phi)$. One has $J(M^4)_{tz} = 1$ and $J_{u\Phi} = 1$. Identify string world sheet via map $(u, \Phi) = (kz, \omega t)$ from M^2 to S^2. The induced CP_2 Kahler form is $J(CP_2)_{tz} = k\omega$. $k\omega = -1$ guarantees $J(M^4)+J(CP_2) = 0$. The strings have necessarily finite length from $L = 1/k \leq z \leq L$. One can perform symplectic transformations of CP_2 and symplectic transformations of M^4 to obtain new string world sheets. In general these are not minimal surfaces and this condition would select some preferred string world sheets.

Number theoretic vision about the analog of brane hierarchy

An alternative - but of course not necessarily equivalent - attempt to formulate SH would be in terms of number theoretic vision. Space-time surfaces would be associative or co-associative depending on whether tangent space or normal space in imbedding space is associative - that is quaternionic. These two conditions would reduce space-time dynamics to associativity and commutativity conditions. String world sheets and partonic 2-surfaces would correspond to maximal commutative or co-commutative sub-manifolds of imbedding space. Commutativity (co-commutativity) would mean that tangent space (normal space as a sub-manifold of space-time surface) has complex tangent space at each point and that these tangent spaces integrate to 2-surface. SH would mean that data at these 2-surfaces plus number theoretic discretization of space-time surface would be enough to construct quantum states. Therefore SH would be thus slightly broken. String world sheet boundaries would in turn correspond to real curves of the complex 2-surfaces intersecting partonic 2-surfaces at points so that the hierarchy of classical number fields would have nice realization at the level of the classical dynamics of quantum TGD.

To sum up, one cannot exclude the possibility that $J(M^4)$ is present implying a universal transversal localization of imbedding space spinor harmonics and the modes of spinor fields in the interior of X^4: this could perhaps relate to somewhat mysterious de-coherence interaction producing locality and to CP breaking and matter-antimatter asymmetry. The moduli space for M^4 Kähler structures proposed by number theoretic considerations would save from the loss of Poincare invariance and the number theoretic vision based on quaternionic and octonionic structure would have rather concrete realization. This moduli space would only extend the notion of WCW.

Chapter 15

Some Questions Related to the Twistor Lift of TGD

15.1 Introduction

During last couple years (I am writing this in the beginning of 2017) a kind of palace revolution has taken place in the formulation and interpretation of TGD. The notion of twistor lift and 8-D generalization of twistorialization have dramatically simplified and also modified the view about what classical TGD and quantum TGD are.

The notion of adelic physics suggests the interpretation of scattering diagrams as representations of algebraic computations with diagrams producing the same output from given input are equivalent. The simplest possible manner to perform the computation corresponds to a tree diagram [L22]. As will be found, it is now possible to even propose explicit twistorial formulas for scattering formulas since the horrible problems related to the integration over WCW might be circumvented altogether.

From the interpretation of p-adic physics as physics of cognition, $h_{eff}/h = n$ could be interpreted dimension of extension dividing the the order of its Galois group. Discrete coupling constant evolution would correspond to phase transitions changing the extension of rationals and its Galois group. TGD inspired theory of consciousness is an essential part of TGD and the crucial Negentropy Maximization Principle in statistical sense follows from number theoretic evolution as increase of the order of Galois group for extension of rationals defining adeles.

In the sequel I consider the questions related to both classical and quantum aspects of twistorialization.

15.1.1 Questions related to the classical aspects of twistorialization

Classical aspects are related to the twistor lift of classical TGD replacing space-time surfaces with their twistor spaces realized as extremals of 6-D analog of Kähler action in the product $T(M^4) \times T(CP_2)$ of twistor space of M^4 and CP_2 such that twistor structure is induced. The outcome is 4-D Kähler action with volume term having interpretation in terms of cosmological constant. Hence the twistorialization has profound physical content rather than being mere alternative formulation for TGD.

1. What does the induction of the twistor structure really mean? What is meant with twistor space. For instance, is the twistor sphere for M^4 time-like or space-like. The induction procedure involves dimensional reduction forced by the condition that the projection of the sum of Kähler forms for the twistor spaces $T(M^4)$ and $T(CP_2)$ gives Kähler form for the twistor sphere of X^4. Better understanding of the details is required.

2. Can the analog of Kähler form $J(M^4)$ assignable to M^4 suggested by the symmetry between M^4 and CP_2 and by number theoretical vision appear in the theory? What would be the physical implications?

The basic objection is the loss of Poincare invariance. This can be however avoided by introducing the moduli space for Kähler forms. This moduli space is actually the moduli space of causal diamonds (CDs) forced in any case by zero energy ontology (ZEO) and playing central role in the generalization of quantum measurement theory to a theory of consciousness and in the explanation of the relationship between geometric and subjective time [K41].

Why $J(M^4)$ would be needed? $J(M^4)$ corresponds to parallel constant electric and magnetic fields in given direction. Constant E and $B = E$ fix directions of quantization axes for energy (rest system) and spin. One implication is transversal localization of imbedding space spinor modes: imbedding space spinor modes are products of harmonic oscillator Gaussians in transversal degrees of freedom very much like quarks inside hadrons.

Also CP breaking is implied by the electric field and the question is whether this could explain the observed CP breaking as appearing already at the level of imbedding space $M^4 \times CP_2$. The estimate for the mass splitting of neutral kaon and anti-kaon is of correct order of magnitude.

Whether stationary spherically symmetric metric as minimal surface allows a sensible physical generalization is a killer test for the hypothesis that $J(M^4)$ is covariantly constant. The question is basically about how large the moduli space of forms $J(M^4)$ can be allowed to be. The mere self duality and closedness condition outside the line connecting the tips of CD allows also variants which are spherically symmetric in either Minkowski coorinates or Robertson-Walker coordinates for light-cone.

3. How does gravitational coupling emerge at fundamental level? The first naive guess is obvious: string area action is scaled by $1/G$ as in string models. The objection is that p-adic mass calculations suggest that string tension is determined by CP_2 size R: the analog of string tension appearing in mass formula given by p-adic mass calculations would be by a factor about 10^{-8} smaller than that estimated from string tension. The discrepancy evaporates by noticing that p-adic mass calculations rely on p-adic thermodynamics at imbedding space level whereas string world sheets appear at space-time level. Furthermore, if the action assignable to string world sheets is effective action expressing 4-D action in 2-D form as strong form of holography (SH) suggests string tension is expected to be function of the parameters appearing in the 4-D action.

4. Could one regard the localization of spinor modes to string world sheets as a localization to Lagrangian sub-manifolds of space-time surface having by definition vanishing induced Kähler form: $J(M^4)+J(CP_2) = 0$. Lagrangian sub-manifolds would be commutative in the sense of Poisson bracket? Could string world sheets be minimal surfaces satisfying $J(M^4)+J(CP_2) = 0$. The Lagrangian condition allows also more general solutions - even 4-D space-time surfaces and one obtains analog of brane hierarchy. Could one allow spinor modes also at these analogs of branes. Is Lagrangian condition equivalent with the original condition that induced W boson fields making the em charge of induced spinor modes ill-defined vanish and allowing also solution with other dimensions. How Lagrangian property relates to the idea that string world sheets correspond to complex (commutative) surfaces of quaternionic space-time surface in octonionic imbedding space.

During the re-processing of the details related to twistor lift, it became clear that the earlier variant for the twistor lift [L24] contained an error. This led to much simpler view about twistor lift, to the conclusion that minimal surface extremals of Kähler action represent only asymptotic situation (external particles in scattering), and also to a re-interpretation for the p-adic evolution of the cosmological constant.

15.1.2 Questions related to the quantum aspects of twistorialization

Also the questions related to the quantum aspects of twistorialization of TGD are discussed.

1. There are several notions of twistor. Twistor space for M^4 is $T(M^4) = M^4 \times S^2$ [B72] (see `http://arxiv.org/pdf/1308.2820.pdf`) having projections to both M^4 and to the standard twistor space $T_1(M^4)$ often identified as CP_3. $T(M^4) = M^4 \times S^2$ is necessary for

the twistor lift of space-time dynamics. CP_2 gives the factor $T(CP_2) = SU(3)/U(1) \times U(1)$ to the classical twistor space $T(H)$. The quantal twistor space $T(M^8) = T_1(M^4) \times T(CP_2)$ assignable to momenta. The possible way out is $M^8 - H$ duality relating the momentum space M^8 (isomorphic to the tangent space H) and H by mapping space-time associative and co-associative surfaces in M^8 to the surfaces which correspond to the base spaces of in H: they construction would reduce to holomorphy in complete analogy with the original idea of Penrose in the case of massless fields.

2. The standard twistor approach has problems. Twistor Fourier transform reduces to ordinary Fourier transform only in signature (2,2) for Minkowski space: in this case twistor space is real RP_3 but can be complexified to CP_3. Otherwise the transform requires residue integral to define the transform (in fact, p-adically multiple residue calculus could provide a nice manner to define integrals and could make sense even at space-time level making possible to define action).

   Also the positive Grassmannian requires (2,2) signature. In $M^8 - H$ relies on the existence of the decomposition $M^2 \subset M^2 = M^2 \times E^2 \subset M^8$. M^2 could even depend on position but $M^2(x)$ should define an integrable distribution. There always exists a preferred M^2, call it M_0^2, where 8-momentum reduces to light-like M^2 momentum. Hence one can apply 2-D variant of twistor approach. Now the signature is (1,1) and spinor basis can be chosen to be real! Twistor space is RP_3 allowing complexification to CP_3 if light-like complex momenta are allowed as classical TGD suggests!

3. A further problem of the standard twistor approach is that in M^4 twistor approach does not work for massive particles. In TGD all particles are massless in 8-D sense. In M^8 M^4-mass squared corresponds to transversal momentum squared coming from $E^4 \subset M^4 \times E^4$ (from CP_2 in H). In particular, Dirac action cannot contain anyo mass term since it would break chiral invariance.

   Furthermore, the ordinary twistor amplitudes are holomorphic functions of the helicity spinors λ_i and have no dependence on $\tilde{\lambda}_i$: no information about particle masses! Only the momentum conserving delta function gives the dependence on masses. These amplitudes would define as such the M^4 parts of twistor amplitudes for particles massive in TGD sense. The simplest 4-fermion amplitude is unique.

Twistor approach gives excellent hopes about the construction of the scattering amplitudes in ZEO. The construction would split into two pieces corresponding to the orbital degrees of freedom in "world of classical worlds" (WCW) and to spin degrees of freedom in WCW: that is spinors, which correspond to second quantized induced spinor fields at space-time surface (actually string world sheets- either at fundamental level or for effective action implied by strong form of holography (SH)).

1. At WCW level there is a perturbative functional integral over small deformations of the 3-surface to which space-time surface is associated. The strongest assumption is that this 3-surface corresponds to maximum for the real part of action and to a stationary phase for its imaginary part: minimal surface extremal of Kähler action would be in question. A more general but number theoretically problematic option is that an extremal for the sum of Kähler action and volume term is in question.

   By Kähler geometry of WCW the functional integral reduces to a sum over contributions from preferred extremals with the fermionic scattering amplitude multiplied by the ration X_i/X, where $X = \sum_i X_i$ is the sum of the action exponentials for the maxima. The ratios of exponents are however number theoretically problematic.

   Number theoretical universality is satisfied if one assigns to each maximum independent zero energy states: with this assumption $\sum X_i$ reduces to single X_i and the dependence on action exponentials becomes trivial! ZEO allow this. The dependence on coupling parameters of the action essential for the discretized coupling constant evolution is only via boundary conditions at the ends of the space-time surface at the boundaries of CD.

   Quantum criticality of TGD [K106, K110, K111] demands that the sum over loops associated with the functional integral over WCW vanishes and strong form of holography (SH) suggests that the integral over 4-surfaces reduces to that over string world sheets and partonic 2-surfaces corresponding to preferred extremals for which the WCW coordinates parametrizing them belong to the extension of rationals defining the adele [L34]. Also the intersections of the real and various p-adic space-time surfaces belong to this extension.

2. Second piece corresponds to the construction of twistor amplitude from fundamental 4-fermion amplitudes. The diagrams consists of networks of light-like orbits of partonic two surfaces, whose union with the 3-surfaces at the ends of CD is connected and defines a boundary condition for preferred extremals and at the same time the topological scattering diagram.

   Fermionic lines correspond to boundaries of string world sheets. Fermion scattering at partonic 2-surfaces at which 3 partonic orbits meet are analogs of 3-vertices in the sense of Feynman and fermions scatter classically. There is no local 4-vertex. This scattering is assumed to be described by simplest 4-fermion twistor diagram. These can be fused to form more complex diagrams. Fermionic lines runs along the partonic orbits defining the topological diagram.

3. Number theoretic universality [K111] suggests that scattering amplitudes have interpretation as representations for computations. All space-time surfaces giving rise to the same computation wold be equivalent and tree diagrams corresponds to the simplest computation. If the action exponentials do not appear in the amplitudes as weights this could make sense but would require huge symmetry based on two moves. One could glide the 4-vertex at the end of internal fermion line along the fermion line so that one would eventually get the analog of self energy loop, which should allow snipping away. An argument is developed stating that this symmetry is possible if the preferred M_0^2 for which 8-D momentum reduces to light-like M^2-momentum having unique direction is same along entire fermion line, which can wander along the topological graph.

   The vanishing of topological loops would correspond to the closedness of the diagrams in what might be called BCFW homology. Boundary operation involves removal of BCFW bridge and entangled removal of fermion pair. The latter operation forces loops. There would be no BCFW bridges and entangled removal should give zero. Indeed, applied to the proposed four fermion vertex entangled removal forces it to correspond to forward scattering for which the proposed twistor amplitude vanishes.

To sum up, the twistorial approach leads to a proposal for an explicit construction of scattering amplitudes for the fundamental fermions. Bosons and fermions as elementary particles are bound states of fundamental fermions assignable to pairs of wormhole contacts carrying fundamental fermions at the throats. Clearly, this description is analogous to a quark level description of hadron. Yangian symmetry with multilocal generators is expected to crucial for the construction of the many-fermion states giving rise to elementary particles. The problems of the standard twistor approach find a nice solution in terms of M^8-H duality, 8-D masslessness, and holomorphy of twistor amplitudes in λ_i and their indepence on $\tilde{\lambda}_i$.

15.2 More details about the induction of twistor structure

The notion of twistor lift of TGD [L22] [L40] has turned out to have powerful implications concerning the understanding of the relationship of TGD to general relativity. The meaning of the twistor lift really has remained somewhat obscure. There are several questions to be answered. What does one mean with twistor space? What does the induction of twistor structure of $H = M^4 \times CP_2$ to that of space-time surface realized as its twistor space mean?

15.2.1 What does one mean with twistor space?

The notion of twistor space has been discussed in [L22] from TGD point of view.

1. In the case of twistor space of M^4 the starting point of Penrose was the isomorphism between the conformal group of Spin(4,2) of 6-D Minkowski space $M^{4,2}$ and the group SU(2,2) acting on 2+2 complex spinors.

6-D twistor space could be identified as 6-D coset space $SU(2,2)/SU(2,1)\times U(1)$. For E^6 this would give projective space $CP_3 = SU(4)/SU(3)\times U(1)$ and in twistor Grassmann approach this definition is indeed used. It is thought that the problems caused by Euclidization are not serious.

2. One can think $SU(2,2)$ as 4×4 complex matrices with orthogonal complex row vector $Z_i = (Z_{i1}, ..., Z_{i4})$, and norms $(1,1,-1-1)$ in the metric $s^2 = \sum \epsilon_i |z_i|^2$, $\epsilon_i \leftrightarrow (1,1,-1,-1)$. The sub-matrices defined by (Z_{k2}, Z_{k3}, Z_{k4}), $k = 2,3,4$, can be regarded apart from normalization elements of $SU(1,2)$. The column vector with components Z_{i1} with $Z_{11} = \sqrt{1+\rho^2}$, $\rho^2 = |Z_{21}|^2 - |Z_{31}|^2 - |Z_{41}|^2$ corresponds to a point of the twistor space. The S^2 fiber for given values of ρ and (Z_{31}, Z_{41}) could be identified as the space spanned by the values of Z_{21}. Note that S^2 would have time-like signature and the signature of twistor space would be (3,3), which conforms with the existence of complex structure. There would be dimensional democracy at this level.

3. The identification of 4-D base of the twistor space is unclear to me. The base space of the this twistor space should correspond to the conformal compactification M^4_c of M^4 having metric defined only apart from conformal scaling. The concrete realization M^4_c would be in terms of $M^{4,2}$ light-cone with points projectively identified. As a metric object this space is ill-defined and can appear only at the level of scattering amplitudes in conformally invariant quantum field theories in M^4.

4. Mathematicians define also a second variant of twistor space with S^2 fiber and this space is just $M^4 \times S^2$ [B72] (see `http://tinyurl.com/yb4bt74l`). This space has a well-defined metric and seems to be the only possible one for the twistor lift of classical TGD replacing space-time surfaces with their twistor spaces. Whether the signature of S^2 is time-like or space-like has remained an open question but time-like signature looks natural. The radius R_P of S^2 has been proposed to be apart from a numerical constant equal to Planck length l_P. Note that the isometry group is 9-D $SO(3,1)\times SU(2)$ rather than 15-D $SU(2,2)$. In TGD light-likeness in 8-D sense replaces light-likeness in 4-D sense: does this somehow replace the conformal symmetry group $SO(4,2)$ with $SO(3,1)\times SO(3)$? Could $SU(2)$ rotate the direction of spin quantization axis.

I must confess that I have found the notions of twistor and twistor sphere very difficult to understand. Perhaps this is not solely due to my restricted mathematical skills. Also the physics of twistors looks confusing to me.

The twistor space assignable to Minkowski space and corresponding twistor sphere have several meanings. Consider first the situation in standard framework.

1. One can define twistor space as complex 8-D space C^4. Given four-momentum corresponds however to projective line so that one can argue that twistor space is 6-D space $T_1(M^4) = CP_3 = SU(4)/SU(3)\times U(1)$ of projective lines of C^4 in C^4. One could also argue that one must take the signature of Minkowski space into account. $SU(2,2)$ acts as symmetries of twistor bilinear form and one would have $T_1(M^4) = SU(2,2)/SU(2,1)\times U(1)$. In this case twistor sphere could correspond to the projective line in C^4.

2. Incidence relations $\mu^{\dot a} = m^{a\dot a}\lambda_a$ relate M^4 points to those of twistor space. In the usual twistor formalism twistor sphere corresponds to the projective line of 8-D C^4. When m is not light-like, it corresponds to a matrix which is invertible and one can solve μ from λ and vice versa. The twistor spheres associated with m_1 and m_2 are said to intersect if $m_1 - m_2$ is a complex light-like vector defining a complexified light ray. One could identify twistor sphere of $T_1(M^4)$ as the Riemann sphere defined by these complex points and going to CP_3 one actually eliminates it altogether, which is somewhat unsatisfactory.

3. When m is light-like and thus expressible as $\mu = \lambda\otimes\tilde\lambda$ one has $\mu = \mu_0 + t\tilde\lambda$, t a complex number. One can say that one has a full Riemann sphere S^2 of solutions. There is also additional degeneracy due to the scaling of both λ and μ. For light-like M^4 points (say momenta) one obtains a Riemann sphere in 6-D twistor space. Which twistor sphere is the correct one: the sphere associated with all points of M^4 and 8-D twistor space or the sphere associated with light-like points of M^4 and 6-D twistor space?

Consider now the situation in TGD.

1. For the twistor lift of Kähler action lifting the dynamics of space-time surfaces to the dynamics of their twistor spaces, the twistor lift of M^4 corresponds to $T(M^4) = M^4 \times CP_2$. This might look strange but the proper mathematical definition of twistor space relies on double fibration involving both views about twistor space discussed in [B72] (see `http://tinyurl.com/yb4bt74l`). This double fibration would be crucially involved with $M^8 - H$ duality. The fiber space is $T(M^4) = M^4 \times CP_1$, where CP_1 corresponds to the projective sphere assignable to complex spinors λ. This fiber is trivially projected both to M^4 and less trivially to a subset of 6-dimensional complex projective space $T_!(M^4) = CP_3$.

   At space-time level $T(M^4)$ is the only correct choice since twistor space must have isometries of M^4. This choices brings into the dynamics Planck length essentially as the radius of S^2 and cosmological constant as volume term resulting in the dimensional reduction of 6-D Kähler action forced by twistor space property of 6-surface.

   At the level of momentum space - perhaps the M^8 appearing in $M^8 - H$ duality identifiable as tangent space of H - the twistor space would correspond to twistor space assignable to momentum space and should relate to the ordinary twistor space $T_1(M^4)$ - whatever it is!

2. In M^8 picture the twistor space is naturally associated with preferred $M^2 \subset M^4$, where M^4 is quaternionic space. The moduli space of $M^2 \subset M^4$ for time direction assigned with real octonion, is parametrized by S^2 and a possible interpretation is as twistor sphere of $M^2 \times CP_1$. Interestingly, $M^2 \subset M^4$ is characterized by light-like vector together with its unique dual light-like vector.

   By restricting 4-D conformal invariance to 2-D situation, one finds that the twistor space becomes RP_3 but can be complexified to CP_3 to allowing complexified M^2 momenta. The signature (1,1) of M^2 and reality of spinor basis gives hopes of resolving the conceptual problems of the ordinary twistor approach. For the real spinor spinor pair (λ, μ) the solutions to the co-incidence relations real M^2 spinors but one can allowing their complex multiples.

3. $M^8 - H$ correspondence allows to map M^4 points to each other: this involves a choice of $M^4 \subset M^8$. $M^8 - H$ correspondence maps quaternionic (and co-quaternionic) surfaces in M^8 to preferred extremals of Kähler in H proposed to correspond to the base bases of of twistor bundles $T(X^4) \subset T(M^4) \times T(CP_2)$ constructible using holomorphic maps. One can thus argue that there should be also a correspondence between the twistor spaces $T(M^4)$ and $T_1(M^4)$ - the correspondence between the twistor spheres would be enough.

   The two M^4:s correspond to each other naturally. What is required is a map of twistorial spheres S^2 to each other. Suppose that the twistorial sphere of H corresponds to that assignable to the choice of $M^2 \subset M^8$ by a choice of quaternionic imaginary unit in M^4 of equivalently by a choice of a light-like vector n of M^2 plane. But by incidence relations the light-like vector n has twistor sphere CP_1 as a pre-image in complexified $T_1(M^2) = CP_3$ characterized by the shifts $\mu \rightarrow \mu + \tilde{\lambda}$. Therefore the two twistor spheres can be identified by mapping n of $S^2(T(M^4)$ to its counterpart of $T_1(M^2)$ isometrically.

It therefore seems that the double fibration is essential in TGD framework and the usual twistor space is assignable to the M^8 interpreted asthe space of complexified octonion momenta subject to the quaternionicity condition. Sharply defined transversed quaternionic momentum eigenstates in $E^2 \times E^4$ are replaced with wave functions in $T(CP_2)$ reducing locally to $CP_2 \times U(2)/U(1) \times U(1)$ with em charge identifiable as the analog of angular momentum for the wave functions in $CP_1 = U(2)/U(1) \times U(1)$. In $M^4 \times CP_2$ picture one has spinor modes labelled by electroweak quantum numbers.

15.2.2 Twistor lift of TGD

In TGD one replaces imbedding space $H = M^4 \times CP_2$ with the product $T = T(M^4) \times T(CP_2)$ of their 6-D twistor spaces, and calls $T(H)$ the twistor space of H. For CP_2 the twistor space is the flag manifold $T(CP_2) = SU(3)/U(1) \times U(1)$ consisting of all possible choices of quantization axis of color isospin and hypercharge.

1. The basic idea is to generalize Penrose's twistor program by lifting the dynamics of space-time surfaces as preferred extremals of Kähler action to those of 6-D Kähler action in twistor space $T(H)$. The conjecture is that field equations reduce to the condition that the twistor structure of space-time surface as 4-manifold is the twistor structure induced from $T(H)$.

   Induction requires that dimensional reduction occurs effectively eliminating twistor fiber $S^2(X^4)$ from the dynamics. Space-time surfaces would be preferred extremals of 4-D Kähler action plus volume term having interpretation in terms of cosmological constant. Twistor lift would be more than an mere alternative formulation of TGD.

2. The reduction would take place as follows. The 6-D twistor space $T(X^4)$ has S^2 as fiber and can be expressed locally as a Cartesian product of 4-D region of space-time and of S^2. The signature of the induced metric of S^2 should be space-like or time-like depending on whether the space-time region is Euclidian or Minkowskian. This suggests that the twistor sphere of M^4 is time-like as also standard picture suggests.

3. Twistor structure of space-time surface is induced to the allowed 6-D surfaces of $T(H)$, which as twistor spaces $T(X^4)$ must have fiber space structure with S^2 as fiber and space-time surface X^4 as base. The Kähler form of $T(H)$ expressible as a direct sum

   $$J(T(H)) = J(T(M^4)) \oplus J(T(CP_2))$$

   induces as its projection the analog of Kähler form in the region of $T(X^4)$ considered.

   There are physical motivations (CP breaking, matter antimatter symmetry, the well-definedness of em charge) to consider the possibility that also M^4 has a non-trivial symplectic/Kähler form of M^4 obtained as a generalization of ordinary symplectic/Kähler form [L40]. This requires the decomposition $M^4 = M^2 \times E^2$ such that M^2 has hypercomplex structure and E^2 complex structures.

   This decomposition might be even local with the tangent spaces $M^2(x)$ and $E^2(x)$ integrating to locally orthogonal 2-surfaces. These decomposition would define what I have called Hamilton-Jacobi structure [K79]. This would give rise to a moduli space of M^4 Kähler forms allowing besides covariantly constant self-dual Kähler forms with decomposition (m^0, m^3) and (m^1, m^2) also more general self-dual closed Kähler forms assignable to integrable local decompositions. One example is spherically symmetric stationary self-dual Kähler form corresponding to the decomposition (m^0, r_M) and (θ, ϕ) suggested by the need to get spherically symmetric minimal surface solutions of field equations. Also the decomposition of Robertson-Walker coordinates to (a, r) and (θ, π) assignable to light-cone M^4_+ can be considered.

   The moduli space giving rise to the decomposition of WCW to sectors would be finite-dimensional if the integrable 2-surfaces defined by the decompositions correspond to orbits of subgroups of the isometry group of M^4 or CD. This would allow planes of M^4, and radial half-planes and spheres of M^4 in spherical Minkowski coordinates and of M^4_+ in Robertson-Walker coordinates. These decomposition could relate to the choices of measured quantum numbers inducing symmetry breaking to the subgroups in question. These choices would chose a sector of WCW [K41] and would define quantum counterpart for a choice of quantization axes as distinct from ordinary state function reduction with chosen quantization axes.

4. The induced Kähler form of S^2 fiber of $T(X^4)$ is assumed to reduce to the sum of the induced Kähler forms from S^2 fibers of $T(M^4)$ and $T(CP_2)$. This requires that the projections of the Kähler forms of M^4 and CP_2 to $S^2(X^4)$ are trivial. Also the induced metric is assumed to be direct sum and similar conditions holds true.These conditions are analogous to those occurring in dimensional reduction.

   Denote the radii of the spheres associated with M^4 and CP_2 as $R_P = kl_P$ and R and the ratio R_P/R by ϵ. Both the Kähler form and metric are proportional to R_p^2 resp. R^2 and satisfy the defining condition $J_{kr}g^{rs}J_{sl} = -g_{kl}$. This condition is assumed to be true also for the induced Kähler form of $J(S^2(X^4)$.

   Let us introduce the following shorthand notations

$$\begin{array}{lll} S_1^2 = S^2(X^4) \ , & S_2^2 = S^2(CP_2) \ , & S_3^2 = S^2(M^4) \ , \\ J_i = \frac{J(S_i^2)}{R^2} \ , & g_i = \frac{g(S_i^2))}{R^2} \ . & \end{array} \tag{15.2.1}$$

This gives the following equations.

$$J_1 = J_2 + \epsilon J_3 \ , \quad g_1 = g_2 + \epsilon g_3 \ , \quad J_1 g_1 J_1 = -g_1 \ . \tag{15.2.2}$$

Projections to $S_1^2 = S^2(X^4)$ are assumed at r.h.s.. The product of the third equation is defined as tensor contraction and involves contravariant form of g.

15.2.3 Solutions to the conditions defining the twistor lift

Consider now solutions to the conditions defining the twistor lift.

1. The simplest solution type corresponds to the situation in which either S_2^2 (S_3^2) equals to S_1^2) and S_3^2 (S_2^2) projection of $T(X^4)$ is single point. In this case the conditions of Eq. are trivially satisfied. These two solutions could correspond to Euclidian and Minkowskian space-time regions. Also the solution for which twistor sphere degenerates to a point must be considered and form $J(M^4) = 0$ this would correspond to the reduction of dimensionally reduced action to Kähler action defining the original variant of TGD. Note that preferred extremals are conjectured to be minimal surfaces extremals of Kähler action always [L19].

2. One can consider also more general solutions. Depending on situation, one can use for $S^2(X^4)$ either the coordinates of S_2^2 or S_3^2. Let us choose S_2^2. One can of course change the roles of the spheres.

   Consider an ansatz for which the projections of J_2 and J_3 to S_1^2 are in constant proportionality to each other. This is guaranteed if the spherical coordinates ($u = cos(\Theta), \Phi$) of S_2^2 and S_3^2 are related by $(u(M^4), \Phi(M^4)) = (u(CP_2), n\Phi(CP_2))$ so that the map between the two spheres has winding number n. With this assumption one has

$$\begin{array}{l} J_1 = (1 + \epsilon n) J_2 \ , \\ g_1 = (1 + \epsilon n^2) g_2 \ , \end{array} \tag{15.2.3}$$

   The third condition of Eq. 1 equation gives

$$(1 + n\epsilon)^2 \quad = \quad (1 + n^2\epsilon)^2 \ . \tag{15.2.4}$$

   This in turn gives

$$1 + n\epsilon = \delta(1 + n^2\epsilon) \ , \quad \delta = \pm 1 \ . \tag{15.2.5}$$

   The only solution for $\delta = +1$ is $n = 0$ or $n = 1$. For $\delta = -1$ there are no solutions.

   One has 3+1 different solutions corresponding to the degenerate solution $(n_1, n_2) = (0, 0)$ and 3 solutions with (n_1, n_2) equal $(1, 0)$, $(0, 1)$ or $(1, 1)$. The conditions are very stringent and it is not clear whether there are any other solutions.

3. The further conditions implying locally direct sum for g and J pose strong restrictions on space-time surfaces. The conjecture that the solutions of these conditions correspond to preferred extremals of 6-D Kähler action leads by dimensional reduction to the conclusion that the 4-D action contains besides 4-D Kähler action also a volume term coming from S^2 Kähler actions and giving rise to cosmological constant.

   What is of special interest is that for the degenerate solution the volume term vanishes, and one has mere 4-D Kähler action with induced Kähler form possibly containing also $J(M^4)$, which leads to a rather sensible cosmology having interpretation as infinite volume limit for causal diamond (CD) inside which space-time surfaces exist. This limit could be appropriate for QFT limit of TGD, which indeed corresponds to infinite-volume limit at which cosmological constant approaches zero.

What could be the physical interpretation of the solutions?

1. Physical intuition suggests that S_1^2 must be space-like for Euclidian signature of space-time region $[(n_1, n_2) = (1, 0)]$ and time-like for Minkowskian signature $[(n_1, n_2) = (0, 1)]$.

2. By quantum classical correspondence one can argue that the non-vanishing of space-time projection of $J(M^4)$ resp. $J(CP_2)$ is necessary to fix local quantization axis of spin resp. weak isospin. If so, then $n_1 = 1/0$ resp. $n_2 = 1/0$ would tell that the projection of $J(CP_2)$ resp. $J(M^2)$ is non-vanishing/vanishes. If both contributions vanish $[(n_1, n_2) = (0, 0)]$ one has generalized Lagrangian 4-surface, which would be vacuum extremal. The products of 2-D Lagrangian manifolds for M^4 and CP_2 would be vacuum extremals. One can wonder whether there exist 4-surfaces representable as a graph of a map $M^4 \to CP_2$ such that the induced Kähler form vanishes. This picture allows only the imbeddings of trivial Robertson-Walker cosmology as vacuum extremal of Kähler action since both M^4 contribution to Kähler action and volume term would be non-vanishing $[(n_1, n_2) = (0, 1)]$.

15.2.4 Twistor lift and the reduction of field equations and SH to holomorphy

It has become clear that twistorialization has very nice physical consequences. But what is the deep mathematical reason for twistorialization? Understanding this might allow to gain new insights about construction of scattering amplitudes with space-time surface serving as analogs of twistor diatrams.

Penrose's original motivation for twistorilization was to reduce field equations for massless fields to holomorphy conditions for their lifts to the twistor bundle. Very roughly, one can say that the value of massless field in space-time is determined by the values of the twistor lift of the field over the twistor sphere and helicity of the massless modes reduces to cohomology and the values of conformal weights of the field mode so that the description applies to all spins.

I want to find the general solution of field equations associated with the Kähler action lifted to 6-D Kähler action. Also one would like to understand strong form of holography (SH). In TGD fields in space-time are are replaced with the imbedding of space-time as 4-surface to H. Twistor lift imbeds the twistor space of the space-time surface as 6-surface into the product of twistor spaces of M^4 and CP_2. Following Penrose, these imbeddings should be holomorphic in some sense.

Twistor lift $T(H)$ means that M^4 and CP_2 are replaced with their 6-D twistor spaces.

1. If S^2 for M^4 has 2 time-like dimensions one has 3+3 dimensions, and one can speak about hyper-complex variants of holomorphic functions with time-like and space-like coordinate paired for all three hypercomplex coordinates. For the Minkowskian regions of the space-time surface X^4 the situation is the same.

2. For $T(CP_2)$ Euclidian signature of twistor sphere guarantees this and one has 3 complex coordinates corresponding to those of S^2 and CP_2. One can also now also pair two real coordinates of S^2 with two coordinates of CP_2 to get two complex coordinates. For the Euclidian regions of the space-time surface the situation is the same.

Consider now what the general solution could look like. Let us continue to use the shorthand notations $S_1^2 = S^2(X^4); S_2^2 = S^2(CP_2); S_3^2 = S^2(M^4)$.

1. Consider first solution of type $(1,0)$ so that coordinates of S^2_2 are constant. One has holomorphy in hypercomplex sense (light-like coordinate $t-z$ and $t+z$ correspond to hypercomplex coordinates).

   (a) The general map $T(X^4)$ to $T(M^4)$ should be holomorphic in hyper-complex sense. $S^2_1$is in turn identified with S^2_3 by isometry realized in real coordinates. This could be also seen as holomorphy but with different imaginary unit. One has analytical continuation of the map $S^2_1 \rightarrow S^2_3$ to a holomorphic map. Holomorphy might allows to achieve this rather uniquely. The continued coordinates of S^2_1 correspond to the coordinates assignable with the integrable surface defined by $E^2(x)$ for local $M^2(x) \times E^2(x)$ decomposition of the local tangent space of X^4. Similar condition holds true for $T(M^4)$. This leaves only $M^2(x)$ as dynamical degrees of freedom. Therefore one has only one holomorphic function defined by 1-D data at the surface determined by the integrable distribution of $M^2(x)$ remains. The 1-D data could correspond to the boundary of the string world sheet.

   (b) The general map $T(X^4)$ to $T(CP_2)$ cannot satisfy holomorphy in hyper-complex sense. One can however provide the integrable distribution of $E^2(x)$ with complex structure and map it holomorphically to CP_2. The map is defined by 1-D data.

   (c) Altogether, 2-D data determine the map determining space-time surface. These two 1-D data correspond to 2-D data given at string world sheet: one would have SH.

2. What about solutions of type $(0,1)$ making sense in Euclidian region of space-time. One has ordinary holomorphy in CP_2 sector.

   (a) The simplest picture is a direct translation of that for Minkowskian regions. The map $S^2_1 \rightarrow S^2_2$ is an isometry regarded as an identification of real coordinates but could be also regarded as holomorphy with different imaginary unit. The real coordinates can be analytically continued to complex coordinates on both sides, and their imaginary parts define coordinates for a distribution of transversal Euclidian spaces $E^2_2(x)$ on X^4 side and $E^2(x)$ on M^4 side. This leaves 1-D data.

   (b) What about the map to $T(M^4)$? It is possible to map the integrable distribution $E^2_2(x)$ to the corresponding distribution for $T(M^4)$ holomorphically in the ordinary sense of the word. One has 1-D data. Altogether one has 2-D data and SH and partonic 2-surfaces could carry these data. One has SH again.

3. The above construction works also for the solutions of type $(1,1)$, which might make sense in Euclidian regions of space-time. It is however essential that the spheres S^2_2 and S^2_3 have real coordinates.

SH thus would thus emerge automatically from the twistor lift and holomorphy in the proposed sense.

1. Two possible complex units appear in the process. This suggests a connection with quaternion analytic functions [L22] suggested as an alternative manner to solve the field equations. Space-time surface as associative (quaterionic) or co-associate (co-quaternionic) surface is a further solution ansatz.

   Also the integrable decompositions $M^2(x) \times E^2(x)$ resp. $E^2_1(x) \times E^2_2(x)$ for Minkowskian resp. Euclidian space-time regions are highly suggestive and would correspond to a foliation by string wold sheets and partonic 2-surfaces. This expectation conforms with the number theoretically motivated conjectures [K111].

2. The foliation gives good hopes that the action indeed reduces to an effective action consisting of an area term plus topological magnetic flux term for a suitably chosen stringy 2-surfaces and partonic 2-surfaces. One should understand whether one must choose the string world sheets to be Lagrangian surfaces for the Kähler form including also M^4 term. Minimal surface condition could select the Lagrangian string world sheet, which should also carry vanishing classical W fields in order that spinors modes can be eigenstates of em charge.

The points representing intersections of string world sheets with partonic 2-surfaces defining punctures would represent positions of fermions at partonic 2-surfaces at the boundaries of CD and these positions should be able to vary. Should one allow also non-Lagrangian string world sheets or does the space-time surface depend on the choice of the punctures carrying fermion number (quantum classical correspondence)?

3. The alternative option is that any choice produces of the preferred 2-surfaces produces the same scattering amplitudes. Does this mean that the string world sheet area is a constant for the foliation - perhaps too strong a condition - or could the topological flux term compensate for the change of the area?

   The selection of string world sheets and partonic 2-surfaces could indeed be also only a gauge choice. I have considered this option earlier and proposed that it reduces to a symmetry identifiable as U(1) gauge symmetry for Kähler function of WCW allowing addition to it of a real part of complex function of WCW complex coordinates to Kähler action. The additional term in the Kähler action would compensate for the change if string world sheet action in SH. For complex Kähler action it could mean the addition of the entire complex function.

15.3 How does the twistorialization at imbedding space level emerge?

An objection against twistorialization at imbedding space level is that M^4-twistorialization requires 4-D conformal invariance and massless fields. In TGD one has towers of particle with massless particles as the lightest states. The intuitive expectation is that the resolution of the problem is that particles are massless in 8-D sense as also the modes of the imbedding space spinor fields are.

To explain the idea, let us select a fixed decomposition $M^8 = M_0^4 \times E_0^4$ and assume that the momenta are complex - for motivations see below.

1. With inspiration coming from M^8-H duality [K74] suppose that for the allowed compositions $M^8 = M^4 \times E^4$ one has $M^4 = M_0^2 \times E^2$ with M_0^2 fixed, and corresponding to real octonionic unit and preferred imaginary unit. Obviously 8-D light-likeness for $M^8 = M_0^4 \times E_0^4$ reduces to 4-D light-likeness for a preferred choice of $M^8 = M^4 \times CP_2$ decomposition.

2. This suggests that in the case of massive M_0^4 momenta one can apply twistorialization to the light-like M^4-momentum and code the information about preferred M^4 by a point of CP_2 and about 8-momentum in $M^8 = M_0^4 \times E_0^4$ by an $SU(3)$ transformation taking M_0^4 to M^4. Pairs of twistors and $SU(3)$ transformations would characterize arbitrary quaternionic 8-momenta. 8-D masslessness gives however 2 additional conditions for the complex 8-momenta probably reducing $SU(3)$ to $SU(3)/U(1) \times U(1)$ - the twistor space of CP_2! This would also solve the basic problem of twistor approach created by the existence of massive particles.

The assumption of complex momenta in previous considerations might raise some worries. The space-time action of TGD is however complex if Kähler coupling strength is complex, and there are reasons to believe that this is the case. Both four-momenta and color quantum numbers - all Noether charges in fact - could be complex. A possible physical interpretation for complex momenta could be in terms of the natural width of states induced by the finite size of CD. Also in twistor Grassmannian approach one encounters complex but light-like four-momenta. Note that complex light-like space-time momenta correspond in general to massive real momenta. It is not clear whether it makes sense to speak about width of color quantum numbers: their reality would give additional constraint. The emergence of M^4 mass in this manner could be involved with the classical description for the emergence of the third helicity.

The observation that octonionic twistors make sense and their restriction to quaternionic twistors produce ordinary M^4 twistors provides an alternative view point to the problem. Also $M^8 - H$ duality proposed to map quaternionic 4-D surfaces in octonionic M^8 to (possibly quaternionic) 4-D surfaces in $M^4 \times CP_2$ is expected to be relevant. The twistor lift of $M^8 - H$ duality would give $T(M^8) - T(H)$ duality.

Twistor Grassmann approach [B33, B27, B26, B38, B39, B20] uses as twistor space the space $T_1(M^4) = SU(2,2)/SU(2,1) \times U(1)$ whereas the twistor lift of classical TGD uses $M^4 \times S^2$. The formulation of the twistor amplitudes in terms of SH using the data assignable to the 2-D surfaces - string world sheets and partonic 2-surfaces perhaps - identified as surfaces in $T(M^4) \times T(CP_2)$ requires the mapping of these twistor spaces to each other - the incidence relations of Penrose indeed realize this map.

15.3.1 $M^8 - H$ duality at space-time level

Twistors emerge as a description of massless particles with spin [B69] but are not needed for spin zero particles. Therefore one can consider first mere momenta.

1. Consider first space-time surfaces of M^8 with Minkowskian signature of the induced metric so that the tangent space is M^4. $M^8 - H$ duality [K74] implies that CP_2 points parameterize quaternionic sub-spaces M^4 of octonions containing fixed $M_0^2 \subset M^4$. Using the decomposition $1+1+3+\overline{3}$ of complexified octonions to representations of $SU(3)$, it is easy to see that this space is indeed CP_2. M^4 correspond to the sub-space $1+1+2$ where 2 is $SU(2) \subset SU(3)$ doublet.

   CP_2 spinor mode would be spinor mode in the space of quaternionic sub-spaces $M^4 \subset M^8$ with $M_0^2 \subset M^4$ with real octonionic unit defining preferred time like direction and imaginary unit defining preferred spin quantization axis. $M^8 - H$ duality allows to map quaternionic 4-surfaces of $M^4 \supset M_0^2$ to 4-surfaces in H. The latter could be quaternionic but need not to.

2. For Euclidian signature of the induced metric tangent space is E^4. In this case co-associative surfaces are needed since the above correspondence make sense only if the tangent space corresponds to M^4. For instance, for CP_2 type exremals tangent space corresponds to E^4. M^4 and E^4 change roles. Also now the space of co-associative tangent spaces is CP_2 since co-associative tangent space is the octonionic orthogonal complement of the associative tangent space. One would have Euclidian variant of the associative case.

$M^8 - H$ correspondence raises the question whether the octonionic M^8 or $M^4 \times CP_2$ represents the level, which deserves to be called fundamental. Or are they just alternative descriptions made possible by the quaternionicity of space-time surface in M^8 and quaternionic momentum space necessitating quaternionicity of the tangent space of X^4? In any case, one should demonstrate that the spectrum of states with $M^4 \times E^4$ with quaternionic light-like 8-momenta is equivalent with the spectrum of states for $M^4 \times CP_2$

15.3.2 Parametrization of light-like quaternionic 8-momenta in terms of $T(CP_2)$

The following argument shows that the twistor space $T(CP_2)$ emerges naturally from $M^8 - H$ correspondence for quaternionic light-like M^8 momenta.

1. Continue to assume a fixed decomposition $M^8 = M_0^4 \times E_0^4$, and that for the allowed compositions $M^8 = M^4 \times E^4$ one has $M^4 = M_0^2 \times E^2$ with M_0^2 fixed. Light-like quaternionic 8-momentum in $M^8 = M_0^4 \times E_0^4$ can be reduced to light-like M^4 momentum and vanishing E^4 momentum for some preferred $M^8 = M^4 \times E^4$ decomposition.

   One can therefore describe the situation in terms of light-like M^4-momentum and $U(2)$ transformation (as it turns out) mapping this momentum to 8-D momentum in given frame and giving the M_0^4 and E_0^4 momenta. The alternative description is in terms M_0^4 massive momentum and the E_0^4 momentum. The space of light-like complex M^4 momenta with fixed M_0^2 part and non-vanishing E^2 part is given by CP_2 as also the space of quaternionic planes. Given quaternionic plane is in turn characterized by massless M^4-momentum.

2. The description of M^4-massive momentum should be based on twistor associated with the light-like M^4 momentum plus something describing the $SU(3)$ transformation leaving the preferred imaginary unit of M_0^2 un-affected. The transformations leaving unaffected the M^4 part of M^8-momentum coded by the $SU(2)$ doublet 2 of color triplet 3 in the color decomposition of complex 8-momentum $1+1+3+\overline{3}$ but acting on E^4 part $1+\overline{3}$ non-trivially correspond to $U(2)$ subgroup. $U(2)$ element thus codes for the E^4 part of the light-like

momentum and $SU(3)$ code for quaternionic 8-momenta, which can be also massive. Massless and complex M^4 momenta are coded by $SU(3)/U(2) = CP_2$ as also the tangent spaces of Minkowskian space-time regions (by $M^8 - H$ duality).

The complexity of particle 8-momenta -and more generally Noether charges - is not in conflict with the hermiticity of quantal Noether charges if total classical and quantal Noether charges are real (and equal by QCC). This would give rise to a kind of confinement condition applying to many-particle states. I have earlier proposed that single particle conformal weights are complex but that conformal confinement holds in the sense that the total conformal weights are real.

3. General complex quaternionic momenta with fixed M^4 part are parameterized by $SU(3)$. Complex light-like 8-momenta satisfy two additional constraints from light-likenes condition, and one expects the reduction of $SU(3)$ to $SU(3)/U(1) \times U(1)$ - the twistor space of CP_2. Therefore the light-like 8-momentum is coded by a twistor assignable to massless M^4-momentum by an point of $SU(3)/U(1) \times U(1)$ giving $T(M^4) \times T(CP_2)$.

By the previous arguments, the inclusion of helicities and electroweak charges gives twistor lift of $M^8 - H$ correspondence.

1. In the case of E^4 the helicities would correspond to two $SO(4)$ spins to be mapped to right and left-handed electroweak spins or weak spin and weak charges. Twistor space $T(CP_2)$ gives hopes about a unified description of color - and electro-weak quantum numbers in terms of partial waves in the space $SU(3)/U(1) \times U(1)$ for selections of quantization axes for color quantum numbers.

2. A possible problem relates to the particles massive in M^4 sense having more helicity states than massless particles. How can one describe the presence of additional helicities. Should one introduce the analog of Higgs mechanism providing the missing massless helicities? Quantum view about twistors describes helicity as a quantum number - conformal weight - of a wave function in the twistor sphere S^2. In the case of massive gauge bosons which would require the introduction of zero helicity as a spin 0 wave function in twistor space.

3. One should relate the description in terms of M^8 momenta to the description in terms of $M^4 \times CP_2$ color partial waves massless in 8-D sense. The number of partial waves for given CP_2 mass squared is finite and this should be the case for quaternionic E^4 momenta. How color quantum numbers determining the M^4 mass relate to complex E^4 momenta parameterized by $U(2)$ plus two constraints coming from complex light-likeness. The number of degrees of freedom is 2 for given $U(2)$ orbit and the quantization suggests dramatic reduction in the number of 8-momenta. This strongly suggests that it is only possible to talk about wave functions in the space of allowed E^4 momenta - that is in the twistor space $T(CP_2)$. Fixing the M^4-part of 8-momentum parameterized by a point of CP_2 leaves only a wave function in the fiber S^2.

The discussion leaves some questions to ponder.

1. $M^8 - H$ correspondence raises the question whether the octonionic M^8 or $M^4 \times CP_2$ represents the fundamental level. Or are they just alternative descriptions made possible by the quaternionicity of space-time surface in M^8 and quaternionic momentum space necessitating quaternionicity of the tangent space of X^4?

2. What about more general $SO(1,7)$ transformations? Are they needed? One could consider the possibility that $SO(1,7)$ acts in the moduli space of octonion structures of M^8. If so, then these additional moduli must be included. Otherwise given 8-D momenta have M_0^2 part fixed and orbit of given M^4 momentum is the smaller, the smaller the E^2 part of M^4 momentum is. It reduces to point if M^4 momentum reduces to M_0^2.

15.3.3 A new view about color, color confinement, and twistors

To my humble opinion twistor approach to the scattering amplitudes is plagued by some mathematical problems. Whether this is only my personal problem is not clear.

1. As Witten shows in [B33], the twistor transform is problematic in signature (1,3) for Minkowski space since the the bi-spinor μ playing the role of momentum is complex. Instead of defining the twistor transform as ordinary Fourier integral, one must define it as a residue integral. In signature (2,2) for space-time the problem disappears since the spinors μ can be taken to be real.

2. The twistor Grassmannian approach works also nicely for (2,2) signature, and one ends up with the notion of positive Grassmannians. Could it be that something is wrong with the ordinary view about twistorialization rather than only my understanding of it?

3. For M^4 the twistor space should be non-compact $SU(2,2)/SU(2,1) \times U(1)$ rather than $CP_3 = SU(4)/SU(3) \times U(1)$, which is taken to be. I do not know whether this is only about short-hand notation or a signal about a deeper problem.

4. Twistorilizations does not force SUSY but strongly suggests it. The super-space formalism allows to treat all helicities at the same time and this is very elegant. This however forces Majorana spinors in M^4 and breaks fermion number conservation in $D = 4$. LHC does not support $\mathcal{N} = 1$ SUSY. Could the interpretation of SUSY be somehow wrong? TGD seems to allow broken SUSY but with separate conservation of baryon and lepton numbers.

In number theoretic vision something rather unexpected emerges and I will propose that this unexpected might allow to solve the above problems and even more, to understand color and even color confinement number theoretically. First of all, a new view about color degrees of freedom emerges at the level of M^8.

1. One can always find a decomposition $M^8 = M_0^2 \times E^6$ so that the possibly complex light-like quaternionic 8-momentum restricts to M_0^2. The preferred octonionic imaginary unit represent the direction of imaginary part of quaternionic 8-momentum. The action of G_2 to this momentum is trivial. Number theoretic color disappears with this choice. For instance, this could take place for hadron but not for partons which have transversal momenta.

2. One can consider also the situation in which one has localized the 8-momenta only to $M^4 = M_0^2 \times E^2$. The distribution for the choices of $E^2 \subset M_0^2 \times E^2 = M^4$ is a wave function in CP_2. Octonionic $SU(3)$ partial waves in the space CP_2 for the choices for $M_0^2 \times E^2$ would correspond ot color partial waves in H. The same interpretation is also behind $M^8 - H$ correspondence.

3. The transversal quaternionic light-like momenta in $E^2 \subset M_0^2 \times E^2$ give rise to a wave function in transversal momenta. Intriguingly, the partons in the quark model of hadrons have only precisely defined longitudinal momenta and only the size scale of transversal momenta can be specified. This would of course be a profound and completely unexpected connection! The introduction of twistor sphere of $T(CP_2)$ allows to describe electroweak charges and brings in CP_2 helicity identifiable as em charge giving to the mass squared a contribution proportional to Q_{em}^2 so that one could understand electromagnetic mass splitting geometrically.

   The physically motivated assumption is that string world sheets at which the data determining the modes of induced spinor fields carry vanishing W fields and also vanishing generalized Kähler form $J(M^4) + J(CP_2)$. Em charge is the only remaining electroweak degree of freedom. The identification as the helicity assignable to $T(CP_2)$ twistor sphere is natural.

4. In general case the M^2 component of momentum would be massive and mass would be equal to the mass assignable to the E^6 degrees of freedom. One can however always find $M_0^2 \times E^6$ decomposition in which M^2 momentum is light-like. The naive expectation is that the twistorialization in terms of M^2 works only if M^2 momentum is light-like, possibly in complex sense. This however allows only forward scattering: this is true for complex M^2 momenta and even in M^4 case.

The twistorial 4-fermion scattering amplitude is however *holomorphic* in the helicity spinors λ_i and has no dependence on $\tilde{\lambda}_i$. Therefore carries no information about M^2 mass! Could M^2 momenta be allowed to be massive? If so, twistorialization might make sense for massive fermions!

M_0^2 momentum deserves a separate discussion.

1. A sharp localization of 8-momentum to M_0^2 means vanishing E^2 momentum so that the action of $U(2)$ would becomes trivial: electroweak degree of freedom would simply disappear, which is not the same thing as having vanishing em charge (wave function in $T(CP_2)$ twistorial sphere S^2 would be constant). Neither M_0^2 localization nor localization to single M^4 (localization in CP_2) looks plausible physically - consider only the size scale of CP_2. For the generic CP_2 spinors this is impossible but covariantly constant right-handed neutrino spinor mode has no electro-weak quantum numbers: this would most naturally mean constant wave function in CP_2 twistorial sphere.

   For the preferred extremals of twistor lift of TGD either M^4 or CP_2 twistor sphere can effectively collapse to a point. This would mean disappearence of the degrees of freedom associated with M^4 helicity or electroweak quantum numbers.

2. The localization to $M^4 \supset M_0^2$ is possible for the tangent space of quaternionic space-time surface in M^8. This could correlate with the fact that neither leptonic nor quark-like induced spinors carry color as a spin like quantum number. Color would emerge only at the level of H and M^8 as color partial waves in WCW and would require de-localization in the CP_2 cm coordinate for partonic 2-surface. Note that also the integrable local decompositions $M^4 = M^2(x) \times E^2(x)$ suggested by the general solution ansätze for field equations are possible.

3. Could it be possible to perform a measurement localization the state precisely in fixed M_0^2 always so that the complex momentum is light-like but color degrees of freedom disappear? This does not mean that the state corresponds to color singlet wave function! Can one say that the measurement eliminating color degrees of freedom corresponds to color confinement. Note that the subsystems of the system need not be color singlets since their momenta need not be complex massless momenta in M_0^2. Classically this makes sense in many-sheeted space-time. Colored states would be always partons in color singlet state.

4. At the level of H also leptons carry color partial waves neutralized by Kac-Moody generators, and I have proposed that the pion like bound states of color octet excitations of leptons explain so called lepto-hadrons [K78]. Only right-handed covariantly constant neutrino is an exception as the only color singlet fermionic state carrying vanishing 4-momentum and living in all possible M_0^2:s, and might have a special role as a generator of supersymmetry acting on states in all quaternionic subs-spaces M^4.

5. Actually, already p-adic mass calculations performed for more than two decades ago [K39, K12, K47], forced to seriously consider the possibility that particle momenta correspond to their projections o $M_0^2 \subset M^4$. This choice does not break Poincare invariance if one introduces moduli space for the choices of $M_0^2 \subset M^4$ and the selection of M_0^2 could define quantization axis of energy and spin. If the tips of CD are fixed, they define a preferred time direction assignable to preferred octonionic real unit and the moduli space is just S^2. The analog of twistor space at space-time level could be understood as $T(M^4) = M^4 \times S^2$ and this one must assume since otherwise the induction of metric does not make sense.

What happens to the twistorialization at the level of M^8 if one accepts that only M_0^2 momentum is sharply defined?

1. What happens to the conformal group $SO(4,2)$ and its covering $SU(2,2)$ when M^4 is replaced with $M_0^2 \subset M^8$? Translations and special conformational transformation span both 2 dimensions, boosts and scalings define 1-D groups $SO(1,1)$ and R respectively. Clearly, the group is 6-D group $SO(2,2)$ as one might have guessed. Is this the conformal group acting at the level of M^8 so that conformal symmetry would be broken? One can of course

ask whether the 2-D conformal symmetry extends to conformal symmetries characterized by hyper-complex Virasoro algebra.

2. Sigma matrices are by 2-dimensionality real (σ_0 and σ_3 - essentially representations of real and imaginary octonionic units) so that spinors can be chosen to be real. Reality is also crucial in signature $(2,2)$, where standard twistor approach works nicely and leads to 3-D real twistor space.

   Now the twistor space is replaced with the real variant of $SU(2,2)/SU(2,1)\times U(1)$ equal to $SO(2,2)/SO(2,1)$, which is 3-D projective space RP^3 - the real variant of twistor space CP_3, which leads to the notion of positive Grassmannian: whether the complex Grassmannian really allows the analog of positivity is not clear to me. For complex momenta predicted by TGD one can consider the complexification of this space to CP_3 rather than $SU(2,2)/SU(2,1)\times U(1)$. For some reason the possible problems associated with the signature of $SU(2,2)/SU(2,1)\times U(1)$ are not discussed in literature and people talk always about CP_3. Is there a real problem or is this indeed something totally trivial?

3. SUSY is strongly suggested by the twistorial approach. The problem is that this requires Majorana spinors leading to a loss of fermion number conservation. If one has $D = 2$ only effectively, the situation changes. Since spinors in M^2 can be chosen to be real, one can have SUSY in this sense without loss of fermion number conservation! As proposed earlier, covariantly constant right-handed neutrino modes could generate the SUSY but it could be also possible to have SUSY generated by all fermionic helicity states. This SUSY would be however broken.

   There is an delicacy involved. If $J(M^4)$ is present, the action of the gauge commutator $[D_k, D_l] = J_{kl}(M^4)$ on right-handed neutrino is non-vanishing and gives rise to the constant term $J^{kl}(M^4)\Sigma_{kl}$ appearing in the square of Dirac equation at imbedding space level. Neutrino would become massive at imbedding space level and also other states receive an additional contribution to mass squared. String world sheets can be however analogs of Lagrangian sub-manifolds so that $J(M^4)$ projected to them vanishes, and one can have massless right-handed neutrino. Also the right- or left M^4-handedness of operator $J^{kl}(M^4)\Sigma_{kl}$ makes it possible to annihilate the spinor mode at string world sheet. The physical interpretation of this picture is still unclear.

4. The selection of M_0^2 could correspond at space-time level to a localization of spinor modes to string world sheets. Could the condition that the modes of induced spinors at string world sheets are expressible using real spinor basis imply the localization? Whether this localization takes place at fundamental level or only for effective action being due to SH, is a question to be settled. The latter options looks more plausible.

To sum up, these observation suggest a profound re-evaluqtion of the beliefs related to color degrees of freedom, to color confinement, and to what twistors really are.

15.3.4 How do the two twistor spaces assignable to M^4 relate to each other?

Twistor Grassmann approach [B33, B27, B26, B38, B39, B20] uses as twistor space the space $T_1(M^4) = SU(2,2)/SU(2,1)\times U(1)$. Twistor lift of classical TGD uses $M^4\times S^2$: this seems to be necessary since $T_1(M^4)$ does not allow M^4 as space-space. The formulation of the twistor amplitudes in terms of SH using the data assignable to the 2-D surfaces - string world sheets and partonic 2-surfaces perhaps - identifed as surfaces in $T(M^4)\times T(CP_2)$ is an attractive idea suggesting a very close correspondence with twistor string theory of Witten and construction of scattering amplitudes in twistor Grassmann approach.

One should be able to relate these two twistor spaces and map the twistor spaces $T(X^4)$ identified as surfaces in $T(H) = T(M^4)\times T(CP_2)$ to those in $T_1(H) = T_1(M^4)\times T(CP_2)$. This map is strongly suggested also by twistor string theory. This map raises hopes about the analogs of twistor Grassmann amplitudes based on introduction of $T(CP_2)$.

At least the projections of 2-surfaces to $T(M^4)$ should be mappable to those in $T_1(M^4)$. A stronger condition is that $T(M^4)$ is mappable to $T_1(M^4)$. Incidence relations for twistors $Z = (\lambda, \mu)$ assigning to given M^4 coordinates twistor sphere, are given by

$$\mu_{\dot{\alpha}} = m_{\alpha\dot{\alpha}}\lambda^{\alpha} \quad .$$

This condition determines a 2-D sub-space - complex light ray - of complexified Minkowski space M^4_c. Also complex scaling of Z determines the same sub-space. Therefore twistor sphere corresponds to a complex light ray M^4_c, whose points differ by a shift by a complex light-like vector (λ is null bi-spinor annihilated by light-like m).

Since twistor line (projective sphere) determines a point of M^4_c, two points of twistor sphere labelled by A and B are needed to determined m:

$$m_{\alpha\dot{\alpha}} = \frac{\lambda_{A,\alpha}\mu_{B,\dot{\alpha}}}{\langle\lambda_A\lambda_B\rangle\rangle} + \frac{\lambda_{B,\alpha}\mu_{A,\dot{\alpha}}}{\langle\lambda_B\lambda_A\rangle} \quad .$$

The solutions are invariant under complex scalings $(\lambda, \mu) \to k(\lambda, \mu)$. Therefore co-incidence relations allow to assign projective line - sphere S^2 - to a point of M^4 in $T(M^4)$. This sphere naturally corresponds to S^2 in $T(M^4) = M^4 \times S^2$. This allows to assign pairs $(m \times S^2)$ in $T(M^4)$ to spheres of $T_1(M^4)$ and one can map the projections of 2-surfaces to $T(M^4)$ to $T_1(M^4)$.

Thus one cannot assign M^4 point to single twistor but can map any pair of points at twistor sphere of $T_1(M^4)$ to the same point of M^4 in $T(M^4) = M^4 \times S^2$ and also identify the twistor sphere with S^2. Twistor spheres are labelled by the base space of $T_1(M^4)$ and therefore base space can be mapped to M^4.

Two M^4 points separated by light-like distance correspond to twistor spheres intersecting at one point as is clear from the fact that the difference $m_1 - m_2$ of the points annihilates the twistor λ. $T_1(M^4)$ is singular as fiber bundle over M^4 since the same point of fiber is projected to two different points of M^4.

Could one replace $T(M^4)$ with $T_1(M^4)$ by modifying the induction procedure suitable?

1. $T_1(M^4) = SU(2,2)/SU(2,1) \times U(1)$ has $SU(2,2)$ invariant metric and $SU(2,2)$ corresponds to the 15-D spin covering group of $SO(4,2)$ having $SO(3,1)$ as sub-group. What does one obtain if one induces the metric of the base space of $T_1(M^4)$ to M^4 via the above identification?

   The induced metric would depend on the choice of the base space, and one would have analog of gauge invariance since for a given point of the base the point of the fiber sphere can be chosen freely. A reasonable guess is that the induced metric is determined apart from conformal scaling. One could fix the gauge by - say - assuming that the S^2 point is constant but it is not clear whether this allows to get the flat M^4 metric with any choice.

2. If the twistor sphere of $T_1(M^4)$ has radius of order Planck length l_P, the overall scaling factor of the metric of $T_1(M^4)$ is of order l_P^2. Also the induced M^4 metric would have this scaling factor. For $T_1(M^4)$ one could not perform this scaling. This need not be a problem in $T(M^4)$ since one scale up the flat metric of M^4 by scaling the coordinates. This kind of scaling would in fact smooth out the possible deviations from flat M^4 metric very effectively. In any case, it seems that one must assume that imbedding space corresponds to $T(M^4)$.

15.3.5 Can the Kähler form of M^4 appear in Kähler action?

I have already earlier considered the question whether the analog of Kähler form assignable to M^4 could appear in Kähler action. Could one replace the induced Kähler form $J(CP_2)$ with the sum $J = J(M^4) + J(CP_2)$ such that the latter term would give rise to a new component of Kähler form both in space-time interior at the boundaries of string world sheets regarded as point-like particles? This could be done both in the Kähler action for the interior of X^4 and also in the topological magnetic flux term $\int J$ associated with string world sheet and reducing to a boundary term giving couplings to U(1) gauge potentials $A_\mu(CP_2)$ and $A_\mu(M^4)$ associated with $J(CP_2)$ and $J(M^4)$. The interpretation of this coupling is an interesting challenge.

Conditions on $J(M^4)$

What conditions one can pose on $J(M^4)$?

1. The simplest possibility is that $J(M^4)$ is covariantly constant and self-dual and satisfies $J^2(M^4) = -g(M^4)$ meaning that $J(M^4)$ *resp.* $g(M^4)$ represents imaginary *resp.* real unit. Hypercomplexity for M^2 would suggest the restriction $J^2(M^2) = g(M^2)$ and $J^2(E^2) = -g(E^2)$. Since complexified octonions are used, it is convenient to include imaginary unit to $J(M^2)$ so that one indeed obtains $J^2(M^4) = -g(M^4)$. $J(M^4)$ would define a global decomposition $M^4 = M^2 \times E^2$ in terms of parallel constant electric and magnetic fields of equal magnitude. CD with this variant of $J(M^4)$ would be naturally associated with planewave like radiative solutions.

2. One could however give up the covariant constancy. In this case spherically symmetric variants of $J(M^4)$ naturally associated with spherically symmetric stationary metric and possible analogs of Robertson-Walker metrics. $J(M^4)$ would be closed except at the world line connecting the tips of CD and carry identical magnetic and electric charges.

3. $J(M^4)$ would define Hamilton Jacobi-structure and an attractive idea is that the orthogonal 2-surfaces associated with the foliation of M^4 are orbits of a subgroup of Poincare group. This structure would characterize quantum measurement at the level of WCW and quantum measurement would involve selection of a sector of WCW characterized by $J(M^4)$ [K41].

   The most plausible assumption is that $J(M^4)$ is covariantly constant.

Objections against $J(M^4)$

Consider now the objections against introducing $J(M^4)$ to the Kähler action at imbedding space level.

1. $J(M^4)$ would would break translational and Lorentz symmetries at the level of imbedding space since $J(M^4)$ cannot be Lorentz invariant. For imbedding space spinor modes this term would bring in coupling to the self-dual Kähler form in M^4. The simplest choice is $A = (A_t = z, A_z = 0, A_x = y, A_y = 0)$ defining decomposition $M^4 = M^2 \times E^2$. For Dirac equation in M^4 one would have free motion in preferred time-like (t,z)-plane plane M^2 in whereas in x- and y-directions (E^2 plane) would one have harmonic oscillator potentials due to the gauge potentials of electric and magnetic fields. One would have something very similar to quark model of hadron: quark momenta would have conserved longitudinal part and non-conserved transversal part. The solution spectrum has scaling invariance $\Psi(m^k) \to \Psi(\lambda m^k)$ so that there is no preferred scale and the transversal scales scale as $1/E$ and $1/k_x$.

2. Since $J(M^4)$ is not Lorentz invariant, Lorentz boosts would produce new $M^2 \times E^2$ decomposition (or its local variant). If one assumes above kind of linear gauge as gauge invariance suggests, the choices with fixed second tip of causal diamond (CD) define finite-dimensional moduli space $SO(3,1)/SO(1,1) \times SO(2)$ having in number theoretic vision an interpretation as a choice of preferred hypercomplex plane and its orthogonal complement. This is the moduli space for hypercomplex structures in M^4 with the choices of origins parameterized by M^4. The introduction of the moduli space would allow to preserve Poincare invariance.

3. If one generalizes the condition for Kähler metric to $J^2(M^4) = -g(M^4)$ fixing the scaling of J, the coupling to $A(M^4)$ is also large and suggests problems with the large breaking of Poincare symmetry for the spinor modes of the imbedding space for given moduli. The transversal localization by the self-dual magnetic and electric fields for $J(M^4)$ would produce wave packets in transversal degrees of freedom: is this physical?

   This moduli space is actually the moduli space introduced for causal diamonds (CDs) in zero energy ontology (ZEO) forced by the finite value of volume action: fixing of the line connecting the tips of CD the Lorentz boost fixing the position for the second tip of CD parametrizes this moduli space apart from division with the group of transformations leaving the planes M^2 and E^2 having interpretation a plane defined by light-like momentum and polarization plane associated with a given CD invariant.

4. Why this kind of symmetry breaking for Poincare invariance? A possible explanation proposed already earlier is that quantum measurement involves a selection of quantization axis. This choice necessarily breaks the symmetries and $J(M^4)$ would be an imbedding space correlate for the selection of rest frame and quantization axis of spin. This conforms with the fact that CD is interpreted as the perceptive field of conscious entity at imbedding space level: the contents of consciousness would be determined by the superposition of space-time surfaces inside CD. The choice of $J(M^4)$ for CD would select preferred rest system (quantization axis for energy as a line connecting tips of CD) via electric part of $J(M^4)$ and quantization axis of spin (via magnetic part of $J(M^4)$. The moduli space for CDs would be the space for choices of these particular quantization axis and in each state function reduction would mean a localization in this moduli space. Clearly, this reduction would be higher level reduction and correspond to a decision of experimenter.

To summarize, for $J(M^4) = 0$ Poincare symmetries are realized at the level of imbedding space but obviously broken slightly by the geometry of CD. The allowance of $J(M^4) \neq 0$ implies that both translational and rotational symmetries are reduced for a given CD: the interpretation would be in terms of a choice of quantization axis in state function reduction. They are however lifted to the level of moduli space of CDs and exact in this more abstract sense. This is nothing new: already the introduction of ZEO and CDs force by volume term in action forced by twistor lift of TGD implies the same. Also the view about state function reduction requires wave functions in the moduli space of CDs. This is also essential for understanding how the arrow of geometric time is inherited from that of subjective time in TGD inspired theory of consciousness [K4, K118].

Situation at space-time level

What about the situation at space-time level?

1. The introduction of $J(M^4)$ part to Kähler action has nice number theoretic aspects. In particular, J selects the preferred complex and quaternionic sub-space of octonionic space of imbedding space. The simplest possibility is that the Kähler action is defined by the Kähler form $J(M^4) + J(CP_2)$.

   Since M^4 and CP_2 Kähler geometries decouple it should be possible to take the counterpart of Kähler coupling strength in M^4 to be much larger than in CP_2 degrees of freedom so that M^4 Kähler action is a small perturbation and slowly varying as a functional of preferred extremal. This option is however not in accordance with the idea that entire Kähler form is induced.

2. Whether the proposed ansätze for general solutions make still sense is not clear. In particular, can one still assume that preferred extremals are minimal surfaces? Number theoretical vision strongly suggests - one could even say demands - the effective decoupling of Kähler action and volume term. This would imply the universality of quantum critical dynamics. The solutions would not depend at all on the coupling parameters except through the dependence on boundary conditions. The coupling between the dynamics of Kähler action and volume term would come also from the conservation conditions at light-like 3-surfaces at which the signature of the induced metric changes.

3. At space-time level the field equations get more complex if the M^4 projection has dimension $D(M^4) > 2$ and also for $D(M^4) = 2$ if it carries non-vanishing induced $J(M^4)$. One would obtain cosmic strings of form $X^2 \times Y^2$ as minimal surface extremals of ordinary Kähler action or X^2 Lagrangian manifold of M^4 as also CP_2 type vacuum extremals and their deformations with M^4 projection Lagrangian manifold. Thus the differences would not be seen for elementary particle and string like objects. Simplest string worlds sheet for which $J(M^4)$ vanishes would correspond to a piece of plane M^2.

   M^4 is the simplest minimal surface extremal of Kähler action necessarily involving also $J(M^4)$. The action in this case vanishes identically by self-duality (in Euclidian signature self-duality does not imply this). For perturbations of M^4 such as spherically symmetric stationary metric the contribution of M^4 Kähler term to the action is expected to be small and the come mainly from cross term mostly and be proportional to the deviation from flat metric. The interpretation in terms of gravitational contribution from M^4 degrees of freedom could make sense.

4. What about massless extremals (MEs)? How the induced metric affects the situation and what properties second fundamental form has? Is it possible to obtain a situation in which the energy momentum tensor T^α and second fundamental form $H^k_{\alpha\beta}$ have in common components which are proportional to light-like vector so that the contraction $T^{\alpha\beta}H^k_{\alpha\beta}$ vanishes?

   Minimal surface property would help to satisfy the conditions. By conformal invariance one would expect that the total Kähler action vanishes and that one has $J^\alpha_{\ \gamma}J^{\gamma\beta} \propto ag^{\alpha\beta} + bk^\alpha k^\beta$. These conditions together with light-likeness of Kähler current guarantee that field equations are satisfied.

   In fact, one ends up to consider a generalization of MEs by starting from a generalization of holomorphy. Complex CP_2 coordinates ξ^i would be functions of light-like M^2 coordinate $u_+ = k \cdot m$, k light-like vector, and of complex coordinate w for E^2 orthogonal to M^2. Therefore the CP_2 projection would 3-D rather than 2-D now.

   The second fundamental form has only components of form $H^k_{u_+w}$, $H^k_{u_+\overline{w}}$ and H^k_{ww}, $H^k_{\overline{w}\overline{w}}$. The CP_2 contribution to the induced metric has only components of form Δg_{u_+w}, $\Delta g_{+\overline{w}}$, and $g_{\overline{w}w}$. There is also contribution $g_{u_+u_-} = 1$, where v is the light-like dual of u in plane M^2. Contravariant metric can be expanded as a power series for in the deviation (Δg_{u_+w}, $\Delta g_{u_+\overline{w}}$) of the metric from $(g_{u_+u_-}, g_{w\overline{w}})$. Only components of form g^{u_+,u_i} and $g^{w,\overline{w}}$ are obtained and their contractions with the second fundamental form vanish identically since there are no common index pairs with simultaneously non-vanishing components. Hence it seems that MEs generalize!

   I have asked earlier whether this construction might generalize for ordinary MEs. One can introduce what I have called Hamilton-Jacobi structure for M^4 consisting of locally orthogonal slicings by integrable 2-surfaces having tangent space having local decomposition $M^2_x \times E^2_x$ with light-like direction depending on point x. An objection is that the direction of light-like momentum depends on position: this need not be inconsistent with momentum conservation but would imply that the total four-momentum is not light-like anymore. Topological condensation for MEs and at MEs could imply this kind modification.

5. There is also a topological magnetic flux type term for string world sheet. Topological term can be transformed to a boundary term coupling classical particles at the boundary of string world sheet to CP_2 Kähler gauge potential (added to the equation for a light-like geodesic line). Now also the coupling to M^4 gauge potential would be obtained. The condition $J(M^4) + J(CP_2) = 0$ at string world sheets [L22] is very attractive manner to identify string world sheets as analogs of Lagrangian manifolds but does not imply the vanishing of the net $U(1)$ couplings at boundary since the induce gauge potentials are in general different.

   Also topological term including also M^4 Kähler magnetic flux for string world sheet contributes also to the modified Dirac equation since the gamma matrices are modified gamma matrices required by super-conformal symmetries and defined as contractions of canonical momentum densities with imbedding space gamma matrices [K88]. This is true both in space-time interior, at string world sheets and at their boundaries. CP_2 (M^4) term gives a contribution proportional to CP_2 (M^4) gamma matrices.

   At imbedding space level transversal localization would be the outcome and a good guess is that the same happens also now. This is indeed the case for M^4 defining the simplest extremal. The general interpretation of M^4 Kähler form could be as a quantum tool for transversal dynamical localization of wave packets in Kähler magnetic and electric fields of M^4. Analog for decoherence occurring in transversal degrees of freedom would be in question. Hadron physics could be one application.

Testing the existence of $J(M^4)$

How to test the idea about $J(M^4)$?

1. It might be possible to kill the assumption that $J(M^4)$ is covariantly constant by showing that one does not obtain spherically symmetric Schwartschild type metric as a minimal surface extremal of generalized Kähler action: these extremals are possible for ordinary Kähler action [L19] [L20]. For the canonical imbedding of M^4 field equations are satisfied since energy momentum tensor vanishes identically. For the small deformations the presence of $J(M^4)$ would reduce rotational symmetry to cylindrical symmetry.

   The question is basically about how large the moduli space of forms $J(M^4)$ can be allowed to be. The mere self duality and closedness condition outside the line connecting the tips of CD allows also variants which are spherically symmetric in either Minkowski coorinates or Robertson-Walker coordinates for light-cone.An attractive proposal is that the pairs of orthogonal 2-surface correspond to Hamilton-Jacobi structures for which the two surfaces are orbits of subgroups of Poincare group.

2. $J(M^4)$ could make its presence manifest in the physics of right-handed neutrino having no direct couplings to electroweak gauge fields. Mixing with left handed neutrino is however induced by mixing of M^4 and CP_2 gamma matrices. The transversal localization of right-handed neutrino in a background, which is a small deformation of M^4 could serve as an experimental signature.

3. CP breaking in hadronic systems is one of the poorly understood aspects of fundamental physics and relates closely to the mysterious matter-antimatter asymmetry. The constant electric part of self dual $J(M^4)$ implies CP breaking. I have earlier consider that Kähler electric fields could cause this breaking but now the electric field is not constant. Second possibility is that matter and antimatter correspond to different values of h_{eff} and are dark relative to each other. The question is whether $J(M^4)$ could explain the observed CP breaking as appearing already at the level of imbedding space $M^4 \times CP_2$ and whether this breaking could explain hadronic CP breaking and matter anti-matter asymmetry. Could M^4 part of Kähler electric field induce different $h_{eff}/h = n$ for particles and antiparticles.

Kerr effect, breaking of T symmetry, and Kähler form of M^4

I encountered in Facebook a link to a very interesting article [D1] (see `http://tinyurl.com/h5lmplw`). Here is the abstract of the article.

We prove an instance of the Reciprocity Theorem that demonstrates that Kerr rotation, also known as the magneto-optical Kerr effect, may only arise in materials that break microscopic time reversal symmetry. This argument applies in the linear response regime, and only fails for nonlinear effects. Recent measurements with a modified Sagnac Interferometer have found finite Kerr rotation in a variety of superconductors. The Sagnac Interferometer is a probe for nonreciprocity, so it must be that time reversal symmetry is broken in these materials.

Magneto-optic Kerr effect (see `http://tinyurl.com/hef8xgv`) occurs when a circularly polarized light beam (plane wave) (often with normal incidence) reflects from a sample. For instance, reflected circular polarized beams suffers a phase change in the reflection: as if they would spend some time at the surface before reflecting. Linearly polarized light reflects as elliptically polarized light. In magneto-optic Kerr effect there are many options depending on the relative directions of the reflection plane (incidence is not normal in the general case so that one can talk about reflection plane) and magnetization.

Kerr angle θ_K is defined as 1/2 of the difference of these phase angle increments caused by reflection for oppositely circularly polarized plane wave beams. As the name tells, magneto-optic Kerr effect is often associated with magnetic materials. Kerr effect has been however observed also for high Tc superconductors and this has raised controversy. As a layman in these issues I can safely wonder whether the controversy is created by the expectation that there are no magnetic fields inside the super-conductor. Anti-ferromagnetism is however important for high Tc superconductivity. In TGD based model for high Tc superconductors the supracurrents would flow along pairs of flux tubes with the members of $S = 0$ ($S = 1$) Cooper pairs at parallel flux tubes carrying magnetic fields with opposite (parallel) magnetic fluxes. Therefore magneto-optic Kerr effect could be in question after all.

The author claims to have proven that Kerr effect in general requires breaking of microscopic time reversal symmetry. Time reversal symmetry breaking (TRSB) caused by the presence of magnetic field and in the case of unconventional superconductors is explained nicely at http://tinyurl.com/jbabcjt. Magnetic field is required. Magnetic field is generated by a rotating current and by right-hand rule time reversal changes the direction of the current and also of magnetic field. For spin 1 Cooper pairs the analog of magnetization is generated, and this leads to T breaking.

This result is very interesting from the point of TGD. The reason is that twistorial lift of TGD requires that imbedding space $M^4 \times CP_2$ has Kähler structure in generalized sense [L24, L38]. M^4 has the analog of Kähler form, call it $J(M^4)$. $J(M^4)$ is assumed to be self-dual and covariantly constant as also CP_2 Kähler form, and contributes to the Abelian electroweak U(1) gauge field (electroweak hypercharge) and therefore also to electromagnetic field. By definition it satisfies $J^2(M^4) = -g(M^4)$ saying that it represents imaginary unit geometrically.

$J(M^4)$ implies breaking of Lorentz invariance since it defines decomposition $M^4 = M^2 \times E^2$ implying preferred rest frame and preferred spatial direction identifiable as direction of spin quantization axis. In zero energy ontology (ZEO) one has moduli space of causal diamonds (CDs) and therefore also moduli space of Kähler forms and the breaking of Lorentz invariance cancels. Note that a similar Kähler form is conjectured in quantum group inspired non-commutative quantum field theories and the problem is the breaking of Lorentz invariance.

What is interesting that the action of P,CP, and T on Kähler form transforms it from self-dual to anti-self-dual form and vice versa. If $J(M^4)$ is self-dual as also $J(CP_2)$, all these 3 discrete symmetries are broken in arbitrarily long length scales. On basis of tensor property of $J(M^4)$ one expects P: $(J(M^2), J(E^2) \to (J(M^2), -J(E^2)$ and T: $(J(M^2), J(E^2) \to (-J(M^2), J(E^2)$. Under C one has $(J(M^2), J(E^2) \to (-J(M^2), -J(E^2)$. This gives CPT: $(J(M^2), J(E^2) \to (J(M^2), J(E^2)$ as expected.

One can imagine several consequences at the level of fundamental physics.

1. One implication is a first principle explanation for the mysterious CP violation and matter antimatter asymmetry not predicted by standard model (see below).

2. A new kind of parity breaking is predicted. This breaking is separate from electroweak parity breaking and perhaps closely related to the chiral selection in living matter.

3. The breaking of T might in turn relate to Kerr effect if the argument of authors is correct. It could occur in high Tc superconductors in macroscopic scales. Also large $h_{eff}/h = n$ scaling up quantum scales in high Tc superconductors could be involved as with the breaking of chiral symmetry in living matter. Strontium ruthenate for which Cooper pairs are in $S = 1$ state is is indeed found to exhibit TRSB (for references and explanation see http://tinyurl.com/jbabcjt).

   In TGD based model of high Tc superconductivity [K57, K58] the members of the Cooper pair are at parallel magnetic flux tubes with the same spin direction of magnetic field. The magnetic fields and thus the direction of spin component in this direction changes under T causing TRSB. The breaking of T for $S = 1$ Cooper pairs is not spontaneous but would occur at the level of physics laws: the time reversed system finds itself experiences in the original self-dual $J(M^4)$) rather than in $(-J(M^2), J(E^2))$ demanded by T symmetry.

15.3.6 What causes CP violation?

CP violation and matter antimatter asymmetry involving it represent white regions in the map provided by recent day physics. Standard model does not predict CP violation necessarily accompanied by the violation of time reflection symmetry T by CPT symmetry assumed to be exact. The violation of T must be distinguished from the emergence of time arrow implies by the randomness associated with state function reduction.

CP violation was originally observed for mesons via the mixing of neutral kaon and antikaon having quark content $n\overline{s}$ and $\overline{n}s$. The lifetimes of kaon and antikaon are different and they transform to each other. CP violation has been also observed for neutral mesons of type $n\overline{b}$. Now it has been observed also for baryons Λ_b with quark composition u-d-b and its antiparticle (see http://tinyurl.com/zyk8w44). Standard model gives the Feynman graphs describing the mixing in standard model in terms of CKM matrix (see http://tinyurl.com/hvpz2su).

The CKM mixing matrix associated with weak interactions codes for the CP violation. More precisely, the small imaginary part for the determinant of CKM matrix defines the invariant coding for the CP violation. The standard model description of CP violation involves box diagrams in which the coupling to heavy quarks takes place. b quark gives rise to anomalously large CP violation effect also for mesons and this is not quite understood. Possible new heavy fermions in the loops could explain the anomaly.

Quite generally, the origin of CP violation has remained a mystery as also CKM mixing. In TGD framework CKM mixing has topological explanation in terms of genus of partonic 2-surface assignable to quark (sphere, torus or sphere with two handles). Topological mixings of U and D type quarks are different and the difference is not same for quarks and antiquarks. But this explains only CKM mixing, not CP violation.

Classical electric field - not necessary electromagnetic - prevailing inside hadrons could cause CP violation. So called instantons are basic prediction of gauge field theories and could cause strong CP violation since self-dual gauge field is involved with electric and magnetic fields having same strength and direction. That this strong CP violation is not observed is a problem of QCD. There are however proposals that instantons in vacuum could explain the CP violation of hadron physics (see `http://tinyurl.com/zptbd4j`).

What says TGD? I have considered this in [L40] and earlier blog posting (see `http://tinyurl.com/hvzqjua`).

1. M^4 and CP_2 are unique in allowing twistor space with Kähler structure (in generalized sense for M^4). If the twistor space $T(M^4) = M^4 \times S^2$ having bundle projections to both M^4 and to the conventional twistor space CP_3, or rather its non-compact version) allows Kähler structure then also M^4 allow the generalized Kähler structure and the analog symplectic structure.

   This boils down to the existence of self-dual and covariantly constant U(1) gauge field $J(M^4)$ for which electric and magnetic fields E and B are equal and constant and have the same direction. This field is not dynamical like gauge fields but would characterize the geometry of M^4. $J(M^4)$ implies violation Lorentz invariance. TGD however leads to a moduli space for causal diamonds (CDs) effectively labelled by different choices of direction for these self-dual Maxwell fields. The common direction of E and B could correspond to that for spin quantization axis. $J(M^4)$ has nothing to do with instanton field. It should be noticed that also the quantum group inspired attempts to build quantum field theories for which space-time geometry is non-commutative introduce the analog of Kähler form in M^4, and are indeed plagued by the breaking of Lorentz invariance. Here there is no moduli space saving the situation.

2. The choice of quantization axis would therefore have a correlate at the level of "world of classical worlds" (WCW). Different choices would correspond to different sectors of WCW. The moduli space for the choices of preferred point of CP_2 and color quantization axis corresponds to the twistor space $T(CP_2) = SU(3)/U(1) \times U(1)$ of WCW. One could interpret also the twistor space $T(M^4) = M^4 \times S^2$ as the space with given point representing the position of the tip of CD and the direction of the quantization axis of angular momentum. This choice requires a characterization of a unique rest system and the directions of quantization axis and time axes defines plane M^2 playing a key role in TGD approach to twstorialization [L24, L38].

3. The prediction would be CP violation for a given choice of $J(M^4)$. Usually this violation would be averaged out in the average over the moduli space for the choices of M^2 but in some situation this would not happen. Why the CP violation does not average out when there is CKM mixing of quarks? Why the parity violation due to the preferred direction is not compensated by C violation meaning that the directions of E and B fields would be exactly opposite for quarks and antiquarks. Could the fact that quarks are not free but inside hadron induce CP violation? Could a more abstract formulation say that the wave function in the moduli space for $J(M^4)$ (wave function for the choices of spin quantization axis!) is not CP symmetric and this is reflected in the CKM matrix.

4. An important delicacy is that $J(M^4)$ can be both self-dual and anti-self-dual depending on whether the magnetic and electric field have same or opposite directions. It will be found

that reflection P and CP transform self-dual $J(M^4)$ to anti-self-dual one. If only self-dual $J(M^4)$ is allowed, one has both parity breaking and CP violations.

Can one understand the emergence of CP violation in TGD framework?

1. Zero energy state is pair of two positive and negative energy parts. Let us assume that positive energy part is fixed - one can call corresponding boundary of CD passive. This state corresponds to the outcome of state function reduction fixing the direction of quantization axes and producing eigenstates of measured observables, for instance spin. Single system at passive boundary is by definition unentangled with the other systems. It can consists of entangled subsystems hadrons are basic example of systems having entanglement in spin degrees of freedom of quarks: only the total spin of hadron is precisely defined.

   The states at the active boundary of CD evolve by repeated unitary steps by the action of the analog of S-matrix and are not anymore eigenstates of single particle observables but entangled. There is a sequence of trivial state function reductions at passive boundary inducing sequence of unitary time evolutions to the state at the active boundary of CD and shifting it. This gives rise to self as a generalized Zeno effect.

   Classically the time evolution of hadron corresponds to a superposition of space-time surfaces inside CD. The passive ends of the space-time surface or rather, the quantum superposition of them - is fixed. At the active end one has a superposition of 3-surfaces defining classical correlates for quantum states at the active end: this superposition changes in each unitary step during repeated measurements not affecting the passive end. Also time flows, which means that the distance between the tips of CD defining clock-time increases as the active boundary of CD shifts farther away.

2. The classical field equations for space-time surface follow from an action, which at space-time level is sum of Kähler action and volume term. If Kähler form at space-time surface is induced (projected to space-time surface) from $J = J(M^4) + J(CP_2)$, the classical time evolution is CP violating. CKM mixing is induced by different topological mixings for U and D type quarks (recall that 3 particle generations correspond to different genera for partonic 2-surfaces: sphere, torus, and sphere with two handles). $J(M^4) + J(CP_2)$ defines the electroweak $U(1)$ component of electric field so that $J(M^4)$ contributes to $U(1)$ part of em field and is thus physically observable.

3. Topological mixing of quarks corresponds to a superposition of time evolutions for the partonic 2-surfaces, which can also change the genus of partonic 2-surface defined as the number of handles attached to 2-sphere. For instance, sphere can transform to torus or torus to a sphere with two handles. This induces mixing of quantum states. For instance, one can say that a spherical partonic 2-surface containing quark would develop to quantum superposition of sphere, torus, and sphere with two handles. The sequence of state function reductions leaving the passive boundary of CD unaffected (generalized Zeno effect) by shifting the active boundary from its position after the first state function reduction to the passive boundary could but need not give rise to a further evolution of CKM matrix.

4. The determinant of CKM matrix is equal to phase factor by unitarity ($UU^\dagger = 1$) and its imaginary part characterizes CP breaking. The imaginary part of the determinant should be proportional to the Jarlskog invariant $J = \pm Im(V_{us}V_{cb}\overline{V}_{ub}\overline{V}_{cs})$ characterizing CP breaking of CKM matrix (see `http://tinyurl.com/kakxwl8`).

If the topological mixings are different for U and D type quarks, one obtains CKM mixing. How could the classical time evolution for quarks and for antiquarks as their CP transforms differ? To answer the question one must look how $J(M^4)$ transforms under C, P, T and CP.

1. $J(M^4) = (J_{0z}, J_{xy} = \epsilon J_{0z})$, $\epsilon = \pm 1$, characterizes hadronic space-time sheet (all space-time sheets in fact). Since $J(M^4)$ is tensor, P changes only the sign of J_{0z} giving $J(M^4) \to (-J_{0z}, J_{xy})$. Since C changes the signs of charges and therefore the signs of fields created by them, one expects $J(M^4) \to -J(M4)$ under C. CP would give $J(M^4) \to (J_{0z}, -J_{xy})$ transforming selfdual $J(M^4)$ to anti-selfdual $J(M^4)$. If WCW has no anti-self-dual sector, CP is violated at the level of WCW.

2. If CPT leaves $J(M^4)$ invariant, one must have $J(M^4) \rightarrow (J_{0z}, -J_{xy})$ under T rather than $J(M^4) \rightarrow (-J_{0z}, J_{xy})$. The anti-unitary character of T could correspond for additional change of sign under T. Otherwise CPT should act as $J(M^4) \rightarrow -J(M^4)$ and only $(CPT)^2$ would correspond to unity.

3. Same considerations apply to $J(CP_2)$ but the difference would be that induced $J(M^4)$ for space-time surfaces, which are small deformations of M^4 covariantly constant in good approximation. Also for string world sheets corresponding to small cosmological constant $J(M^4) \times J(M^4) - 2 \simeq 0$ holds true in good approximation and induced $J(M^4)$ at string world sheet is in good approximation covariantly constant. If the string world sheet is just M^2 characterizing $J(M^4)$ the condition is exact and was has Kähler electric field induced by $J(M^4)$ but no corresponding magnetic field. This would make the CP breaking effect large.

If CP is not violated, particles and their CP transforms correspond to different sectors of WCW with self dual and anti-self dual $J(M^4)$. If only self-dual sector of WCW is present then CP is violated. Also P is violated at the level of WCW and this parity breaking is different from that associated with weak interactions and could relate to the geometric parity breaking manifesting itself via chiral selection in living matter. Classical time evolutions induce different CKM mixings for quarks and antiquarks reflecting itself in the small imaginary part of the determinant of CKM matrix. CP breaking at the level of WCW could explain also matter-antimatter asymmetry. For instance, antimatter could be dark with different value of $h_{eff}/h = n$.

What is interesting that P is badly broken in long length scales as also CP. The same could be true for T. Could this relate to the thermodynamical arrow of time? In ZEO state function reductions to the opposite boundary change the direction of clock time. Most physicist believe that the arrow of thermodynamical time and thus also clock time is always the same. There is evidence that in living matter both arrows are possible. For instance, Fantappie has introduced the notion of syntropy as time reversed entropy [J3]. This suggests that thermodynamical arrow of time could correspond to the dominance of the second arrow of time and be due to self-duality of $J(M^4)$ leading to breaking of T. For instance, the clock time spend in time reversed phase could be considerably shorter than in the dominant phase. A quantitative estimate for the ratio of these times might be given some power of the ratio $X = l_P/R$.

15.3.7 Quantitative picture about CP breaking in TGD

One must specify the value of α_1 and the scaling factor transforming $J(CD)$ having dimension length squared as tensor square root of metric to dimensionless $U(1)$ gauge field $F = J(CD)/S$. This leads to a series of questions.

How to fix the scaling parameter S?

1. The scaling parameter relating $J(CD)$ and F is fixed by flux quantization implying that the flux of $J(CD)$ is the area of sphere S^2 for the twistor space $M^4 \times S^2$. The gauge field is obtained as $F = J/S$, where $S = 4\pi R^2(S^2)$ is the area of S^2.

2. Note that in Minkowski coordinates the length dimension is by convention shifted from the metric to linear Minkowski coordinates so that the magnetic field B_1 has dimension of inverse length squared and corresponds to $J(CD)/SL^2$, where L is naturally be taken to the size scale of CD defining the unit length in Minkowski coordinates. The $U(1)$ magnetic flux would the signed area using L^2 as a unit.

How $R(S^2)$ relates to Planck length l_P? l_P is either the radius $l_P = R$ of the twistor sphere S^2 of the twistor space $T = M^4 \times S^2$ or the circumference $l_P = 2\pi R(S^2)$ of the geodesic of S^2. Circumference is a more natural identification since it can be measured in Riemann geometry whereas the operational definition of the radius requires imbedding to Euclidian 3-space.

How can one fix the value of $U(1)$ coupling strength α_1? As a guideline one can use CP breaking in K and B meson systems and the parameter characterizing matter-antimatter symmetry.

1. The recent experimental estimate for so called Jarlskog parameter characterizing the CP breaking in kaon system is $J \simeq 3.0 \times 10^{-5}$. For B mesons CP breading is about 50 times larger than for kaons and it is clear that Jarlskog invariant does not distinguish between different meson so that it is better to talk about orders of magnitude only.

2. Matter-antimatter asymmetry is characterized by the number $r = n_B/n_\gamma \sim 10^{-10}$ telling the ratio of the baryon density after annihilation to the original density. There is about one baryon 10 billion photons of CMB left in the recent Universe.

Consider now the identification of α_1.

1. Since the action is obtained by dimensional reduction from the 6-D Kähler action, one could argue $\alpha_1 = \alpha_K$. This proposal leads to unphysical predictions in atomic physics since neutron-electron $U(1)$ interaction scales up binding energies dramatically.

   $U(1)$ part of action can be however regarded a small perturbation characterized by the parameter $\epsilon = R^2(S^2)/R^2(CP_2)$, the ratio of the areas of twistor spheres of $T(M^4)$ and $T(CP_2)$. One can however argue that since the relative magnitude of $U(1)$ term and ordinary Kähler action is given by ϵ, one has $\alpha_1 = \epsilon \times \alpha_K$ so that the coupling constant evolution for α_1 and α_K would be identical.

2. ϵ indeed serves in the role of coupling constant strength at classical level. α_K disappears from classical field equations at the space-time level and appears only in the conditions for the super-symplectic algebra but ϵ appears in field equations since the Kähler forms of J resp. CP_2 Kähler form is proportional to $R^2(S^2)$ resp. $R^2(CP_2)$ times the corresponding $U(1)$ gauge field. $R(S^2)$ appears in the definition of 2-bein for $R^2(S^2)$ and therefore in the modified gamma matrices and modified Dirac equation. Therefore $\sqrt{\epsilon} = R(S^2)/R(CP_2)$ appears in modified Dirac equation as required by CP breaking manifesting itself in CKM matrix.

   NTU for the field equations in the regions, where the volume term and Kähler action couple to each other demands that ϵ and $\sqrt{\epsilon}$ are rational numbers, hopefully as simple as possible. Otherwise there is no hope about extremals with parameters of the polynomials appearing in the solution in an arbitrary extension of rationals and NTU is lost. Transcendental values of ϵ are definitely excluded. The most stringent condition $\epsilon = 1$ is also unphysical. $\epsilon = 2^{2r}$ is favoured number theoretically.

Concerning the estimate for ϵ it is best to use the constraints coming from p-adic mass calculations.

1. p-Adic mass calculations [K39] predict electron mass as

$$m_e = \frac{\hbar}{R(CP_2)\sqrt{5+Y}} \quad .$$

   Expressing m_e in terms of Planck mass m_P and assuming $Y = 0$ ($Y \in (0,1)$) gives an estimate for $l_P/R(CP_2)$ as

$$\frac{l_P}{R(CP_2)} \simeq 2.0 \times 10^{-4} \quad .$$

2. From $l_P = 2\pi R(S^2)$ one obtains estimate for ϵ, α_1, $g_1 = \sqrt{4\pi\alpha_1}$ assuming $\alpha_K \simeq \alpha \simeq 1/137$ in electron length scale.

$$\begin{aligned} \epsilon &= 2^{-30} \simeq 1.0 \times 10^{-9} \quad , \\ \alpha_1 &= \epsilon\alpha_K \simeq 6.8 \times 10^{-12} \quad , \\ g_1 &= \sqrt{4\pi\alpha_1} \simeq 9.24 \times 10^{-6} \quad . \end{aligned}$$

There are two options corresponding to $l_P = R(S^2)$ and $l_P = 2\pi R(S^2)$. Only the length of the geodesic of S^2 has meaning in the Riemann geometry of S^2 whereas the radius of S^2 has operational meaning only if S^2 is imbedded to E^3. Hence $l_P = 2\pi R(S^2)$ is more plausible option.

For $\epsilon = 2^{-30}$ the value of $l_P^2/R^2(CP_2)$ is $l_P^2/R^2(CP_2) = (2\pi)^2 \times R^2(S^2)/R^2(CP_2) \simeq 3.7 \times 10^{-8}$. $l_P/R(S^2)$ would be a transcendental number but since it would not be a fundamental constant but appear only at the QFT-GRT limit of TGD, this would not be a problem.

One can make order of magnitude estimates for the Jarlskog parameter J and the fraction $r = n(B)/n(\gamma)$. Here it is not however clear whether one should use ϵ or α_1 as the basis of the estimate

1. The estimate based on ϵ gives

$$J \sim \sqrt{\epsilon} \simeq 3.2 \times 10^{-5} \quad , \quad r \sim \epsilon \simeq 1.0 \times 10^{-9} \quad .$$

The estimate for J happens to be very near to the recent experimental value $J \simeq 3.0 \times 10^{-5}$. The estimate for r is by order of magnitude smaller than the empirical value.

2. The estimate based on α_1 gives

$$J \sim g_1 \simeq 0.92 \times 10^{-5} \quad , \quad r \sim \alpha_1 \simeq .68 \times 10^{-11} \quad .$$

The estimate for J is excellent but the estimate for r by more than order of magnitude smaller than the empirical value. One explanation is that α_K has discrete coupling constant evolution and increases in short scales and could have been considerably larger in the scale characterizing the situation in which matter-antimatter asymmetry was generated.

There is an intriguing numerical co-incidence involved. $h_{eff} = \hbar_{gr} = GMm/v_0$ in solar system corresponds to $v_0 \simeq 2^{-11}$ and appears as coupling constant parameter in the perturbative theory obtained in this manner [K66]. What is intriguing that one has $\alpha_1 = v_0^2/4\pi^2$ in this case. Where does the troublesome factor $(1/2\pi)^2$ come from? Could the p-adic coupling constant evolutions for v_0 and α_1 correspond to each other and could they actually be one and the same thing? Can one treat gravitational force perturbatively either in terms of gravitational field or $J(CD)$? Is there somekind of duality involved?

Atomic nuclei have baryon number equal the sum $B = Z+N$ of proton and neutron numbers and neutral atoms have $B = N$. Only hydrogen atom would be also $U(1)$ neutral. The dramatic prediction of $U(1)$ force is that neutrinos might not be so weakly interacting particles as has been thought. If the quanta of $U(1)$ force are not massive, a new long range force is in question. $U(1)$ quanta could become massive via $U(1)$ super-conductivity causing Meissner effect. As found, $U(1)$ part of action can be however regarded a small perturbation characterized by the parameter $\epsilon = R^2(S^2)/R^2(CP_2)$. One can however argue that since the relative magnitude of $U(1)$ term and ordinary Kähler action is given by ϵ, one has $\alpha_1 = \epsilon \times \alpha_K$.

Quantal $U(1)$ force must be also consistent with atomic physics. The value of the parameter α_1 consistent with the size of CP breaking of K mesons and with matter antimatter asymmetry is $\alpha_1 = \epsilon\alpha_K = 2^{-30}\alpha_K$.

1. Electrons and baryons would have attractive interaction, which effectively transforms the em charge Z of atom $Z_{eff} = rZ$, $r = 1 + (N/Z)\epsilon_1$, $\epsilon_1 = \alpha_1/\alpha = \epsilon \times \alpha_K/\alpha \simeq \epsilon$ for $\alpha_K \simeq \alpha$ predicted to hold true in electron length scale. The parameter

$$s = (1 + (N/Z)\epsilon)^2 - 1 = 2(N/Z)\epsilon + (N/Z)^2\epsilon^2$$

would characterize the isotope dependent relative shift of the binding energy scale.

The comparison of the binding energies of hydrogen isotopes could provide a stringent bounds of the value of α_1. For $l_P = 2\pi R(S^2)$ option one would have $\alpha_1 = 2^{-30}\alpha_K \simeq .68 \times 10^{-11}$ and $s \simeq 1.4 \times 10^{-10}$. s is by order of magnitude smaller than $\alpha^4 \simeq 2.9 \times 10^{-9}$ corrections from QED (see `http://tinyurl.com/kk9u4rh`). The predicted differences between the binding energy scales of isotopes of hydrogen might allow to test the proposal.

2. $B = N$ would be neutralized by the neutrinos of the cosmic background. Could this occur even at the level of single atom or does one have a plasma like state? The ground state binding energy of neutrino atoms would be $\alpha_1^2 m_\nu/2 \sim 10^{-24}$ eV for $m_\nu = .1$ eV! This is many many orders of magnitude below the thermal energy of cosmic neutrino background estimated to be about 1.95×10^{-4} eV (see `http://tinyurl.com/ldu95o9`). The Bohr radius would be $\hbar/(\alpha_1 m_\nu) \sim 10^6$ meters and same order of magnitude as Earth radius. Matter should be $U(1)$ plasma. $U(1)$ superconductor would be second option.

15.4 About the interpretation of the duality assignable to Yangian symmetry

The $D = 4$ conformal generators acting on twistors have a dual representation in which they act on momentum twistors: one has dual conformal symmetry, which becomes manifest in this representation. These two separate symmetries extend to Yangian symmetry providing a powerful constraint on the scattering amplitudes.

In TGD the conformal Yangian extends to super-symplectic Yangian - actually, all symmetry algebras have a Yangian generalization with multi-locality generalized to multi-locality with respect to partonic 2-surfaces. The generalization of the dual conformal symmetry has remained obscure. In the following I describe what the generalization of the two conformal symmetries and Yangian symmetry would mean in TGD framework. I also propose an information theoretic duality between Euclidian and Minkowskian regions of space-time surface. I am not algebraist and apologize for the unavoidable inaccuracies.

15.4.1 Formal definition associated with Yangian

The notion of Yangian appears as two very different looking variants. The first variant can be found from Wikipedia (see `goo.gl/q1twRZ`) and second variant assignable to gauge theories can be found from [B30, B31].

Consider first the Wikipedia definition. The definition is in terms of quantum group notion in which the elements of matrix representing group element are made non-commuting operators.

1. The generators of Yangian algebra are labelled by an integer $n \geq -1$ with $n = -1$ generator identified as unit matrix. $n \geq 1$ generators generate the algebra and commutators with $n = 1$ generators preserving the weight allow to assign quantum numbers to them. From the Wikipedia article one learns that Yangian is generated by elements $t_{ij}^{(p)}$, $1 \leq i, j \leq N$, $p \geq 0$ of quantum matrices satisfy the relations

   $$\left[t_{ij}^{(p+1)}, t_{kl}^{(q)}\right] - \left[t_{ij}^{(p)}, t_{kl}^{(q+1)}\right] = -(t_{kj}^{(p)} t_{il}^{(q)} - t_{kj}^{(q)} t_{il}^{(p)}) \ . \tag{15.4.1}$$

   Note there are two operations involved: commutator and operator product. The formula here is not consistent with the formula used in Yang-Mills theories for the commutators between $m = 0$ generators and generators with generators having $n \in \{0, 1\}$, and it seems that this formula suggesting $m, n \to m + n - 1$ in commutator cannot hold true for the commutators with $m = 0$ generators.

   By defining $t_{ij}^{(-1)} = \delta_{ij}$ and setting

   $$T(z) \quad = \quad \sum_{p \geq -1} t_{ij}^{(p)} z^{-p+1} \ . \tag{15.4.2}$$

   $T(z)$ is thus a quantum matrix depending on the point of 2-D space.

2. Introduce R-matrix $R(z) = 1 + z^{-1}P$ acting on $C^N \otimes C^N$, where P is the operator permuting the tensor factors. This allows to write the defining relations as Yang-Baxter equation (see `http://tinyurl.com/gogn75s`):

   $$R_{12}(z - w) T_1(z) T_2(w) = T_2(w) T_1(z) R_{12}(z - w) \ . \tag{15.4.3}$$

   R_{12}, which depends only on the difference $z - w$, performs the permutation of the generators $T_1(z)$ and $T_2(w)$.

Yangian is a Hopf algebra with co-multiplication Δ mapping $T(z)$ acting in V to operator acting in $V \otimes V$, co-unit ϵ and antipode s given by

$$(\Delta \otimes id)T(z) = T_{12}(z)T_{13}(z) \ , \quad (\epsilon \otimes id)T(z) = I \ , \quad (s \otimes id)T(z) = T(z)^{-1} \ . \tag{15.4.4}$$

Δ taking generator $T(z)$ acting in V to generator $\Delta(T) = T_{12}(z)$ acting in $V \otimes V$. Δ transforms a generator acting on single-particle states to a generator acting on 2-particles states.

3. The Yangian weight of the commutator of elements with weights m and n is $m + n - 1$ rather than $m + n$ as for Virasoro and Kac-Moody algebras. This means that generators with conformal weight 1 do not affect the conformal weight and Cartan algebra elements defining quantum numbers of generators have weight 1. For conformal algebras the Cartan algebra defining quantum numbers has conformal weight 0.

   For Virasoro algebra having integer valued conformal weights the scaling $L_0 = zd/dz$ appears as basic derivative operation and generators are products $L_n = z^n zd/dz$. By taking translation operator $T = d/dz$ as the derivative operator and writing $K_n = z^n d/dz$, the weight of commutator becomes $m + n - 1$. This is a trivial change. The map $u = exp(z)$ relates these two representations. That $n \leq 2$ appear in generators distinguishes the representations from Virasoro and Kac-Moody representations - note however that also for these algebras the generators with positive weight generate physical states.

What bothers me in this definition is that only the action of the generators with $p = 1$ leaves the weight unaffected whereas for the dual conformal symmetry generators with both $p = 0$ and $p = 1$ do this and define conformal symmetry and its dual.

15.4.2 Dual conformal symmetry in $\mathcal{N} = 4$ SUSY

Yangian symmetry appears also in gauge theories and the definition looks very different from the Wikipedia definition. In $\mathcal{N} = 4$ SUSY conformal symmetry (in 4-D sense) has two representations. There is a duality between two representations of conformal generators crucial for twistor Grassmannian approach [B30, B31] (see `http://tinyurl.com/n22lwuy`).

1. In the first representation conformal symmetry generators $J_a^{(0)}$ are local and act in the space of external momenta. This induces a local and linear action in twistor space.

2. The generators $J_a^{(1)}$ of the dual conformal symmetry act in a local manner in the space of region momenta and associated momentum twistor space whereas the action of $J_a^{(1)}$ is bi-local in the momentum space and corresponding twistor space.

   Region momenta can be assigned with a twistor diagram defined by a closed polygon of Minkowski space having region momenta (, which need not be light-like) as edges having external light-like momenta emitted at the corners. The dual of this representation is the representation in which the light-like external momenta summing up to zero form a closed polygon.

Yangian is generated by ordinary generators $J_a^{(0)}$ and bi-local dual generators $J_a^{(1)}$.

1. They satisfy the commutations

$$\left[J_a^{(0)}, J_b^{(1)}\right] = f_{ab}{}^c J_c^{(1)} \ . \tag{15.4.5}$$

   This condition is perfectly sensible physically but is not consistent with the above general consistency condition pf Eq. 15.4.1 from R-matrix requiring that the commutator has vanishing weight. Now the weights are additive in commutator.

2. The generators $J_a^{(1)}$ have an easy-to-guess representation:

$$J_a^{(1)} = f_a{}^{cb} \sum_{0\leq i<j\leq n} J_{ib}^{(0)} J_{jc}^{(0)} \tag{15.4.6}$$

making explicit the bi-locality. The commutators of these generators have also weight 1. This is consistent with the above general formula unlike the formula the commutators of generators with vanishing weight. Both generators form a closed sub-algebra of Yangian and this must be behind the possibility to represent $J_a^{(1)}$ locally.

3. Also so called Serre relations are satisfied. They look rather complex and look different from the relations associated with R-matrix.

$$\begin{array}{lll} X(a,b,c) & + & \epsilon(a,b,c)X(b,c,a) + \epsilon(c,a,b)X(c,a,b) = h\epsilon_{rm,tn}Y(l,m,n)f_{ar}^l f_{bs}^m f_{ct}^n f^{rst} \quad , \\ X(a,b,c) & = & \left[J_a^{(1)}, \left[J_b^{(1)}, J_c^{(0)}\right\}\right\} \quad , \quad Y(l,m,n) = \{J_l^{(0)}, J_m^{(0)}, J_n^{(0)}\right] \\ \epsilon(a,b,c) & = & (-1)^{|a|(|b|+|c|)} \quad , \quad \epsilon_{rm,tn} = (-1)^{|r|m|+|t|n|} \quad . \end{array} \tag{15.4.7}$$

Here the mixed brackets the $[.,\}$ denote the graded commutator, and $\{.,..]$ denotes the graded symmetrizer. h is a parameter characterizing the Yangian and should correspond to the parameter characterizing quantum group.

These conditions are sufficient to give a representation of graded Yangian if the tensor product $\mathcal{R}\otimes\overline{\mathcal{R}}$ of the representation $\mathcal{R}$ and its conjugate $\overline{\mathcal{R}}$ contains adjoint representation only once. The higher generators can be generate by applying co-product operation to the generators.

4. Both local and bi-local generators form two closed sub-algebras. This is not consistent with the consistency conditions of appearing in Wikipedia definition. The Wikipedia definition seems to be wrong for commutators of generators $[J_A^{(m)}, J_B^{(n)}]$ with weights $(m,n) \in \{(0,0),(0,1),(1,0)\}$.

5. Co-product Δ has representation

$$\begin{array}{lll} \Delta(J^A) & = & J^A\otimes 1 + 1\otimes J^A \quad , \\ \Delta(Q^A) & = & Q^A\otimes 1 + 1\otimes Q^A + f_{BC}^A J^B\otimes J^C \quad . \end{array} \tag{15.4.8}$$

The first formula is obvious. Single particle generator lifted to a tensor product is sum of the single particle generators acting on the tensor factors. When Q^A annihilates single spin representations, one obtains just the defining formula for the bi-local generators.

One could have a situation in which single particle states are actually many-particle states annihilated by Q^A and satisfying the condition that adjoint is contained only once in $\mathcal{R}\otimes\overline{\mathcal{R}}$. In TGD framework one might argue that this kind of effective single particle states could quite generally define bound states behaving like single particle states physically. One would obtain infinite hierarchy of this kind of states realizing concretely the vision about fractal hierarchy.

15.4.3 Possible TGD based interpretation of Yangian symmetries

In TGD partonic 2-surfaces replace point-like objects and multi-locality is with respect to these. The proposal is that the TGD counterpart of the Yangian algebra [B31] of gauge theories could act as symmetries of many-parton states characterized by n partonic 2-surfaces assignable to the same 3-D surface at the boundary of causal diamond (CD). What is remarkable that this symmetry would relate particle states with different particle numbers to each other unlike the usual single particle symmetries.

1. This condition forces the partons to form a bound state with partonic 2-surfaces having *space-like* separations. Note that the separations along orbits of wormhole throats at opposite ends of CD are space-like or light-like. This must be taken into account when correlation functions are calculated. In QFT there is no description of this kind and this could explain the general failure of QFT in the description of bound states already in QED, where Bethe-Salpeter equation predicts large numbers of non-existing states.

2. Yangian algebra involves complex (hypercomplex) coordinate z which could be associated with the boundaries of string world sheets connecting partonic surfaces at the same boundary (at opposite boundaries) of CD. One can also assign complex coordinate with partonic 2-surfaces and the braiding of fermionic lines would be described by the matrix R assignable to the Yangian. The Cartan algebra of local and bi-local string like operators define quantum numbers for states. That point-like and string-like operators generate the algebra conforms with the idea about tensor networks with nodes connected by edges.

   On can think that partonic 2-surfaces form a single connected unit consisting of partonic surfaces connected by boundaries of string world sheets assignable to the topological Feynman diagram defined by the light-like 3-surface defining the boundary between Euclidian and Minkowskian regions of the space-time surface.

3. The operation Δ for Yangian would assign to the generators acting on single parton states generators acting on 2-parton states. R_{12} would act as an exchange operation for parton states, which could reduces to many-fermion states at partonic 2-surfaces.

4. R_{12} can appear in many contexts in TGD. It can be associated with braiding of fermionic lines inside partonic orbits or magnetic flux tubes at the ends of space-time surfaces. It can be also associated with the fermionic lines in the preferred plane M^2 associated with twistor scattering amplitudes.

   From the twistorial point of view the preferred M^2 defined by light-like quaterionic 8-momentum is of special interest. M^2 identified as octonionic complex plane and its complexification brings in mind integrable field theories in M^2 allowing Yangian symmetry characterized by R-matrix. The scattering matrix is trivial for these field theories: scattering involves only a phase shift. In twistorial approach to TGD scattering is non-trivial. The R-matrix would be present also now and exchange the momentum projections in preferred M^2 plane. If the entire scattering diagram -apart from external lines corresponds to the same M^2, the braiding operation permutes also fermions at different partonic 2-surfaces located at the ends of string.

The possibility to localize the action of generators $J^{(1)}$ in momentum twistor representation leads to ask whether the stringy generators appearing TGD framework could allow local action using the analog of the space of region momenta. Could M^8-H duality [K74, K110] make this possible? At M^8 level the light-like momenta (in 8-D sense) would correspond to differences of region momenta assignable to strings connecting the partonic 2-surfaces. The 8-D region momenta should be quaternionic. They cannot be light-like as is easy to see.

The notion of region momentum and thus localization would make sense only in M^8, where the wave functions are completely localizable to quaternionic light-like momenta in M^8, whereas in H one has localization to light-like momenta only in preferred M^2 plus wave functions in the space of planes M^4 and in the space of transverse momenta in $E^2 \subset M^4$. This would suggest that M^8-H duality corresponds to the duality of twistor and momentum twistor representations.

What would be new that this duality would be realized also at the level of space-time surfaces. One would have associative/quaternionic space-time surfaces in M^8 and preferred extremals of dimensionally reduced Kähler action in H identifiable as 6-D holomorphic surfaces representing twistor spaces of space-time surfaces.

Note that $M^8 - H$ duality could be seen as a number-theoretic analog of spontaneous compactification. Non-perturbative effects would force a delocalization in the space of light-like 8-momenta in M^8 to give states having interpretation as wave functions in H. Nothing would happen to the topology of M^8. Only the state space would be compactified.

15.4.4 A new kind of duality of old duality from a new perspective?

$M^8 - H$ duality [K74, K110] maps the preferred extremals in H to those $M^4 \times CP_2$ and vice versa. The tangent spaces of an associative space-time surface in M^8 would be quaternionic (Minkowski) spaces.

In M^8 one can consider also co-associative space-time surfaces having associative *normal* space [K74]. Could the co-associative normal spaces of associative space-time surfaces in the case of preferred extremals form an integrable distribution therefore defining a space-time surface in M^8 mappable to H by $M^8 - H$ duality? This might be possible but the associative tangent space and the normal space correspond to the same CP_2 point so that associative space-time surface in M^8 and its possibly existing co-associative companion would be mapped to the same surface of H.

This dead idea however inspires an idea about a duality mapping Minkowskian space-time regions to Euclidian ones. This duality would be analogous to inversion with respect to the surface of sphere, which is conformal symmetry. Maybe this inversion could be seen as the TGD counterpart of finite-D conformal inversion at the level of space-time surfaces. There is also an analogy with the method of images used in some 2-D electrostatic problems used to reflect the charge distribution outside conducting surface to its virtual image inside the surface. The 2-D conformal invariance would generalize to its 4-D quaterionic counterpart. Euclidian/Minkowskian regions would be kind of Leibniz monads, mirror images of each other.

1. If strong form of holography (SH) holds true, it would be enough to have this duality at the informational level relating only 2-D surfaces carrying the holographic information. For instance, Minkowskian string world sheets would have duals at the level of space-time surfaces in the sense that their 2-D normal spaces in X^4 form an integrable distribution defining tangent spaces of a 2-D surface. This 2-D surface would have induced metric with Euclidian signature.

   The duality could relate either a) Minkowskian and Euclidian string world sheets or b) Minkowskian/Euclidian string world sheets and partonic 2-surfaces common to Minkowskian and Euclidian space-time regions. a) and b) is apparently the most powerful option information theoretically but is actually implied by b) due to the reflexivity of the equivalence relation. Minkowskian string world sheets are dual with partonic 2-surfaces which in turn are dual with Euclidian string world sheets.

   (a) Option a): The dual of Minkowskian string world sheet would be Euclidian string world sheet in an Euclidian region of space-time surface, most naturally in the Euclidian "wall neighbour" of the Minkowskian region. At parton orbits defining the light-like boundaries between the Minkowskian and Euclidian regions the signature of 4-metric is $(0,-1,-1,-1)$ and the induced 3-metric has signature $(0,-1,-1)$ allowing light-like curves. Minkowskian and Euclidian string world sheets would naturally share these light-like curves aas common parts of boundary.

   (b) Option b): Minkowskian/Euclidian string world sheets would have partonic 2-surfaces as duals. The normal space of the partonic 2-surface at the intersection of string world sheet and partonic 2-surface would be the tangent space of string world sheets so that this duality could make sense locally. The different topologies for string world sheets and partonic 2-surfaces force to challenge this option as global option but it might hold in some finite region near the partonic 2-surface. The weak form of electric-magnetic duality [K105] could closely relate to this duality.

   In the case of elementary particles regarded as pairs of wormhole contacts connected by flux tubes and associated strings this would give a rather concrete space-time view about stringy structure of elementary particle. One would have a pair of relatively long (Compton length) Minkowskian string sheets at parallel space-time sheets completed to a parallelepiped by adding Euclidian string world sheets connecting the two space-time sheets at two extremely short (CP_2 size scale) Euclidian wormhole contacts. These parallelepipeds would define lines of scattering diagrams analogous to the lines of Feynman diagrams.

This duality looks like new but as already noticed is actually just the old electric-magnetic duality [?]een from number-theoretic perspective.

15.5 TGD view about construction of twistor amplitudes

In the following TGD view about twistorialization and its relation to other visions about TGD is discussed. I start with a brief summary of twistor approach to scattering amplitudes and then describe the application of this approach TGD.

15.5.1 Some key ideas of the twistor Grassmann approach

In the following I summarize the basic technical ideas of twistor Grassmann approach. I am not a specialist. On the other hand, my views about twistorialization of TGD differ in many aspects about those applied in the twistorialization of gauge theories, and my own attention is directed towards the physical interpretation and mathematical consistency rather than calculational techniques.

Variants of twistor formalism

The reader can find details about twistors in the article of Witten [B33] and in the thesis of Trnka [B75] (see `http://tinyurl.com/zbj9ad7`).

1. Helicity spinor formalism assigns to light-like momentum pair of conjugate spinors $(\lambda_a, \tilde{\lambda}_{\dot{a}})$ transforming in conjugate representations of Lorentz group $SL(2,C)$. Light-like momentum is expressible as $p^k\sigma_k$ using Pauli sigma matrices and this gives the representation as matrix components $p^{a\dot{a}} = \lambda^a\lambda^{\dot{a}}$. The determinant of the matrix equals to $p^kp_k = 0$ since its rows are linearly dependent.

   One can introduce the bilinears $[\tilde{\lambda}_1, \tilde{\lambda}_2] = -[\tilde{\lambda}_2, \tilde{\lambda}_1]$ and $\langle\lambda_1, \lambda_2\rangle = -\langle\lambda_2, \lambda_1\rangle$ using the antisymmetric Lorentz invariant bilinear defined by permutation symbols ϵ^{ab} and $\epsilon^{\dot{a}\dot{b}}$. The inner product $p_1 \cdot p_2$ is expressible as $p_1 \cdot p_2 = \langle\lambda_1, \lambda_2\rangle[\tilde{\lambda}_1, \tilde{\lambda}_2]$.

   One could express also polarization vectors of massless bosons using pair $(\lambda, \tilde{\mu})$ of helicity spinors. There is however a more elegant approach available. The spinors $(t\lambda, \tilde{\lambda}/t)$ correspond to same momentum for all non-vanishing complex values of t. t represents an element of little group of Lorentz group leaving the helicity state invariant. The helicity dependence of the scattering amplitude is fixed by the transformation property under little group and coded to the weight under the scalings by t: $A(t_a\lambda, t_a^{-1}\tilde{\lambda}_a) = t_a^{-2h_a}A(\lambda, \tilde{\lambda})$. Thus the formalism allows very elegant description of spin and can be applied in SUSYs.

   For Minkowski signature (2,2) the spinors are real and this makes this signature preferred. Personally I see this as a basic problem of twistorialization. A possible TGD inspired solution of the problem is provided by the effective replacement of M^4 with M^2 with signature $(1,1)$ and thus allowing real spinors.

2. Twistors $(\lambda_a, \mu_{\dot{a}})$ are obtained by performing a twistor Fourier transform of scattering amplitude $A(\lambda, \tilde{\lambda})$ with respect to $\tilde{\lambda}$.

   At local level [B33] the twistor transform corresponds to Fourier transform

$$\tilde{\lambda}_{\dot{a}} \to i\frac{\partial}{\mu^{\dot{a}}} \ ,$$
$$-i\frac{\partial}{\tilde{\lambda}^{\dot{a}}} \to \mu_{\dot{a}} \ .$$

The action of little group corresponds now to the scaling $(\lambda,\mu)\rightarrow t(\lambda,\mu)$ and does not affect the helicity state. For this reason twistors differing by complex scaling can be identified. The proper twistor space is CP_3 rather than C^4.

The twistor transform of the amplitude transforms as $A(t_a\lambda, t_a\tilde{\lambda}_a)=t_a^{-2h_a-2}A(\lambda,\mu)$.

In signature (2,2) the helicity spinors $(\lambda,\tilde{\lambda})$ are real so that the twistor Fourier transform reduces to an ordinary Fourier transform. In signature (1,3) the rigorous definition is rather challenging and is discussed by Penrose [B69]. One manner to define the transform is by using residue integral. Residue integral is also p-adically attractive.

The incidence relation of Penrose given by

$$\mu_{\dot{a}} = -x_{a\dot{a}}\lambda^a$$

relates M^4 coordinates to λ,μ. By little group invariance entire complex twistor line corresponds to a given point of M^4.

The twistor transform of plane wave allows to construct the twistor transform of momentum space wave function, and is given by $\delta^2(\mu_{\dot{a}}+x_{a\dot{a}}\lambda^a)$, which is non-vanishing at complex light ray. Twistor Fourier transform in real Minkowski space is therefore non-vanishing at light ray and maps light rays to twistors.

If the incidence relation for given (λ,μ) is satisfied at two space-time points m_1,m_2, the difference m_1-m_2 is a light-like vector since corresponding matrix has vanishing determinant. Two intersecting twistor lines correspond to M^4 points with light-like distance. This allows to develop geometric picture about twistor diagrams in which the external light-like momenta correspond to intersections of twistor lines assignable to the internal lines of graph.

3. Momentum twistors define a third basic notion. It is convenient to describe particle scattering with external light-like momenta in terms of a diagram in which the external momenta are assigned with the vertices of a polygon such that the lines carry possibly complex momenta. Clearly, the polygon like object is obtained by repeatedly adding light-like momenta to the polygon and since the sum of the external momenta vanishes, the polygon closes.

   The vertices of polygon correspond to intersections of twistor lines defining light-like momenta as differences of the momenta associated with the lines meeting at the vertex. One can assign to the complex momenta of internal lines twistors known as momentum twistors.

   Dual momentum twistor is a further variant of twistor concept being defined in terms of three adjacent momentum twistors contracting them with the 4-D permutation symbol defined in the representation of twistor as a point of C^4 [B75].

Leading singularities

Twistor Grassmann approach to planar loop amplitudes relies on the idea that the discontinuities associated with the singularities of the scattering amplitudes carry all information about the amplitudes. This of course holds true already for the tree diagrams having only poles as singularities.

The idea is same as in the case of analytic continuation: 1-D data at poles and cuts allows to construct the functions. This idea generalizes to functions of several variables and leads to a generalization of residue calculus. At space-time level strong form of holography (SH) relies on the same idea: the 3-D data determine 4-D dynamics and in TGD allowing strong form of holography 2-D data is almost enough.

The discontinuities assignable to singularities can have lower-dimensional singularities so that a hierarchical structure is obtained. The leading singularities are those for which maximal number of propagators are on mass shell and the diagram decomposes to a product of diagrams with virtual particle on mass shell. For one loop diagrams the maximal number of propagators is $N=4$ corresponding to the fixing of four components of loop momentum. For L loops it is $N=4L$.

Non-leading singularities have less than the maximal number of propagators on shell and this leaves integral over a subset of loop momenta. If the number of propagator is larger than $4L$, one can have kinematical singularities for some combinations of external momenta.

In the case of scattering amplitudes in twistor Grassmann formulation one encounters a similar situation. In twistor Grassmann approach one defines also the loop integrals in momentum space as residue integrals in the space of complexified momenta. If the functions involved are rational functions the residue integrals are well-defined.

One of the surprising findings is that the leading singularities of MHV loop amplitudes always proportional to tree amplitudes. Second finding is that for $\mathcal{N} = 4$ theory the leading singularities determine completely the scattering amplitudes [B75].

In TGD framework quantum criticality suggests that locally all loop corrections vanish and coupling constant evolution is discrete. This would mean that the only singularities correspond to poles of propagators and this indeed leads to diagrams in which internal lines have complex on mass shell momenta. If this vision is correct, this part of twistor Grassmann approach does not look relevant from TGD point of view.

BCFW recursion formula

The original form of BCFW recursion formula [B26] was derived for tree diagrams. The finding was that the diagrams can be decomposed to two pieces containing with a propagator line connecting them.

1. The proof of this result was rather simple in spinor helicity formalism and based on modification of two momenta p_k and p_n by BCFW shift:

$$\begin{aligned} p_k(z) &= \lambda_k(\tilde{\lambda}_k - z\tilde{\lambda}_n) \ , \\ p_n(z) &= (\lambda_n + z\lambda_k)\tilde{\lambda}_n) \ , \end{aligned} \tag{15.5.1}$$

Obviously, the modification is induced by modifications $\tilde{\lambda}_k$ and λ_n. With some assumptions about asymptotic behaviour of scattering amplitude A, one can express the original amplitude $A = A(z = 0)$ as residue integral

$$A(z=0) = \frac{1}{2\pi}\oint_C dz \frac{A(z)}{z} \ . \tag{15.5.2}$$

Here C does not close any other poles than $z = 0$. This integral is the negative of the residue integral around the complement of the region closed by C.

2. It is assumed that poles are the only singularities in this region. Hence one can express $A(z)$ as sum of its poles

$$A(z) = \sum_i \frac{c_i}{z - z_i} \ . \tag{15.5.3}$$

3. With these assumptions the residue integral gives

$$A = A(0) = \frac{1}{2\pi}\sum_i \frac{c_i}{z_i} \ . \tag{15.5.4}$$

This leads to the desired factorization with c_i reducing to a product of amplitudes and z_i identifiable as a complex pole for the propagator connecting the sub-diagrams in the decomposition.

In [B35] details of the BCFW shift in the general case are given. One assumes a more general shift $p_i \to \hat{p}_i = p_i + zr_i$ such that r_i are light-like, mutually orthogonal, orthogonal to p_i, and sum up to zero. The modified momenta are complex massless and sum up to zero. One can define $P_I = \sum_{i<I} p_i$ and $R_I = \sum_{i<I} r_i$. The shifted variant $\hat{P_I}^2 = P_I^2 + 2zP \cdot R_I$ is linear in z and vanishes for $z = z_I = -P_I^2/P_I \cdot R_I$. Z_I define the counterparts z_i. Performing the residue integral one obtains $A(0) = \frac{1}{2\pi}\sum_I \frac{c_I}{z_I}$.

This formula allows a recursive construction of tree diagrams by starting from the basic vertices of YM theory. BCFW recursion formula was later generalized to a recursion for the sum planar loops diagrams in terms of diagrams with lower number of loops [B35, B75].

Scattering amplitudes in terms of Yangian invariants defined as multiple residue integrals in Grassmannian manifolds

The generators of Yangian are ordinary conformal generators with conformal weight 0 and dual generators with conformal weight 1. The latter generators act in simple manner in momentum twistor space.

Twistor Grassmannian approach utilizing either twistors or momentum twistors allows to demonstrate that these both conformal symmetry and its dual are present.

The construction of Yangian invariants is summarize in [B75]. Grassmannian residues are Yangian invariants. Yangian transformation introduces total divergence and is exact if its integral vanishes. The operations producing new Yangian invariant can change n or k or both.

1. There are several relatively trivial manners to construct Yangian invariants. One can take the integrand of n-1-D invariant and formally interpret it as integrand of n-D invariant. One can integrate over one twistor variable so that n decreases by one unit.

   Invariants can be multiplied. One can a merge invariants by identifying the twistors in the factors of the product. For instance, one can take the fundamental invariants defining 3-vertices and multiply them to build twistor box giving rise to four particles. One can also merge invariants by integrating over the identified invariants.

2. Inverse soft factor [B60] adds to the diagram expressed in terms of spinor helicity formalism one new particle but keeps k constant. Therefore this operation does cannot be applied in TGD where one has only fermions as external particles. The operation can be formulated as a linear shift for $\tilde{\lambda}_a$ and $\tilde{\lambda}_b$.

3. One can prove the BCFW recursion formula for tree diagrams [B26] by using a deformation of the twistor amplitude in helicity spinor formalism allowing to deduce the factorized formula of the amplitude, two adjacent external lines and deform the twistors λ and $\tilde{\lambda}$ in helicity spinor representation by performing the BCFW shift [B68].

   This deformation describes interaction between the external lines, and is essential in the construction of the scattering amplitudes using BCFW recursion. One takes the sum over the products of diagrams with left and right helicities obtained by putting internal particle on mass shell and adds BCFW bridge. BCFW allows to construct all tree amplitudes by starting from fundamental 3-particle amplitudes.

4. Entangled removal [?, B75, B35] removing two external particles producing a loop in the sense of Feynman diagrammatics but residue of the pole of the propagator is possible and appears as part of the boundary operation for the diagrams. The resulting recursion formula allows to deduce loop corrections.

Twistor Grassmann diagrams are known to allow "moves" [B75, B37]. For instance, moves can be used to remove boxes: it is known that apart from scaling factors depending on momenta the diagrams are reducible to ordinary tree diagrams [B75] (`http://tinyurl.com/zbj9ad7`). This allows to consider the possibility that twistor trees could allow to construct all diagrams. Note however that the moves reducing the twistor diagram to a counterpart of tree diagram gives an overall multiplicative factor depending on momenta and helicities.

From TGD point the definition of loop integrals and Grassmannian integrals as residue integrals is of great potential importance. Scattering amplitudes should be number theoretically universal but in p-adic context the definition of definite integral is very difficult. Residue integral provides however a manner to define multiple residue integrals using only holomorphy and the notion of pole. This could be the deep reason for why one should be able to reduce loop integrals to residue integrals.

There is however a potential problem involved related to number theoretic universality. 2π does not exist p-adically in any reasonable sense (if one wants to define it one must introduce infinite-D extension of rationals by powers of 2π. One might hope that 2π cancels from the scattering amplitudes by normalization. Another possibility is that for an extension containing $exp(i2\pi/N)$ as the highest root of unity, one can define π approximately as $i\pi \equiv N\times(exp(i\pi/N)-1)$. An alternative option is that only the analogs of tree diagrams having only poles as singularities are possible

Linearization of the twistorial representation of overall momentum delta function

An little but not insignificant technical detail [B38] is the linearization of the constraint expressing the overall momentum conservation by interpreting it as a condition in Grassmannian $G(k,n)$, where k is the number of negative helicities and n is the number of particles, and allowing to reduce integrations over $G(k,n)$ to those over $G(k-2,n-4)$.

Spinor helicity diagrams and twistor diagrams are proportional to a delta function expressing overall momentum conservation. Dropping twistor indices this delta function one reads as $\delta(\sum_k P_k) = \delta(\lambda_i\tilde{\lambda}_i)$. One can combine the 2 components of λ_i and $\tilde{\lambda}_i$ to form 2+2 n-component vectors and interpret momentum conservation as orthogonality conditions for the 2-planes spanned by λ_a and $\tilde{\lambda}_{\dot{a}}$ for $k>2$. These plane spanned by 2 n-component λ vectors can be interpreted as 2 vectors in $G(k,n-k)$ defining rows of $G(k,n-k)$ matrix. $\tilde{\lambda}$ defines a similar plane in $G(n-k,k)$.

These conditions are equivalent with the condition that there exists in $G(k,n)$ a 2-D C and its $n-k$-dimensional orthogonal complement $\tilde{C}$ such that the 2-plane spanned by λ_a is orthogonal to $\tilde{C}$ and the two-plane spanned by $\tilde{\lambda}_{\dot{a}}$ is orthogonal to C. These conditions can be expressed as a product of delta functions $\delta(C\cdot\tilde{\lambda})$ and $\delta(\tilde{C}\cdot\lambda)$.

Since $G(k)$ acts as a "gauge symmetry" for $G(k,n)$, the first $k\times k$ block of the $k\times n$ matrix representing a point of C can be transformed to a unit matrix so that $k\times(n-k)$ variables remain.. Same can be carried out for the last $n\times(n-k)$ block of $\tilde{C}$ by $G(n)$ "gauge invariance" so that $(n-k)\times n$ variables remain. With these gauge choices the orthogonality conditions can be solved explicitly and corresponding integrations can be carried out. The integration over delta functions leaves $(k-2)(n-k-2)$ variables, the dimension of $G(k-2,n-4)$. $G(k,n)$ reduces to $G(k-2,n-4)$ by momentum conservation.

15.5.2 Basic vision behind scattering amplitudes

It is good to summarize the basic vision about TGD first.

Separation of WCW functional integral and fermionic dynamics

The works of Penrose and Witten have served as inspiration in the attempts to twistorialize TGD and led to the conjecture that the twistor lift of TGD is possible and means that space-time surfaces are replaced with their twistor spaces representable as 6-D surfaces in 12-D product of twistor spaces of M^4 and CP_2. What makes this idea so attractive is that S^4 and CP_2 are the only 4-D compact manifolds with Euclidian signature having twistor space with Kähler structure [A63]. TGD would be unique both from the existence of the lift of Kähler action to the product of twistor spaces of M^4 and CP_2!

What the twistor space of M^4 is, is however not at all clear. It can be defined in two manners: as the usual CP_3 very natural at the level of momentum space or as the trivial bundle $T(M^4) = M^4\times S^2$ natural in the twistorialization at classical space-time level. Standard twistorialization has however problems.

1. There is problem associated with the signature. Twistorialization works best at signature $(2,2)$ for Minkowski space and gives rise to real projective space P^3.

2. Second problem is that CP_3 should be actually $SU(2,2)/SU(2,1) \times U(1)$. There is clearly something not so well understood.

In the number theoretic vision about TGD twistor space would be replaced with commutative hyper-complex $M_2 \subset M^4 \subset M^8$ and this space is just RP^3 and problems wth signature disappear since 2-D spinors can be chosen to have real basis. For complex momenta this extends to CP_3. Number theory would also justify the identification of geometric twistor sphere as $M^4 \times S^2$.

In TGD the dynamics of fields is replaced with that for 4-surfaces. Penrose's idea about generalization of holomorphy of field modes in twistor space generalizes to the holomorphy of the representation of 6-surface representing twistor bundle of space-time leads to a concrete ansatz for space-time surfaces as preferred extremals [L24] [L40].

SH leads to the proposal that the data determining space-surfaces are preferred extremals is given at 2-D surfaces and these 2-D surfaces bring in mind Witten's twistor strings [B33]. By SH the functional integral over them would correspond to that over WCW and twistor amplitudes asignable to given space-time surface would be constructed at fermionic level by the analog of twistor Grassmannian approach. This integral over 2-surfaces corresponds to the deviation of TGD from QFT in fixed background and cannot be equivalent with the introduction of twistor strings.

Adelic physics and scattering diagram as a representation of computation

Adelic physics [L34] suggested to provide quantum physical correlates also for cognition is in a central role. Adelic physics predicts the hierarchy $h_{eff} = n \times h$, where n as dimension of the extension is divisor of the order its Galois group identified in terms of dark matter regarded as a phase of ordinary matter. p-Adic physics and p-adic length scale hypothesis could be also understood.

The number theoretic universality of scattering amplitudes suggests that all loops vanish identically and the evolution of various couplings constants is discrete occurring by phase transitions changing the extension of rationals and values of various coupling parameters.

1. The vanishing of loops at the level of space-time action would mean that the loops associated with the functional integral defined by the action, which is sum of Kähler action and volume term. This vanishing would state essentially local quantum criticality as invariance of coupling parameters under local renormalization group evolution. One would obtain only a sum of action exponentials since Gaussian and metric determinants cancel each other in Kähler metric.

2. Exponents of Kähler action represent a number theoretical nightmare.

   (a) The functional integral expressions for scattering amplitudes are normalized by a functional integral for for the vacuum state. This implies that only the ratios X_i/X of the exponents X_i for the extrema and sum $\sum X_i$ appear in the amplitudes [L34] so that there are slightly better hopes of achieving number theoretic universality.

   (b) Number theoretical universality forces to imagine even more attractive option making sense in ZEO but not in standard ontology. If the amplitude is sum over the contributions normalized by corresponding exponentials X_i rather than $\sum X_i$, exponentials cancel altogether and the couplings constants appear only in boundary conditions. In this case one could speak of a basis of zero energy states assignable to various extrema of the action. The real part of the action is maximum and the the imaginary part of the action saddle point if preferred extrema are minimal surface extremals of Kähler action [L24]. Number-theoretical universality more or less forces this option.

3. An even stronger proposal is based on the idea that that the TGD analogs of stringy diagrams. The lines of these diagrams correspond to light-like parton orbits carrying fermion lines and meeting at vertices which are partonic 2-surfaces. The proposal is that the topological diagrams involving analogs of loops represent algebraic computations so that all diagrams with given initial and final collection of algebraic objects are equivalent.

If this is the case, all topological diagrams should reduce to topological tree diagrams by a generalization of the duality symmetry of the old-fashioned hadronic string model stating that the sum of s-channel resonances equals to the sum of t-channel exchanges and that these diagrams can be constructed as twistor Grassmann diagrams by allowing on mass shell fermions with complex momenta at internal lines. For external particles the momenta could be real and light-like in 8-D sense. A weaker condition is that real and imaginary parts of complex momenta 8-D momenta are separately light-like and orthogonal.

One could indeed argue that one cannot allow loops of this kind since it would be impossible to decide which kind graph experimental scattering situation corresponds if all these graphs are different since one observes only the initial and final states. Therefore all scattering diagrams with same real particles in the final states correspond to identical scattering amplitudes.

These diagrams would correspond to the same amplitude but it might be possible to perform a localization to any of them. p-Adically however the corresponding space-time surface would be different by p-adic non-determinism (the number theoretic discretization - cognitive representation - defined by the common points of reality and p-adicities as space-time surfaces would be different): one might say that the tree representation involves smallest cognitive representation and is therefore the shortest one.

If the action exponentials X_i cancel from the scattering amplitudes, this option can indeed make sense. Otherwise it is extremely implausible since different contributions would have different vacuum weights.

4. If only the twistor analogs from tree diagrams in Feynman sense are allowed, the scattering amplitudes are rational functions of external momenta as strongly suggested by the number theoretic universality and by the requirement that the diagrams can be interpreted in terms of algebraic computations so that the simplest manner to do the computation corresponds to a tree diagram. Even tree diagrams in Feynman sense are planar so that one would get rid of the basic problem of the twistor approach to SUSY.

Quantum classical correspondence (QCC) states that scattering diagrams have classical counterparts in the sense that fermion lines correspond to the boundaries of string worlds sheets assignable to the light-like orbits of partonic 2-surfaces and topological 3.vertices correspond to 2-surfaces at which the ends of light-like orbits meet. This correlation is extremely restrictive and it is not at all clear whether it leaves room for loops.

In the most general case one would have a superposition of allowed space-time surfaces realizing scattering diagram with given initial and final quantum numbers identified as corresponding classical charges.

The idea about diagram as computation suggests that the simplest possible diagram - tree diagram - is realized together with the corresponding space-time topology. If diagrams with topological loops are possible this requires the existence of moves transforming diagrams to each other. This condition might be not consistent with the condition that the move acts on the space-time surface too. Very simple diagrammatics - even twistor tree diagrammatics - could follow from mere QCC.

Classical number fields and M^8-H duality

Quaternionicity and octonionicity is second central aspect of number theoretical vision.

1. The key concept is $M^8-M^4\times CP_2$ duality allowing to see space-time surfaces quaternionic surface in M^8 or as holomorphic surfaces in the twistor space $T(M^4)\times T(CP_2)$. This would realize SH. Physical states are characterized by quaternionic (possibly complexified-) octonion valued 8-momenta in accordance with the vision that tangent space Minkowskian region of space-time surface is quaternionic and contains preferred hyper-complex M^2, which can depend on point provided that tangent spaces $M^2(x)$ integrate to 2-D surface. This view leads to a new view about QCD color as octonionic color.

2. Twistor space reduces to that associated with M^2 and 2-D variant of conformal invariance corresponds to SO(2,2) and leads to the identification real projective space P^3 as twistor

space. One can however complexify it to CP_3 since momenta are in general complex. The signature is (1,1) so that bi-spinors λ, $\tilde{\lambda}$ have real basis and twistor Fourier transform can be defined as ordinary Fourier transform. The reality of M^2 or induced spinors at string world sheets might allow to have SUSY without Majorana spinors.

The reduction of external momenta to M^2 implies that real and imaginary parts are parallel and light-like. At classical level this poses strong conditions on preferred extremals. This does not require that color and electroweak quantum numbers are complex. The reason is that they emerge as labels of wave functions in twistor space $T(CP_2)$ representing wave functions in the moduli space of transversal E^2s with corresponding helicity identifiable as em charge.

Localization of the light-like 8-momentum is possible to preferred M_0^2. Localization does not imply the disappearance of color wave function. The transversal E^2 momentum degrees of freedom however disappear. In the case of leptons and hadrons complete localization could be a good approximation but not in the case of quarks.

Elementary particles have fundamental fermions as building bricks

The assumption that the physics of elementary particles reduces at fundamental level to that of fundamental fermions has strong implications, when combined with the twistor Grassmann approach.

1. In TGD elementary particle would correspond to a pair of wormhole throats of wormhole connecting two space-time sheets with Minkowski signature. Wormhole itself would have Euclidian signature. Wormhole contacts would be connected by monopole flux tube with fermionic quantum numbers at the 4 wormhole throats defining the partonic 2-surfaces.

2. Fundamental vertices are associated with 2-surfaces at which light-like 3-surfaces carrying fermions and antifermions as string world sheet boundaries are glued together along their ends. Note that these surfaces are analogous to vertices of Feynman diagrams and singular as 4-surfaces but 3-surfaces are smooth unlike for stringy vertices.

3. Fermion lines correspond to the boundaries of string world sheets at the light-like orbits of partonic 2-surface at which the signature of the induced metric changes. At momentum space M^8 this picture should also make sense since space-time surfaces in M^8 and H would correspond to each other by M^8-H duality. At the level of M^8 the orbits of fermion lines could be seen as light-like geodesics along with twistor spheres move. At the edges of string world sheets they would intersect at single point and give rise to external massless particle.

4. The basic vertex is 4-fermion vertex in which fermions scatter classically and assignable to the 2-surface at which the ends of light-like 3-surfaces representing partonic orbits intersect. There would be no local 4-fermion vertex. Fermions would move as free particles in the background and the background would gives rise to the interaction between fermions at partonic vertices analogous to vertices of Feynman diagrams. This would automatically resolve possible problems caused by divergences and would be analogous to the vanishing of bosonic loops from WCW functional integration.

5. FFB couplings could be identified in terms of $FF(F\overline{F})$ couplings, where $F\overline{F}$ is associated with the same partonic orbit. These couplings would not be fundamental.

What could SUSY mean in TGD?

Extended super-conformal invariance is basic symmetry of TGD but it is not whether it possible to have SUSY (space-time supersymmetry) in TGD framework. Certainly the SUSY in question is not $\mathcal{N}=1$ SUSY since Majorana spinors are definitely excluded. $\mathcal{N}=2$ SUSY generated by right-handed neutrino and antineutrino can be however considered.

1. If one allows the boundaries of string world sheets carry fermion number bounded only by statistics (all spin-charge states for quarks and leptons would define maximal $\mathcal{N}$ for SUSY). This would allow local vertices for fermions and does not look like an attractive option unless SUSY manages to cancel the divergences.

2. SUSY could mean addition of fermions as separate lines to the orbits of wormhole throat. This SUSY would be broken and only approximately local. The question what the propagator for the many-fermion state at same string line is, is not quite obvious. SUSY would suggest propagator determined by the total spin of the state. I have also considered the possibility that the propagator is just the product of fermionic propagators acting on tensor power of single fermion spaces. The propagator behaves as $1/p^N$ for N fermion state and only for $N = 1, 2$ one would have the usual behavior. This option is not attractive.

3. SUSY could mean addition of right-handed neutrino or its antiparticle to the throat. The short range of weak interactions is explained by assuming that pair of right-handed neutrino and left-handed neutrino compensates the weak isospin at the second wormhole throat carrying quantum numbers of quark or lepton.

   Addition of right-handed neutrino or its antiparticle or both to a given boundary component could give rise to $\mathcal{N} = 2$ SUSY. The breaking of SUSY could correspond to different p-adic length scales for spartners. Mass formula could be exactly the same and provided by p-adic thermodynamics. Why the p-adic mass scale would depend so much on the presence of covariantly constant ν_R having no color and ew interactions nor even gravitational interaction, remains to be understood. If the extensions of rationals are different for the members of SUSY multiplet, the corresponding preferred p-adic primes would be different and this could explain the widely different p-adic mass scales. One can of course ask the covariant constancy means that ν_R does not have any coupling to anything and its presence is undetectable.

15.5.3 Options for the construction of scattering amplitudes

There are several guidelines in the construction of scattering amplitudes.

1. SH in strongest form would mean that string word sheets and partonic 2-surfaces are all that is needed. In number theoretical vision also fixing the extension of rationals associated with the intersection of realities and p-adicities is needed and leads to a hierarchy of extensions which could realized discrete coupling constant evolution. SH would suggest that hybrids for analogs of string diagrams and Feynman diagrams code for the scattering amplitudes.

2. QCC suggests that the eigenvalues of the Cartan algebra generators of symmetries are equal to classical Noether charges. A weaker condition is that the eigenvalues of fermionic generators not affecting space-time surfaces are equal to the classical Noether charges. The generators have also bosonic parts acting in WCW.

   A prediction following from the condition that there is charge transfer between Euclidian and Minkowskian space-time regions is that the classical charges must be complex valued guaranteed if Kähler coupling strength as a spectrum of complex values. One proposal is that the spectrum of zeros of Riemann zeta determines if [L16]. This supports the twistorial view that momenta in the internal lines can be regarded as complex light-like on mass shell momenta.

3. QCC also suggests that scattering diagrams have space-time correlates. The lines of diagrams correspond to light-like orbits of partons at which the signature of induced metric changes. Vertices correspond to partonic 2-surfaces at which these 3-D lines meet. At fermion level fermion lines at partonic orbits correspond to boundaries of string world sheets.

This however leaves several alternative visions concerning the construction of scattering amplitudes.

What scattering diagrams are?

What does one mean with scattering diagrams is not at alle clear.

1. Are they counterparts of Feynman diagrams so that one would have a superposition of all space-time topologies corresponding to these diagrams? Probably not.

2. Or are they counterparts of twistor Grassmannian diagrams in which all particles are on mass shell but with possibly complex light-like quaternionic 8-momenta in $M^8 = M^4 \times E^4$ with $M^4 = M_0^2 \times E^2$. Why this option is interesting is that twistor Grassmann diagrams allow large number of moves reducing their number.

   This would translate to a conserved and massive longitudinal M^2-momentum; which for a special choice of M^2 is light-like, a wave function in the space of transversal E^2 momenta; color partial wave in the moduli space of E^2 planes for given M_0^2; and em charge describable as CP_2 helicity and allowing twistorialization.

   There is however a problem: the transverse E^6-momentum makes M^2 momentum massive and twistorialization fails. But what if the 8-momenta are real and in twistorial description M^2 momentum becomes complex but light-like. The square for the real part of M^2 momentum would be equal to the square of real E^6 momentum and twistor approach would apply! This map would be define the essence of M^2-twistorialization.

In ZEO one can interpret the construction of preferred extremals as a boundary value problem with ends of space-time surfaces at the boundaries of CD and the light-like orbits of partonic 2-surfaces defining a closed 3-surface and defining the scattering diagram as 3-D boundary. If so, it might be possible to construct rather large number of diagrams, even counterpartz of loop diagrams.

The situation would be analogous to the construction of soap films spanned by wires with wire network analogous to the network formed by the partonic orbits. Also an analogy with 4-D tensor network suggests strongly itself and scattering diagrams representing zero energy states would correxpond to the states of the tensor network.

The basic space-time vertex would be 3-vertex defined by partonic 2-surface.The basic fermionic vertex would be 4-fermion vertex in which fermions do not exchange gauge boson but interact classically at the 2-D vertex. All particles emerge as bound states of fundamental fermions at boundaries of string world sheets.

1. The basic view would be that M^2 momenta, and transversal momenta correspond to M^4-momenta. The moduli space for $M_0^2 \times E^2$ planes corresponds to CP_2 and color quantum numbers. M^2 helicities and electroweak quantum numbers would be coded to the weights twistor wave functions in twistor space if $M^2 \times CP_2$.

2. One approach to scattering amplitudes relies on symmetries. Twistor Grassmannian approach suggest strongly Yangian symmetry. The diagrams should be representations of multi-local Yangian algebra with basic algebra being that of the conformal group of M^4 restricted to M^2.

   This would give nicely real projective space RP^3 allowing to solve some problems of the standard twistor approach. In color degrees of freedom one would have color Yangian: hadrons could correspond to the multilocal generators created by multi-local Yangian generators. The E^2 degrees of freedom would correspond to states generated by Kac-Moody algebra and also now one could have Yangian algebra. The states for the representation of Yangian itself would be singlets.

   Besides fermionic lines there are string world sheets. Infinite-D 2-D conformal group and Kac-Moody symmetries act as symmetries for string world sheets. The super-symplectic group would the isometry group of WCW and would give rise to conditions analogous to Super Virasoro conditions. These conditions would be satisfied by preferred extremals realizing number theoretic variant of SH. Also these symmetries would be extended to their Yangian versions naturally.

3. One can argue that classical field equations do not allow all possible diagrams. More precisely, for a given extension of rationals adelic physics allows only finite number diagrams and the extension induces a natural cutoff as minimal distance between points with coordinates in the extension representing intersection of reality and p-adicities [L34].

   The assumption that the end points of fermionic lines at partonic 2-surfaces at ends of CD and at the vertices carry fermions would give an immediate connection with the adelic physics. As the dimension of the extension increases, the number of the points in the intersection increases and more lines appear in the allowed diagrams. This would give rise to a discrete coupling constant evolution, hierarchy of Planck constants, and p-adic length scale hypothesis.

Quantum criticality strongly suggests that coupling constant evolution is locally trivial and is discretized with discrete steps realized as phase transitions changing the extension. Galois group would be the fundamental number theoretic symmetry group acting on the intersection and its order would correspond to $h_{eff}/h = n$ allowing to realize the analogs of perturbative phases of gauge theories as perturbative phases.

4. The discreteness of coupling constant evolution demands that loop corrections vanish. This makes perfect sense for the functional integral over WCW. But what about fermionic degrees of freedom and topological counterparts of scattering diagrams, which very probably do not correspond to Feynman diagrams but could be analogous to twistor diagrams? For fermions there is actually no perturbation theory since effective 4-fermion vertices correspond to classical scattering of external fermions at partonic 2-surfaces defining the vertices. This is not a problem since thanks to h_{eff} guaranteeing the existence of perturbative expansion.

Three roads to follow

In ZEO construction of scattering amplitudes is basically a construction of zero energy states and one must be very cautious in applying QFT intuitions relying on positive energy ontology. One ends up to to a road fork.

Option I: Can one interpret the topological space-time diagrams as analogs of Feynman diagrams and assume that by quantum criticality the sum over the topological loops vanish? This option looks rather ad hoc.

Option II: Can one assume - with inspiration coming from adelic physics - that the number of these loops with fixed states at the boundaries of CD is finite and one just sums over these states with weights given by the exponential of the space-time action?

Here one encounters problems with number theoretical universality [L34]. One has superposition of vacuum exponentials over the diagrams and number theoretical universality demands that the ratio of given exponential to the sum is in the extension of rationals involved. This is very tough order - perhaps too tough.

Option III: Can one follow number theoretical vision suggesting that scattering diagrams correspond to computations in some sense [L22]. This leads to a new road fork.

1. Option IIIa): Could one generalize the old-fashioned string duality and require that there exist a huge symmetry allowing to transform the scattering diagrams using basic moves to tree diagrams? The basic moves would allow to shift the end of line past vertex and to remove self energy loop and hence the transformation to tree diagrams would become possible. Originally it was inspired by the idea that the vertices of the scattering diagram correspond to products and co-products in quantum algebra and that the condition involved can be interpreted as algebraic identities.

   Twistor Grassmannian diagrams indeed allow moves allowing surprising simplification allowing to show that all loop corrections with a given number of loops sum up to something proportional to a tree diagram [B75].

   The assumption that the states moving in the internal lines have light-like quaternionic M^8 momenta gives very strong constraints on the moves and it might well be that the moves are not possible in the general case. Even if the move is possible, the value of the action exponential can change so that this option seems to demand mathematical miracles. The proposed manner to achieve number theoretical universality however eliminates action exponentials.

   The mathematical miracle might be made possible by the possibility to find preferred M_0^2 in which the 2-momentum of fermion line is light-like. If M_0^2 is constant along entire fermion line, it seems to be possible perform the gliding operation past vertices as will be found. Note that ach fermion can wander around the network formed by the partonic orbits.

Note that the different space-time surface realizing equivalent computations would be cognitively non-equivalent since the cognitive representation defined by the points in extension of rationals would be different. Optimum computation would have smallest number of points and would correspond to tree diagram.

2. Option IIIb): Should one sum over the possible diagrams so that one would have quantum superposition of computations. This is done for loop diagrams in twistor Grassmann approach. Infinite sum is however awkward number theoretically. Adelic vision suggests that the number of loops is finite. The action exponentials would not disappear from the scattering amplitudes and are very problematic from the point of view of number theoretical universality.

3. Option IIIc): Could one regard the light-like partonic orbits as part of the dynamical system - this is what effectively is done if they form part of connected 3-surface defining the topological scattering diagram - and assume that each such diagram corresponds to a different physical situation analogous to a computation?

   One can argue that one must be also able to localize the zero energy state to single computation by state function reduction [L39]! State function reduction to single diagram should be possible. A rather classical picture about space-time would emerge: one would have just a superposition of space-time surfaces with the same topology and same action apart from quantum fluctuations around the point which is maximum with stationary phase. One would also have color wave functions and momentum wave functions in cm degrees of freedom of partonic 2-surfaces as WCW degrees of freedom.

   The action exponential, which is very problematic from the point of view of number theoretic vision, would be cancelled from the functional integral since it is normalized by the action exponential. The dependence on coupling parameters is however visible in the boundary conditions at boundaries of CD stating the vanishing of most supersymplectic charges and identifying the remaining super-symplectic charges and also isometry charge with the fermionic counterparts.

   This picture would be extremely simple and would be analogous to that of integrable quantum field theories in which the integral over small fluctuations gives Gaussian determinant and action exponential (now Gaussian determinant is cancelled by the metric determinant coming the Kähler metric of WCW) [K110].

   One can argue that the absence of loops makes it impossible to have non-perturbative effects. This is not true in adelic physics. Recall that the original motivation for $h_{eff} = n \times h$ was that this phase is generated with perturbation theory ceases to converge [K106]. The large value of h_{eff} scales down the coupling strengths proportional to $1/h_{eff}$ and perturbation theory works again.

It must be admitted that one must accept all these options. Number theoretical universality of scattering amplitudes would select $IIIa$) and the need to realize given topological diagram using complex enough extension of rationals supports Option IIIc). I believe that the large number of the options reflects my limited mathematical understanding of the situation a careful analysis of the general implications of the options allows to pinpoint the most feasible one.

15.5.4 About problems related to the construction of twistor amplitudes

The dream is to construct twistorially fermionic scattering amplitudes and this requires the identification of fermionic 4-vertex. There are however several conceptual problems to be solved.

Could M^2 momenta be massive?

The naive objection against massive particles is that one loses the twistorial description both in M^4 sense and M^2 sense. Real quaternionic M^8 momenta are massless but the transversal momentum in E^6 degrees of freedom makes M^2 momenta and M^4 momenta for arbitrary choice of M^4 are massive, and one cannot describe the M^2 and M^4 momenta using the helicity spinor pair $(\lambda, \tilde{lambda})$. The beautiful formalism seems to be lost.

1. The naive argument is however wrong in TGD framework where particles are massless in M^8 sense. This means that mass does not correspond to $\overline{\Psi}\Psi$ in Dirac action but to comes from E^4 momentum (CP_2 "momentum"). 8-D chiral symmetry is unbroken as required by separate conservation of lepton and baryon numbers. In preferred M_0^2 one can indeed make M^2-momentum light-like.

2. Furthermore, 4-fermion twistor amplitudes are *holomorphic* functions of λ_i . There is no dependence of $\tilde{\lambda}$ and therefore no information about light-likeness! Why this amplitude could not describe the scattering of fermions only apparently massive in TGD Universe? Note that the momentum conserving delta function depends on the masses of the particles so that mass-dependence would be purely kinematical and analogous to the dependence on transverse momentum squared. Note that this argument makes sense also for M^4 twistorialization. If this view is correct then twistors are something more profound than momenta.

3. For M^2 twistorialization end would end up to effective (2,2) signature favored by twistorialization. (1,1) signature of real M^2 becomes (2,2) signature for complexified M^2 and real twistor space RP^3 is replaced with CP_3. This looks attractive description. If this picture is correct, all the nice results such as the possibility to assume reduction of amplitudes to positive Grassmannian remain unaffected.

Momentum conservation and mass shell conditions in 4-vertex

What is the exact meaning of the mass shell condition?

1. $H = M^4 \times CP_2$ harmonics would suggest that it mass squared in M^4 is eigenvalue of spinor d'Alembertian plus possible super-conformal contribution from Super Virasoro algebra, which is integer valued in suitable units. M^4-momentum decomposes to longitudinal M_0^2 momentum and transversal E^2 momentum. Super Virasoro algebra in transversal degrees of freedom suggests quantization of E^2 mass squared in integer multiples of a basic unit.

2. The CP_2 part of wave function in H corresponds in M^8 to a wave function in the moduli space of transversal planes E^2 assignable to M_0^2 and is involved only if the deformations of M^4 (or equivalently E^2) are present.

3. In the preferred frame M_0^4 the wave function would be strictly localized in single point of CP_2 and have maximally uncertain color quantum numbers. This kind of localization does look feasible physically. For instance, for color singlet CP_2 wave function of right-handed neutrino there is no localization. For sharp localization of 8-momentum to M_0^2 both color degrees and transvervsal E^2 degrees of freedom would effectively disappear.

4. The wave function in transversal E^2 momentum space with interpretation in terms of transversal momentum distribution - this at least in the case of hadrons.

5. The physically motivated assumption is that string world sheets at which the data determining the modes of induced spinor fields carry vanishing W fields and also vanishing generalized Kähler form $J(M^4)+J(CP_2)$. Em charge would be the only remaining electroweak degree of freedom. The identification as the helicity assignable to $T(CP_2)$ twistor sphere looks therefore natural. Note that the contribution to mass squared would be proportional to Q_{em}^2 so that one would obtain the electroweak mass splitting automatically. This is true also for CP_2 spinor harmonics.

How plausible topological loops are?

Topological loops are associated with the networks formed from the orbits of partonic 2-surfaces meeting at their ends (this would define topological 3-vertex containing fermionic 4-vertex). The tree topologies would provide a nice space-time description of particle reactions but loops could be possible? The original vision about construction of WCW geometry indeed was that the space-time surfaces with fixed ends are unique.

In the original vision the non-determinism of Kähler action inspired the hypothesis that loops are possible but volume term removes to high extent this non-determinism. In the recent vision the fusion of 3-surfaces at the ends of CD with light-like parton orbits to single 3-surface as a boundary condition (analogous to a fixing of a frame for soap films) would define the scattering diagram classically. There is no reason why it could not contain topological loops. Option IIIa) assuming that one can transform the diagrams ot tree diagrams, is therefore attractive.

1. There are also conditions from space-time dynamics. Twistor graph topologies correlate with space-time topologies since fermion line are inside the parton orbits and at vertices the ends of the orbits meet. Topological vertices would be basically 3-vertices for partonic 2-surfaces. The fermion and anti-fermion lines associated with the effective boson exchange would be naturally associated with opposite throats of wormhole contact.

   By above argument one can in ZEO pose at space-time level conditions fixing the vertices and identify the graph topology as a topology of the network of light-like 3-surfaces defining the diagram as boundary of 3-surface defined by the union of the ends of space-time and by parton orbits forming a connected surface.

2. There is a further delicacy to be taken into account - measurement resolution coded by the extension of rationals involved. This might allow to interpret addition of loops as in quantum field theories: as a result of increased measurement resolution determined dynamically by the intersection of reality and p-adicities. Different computation yielding the same result would not be cognitively equivalent since these intersections would be different.

3. If this view is correct, one can obtain also loops but non-negativity of energy for a given arrow of time for quantum state would allow only loops resulting from the decay and re-fusion of partonic 2-surfaces. Tadpoles appearing in BCFW recursion formula are impossible if the energy is non-negative. One can of course ask whether the sign of energy could be also negative if complex four-momenta are allowed. If so, one could have also tadpoles classically.

Identification of the fundamental 4-fermion vertex

The fundamental 4-fermion vertex would not be local 4-fermion vertex but correspond to classical scattering at partonic 2-surface. This saves from the TGD counterparts of the problems of QFT approach produced by non-renormalizability.

What would be this 4-fermion vertex? Yangian invariance suggests that the classical interaction between fermions must be expressible in terms of fictive 3-vertex of SUSY theories describing classical interaction as exchange of a fictive boson. This leaves 3 options.

Option I: 4-fermion vertex could be fusion of two 3-vertices with complex massless 8-momenta in M^8 picture. For instance, the exchanged momentum could be complex massless momentum and external momenta real on-mass-shell momenta. This vertex does not have QFT counterpart as such.

Loops could be absent either in the strong sense twistorial loops are absent (Option Ia) or in the sense that corresponding Feynman diagrams contain no loops (Option Ib). In particular, formation of BCFW bridge would not be allowed for Option Ia). Given diagram would be twistorial tree diagram obtained by replacing the vertices of ordinary tree diagram with these 4-vertices with complex massless fermions in 8-D sense.

Option II: 4-fermion could be identified as BCFW bridge associated with a tree Feynman diagram describing an exchange of a fictive boson. This 4-vertex would be analogous to an exchange of ordinary boson and counterpart for a QFT tree diagram. One can even forget the presence of the fictive boson exchange and write the formula for the simplest Yangian invariant as a candidate for four-fermion vertex.

Option III: If one allows higher fermion numbers at the same line, it is also natural to allow branching of lines. This requires allowance of 3-vertex as branching of fermion line as analog of splitting of open string (now strings are actually closed if they continue to another space-time sheet through wormhole contact). The situation would resemble that in SUSY. One cannot completely exclude this possibility.

Consider now the construction of 4-fermion vertex in more detail.

1. The helicities of fermions are $h_i = \pm 1$ and the general conjecture for the 4-fermion twistorial scattering amplitude is the simplest possible holomorphic rational function in λ_i, which does

not depend on $\tilde{\lambda}_i$, and satisfies the condition that the scaling $\lambda_i \to t\lambda_i$ introduces the scaling factor t^{-2}.

2. The rule is that fermions correspond to 2 positive powers of λ_i and antifermions to 2 negative powers in λ_i: schematically the $F_1F_2\bar{F}_3\bar{F}_4$ vertex is of form $\lambda_1^2\lambda_2^1/\lambda_3^2\lambda_4^2$ and constructible from $\langle\lambda_i, \lambda_j$. One can multiply any term in the expression of vertex by a rational function of for which the weights associated with λ_i vanish. Ratios $P_i(f)/P_j(f)$ of functions $P(f)$ obtained by via odd permutations P of the arguments λ_i of function

$$f(\lambda_1, \lambda_2, \lambda_3, \lambda_4) = \langle\lambda_1, \lambda_2\rangle\langle\lambda_2, \lambda_3\rangle\langle\lambda_3, \lambda_4\rangle\langle\lambda_4, \lambda_1\rangle$$

3. invariant under 4 cyclic permutations. The number of these functions would be $4!/4 = 3! = 6$ corresponding to the 6 orbits of an odd permutation under the cyclic group Z_4. The simplest assumption is that these functions are not involved.

   The simplest guess for the 4-fermion scattering amplitude would be following:

$$T(F_1, F_2, \overline{F}_3, \overline{F}_4) = J \times \frac{\langle\lambda_1, \lambda_2\rangle^2}{\langle\lambda_3, \lambda_4\rangle^2} \quad . \tag{15.5.5}$$

   Charge conjugation would take the function to itse inverse. J is constant.

4. In 4-fermion vertex one has exchange of fictive boson and annihilation to fictive boson and the particles i, j in the vertex should contribute $\langle\lambda_i, \lambda_j\rangle$ to the scattering amplitudes.

Remarkably, this amplitude is holomorphic in λ_i and has no dependence on $\tilde{\lambda}_i$ and therefore carries no information about whether the momenta are light-like or not. It seems that one could allow massive fermions characterized by (λ_i, μ_i) and fermion masses would not be a problem! As already explained in TGD mass is not M^8-scalar and states are massless in 8-D sense: hence twistorialization should work!

One could construct more complex diagrams in very simple manner using these basic diagrams as building bricks just as in the twistor Grassmann approach. One could form product of diagrams A and B using merge operation [B75] identifying twistor variables Z_a and Z_b belonging to the two diagrams A and B to be fused.

For Option Ia) the diagram would represent repeated on mass shell 4-fermion scatterings but with of mass shell particles having complex momenta in 8-D sense. Real on mass shell particles would have massless but real 8-D momenta and physical polarizations.

The conservation of baryon and lepton numbers implies for all options that only $G(m, n = 2\times m)$ Grassmannians are needed. This simplifies considerably the twistor Grassmannian approach.

Why fermions as fundamental particles (to be distinguished from elementary particles in TGD) are so special?

1. The mass of the fundamental fermion is not visible in the holomorphic basic amplitude being visible only via momentum conserving delta function $\delta(\sum_i \lambda_i\tilde{\mu}_i)$. This property holds true also for more complex diagrams. Massivation does not require in TGD framework $\bar{\Psi}\Psi$ term in Dirac action since M^4-massive fermions are M^8-massless and have only chiral couplings in 8-D sense. Scalar coupling would also break separate baryon and lepton conservation. Mass term correspond to a momentum in $E^4 \subset M^4 \times E^4 = M^8$ degrees of freedom. Massivation without losing 8-D light-likeness is consistent with conformal symmetry and with 8-D twistor approach.

2. Fermions are exceptional in the sense that the number of helicities is same for both massive and massless fermions. In particular, 4-fermion amplitude has $k = n/2$ and positive Grassmannian $G(n/2, n)$ with special symmetry property that one can take either negative or positive helicities in preferred role, could be important. For massless states with higher spin the number of helicities is 2 and maximal spin is $J_{max} = h_{max}/2$. For M^4 -massive states also the lower helicities $h_{max} - 2k$ are possible. The scattering amplitudes remain holomorphic.

3. For SUSY one would have all helicities $h(k) = h_{max} - k$ and the general form of amplitude could be written from the knowledge of $h(k)$. The number of fermions at the boundary of string world sheets could be maximal allowed by statistics. This would give SUSY in TGD sense but would require splitting of string boundaries: it is not clear whether this can be allowed. For light-like orbits of partonic 2-surface it has been assumed.

   Sparticles could correspond to states with higher fermion number at given partonic orbits. In this case one expects only approximate SUSY: the p-adic primes characterizing different SUSY states could be different. In adelic physics different p-adic prime could correspond to a different extension of rationals: one might say that the particles inside super-multiplets are at different levels in number theoretic evolution!

BCFW recursion formula as a consistency condition: BCFW homology

The basic consistency condition is that the boundary operation in the BCFW recursion formula gives zero so that the recursion formula can be solved without introducing sum over topological loops. The twistorial trees would have no boundaries but would not be boundaries and would be therefore closed in what might be called BCFW homology. Diagrams would correspond to closed forms.

Consider first the proposal assuming that all diagrams are equivalent with twistorial string diagrams with fermionic 4-vertex as the basic vertex. The boundary operation appearing in BCFW formula gives two terms [B38, B75, B35]. Recall that options I, II, and III correspond to twistorial diagrams without loops created by BCFW bridges, to twistor diagrams assignable to Feynman diagrams without loops, and to diagrams analogous to SUSY diagrams for which fermion lines carry also higher fermion number and can split.

1. The first term results as one BCFW bridge by contracting the three lines connecting the external particles to a larger diagram to a point in all possible manners. The non-vanishing of this term does not force loops in the sense of Feynman diagrams. For Option Ia) (no twistorial loops) there are no BCFW boxes to be reduced so that the outcome is zero.

   For option Ib) (no Feynman loops) a BCFW box diagram for which the two outward direct lines of the bridge are fictive, this operation makes sense and reduces the box to that describing the basic 4-fermion vertex. Same is true for the option II. For option III the operation would be essentially the same as in SUSY.

2. Second term corresponds to entangled removal of a fermion and anti-fermion and if it is non-vanishing, loops are unavoidable. This operation creates a closed fermionic loop to which several internal lines couple. By QCC the fermionic loop would be associated with a topological loop. One can argue that the topological tadpole loop must be closed time loop and that this is not possible since the sign of energy must change at the top and bottom of the loop, where the arrow of time changes: actually the energy should vanish. The same result would obtained if one requires that the energy identified as real part of complexified energy is non-negative for all on mass shell particles.

   Consider the 4-fermion vertex to which the fermionic tadpole loop is associated. Entangled removal gives for the members of a pair of external lines opposite momenta and helicities in twistor-diagrammatics. If so, there exist a vertex for which one fermion scatters in forward direction. Momentum conservation implies the same for the second fermion. One would obtain amplitude, which equals to unity rather than vanishing! Integration over four-momenta would give divergence. However, if the 4-momentum in the tadpole vanishes, the corresponding helicity spinor and also the amplitude vanishes. QCC indeed demands that fermionic loop corresponds to a time loop possible only only if the energy and by time-likeness also 3-momentum vanishes.

It seems that only the simplest option - Option Ia) - is consistent with the BCFW reduction formula. One can say that scattering diagrams are closed objects in the BCFW cohomology. Closedness condition might allow also topological loops, which are not tadpole loops: say decay of fermion to 3 fermions fusing back to the fermion.

Under what conditions fermionic self energy loop is removable?

Scattering diagram as a representation of computation demands that the fermionic "self energy" loop involving two external fermions gives free propagator. The situation in which the vertex contains only *light-like* complex momenta in M_0^2 can be considered as an example. In fact, one can always choose in M^8 the frame for given component of state in this manner.

1. The three fermion/antifermion internal lines in the loop would be light-like in complex 2-D sense as also external momentum. For external momenta $Re(p(M^2))$ would be light-like and orthogonal to light-like $Im(p(M^2))$: it is not clear whether $Im(p(M^2))$ vanishes.

   Light-likeness condition gives $Re(k)^2 - Im(k)^2 = 0$ and $Re(k) \cdot Im(k) = 0$, and $Re(k) = \pm Im(k)$ as a solution meaning that $Re(k)$ is proportional to a light-like vector $(1,1)$ or $(1-1)$. This applies to p, k_1,k_2, and $p - k_1 - k_2$. All these vectors are proportional to the same light-like vector in M^2.

   Apart from the degeneracy for sign factors the situation is equivalent with real 2-D case and one has from momentum conservation that the real parts of the virtual momenta are light-like and parallel and one has $Re(k_i) = \lambda_i p$ leaving two real parameters λ_i.

2. The only possible outcome from the integral is proportional to $D_F(p)$. The outcome is non-vanishing if the proportionality constant is proportional to $1/p^2$. This dependence should come from 4-fermion vertices. The integrand is proportional to the product $\lambda_1\lambda_2(1-\lambda_1-\lambda_2)$ and involves times the $D_F(p)$. Vertices give the inverses of these scaling factors. Since the outcome should be proportional to $1/D_F$ and lines are proportional to p^3, the 4- vertices should give a factor $1/p^2$ each.

   Assuming this one obtains integrand $1/(\lambda_1\lambda_2(1-(\lambda_1-\lambda_2)^2$. The integral over λ_i is of proportional to

   $$I = \int d\lambda_1 d\lambda_2/\lambda_1\lambda_2(1-\lambda_1-\lambda_2) \ .$$

   The ranges of integration are from $(-\infty, \infty)$.

   One can decompose the integral to four parts so that integration ranges are positive. The outcome is

   $$I = \int dlog(\lambda_1)dlog(\lambda 2)\left[\frac{1}{1-\lambda_1-\lambda_2} + \frac{1}{1+\lambda_1+\lambda_2} - \frac{1}{1+\lambda_1-\lambda_2} - \frac{1}{1-\lambda_1+\lambda_2}\right] \ .$$

   The change of variables $(u,v) = (\lambda_1+\lambda_2, \lambda_1-\lambda_2)$ transforms the integral to a product of integrals

   $$I = \int dudv\frac{1}{1-u^2}\int dv\frac{1}{1-v^2} \ .$$

   The interpretation as residue integral gives the outcome $I = (4\pi)^2$.

   Residue integration gives finite result for this integrals. One can worry about the singularity of the vertices for M_0^2 on mass shell momenta. The problem is that p is on mass shell so that the outcome from loop diverges. The outcome is $D_F$would be however finite.

Gliding conditions for 4-vertices

One can construct also loop diagrams with loops understood in twistorial sense. The interpretation of twistor diagram as computation requires that there exist moves reducing general loopy diagrams to tree diagrams. This requires that the vertices connected by a fermionic loop lines can be glided along fermion lines such that they become nearest neighbors and that these loops can be removed without affective the diagram.

If these diagrams are acceptable mathematically, moves reducing these loop diagrams to twistorial tree diagrams should exist. Could the basic rule be following?

1. One can glide the vertices past each other along fermion lines and reduce loops connecting points at different part of graph to the analogs of self-energy loops located at single fermion lines. These loops involve decay of fermion to 2 fermions and 1 anfifermion which then fuse to single fermion. All fermions are on mass shell in complex sense. The situation thus reduces to single fermion self energy loop if the gliding is possible always. Mass shell conditions could however prevent this.

2. To single fermion line one can assign D_F - the inverse of massless fermion propagator - having formal interpretation as a density matrix. The loop would not vanish but would give rise to a inverse of fermionic propagator so that the overall outcome should be just D_F. Is it possible to achieve this?

Under what conditions the gliding is possible?

1. Suppose that the 4-vertex V_1 is glided along fermion line past second 4-vertex V_1. V_1 corresponds to momenta $(P_{i,in}, P_{i1,in} - P, P_{i,1}, P_{i,2})$. The momentum $P_i = \sum_{k=1}^{2} P_{i,k}$ of 2 particles emanates from V_i so that the outgoing and incoming momenta are $P_{i,in} - P_i$, and $P_{i,in}$ $i = 1, 2$. Furthermore $P_{1,in} = P_{2,in} - P_2$. These complex momenta are on M^2 mass shell in the proposed sense.

2. Can one perform the gliding without changing the M_0^2-momenta $P_{i,1}$ and $P_{i,2}$? Gliding is possible if the on mass shell condition is satisfied also for $P_{2,in} - P_1 + P_2$ rather than only $P_{2,in} + P_2$. If the mass squared spectrum is integer valued in suitable units the condition reduces to the requirement that $2P_{2,in} \cdot P_1$ is real and integer valued.

   These conditions are independent of the conditions for $2P_{2,in} \cdot P_2$ coming from V_2, the conditions would correlate P_1 and P_2. The construction of the amplitude would involve non-local conditions on vertices rather than only momentum conservation and mass shell conditions at vertices as expected.

   M^2-momentum is however light-like for a special choice$M^2 = M_0^2$. If M_0^2 same along connected fermion lines, the gliding condition would make sense. M_0^2 is constant of motion along fermion line which can wander along the network formed by partonic orbits.

   In fact, M_0^2 must be same for all fermions in given vertex so that its is constant for all connected regions of fermionic part of the graph. Is there any hope of having non-trivial scattering amplitude or must all momenta be light-like and parallel in plane M_0^2? Tree diagrams certainly give rise to non-trivial scattering. One can also assign to all internal lines this kind of networks with M_0^2 that assignable to the internal line. It is quite possible that for general graphs allowing different M_0^2s in internal lines and loops, the reduction to tree graph is not possible.

3. The analogs of these conditions apply also to tree graphs. So that one must either sum over trees with different orderings of vertices or pose additional conditions on the M^2-momenta say the assumption that they are light-like and proportional to the same real momentum $(1, \pm 1)$ along the fermion line.

To conclude: if M_0^2 is constant of motion along the connected networks of fermion lines, the gliding conditions could be satisfied. Action exponentials do not produce trouble if one identifies the basis of zero energy states in such a manner that every maximum of action gives its own separate amplitude (state) as also number theoretic universality demands. The most attractive option number theoretically is the option IIIa) assuming that localization of zero energy state to single computation is possible as quantum measurement: different localizations would have different intersections between reality and p-adicicties and would correspond to different computation sequences as cognitive processes. The idea that twistor diagrams are closed forms in the sense that tadpole diagrams vanish is also very attractive and natural in this framework.

Permutation as basic data for a scattering diagram

In twistor Grassmannian approach to $\mathcal{N} = 4$ SUSY the data determining the Yangian invariants defining the basic building bricks of the amplitudes can be constructed using two 3-vertices. For the first (second) kind of vertex the helicity spinors λ_i ($\tilde{\lambda}_i$) are parallel that is $\lambda_1 \propto \lambda_2 \propto \lambda_3$ ($\overline{\lambda}_1 \propto \overline{\lambda}_2 \propto \overline{\lambda}_3$) and can be chosen to be identical by complex scaling invariant: momentum conservation reduces to that for $\tilde{\lambda}_i$ (λ_i). The graphical notation for the two vertices is as a small white *resp.* black disk [B75, B35] (see Fig. 3.3.35 `http://tinyurl.com/zbj9ad7`).

There are two basic moves leaving the amplitude unaffected (see Fig. 3.3.38 at `http://tinyurl.com/zbj9ad7`). Merging symmetry implies that 4-vertices satisfy a symmetry analogous to the duality of old-fashioned hadron physics: an internal line connecting black (white) vertices as exchange in s-channel can be transformed to an exchange in t-channel: $1{+}2 \rightarrow 3{+}4 \equiv 1{+}3 \rightarrow 2{+}4$. Merging symmetry allows to transform the diagram into a form in which neighboring vertices have opposite colors. Square move symmetry follows from the cyclic symmetry of the 4-particle amplitude and means black$\leftrightarrow$white replacement in 4-vertex.

These two moves do not affect the permutation defining the diagram. A given diagram is represented as a disk with external lines ordered cyclically along its boundary. The permutation of the n external particles associated with the diagram is constructed from the two 3-particle diagrams is defined by the following rule.

Start from k:th point at boundary end and go to the left in each white vertex and to the right in each black vertex (see Fig. 3.3.35 at `http://tinyurl.com/zbj9ad7`).

This leads to a particle $P(k)$ and the outcome is a permutation $P : k \rightarrow P(k)$ charactering the twistor diagram.

Moves do not affect the permutation associated with the diagram and leave the amplitude unaffected. BCFW bridge can be interpreted as a permutation of two neighboring external lines and allows to generate non-equivalent diagrams.

This permutation symmetry generalizes to 4-D SUSY the role of permutations in 1+1-D integrable field theories, where the scattering S-matrix induces only a phase shift of the wave functions of identical particles. The scattering diagram depends only on the permutation of particles induced by the scattering event. Yang-Baxter relation expresses this . Scattering corresponds to particles passing by each other and diagram is drawn in M^2 plane.

1. In 1+1-D integrable theory 3+3 scattering reduces to 2 particle scatterings. This can be illustrated using world lines in M^2 plane (see the illustration of `http://tinyurl.com/gogn75s`). The particle 2 can be taken to be at rest and 1 and 3 move with opposite velocities. There are three 2-particle scatterings of i and j as crossings of world-lines of i and j (pass-by spatially): denote the crossing by ij.

   For the diagram on the left hand side one has crossings 12, 13 and 23 with this time order. For the second case one has crossings 23, 13, and 12 in this time order. Graphically YB relation (see the illustration of `http://tinyurl.com/gogn75s`) says that the scattering amplitude fo 3+3 scattering does not depend on whether the position of the stationary particle 2 is to the left or right from the point at which the second scattering occurs: the time order of scatterings 12 and 23 does not matter.

2. Mathematically the two-particle scatterings are described by operators $R_{12}(u)$, $R_{13}(u+v)$, and $R_{23}(v)$ representing basic braiding operation $ij \rightarrow ji$. u, $u+v$, and v are parameters characterizing the Lorentz boosts determining the velocities of particles. YB equation reads as

$$R_{12}(u)R_{13}(u+v)R_{23}(v) = R_{23}(v)R_{13}(u+v)R_{12}(u) \ .$$

   For a graphical illustration see `http://tinyurl.com/gogn75s`. The first and third R-matrices are permuted and the outcome is trivial. In pass-by interpretation YB equation states that the two manners to realize $123 \rightarrow 321$ give the same amplitude.

   Instead of pass-by one could assume a reconnection of the world lines at the intersection: world lines are split and future pieces are permuted and connected to the past pieces again. With this interpretation one has $123 \rightarrow 123$ (the illustration of Wikipedia article corresponds to this interpretation).

3. At the static limit $u, v \rightarrow 0$ YB equation gives rise to an identity satisfied by braiding matrices. The pass-by at this limit can be interpreted as permutation lifted to braiding (braid groups is covering group of permutation group).

2+2 vertices are fundamental in integrable theories in M^2. Also in TGD 2+2 vertices for fundamental fermions are proposed to be fundamental, and the effective reduction to M^2 is crucial in many respects and reflects $M^8 - CP_2$ duality and 8-D quaternionic light-likeness implying that 2+2 fermion vertices reduce to vertices in M^2. TGD could be an integrable theory able to circumvent the limitations of integrable QFTs in M^2.

1. How could the 2+2-fermionic scattering matrix relate to the R-matrix? In TGD framework the scattering involves momentum transfer even in M_0^2 frame: the parallel light-like M^2 momenta are rescaled in momentum conserving manner. Could R matrix appear as additional factor in the scattering? The earlier picture indeed is that the fermion lines at partonic orbits can experience braiding described by R-matrix at the static limit (string world sheet boundaries would braid!).

2. In TGD the scattering of 2 fermions could occur in two manners by classical interactions at partonic 2-surface. The world lines either cross each other or not. In M^2 the first contribution is planar and second one non-planar. Both options should contribute to the 4-fermion amplitude but this is not be visible in the proposed form of the amplitude. Does the proposed 4-fermion scattering amplitude allow this interpretation?

   In $\mathcal{N} = 4$ SUSY the addition of BCFW bridge would permute the two external particles. In TGD the introduction of BCFW bridge would force to have bosonic lines in the BCFW bridge. This is not possible. The only manner to have BCFW diagram is to allow SUSY perhaps realized as and addition right-handed neutrinos to the fermion lines but this would force to allow splitting of fermion lines requiring splitting of strings.

3. Annihilations of fermion-antifermion pairs to bosons are not possible in 1+1-D QFTs but in TGD topological 3-vertices allow them. Boson would correspond to the final $B \equiv F\overline{F}$ pair at same parton orbit. There are two manners to achieve the annihilation. In s-channel $F\overline{F} \rightarrow vacuum \rightarrow F\overline{F} \equiv B$ is possible. Both F_1 coming from past and F_2 from future scatter classically backwards in time to give $\overline{F}_1$ travelling back to past and $\overline{F}_2$ travelling back to future. In t-channel one can have braiding ($F\overline{F} \rightarrow \overline{F}F \equiv B$.

About unitarity for scattering amplitudes

The first question is what one means with S-matrix in ZEO. I have considered several proposals for the counterparts of S-matrix [K91].In the original U-matrix, M-matrix and S-matrix were introduced but it seems that U-matrix is not needed.

1. The first question is whether the unitary matrix is between zero energy states or whether it characterizes zero energy states themselves as time-like entanglement coefficients between positive and negative energy parts of zero energy states associated with the ends of CD. One can argue that the first option is not sensible since positive and negative energy parts of zero energy states are strongly correlated rather than forming a tensor product: the S-matrix would in fact characterize this correlation partially.

   The latter option is simpler and is natural in the proposed identification of conscious entity - self - as a generalized Zeno effect, that is as a sequence of repeated state function reductions at either boundary of CD shifting also the boundary of CD farther away from the second boundary so that the temporal distance between the tips of CD increases. Each shift of this kind is a step in which superposition of states with different distances of upper boundary from lower boundary results followed by a localization fixing the active boundary and inducing unitary transformation for the states at the original boundary.

2. The proposal is that the the proper object of study for given CD is M-matrix. M-matrix is a product for a hermitian square root of diagonalized density matrix ρ with positive elements and unitary S-matrix $S : M = \sqrt{\rho}S$. Density matrix ρ could be interpreted in this approach as a non-trivial Hilbert space metric. Unitarity conditions are replaced with the conditions $MM^\dagger = \rho$ and $M^\dagger M = \rho$. For the single step in the sequence of reductions at active boundary of CD one has $M \rightarrow MS(\Delta T)$ so that one has $S \rightarrow SS(\Delta T)$. $S(\Delta T)$ depends on the time interval ΔT measured as the increase in the proper time distance between the tips of CD assignable to the step.

What does unitarity mean in the twistorial approach?

1. In accordance with the idea that scattering diagrams is a representation for a computation, suppose that the deformations of space-time surfaces defining a given topological diagram as a maximum of the exponent of Kähler function, are the basic objects. They would define different quantum phases of a larger quantum theory regarded as a square root of thermodynamics in ZEO and analogous to those appearing also in QFTs. Unitarity would hold true for each phase separately.

   The topological diagrams would not play the role of Feynman diagrams in unitarity conditions although their vertices would be analogous to those appearing in Feynman diagrams. This would reduce the unitarity conditions to those for fermionic states at partonic 2-surfaces at the ends of CDs, actually at the ends of fermionic lines assigned to the boundaries of string world sheets.

2. The unitarity conditions be interpreted stating the orthonormality of the basis of zero energy states assignable with given topological diagram. Since 3-surfaces as points of WCW appearing as argument of WCW spinor field are pairs consisting of 3-surfaces at the opposite boundaries of CD, unitarity condition would state the orthonormality of modes of WCW spinor field. If might be even that no mathematically well-defined inner product assignable to either boundary of CD exists since it does not conform with the view provided by WCW geometry. Perhaps this approach might help in identifying the correct form of S-matrix.

3. If only tree diagrams constructed using 4-fermion twistorial vertex are allowed, the unitarity relations would be analogous to those obtained using only tree diagrams. They should express the discontinuity for T in $S = 1 + iT$ along unitary cut as $Disc(T) = TT^\dagger$. T and $T^\dagger$ would be T-matrix and its time reversal.

4. The correlation between the structure of the fermionic scattering diagram and topological scattering diagrams poses very strong restrictions on allowed scattering reactions for given topological scattering diagram. One can of course have many-fermion states at partonic 2-surfaces and this would allow arbitrarily high fermion numbers but physical intuition suggests that for given partonic 2-surface (throat of wormhole contact) the fermion number is only 0, 1, or perhaps 2 in the case of supersymmetry possibly generated by right-handed neutrino.

   The number of fundamental fermions both in initial and final states would be finite for this option. In quantum field theory with only masive particles the total energy in the final state poses upper bound on the number of particles in the final state. When massless particles are allowed there is no upper bound. Now the complexity of partonic 2-surface poses an upper bound on fermions.

   This would dramatically simplify the unitarity conditions but might also make impossible to satisfy them. The finite number of conditions would be in spirit with the general philosophy behind the notion of hyper-finite factor. The larger the number of fundamental fermions associated with the state, the higher the complexity of the topological diagram. This would conform with the idea about QCC. One can make non-trivial conclusions about the total energy at which the phase transitions changing the topology of space-time surface defined by a topological diagram must take place.

15.5.5 Criticism

One can criticize the proposed vision.

What about loops of QFT?

The idea about cancellation of loop corrections in functional integral and moves allowing to transform scattering diagrams represented as networks of partonic orbits meeting at partonic 2-surfaces defining topological vertices is nice.

Loops are however unavoidable in QFT description and their importance is undeniable. Photon-photon (see `http://tinyurl.com/lqhdujm`) scattering is described by a loop diagram in which fermions appear in box like loop. Magnetic moment of muon see http://tinyurl.com/p7znfmd) involves a triangle loop. A further, interesting case is CP violation for mesons (see http://tinyurl.com/oop4apy) involving box-like loop diagrams.

Apart from divergence problems and problems with bound states, QFT works magically well and loops are important. How can one understand QFT loops if there are no fundamental loops? How could QFT emerge from TGD as an approximate description assuming lengths scale cutoff?

The key observation is that QFT basically replaces extended particles by point like particles. Maybe loop diagrams can be "unlooped" by introducing a better resolution revealing the non-point like character of the particles. What looks like loop for a particle line becomes in an improved resolution a tree diagram describing exchange of particle between sub-lines of line of the original diagram. In the optimal resolution one would have the scattering diagrams for fundamental fermions serving as building bricks of elementary particles.

To see the concrete meaning of the "unlooping" in TGD framework, it is necessary to recall the qualitative view about what elementary particles are in TGD framework.

1. The fundamental fermions are assigned to the boundaries of string world sheets at the light-like orbits of partonic 2-surfaces: both fermions and bosons are built from them. The classical scatterings of fundamental fermions at the 2-D partonic 2-surface defining the vertices of topological scattering diagrams give rise to scattering amplitudes at the level of fundamental fermions and twistor lift with 8-D light-likeness suggests essentially unique expressions for the 4-fermion vertex.

2. Elementary particle is modelled as a pair of wormhole contacts (Euclidian signature of metric) connecting two space-time sheets with throats at the two sheets connected by monopole flux tubes. All elementary particles are hadronlike systems but at recent energies the substructure is not visible. The fundamental fermions at the wormhole throats at given space-time sheet are connected by strings. There are altogether 4 wormhole throats per elementary particle in the simplest model.

   Elementary boson corresponds to fundamental fermion and antifermion at opposite wormhole throats with very small size (CP_2 size). Elementary fermion has only single fundamental fermion at either throat. There is $\nu_L\overline{\nu}_R$ pair or its CP conjugate at the other end of the flux tube to neutralize the weak isospin. The flux tube has length of order Compton length (or elementary particle or of weak boson) gigantic as compared to the size of the wormhole contact.

3. The vertices of topological diagram involve joining of the stringy diagrams associated with elementary particles at their ends defined by wormhole contacts. Wormhole contacts defining the ends of partonic orbits of say 3 interacting particles meet at the vertex - like lines in Feynman diagram - and fundamental fermion scattering redistributes fundamental fermions between the outgoing partonic orbits.

4. The important point is that there are $2 \times 2 = 4$ manners for the wormhole contacts at the ends of two elementary particle flux tubes to join together. This makes a possible a diagrams in which particle described by a string like object is emitted at either end and glued back at the other end of string like object. This is basically tree diagram at the level of wormhole contacts but if one looks it at a resolution reducing string to a point, it becomes a loop diagram.

5. Improvement of the resolution reveals particles inside particles, which can scatter by tree diagrams. This allows to "unloop" the QFT loops. By increasing resolution new space-time sheets with smaller size emerge and one obtains "unlooped" loops in shorter scales. The space-time sheets are characterized by p-adic length scale and primes near powers of 2 are favored. p-Adic coupling constant evolution corresponds to the gradual "unlooping" by going to shorter and shorter p-adic length scales revealing smaller and smaller space-time sheets.

The loop diagrams of QFTs could thus be seen as a direct evidence of the fractal many-sheeted space-time and quantum criticality and number theoretical universality (NTU) of TGD Universe. Quantum critical dynamics makes the dynamics universal and this explains the unreasonable success of QFT models as far as length scale dependence of couplings constants is considered. The weak point of QFT models is that they are not able to describe bound states: this indeed requires that the extended structure of particles as 3-surfaces is taken into account.

Can action exponentials really disappear?

The disappearance of the action exponentials from the scattering amplitudes can be criticized. In standard approach the action exponentials associated with extremals determine which configurations are important. In the recent case they should be the 3-surfaces for which Kähler action is maximum and has stationary phase. But what would select them if the action exponentials disappear in scattering amplitudes?

The first thing to notice is that one has functional integral around a maximum of vacuum functional and the disappearance of loops is assumed to follow from quantum criticality. This would produce exponential since Gaussian and metric determinants cancel, and exponentials would cancel for the proposal inspired by the interpretation of diagrams as computations. One could in fact *define* the functional integral in this manner so that a discretization making possible NTU would result.

Fermionic scattering amplitudes should depend on space-time surface somehow to reveal that space-time dynamics matters. In fact, QCC stating that classical Noether charges for bosonic action are equal to the eigenvalues of quantal charges for fermionic action in Cartan algebra would bring in the dependence of scattering amplitudes on space-time surface via the values of Noether charges. For four-momentum this dependence is obvious. The identification of $h_{eff}/h = n$ as the dimension of the extension dividing the order of its Galois group would mean that the basic unit for discrete charges depends on the extension characterizing the space-time surface. Also the cognitive representations defined by the set of points for which preferred imbedding space coordinates are in this extension. Could the cognitive representations carry maximum amount of information for maxima? For instance, the number of the points in extension be maximal. Could the maximum configurations correspond to just those points of WCW, which have preferred coordinates in the extension of rationals defining the adele? These 3-surfaces would be in the intersection of reality and p-adicities and would define cognitive representation.

These ideas suggest that the usual quantitative criterion for the importance of configurations could be equivalent with a purely number theoretical criterion. p-Adic physics describing cognition and real physics describing matter would lead to the same result. Maximization for action would correspond to maximization for information.

Irrespective of these arguments, the intuitive feeling is that the exponent of the bosonic action must have physical meaning. It is number theoretically universal if action satisfies $S = q_1 + iq_2\pi$. This condition could actually be used to fix the dependence of the coupling parameters on the extension of rationals [L24]. By allowing sum over several maxima of vacuum functional these exponentials become important. Therefore the above ideas are interesting speculations but should be taken with a big grain of salt.

15.6 Appendix: Some background about twistors

In the following I try to summarize my view about how the ideas related to the twistor approach to scattering amplitudes evolved. A readable summary of specialist about twistor approach is given in the article *Scattering amplitudes* of Elvang and Huang [B35]. Also the thesis *Grassmannian Origin of Scattering Amplitudes* of Trnka [B75] gives a good summary about the work done in association with Nima Arkani-Hamed. I am not a specialist and have not been endowed with practical calculations so that my representation considers only the basic ideas and their relationship to TGD. In the following I summarize my very partial view about the development of ideas.

15.6.1 The pioneering works of Penrose and Witten

The pioneering work of Penrose discussed in *The Central Programme of Twistor Theory* [B69] on twistors initiated the twistor program, which had already had applications in Yang-Mills theories int he description of instantons. The key vision is that massless field equations reduce to holomorphy in twistor formulation.

Witten's *Perturbative Gauge Theory As a String Theory In Twistor Space* [B33] in 2003 initiated the progress leading to dramatic understanding of the planar scattering amplitudes of $\mathcal{N} = 4$ SUSY and eventually to the notion of amplituhedron. The abstract gives some idea about the key ideas.

Perturbative scattering amplitudes in Yang-Mills theory have many unexpected properties, such as holomorphy of the maximally helicity violating amplitudes. To interpret these results, we Fourier transform the scattering amplitudes from momentum space to twistor space, and argue that the transformed amplitudes are supported on certain holomorphic curves. This in turn is apparently a consequence of an equivalence between the perturbative expansion of $\mathcal{N} = 4$ super Yang-Mills theory and the D-instanton expansion of a certain string theory, namely the topological B model whose target space is the Calabi-Yau supermanifold $CP_{3|4}$.

Witten's observation was that the twistor Fourier transform of the scattering amplitudes of YM theories seem to be localized at 2-dimensional complex surfaces of twistor space and this led him to propose that twistor string theory in the twistor space CP_3 could allow to describe the scattering amplitudes. The basic problem of the twistor approach relates to space-time signature: all works nicely in signature (2,2), which suggests that something might be wrong in the basic assumptions.

15.6.2 BCFW recursion formula

BCFW recursion was first derived for tree amplitudes and later generalized to planar loop diagrams.

1. *Twistor diagram recursion for all gauge-theoretic tree amplitudes* by Hodges [B15] in 2005 and *Direct Proof of Tree-Level Recursion Relation in Yang- Mills Theory* by Britto, Cachazo, Feng, and Witten [B26] in 2005 proposed at tree level a recursion formula for the tree level MHV amplitudes of Yang-Mills theory in twistor space.

2. *Scattering Amplitudes and BCFW Recursion in Twistor Space* By Mason and Skinner [B26] discussed BCFW recursion relations for tree diagrams of YM theories.

3. *The S-Matrix in Twistor Space* by Arkani-Hamed, Cachazo, Cheung and Kaplan [B36] in 2009 discussed NkMHV amplitudes with more than two negative helicities (MHV amplitudes have 2 negative helicities are are extremely simple).

This work is carried out in metric signature (2,2), where the twistor transform reduces to ordinary Fourier transform. The other signatures are problematic. Only planar diagrams are considered. *On-Shell Structures of MHV Amplitudes Beyond the Planar Limit* [B41] in 2014 of Arkani-Hamed et al consider the problem posed by the non-planar diagrams.

15.6.3 Yangian symmetry and Grassmannian

The discovery of dual super-conformal invariance is one of the key steps of progress. This symmetry means extension of the conformal algebra from space-time level to the level of twistor space so that the dual superconformal invariance acts also on so called momentum twistors assigned with the twistor diagram. These dual conformal symmetries extend to a Yangian algebra containing besides local generators also multilocal generators. The dual conformal generators are bi-local generators and have weight $n = 1$. The Yangian symmetry is completely general and expected to generalize.

In the following I list the abstracts of some important articles.

1. *Magic identities for conformal four-point integrals* by Drummond, Henn, Smirnov, and Sokatchev [B42] in 2006 initiated the development of ideas. The interpretation is as dual conformal invariance generator by the weight 1 generators of Yangian.

   We propose an iterative procedure for constructing classes of off-shell four-point conformal integrals which are identical. The proof of the identity is based on the conformal properties of a sub-integral common for the whole class. The simplest example are the so-called "triple scalar box" and "tennis court" integrals. In this case we also give an independent proof using the method of Mellin-Barnes representation which can be applied in a similar way for general off-shell Feynman integrals.

2. *Yangian symmetry of scattering amplitudes in $\mathcal{N} = 4$ super Yang-Mills theory* [B31] by Drummond, Henn, and Plefka in 2009 continued this work and discussed Yangian algebra as as a symmetry having besides local generators also multilocal generators.

   Tree-level scattering amplitudes in $\mathcal{N} = 4$ super Yang-Mills theory have recently been shown to transform covariantly with respect to a "dual" superconformal symmetry algebra, thus extending the conventional superconformal symmetry algebra psu(2, 2|4) of the theory. In this paper we derive the action of the dual superconformal generators in on-shell superspace and extend the dual generators suitably to leave scattering amplitudes invariant. We then study the algebra of standard and dual symmetry generators and show that the inclusion of the dual superconformal generators lifts the psu(2,2|4) symmetry algebra to a Yangian. The non-local Yangian generators acting on amplitudes turn out to be cyclically invariant due to special properties of psu(2,2|4). The representation of the Yangian generators takes the same form as in the case of local operators, suggesting that the Yangian symmetry is an intrinsic property of planar $\mathcal{N} = 4$ super Yang-Mills, at least at tree level.

3. *Dual Superconformal Invariance, Momentum Twistors and Grassmannians* [B63] by Mason and Skinner introduces momentum twistors and Grassmannians.

   Dual superconformal invariance has recently emerged as a hidden symmetry of planar scattering amplitudes in $\mathcal{N} = 4$ super Yang-Mills theory. This symmetry can be made manifest by expressing amplitudes in terms of "momentum twistors", as opposed to the usual twistors that make the ordinary superconformal properties manifest. The relation between momentum twistors and on-shell momenta is algebraic, so the translation procedure does not rely on any choice of space-time signature. We show that tree amplitudes and box coefficients are succinctly generated by integration of holomorphic delta-functions in momentum twistors over cycles in a Grassmannian. This is analogous to, although distinct from, recent results obtained by Arkani-Hamed et al. in ordinary twistor space. We also make contact with Hodges polyhedral representation of NMHV amplitudes in momentum twistor space.

4. *A Duality For The S Matrix* [B38] in 2009 by Arkani-Hamed et al discusses also Yangian invariance and introduces central ideas in algebraic geometry: Grassmannians, higher-dimensional residue theorems, intersection theory, and the Schubert calculus.

   We propose a dual formulation for the S Matrix of $\mathcal{N} = 4$ SYM. The dual provides a basis for the leading singularities of scattering amplitudes to all orders in perturbation theory, which are sharply defined, IR safe data that uniquely determine the full amplitudes at tree level and 1-loop, and are conjectured to do so at all loop orders. The scattering amplitude for n particles in the sector with k negative helicity gluons is associated with a simple integral over the space of k planes in n dimensions, with the action of parity and cyclic symmetries manifest. The residues of the integrand compute a basis for the leading singularities. A given leading singularity is associated with a particular choice of integration contour, which we explicitly identify at tree level and 1-loop for all NMHV amplitudes as well as the 8 particle N^2MHV amplitude. We also identify a number of 2-loop leading singularities for up to 8 particles. There are a large number of relations among residues which follow from the multi-variable generalization of Cauchys theorem known as the "global residue theorem". These relations imply highly non-trivial identities guaranteeing the equivalence of many different representa- tions of the same amplitude. They also enforce the cancellation of non-local poles as well as consistent infrared structure at loop level. Our conjecture connects the physics of scattering amplitudes to a particular subvariety in a Grassmannian; space-time locality is reflected in the topological properties of this space.

5. *The All-Loop Integrand For Scattering Amplitudes in Planar $\mathcal{N} = 4$ SYM* [B39] by Arkani-Hamed et al in 2010.

   We give an explicit recursive formula for the all L-loop integrand for scattering amplitudes in $\mathcal{N} = 4$ SYM in the planar limit, manifesting the full Yangian symmetry of the theory. This generalizes the BCFW recursion relation for tree amplitudes to all loop orders, and extends the Grassmannian duality for leading singularities to the full amplitude. It also provides a new physical picture for the meaning of loops, associated with canonical operations for removing particles in a Yangian-invariant way. Loop amplitudes arise from the "entangled" removal of *pairs of particles, and are naturally presented as an integral over lines in momentum-twistor space. As expected from manifest Yangian-invariance, the integrand is given as a sum over non-local terms, rather than the familiar decomposition in terms of local scalar integrals with rational coefficients. Knowing the integrands explicitly, it is straightforward to express them in local forms if desired; this turns out to be done most naturally using a novel basis of chiral, tensor integrals written in momentum-twistor space, each of which has unit leading singularities. As simple illustrative examples, we present a number of new multi-loop results written in local form, including the 6- and 7-point 2-loop NMHV amplitudes. Very concise expressions are presented for all 2-loop MHV amplitudes, as well as the 5-point 3-loop MHV amplitude. The structure of the loop integrand strongly suggests that the integrals yielding the physical amplitudes are "simple", and determined by IR-anomalies. We briefly comment on extending these ideas to more general planar theories.*

15.6.4 Amplituhedron

The latest development in twistorial revolution was the notion of amplituhedron. Since I do not have intuitive understanding about amplituhedron and since amplituhedron does not have role in the twistorialization of TGD as I understand it now, I provide only abstracts about two articles to it.

1. *The Amplituhedron* [B20] by Arkani-Hamed and Trnka in 2013.

   Perturbative scattering amplitudes in gauge theories have remarkable simplicity and hidden infinite dimensional symmetries that are completely obscured in the conventional formulation of field theory using Feynman diagrams. This suggests the existence of a new understanding for scattering amplitudes where locality and unitarity do not play a central role but are derived consequences from a different starting point. In this note we provide such an understanding for $\mathcal{N} = 4$ SYM scattering amplitudes in the planar limit, which we identify as "the volume" of a new mathematical object–the Amplituhedron–generalizing the positive Grassmannian. Locality and unitarity emerge hand-in-hand from positive geometry.

2. *Positive Amplitudes in the Amplituhedron* [B19] by Arkani-Hamed et al in 2014.

   The all-loop integrand for scattering amplitudes in planar $\mathcal{N} = 4$ SYM is determined by an "amplitude form" with logarithmic singularities on the boundary of the amplituhedron. In this note we provide strong evidence for a new striking property of the superamplitude, which we conjecture to be true to all loop orders: the amplitude form is positive when evaluated inside the amplituhedron. The statement is sensibly formulated thanks to the natural "bosonization" of the superamplitude associated with the amplituhedron geometry. However this positivity is not manifest in any of the current approaches to scattering amplitudes, and in particular not in the cellulations of the amplituhedron related to on-shell diagrams and the positive Grassmannian. The surprising positivity of the form suggests the existence of a "dual amplituhedron" formulation where this feature would be made obvious. We also suggest that the positivity is associated with an extended picture of amplituhedron geometry, with the amplituhedron sitting inside a co-dimension one surface separating "legal" and "illegal" local singularities of the amplitude. We illustrate this in several simple examples, obtaining new expressions for amplitudes not associated with any triangulations, but following in a more invariant manner from a global view of the positive geometry.

Chapter 16

The Recent View about Twistorialization in TGD Framework

16.1 Introduction

The construction of scattering amplitudes is a dream that I have had since the birth of TGD for four decades ago. Various ideas have gradually emerged, some of them have turned out to be wrong, and some of them have survived. At this age I must admit that the dream about explicit algorithms that any graduate student could apply to construct the scattering amplitudes, would require a collective effort and probably will not be realized during my lifetime.

I have however identified a set of general powerful principles leading to a generalization of the recipes for constructing twistorial amplitudes and already now these principles suggest the possibility of rather concrete realizations. In the sequel several additional insights are developed in more detail. Some of them are discussed already earlier in the formulation of $M^8 - H$ duality [L33] in adelic framework [L35, L36] and in the chapters developing the TGD based generalization of twistor Grasmannian approach [K76, L22, L24, L38].

1. A proposal made already earlier [L38] is that scattering diagrams as analogs of twistor diagrams are constructible as tree diagrams for CDs connected by free particle lines. Loop contributions are not even well-defined in zero energy ontology (ZEO) and are in conflict with number theoretic vision. The coupling constant evolution would be discrete and associated with the scale of CDs (p-adic coupling constant evolution) and with the hierarchy of extensions of rationals defining the hierarchy of adelic physics.

2. Logarithms appear in the coupling constant evolution in QFTs. The identification of their number theoretic versions as rational number valued functions required by number-theoretical universality for both the integer characterizing the size scale of CD and for the hierarchy of Galois groups leads to an answer to a long-standing question what makes small primes and primes near powers of them physically special. The primes $p \in \{2, 3, 5\}$ indeed turn out to be special from the point of view of number theoretic logarithm.

3. The reduction of the scattering amplitudes to tree diagrams is in conflict with unitarity in 4-D situation. The imaginary part of the scattering amplitude would have discontinuity proportional to the scattering rate only for many-particle states with light-like total momenta. Scattering rates would vanish identically for the physical momenta for many-particle states.

   In TGD framework the states would be however massless in 8-D sense. Massless pole corresponds now to a continuum for M^4 mass squared and one would obtain the unitary cuts from a pole at $P^2 = 0$! Scattering rates would be non-vanishing only for many-particle states having light-like 8-momentum, which would pose a powerful condition on the construction of many-particle states. Single particle momenta cannot be however light-like for this kind of states unless they are parallel. They must be also complex as they indeed are already in classical TGD.

In fact, BCFW deformation $p_i \rightarrow p_i + zr_i$, $r_i \cdot r_j = 0$ creates at z-poles of the resulting amplitude pairs of zero energy states for which complex single particle momenta are not light-like but sum up to massless momentum. One can interpret these zero energy analogs of resonances, states inside CDs formed from massless external particles as they arrive to CD. This strong form of conformal symmetry has highly non-trivial implications concerning color confinement.

4. The key idea is number theoretical discretization [L35] in terms of "cognitive representations" as space-time time points with M^8-coordinates in an extension of rationals and therefore shared by both real and various p-adic sectors of the adele. Discretization realizes measurement resolution, which becomes an inherent aspect of physics rather than something forced by observed as outsider. This fixes the space-time surface completely as a zero locus of real or imaginary part of octonionic polynomial.

   This must imply the reduction of "world of classical worlds" (WCW) corresponding to a fixed number of points in the extension of rationals to a finite-dimensional discretized space with maximal symmetries and Kähler structure [K34, K15, K110].

   The simplest identification for the reduced WCW would be as complex Grassmannian - a more general identification would be as a flag manifold. More complex options can of course be considered. The Yangian symmetries of the twistor Grassmann approach known to act as diffeomorphisms respecting the positivity of Grassmannian and emerging also in its TGD variant would have an interpretation as general coordinate invariance for the reduced WCW. This would give a completely unexpected connection with supersymmetric gauge theories and TGD.

5. M^8 picture [L33] implies the analog of SUSY realized in terms of polynomials of super-octonions whereas H picture suggests that supersymmetry is broken in the sense that many-fermion states as analogs of components of super-field at partonic 2-surfaces are not local. This requires breaking of SUSY. At M^8 level the breaking could be due to the reduction of Galois group to its subgroup G/H, where H is normal subgroup leaving the point of cognitive representation defining space-time surface invariant. As a consequence, local many-fermion composite in M^8 would be mapped to a non-local one in H by $M^8 - H$ correspondence.

16.2 General view about the construction of scattering amplitudes in TGD framework

Before twistorial considerations a general vision about the basic principles of TGD and construction of scattering amplitudes in TGD framework is in order.

16.2.1 General principles behind S-matrix

Although explicit formulas for scattering amplitudes are probably too much to hope, one can try to develop a convincing general view about principles behind the S-matrix.

World of Classical Worlds

The first discovery was what I called the "world of classical worlds" (WCW) [K34, K15, K110] as a generalization of loop space allowing to replace path integral approach failing in TGD work. This led to a generalization of Einstein's geometrization program to an attempt to geometrize entire quantum physics. The geometry of WCW would be essentially unique from its mere existence since the existence of Riemann connection requires already in the case of loop spaces maximal isometries. Super-symplectic and super-conformal symmetries generalizing the 2-D conformal symmetries by replacing 2-D surfaces with light-like 3-surfaces (metrically 2-D!) would define the isometries.

Physical states would be classical spinor fields in the infinite-dimensional WCW and spinors at given point of WCW would be fermionic Fock states. Gamma matrices would be linear combinations of fermionic oscillator operators associated with the analog of massless Dirac equation at space-time surface determined by the variational principle whose preferred extremals the space-time surfaces are. Strong form of holography implied by strong form of general coordinate invariance would imply that it is enough to consider the restrictions of the induced spinor fields at string world sheets and partonic 2-surfaces (actually at discrete points at them defining the ends of boundaries of string world sheets) [K88, K110].

Zero Energy Ontology and generalization of quantum measurement theory to a theory of consciousness

The attempts to understand S-matrix led to the question about what does state function reduction really mean. This eventually led to the discovery of Zero Energy Ontology (ZEO) in which time=constant snapshot as a physical state is replaced with preferred extremal satisfying infinite number of additional gauge conditions [L39]. Temporal pattern becomes the fundamental entity: this conforms nicely with the view neuroscientists and computational scientists for whom behavior and program are basic notions. One can say that non-deterministic state function reduction replaces this kind time evolution with new one. One gets rid of the basic difficulty of ordinary quantum measurement theory.

Causal diamond (CD) is the basic geometric object of ZEO. The members of the state pair defining zero energy state - the analog of physical event characterized by initial and final states - have opposite total conserved quantum numbers and reside at the opposite light-like boundaries of CD being associated with 3-surfaces connected by a space-time surface, the preferred extremal. CDs form a fractal hierarchy ordered by their discrete size scale.

One ends up to a quite radical prediction: the arrow of time changes in "big" state function reduction changing the roles of active and passive boundaries of CD. The state function reductions occurring in elementary reactions represent an example of "big" state function reduction. The sequence of "small" state function reductions - analogs of so called weak measurements - defines self as a conscious entity having CD as imbedding space correlate [L39].

In ZEO based view about WCW 3-surfaces X^3 are pairs of 3-surfaces at boundaries of CD connected by preferred extremals of the action principle. WCW spinors are pairs of fermionic Fock states at these 3-surfaces and WCW spinor fields are WCW spinors depending on X^3 . They satisfy the analog of massless Dirac equation which boils down to the analogs of Super Virasoro conditions including also gauge conditions for a sub-algebra of super-symplectic algebra. S-matrix describing time evolution followed by "small" state function reduction relates two WCW spinor fields of this kind.

Generalization of twistor Grassmannian approach to TGD framework

Twistorial approach generalizes from M^4 to $H = M^4 \times CP_2$. One possible motivation could be the fact that ordinary twistor approach describes only scattering of massless particles. In the proposed generalization particles are massless in 8-D sense and in general massive in 4-D sense [K76, L22, L24, L38].

1. The existence of twistor lift of Kähler action as 6-D analog of Kähler action fixes the choice of H uniquely: only M^4 and CP_2 allow twistor space with Kähler structure. The 12-D product of the twistor spaces of M^4 and CP_2 induces twistor structure for 6-D surface X^6 under additional conditions guaranteeing that the X^6 is twistor space of 4-D surface X^4 (S^2 bundle over X^4) - its twistor lift. The conjecture that 6-D Kähler action indeed gives rise to twistor spaces of X^4 as preferred extremals.

2. This conjecture is the analog for Penrose's original twistor representation of Maxwellian fields reducing dynamics of massless fields to homology. There is also an analogy with massless fields. Dimensional reduction of Kähler action occurs for 6-surfaces, which represent twistor spaces and the external particles entering CD would be minimal surfaces defining simultaneous preferred extremals of Kähler action satisfying infinite number of additional gauge conditions. Minimal surfaces indeed satisfy generalization of massless field equations.

In the interior of CD defining interaction region there is a coupling to Kähler 4-force and one has analog of massless particle coupling to Maxwellian field.

3. 6-D Kähler action would give the preferred extremals via the analog of dimensional reduction essential for the twistor space property requiring that one has S^2 bundle over space-time surface. I have considered the generalization of the standard twistorial construction of scattering amplitudes of $\mathcal{N}=4$ SUSY to TGD context. In particular, the crucial Yangian invariance of the amplitudes holds true also now in both M^4 and CP_2 sectors.

4. Skeptic could argue that TGD generalization of twistors does not tell anything about the origin of the Yangian symmetry. During writing of this contribution I however realized that the hierarchy of Grassmannians realizing the Yangian symmetries could be seen as a hierarchy of reduced WCWs associated with the hierarchy of adeles defined by the hierarchy of extensions of rationals. The isometries of Grassmannian would emerge in the reduction of the isometry group of WCW to a finite-D isometry group of Grassmannian and would be caused by finite measurement resolution described number theoretically. Of course, one can consider also more general flag manifolds with Kähler property as candidates for the analogs of Grassmannians. I will represent the argument in more detail later.

   This could also relate to the postulated infinite hierarchy of hyper-finite factors of type II_1 (HFFs) [K87, K26] as a correlate for the finite measurement resolution with included sub-factor inducing transformations which act trivially in the measurement resolution used.

Remark: There is an amusing connection with empiria. Topologist Barbara Shipman observed that honeybee dance allows a description in terms of flag manifold $F = SU(3)/U(1) \times U(1)$, which is the space for the choices of quantization axes of color quantum numbers and also the twistor space in CP_2 degrees of freedom [A37]. This suggest that QCD type physics might make sense in macroscopic length scales. p-Adic length scale hypothesis and the predicted long range classical color gauge fields suggest a hierarchy of QCD type physics. One can indeed construct a TGD based model of honeybee dance with aconcrete interpretation and representation for the points of F at space-time level [L42].

M^8-H duality

M^8-H duality provides two equivalent manners to see the dynamics with either M^8 or $H = M^4\times CP_2$ as imbedding space [L33]. One might speak of number theoretic compactification which is a completely non-dynamical analog for spontaneous compactification.

1. In M^8 picture the space-time corresponds to a zero locus for either imaginary part $IM(P)$ or real part $RE(P)$ of octonionic polynomial ($RE(o)$ and $IM(o)$ are defined by the decomposition $o = RE(o) + I_4IM(o)$, where I_4 is octonion unit orthogonal to quaternionic subalgebra). The dynamics is purely algebraic and ultra-local.

2. At the level of H the dynamics is dictated by variational principle and partial differential equations. Space-time surfaces are preferred extremals of the twistor lift of Kähler action reduced to a sum of 4-D Kähler action and volume term analogous to cosmological term in GRT. The equivalence of these descriptions gives powerful constraints and should follow from the infinite number of gauge conditions at the level of H associated with a sub-algebra of supersymplectic algebra implying the required dramatic reduction of degrees of freedom [K15, K110]. One has a hierarchy of these sub-algebras, which presumably relates to the hierarchy of HFFs and hierarchy of extensions of rationals.

   H picture works very nicely in applications. For instance, the notions of field body and magnetic body are crucial in all applications.

The notion of quaternionicity, which is a central element of M^8-H duality has a deep connection with causality which I have not noticed earlier. At the level of momentum space quaternionicity means that 8-momenta -, which by M^8-H-duality correspond to 4-momenta at level of M^4 and color quantum numbers at the level of CP_2 - are quaternionic. Quaternionicity means that the time component of 8-momentum, which is parallel to real octonion unit, is non-vanishing. The 8-momentum itself must be time-like, in fact light-like. In this case one can always regard the momentum as momentum in some quaternionic sub-space. Causality requires a fixed sign for the time component of the momentum.

It must be however noticed that 8-momentum can be complex: also the 4-momentum can be complex at the level of $M \times CP_2$ already classically. A possible interpretation is in terms of decay width as part of momentum as it indeed is in phenomenological description of unstable particles.

Could one require that the quaternionic momenta form a linear space with respect to octonionic sum? This is the case if the energy - that is the time-like part parallel to the real octonionic unit - has a fixed sign. The sum of the momenta is quaternionic in this case since the sum of light-like momenta is in general time-like and in special case light-like. If momenta with opposite signs of energy are allowed, the sum can become space-like and the sum of momenta is co-quaternionic.

This result is technically completely trivial as such but has a deep physical meaning. Quaternionicity at the level of 8-momenta implies standard view about causality: only time-like or at most light-like momenta and fixed sign of time-component of momentum.

Adelic physics

The adelization of ordinary physics fusing real number based physics and various p-adic variants of physics in order to describe cognition.

1. Adelic physics [L35, L36] gives powerful number theoretic constraints when combined with $M^8 - H$ duality and leads to the vision about evolutionary hierarchy defined by extensions of rationals. The higher the level in the hierarchy, the higher the dimension n of the extension identified in terms of Planck constant $h_{eff}/h = n$ labelling the levels of dark matter hierarchy.

2. Adelic hypothesis allows to sharpen the strong form of holography to a statement that discrete cognitive representations consisting of a finite number of points identified as points of space-time surface with M^8 coordinates in the extension of rationals fixes the space-time surface itself. This dramatic reduction would be basically due to finite measurement resolution realized as an inherent property of dynamics. Cognitive representation in fact gives the WCW coordinates of the space-time surface in WCW! WCW reduces to a number theoretic discretization of a finite-dimensional space with Kähler structure and presumably maximal isometries.

3. In ZEO space-time surface becomes analogous to a computer program determined in terms of finite net of numbers! Of course, at the QFT limit of TGD giving standard model and GRT space-time is locally much more complex since one approximates the many-sheeted space-time with single slightly curved region of M^4. This is the price paid for getting rid (or losing) the topological richness of the many-sheeted space-time crucial for the understanding living matter and even physics in galactic scales.

4. Skeptic can argue that this discretization of WCW leads to the loss of WCW geometry based on real numbers. One can however consider also continuous values for the points of cognitive representations and assigning metric to the points of cognitive representation. Metric could be defined as kind of induced metric. One slices CD by parallel CDs by shift the CD along the axis connecting its tips. This allows to see the point of cognitive representation as point at one particular CD. One shifts slightly the point along its CD. Imbedding space metric allows to deduce the infinitesimal line element ds^2 and to deduce the metric components. This allows a definition of differential geometry so that the analog of WCW metric makes sense as a hierarchy of finite-dimensional metrics for space-time surfaces characterize by the cognitive representations.

   The interpretation in real context would be in terms of finite measurement resolution and the hierarchy would correspond to a hierarchy of hyper-finite factors (HFFs) [K87, K26], whose defining property is that they allow arbitrarily precise finite-dimensional approximations. What would be new is that the hierarchy of extensions of rationals would define a hierarchy of discretizations and hierarchy of HFFs.

Thabove list involves several unproven conjectures, which I can argue to be intuitively obvious with the experience of four decades: I cannot of course expect that a colleague reading for the first time about TGD would share these intuitions.

16.2.2 Classical TGD

Classical TGD is now rather well understood both in both $H = M^4 \times CP_2$ and M^8 pictures. Applications of classical TGD are in H picture and rather detailed phenomenology has emerged. M^8 picture has led to a rather precise vision about adelic physics and to understanding of finite measurement resolution.

Classical TGD in M^8 picture

Classical TGD in M^8 picture is discussed in [L33].

1. In M^8 picture one ends to an extremely simple number theoretic construction of space-time surfaces fixing only discrete or even finite number of space-time points to obtain space-time surface for a given extension of rationals. The reason is that space-time surfaces are zero loci for $RE(P)$ or $IM(P)$ of octonionic polynomials obtained by continuing real polynomial with coefficients in an extension of rationals to an octonionic polynomial.

   Needless to say, the hierarchy of algebraic extensions of rationals is what makes the dynamics at given level so simple. The coordinates of space-time surface as a point of WCW must be in the extension of rationals. As noticed, the points of space-time surface defining the cognitive representation determining the space-time surface serve as its natural WCW coordinates.

2. The highly non-trivial point is that no variational principle is involved with M^8 construction. Therefore it seems that neither WCW metric nor Kähler function is needed. If this is the case, the exponential of Kähler function definable as action exponential does not appear in scattering amplitudes and must disappear also at H-side from the scattering amplitudes.

3. Skeptic could argue that one loses general coordinate invariance in this approach. This is not true. Linear M^8 coordinates are the only possible option and forced already by symmetries. The choice octonionic and quaternionic structures fixes the linear M^8 coordinates almost uniquely since time direction is associated with real octonion unit and one spatial direction to special imaginary unit defining spin quantization axis. In algebraic approach identifying space-time surface as a zero locus of $RE(P)$ or $IM(P)$ these coordinates define space-time coordinates highly uniquely.

   Skeptic could also argue that number theoretic discretization implies reduction of the basic symmetry groups to their discrete sub-groups. This is true and one can argue that this loss of symmetry is due to the use of cognitive representations with finite resolution. Points with algebraic coordinates could be seen as a choices of representatives from a set of points, which are equivalent as far as measurement resolution is considered.

4. A physically important complication related to M^8 dynamics is the possibility of different octonionic and quaternionic structures. For instance, external particles arriving into CD correspond to different octonionic and quaternionic structures in general since Lorentz boost affects the octonionic structure changing the direction of time axis, which corresponds to the real octonionic unit. In color degrees of freedom one has wave function over different quaternionic structures: essentially color partial waves labelled by color quantum numbers [K39].

   One can apply Poincare transformations and color rotations (or transformation in sub-groups of these groups if one requires that the image points belong to the same extension) to the discrete cognitive representation defining space-time surface. The moduli spaces for these structures are essential for the understanding the standard Poincare and color quantum numbers and standard conservation laws in M^8 picture. Also the size scales of CDs define moduli as also Lorentz boosts leaving either boundary of CD unaffected.

Classical TGD in H picture

At the H side one action principle has partial differential equations and infinite number of gauge conditions associated with a sub-algebra of super-symplectic algebra selecting only extremely few preferred extremals of the action principle in terms of gauge conditions for a sub-algebra of super-symplectic algebra. This dynamics is conjectured to follow from the assumption that 6-D lift of space-time surface X^4 to a CP_1 bundle over X^4 is twistor space of X^4. This condition requires the analog of dimensional reduction since S^2 fiber is dynamically trivial.

For 6-D preferred extremals identifiable as twistor spaces of space-time surfaces the 6-D Kähler action in the product of twistor spaces of M^4 and CP_2 is assumed to dimensionally reduce to 4-D Kähler action plus volume term identifiable as the analog of cosmological constant term. This picture reproduces a description of scattering events highly analogous to that emerging in M^8. External particles correspond to minimal surfaces as analogs of free massless fields and all couplings disappear from the value of the action. The interior of CD corresponds to non-trivial coupling to Kähler 4-force which does not vanish. In M^8 picture one has associative and non-associative regions as counterparts of these regions.

What is remarkable is that the dynamics determined by partial differential equations plus gauge conditions would be equivalent with the number theoretic dynamics determined in terms of zero loci for real or imaginary parts of octonionic polynomials.

16.2.3 Scattering amplitudes in ZEO

The construction of scattering amplitudes even at the level of principle is far from well-understood. I have discussed rather concrete proposals for the twistorial construction but the feeling is that something is still missing [K76, L22, L24, L38]. This feeling might well reflect my quite too limited mathematical understanding of twistors and experience about practical construction of the scattering amplitudes. Later I will discuss possible identification of the missing piece of puzzle.

Consider first the general picture about the construction of scattering amplitudes suggested by ZEO inspired theory of quantum measurement theory defining also a theory of consciousness.

1. The portions of space-time surfaces outside CD correspond to external particles. They satisfy associativity conditions at M^8 side making possible to map them to minimal surfaces in $H = M^4 \times CP_2$ satisfying various infinite number of gauge conditions for a sub-algebra of super-symplectic algebra isomorphic with it.

   Remark: There is an additional condition requiring that associative tangent space or normal space contains fixed complex subspace of quaternions. It is not quite clear whether this condition can be generalized so that the distribution of these spaces is integrable.

   At both sides the dynamics of external particles is in a well-defined sense critical at both sides and does not depend at all on coupling constants.

2. Inside CDs associativity conditions break down in M^8 and one cannot map this spacetime region - call it X^4 - to H [L33]. It is however possible to construct counterpart of X^4 in H as a preferred extremal for the twistor lift of Kähler action by fixing the 3-surfaces at the boundaries of CD (boundary conditions). The dependence on couplings at the level of H would come from the vanishing conditions for classical Noether charges, which depend on coupling parameters.

3. If the two descriptions of the scattering amplitudes are equivalent, the dependence on coupling parameters in H should have a counterpart in M^8. Coupling constants making sense only at H side are expected to depend on the size scale of CD and on the extension of rationals defining the adele [L35, L36]. Coupling constants should be determined completely by the boundary values of Noether charges at the ends of space-time surface, and therefore by the 3-D ends of associative space-time regions representing external particles at M^8 side. This would suggest that coupling constants are functions of the coefficients of the polynomials and the points of cognitive representation.

Zero energy ontology and the life cycle of self

ZEO meant a decisive step in the understanding of quantum TGD since it solved the basic paradox of quantum measurement problem by forcing to realize that subjective and geometric time are not the same thing [L39].

1. Both the passive boundary of CD and the members of state pairs at it are unaffected during the sequence of state reductions analogous to weak measurements (see http://tinyurl.com/zt36hpb) defining self as a generalized Zeno effect. The members of state pairs associated with the active boundary change and the active boundary itself drifts farther away from the passive one in the sequence of "small" state function reductions.

   Also the space-time surfaces connecting passive and active boundaries change during the sequence of weak measurements. Only the 3-surfaces at the passive boundary are unaffected. Hence the geometric past relative to the active boundary changes during the life cycle of self. In positive energy ontology (PEO) this is not possible.

2. In "big" state function reduction the roles of passive and active boundary are changed and the arrow of time identifiable as the direction in which CD grows changes. In consciousness theory "big" state function reduction corresponds to the death of self and subsequent re-incarnations as a self with an opposite arrow of geometric time.

3. In ZEO the life cycle of self corresponds to a sequence of steps. Single step begins with a unitary time evolution in which a superposition of states associated with CDs larger than the original CD emerges. Then follows the analog of weak measurement leading to a localization to a CD in the moduli space of CDs so that it has a fixed and in general larger size. A measurement of geometric time occurs and gives rise to an experience about the flow of time.

   This option would allow to identify the total S-matrix as a product of the S-matrices associated with various steps in spirit with the interpretation as a generalized Zeno effect.

   Remark: In the usual description one fixes the time interval to which one assigns the S-matrix. There is no division to steps giving rise to the experience of time flow.

4. The measurement of geometric time would be a partial measurement reducing more general unitary time evolution to a unitary time evolution in the standard sense. Can one generalize the notion of partial measurement to other observables so that one would still have unitary time evolution albeit in more restricted sense? Or should one consider giving up the unitary time evolution?

   These observables should commute with the observables having the states at passive boundary as eigenstates: otherwise the state at passive boundary would change. If this picture makes sense, the "big" reduction to the opposite boundary meaning the death of self would necessarily occur when all observables commuting with the eigen observables at the passive boundary have been measured. It could of course occur already earlier.

   Should one allow measurements of all observables commuting with the eigen observables at the passive boundary. This would lead to partial de-coherence of the zero energy state. In TGD inspired quantum biology this could allow to understand ageing as an unavoidable gradual loss of the quantum coherence.

More detailed interpretation of ZEO

There are several questions related to the detailed interpretation of ZEO. The intuitive picture is that inside CD representing self one has collection of sub-CDs representing sub-selves identified as mental images of self. On can loosely say, that sub-CDs represent mind. The sub-CDs are connected by on mass shell lines, which correspond to external particles - matter. Sub-CDs can also have sub-CDs and the hierarchy can have several levels.

The states at the boundaries of CD have opposite total quantum numbers. One can consider two interpretations.

1. In positive energy ontology (PEO) the notion of zero energy state could be seen only as an elegant manner to express conservation laws. This is done in QFT quite generally - also in twistor approach. Also the largest CD would have external particles emanating from its boundaries travelling to the geometric past and future. One would have however have only information about the interior of the CD possessed by conscious entity for which CD plus its sub-CDs (mental images) serve as correlates.

In this picture the arrow of time is fixed since it must be same for all sub-CDs in order to void inconsistency with the basic idea about self as generalized Zeno effect realized as a sequence of weak measurements.

2. ZEO suggest a more radical interpretation. Zero energy state defines an event. There would be the largest CD defining self and sub-CDs would correspond to mental images. There would be no external particles emanating from the boundaries of the largest CD. In this framework it becomes possible to speak about the death of self as the first state function reduction to the opposite boundary changing the roles of active and passive boundaries of self.

This picture should be consistent with what we know about arrow of time and in TGD framework with the idea that the arrow of time can also change - in particular in living matter.

1. How would the standard arrow of time emerge in ZEO? One could see the emergence of the global arrow of geometric time as a process in which the size of the largest CD increases: the sub-CDs are forced to have the same arrow of time as the largest CD and cannot make state function reductions on opposite boundary (die) independently of it. During evolution the size of the networks with the same arrow of geometric time increases and fixed arrow of geometric time is established in longer scales.

2. This picture cannot be quite correct. The applications of TGD inspired consciousness require that the mental images of self can have arrow of geometric time opposite to that of self. For instance, motor actions could be sensory perceptions in non-standard arrow of time. Memory could be communications with brain of geometric past - seeing in time direction - involving signals to geometric past requiring temporary reversals of the arrow of time at some level of self-hierarchy. Hence space-time regions with different arrows of time but forming a connected space-time surface ought to be possible.

   Many-sheeted space-time means a hierarchy of space-time sheets connected by what I call wormhole contacts having Euclidian signature of the induced metric. Space-time sheets at different levels of the hierarchy are not causally connected in the sense that one cannot speak of signal propagation in the regions of Euclidian signature. This suggests that the space-time sheets connected by wormhole contacts can have different arrows of geometric time and are associated with their own CDs.

   In this manner one would avoid the paradox resulting when sub-self - mental image - dies so that its passive boundary becomes active and the particles emanating from it end up to the passive boundary of CD, where no changes are allowed during the life cycle of self. If the particles emanating from time-reversed sub-self and up to boundaries of parallel CD, the problem is circumvented.

3. Wormhole contacts induce an interaction between Minkowskian space-time sheets that they connect. The interaction is not mediated by classical signals but by boundary conditions at the boundaries between Minkowskian regions and Euclidian wormhole contact. These two boundaries are light-like orbits of opposite wormhole throats (partonic 2-surfaces).

   In number theoretic picture the presence of wormhole contact is reflected in the properties set of points in extension of rationals defining the cognitive representation in turn defining the space-time surface. In particular, the points associated with wormhole contact have space-like distance although they are at opposite boundaries of CD and have time-like distance in the metric of imbedding space. This kind of point pairs associated with wormhole contacts serve serve as a tell-tale signature for them.

16.3 The counterpart of the twistor approach in TGD

The analogs of twistor diagrams could emerge in TGD [L22, L38] in the following manner in ZEO.

1. Portions of space-time surfaces inside CDs would appear as analogs of vertices and the spacetime surfaces connecting them as analogs of propagator lines. The "lines" connecting sub-CDs would carry massless on mass shell states but possibly with complex momenta

analogous to those appearing in twistor diagrams. This is true also classically at level of H: the coupling constants appearing in the action defining classical dynamics - at least Kähler coupling strength - are complex so that also conserved quantities have also imaginary parts.

Remark: At the level of M^8 one does not have action principle and cannot speak of Noether charges. Here the conserved charged are associated with the symmetries of the moduli spaces such as the moduli spaces for octonion and quaternion structures [L33]. The identification of the classical charges in Cartan algebra at H level with the quantum numbers labeling wave functions in moduli space at M^8 level could be seen as a realization of quantum classical correspondence.

2. At space-time level the vertices of twistor diagrams correspond to partonic 2-surfaces in the interior of given CD. In H description fermionic lines along the light-like orbits of partonic 2-surfaces scatter at partonic 2-surfaces. If each partonic 2-surface defining a vertex is surrounded by a sub-CD, these two views about TGD variants of twistor diagrams are unified. Sub-CD can of course contain more complex structures such as pair of wormhole contacts assignable to an elementary particle.

16.3.1 Could the classical number theoretical dynamics define the hard core of the scattering amplitudes?

The natural hope is that the simple picture about classical dynamics at the level of M^8 should have similar counterpart at the level of scattering amplitudes in M^8. The above arguments suggest that the scattering diagrams correspond to CDs connected by external particle lines representing on mass shell particles. These surfaces are associative at the level of M^8 and minimal surfaces at the level of H. This suggests that scattering amplitude for single CD serves as a building brick for scattering amplitudes: the rest would be "just kinematics" dictated by the enormous symmetries of WCW.

1. Everything in the construction should reduce to a hard core around which one would have integrations (or sums for number theoretic realization of finite measurement resolution) over various moduli characterizing the standard quantum numbers. Twistors for M^4 and CP_2 and the moduli for the choices of CDs should correspond to essentially kinematic contribution involving no genuine dynamics.

2. The scattering amplitudes should make sense in all sectors of adele. This poses powerful constraints on them. The exponential of Kähler function reducing to action exponential can in principle appear in the description at H-side but cannot be present at M^8 side. Therefore it should disappear also at the level of H.

   If the scattering amplitude at the level of H is sum over contributions with the same value of the action exponential, the exponentials indeed cancel and I have proposed that this condition holds true. In perturbative quantum field theory it holds practically always and in integrable theories is exact. This would mean enormous simplification since all information about the action principle in H would appear in the vanishing conditions for the Noether charges of the subalgebra of super-symplectic algebra at the ends of the space-time surface. These Noether changes indeed depend on the action principle and thus on coupling constants.

3. Could the hard core in the construction of the scattering amplitudes be just the choice of the cognitive representation as points if M^8 belonging to the algebraic extension defining the adele and determining space-time surface in terms of octonionic polynomial inside this CD defining the interaction region?

   The set of points of extension of rationals in the cognitive representation defines space-time surface and also its WCW coordinates. The restriction to a cognitive representation with given number of points in given extension of rationals would mean a reduction of WCW to a finite-dimensional sub-space.

   The first wild guess is that this space is Kähler manifold with maximal symmetries - just as WCW is. A further wild guess is that these reduced WCWs are Grassmannians and correspond to those appearing in the twistor Grassmannian approach. A more general conjecture is inspired by the vision that super-symplectic gauge conditions effectively reduce the super-symplectic algebra to a Kac-Moody algebra of a finite-dimensional Lie group - perhaps belonging to ADE hierarchy. The flag manifolds associated with these Lie groups define more general homogenous spaces as candidates for the reduced WCWs.

4. One must allow the action of Galois group and this gives several options for given set X of points in algebraic extension.

   (a) One can construct $X^4(X)$ in terms of octonionic polynomial and construct a representation of Galois group as superposition of space-time surfaces obtained from space-time surface by the action of Galois group on X giving rise to new sets $X_g = g(X)$.

   (b) One can also consider the action of Galois group on X and get larger set Y of points and construct single multi-sheeted surface $X^4(Y)$. This surface corresponds to Planck constant $h_{eff}/h = n$, where n is the dimension of algebraic extension.

   (c) One can also consider the actions of sub-groups of $H \subset Gal$ to X to get space-time surface with $h_{eff}/h = m$ dividing n. There are several options corresponding to representations for all sub-groups of Galois group. A hierarchy of symmetry breakings seems to be involved with unbroken symmetry associated with the largest value of h_{eff}/h.

5. In this picture the hard core would reduce to the classical number theoretical dynamics of space-time surface in M^8. The additional degrees of freedom would be due to the possibility of different octonionic and quaternionic structures and choices of size scales and Lorentz boosts and translations of CDs. The symmetries would dictate the S-matrix in the moduli degrees of freedom: the dream is that this part of the dynamics reduces to kinematics, so to say.

   The discrete coupling constant evolution would be determined by the hierarchy of extensions of rationals and by the hierarchy of p-adic length scales. The cancellation of radiative corrections in the sense of sub-CDs inside CDs could be achieved by replacing coupling constant evolution with its discrete counterpart.

If this dream has something to do with reality, the construction of scattering amplitudes would reduce to their construction in moduli degrees of freedom and here the generalization of twistorial approach relying on Yangian symmetry allowing to identify scattering amplitudes as Yangian invariants might "trivialize" the situation. It will be found that the Yangian symmetry could corresponds to general coordinate transformations for the reduced WCW forced by the restriction of the spacetime surfaces to those allowed by octonionic polynomials with coefficients in the extensiom of rationals.

16.3.2 Do loop contributions to the scattering amplitudes vanish in TGD framework?

In TGD scattering amplitudes interpreted as zero energy states would correspond at imbedding space level to collections of space-time surfaces inside CDs analogous to vertices and connected by lines defined by the space-time surfaces representing on-mass-shell particles. One would have massless particles in 8-D sense. The quaternionicity of 8-momentum leads to $M^4 \times CP_2$ picture and CP_2 twistors should replace E^4 twistors of M^8 approach.

Why loop corrections should vanish?

There are several arguments suggesting that the loop contributions should vanish in TGD framework. This would give rise to a discrete coupling constant evolution analogous to a sequence of phase transitions between different critical coupling parameters. Amplitudes would be obtained as tree diagrams.

1. In ZEO it is far from clear what the basic operation defining the loop contribution could even mean. One would have zero energy state for which the members of added particle pair have

opposite but momenta but the amplitude is superposition of states with varying momenta. Why should one allow zero energy states containing one particle which is not an eigenstate of momentum? This suggests that ZEO does not allow loop contributions at all: the distinction between PEO and ZEO would make itself visible in rather dramatic manner.

2. The restriction of the BCFW to tree diagrams is internally consistent since the loop term is identically vanishing in this case. The first term in the BCFW for diagram with l loops involves a factor with $l > 0$ loops which vanishes. In $l = 1$ case the second term is obtained from $(n + 2, l - 1 = 0)$ diagram by generating loop but this vanishes by assumption.

3. Number theoretic vision does not favor the decomposition of the amplitude to an infinite sum of amplitudes since this is expected to lead to the emergence of transcendental numbers and functions in the amplitude in conflict with the number theoretical universality.

   Loops indeed give logarithms and poly-logarithms of rational functions of external momenta in Grassmannian approach. This violates the number theoretical universality since the p-adic counterpart of logarithm exist only for the argument of form $x = 1 + O(p)$. This condition cannot hold true for all primes simultaneously.

Discrete coupling constant evolution suggests the vanishing of loops. One can imagine two alternative mechanisms for the vanishing of loop contributions. Either the loop contributions do not make sense at all in ZEO, or the sum of loop contributions for the critical values of coupling constants vanishes. The summing up of loop contributions to zero for critical values of couplings should happen for all values of external momenta and other quantum numbers: this does not look plausible.

General number theoretic ideas about coupling constant evolution

The discrete coupling constant evolution would be associated with the scale hierarchy for CDs and the hierarchy of extensions of rationals.

1. Discrete p-adic coupling constant evolution would naturally correspond to the dependence of coupling constants on the size of CD. For instance, I have considered a concrete but rather ad hoc proposal for the evolution of Kähler couplings strength based on the zeros of Riemann zeta [L16]. Number theoretical universality suggests that the size scale of CD identified as the temporal distance between the tips of CD using suitable multiple of CP_2 length scale as a length unit is integer, call it l. The prime factors of the integer could correspond to preferred p-adic primes for given CD.

2. I have also proposed that the so called ramified primes of the extension of rationals correspond to the physically preferred primes. Ramification is algebraically analogous to criticality in the sense that two roots understood in very general sense co-incide at criticality. Could the primes appearing as factors of l be ramified primes of extension? This would give strong correlation between the algebraic extension and the size scale of CD.

In quantum field theories coupling constants depend in good approximation logarithmically on mass scale, which would be in the case of p-adic coupling constant evolution replaced with an integer n characterizing the size scale of CD or perhaps the collection of prime factors of n (note that one cannot exclude rational numbers as size scales). Coupling constant evolution could also depend on the size of extension of rationals characterized by its order and Galois group.

In both cases one expects approximate logarithmic dependence and the challenge is to define "number theoretic logarithm" as a rational number valued function making thus sense also for p-adic number fields as required by the number theoretical universality.

1. Coupling constant evolution with respect to CD size scale

Consider first the coupling constant as a function of the length scale $l_{CD}(n)/l_{CD}(1) = n$.

1. The number $\pi(n)$ of primes $p \leq n$ behaves approximately as $\pi(n) = n/log(n)$. This suggests the definition of what might be called "number theoretic logarithm" as $Log(n) \equiv n/\pi(n)$. Also iterated logarithms such $log(log(x))$ appearing in coupling constant evolution would have number theoretic generalization.

2. If the p-adic variant of $Log(n)$ is mapped to its real counterpart by canonical identification involving the replacement $p \to 1/p$, the behavior can very different from the ordinary logarithm. $Log(n)$ increases however very slowly so that in the generic case one can expect $Log(n) < p_{max}$, where p_{max} is the largest prime factor of n, so that there would be no dependence on p for p_{max} and the image under canonical identification would be number theoretically universal.

   For $n = p^k$, where p is small prime the situation changes since $Log(n)$ can be larger than small prime p. Primes p near primes powers of 2 and perhaps also primes near powers of 3 and 5 - at least - seem to be physically special. For instance, for Mersenne prime $M_k = 2^k - 1$ there would be dramatic change in the step $M_k \to M_k + 1 = 2^k$, which might relate to its special physical role.

3. One can consider also the analog of $Log(n)$ as

$$Log(n) = \sum_p k_p Log(p) \ ,$$

   where p^{k_i} is a factor of n. $Log(n)$ would be sum of number theoretic analogs for primes factors and carry information about them.

   One can extend the definition of $Log(x)$ to the rational values $x = m/n$ of the argument. The logarithm $Log_b(n)$ in base $b = r/s$ can be defined as $Log_b(x) = Log(x)/Log(b)$.

4. For $p \in \{2, 3, 5\}$ one has $Log(p) > log(p)$, where for larger primes one has $Log(p) < log(p)$. One has $Log(2) = 2 > log(2) = .693...$, $Log(3) = 3k/2 > log(3) = 1.099$, $Log(5) = 5/3 = 1.666.. > log(5) = 1.609$. For $p = 7$ one has $Log(7) = 7/4 \simeq 1.75 < log(7) \simeq 1.946$. Hence these primes and CD size scales n involving large powers of $p \in \{2, 3, 5\}$ ought to be physically special as indeed conjectured on basis of p-adic calculations and some observations related to music and biological evolution [K48, K51, K59, K113].

   In particular, for Mersenne primes $M_k = 2^k - 1$ one would have $Log(M_k) \simeq klog(2)$ for large enough k. For $Log(2^k)$ one would have $k \times Log(2) = 2k > log(2^k) = klog(2)$: there would be sudden increase in the value of $Log(n)$ at $n = M_k$. This jump in p-adic length scale evolution might relate to the very special physical role of Mersenne primes strongly suggested by p-adic mass calculations [K39].

5. One can wonder whether one could replace the $log(p)$ appearing as a unit in p-adic negentropy [K41] with a rational unit $Log(p) = p/\pi(p)$ to gain number theoretical universality? One could therefore interpret the p-adic negentropy as real or p-adic number for some prime. Interestingly, $|Log(p)|_p = 1/p$ approaches zero for large primes p (eye cannot see itself!) whereas $|Log(p)|_q = 1/|\pi(p)|_q$ has large values for the prime power factors q^r of $\pi(p)$.

2. The dependence of $1/\alpha_K$ on the extension of rationals

Consider next the dependence on the extension of rationals. The natural algebraization of the problem is to consider the Galois group of the extension.

1. Consider first the counterparts of primes and prime factorization for groups. The counterparts of primes are simple groups, which do not have normal subgroups H satisfying $gH = Hg$ implying invariance under automorphisms of G. Simple groups have no decomposition to a product of sub-groups. If the group has normal subgroup H, it can be decomposed to a product $H \times G/H$ and any finite group can be decomposed to a product of simple groups.

   All simple finite groups have been classified (see `http://tinyurl.com/jn44bxe`). There are cyclic groups, alternating groups, 16 families of simple groups of Lie type, 26 sporadic groups. This includes 20 quotients G/H by a normal subgroup of monster group and 6 groups which for some reason are referred to as pariahs.

2. Suppose that finite groups can be ordered so that one can assign number $N(G)$ to group G. The roughest ordering criterion is based on $ord(G)$. For given order $ord(G) = n$ one has all

groups, which are products of cyclic groups associated with prime factors of n plus products involving non-Abelian groups for which the order is not prime. $N(G) > ord(G)$ thus holds true. For groups with the same order one should have additional ordering criteria, which could relate to the complexity of the group. The number of simple factors would serve as an additional ordering criterion.

If its possible to define $N(G)$ in a natural manner then for given G one can define the number $\pi_1(N(G))$ of simple groups (analogs of primes) not larger than G. The first guess is that that the number $\pi_1(N(G))$ varies slowly as a function of G. Since Z_i is simple group, one has $\pi_1(N(G)) \geq \pi(N(G))$.

3. One can consider two definitions of number theoretic logarithm, call it Log_1.

$$\begin{array}{ll} a) & Log_1(N(G)) = \frac{N(G)}{\pi_1(N(G))} \quad , \\ b) & Log_1(G) = \sum_i k_i Log_1(N(G_i)) \quad , \quad Log_1(N(G_i)) = \frac{N(G_i)}{\pi_1(N(G_i))} \quad . \end{array} \tag{16.3.1}$$

Option a) does not provide information about the decomposition of G to a product of simple factors. For Option b) one decomposes G to a product of simple groups G_i: $G = \prod_i G_i^{k_i}$ and defines the logarithm as Option b) so that it carries information about the simple factors of G.

4. One could organize the groups with the same order to same equivalence class. In this case the above definitions would give

$$\begin{array}{ll} a) & Log_1(ord(G)) = \frac{ord(G)}{\pi_1(ord(G))} < Log(ord(G)) \quad , \\ b) & Log_1(ord(G)) = \sum_i k_i Log(ord(G_i)) \quad , \quad Log_1(ord(G_i)) = \frac{ord(G_i)}{\pi_1(ord(G_i))} \quad . \end{array} \tag{16.3.2}$$

Besides groups with prime orders there are non-Abelian groups with non-prime orders. The occurrence of same order for two non-isomorphic finite simple groups is very rare (see `http://tinyurl.com/ydd6uomb`). This would suggests that one has $\pi_1(ord(G)) < ord(G)$ so that $Log_1(ord(G))/ord(G) < 1$ would be true.

5. For orders $n(G) \in \{2,3,5\}$ one has $Log_1(n(G)) = Log(n(G)) > log(n(G))$ so that the ordes $n(G)$ involving large factors of $p \in \{2,3,5\}$ would be special also for the extensions of rationals. S_3 with order 6 is the first non-abelian simple group. One has $\pi(S_3) = 4$ giving $Log(6) = 6/4 = 1.5 < log(6) = 1.79$ so that S_3 is different from the simple groups below it.

To sum up, number theoretic logarithm could provide answer to the long-standing question what makes Mersenne primes and also other small primes so special.

Considerations related to coupling constant evolution and Riemann zeta

I have made several number theoretic peculations related to the possible role of zeros of Riemann zeta in coupling constant evolution. The basic problem is that it is not even known whether the zeros of zeta are rationals, algebraic numbers or genuine transcendentals or belong to all these categories. Also the question whether number theoretic analogs of ζ defined for p-adic number fields could make sense in some sense is interesting.

1. Is number theoretic analog of ζ possible using $Log(p)$ instead of $log(p)$?

The definition of $Log(n)$ based on factorization $Log(n) \equiv \sum_p k_p Log(p)$ allows to define the number theoretic version of Riemann Zeta $\zeta(s) = \sum n^{-s}$ via the replacement $n^{-s} = exp(-log(n)s) \rightarrow exp(-Log(n)s)$.

1. In suitable region of plane number-theoretic Zeta would have the usual decomposition to factors via the replacement $1/(1-p^{-s}) \rightarrow 1/(1-exp(-Log(p)s)$. p-Adically this makes sense for $s=O(p)$ and thus only for a finite number of primes p for positive integer valued s: one obtains kind of cut-off zeta. Number theoretic zeta would be sensitive only to a finite number of prime factors of integer n.

2. This might relate to the strong physical indications that only a finite number of cognitive representations characterized by p-adic primes are present in given quantum state: the ramified primes for the extension are excellent candidates for these p-adic primes. The size scale n of CD could also have decomposition to a product of powers of ramified primes. The finiteness of cognition conforms with the cutoff: for given CD size n and extension of rationals the p-adic primes labelling cognitive representations would be fixed.

3. One can expand the regions of converge to larger p-adic norms by introducing an extension of p-adics containing e and some of its roots (e^p is automatically a p-adic number). By introducing roots of unity, one can define the phase factor $exp(-iLog(n)Im(s))$ for suitable values of $Im(s)$. Clearly, $exp(-ipIm(s))/\pi(p))$ must be in the extension used for all primes p involved. One must therefore introduce prime roots $exp(i/\pi(p))$ for primes appearing in cutoff. To define the number theoretic zeta for all p-adic integer values of $Re(s)$ and all integer values of $Im(s)$, one should allow all roots of unity ($ep(i2\pi/n)$) and all roots $e^{1/n}$: this requires infinite-dimensional extension.

4. One can thus define a hierarchy of cutoffs of zeta: for this the factorization of Zeta to a finite number of "prime factors" takes place in genuine sense, and the points $Im(s)=ik\pi(p)$ give rise to poles of the cutoff zeta as poles of prime factors. Cutoff zeta converges to zero for $Re(s) \rightarrow \infty$ and exists along angles corresponding to allowed roots of unity. Cutoff zeta diverges for $(Re(s)=0, Im(s)=ik\pi(p))$ for the primes p appearing in it.

Remark: One could modify also the definition of ζ for complex numbers by replacing $exp(log(n)s)$ with $exp(Log(n)s)$ with $Log(n)=\sum_p k_p Log(p)$ to get the prime factorization formula. I will refer to this variant of zeta as modified zeta ($\tilde{\zeta}$) below. $\tilde{\zeta}$ would carry explicit number theoretic information via the dependence of its "prime factors" $1/(1-exp(-Log(p)s))$.

2. Could the values of $1/\alpha_K$ be given as zeros of ζ or of $\tilde{\zeta}$

In [L16] I have discussed the possibility that the zeros $s=1/2+iy$ of Riemann zeta at critical line correspond to the values of complex valued Kähler coupling strength α_K: $s=i/\alpha_K$. The assumption that p^{iy} is root of unity for some combinations of p and y [$log(p)y=(r/s)2\pi$] was made. This does not allow s to be complex rational. If the exponent of Kähler action disappears from the scattering amplitudes as M^8-H duality requires, one could assume that s has rational values but also algebraic values are allowed.

1. If one combines the proposed idea about the Log-arithmic dependence of the coupling constants on the size of CD and algebraic extension with $s=i/\alpha_K$ hypothesis, one cannot avoid the conjecture that the zeros of zeta are complex rationals. It is not known whether this is the case or not. The rationality would not have any strong implications for number theory but the existence irrational roots would have (see `http://tinyurl.com/y8bbnhe3`). Interestingly, the rationality of the roots would have very powerful physical implications if TGD inspired number theoretical conjectures are accepted.

   The argument discussed below however shows that complex rational roots of zeta are not favored by the observations [A66] about the Fourier transform for the characteristic function for the zeros of zeta. Rather, the findings suggest that the imaginary parts [L15] should be rational multiples of 2π, which does not conform with the vision that $1/\alpha_K$ is algebraic number. The replacement of $log(p)$ with $Log(p)$ and of 2π with is natural p-adic approximation in an extension allowing roots of unity however allows $1/\alpha_K$ to be an algebraic number. Could the spectrum of $1/\alpha_K$ correspond to the roots of ζ or of $\tilde{\zeta}$?

2. A further conjecture discussed in [L16] was that there is 1-1 correspondence between primes $p \simeq 2^k$, k prime, and zeros of zeta so that there would be an order preserving map $k \rightarrow s_k$. The

support for the conjecture was the predicted rather reasonable coupling constant evolution for α_K. Primes near powers of 2 could be physically special because $Log(n)$ decomposes to sum of $Log(p)$:s and would increase dramatically at $n = 2^k$ slightly above them.

In an attempt to understand why just prime values of k are physically special, I have proposed that k-adic length scales correspond to the size scales of wormhole contacts whereas particle space-time sheets would correspond to $p \simeq 2^k$. Could the logarithmic relation between L_p and L_k correspond to logarithmic relation between p and $\pi(p)$ in case that $\pi(p)$ is prime and could this condition select the preferred p-adic primes p?

3. The argument of Dyson for the Fourier transform of the characteristic function for the set of zeros of ζ

Consider now the argument suggesting that the roots of zeta cannot be complex rationals. On basis of numerical evidence Dyson [A66] (`http://tinyurl.com/hjbfsuv`) has conjectured that the Fourier transform for the characteristic function for the critical zeros of zeta consists of multiples of logarithms $log(p)$ of primes so that one could regard zeros as one-dimensional quasi-crystal.

This hypothesis makes sense if the zeros of zeta decompose into disjoint sets such that each set corresponds to its own prime (and its powers) and one has $p^{iy} = U_{m/n} = exp(i2\pi m/n)$ (see the appendix of [L15]). This hypothesis is also motivated by number theoretical universality [K111, L35].

1. One can re-write the discrete Fourier transform over zeros of ζ at critical line as

$$f(x) = \sum_y exp(ixy)) \ , \ y = Im(s) \ .$$

The alternative form reads as

$$f(u) = \sum_s u^{iy} \ , \ u = exp(x) \ .$$

$f(u)$ is located at powers p^n of primes defining ideals in the set of integers.

For $y = p^n$ one would have $p^{iny} = exp(inlog(p)y)$. Note that $k = nlog(p)$ is analogous to a wave vector. If $exp(inlog(p)y)$ is root of unity as proposed earlier for some combinations of p and y, the Fourier transform becomes a sum over roots of unity for these combinations: this could make possible constructive interference for the roots of unity, which are same or at least have the same sign. For given p there should be several values of $y(p)$ with nearly the same value of $exp(inlog(p)y(p))$ whereas other values of y would interfere deconstructively.

For general values $y = x^n$ $x \neq p$ the sum would not be over roots of unity and constructive interference is not expected. Therefore the peaking at powers of p could take place. This picture does not support the hypothesis that zeros of zeta are complex rational numbers so that the values of $1/\alpha_K$ correspond to zeros of zeta and would be therefore complex rationals as the simplest view about coupling constant evolution would suggest.

Remark: Mumford has argued (`http://tinyurl.com/zemw27o`) that the Fourier transform should include also the trivial zeros at $s = -2, -4, -6...$ giving and exponentially small contributions and providing a slowly varying background to the Fourier transform.

2. What if one replaces $log(p)$ with $Log(p) = p/\pi(p)$, which is rational and thus ζ with $\tilde{\zeta}$? For large enough values of p $Log(p) \simeq log(p)$ finite computational accuracy does not allow distinguish $Log(p)$ from $log(p)$. For $Log(p)$ one could thus understand the finding in terms of constructive interference for the roots of unity if the roots of zeta are of form $s = 1/2 + i(m/n)2\pi$. The value of y cannot be rational number and $1/\alpha_K$ would have real part equal to y proportional to 2π which would require infinite-D extension of rationals. In p-adic sectors infinite-D extension does not conform with the finiteness of cognition.

Remark: It is possible to check by numerical calculations whether the locus of complex zeros of $\tilde{\zeta}$ is at line $Res(2) = 1/2$. If so, then Fourier transform would make sense. One can also check whether the peaks at $nlog(p)$ are shifted to $nLog(p)$: for $p = 2$ one would have $Log(2) = 2 > log(2)$. The positions of peaks should shift to the right for $p = 2, 3, 5$ and to the left for $p > 5$. This should be easy to check by numerical calculations.

3. Numerical calculations have however finite accuracy, and allow also the possibility that y is algebraic number approximating rational multiple of 2π in some natural manner. In p-adic sectors would obtain the spectrum of y and $1/\alpha_K$ as algebraic numbers by replacing 2π in the formula $is = \alpha_K = i/2 + q \times 2\pi$, $q = r/s$, with its approximate value:

$$2\pi \rightarrow sin(2\pi/n)n = i\frac{n}{2}(exp(i2\pi/n) - exp(-i2\pi/n))$$

for an extension of rationals containing n:th of unity. Maximum value of n would give the best approximation. This approximation performed by fundamental physics should appear in the number theoretic scattering amplitudes in the expressions for $1/\alpha_K$ to make it algebraic number.

y can be approximated in the same manner in p-adic sectors and a natural guess is that $n = p$ defines the maximal root of unity as $exp(i2\pi/p)$. The phase $exp(ilog(p)y)$ for $y = qsin(2\pi/n(y))$, $q = r/s$, is replaced with the approximation induced by $log(p) \rightarrow Log(p)$ and $2\pi \rightarrow sin(2\pi/n)n$ giving

$$exp(ilog(p)y) \rightarrow exp(iq(y)sin(2\pi/n(y))\frac{p}{\pi(p)}) \quad .$$

If s in $q = r/s$ does not contain higher powers of p, the exponent exists p-adically for this extension and can can be expanded in positive powers of p as

$$\sum_n i^n q^n sin(2\pi/p)^n (p/\pi(p))^n \quad .$$

This makes sense p-adically.

Also the actual complex roots of ζ could be algebraic numbers:

$$s = i/2 + q \times sin(\frac{2\pi}{n(y)})n(y) \quad .$$

If the proposed correlation between p-adic primes $p \simeq 2^k$, k prime and zeros of zeta predicting a reasonable coupling constant evolution for $1/\alpha_K$ is true, one can have naturally, $n(y) = p(y)$, where p is the p-adic prime associated with y: the accuracy in angle measurement would increase with the size scale of CD. For given p there could be several roots y with same $p(y)$ but different $q(y)$ giving same phases or at least phases with same sign of real part.

Whether the roots of $tilde\zeta$ are algebraic numbers and at critical line $Re(s) = 1/2$ is an interesting question.

Remark: This picture allows many variants. For instance, if one assumes standard zeta, one could consider the possibility that the roots y_p associated with p and giving rise to constructive interference are of form $y = q \times (Log(p)/log(p)) \times sin(2\pi/p)p$, $q = r/s$.

4. Could functional equation and Riemann hypothesis generalize?

It is interesting to list the elementary properties of the $\tilde{\zeta}$ before trying to answer to the questions of the title.

1. The replacement $log(n) \rightarrow Log(n) \equiv sum_p k_p Log(p)$ implies that $\tilde{\zeta}$ codes explicitly number theoretic information. Note that $Log(n)$ satisfies the crucial identity $Log(mn) = Log(m) + Log(n)$. $\tilde{\zeta}$ is an analog of partition function with rational number valued $Log(n)$ taking the role of energy and $1/s$ that of a complex temperature. In ZEO this partition function like entity could be associated with zero energy state as a "square root" of thermodynamical partition function: in this case complex temperatures are possible. $|\tilde{\zeta}|^2$ would be the analog of ordinary partition function.

2. Reduction of $\tilde{\zeta}$ to a product of "prime factors" $1/[1-exp(-Log(p)s)]$ holds true by $Log(n) \equiv sum_p k_p Log(p)$, $Log(p) = p/\pi(p)$.

3. $\tilde{\zeta}$ is a combination of exponentials $exp(-Log(n)s)$, which converge for $Re(s) > 0$. For ζ one has exponentials $exp(-log(n)s)$, which also converge for $Re(s) > 0$: the sum $\sum n^{-s}$ does not however converge in the region $Re(s) < 1$. Presumably $\tilde{\zeta}$ fails to converge for $Re(s) \leq 1$. The behavior of terms $exp(-Log(n)s)$ for large values of n is very similar to that in ζ.

4. One can express ζ o in terms of η function defined as

$$\eta(s) = \sum (-1)^n n^{-s} \quad .$$

The powers $(-1)^n$ guarantee that η converges (albeit not absolutely) inside the critical strip $0 < s < 1$.

By using a decomposition of integers to odd and even ones, one can express ζ in terms of η:

$$\zeta = \frac{\eta(s)}{(-1 + 2^{-s+1})} \quad .$$

This definition converges inside critical strip. Note the pole at $s = 1$ coming from the factor.

One can define also $\tilde{\eta}$:

$$\tilde{\eta}(s) = \sum (-1)^n e^{-Log(n)s} \quad .$$

The formula relating $\tilde{\zeta}$ and $\tilde{\eta}$ generalizes: 2^{-s} is replaced with $exp(-2s)$ ($Log(2) = 2$):

$$\tilde{\zeta} = \frac{\tilde{\eta}(s)}{-1 + 2exp^{-2s}} \quad .$$

This definition $\tilde{\zeta}$ converges in the critical strip $Re(s) \in (0,1)$ and also for $Re(s) > 1$. $\tilde{\zeta}(1-s)$ converges for $Re(s) < 1$ so that in $\tilde{\eta}$ representation both converge.

Note however that the poles of ζ at $s = 1$ has shifted to that at $s = log(2)/2$ and is below $Re(s) = 1/2$ line. If a symmetrically posioned pole at $s = 1 - log(2)/2$ is not present in $\tilde{\eta}$, functional equation cannot be true.

5. $Log(n)$ approaches $log(n)$ for integers n not containing small prime factors p for which $\pi(n)$ differs strongly from $p/log(p)$. This suggests that allowing only terms $exp(-Log(n)s)$ in the sum defining $\tilde{\zeta}$ not divisible by primes $p < p_{max}$ might give a cutoff $\tilde{\zeta}^{cut,p_{max}}(s)$ behaving very much like ζ from which "prime factors" $1/(1 - exp(-Log(p)s)$, $p < p_{max}$ are dropped of. This is just division of $\tilde{\zeta}$ by these factors and at least formally, this does not affect the zeros of $\tilde{\zeta}$. Arbitrary number of factors can be droped. Could this mean that $\tilde{\zeta}^{cut}$ has same or very nearly same zeros as ζ at critical line? This sounds paradoxical and might reflect my sloppy thinking: maybe the lack of the absolute implies that the conclusion is incorrect.

The key questions are whether $\tilde{\zeta}$ allows a generalization of the functional equation $\xi(s) = \xi(1-s)$ with $\xi(s) = \frac{1}{2}s(s-1)\Gamma(s/2)\pi^{-s/2}\zeta(s)$ and whether Riemann hypothesis generalizes. The derivation of the functional equation is quite a tricky task and involves integral representation of ζ .

1. One can start from the integral representation of ζ true for $s > 0$.

$$\zeta(s) = \frac{1}{(1-2^{1-s})\Gamma(s)} \int_0^\infty \frac{t^{s-1}}{e^t + 1} dt \quad , \quad Re(s) > 0 \quad .$$

deducible from the expression in terms of $\eta(s)$. The factor $1/(1 + e^t)$ can be expanded in geometric series $1/(1 + e^t) = \sum (-1)^n exp(nt)$ converning inside the critical strip. One formally performs the integrations by taking nt as an integration variable. The integral gives the result $\sum(-1)^n/n^z)\Gamma(s)$.

The generalization of this would be obtained by a generalization of geometric series:

$$1/(1+e^t) = \sum(-1)^n exp(nt) \rightarrow \sum(-1)^n e^{exp(Log(n))t}$$

in the integral representation. This would formally give $\tilde{\zeta}$: the only difference is that one takes $u = exp(Log(n))t$ as integration variable.

One could try to prove the functional equation by using this representation. One proof (see `http://tinyurl.com/yak93hyr`) starts from the alternative expression of ζ as

$$\zeta(s) = \frac{1}{\Gamma(s)}\int_1^\infty \frac{t^{s-1}}{e^t-1}dt \quad , \quad Re(s) > 1 \quad .$$

One modifies the integration contour to a contour C coming from $+\infty$ above positive real axis, circling the origin and returning back toc$+\infty$ below the real axes to get a modified representation of ζ:

$$\zeta(s) = \frac{1}{2isin(\pi s)\Gamma(s)}\int_1^\infty \frac{(-w)^{s-1}}{e^w-1}dw \quad , \quad Re(s) > 1 \quad .$$

One modifies the C further so that the origin is circled around a square with vertices at $\pm(2n+1)\pi$ and $\pm i(2n+1)\pi$.

One calculates the integral the integral along C as a residue integral. The poles of the integrand proportional to $1/(1-e^t)$ are at imaginary axis and correspond to $w = ir2\pi$, $r \in Z$. The residue integral gives the other side of the functional equation.

2. Could one generalize this representation to the recent case? One must generalize the geometric series defined by $1/(e^w-1)$ to $-\sum e^{exp(Log(n))w}$. The problem is that one has only a generalization of the geometric series and not closed form for the counterpart of $1/(exp(w)-1)$ so that one does not know what the poles are. The naive guess is that one could compute the residue integrals term by term in the sum over n. An equally naive guess would be that for the poles the factors in the sum are equal to unity as they would be for Riemann zeta. This would give for the poles of n:th term the guess $w_{n,r} = r2\pi/exp(Log(n)$, $r \in Z$. This does not however allow to deduce the residue at poles.Note that the poles of $\tilde{\eta}$ at $s = log(2)/2$ suggests that functional equation is not true.

There is however no need for a functional equation if one is only interested in $F(s) \equiv \tilde{\zeta}(s) + \tilde{\zeta}(1-s)$ at the critical line! Also the analog of Riemann hypothesis follows naturally!

1. In the representation using $\tilde{\eta}$ $F(s)$ converges at critical striple and is *real*(!) at the critical line $Re(s) = 1/2$ as follows from the fact that $1 - s = \overline{s}$ for $Re(s) = 1/2$! Hence $F(s)$ is expected to have a large number of zeros at critical line. Presumably their number is infinite, since $F(s)^{cut,p_{max}}$ approaches $2\zeta^{cut,p_{max}}$ for large enough p_{max} at critical line.

2. One can define a different kind of cutoff of $\tilde{\zeta}$ for given n_{max}: $n < n_{max}$ in the sum over $e^{-Log(n)s}$. Call this cutoff $\tilde{\zeta}^{cut,n_{max}}$. This cutoff must be distinguished from the cutoff $\tilde{\zeta}^{cut,p_{max}}$ obtained by dropping the "prime factors" with $p < pmax$. The terms in the cutoff are of the form $u^{\sum k_p p/\pi(p)}$, $u = exp(-s)$. It is analogous to a polymomial but with fractional powers of u. It can be made a polynomial by a change of variable $u \rightarrow v = exp(-s/a)$, where a is the product of all $\pi(p)$:s associated with all the primes involved with the integers $n < n_{max}$.

   One could solve numerically the zeros of $\zeta(\tilde{s}) + \zeta(\tilde{s})$ using program modules calculating $\pi(p)$ for a given p and roots of a complex polynomial in given order. One can check whether also all zeros of $\zeta(\tilde{s}) + \zeta(\tilde{s})$ might reside at critical line.

3. One an define also $F(s)^{cut,n_{max}}$ to be distinguished from $F(s)^{cut,p_{max}}$. It reduces to a sum of terms $exp(-Log(n)/2)cos(-Log(n)y)$ at critical line, $n < n_{max}$. Cosines come from roots of unity. $F(s)$ function is not sum of rational powers of $exp(-iy)$ unlike $\tilde{\zeta(s)}$. The existence of zero could be shown by showing that the sign of this function varies as function of y. The functions $cos(-Log(n)y)$ have period $\Delta y = 2\pi/Log(n)$. For small values of n the exponential terms $exp(-Log(n)/2)$ are largest so that they dominate. For them the periods Δy are smallest so that one expected that the sign of both $F(s)$ and $F(s)^{cut,n_{max}}$ varies and forces the presence of zeros.

   One could perhaps interpret the system as quantum critical system. The rather large rapidly varying oscillatory terms with $n < n_{max}$ with small $Log(n)$ give a periodic infinite set of approximate roots and the exponentially smaller slowly varying higher terms induce small perturbations of this periodic structure. The slowly varying terms with large $Log(n)$ become however large near the $Im(s) = 0$ so that here the there effect is large and destroys the period structure badly for small root of $\hat{\zeta}$.

Is the vanishing of the loop corrections consistent with unitarity?

Skeptic could argue that the vanishing of loop corrections is not consistent with unitarity. The following argument however shows that the fact that momenta in TGD framework are 8-D light-like momenta could save the situation. If not only single particle states but also *many-particle states* have light-like 8-momenta, the discontinuity of the amplitude at pole $P^2(M^8) = 0$ implies the discontinuity of the amplitude as function of $s \equiv P^2(M^4)$ along s-axis.

Minkowskian contribution to mass squared would essentially the sum of conformal (stringy) contribution from vibrational degrees of freedom and color contribution from CP_2 degrees of freedom. This suggests a weak form of color confinement: many-particle states could have vanishing color hyper charge and isospin but the eigenvalue value of color Casimir operator would be non-vanishing.

To get more concrete view about the situation the reader is encouraged to study the slides of Jaroslav Trnka explaining BCFW recursion formula [B54] (see `http://tinyurl.com/pqjzffj`) or the article [B35] of Elvang and Huang (see `http://tinyurl.com/y9rhbzhk`).

1. Unitarity condition $SS^\dagger = Id$ for S-matrix $S = 1 + iT$ gives $i(T - T^\dagger) = TT^\dagger$. For forward scattering the physical interpretation is that the discontinuity of $-2Im(T) = i(T - T^\dagger)$ in forward scattering as a function of total mass s above kinematical threshold along real axis is essentially the total scattering rate.

2. For a given tree amplitude, which is rational function, one replaces external momenta p_i with $\hat{p}_i = p_i + zr_i$. r_i real, light-like and orthogonal to each other and their sum vanishes. This gives on mass shell scattering amplitude with complex light-like momenta satisfying conservation conditions.

3. One can consider any non-trivial subset I of momenta and for this set one has $\hat{P}_I^2 = P_i^2 + 2zP \cdot R_I$, where one has $P_I = \sum_i p_i$ and $R_I = \sum_i r_i$. This gives

$$\hat{P}_I^2 = -P_I^2 \frac{(z - z_I)}{z_I} \quad , \quad z_I = \frac{P_I^2}{2P_I \cdot R_I} \quad .$$

   The poles of the modified amplitude $\hat{A}_n(z)$ come from the propagators at $\hat{P}_I^2 = 0$ and correspond to the points $z = z_I$.

4. From the modified scattering amplitude $\hat{A}_n(z)$ one can obtain the original scattering amplitude by performing a residue integral for $\hat{A}_n(z)/z$ along a curve enclosing the poles z_I. This gives

$$A_n = \hat{A}_n(z = 0) + \sum_{z_I} Res_{z=z_I}(\frac{\hat{A}_n(z)}{z}) + B_n \quad .$$

B_n comes from the possible pole at $z = \infty$ and is often assumed to vanish. If so, the amplitude factorizes into a sum of products

$$Res_{z=z_I}\frac{\hat{A}_n(z)}{z} = \sum_I \hat{A}_L(z_I)\frac{1}{P_I^2}\hat{A}_R(z_I) \ .$$

The amplitudes appearing in the product are for modified complex momenta.

The vanishing of loop corrections thus implies that the product terms $\hat{A}_L(1/P^2)\hat{A}_R$ in the BCFW formula give rational functions having no cuts just as the number theoretical vision demands. The discontinuities of the imaginary part of the amplitude are at poles and reduce to the products $\hat{A}_L\hat{A}_R$ with complex on-mass- shell light-like momenta as unitarity demands.

For forward scattering the discontinuity would be essentially positive definite total scattering rate. It would be however non-vanishing only at $P^2 = 0$ so that scattering rate could be non-vanishing only for $P^2 = 0$! This does not make sense in 4-D physics. Is it possible to overcome this difficulty in TGD framework?

1. The first thing to notice is that classical TGD predicts complex Noether charges since for instance Kähler coupling strength has imaginary part. This would suggest that the momenta of incoming particles could be complex. Could complex value of $P(M^4) \equiv P$ implying

$$P^2 = Re(P)^2 - Im(P)^2 + i2Re(P) \cdot Im(P) = 0$$

save the situation? The condition requires that $Re(P)$ and $Im(P)$ are light-like and parallel so that one would obtain only light-like four-momenta as total M^4 momenta.

2. However, in TGD light-likeness holds true in 8-D sense for single particle states: this led to the proposed generalization of twistor approach allowing particles to be massive in 4-D sense. $M^8 - H$ duality allows to speak about light-like M^8 momenta satisfying quaternionicity condition. The wave functions in CP_2 degrees of freedom emerge from momentum wave functions in M^8 degrees of freedom respecting quaternionicity. The condition $P^2(M^8) = 0$ implies that $Re[P(M^8)]$ and $Im[P(M^8)]$ are light-like and parallel. $Im[P(M^8)]$ can be arbitrarily small. One has also $Re[P(M^4)]^2 = Re[P(E^4)]^2$ and $Im[P(M^4)]^2 = Im[P(E^4)]^2$.

3. Could one pose the condition $P^2(M^8) = 0$ also on *many-particle states* or only to the many-particle states appearing as complex massless poles in the BCFW conditions? Kind of strong form of conformal invariance would be in question: not only single-particle states but also many-particle states would be massless in 8-D sense. Now $s = Re[P(M^4))]^2 = Re[P(E^4))]^2$ could have a continuum of values. The discontinuity along s-axis required by unitarity would would emerge from the discontinuity due to the pole at $P^2(M^8) = 0$! Hence 8-dimensional light-likeness in strong sense would be absolutely essential for having vanishing loop corrections together with non-vanishing scattering rates!

   Here one must be however extremely careful.

   (a) In BCFW approach the expression of residue integral as sum of poles in the variable z associated with the amplitude obtained by the deformation $p_i \rightarrow p_i + zr_i$ of momenta ($\sum r_i = 0$, $r_i \cdot r_j = 0$) leads to a decomposition of the tree scattering amplitude to a sum of products of amplitudes in resonance channels with complex momenta at poles. The products involve $1/P^2$ factor giving pole and the analog of cut in unitary condition. Proof of tree level unitarity is achieved by using complexified momenta as a mere formal trick and complex momenta are an auxiliary notion. The complex massless poles are associated with groups I of particles whereas the momenta of particles inside I are complex and non-light-like.

   (b) Could BCFW deformation give a description of massless bound states massless particles so that the complexification of the momenta would describe the effect of bound state formation on the single particle states by making them non-light-like? This makes sense if one assumes that all 8-momenta - also external - are complex. The classical charges are indeed complex already classically since Kähler coupling strength is complex [L16]. A possible interpretation for the imaginary part is in terms of decay width characterizing the life-time of the particle and defining a length of four-vector.

(c) The basic question in the construction of scattering amplitudes is what happens inside CD for the external particles with light-like momenta. The BCFW deformation leading to factorization suggests an answer to the question. The factorized channel pair corresponds to two CDs inside which analogs of M and $N-M$ particle bound states of external massless particles would be formed by the deformation $p_i \rightarrow p_i + zr_i$ making particle momenta non-light-like. The allowed values of z would correspond to the physical poles. The factorization of BCFW scattering amplitude would correspond to a decomposition to products of bound state amplitudes for pairs of CDs. The analogs of bound states for zero energy states would be in question. BCFW factorization could be continued down to the lowest level below which no factorization is possible.

(d) One can of course worry about the non-uniqueness of the BCFW deformation. For instance, the light-like momenta r_i must be parallel ($r_i = \lambda_i r$) but the direction of r is free. Also the choice of λ_i is free to a high extent. BCFW expression for the amplitude as a residue integral over z is however unique. What could this non-uniqueness mean?

Suppose one accepts the number theoretic vision that scattering amplitudes are representations for sequences of algebraic manipulations. These representations are bound to be highly non-unique since very many sequences can connect the same initial and final expressions. The space-time surface associated with given representation of the scattering amplitude is not unique since each computation corresponds to different space-time surface. There however exists a representation with maximal simplicity.

Could these two kinds of non-uniqueness relate?

It is indeed easy to see that many-particle states with light-like single particle momenta cannot have light-like momenta unless the single-particle momenta are parallel so that in non-parallel case one must give up light-likeness condition also in complex sense.

(a) The condition of light-likeness in complex sense allows the vanishing of real and imaginary mass squared for individual particles

$$Im(p_i) = \lambda_i Re(p_i) \ , \quad (Re(p_i))^2 = (Im(p_i))^2 = 0 \ . \tag{16.3.3}$$

Real and imaginary parts are parallel and light-like in 8-D sense. All λ_i have same sign and p_i has positive or negative time component depending on whether positive or negative energy part of zero energy state is in question.

(b) The remaining two conditions come from the vanishing of the real and imaginary parts of the total mass squared:

$$\sum_{i\neq j} Re(p_i)\cdot Re(p_j) - Im(p_i)\cdot Im(p_j) = 0 \ , \quad \sum_{i\neq j} Re(p_i)\cdot Im(p_j) = 0 \tag{16.3.4}$$

By using proportionality of $Im(p_i)$ and $Re(p_i)$ one can express the conditions in terms of the real momenta

$$\sum_{i\neq j}(1-\lambda_i\lambda_j) Re(p_i)\cdot Re(p_j) = 0 \ , \quad \sum_{i\neq j} \lambda_j Re(p_i)\cdot Re(p_j) = 0 \ . \tag{16.3.5}$$

For positive/negative energy part of zero energy state the sign of time component of momentum is fixed and therefore λ_i have fixed sign. Suppose that λ_i have fixed sign. Since the inner products $p_i \cdot p_j$ of time-like vectors with fixed sign of time compomemet are all positive or negative the second term can vanish only if one has $p_i \cdot p_j = 0$. If the sign of λ_i can vary, one can satisfy the condition linear in λ_i but not the first condition as is easy to see in 2-particle case.

(c) States with light-like parallel 8-momenta are allowed and one can ask whether this kind of states might be realized inside magnetic flux tubes identified as carriers of dark matter in TGD sense. The parallel light-like momenta in 8-D sense would give rise to a state analogous to super-conductivity. Could this be true also for quarks inside hadrons assumed to move in parallel in QCD based model. This also brings in mind the earlier intuitive proposal that the momenta of fermions and antifermions associated with partonic 2-surfaces must be parallel so that the propagators for the states containing altogether n fermions and antifermions would behave like $1/(p^2)^{n/2}$ and would not correspond to ordinary particles.

These arguments are formulated in M^8 picture. What could this mean in $M^4 \times CP_2$ picture?

(a) The intuitive expectation is that $Re[P(E^4)]^2$ corresponds to the eigenvalue Λ of CP_2 d'Alembertian so that the higher the momentum, the larger the value of Λ. CP_2 d'Alembertian would be essentially the M^4 mass squared of the state. This would allow vanishing color quantum numbers Y and I_3 but force symmetry breaking $SU(3) \rightarrow SU(2) \times U(1)$. This picture is not quite accurate: also the vibrational degrees of freedom contribute to the mass squared what might be called stringy contribution.

(b) Could the geometry of CP_2 induce this symmetry breaking? For instance, Kähler gauge potential depends on the $U(2)$ invariant "radial" coordinate of CP_2 and is invariant only under $U(2)$ rotations and changes by gauge transformation in other color rotations. Could one assign the symmetry breaking to the choice of color quantization axes boiling down at the classical level to the fixing of CP_2 Kähler function would?

   One would have color confinement in weak sense: in QCD picture physical states correspond to color singlet representations. This is certainly very strong statement in a sharp conflict with the standard view about color confinement. It would make sense in TGD framework, where color as a spin like quantum number is replaced with angular momentum like quantum number. One could say that macroscopic systems perform macroscopic color rotation. The model for the honeybee dance [L42] conforms with this view and actually led to the proposal for a modification of cosmic string type extremals $X^4 = X^2 \times Y^2 \subset M^4 \times CP_2$ by putting Y^2 in 2-D rigid body color rotation along both time axis and spatial axis of the string world sheet X^2.

(c) This picture raises again the old question about the relationship of color and electroweak quantum numbers in TGD framework. Could one regard electroweak quantum numbers as a spin related to color group $SU(3)$ just as one can relate ordinary spin with Lorentz transformations? Color quantum numbers of say quarks would be analogous to orbital angular momentum. The realization of the action of the electroweak $U(2)_{ew}$ on CP_2 spinors indeed involves also geometric color rotation affecting the gauge potentials in the general case and $U(2)_{ew}$ can be identified as holonomy group of CP_2 spinor connection and sugroup of $SU(3)$. One could also see electroweak symmetry breaking as a further symmetry breaking $U(2) \rightarrow U(1) \times U(1)$ assignable with the flag manifold $SU(3)/U(1) \times U(1)$ parameterizing different choices of color quantization axes and having interpretation as CP_2 twistor space.

Remark: Number theoretic vision means that the quaternionic M^8-momenta are discrete with components having values in the extension of rationals. $P^2(M^4)$ becomes discrete if one poses $P^2(M^8) = 0$ condition for all states. The values of discontinuity of $Im(T)$ correspond now to a discrete sequence of poles along s-axis approximating cut. At the continuum limit this discrete sequence of poles becomes cut. Continuum limit would correspond to a finite measurement resolution in which one cannot distinguish the poles from each other.

16.3.3 Grassmannian approach and TGD

Grassmannian approach has provided besides technical progress deeper views about twistorialization and also led to the understanding of the Yangian symmetry.

Grassmannian twistorialization - or what I understand about it

The twistorialization of the scattering amplitudes works for planar amplitudes in massless theories and involves the following ingredients.

(a) All scattering amplitudes are expressible in terms of on-mass-shell scattering amplitudes with massless on-mass-shell particles in complex sense.

(b) The scattering amplitude is sum over contributions with varying number of loops. BCFW recursion relation allows to construct scattering amplitudes from their singularities using 3-particle amplitudes as building brick amplitudes. There are two types of singularities.

For the first type of singularity one has on-shell internal line and one obtains a sum over all possible decompositions of the scattering amplitude to a product of on-mass-shell scattering amplitudes multiplied by delta function for momentum squared of the internal line. Second type of singularity corresponds to the so called forward limit and is obtained from $(n+2,k)$ amplitude by contracting two added adjacent particles to form a loop so that their momenta are opposite and integrating over the momentum.

(c) The singular term is algebraically analogous to an exterior derivative of the scattering amplitude and can be integrated explicitly: the integration adds BCFW bridge to the both terms such that the forward limit loop in the second term is under the bridge. The outcome is BCFW formula for l-loop amplitude with n external particles with k negative helicities consisting of these two terms.

Twistor Grassmannian approach expresses the on mass shell scattering amplitudes appearing as building bricks as residue integrals over Grassmannian $Gr(n,k)$, where n is the number of particles and k is the the number of negative helicities. The Grassmannian approach is described in a concise form in the slides by Jaroslav Trnka [B54] (see `http://tinyurl.com/pqjzffj`).

(a) The construction of the on-mass-shell scattering amplitudes appearing in BCFW formula as residue integrals in Grassmannians follows by expressing the momentum conserving delta functions in twistor description in terms of auxiliary variables serving as coordinates of Grassmannian $G(n,k,C)$ for the on mass shell tree amplitude with n external particles having k negative helicities. Grassmannian has dimension $d=(n-k)k$ and can be identified as the space of k-planes - or equivalently $n-k$-planes in C^N. Grassmannian has a representation as homogenous space $G(n,k,C)=U(n)/U(n-k)\times U(k)$ having $SU(n)$ as the group of isometries. For $k=1$ one obtains projective space which is also symmetric space (allowing reflection along geodesic lines as isometries).

(b) Grassmannians emerge as an auxiliary construct, and the multiple residue integral over Grassmannian gives sum of residues so that the introduction of Grassmannians might look like un-necessary complication. The selection of points of Grassmannian for given external quantum numbers by residue integral given at the same time the value of the amplitude might however have some deeper meaning.

The construction involves standard mathematics, which is however new for physicists. For instance, notions such as Plücker coordinates, Schubert cells and cell decomposition appear. One can relate to each other various widely different looking expressions for the amplitudes as being associated with different cell decompositions of Grassmannian. The singularities of the integrand of the scattering amplitude defined as a multiple residue integral over $G(k,n)$ define a hierarchy of Schubert cells.

(c) The so called positive Grassmannian [B37] defines a subset of singularities appearing in the scattering amplitudes of $\mathcal{N}=4$ SUSY. The points of positive Grassmannian $Gr_+(k,n)$ are representable as $k\times n$ matrices with positive $k\times k$ determinants. The singularities correspond to the boundaries of $Gr_+(k,n)$ with some $k\times k$ determinants vanishing. For tree diagrams the singularities correspond to poles appearing in the factorized term of the BCFW decomposition of the scattering amplitude. The positivity conditions hold true also for the twistors representing external particles.

(d) Positivity conditions guarantee the convexity of the integration region determined by the C-matrix as point of $Gr_+(k,n)$ appearing in the conditions dictating the integration region.

To better understand the meaning of positivity one can first consider triangle call it T - as a representation of positive Grassmannian $Gr_+(1,3) = P^2_+$. Any interior points of T can be regarded as center of mass for suitable positive masses at the vertices of the triangle. These conditions generalizes to the case of general polygons, which must be convex. If the number of vertices of the polygon is larger than 3, convexity is not automatically satisfied, and requires additional conditions.

This description generalizes to Grassmannians $Gr_+(k,n)$. Masses define the analog of C-matrix as element of $Gr_+(k,n)$ appearing in the twistor approach and the vertices of the triangle are analogous to the twistors associated with external particles combining to form a point of $Gr(4,n)$. Positivity condition is generalized to the condition that $k \times k$ minors of the $k \times n$ matrix are positive.

(e) Also the twistors associated with the external particles must satisfy analogs of the positivity conditions. This involves the replacement of $Gr(4,n)$ associated with twistors of the external particles with $Gr_+(k+4,n)$. The additional k components of the twistors are Grassman numbers and determined by the superparts of the twistors (see the slides of Trnka at `http://tinyurl.com/pqjzffj`. I must admit that I did not understand this.

(f) Residue integral can be defined in terms of what is called canonical form Ω - analog of volume form - having logarithmic singularities at the boundaries of the $Gr_+(k,n)$. Hence one can perform a reduction of the residue integral to a sum of integrals over $G(k,k+4)$ instead of $G(k,n)$ (actually not so surprising since the residue integrals give as outcome the residues at discrete points!).

This leads to a reduction of the residue integral over $Gr_+(k,n)$ to a sum of lower dimensional residue integrals over triangulation defined by $Gr_+(k,k+4)$ represented as surfaces of $Gr_+(k,n)$ glued together along sides. The geometric analog would be decomposition of polygon to a union of triangles.

This simplifies the situation dramatically [B75, B54, B37] and leads to the notion of amplituhedron [B20, B19]. What is so remarkable, is the simplicity of the expressions for all-loop amplitudes and the fact that positivity implies locality and unitarity for $\mathcal{N}=4$ SUSY.

(g) It should be possible to construct Ω explicitly having the desired singularities which would be in TGD framework poles with $P^2(M^8) = P^2(M^4 \times CP_2) = 0$ if the proposed realization of unitary makes sense? Could one just assumes that Ω vanishes for that part of the boundary of $Gr_+(k,n)$, which gives loop singularities? Could these points $Gr_+(k,n)$ be transcendental and excluded for this reason?

If loop corrections are vanishing as ZEO strongly suggests, only tree amplitudes are needed. Therefore it is appropriate to summarize what I have managed to understand about the construction of the tree amplitudes with general value of k in the amplituhedron approach.

(a) The notion of amplituhedron relies on the mapping of $G(k,n)$ to $G_+(k,k+m)$ $n \geq k+m$. Actually a map from $G(k,n) \times G(k+4,n) \to G_+(k,k+m)$ is in question. $m=4$ identifiable as the apparent dimension of twistor space without projective identification giving the actual dimension $d=3$. n is the number of external particles and k the number of negative helicities.

The value of m is $m=4$ and follows from the conditions that amplitudes come out correctly. The constraint $Y = C \cdot Z$, where Y corresponds to point of $G_+(k,k+4)$ and Z to the point of $G(k+4,n)$ performs this mapping, which is clearly many-to one. One can decompose integral over $G_+(k,n)$ to integrals over positive regions $G_+(k,k+4)$ intersecting only along their common boundary portions. The decomposition of a convex polygon in plane to triangles represent the basic example of this kind of decomposition. Obviously there are several decompositions of this kind.

(b) Each decomposition defines a sum of contributions to the scattering amplitude involving integration of a projectively invariant volume form over the positive region in question. The form has a logarithmic singularity at the boundaries of the integration region but spurious singularities cancel so that only the contribution of the genuine boundary of $G_+(k, k+4)$ remains. There are additional delta function constraints fixing the integral completely in real case.

(c) In complex case one has residue integral. The proposed generalization to the complex case is by analytic continuation. TGD inspired proposal is that the positivity condition in the real case is generalized to the condition that the positive coordinates are replaced by complex coordinates of hyperbolic space representable as upper half plane or equivalently as the unit disk obtained from upper half plane by exponential mapping $w = exp(iz)$. The measure $d\alpha/\alpha$ would correspond to $dz = dw/w$. If taken over boundary circle labelled by discrete phase factors $exp(i\phi)$ given by roots of unity the integral would be numerically a discrete Riemann sum making no sense p-adically but residue theorem could allow to avoid the discretizaton and to define the p-adic variant of the integral by analytic continuation. These conditions would be completely general conditions on various projectively invariant moduli involved.

(d) One must extend the bosonic twistors Z_a of external particles by adding k coordinates. This extension looks very difficult to understand intuitively. Somewhat surprisingly, these coordinates are anti-commutative super-coordinates expressible as linear combinations of fermionic parts of super-twistor using coefficients, which are also Grassmann numbers. Integrating over these one ends up with the standard expression of the amplitude using canonical integration measure for the regions in the decomposition of amplituhedron. An interesting question is whether the addition of k-dimensional anti-commutative parts to Z_a expressible in terms of super-coordinates is only a trick or whether it could have some physical interpretation.

Grassmannians as reduced WCWs?

Grassmannians appear as auxiliary spaces in twistor approach. Could Grassmannians and the procedure assigning to external momenta and helicities discrete set of points of Grassmannian and scattering amplitude have some concrete interpretation in TGD framework?

(a) The points of cognitive representation define WCW coordinates for space-time surface. For a fixed number of points in cognitive representation WCW is effectively replaced with a finite-dimensional reduced WCW. These points would naturally correspond to the points defining ends of fermionic lines at partonic 2-surfaces. WCW has Kähler metric with Euclidian signature. This could be true also for its reduction.

(b) The experience with twistorialization suggests that these spaces could be simply Grassmannians $Gr(n, r, C)$ consisting or r-dimensional complex planes of n-dimensional complex space representable as coset spaces $U(n)/U(n-r) \times U(r)$ appearing as auxiliary spaces in the construction of twistor amplitudes.

Note that the correlation between quantum states and geometry would be present since n corresponds to the number of external particles and r to those with negative helicity in ordinary twistor Grassmann approach. In TGD framework discretized variants of these spaces corresponding to the extension of rationals used would appear. Yangian symmetries could correspond to general coordinate transformations for the reduced WCW acting as gauge symmetry. These transformations act as diffeomorphisms for so called positive Grassmannians also in the standard twistorialization. If the reduced WCWs indeed correspond to twistor Grassmannians, one would have a completely unexpected connection with supersymmetric QFTs.

(c) The reduction of WCW to a finite dimensional Kähler manifold suggests that also WCW spinors become ordinary spinors for Kähler manifold so that gamma matrices form a finite-D fermionic oscillator operator algebra. WCW has maximal symmetries and it would not be surprising if also the finite-D Kähler manifold would possess maximal symmetries. Note that WCW gamma matrices together with isometry generators of WCW give rise to a super-symplectic algebra involving a generalization of 2-D conformal invariance replacing 2-D surfaces with light-like 3-surfacs.

(d) The interpretation of supersymmetry would be different from the standard one. Kähler structure implies that $\mathcal{N}$ is even and Majorana spinors are absent and both baryon and lepton number can be conserved separately. The ordinary fermionic oscillator algebra is a Clifford algebra and could be interpreted in terms of a broken supersymmetry.

Also more general flag manifolds than Grassmannians can be considered. If these spaces are homogenous spaces they have maximal isometries. They should have also Kähler structure. Compactness looks also a highly desirable property. The gauge conditions for the subalgebra of super-symplectic algebra state that the sub-algebra and its commutator with the entire algebra annihilate physical states and give rise to vanishing classical Noether charges. This would effectively reduce the super-symplectic algebra to a finite-D Lie group or Kac-Moody algebra of a finite-dimensional Lie group - perhaps belonging to the ADE hierarchy as the hierarchy of inclusions of HFFs as an alternative correlate for the realization of finite measurement resolution suggests. The flag manifolds associated with these Lie groups define more general homogenous spaces as candidates for the reduced WCWs.

Interpretation for Grassmannian residue integrations

The identification of Grassmannians (or possibly more general spaces) as reduced WCWs would give a genuine physical interpretation for the Grassmannian integrations as residue integrations over reduced WCW. What looks mysterious and maybe even frustrating is that the outcome of the entire process is sum over discrete residues: what does this mean?

(a) The residue integration is only over a surface of reduced WCW with dimension equal to one half of that of WCW. One has integrand, which depends on the external quantum numbers coded in terms of twistors and on coordinates of reduced WCW. The residue integration is analogous to summation over amplitude associated with space-time surfaces coded by different cognitive representations.

(b) One can argue that a continuous residue integral over Grassmannian is not consistent with the number theoretic discretization. The outcome is however discrete set of space-time surfaces labelled by cognitive representations as points of Grassmannian. Of the points in question are in the extension and if this is equivalent with the corresponding property for the coordinates of Grassmannian, there should be no problems. The restriction of external momenta to the extension of rationals might guarantee this.

(c) The full multiple residue integral leaves only pole contributions, which correspond to a discrete collection of space-time surfaces (at least the set of space-time surfaces obtained by the action of Galois group), that is discrete set of points of reduced WCW. It seems that the entire residue integration is just a manner to realize quantum classical correspondence by associating to the external quantum numbers space-time surfaces and corresponding cognitive representations - and of course, also the scattering amplitude.

(d) One can also ask whether the positivity of Grassmannian might relate to the fact that p-adic numbers as ordinary integers are always non-negative (most of them infinite). The positivity might be necessary in order to have number theoretic universality. If the minors associated with the C-matrix serve as coordinates for $Gr_+(k,n)$ they could be interpreted also as p-adic numbers. If they are allowed to be negative, one encounters problems since p-adic numbers are not well-ordered and one cannot say whether p-adic number is negative or positive.

Posible description of SUSY and its breaking in TGD framework

Although twistor description make sense also in the absence of supersymmetry, super-symmetry is an essential part of the elegance of the Grassmannian approach. For the ordinary SUSY one has gluons and their superpartners characterized in terms of super-twistors. In TGD one has two pictures [L33, L38].

(a) At the level H fermions as fundamental particles are described in terms of second quantized induced spinor fields, whose oscillator operators can be used to build gamma matrices for WCW [K88, K110]. In TGD universe all known elementary particles would be composites of fundamental fermions represented as lines at the light-like orbits of partonic 2-surfaces (wormhole throats) and ordinary elementary particles involve a pair of wormhole contacts with throats containing these fermion lines. It is assumed that the fermions are at different points: this allows to avoid problems due to infinities.

In the proposed generalization of twistor approach $2 \rightarrow 2$ fermion scattering in the classical fields at partonic 2-surface would define the basic $2 \rightarrow 2$-vertex replacing 3-vertices of twistorial SUSY. Essentially one has only two-vertices describing the redistribution of fermions at partonic 2-surface between orbits of the partonic 2-surfaces meeting at it. This is different from $\mathcal{N} = 4$ SUSY [L22]. If one allows completely local multi-fermion states at the level of H one cannot avoid fermionic contact interactions.

The many-fermion states associated with partonic 2-surfaces would define the analogs of super-multiplets. One can wonder whether a SUSY type description could exist as a limit when the partonic 2-surface is approximated with single point so that also positions of fermions are approximated as single point. SUSY would be only approximate.

(b) At the level of M^8 I have proposed the use of polynomials P of super-octonion serving as analogs of super-gluon fields to construct scattering amplitudes [L33]. This allows geometric description of all particles using super-multiplets. Each monomial of theta parameters would give rise to its own space-time surface by the condition that either $IM(P)$ or $RE(P)$ vanishes for the corresponding polynomial P. This condition would reduce the components of super-field to algebraic surfaces.

There is however an important difference from H picture. The members of super-multiplet defined by P correspond to the coefficients of monomials of theta parameters having interpretation as analogs of oscillator operators. Super-partners would be in this sense point-like objects unlike in H approach, where this can hold true only approximately.

Could H- and M^8 pictures be equivalent and could one understand the breaking of SUSY in this framework?

(a) $M^8 - H$ correspondence as a map of associative space-time regions from M^8 to minimal surfaces in H makes sense for the external particles and thus at boundaries of CDs. It assigns to a point of the partonic 2-surface $X^2 \subset X^4 \subset M^8$ the quaternionic tangent space of X^4 at it characterized by a point of CP_2. M^4 point is mapped to itself. There is additional condition requiring that quaternionic tangent space contains fixed complex sub-space but this is not relevant now.

(b) Could this map be one-to-many so that super-field component describing purely many-fermion state would be mapped to several points at the image of X^2 in H describing multi-local many-fermion state? This is possible if the points in M^8 are singular in the sense that the action of a normal subgroup H of Galois group Gal leaves the point invariant so that Gal reduces to Gal/H: symmetry breaking takes place.

The tangent spaces of the degenerate points are however different and are mapped to different points of CP_2 in $M^8 - H$ correspondence making sense at boundaries of CDs but not in their interiors. One would have several fermions with same M^4 coordinates but different CP_2 coordinates and the outcome would be many-fermion state. In the case of 2-fermion state the different values of CP_2 coordinates would be associated with the opposite throats of a wormhole contact whose orbit defines light-like 3-surface. Could light-likeness inducing the reduction of the metric dimension of the tangent space from 4 to 3 somehow induce also this degeneration?

(c) Could symmetry breaking as a degeneration of Gal action to that for Gal/H take place for the conditions defining the 4-surfaces associated with the higher components of super-octonion and induce the breaking of SUSY at the level of M^8 manifesting as the non-locality of the fermion state at the level of H? This degeneration would be a typical manifestation of quantum criticality: criticality in general means co-incidence of two roots.

Comments about coupling constant evolution

16.3.4 Summary

Since the contribution means in well-defined sense a breakthrough in the understanding of TGD counterparts of scattering amplitudes, it is useful to summarize the basic results deduced above as a polished answer to a Facebook question.

There are two diagrammatics: Feynman diagrammatics and twistor diagrammatics.

(a) Virtual state is an auxiliary mathematical notion related to Feynman diagrammatics coding for the perturbation theory. Virtual particles in Feynman diagrammatics are off-mass-shell.

(b) In standard twistor diagrammatics one obtains counterparts of loop diagrams. Loops are replaced with diagrams in which particles in general have complex four-momenta, which however light-like: on-mass-shell in this sense. BCFW recursion formula provides a powerful tool to calculate the loop corrections recursively.

(c) Grassmannian approach in which Grassmannians $Gr(k,n)$ consisting of k-planes in n-D space are in a central role, gives additional insights to the calculation and hints about the possible interpretation.

(d) There are two problems. The twistor counterparts of non-planar diagrams are not yet understood and physical particles are not massless in 4-D sense.

In TGD framework twistor approach generalizes.

(a) Massless particles in 8-D sense can be massive in 4-D sense so that one can describe also massive particles. If loop diagrams are not present, also the problems produced by non-planarity disappear.

(b) There are no loop diagrams- radiative corrections vanish. ZEO does not allow to define them and they would spoil the number theoretical vision, which allows only scattering amplitudes, which are rational functions of data about external particles. Coupling constant evolution - something very real - is now discrete and dictated to a high degree by number theoretical constraints.

(c) This is nice but in conflict with unitarity if momenta are 4-D. But momenta are 8-D in M^8picture (and satisfy quaternionicity as an additional constraint) and the problem disappears! There is single pole at zero mass but in 8-D sense and *also many-particle states* have vanishing mass in 8-D sense: this gives all the cuts in 4-D mass squared for all many-particle state. For many-particle states not satisfying this condition scattering rates vanish: these states do not exist in any operational sense! This is certainly the most significant new discovery in the recent contribution.

    BCFW recursion formula for the calculation of amplitudes trivializes and one obtains only tree diagrams. No recursion is needed. A finite number of steps are needed for the calculation and these steps are well-understood at least in 4-D case - even I might be able to calculate them in Grassmannian approach!

(d) To calculate the amplitudes one must be able to explicitly formulate the twistorialization in 8-D case for amplitudes. I have made explicit proposals but have no clear understanding yet. In fact, BCFW makes sense also in higher dimensions unlike Grassmannian approach and it might be that the one can calculate the tree diagrams in TGD framework using 8-D BCFW at M^8 level and then transform the results to $M^4 \times CP_2$.

What I said above does yet contain anything about Grassmannians.

(a) The mysterious Grassmannians $Gr(k,n)$ might have a beautiful interpretation in TGD: they could correspond at M^8 level to reduced WCWs which is a highly natural notion at $M^4 \times CP_2$ level obtained by fixing the numbers of external particles in diagrams and performing number theoretical discretization for the space-time surface in terms of cognitive representation consisting of a finite number of space-time points.

Besides Grassmannians also other flag manifolds - having Kähler structure and maximal symmetries and thus having structure of homogenous space G/H - can be considered and might be associated with the dynamical symmetries as remnants of super-symplectic isometries of WCW.

(b) Grassmannian residue integration is somewhat frustrating procedure: it gives the amplitude as a sum of contributions from a finite number of residues. Why this work when outcome is given by something at finite number of points of Grassmannian?!

In M^8 picture in TGD cognitive representations at space-time level as finite sets of points of space-time determining it completely as zero locus of real or imaginary part of octonionic polynomial would actually give WCW coordinates of the space-time surface in finite resolution.

The residue integrals in twistor diagrams would be the manner to realize quantum classical correspondence by associating a space-time surface to a given scattering amplitude by fixing the cognitive representation determining it. This would also give the scattering amplitude.

Cognitive representation would be highly unique: perhaps modulo the action of Galois group of extension of rationals. Symmetry breaking for Galois representation would give rise to supersymmetry breaking. The interpretation of supersymmetry would be however different: many-fermion states created by fermionic oscillator operators at partonic 2-surface give rise to a representation of supersymmetry in TGD sense.

16.4 New insights about quantum criticality for twistor lift inspired by analogy with ordinary criticality

Quantum criticality (QC) is one of the basic ideas of TGD. Zero energy ontology (ZEO) is second key notion and leads to a theory of consciousness as a formulation of quantum measurement theory making observer part of the quantum system in terms of notion of self identified as a generalized Zeno effect or analog for a sequence of weak measurements, and solving the basic paradox of standard quantum measurement theory, which one usually tries to avoid by introducing some "interpretation".

ZEO allows to see quantum theory could be seen as "square root" of thermodynamics. It occurred to me that it would be interesting to apply this vision in the case of quantum criticality to perhaps gain additional insights about its meaning. We have a picture about criticality in the framework of thermodynamics: what would be the analogy in ZEO based interpretation of Quantum TGD? Could it help to understand more clearly the somewhat poorly understood views about the notion of self, which as a quantum physical counterpart of observer becomes in ZEO a key concept of fundamental physics?

The basic ingredients involved are discrete coupling constant evolution, zero energy ontology (ZEO) implying that quantum theory is analogous to "square root" of thermodynamics, self as generalized Zeno effect as counterpart of observer made part of the quantum physical system, $M^8 \leftrightarrow M^4 \times CP_2$ duality, and quantum criticality. A further idea is that vacuum functional is analogous to a thermodynamical partition function as exponent of energy $E = TS - PV$.

The correspondence rules are simple. The mixture of phases with different 3-volumes per particle in a critical region of thermodynamical system is replaced with a superposition of space-time surfaces of different 4-volumes assignable to causal diamonds (CDs) with different sizes. Energy E is replaced with action S for preferred extremals defining Kähler function in the "world of classical worlds" (WCW). S is sum of Kähler action and 4-volume term, and these terms correspond to entropy and volume in the generalization $E = TS - PV \rightarrow S$. P resp. T corresponds to the inverse of Kähler coupling strength α_K resp. cosmological constant Λ. Both have discrete spectrum of values determined by number theoretically determined discrete coupling constant evolution. Number theoretical constraints force the analog of micro-canonical ensemble so that S as the analog of E is constant for all 4-surfaces appearing in the quantum superposition. This implies quantization rules for Kähler action and volume, which are very strong since α_K is complex.

This kind of quantum critical zero energy state is created in unitary evolution created in single step in the process defining self as a generalized Zeno effect. This unitary process implying time de-localization is followed by a weak measurement reducing the state to a fixed CD so that the clock time identified as the distance between its tips is well-defined. The condition that the action is same for all space-time surfaces in the superposition poses strong quantization conditions between the value of Kähler action (Kähler coupling strength is complex) and volume term proportional to cosmological constant. The outcome is that after sufficiently large number of steps no space-time surfaces satisfying the conditions can be found, and the first reduction to the opposite boundary of CD must occur - self dies. This is the classical counterpart for the fact that eventually all state function reduction leaving the members of state pairs at the passive boundary of CD invariant are made and the first reduction to the opposite boundary remains the only option.

The generation of magnetic flux tubes provides a manner to satisfy the constancy conditions for the action so that the existing phenomenology as well as TGD counterpart of cyclic cosmology as re-incarnations of cosmic self follows as a prediction. This picture allows to add details to the understanding of the twistor lift of TGD at classical level and allows an improved understanding of the p-adic length scale evolution of cosmological constant solving the standard problem caused by the huge value of Λ. The sign of Λ is predicted correctly.

This picture generalizes to the twistor lift of TGD and cosmology provides an interesting application. One ends up with a precise model for the p-adic coupling constant evolution of the cosmological constant Λ explaining the positive sign and smallness of Λ in long length scales as a cancellation effect for M^4 and CP_2 parts of the Kähler action for the sphere of twistor bundle in dimensional reduction, a prediction for the radius of the sphere of M^4 twistor bundle as Compton length associated with Planck mass (2π times Planck length), and a prediction for the p-adic coupling constant evolution for Λ and coupling strength of M^4 part of Kähler action giving also insights to the CP breaking and matter antimatter asymmetry. The observed two values of Λ could correspond to two different p-adic length scales differing by a factor of $\sqrt{2}$.

16.4.1 Some background

Some TGD background is needed to understand the ideas proposed in the sequel.

Discrete coupling constant evolution

The most obvious implication is discrete coupling constant evolution in which the set of values for coupling constants is discrete and analogous to the set of the critical values of temperature [L44] (see `http://tinyurl.com/y9hlt3rp`). Zeros of Riemann Zeta or its slight modification suggest themselves as the spectrum for the Kähler coupling strength. This discrete coupling constant evolution requires that loop corrections vanish. This vision is realized concretely in the generalization of the twistorial approach to the construction of scattering amplitudes [L44].

Non-manifest unitarity is the basic problem of the twistor Grassmann approach. A generalization of the BCFW formula without the loop corrections gives scattering amplitudes satisfying unitary constraints. The needed cuts are be replaced by sequences of massless poles in 8-D sense and cuts approximate these sequences (consider electrostatic analogy in which line charge approximates a discrete sequences of poles). The replacement cuts with sequences of poles is forced by the number theoretic discretization of momenta so that they belong to an extension of rationals defining the adele [L35] (see `http://tinyurl.com/ycbhse5c`).

Non-planar loop diagrams are a chronic problem of twistor approach since there is no general rule loop integrations allowing to combine them neatly. Also this problem disappears now.

M^8-H duality plays key role in the twistorial approach [L33] (see `http://tinyurl.com/yd43o2n2`). In the ordinary twistor approach all momenta are light-like so that it does not apply to massive particles. TGD solves this problem: at M^8 level one has quaternionic light-like 8-D momenta, which correspond to massive 4-D momenta in M^8 picture. In $H=M^4\times CP_2$ picture ground states of super-conformal representations are constructed in terms of spinor harmonics of in $M^4\times CP_2$, which are products plane-waves characterized by massive 4-momenta and color wave functions associated with massless Dirac equation in H. Also the analog of Dirac equation for the induced spinor fields at space-time surface is massless [K88] (see `http://tinyurl.com/yc2po5gf`).

ZEO and self as generalized Zeno effect

ZEO allows to see self as generalized Zeno effect [L39](see `http://tinyurl.com/ycxm2tpd`).

(a) Generalized Zeno effect can be regarded as a sequence of "small" state function reductions analogous to weak measurements performed at active boundary of causal diamond (CD). In usual Zeno effect the state is unaffected under repeated measurements: now the same is true at passive boundary of CD whereas the members of state pairs at the active boundary change. The unitary evolutions followed by these evolutions leave thus passive boundary and states at it invariant whereas active boundary shifts farther away from the passive boundary and the members of state pairs at it are affected. This gives rise to the experienced flow of time.

The change of states is characterized unitary S-matrix. Each unitary evolution involves de-localization in the space of CDs so that one has quantum superposition of CDs with sizes not smaller than the CD to which the state was localized at previous reduction. This gives rise to a steady increase of clock time defined as the distance between the tips of CD. Self dies and reincarnates as a self with opposite direction of clock time when the first unitary evolution at the passive boundary followed by a weak measurement at it takes place. Self dies when all observables leaving the states at passive boundary invariant are measured. There are no choices to be made anymore.

(b) Quantum TGD as "square root " of thermodynamics means that the partition function of thermodynamics is replaced by its "square root" defined by the vacuum functional identified as exponent of Kähler function of "world of classical worlds" (WCW). Kähler function is analogous to energy $E=TS-PV$ in thermodynamics with T replaced with the inverse of complex Kähler coupling strength and P with cosmological constant, which have discrete spectrum of values.

One has the analog of micro-canonical ensemble for which only states with given energy are possible. Now the action (Kähler function) is same for the space-time surfaces assignable to the zero energy states involved. This condition allows to get rid of the exponentials defining the vacuum functional otherwise appearing in the scattering amplitudes. This condition is strongly suggested by number theoretic universality for which these exponentials are extremely troublesome since both the exponent and exponential should belong to the extension of rationals used.

This implies a huge simplification in the construction of the amplitudes [L33] (see `http://tinyurl.com/yd43o2n2`) because finite measurement resolution effectively replaces space-time surfaces with their cognitive representation defined by a discrete set of space-time points with imbedding space coordinates in the extension of rationals defining the adele.This representation codes for the space-time surface if it corresponds to zero locus of real or imaginary part (in quaterionic sense) of an octonionic polynomial with real coefficients. WCW coordinates are given by the cognitive representation and are discrete. One is led to enumerative algebraic geometry.

$M^8 - H$ duality

$M^8 - H$ duality [L33] (see `http://tinyurl.com/yd43o2n2`) states that the purely algebraic dynamics determined by the vanishing of real or imaginary part for octonionic polynomial is dual to the dynamics dictated by partial differential equations for an action principle.

(a) The M^8 counterparts of space-time surfaces are obtained as M^8 projections of algebraic surfaces in the complexification M_c^8 by imaginary unit commuting with octonionic units. One can decompose these surfaces to regions with associative (quaternionic) tangent space or normal space and they are analogous to external particles of a twistor diagram entering CD and to interaction regions in which associativity does not hold true and which correspond to interiors of CD.

(b) At the level of H external particles correspond to minimal surfaces, which are also extremals of Kähler action and in accordance with the number theoretical universality and quantum criticality do not depend on the coupling parameters at all. They are obtained by a map taking the 4-surfaces in M^8 to those in H. These conditions should be equivalent with the condition that the 6-D surfaces X^6 in 12-D twistor space of H define twistor bundles of space-time surfaces X^4.

(c) The space-time regions in the interiors of CDs are not minimal surfaces so that Kähler action and volume term couple dynamically and coupling parameters characterize the extremals. The analog is motion of point like particle in the Maxwell field defined by induced Kähler form: this is generalize to the motion of 3-D object with purely internal Kähler field and that associated with wormhole contacts and mediating interaction with larger and smaller space-time sheets.

    In these regions the map mediating $M^8 - H$ duality does not exist since one cannot label the tangent spaces of space-time surface by points of CP_2. The non-existence of this map is due to the failure of either associativity of tangent space or normal space at M^8 level. The initial values at boundaries of CD for the incoming preferred extremals however allows to fix the time evolution in the interior of CD. This is essentially due to the infinite number of gauge conditions for the super-symplectic algebra.

Quantum criticality

Quantum criticality is a further key notion of TGD and was originally motivated by the idea that Kähler coupling strength must be unique in order that the theory is unique.

(a) The first implication of quantum criticality is quantization of various coupling strengths as analogs of critical temperature and of other critical parameters such as pressure. This quantization is required also by number theoretical universality in the adelic approach: coupling constant parameters must belong to the extension of rationals used.

(b) Second implication of quantum criticality is a huge generalization of conformal symmetries to their 4-D analogs. The key observation is that 3-D light-like surfaces allow a generalization of conformal invariance to get the Kac-Moody algebra associated with the isometries of H (at least) as symmetries. In the case of boundary of CD this leads to what I call supersymplectic invariance: the symplectic transformations of the two components of $\delta CD \times CP_2$ act as isometries of WCW. This algebra allows a fractal hierarchy of sub-algebras isomorphic to the algebra itself and gauge conditions state that this kind of sub-algebra and its commutator with the entire algebra annihilate physical states and classical Noether charges for them vanish [L44] (see `http://tinyurl.com/y9hlt3rp`). By quantum classical correspondence (QCC) the eigenvalues of quantum charges are equal to the classical Noether charges in Cartan algebra of supersymplectic algebra.

(c) The third implication is the understanding of preferred extremals in $H = M^4 \times CP_2$ and their counterparts at the level of M^8. Associativity condition at the level of M^8 satisfied by the spacetime surfaces representing external particles arriving into CD corresponds to quantum criticality posing conditions on the coefficients of octonionic polynomials.

The space-time regions inside CD the space-time surfaces do not satisfy associativity conditions and are not critical.

(d) TGD as "square root" of thermodynamics idea suggests a fourth application of quantum criticality. This analogy might allow a better understanding of self as Zeno effect. This application will be studied in the sequel.

16.4.2 Analogy of the vacuum functional with thermodynamical partition function

Consider first the thermodynamical view about criticality. I have discussed criticality from slightly different perspective in [L43] (see `http://tinyurl.com/ydhknc2c`).

(a) Thermodynamical states in critical region, where phases with different densities - say liquid and gas - are present serves as a basic example. This situation is actually a problem of the approach relying on partition function as van der Waals equation predicting 3 different densities for the density of molecules as function of pressure and temperature. Cusp catastrophe gives a view about situation: number density n is behavior variable and P and T are the control variables.

(b) The experimental fact is that the density is constant as function of volume V for fixed temperature T whereas van der Waals predicts dependence on V. The phase corresponding to the middle sheet of the cusp is not at all present and the portions of liquid and gas phases vary. Maxwell's rules (area rule and lever rule) allow to solve the problem plaguing actually all approaches based on partition function. Lever rule assumes that there are actually two kinds of "elements" present. Molecules are the first element but what the second element could be? TGD identification is as magnetic tubes [L43].

(c) In the more general case in which the catastrophe is more general than cusp and has more sheets, two or more phases with different volumes are present and their volumes and possibly other behavior variables analogous to volume vary at criticality.

(d) If one applies criticality in stronger sense by requiring that the function which has extremum as function of n at the surface represented by cusp catastrophe has same value at different sheets of the cusp, only the boundary line of the cusp having V-shaped projection in (p, T)-plane remains.

Generalization of thermodynamical criticality to TGD context

The generalization of this picture to TGD framework replaces the mixture of thermodynamical phases with different volumes with quantum superposition of space-time surfaces with different 4-volumes assignable to CDs with different quantized sizes (by number theoretical constraints).

(a) Vacuum functional, which is exponent of Kähler function of WCW expressible as Kähler action for its preferred extremal, can be regarded as a complex "square root" of thermodynamical partition function Z meaning that its real valued modulus squared is analogous to partition function [K76, L22, L24, L38].

Action S, whose value for preferred extremal defines Kähler function of WCW serves as the analog of energy assumed to have expression $E = PV - TS$, which is not generally true but implied by the condition that E is homogenous as function of conjugate variable pairs P, V and T, S. The analogs of P and T correspond to coupling constant parameters. Pressure p is replaced with the coefficient of volume term in action - essentially cosmological constant. T is replaced with the coefficient $1/\alpha_K$ of Kähler action representing entropy (or negentropy depending on situation).

Remark: Note that T corresponds now to $1/\alpha_K$ rather than α_K analogous to temperature when Kähler action S_K is regarded as analog of energy E rather than entropy S.

(b) Quantum criticality in the sense of ZEO is the counterpart for the criticality in thermodynamics. The mixture of thermodynamical phases with different 3-volumes is replaced with quantum superposition of zero energy states with 4-surface having same action S but different 4-volumes assignable to different CDs. Critical system consists of several phases with same values of coupling parameters α_K and Λ but different 4-volume.

There is also a number theoretic constraint identifiable as the counterpart of the constant energy condition defining micro-canonical ensemble. The exponent of action S must cancel from the scattering amplitudes to avoid serious existence problems in the p-adic sectors of adele associated with given extension of rationals. Criticality means thus that $exp(S)$ has same value for all preferred extremals involved. Real parts are same for all of them and imaginary parts of the action exponential are fixed modulo multiple of 2π. The analog in the case of van der Waals equation of state that the allowed states are associated with the boundary of the projection of the cusp catastrophe to (p,T) plane.

Critical quantum states are superpositions of space-time surfaces with different 4-volumes associated with CDs with quantized size scales (distance between tips) and are generated by unitary evolution. The value of time as size of CD (distance between its tips) is not well-defined in these states.

Remark: Quantum critical states are "timeless" as meditative practices would express it.

This kind of superposition is created by unitary evolution operator at each step in the sequence of unitary evolutions followed by a state function reduction measuring clock time as the distance between the tips of CD. Localization to single CD is the outcome and only superposition with same time-scale and same S but possibly different 4-volumes.

(c) The condition that action is same is very strong and applies to both real and imaginary parts of action (α_K is complex). The proposal [L16, L44] (see `http://tinyurl.com/yas6ofhv` and `http://tinyurl.com/y9hlt3rp`) is that the coupling constant evolution as p-adic length scale $p \simeq 2^k$, k prime corresponds to zero of Riemann ζ for $1/\alpha_K$ or is proportional to it by rational multiplier q. For $q = 1$ $Re(1/\alpha_K)$ analogous to the ordinary temperature would be equal to $Re(s) = 1/2$ for the zeros at the critical line and imaginary parts would correspond to the imaginary parts $Im(s)$ of the zeros. Constancy of the action S would boil down to the conditions

$$Re(S_K) + Re(S_{vol}) = constant \ \ , \quad Im(S_K) + Im(S_{vol}) = constant \ \ mod \ \ 2\pi \tag{16.4.1}$$

Note that the condition for imaginary part is a typical quantization condition.

4-volume can can have arbitrary large values but for S_K this is probably not the case - this already by the quantization conditions. Hence one expects that there is some maximal possible volume for preferred extremals and thus maximal distance between the tips of CDs involved.

When the zero energy state is a superposition of only space-time surfaces with this maximal volume, further unitary evolutions are not possible and the first state function reduction to the opposite boundary of CD happens (death of self and reincarnation with opposite direction of clock time). Self has finite lifetime! This would be the classical correlate for the situation in which no quantum measurements leaving invariant the members of state pairs at the passive boundary of CD are possible.

The constancy of $Re(S)$

How the cancellation of real part of $\Delta(Re(S_K)) + \Delta(Re(S_{vol}))$ could take place?

(a) The physical picture is that the time evolution giving rise to self starts from flux tube dominated phase obtained in the first state function reduction to the opposite boundary of CD and that also asymptotically one obtains flux tube dominated phase again but the flux tubes are scaled up. This is the TGD view about quantum cosmology as a sequences of selves and of their time reversals [K67] [L21] (see `http://tinyurl.com/y7fmaapa`).

This picture suggests that the generation of magnetic flux tubes allows to satisfy the $\Delta Re(S_K)+\Delta Re(S_{vol}) = 0$ condition: in Minkowskian regions the change magnetic part of $\Delta Re(S_K)$ tends to cancel $\Delta Re(S_{vol})$ whereas the electric part is of the same sign. Therefore magnetic flux tubes are favored.

If the sign of the volume term is negative the exponential defining the vacuum functional decreases with volume. If the relative sign of S_K and S_{vol} is negative, the magnetic part of the action is positive. The generation of flux tubes generates positive magnetic action ΔS_K helping to cancel the change ΔS_{vol}.

The additional conditions coming from the imaginary parts are analogous to semiclassical quantization conditions.

(b) The proposed picture can be realized by a proper choice of the relative signs of volume term and Kähler action term. The relative sign comes automatically correct for a positive value of cosmological constant Λ. For this choice the total action density is

$$L_{tot} = (L_K + \frac{\Lambda}{8\pi G})\sqrt{g_4} \ . \tag{16.4.2}$$

This choice gives positive vacuum energy density associated with the volume term.

(c) The density of Kähler action associated with CP_2 degrees of freedom is

$$L_{K,CP_2} = -\frac{1}{4g^2} J^{\mu\nu} J_{\mu\nu} \ . \tag{16.4.3}$$

The action is proportional to $E^2 - B^2$ in Minkowskian regions and magnetic term has sign opposite to that of volume term so that these terms can compensate with the condition guaranteeing constant action. The overall sign of action in the exponent can be chosen so that the exponential vanishes for large volumes. This suggests that the volume term is negative in the vacuum functional (Kähler function as negative of the action for preferred extremal). Euclidian regions, where CP_2 part of Kähler action is of form $B^2 + E^2$ and tends to cancel the volume term.

(d) There is also Kähler action in M^4 degrees of freedom. In twistor lift dimensional reduction occurs for 6-D Kähler action and M^4 part and CP_2 part contribute to Kähler action. The S^2 parts of these actions must give rise to a cosmological constant decreasing like the inverse of p-adic length scale squared. This is achieved if the Kähler contributions have opposite signs so that M^4 contribution has a non-standard sign. This is possible if M^4 Kahler form is proportional to imaginary unit and M^4 Kähler coupling strength contains additional scaling factor.

The induced Kähler form must be sum of the M^4 parts and CP_2 parts and also the action must be sum of M^4 and CP_2 parts. This is achieved if the charge matrices of these two Kähler forms are orthogonal (the trace of their product vanishes). Since CP_2 part couples to both 1 and Γ_9 giving rise to Kähler charges proportional to 1 for quarks and 3 for leptons having opposite chiralities, the corresponding charges would be proportional to 3 for quarks and -1 for leptons.

The imaginary unit multiplying M^4 Kähler form disappears in action and field equations and one obtains

$$L_K = -\frac{1}{4g_K^2}(\epsilon^2 J^2(M^4) + J^2(CP_2) \ , \tag{16.4.4}$$

where ϵ is purely imaginary so that one has $\epsilon^2 < 0$. Since the fields are induced, negative sign for M^4 Kähler action is not expected to lead to difficulties if M^4 term is small.

Some examples are in order.

(a) For cosmic string extremals Kähler action is multiple of volume action. The condition that the two actions cancel would give a constraint between Λ and α_K. Net string tension would be reduced from the value determined by CP_2 scale to a rather small value. This need not occur generally but might be true for very short p-adic length scales, where Λ is large as required by the large value of string tension associated with Kähler action. For thickened cosmic strings (magnetic flux tubes) the value of string tension assignable to Kähler action is reduced and the condition can be satisfied for smaller values of Λ.

(b) For CP_2 type extremals assignable to wormhole contacts serving as basic building bricks of elementary particles the action would be finite for all size scales of CD. Both magnetic and electric contribution to the action are of same sign. For Euclidian regions with 4-D space-time projection with so strong electric field that it changes the signature of the induced metric the same is true.

(c) One can ask whether blackhole interiors as Euclidian regions correspond to these Euclidian space-time sheets or to highly tangled magnetic flux tubes with length considerably longer than Schwartschild radius for which cancellation also can occur (see `http://tinyurl.com/ydhknc2c`). Both pictures are consistent in many-sheeted space-time: magnetic flux tube tangle could topologically condense to a space-time sheet with Euclidian signature. Cancellation cannot last for ever so that also blackholes are unstable against big state function reduction changing the arrow of time. Blackhole evaporation might relate to this instability.

The constancy of $Im(S)$ modulo 2π

If cosmological constant is real, the condition for the constancy of imaginary part of ΔS modulo 2π applies only to the case of S_K and implies that ΔS_K is fixed modulo 2π in the superposition of space-time surfaces. If zeros of ζ [L16] (see `http://tinyurl.com/yas6ofhv`) or its modification) [L44]) (see `http://tinyurl.com/y9hlt3rp`) give the spectrum of $1/\alpha_K$ the value of $\Delta S_{K,red} = \int Tr(J^2)dV$ is given as multiples of $2\pi n/y$, where y is imaginary part for a zero of zeta. The constancy of $Re(S)$ implies that the 4-volume ΔV is quantized as multiples of $2\pi n/\Lambda$. These conditions bring in mind semiclassical quantization of the action in multiples of $\hbar$.

It however turns out that twistor lift forces same phase for M^4 and CP_2 parts of the Kähler action so that the quantization condition for volume is lost. The reason is that $1/\alpha_K(M^4)$ and $1/\alpha_K(CP_2)$ are proportional to

$$\frac{1}{\alpha_{K,6}} = \frac{1}{\alpha_{K,4}R^2} \ , \tag{16.4.5}$$

where R^2 has dimensions of length squared.

16.4.3 Is the proposed picture consistent with twistor lift of Kähler action?

Is it possible to realize the cancellation of real parts of ΔS_{vol} and ΔS_K (modulo 2π for imaginary part) for the twistor lift of Kähler action? Does the sign of the cosmological constant Λ come out correctly (wrong sign of Λ is the probably fatal problem of M-theory)? Can one understand the p-adic evolution of the cosmological constant Λ implying that Λ becomes small in long p-adic length scales and thus solving the key problem related to Λ?

Dimensional reduction of the twistor lift

The condition that the induction of the product of twistor bundles of M^4 and CP_2 to the space-time surface gives the twistor bundle of the space-time surface is conjectured to determine the dynamics of the space-time surfaces. A generalization of 4-D Kähler action to 6-D Kähler action is proposed to give this dynamics, and to dimensionally reduce to a sum of Kähler actions associated with M^4 and CP_2 Kähler forms plus cosmological term.

(a) Twistor bundles are sphere bundles. For the extremals of 6-D Kähler action dimensional reduction takes place since 6-D extremals must be twistor bundle of corresponding space-time surface. Therefore S^2 degrees of freedom are frozen and become non-dynamical.

One could say that the spheres appearing as fibers of twistor bundles of M^4 and CP_2 are identified in the imbedding map. The simplest correspondence between $S^2(M^4)$ and $S^2(CP_2)$ identifies (θ_1, ϕ_1) for $S^2(M^4)$ with (θ_2, ϕ_2) for $S^2(CP_2)$. This means that $S^2(X^6)$ is mapped in the same manner to $S^2(M^4)$ and $S^2(CP_2)$.

One can imagine also correspondence with n-fold winding based on the identification $(\theta_1, \phi_1) = (\theta_2, n\phi_2)$. The area of $S^2(M^4)$ are becomes n-fold and the S^2 part of the Kähler action using θ_2 as coordinate transforms as $S_K(S^2(M^4)n = 1) \rightarrow S_K(S^2(M^4)n) = n^2 S_K(S^2(M^4))$. $n = 1$ is the most plausible option physically.

(b) What the proposed general vision implies for cosmological constant as a sum of $S^2(M^4)$ and $S^2(CP_2)$ parts of 6-D Kähler action giving in dimensional reduction 4-D volume term responsible for the cosmological constant and 4-D Kähler action. If the charge matrices of M^4 and CP_2 parts of Kähler form are orthogonal one can induce Kähler form. If the coupling to M^4 Kähler form is imaginary, M^4 and CP_2 contributions to the total Kähler action have opposite signs. M^4 and CP_2 parts have opposite signs of magnetic terms and the sign of CP_2 magnetic part is opposite to the volume term.

(c) The dimensionally reduced action is obtained by integrating the 6-D Kähler action over S^2 fiber. The integration gives the area $A(S^2)$ of the S^2 fiber, which in the metric induced from the spheres of twistor space of X^4 is given by

$$A(S^2) = (1 + r^2) 4\pi R^2(S^2(CP_2)) \ , \quad r = \frac{R(S^2(CP_2))}{R(S^2(M^4))} \ . \tag{16.4.6}$$

The very natural but un-checked assumption is that the radius of $S^2(CP_2)$ equals to the radius $R(CP_2)$ of the geodesic sphere of CP_2:

$$R(S^2(CP_2)) = R(CP_2) \ . \tag{16.4.7}$$

One obtains

$$L = -\frac{1}{16\pi\alpha_{K,6}} \left[J^2(CP_2) + \epsilon^2 J^2(M^4) + J^2(S^2(CP_2)) + \epsilon^2 J^2(S^2(M^4)) \right] A(S^2) \tag{16.4.8}$$

The immediate conclusion is that the phases of Kähler action and volume term are same so that the quantization condition for imaginary part of the action is not obtained.

(d) The Kähler coupling strengths $\alpha_K(CP_2)$ and $\alpha_K(M^4)$ can be read from the first term

$$\begin{aligned} \frac{1}{\alpha_K(CP_2)} &= \frac{1}{\alpha_{K,4} 4\pi(1+r^2)} \frac{R^2(CP_2)}{R^2} \ , \\ \frac{1}{\alpha_K(M^4)} &= \frac{\epsilon^2}{\alpha_K(CP_2)} \ . \end{aligned} \tag{16.4.9}$$

One can choose the factor R^2 to be the area of S^2 by suitably renormalizing $1/\alpha_K$. This would give simpler expression

$$\begin{aligned} \frac{1}{\alpha_K(CP_2)} &= \frac{1}{\alpha_{K,4}} \quad , \\ \frac{1}{\alpha_K(M^4)} &= \frac{\epsilon^2}{\alpha_K(CP_2)} \quad . \end{aligned} \tag{16.4.10}$$

(e) One can deduce constraints on the value of the ϵ^2 from the smallness of the contributions of the corresponding $U(1)$ gauge potential to the ordinary Coulomb potential affecting the energies of atoms by a coupling proportional to mass number A rather than Z as for Coulomb potential. This allows to distinguish between isotopes. This gives very stringent bounds on ϵ^2. I have earlier derived an upper bound treating this term as a perturbation and by considering the contribution to the Coulomb energy of hydrogen atom [L31] (see `http://tinyurl.com/y8xcem2d`). One obtains $\epsilon^2 \leq 10^{-10}$. The upper bound is also the size scale of CP breaking induced by M^4 part and characterizes also matter-antimatter asymmetry.

Cosmological constant

Consider next the prediction for the cosmological constant term.

(a) The S^2 parts of the actions have constant values. The natural normalization of Kähler form of $J(S^2(X))$, $X = M^4, CP_2$ is as $J^2 = -2$. This a convention is the overall scale of normalization can be chosen freely by rescaling $1/\alpha_{K,4}$. Taking into account the fact that index raising is carried out by induced metric one finds that the cosmological term given the sum of M^4 and CP_2 contributions to S^2 part of Kähler action multiplied by $A(S^2)$

$$\Lambda = \frac{1}{16\pi\alpha_K}\frac{2}{(1+r^2)R^2(CP_2)}(1+\frac{\epsilon^2}{r^4}) \quad . \tag{16.4.11}$$

If ϵ is imaginary one can achieve the cancellation giving rise to small cosmological constant.

(b) The empirical condition on cosmological constant (see `https://en.wikipedia.org/wiki/Cosmological_constant`) can be expressed in terms of critical mass density corresponding to flat 3-space as

$$\begin{aligned} \Lambda &= 3\Omega_\Lambda H^2 \quad , & \Omega &\simeq .691 \quad , \\ H &= \frac{da}{dt}a & \frac{da}{dt} &= \frac{1}{\sqrt{g_{aa}}} \quad . \end{aligned} \tag{16.4.12}$$

Here a corresponds to the proper time for the light-cone M^4_+ and t for the proper time for the space-time surface, which is Lorentz invariant under the Lorentz group leaving the boundary δM^4_+.

From this one obtains a condition for allowing to get idea about the discrete evolution of Λ with p-adic length scale occurring in jumps:

$$1+\frac{\epsilon^2}{r^4} = 24\pi\alpha_K(1+r^2)R^2(CP_2)\times \Omega_\Lambda H^2 \quad . \tag{16.4.13}$$

In an excellent approximation one must have $\epsilon \simeq r^2$, $r = R(M^4)/(CP_2)$. One can consider two obvious guesses. One has either $R(M^4) = L_{Pl} = \sqrt{G}$ - that is Planck length - or one has the Compton length associated with Planck mass given by $R(M^4) = 2\pi l_{Pl}$. The first option gives in reasonable approximation $r = 2^{-11}$ and $\epsilon^2 = r^4 = 2^{-44} \sim .6\times 10^{-13}$. The second option gives $\epsilon^2 \simeq .9\times 10^{-10}$. This values corresponds roughly to the CP_2 breaking parameter and matter-antimatter asymmetry and M^4 part of the

Kähler action indeed gives rise to CP_2 breaking. I have earlier derived an upper bound for ϵ by demanding that the Kähler U(1) forces does not give rise to observable effects in the energy levels of hydrogen atom. The upper bound is of the same magnitude as the estimate for ϵ^2 for the Compton scale option.

(c) If one accepts p-adic length scale hypothesis $L_p \propto \sqrt{p}$, $p \simeq 2^k$ [K100], one expects $\Lambda(k) \propto 1/L(k)^2$ [L24] (see `http://tinyurl.com/ybrhguux`). How to achieve this? The only possibility is that the parameter ϵ^2 is subject to coupling constant evolution. One would have for the cosmological constant

$$\Lambda(k) \propto \frac{\epsilon^2}{r^4} - 1 \propto \frac{1}{L^2(k)} \propto 2^{-k} \quad . \tag{16.4.14}$$

This would suggest for the 2-adic coupling constant evolution of ϵ the expression

$$\epsilon^2 = -r^4(1-X) \ , \quad X = 24\pi\alpha_K(1+r^2)R^2(CP_2) \times \Omega_\Lambda H^2 = q \times 2^{-k} \quad . \tag{16.4.15}$$

where q is rational number. Note that from p-adic length scale hypothesis one has $2^{-k} \propto 1/L^2(k)$. One can consider also p-adic primes near powers of small prime in which case one obtains different evolution.

(d) For Ω_Λ constant this would predict quantization of Hubble constant as $\Omega_\Lambda H^2 \propto 1/L(k)^2$ determined by naive scaling dimension. The ratio of Hubble constants for two subsequent scales would be $H(k)/H(k+1) = \sqrt{2}$ if Ω is constant. The observed - and poorly understood - variation of Hubble constant from cosmological studies and distance ladder studies is in the range $50 - 73.2$ km/s/Mpc. Cosmological studies correspond to longer scales so that the smaller value of H is consistent with the decrease of H. The ratio of these upper and lower bounds is $1.46 < \sqrt{2} \simeq 1.141$ (see `http://tinyurl.com/yd6m8sca` and `http://tinyurl.com/ycr4ffm4`).

Remark: The uncertainty in the value of Hubble constant is reflected as uncertainty in the distances D deduced from cosmic redshift $z \simeq HD/c$. This is taken into account in the definition of cosmological distant unit $h^{-1}Mpc$, where h is in the range $.5 - .75$ corresponding to a scale factor 1.5 rather near to $\sqrt{2}$.

(e) Piecewise constant evolution means that acceleration parameter is positive since constant value of H gives

$$\frac{d^2a}{dt^2} = \frac{(da/dt)^2}{a} = aH^2 > 0 \quad . \tag{16.4.16}$$

If the phase transitions reducing H by factor 1/2 occur at $a(k) = 2^{k/2}a_0$, one has

$$\frac{d^2a}{dt^2} \propto 2^{-k/2} \quad . \tag{16.4.17}$$

Acceleration would be reduced gradually with rate determined by its naive scaling dimension.

Part IV

CATEGORY THEORTY AND QUANTUM TGD

Chapter 17

Category Theory and Quantum TGD

17.1 Introduction

TGD predicts several hierarchical structures involving a lot of new physics. These structures look frustratingly complex and category theoretical thinking might help to build a bird's eye view about the situation. I have already earlier considered the question how category theory might be applied in TGD [K13, K11]. Besides the far from complete understanding of the basic mathematical structure of TGD also my own limited understanding of category theoretical ideas have been a serious limitation. During last years considerable progress in the understanding of quantum TGD proper has taken place and the recent formulation of TGD is in terms of light-like 3-surfaces, zero energy ontology and number theoretic braids [K84, K82]. There exist also rather detailed formulations for the fusion of p-adic and real physics and for the dark matter hierarchy. This motivates a fresh look to how category theory might help to understand quantum TGD.

The fusion rules for the symplectic variant of conformal field theory, whose existence is strongly suggested by quantum TGD, allow rather precise description using the basic notions of category theory and one can identify a series of finite-dimensional nilpotent algebras as discretized versions of field algebras defined by the fusion rules. These primitive fusion algebras can be used to construct more complex algebras by replacing any algebra element by a primitive fusion algebra. Trees with arbitrary numbers of branches in any node characterize the resulting collection of fusion algebras forming an operad. One can say that an exact solution of symplectic scalar field theory is obtained.

Conformal fields and symplectic scalar field can be combined to form symplecto-formal fields. The combination of symplectic operad and Feynman graph operad leads to a construction of Feynman diagrams in terms of n-point functions of conformal field theory. M-matrix elements with a finite measurement resolution are expressed in terms of a hierarchy of symplecto-conformal n-point functions such that the improvement of measurement resolution corresponds to an algebra homomorphism mapping conformal fields in given resolution to composite conformal fields in improved resolution. This expresses the idea that composites behave as independent conformal fields. Also other applications are briefly discussed.

Years after writing this chapter a very interesting new TGD related candidate for a category emerged. The preferred extremals would form a category if the proposed duality mapping associative (co-associative) 4-surfaces of imbedding space respects associativity (co-associativity) [K74]. The duality would allow to construct new preferred extremals of Kähler action.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found at `http://tgdtheory.`

fi/cmaphtml.html [L11]. Pdf representation of same files serving as a kind of glossary can be found at http://tgdtheory.fi/tgdglossary.pdf [L12].

17.2 S-Matrix As A Functor

John Baez's [A71] discusses in a physicist friendly manner the possible application of category theory to physics. The lessons obtained from the construction of topological quantum field theories (TQFTs) suggest that category theoretical thinking might be very useful in attempts to construct theories of quantum gravitation.

The point is that the Hilbert spaces associated with the initial and final state n-1-manifold of n-cobordism indeed form in a natural manner category. Morphisms of Hilb in turn are unitary or possibly more general maps between Hilbert spaces. TQFT itself is a functor assigning to a cobordism the counterpart of S-matrix between the Hilbert spaces associated with the initial and final n-1-manifold. The surprising result is that for $n \leq 4$ the S-matrix can be unitary S-matrix only if the cobordism is trivial. This should lead even string theorist to raise some worried questions.

In the hope of feeding some category theoretic thinking into my spine, I briefly summarize some of the category theoretical ideas discussed in the article and relate it to the TGD vision, and after that discuss the worried questions from TGD perspective. That space-time makes sense only relative to imbedding space would conform with category theoretic thinking.

17.2.1 The *-Category Of Hilbert Spaces

Baez considers first the category of Hilbert spaces. Intuitively the definition of this category looks obvious: take linear spaces as objects in category Set, introduce inner product as additional structure and identify morphisms as maps preserving this inner product. In finite-D case the category with inner product is however identical to the linear category so that the inner product does not seem to be absolutely essential. Baez argues that in infinite-D case the morphisms need not be restricted to unitary transformations: one can consider also bounded linear operators as morphisms since they play key role in quantum theory (consider only observables as Hermitian operators). For hyper-finite factors of type II_1 inclusions define very important morphisms which are not unitary transformations but very similar to them. This challenges the belief about the fundamental role of unitarity and raises the question about how to weaken the unitarity condition without losing everything.

The existence of the inner product is essential only for the metric topology of the Hilbert space. Can one do without inner product as an inherent property of state space and reduce it to a morphism? One can indeed express inner product in terms of morphisms from complex numbers to Hilbert space and their conjugates. For any state Ψ of Hilbert space there is a unique morphisms T_Ψ from C to Hilbert space satisfying $T_\Psi(1) = \Psi$. If one assumes that these morphisms have conjugates T_Ψ^* mapping Hilbert space to C, inner products can be defined as morphisms $T_\Phi^* T_\Psi$. The Hermitian conjugates of operators can be defined with respect to this inner product so that one obtains *-category. Reader has probably realized that T_Ψ and its conjugate correspond to ket and bra in Dirac's formalism.

Note that in TGD framework based on hyper-finite factors of type II_1 (HFFs) the inclusions of complex rays might be replaced with inclusions of HFFs with included factor representing the finite measurement resolution. Note also the analogy of inner product with the representation of space-times as 4-surfaces of the imbedding space in TGD.

17.2.2 The Monoidal *-Category Of Hilbert Spaces And Its Counterpart At The Level Of Ncob

One can give the category of Hilbert spaces a structure of monoid by introducing explicitly the tensor products of Hilbert spaces. The interpretation is obvious for physicist. Baez describes the details of this identification, which are far from trivial and in the theory of quantum groups very interesting things happen. A non-commutative quantum version of the tensor product implying braiding is possible and associativity condition leads to the celebrated Yang-Baxter equations: inclusions of HFFs lead to quantum groups too.

At the level of nCob the counterpart of the tensor product is disjoint union of n-1-manifolds. This unavoidably creates the feeling of cosmic loneliness. Am I really a disjoint 3-surface in emptiness which is not vacuum even in the geometric sense? Cannot be true!

This horrifying sensation disappears if n-1-manifolds are n-1-surfaces in some higher-dimensional imbedding space so that there would be at least something between them. I can emit a little baby manifold moving somewhere perhaps being received by some-one somewhere and I can receive radiation from some-one at some distance and in some direction as small baby manifolds making gentle tosses on my face!

This consoling feeling could be seen as one of the deep justifications for identifying fundamental objects as light-like partonic 3-surfaces in TGD framework. Their ends correspond to 2-D partonic surfaces at the boundaries of future or past directed light-cones (states of positive and negative energy respectively) and are indeed disjoint but not in the desperately existential sense as 3-geometries of General Relativity.

This disjointness has also positive aspect in TGD framework. One can identify the color degrees of freedom of partons as those associated with CP_2 degrees of freedom. For instance, SU(3) analogs for rotational states of rigid body become possible. 4-D space-time surfaces as preferred extremals of Kähler action connect the partonic 3-surfaces and bring in classical representation of correlations and thus of interactions. The representation as sub-manifolds makes it also possible to speak about positions of these sub-Universes and about distances between them. The habitants of TGD Universe are maximally free but not completely alone.

17.2.3 TSFT As A Functor

The category theoretic formulation of TQFT relies on a very elegant and general idea. Quantum transition has as a space-time correlate an n-dimensional surface having initial final states as its n-1-dimensional ends. One assigns Hilbert spaces of states to the ends and S-matrix would be a unitary morphism between the ends. This is expressed in terms of the category theoretic language by introducing the category nCob with objects identified as n-1-manifolds and morphisms as cobordisms and *-category Hilb consisting of Hilbert spaces with inner product and morphisms which are bounded linear operators which do not however preserve the unitarity. Note that the morphisms of nCob cannot anymore be identified as maps between n-1-manifolds interpreted as sets with additional structure so that in this case category theory is more powerful than set theory.

TQFT is identified as a functor nCob $\rightarrow$ Hilb assigning to n-1-manifolds Hilbert spaces, and to cobordisms unitary S-matrices in the category Hilb. This looks nice but the surprise is that for $n \leq 4$ unitary S-matrix exists only if the cobordism is trivial so that topology changing transitions are not possible unless one gives up unitarity.

This raises several worried questions.

(a) Does this result mean that in TQFT sense unitary S-matrix for topology changing transitions from a state containing n_i closed strings to a state containing $n_f \neq n_i$ strings does not exist? Could the situation be same also for more general non-topological stringy S-matrices? Could the non-converging perturbation series for S-matrix with finite individual terms matrix fail to no non-perturbative counterpart? Could it be that M-theory is doomed to remain a dream with no hope of being fulfilled?

(b) Should one give up the unitarity condition and require that the theory predicts only the relative probabilities of transitions rather than absolute rates? What the proper generalization of the S-matrix could be?

(c) What is the relevance of this result for quantum TGD?

17.2.4 The Situation Is In TGD Framework

The result about the non-existence of unitary S-matrix for topology changing cobordisms allows new insights about the meaning of the departures of TGD from string models.

Cobordism cannot give interesting selection rules

When I started to work with TGD for more than 28 years ago, one of the first ideas was that one could identify the selection rules of quantum transitions as topological selection rules for cobordisms. Within week or two came the great disappointment: there were practically no selection rules. Could one revive this naive idea? Could the existence of unitary S-matrix force the topological selection rules after all? I am skeptic. If I have understood correctly the discussion of what happens in 4-D case [A47] only the exotic diffeo-structures modify the situation in 4-D case.

Light-like 3-surfaces allow cobordism

In the physically interesting GRT like situation one would expect the cobordism to be mediated by a space-time surface possessing Lorentz signature. This brings in metric and temporal distance. This means complications since one must leave the pure TQFT context. Also the classical dynamics of quantum gravitation brings in strong selection rules related to the dynamics in metric degrees of freedom so that TQFT approach is not expected to be useful from the point of view of quantum gravity and certainly not the limit of a realistic theory of quantum gravitation.

In TGD framework situation is different. 4-D space-time sheets can have Euclidian signature of the induced metric so that Lorentz signature does not pose conditions. The counterparts of cobordisms correspond at fundamental level to light-like 3-surfaces, which are arbitrarily except for the light-likeness condition (the effective 2-dimensionality implies generalized conformal invariance and analogy with 3-D black-holes since 3-D vacuum Einstein equations are satisfied). Field equations defined by the Chern-Simons action imply that CP_2 projection is at most 2-D but this condition holds true only for the extremals and one has functional integral over all light-like 3-surfaces. The temporal distance between points along light-like 3-surface vanishes. The constraints from light-likeness bring in metric degrees of freedom but in a very gentle manner and just to make the theory physically interesting.

Feynman cobordism as opposed to ordinary cobordism

In string model context the discouraging results from TQFT hold true in the category of nCob, which corresponds to trouser diagrams for closed strings or for their open string counterparts. In TGD framework these diagrams are replaced with a direct generalization of Feynman diagrams for which 3-D light-like partonic 3-surfaces meet along their 2-D ends at the vertices. In honor of Feynman one could perhaps speak of Feynman cobordisms. These surfaces are singular as 3-manifolds but vertices are nice 2-manifolds. I contrast to this, in string models diagrams are nice 2-manifolds but vertices are singular as 1-manifolds (say eye-glass type configurations for closed strings).

This picture gains a strong support for the interpretation of fermions as light-like throats associated with connected sums of CP_2 type extremals with space-time sheets with Minkowski signature and of bosons as pairs of light-like wormhole throats associated with CP_2 type extremal connecting two space-time sheets with Minkowski signature of induced metric. The space-time sheets have opposite time orientations so that also zero energy ontology emerges unavoidably. There is also consistency TGD based explanation of the family replication phenomenon in terms of genus of light-like partonic 2-surfaces.

One can wonder what the 4-D space-time sheets associated with the generalized Feynman diagrams could look like? One can try to gain some idea about this by trying to assign 2-D surfaces to ordinary Feynman diagrams having a subset of lines as boundaries. In the case of

2$\rightarrow$2 reaction open string is pinched to a point at vertex. 1$\rightarrow$2 vertex, and quite generally, vertices with odd number of lines, are impossible. The reason is that 1-D manifolds of finite size can have either 0 or 2 ends whereas in higher-D the number of boundary components is arbitrary. What one expects to happen in TGD context is that wormhole throats which are at distance characterized by CP_2 fuse together in the vertex so that some kind of pinches appear also now.

Zero energy ontology

Zero energy ontology gives rise to a second profound distinction between TGD and standard QFT. Physical states are identified as states with vanishing net quantum numbers, in particular energy. Everything is creatable from vacuum - and one could add- by intentional action so that zero energy ontology is profoundly Eastern. Positive *resp.* negative energy parts of states can be identified as states associated with 2-D partonic surfaces at the boundaries of future *resp.* past directed light-cones, whose tips correspond to the arguments of n-point functions. Each incoming/outgoing particle would define a mini-cosmology corresponding to not so big bang/crunch. If the time scale of perception is much shorter than time interval between positive and zero energy states, the ontology looks like the Western positive energy ontology. Bras and kets correspond naturally to the positive and negative energy states and phase conjugation for laser photons making them indeed something which seems to travel in opposite time direction is counterpart for bra-ket duality.

The new element would be quantum measurements performed separately for observables assignable to positive and negative energy states. These measurements would be characterized in terms of Jones inclusions. The state function reduction for the negative energy states could be interpreted as a detection of a particle reaction.

Finite temperature S-matrix defines genuine quantum state in zero energy ontology

In TGD framework one encounters two S-matrix like operators.

(a) U-matrix is the analog of the ordinary S-matrix and constructible in terms of it and orthonormal basis of square roots of density matrices expressible as products of hermitian operators multiplied by unitary S-matrix [K91].

(b) The S-matrix like operator describing what happens in laboratory corresponds to the time-like entanglement coefficients between positive and negative energy parts of the state. Measurement of reaction rates would be a measurement of observables reducing time like entanglement and very much analogous to an ordinary quantum measurement reducing space-like entanglement. There is a finite measurement resolution described by inclusion of HFFs and this means that situation reduces effectively to a finite-dimensional one.

p-Adic thermodynamics strengthened with p-adic length scale hypothesis predicts particle masses with an amazing success. At first the thermodynamical approach seems to be in contradiction with the idea that elementary particles are quantal objects. Unitarity is however *not* necessary if one accepts that only relative probabilities for reductions to pairs of initial and final states interpreted as particle reactions can be measured.

The beneficial implications of unitarity are not lost if one replaces QFT with thermal QFT. Category theoretically this would mean that the time-like entanglement matrix associated with the product of cobordisms is a product of these matrices for the factors. The time parameter in S-matrix would be replaced with a complex time parameter with the imaginary part identified as inverse temperature. Hence the interpretation in terms of time evolution is not lost.

In the theory of hyper-finite factors of type III_1 the partition function for thermal equilibrium states and S-matrix can be neatly fused to a thermal S-matrix for zero energy states and one could introduce p-adic thermodynamics at the level of quantum states. It seems that this picture applies to HFFs by restriction. Therefore the loss of unitarity S-matrix might after all turn to a victory by more or less forcing both zero energy ontology and p-adic thermodynamics. Note that also the presence of factor of type I coming from imbedding space degrees of freedom forces thermal S-matrix.

Time-like entanglement coefficients as a square root of density matrix?

All quantum states do not correspond to thermal states and one can wonder what might be the most general identification of the quantum state in zero energy ontology. Density matrix formalism defines a very general formulation of quantum theory. Since the quantum states in zero energy ontology are analogous to operators, the idea that time-like entanglement coefficients in some sense define a square root of density matrix is rather natural. This would give the defining conditions

$$\begin{aligned} \rho^+ &= SS^\dagger \ , \rho^- = S^\dagger S \ , \\ Tr(\rho^\pm) &= 1 \ . \end{aligned} \tag{17.2.1}$$

$\rho^\pm$ would define density matrix for positive/negative energy states. In the case HFFs of type II_1 one obtains unitary S-matrix and also the analogs of pure quantum states are possible for factors of type I. The numbers $p^+_{m,n} = |S^2_{m,n}|/\rho^+_{m,m}$ and $p^-_{m,n} = |S^2_{n,m}|/\rho^-_{m,m}$ give the counterparts of the usual scattering probabilities.

A physically well-motivated hypothesis would be that S has expression $S = \sqrt{\rho}S_0$ such that S_0 is a universal unitary S-matrix, and $\sqrt{\rho}$ is square root of a state dependent density matrix. Note that in general S is not diagonalizable in the algebraic extension involved so that it is not possible to reduce the scattering to a mere phase change by a suitable choice of state basis.

What makes this kind of hypothesis aesthetically attractive is the unification of two fundamental matrices of quantum theory to single one. This unification is completely analogous to the combination of modulus squared and phase of complex number to a single complex number: complex valued Schrödinger amplitude is replaced with operator valued one.

S-matrix as a functor and the groupoid structure formed by S-matrices

In zero energy ontology S-matrix can be seen as a functor from the category of Feynman cobordisms to the category of operators. S-matrix can be identified as a "square root" of the positive energy density matrix $S = \rho_+^{1/2}S_0$, where S_0 is a unitary matrix and ρ_+ is the density matrix for positive energy part of the zero energy state. Obviously one has $SS^\dagger = \rho_+$. $S^\dagger S = \rho_-$ gives the density matrix for negative energy part of zero energy state. Clearly, S-matrix can be seen as matrix valued generalization of Schrödinger amplitude. Note that the "indices" of the S-matrices correspond to WCW spinor s (fermions and their bound states giving rise to gauge bosons and gravitons) and to WCW degrees of freedom. For hyper-finite factor of II_1 it is not strictly speaking possible to speak about indices since the matrix elements are traces of the S-matrix multiplied by projection operators to infinite-dimensional subspaces from right and left.

The functor property of S-matrices implies that they form a multiplicative structure analogous but not identical to groupoid [A6]. Recall that groupoid has associative product and there exist always right and left inverses and identity in the sense that ff^{-1} and $f^{-1}f$ are always defined but not identical and one has $fgg^{-1} = f$ and $f^{-1}fg = g$.

The reason for the groupoid like property is that S-matrix is a map between state spaces associated with initial and final sets of partonic surfaces and these state spaces are different so that inverse must be replaced with right and left inverse. The defining conditions for groupoid are replaced with more general ones. Also now associativity holds but the role of inverse is taken by hermitian conjugate. Thus one has the conditions $fgg^\dagger = f\rho_{g,+}$ and $f^\dagger fg = \rho_{f,-}g$, and the conditions $ff^\dagger = \rho_+$ and $f^\dagger f = \rho_-$ are satisfied. Here $\rho_\pm$ is density matrix associated with positive/negative energy parts of zero energy state. If the inverses of the density matrices exist, groupoid axioms hold true since $f_L^{-1} = f^\dagger \rho_{f,+}^{-1}$ satisfies $ff_L^{-1} = Id_+$ and $f_R^{-1} = \rho_{f,-}^{-1}f^\dagger$ satisfies $f_R^{-1}f = Id_-$.

There are good reasons to believe that also tensor product of its appropriate generalization to the analog of co-product makes sense with non-triviality characterizing the interaction between the systems of the tensor product. If so, the S-matrices would form very beautiful mathematical structure bringing in mind the corresponding structures for 2-tangles and N-tangles. Knowing how incredibly powerful the group like structures have been in physics one has good reasons to hope that groupoid like structure might help to deduce a lot of information about the quantum dynamics of TGD.

A word about nomenclature is in order. S has strong associations to unitarity and it might be appropriate to replace S with some other letter. The interpretation of S-matrix as a generalized Schrödinger amplitude would suggest Ψ-matrix. Since the interaction with Kea's M-theory blog at (see `http://tinyurl.com/yb3lsbjq` (M denotes Monad or Motif in this context) was led ot the realization of the connection with density matrix, also M-matrix might be considered. S-matrix as a functor from the category of Feynman cobordisms in turn suggests C or F. Or could just Matrix denoted by M in formulas be enough? Certainly it would inspire feeling of awe!

17.3 Further Ideas

The work of John Baez and students has inspired also the following ideas about the role of category theory in TGD.

17.3.1 Operads, Number Theoretical Braids, And Inclusions Of HFFs

The description of braids leads naturally to category theory and quantum groups when the braiding operation, which can be regarded as a functor, is not a mere permutation. Discreteness is a natural notion in the category theoretical context. To me the most natural manner to interpret discreteness is - not something emerging in Planck scale- but as a correlate for a finite measurement resolution and quantum measurement theory with finite measurement resolution leads naturally to number theoretical braids as fundamental discrete structures so that category theoretic approach becomes well-motivated. Discreteness is also implied by the number theoretic approach to quantum TGD from number theoretic associativity condition [L7] central also for category theoretical thinking as well as from the realization of number theoretical universality by the fusion of real and p-adic physics to single coherent whole.

Operads are formally single object multi-categories [A15, A77]. This object consist of an infinite sequence of sets of n-ary operations. These operations can be composed and the compositions are associative (operations themselves need not be associative) in the sense that the is natural isomorphism (symmetries) mapping differently bracketed compositions to each other. The coherence laws for operads formulate the effect of permutations and bracketing (association) as functors acting as natural isomorphisms. A simple manner to visualize the composition is as an addition of $n_1, ...n_k$ leaves to the leaves $1, ..., k$ of k-leaved tree.

An interesting example of operad is the braid operad formulating the combinatorics for a hierarchy of braids formed from braids by grouping subsets of braids having $n_1, ...n_k$ strands and defining the strands of a k-braid. In TGD framework this grouping can be identified in terms of the formation bound states of particles topologically condensed at larger space-time sheet and coherence laws allow to deduce information about scattering amplitudes. In conformal theories braided categories indeed allow to understand duality of stringy amplitudes in terms of associativity condition.

Planar operads [A39] define an especially interesting class of operads. The reason is that the inclusions of HFFs give rise to a special kind of planar operad [A16]. The object of this multi-category [A13] consists of planar k-tangles. Planar operads are accompanied by planar algebras. It will be found that planar operads allow a generalization which could provide a description for the combinatorics of the generalized Feynman diagrams and also rigorous formulation for how the arrow of time emerges in TGD framework and related heuristic ideas challenging the standard views.

17.3.2 Generalized Feynman Diagram As Category?

John Baez has proposed a category theoretical formulation of quantum field theory as a functor from the category of n-cobordisms to the category of Hilbert spaces [A71, A38]. The attempt to generalize this formulation looks well motivated in TGD framework because TGD can be regarded as almost topological quantum field theory in a well defined sense and braids appear as fundamental structures. It however seems that formulation as a functor from nCob to Hilb is not general enough.

In zero energy ontology events of ordinary ontology become quantum states with positive and negative energy parts of quantum states localizable to the upper and lower light-like boundaries of causal diamond (CD).

(a) Generalized Feynman diagrams associated with a given CD involve quantum superposition of light-like 3-surfaces corresponding to given generalized Feynman diagram. These superpositions could be seen as categories with 3-D light-like surfaces containing braids as arrows and 2-D vertices as objects. Zero energy states would represent quantum superposition of categories (different topologies of generalized Feynman diagram) and M-matrix defined as Connes tensor product would define a functor from this category to the Hilbert space of zero energy states for given CD (tensor product defines quite generally a functor).

(b) What is new from the point of view of physics that the sequences of generalized lines would define compositions of arrows and morphisms having identification in terms of braids which replicate in vertices. The possible interpretation of the replication is in terms of copying of information in classical sense so that even elementary particles would be information carrying and processing structures. This structure would be more general than the proposal of John Baez that S-matrix corresponds to a function from the category of n-dimensional cobordisms to the category Hilb.

(c) p-Adic length scale hypothesis follows if the temporal distance between the tips of CD measured as light-cone proper time comes as an octave of CP_2 time scale: $T = 2^n T_0$. This assumption implies that the p-adic length scale resolution interpreted in terms of a hierarchy of increasing measurement resolutions comes as octaves of time scale. A weaker condition would be $T_p = pT_0$, p prime, and would assign all p-adic time scales to the size scale hierarchy of CDs.

This preliminary picture is of course not far complete since it applies only to single CD. There are several questions. Can one allow CDs within CDs and is every vertex of generalized Feynman diagram surrounded by this kind of CD. Can one form unions of CDs freely?

(a) Since light-like 3-surfaces in 8-D imbedding space have no intersections in the generic position, one could argue that the overlap must be allowed and makes possible the interaction of between zero energy states belonging to different CDs. This interaction would be something new and present also for sub-CDs of a given CD.

(b) The simplest guess is that the unrestricted union of CDs defines the counterpart of tensor product at geometric level and that extended M-matrix is a functor from this category to the tensor product of zero energy state spaces. For non-overlapping CDs ordinary tensor product could be in question and for overlapping CDs tensor product would be non-trivial. One could interpret this M-matrix as an arrow between M-matrices of zero energy states at different CDs: the analog of natural transformation mapping two functors to each other. This hierarchy could be continued ad infinitum and would correspond to the hierarchy of n-categories.

This rough heuristics represents of course only one possibility among many since the notion of category is extremely general and the only limits are posed by the imagination of the mathematician. Also the view about zero energy states is still rather primitive.

17.4 Planar Operads, The Notion Of Finite Measurement Resolution, And Arrow Of Geometric Time

In the sequel the idea that planar operads or their appropriate generalization might allow to formulate generalized Feynman diagrammatics in zero energy ontology will be considered. Also a description of measurement resolution and arrow of geometric time in terms of operads is discussed.

17.4.1 Zeroth Order Heuristics About Zero Energy States

Consider now the existing heuristic picture about the zero energy states and coupling constant evolution provided by CDs.

(a) The tentative description for the increase of the measurement resolution in terms CDs is that one inserts to the upper and/or lower light-like boundary of CD smaller CDs by gluing them along light-like radial ray from the tip of CD. It is also possible that the vertices of generalized Feynman diagrams belong inside smaller CD: s and it turns out that these CD: s must be allowed.

(b) The considerations related to the arrow of geometric time suggest that there is asymmetry between upper and lower boundaries of CD. The minimum requirement is that the measurement resolution is better at upper light-like boundary.

(c) In zero energy ontology communications to the direction of geometric past are possible and phase conjugate laser photons represent one example of this.

(d) Second law of thermodynamics must be generalized in such a manner that it holds with respect to subjective time identified as sequence of quantum jumps. The arrow of geometric time can however vary so that apparent breaking of second law is possible in shorter time scales at least. One must however understand why second law holds true in so good an approximation.

(e) One must understand also why the contents of sensory experience is concentrated around a narrow time interval whereas the time scale of memories and anticipation are much longer. The proposed mechanism is that the resolution of conscious experience is higher at the upper boundary of CD. Since zero energy states correspond to light-like 3-surfaces, this could be a result of self-organization rather than a fundamental physical law.

   i. CDs define the perceptive field for self. Selves are curious about the space-time sheets outside their perceptive field in the geometric future of the imbedding space and perform quantum jumps tending to shift the superposition of the space-time sheets to the direction of geometric past (past defined as the direction of shift!). This creates the illusion that there is a time=snapshot front of consciousness moving to geometric future in fixed background space-time as an analog of train illusion.

   ii. The fact that news come from the upper boundary of CD implies that self concentrates its attention to this region and improves the resolutions of sensory experience and quantum measurement here. The sub-CD: s generated in this manner correspond to mental images with contents about this region. As a consequence, the contents of conscious experience, in particular sensory experience, tend to be about the region near the upper boundary.

iii. This mechanism in principle allows the arrow of the geometric time to vary and depend on p-adic length scale and the level of dark matter hierarchy. The occurrence of phase transitions forcing the arrow of geometric time to be same everywhere are however plausible for the reason that the lower and upper boundaries of given CD must possess the same arrow of geometric time.

iv. If this is the mechanism behind the arrow of time, planar operads can provide a description of the arrow of time but not its explanation.

This picture is certainly not general enough, can be wrong at the level of details, and at best relates to the whole like single particle wave mechanics to quantum field theory.

17.4.2 Planar Operads

The geometric definition of planar operads [A17, A15, A16, A39] without using the category theoretical jargon goes as follows.

(a) There is an external disk and some internal disks and a collection of disjoint lines connecting disk boundaries.

(b) To each disk one attaches a non-negative integer k, called the color of disk. The disk with color k has k points at each boundary with the labeling $1, 2, ...k$ running clockwise and starting from a distinguished marked point, decorated by "*". A more restrictive definition is that disk colors are correspond to even numbers so that there are $k = 2n$ points lines leaving the disk boundary boundary. The planar tangles with $k = 2n$ correspond to inclusions of HFFs.

(c) Each curve is either closed (no common points with disk boundaries) or joins a marked point to another marked point. Each marked point is the end point of exactly one curve.

(d) The picture is planar meaning that the curves cannot intersect and diks cannot overlap.

(e) Disks differing by isotopies preserving *'s are equivalent.

Given a planar k-tangle-one of whose internal disks has color k_i- and a k_i-tangle S, one can define the tangle $T \circ_i S$ by isotoping S so that its boundary, together with the marked points and the *'s co-incides with that of D_i and after that erase the boundary of D_i. The collection of planar tangle together with the composition defined in this manner- is called the colored operad of planar tangles.

One can consider also generalizations of planar operads.

(a) The composition law is not affected if the lines of operads branch outside the disks. Branching could be allowed even at the boundaries of the disks although this does not correspond to a generic situation. One might call these operads branched operads.

(b) The composition law could be generalized to allow additional lines connecting the points at the boundary of the added disk so that each composition would bring in something genuinely new. Zero energy insertion could correspond to this kind of insertions.

(c) TGD picture suggests also the replacement of lines with braids. In category theoretical terms this means that besides association one allows also permutations of the points at the boundaries of the disks.

The question is whether planar operads or their appropriate generalizations could allow a characterization of the generalized Feynman diagrams representing the combinatorics of zero energy states in zero energy ontology and whether also the emergence of arrow of time could be described (but probably not explained) in this framework.

17.4.3 Planar Operads And Zero Energy States

Are planar operads sufficiently powerful to code the vision about the geometric correlates for the increase of the measurement resolution and coupling constant evolution formulated in terms of CDs? Or perhaps more realistically, could one improve this formulation by assuming that zero energy states correspond to wave functions in the space of planar tangles or of appropriate modifications of them? It seems that the answer to the first question is almost affirmative.

(a) Disks are analogous to the white regions of a map whose details are not visible in the measurement resolution used. Disks correspond to causal diamonds (CDs) in zero energy ontology. Physically the white regions relate to the vertices of the generalized Feynman diagrams and possibly also to the initial and final states (strictly speaking, the initial and final states correspond to the legs of generalized Feynman diagrams rather than their ends).

(b) The composition of tangles means addition of previously unknown details to a given white region of the map and thus to an increase of the measurement resolution. This conforms with the interpretation of inclusions of HFFs as a characterization of finite measurement resolution and raises the hope that planar operads or their appropriate generalization could provide the proper language to describe coupling constant evolution and their perhaps even generalized Feynman diagrams.

(c) For planar operad there is an asymmetry between the outer disk and inner disks. One might hope that this asymmetry could explain or at least allow to describe the arrow of time. This is not the case. If the disks correspond to causal diamonds (CDs) carrying positive *resp.* negative energy part of zero energy state at upper *resp.* lower light-cone boundary, the TGD counterpart of the planar tangle is CD containing smaller CD: s inside it. The smaller CD: s contain negative energy particles at their upper boundary and positive energy particles at their lower boundary. In the ideal resolution vertices represented 2-dimensional partonic at which light-like 3-surfaces meet become visible. There is no inherent asymmetry between positive and negative energies and no inherent arrow of geometric time at the fundamental level. It is however possible to model the arrow of time by the distribution of sub-CD: s. By previous arguments self-organization of selves can lead to zero energy states for which the measurement resolution is better near the upper boundary of the CD.

(d) If the lines carry fermion or anti-fermion number, the number of lines entering to a given CD must be even as in the case of planar operads as the following argument shows.

   i. In TGD framework elementary fermions correspond to single wormhole throat associated with topologically condensed CP_2 type extremal and the signature of the induced metric changes at the throat.

   ii. Elementary bosons correspond to pairs of wormhole throats associated with wormhole contacts connecting two space-time sheets of opposite time orientation and modellable as a piece of CP_2 type extremal. Each boson therefore corresponds to 2 lines within CP_2 radius.

   iii. As a consequence the total number of lines associated with given CD is even and the generalized Feynman diagrams can correspond to a planar algebra associated with an inclusion of HFFs.

(e) This picture does not yet describe zero energy insertions.

   i. The addition of zero energy insertions corresponds intuitively to the allowance of new lines inside the smaller CD: s not coming from the exterior. The addition of lines connecting points at the boundary of disk is possible without losing the basic geometric composition of operads. In particular one does not lose the possibility to color the added tangle using two colors (colors correspond to two groups G and H which characterize an inclusion of HFFs [A39]).

   ii. There is however a problem. One cannot remove the boundaries of sub-CD after the composition of CDs since this would give lines beginning from and ending to the interior of disk and they are invisible only in the original resolution. Physically this is of course what one wants but the inclusion of planar tangles is expected to fail in its original form, and one must generalize the composition of tangles to that of CD: s so that the boundaries of sub-CD: s are not thrown away in the process.

iii. It is easy to see that zero energy insertions are inconsistent with the composition of planar tangles. In the inclusion defining the composition of tangles both sub-tangle and tangle induce a color to a given segment of the inner disk. If these colors are identical, one can forget the presence of the boundary of the added tangle. When zero energy insertions are allowed, situation changes as is easy to see by adding a line connecting points in a segment of given color at the boundary of the included tangle. There exists no consistent coloring of the resulting structure by using only two colors. Coloring is however possible using four colors, which by four-color theorem is the minimum number of colors needed for a coloring of planar map: this however requires that the color can change as one moves through the boundary of the included disk - this is in accordance with the physical picture.

iv. Physical intuition suggests that zero energy insertion as an improvement of measurement resolution maps to an improved color resolution and that the composition of tangles generalizes by requiring that the included disk is colored by using new nuances of the original colors. The role of groups in the definition of inclusions of HFFs is consistent with idea that G and H describe color resolution in the sense that the colors obtained by their action cannot be resolved. If so, the improved resolution means that G and H are replaced by their subgroups $G_1 \subset G$ and $H_1 \subset H$. Since the elements of a subgroup have interpretation as elements of group, there are good hopes that by representing the inclusion of tangles as inclusion of groups, one can generalize the composition of tangles.

(f) Also CD: s glued along light-like ray to the upper and lower boundaries of CD are possible in principle and -according the original proposal- correspond to zero energy insertions according. These CD: s might be associated with the phase transitions changing the value of $\hbar$ leading to different pages of the book like structure defined by the generalized imbedding space.

(g) p-Adic length scale hypothesis is realized if the hierarchy of CDs corresponds to a hierarchy of temporal distances between tips of CDs given as $a = T_n = 2^{-n}T_0$ using light-cone proper time.

(h) How this description relates to braiding? Each line corresponds to an orbit of a partonic boundary component and in principle one must allow internal states containing arbitrarily high fermion and anti-fermion numbers. Thus the lines decompose into braids and one must allow also braids of braids hierarchy so that each line corresponds to a braid operad in improved resolution.

17.4.4 Relationship To Ordinary Feynman Diagrammatics

The proposed description is not equivalent with the description based on ordinary Feynman diagrams.

(a) In standard physics framework the resolution scale at the level of vertices of Feynman diagrams is something which one is forced to pose in practical calculations but cannot pose at will as opposed to the measurement resolution. Light-like 3-surfaces can be however regarded only locally orbits of partonic 2-surfaces since generalized conformal invariance is true only in 3-D patches of the light-like 3-surface. This means that light-like 3-surfaces are in principle the fundamental objects so that zero energy states can be regarded only locally as a time evolutions. Therefore measurement resolution can be applied also to the distances between vertices of generalized Feynman diagrams and calculational resolution corresponds to physical resolution. Also the resolution can be better towards upper boundary of CD so that the arrow of geometric time can be understood. This is a definite prediction which can in principle kill the proposed scenario.

(b) A further counter argument is that generalized Feynman diagrams are identified as light-like 3-surfaces for which Kähler function defined by a preferred extremal of Kähler action is maximum. Therefore one cannot pose any ad hoc rules on the positions of the vertices. One can of course insist that maximum of Kähler function with the constraint posed by $T_n = 2^n T_0$ (or $T_p = p^n T_0$) hierarchy is in question.

It would be too optimistic to believe that the details of the proposal are correct. However, if the proposal is on correct track, zero energy states could be seen as wave functions in the operad of generalized tangles (zero energy insertions and braiding) as far as combinatorics is involved and the coherence rules for these operads would give strong constraints on the zero energy state and fix the general structure of coupling constant evolution.

17.5 Category Theory And Symplectic QFT

Besides the counterpart of the ordinary Kac-Moody invariance quantum TGD possesses so called super-symplectic conformal invariance. This symmetry leads to the proposal that a symplectic variant of conformal field theory should exist. The n-point functions of this theory defined in S^2 should be expressible in terms of symplectic areas of triangles assignable to a set of n-points and satisfy the duality rules of conformal field theories guaranteeing associativity. The crucial prediction is that symplectic n-point functions vanish whenever two arguments co-incide. This provides a mechanism guaranteeing the finiteness of quantum TGD implied by very general arguments relying on non-locality of the theory at the level of 3-D surfaces.

The classical picture suggests that the generators of the fusion algebra formed by fields at different point of S^2 have this point as a continuous index. Finite quantum measurement resolution and category theoretic thinking in turn suggest that only the points of S^2 corresponding the strands of number theoretic braids are involved. It turns out that the category theoretic option works and leads to an explicit hierarchy of fusion algebras forming a good candidate for so called little disk operad whereas the first option has difficulties.

17.5.1 Fusion Rules

Symplectic fusion rules are non-local and express the product of fields at two points s_k an s_l of S^2 as an integral over fields at point s_r, where integral can be taken over entire S^2 or possibly also over a 1-D curve which is symplectic invariant in some sense. Also discretized version of fusion rules makes sense and is expected serve as a correlate for finite measurement resolution.

By using the fusion rules one can reduce n-point functions to convolutions of 3-point functions involving a sequence of triangles such that two subsequent triangles have one vertex in common. For instance, 4-point function reduces to an expression in which one integrates over the positions of the common vertex of two triangles whose other vertices have fixed. For n-point functions one has n-3 freely varying intermediate points in the representation in terms of 3-point functions.

The application of fusion rules assigns to a line segment connecting the two points s_k and s_l a triangle spanned by s_k, s_l and s_r. This triangle should be symplectic invariant in some sense and its symplectic area A_{klm} would define the basic variable in terms of which the fusion rule could be expressed as $C_{klm} = f(A_{klm})$, where f is fixed by some constraints. Note that A_{klm} has also interpretations as solid angle and magnetic flux.

17.5.2 What Conditions Could Fix The Symplectic Triangles?

The basic question is how to identify the symplectic triangles. The basic criterion is certainly the symplectic invariance: if one has found N-D symplectic algebra, symplectic transformations of S^2 must provide a new one. This is guaranteed if the areas of the symplectic triangles remain invariant under symplectic transformations. The questions are how to realize this condition and whether it might be replaced with a weaker one. There are two approaches to the problem.

Physics inspired approach

In the first approach inspired by classical physics symplectic invariance for the edges is interpreted in the sense that they correspond to the orbits of a charged particle in a magnetic field defined by the Kähler form. Symplectic transformation induces only a $U(1)$ gauge transformation and leaves the orbit of the charged particle invariant if the vertices are not affected since symplectic transformations are not allowed to act on the orbit directly in this approach. The general functional form of the structure constants C_{klm} as a function $f(A_{klm})$ of the symplectic area should guarantee fusion rules.

If the action of the symplectic transformations does not affect the areas of the symplectic triangles, the construction is invariant under general symplectic transformations. In the case of uncharged particle this is not the case since the edges are pieces of geodesics: in this case however fusion algebra however trivializes so that one cannot conclude anything. In the case of charged particle one might hope that the area remains invariant under general symplectic transformations whose action is induced from the action on vertices. The equations of motion for a charged particle involve a Kähler metric determined by the symplectic structure and one might hope that this is enough to achieve this miracle. If this is not the case - as it might well be - one might hope that although the areas of the triangles are not preserved, the triangles are mapped to each other in such a manner that the fusion algebra rules remain intact with a proper choice of the function $f(A_{klm})$. One could also consider the possibility that the function $f(A_{klm})$ is dictated from the condition that the it remains invariant under symplectic transformations. It however turns that this approach does not work as such.

Category theoretical approach

The second realization is guided by the basic idea of category theoretic thinking: the properties of an object are determined its relationships to other objects. Rather than postulating that the symplectic triangle is something which depends solely on the three points involved via some geometric notion like that of geodesic line of orbit of charged particle in magnetic field, one assumes that the symplectic triangle reflects the properties of the fusion algebra, that is the relations of the symplectic triangle to other symplectic triangles. Thus one must assign to each triplet (s_1, s_2, s_3) of points of S^2 a triangle just from the requirement that braided associativity holds true for the fusion algebra.

All symplectic transformations leaving the N points fixed and thus generated by Hamiltonians vanishing at these points would give new gauge equivalent realizations of the fusion algebra and deform the edges of the symplectic triangles without affecting their area. One could even say that symplectic triangulation defines a new kind geometric structure in S^2. The quantum fluctuating degrees of freedom are parameterized by the symplectic group of $S^2 \times CP_2$ in TGD so that symplectic the geometric representation of the triangulation changes but its inherent properties remain invariant.

The elegant feature of category theoretical approach is that one can in principle construct the fusion algebra without any reference to its geometric realization just from the braided associativity and nilpotency conditions and after that search for the geometric realizations. Fusion algebra has also a hierarchy of discrete variants in which the integral over intermediate points in fusion is replaced by a sum over a fixed discrete set of points and this variant is what finite measurement resolution implies. In this case it is relatively easy to see if the geometric realization of a given abstract fusion algebra is possible.

The notion of number theoretical braid

Braids -not necessary number theoretical- provide a realization discretization as a space-time correlate for the finite measurement resolution. The notion of braid was inspired by the idea about quantum TGD as almost topological quantum field theory. Although the original form of this idea has been buried, the notion of braid has survived: in the decomposition of space-time sheets to string world sheets, the ends of strings define representatives for braid strands at light-like 3-surfaces.

The notion of number theoretic universality inspired the much more restrictive notion of number theoretic braid requiring that the points in the intersection of the braid with the partonic 2-surface correspond to rational or at most algebraic points of H in preferred coordinates fixed by symmetry considerations. The challenge has been to find a unique identification of the number theoretic braid or at least of the end points of the braid. The following consideration suggest that the number theoretic braids are not a useful notion in the generic case but make sense and are needed in the intersection of real and p-adic worlds which is in crucial role in TGD based vision about living matter [K41].

It is only the braiding that matters in topological quantum field theories used to classify braids. Hence braid should require only the fixing of the end points of the braids at the intersection of the braid at the light-like boundaries of CDs and the braiding equivalence class of the braid itself. Therefore it is enough is to specify the topology of the braid and the end points of the braid in accordance with the attribute "number theoretic". Of course, the condition that all points of the strand of the number theoretic braid are algebraic is impossible to satisfy.

The situation in which the equations defining X^2 make sense both in real sense and p-adic sense using appropriate algebraic extension of p-adic number field is central in the TGD based vision about living matter [K41]. The reason is that in this case the notion of number entanglement theoretic entropy having negative values makes sense and entanglement becomes information carrying. This motivates the identification of life as something in the intersection of real and p-adic worlds. In this situation the identification of the ends of the number theoretic braid as points belonging to the intersection of real and p-adic worlds is natural. These points -call them briefly algebraic points- belong to the algebraic extension of rationals needed to define the algebraic extension of p-adic numbers. This definition however makes sense also when the equations defining the partonic 2-surfaces fail to make sense in both real and p-adic sense. In the generic case the set of points satisfying the conditions is discrete. For instance, according to Fermat's theorem the set of rational points satisfying $X^n + Y^n = Z^n$ reduces to the point $(0, 0, 0)$ for $n = 3, 4, ...$ Hence the constraint might be quite enough in the intersection of real and p-adic worlds where the choice of the algebraic extension is unique.

One can however criticize this proposal.

(a) One must fix the number of points of the braid and outside the intersection and the non-uniquencess of the algebraic extension makes the situation problematic. Physical intuition suggests that the points of braid define carriers of quantum numbers assignable to second quantized induced spinor fields so that the total number of fermions anti-fermions would define the number of braids. In the intersection the highly non-trivial implication is that this number cannot exceed the number of algebraic points.

(b) In the generic case one expects that even the smallest deformation of the partonic 2-surface can change the number of algebraic points and also the character of the algebraic extension of rational numbers needed. The restriction to rational points is not expected to help in the generic case. If the notion of number theoretical braid is meant to be practical, must be able to decompose WCW to open sets inside which the numbers of algebraic points of braid at its ends are constant. For real topology this is expected to be impossible and it does not make sense to use p-adic topology for WCW whose points do not allow interpretation as p-adic partonic surfaces.

(c) In the intersection of real and p-adic worlds which corresponds to a discrete subset of WCW , the situation is different. Since the coefficients of polynomials involved with the definition of the partonic 2-surface must be rational or at most algebraic, continuous deformations are not possible so that one avoids the problem.

(d) This forces to ask the reason why for the number theoretic braids. In the generic case they seem to produce only troubles. In the intersection of real and p-adic worlds they could however allow the construction of the elements of M-matrix describing quantum transitions changing p-adic to real surfaces and vice versa as realizations of intentions and generation of cognitions. In this the case it is natural that only the data from the intersection of the two worlds are used. In [K41] I have sketched the idea about number theoretic quantum field theory as a description of intentional action and cognition.

There is also the problem of fixing the interior points of the braid modulo deformations not affecting the topology of the braid.

(a) Infinite number of non-equivalent braidings are possible. Should one allow all possible braidings for a fixed light-like 3-surface and say that their existence is what makes the dynamics essentially three-dimensional even in the topological sense? In this case there would be no problems with the condition that the points at both ends of braid are algebraic.

(b) Or should one try to characterize the braiding uniquely for a given partonic 2-surfaces and corresponding 4-D tangent space distributions? The slicing of the space-time sheet by partonic 2-surfaces and string word sheets suggests that the ends of string world sheets could define the braid strands in the generic context when there is no algebraicity condition involved. This could be taken as a very natural manner to fix the topology of braid but leave the freedom to choose the representative for the braid. In the intersection of real and p-adic worlds there is no good reason for the end points of strands in this case to be algebraic at both ends of the string world sheet. One can however start from the braid defined by the end points of string world sheets, restrict the end points to be algebraic at the end with a smaller number of algebraic pointsandthen perform a topologically non-trivial deformation of the braid so that also the points at the other end are algebraic? Non-trivial deformations need not be possible for all possible choices of algebraic braid points at the other end of braid and different choices of the set of algebraic points would give rise to different braidings. A further constraint is that only the algebraic points at which one has assign fermion or anti-fermion are used so that the number of braid points is not always maximal.

(c) One can also ask whether one should perform the gauge fixing for the strands of the number theoretic braid using algebraic functions making sense both in real and p-adic context. This question does not seem terribly relevant since since it is only the topology of the braid that matters.

Symplectic triangulations and braids

The identification of the edges of the symplectic triangulation as the end points of the braid is favored by conceptual economy. The nodes of the symplectic triangulation would naturally correspond to the points in the intersection of the braid with the light-like boundaries of CD carrying fermion or anti-fermion number. The number of these points could be arbitrarily large in the generic case but in the intersection of real and p-adic worlds these points correspond to subset of algebraic points belonging to the algebraic extension of rationals associated with the definition of partonic 2-surfaces so that the sum of fermion and anti-fermion numbers would be bounded above. The presence of fermions in the nodes would be the physical prerequisite for measuring the phase factors defined by the magnetic fluxes. This could be understood in terms of gauge invariance forcing to assign to a pair of points of triangulation the non-integrable phase factor defined by the Kähler gauge potential.

The remaining problem is how uniquely the edges of the triangulation can be determined.

(a) The allowance of all possible choices for edges would bring in an infinite number of degrees of freedom. These curves would be analogous to freely vibrating strings. This option is not attractive. One should be able to pose conditions on edges and whatever the manner to specify the edges might be, it must make sense also in the intersection of real and p-adic worlds. In this case the total phase factor must be a root of unity in the algebraic extension of rationals involved and this poses quantization rules analogous to those for magnetic flux. The strongest condition is that the edges are such that the non-integrable phase factor is a root of unity for each edge. It will be found that similar quantization is implied also by the associativity conditions and this justifies the interpretation of phase factors defining the fusion algebra in terms of the Kähler magnetic fluxes. This would pose strong constraints on the choice of edges but would not fix completely the phase factors, and it seems that one must allow all possible triangulations consistent with this condition and the associativity conditions so that physical state is a quantum superposition over all possible symplectic triangulations characterized by the fusion algebras.

(b) In the real context one would have an infinite hierarchy of symplectic triangulations and fusion algebras satisfying the associativity conditions with the number of edges equal to the total number N of fermions and anti-fermions. Encouragingly, this hierarchy corresponds also to a hierarchy of $\mathcal{N} = N$ SUSY algebras [K24] (large values of $\mathcal{N}$ are not a catastrophe in TGD framework since the physical content of SUSY symmetry is not the same as that in the standard approach). In the intersection of real and p-adic worlds the value of $\mathcal{N}$ would be bounded by the total number of algebraic points. Hence the notion of finite measurement resolution, cutoff in $\mathcal{N}$ and bound on the total fermion number would make physics very simple in the intersection of real and p-adic worlds.

Two kinds of symplectic triangulations are possible since one can use the symplectic forms associated with CP_2 and $r_M = constant$ sphere S^2 of light-cone boundary. For a given collection of nodes the choices of edges could be different for these two kinds of triangulations. Physical state would be proportional to the product of the phase factors assigned to these triangulations.

17.5.3 Associativity Conditions And Braiding

The generalized fusion rules follow from the associativity condition for n-point functions modulo phase factor if one requires that the factor assignable to n-point function has interpretation as n-point function. Without this condition associativity would be trivially satisfied by using a product of various bracketing structures for the n fields appearing in the n-point function. In conformal field theories the phase factor defining the associator is expressible in terms of the phase factor associated with permutations represented as braidings and the same is expected to be true also now.

(a) Already in the case of 4-point function there are three different choices corresponding to the 4 possibilities to connect the fixed points s_k and the varying point s_r by lines. The options are (1-2, 3-4), (1-3, 2-4), and (1-4, 2-3) and graphically they correspond to s-, t-, and u-channels in string diagrams satisfying also this kind of fusion rules. The basic condition would be that same amplitude results irrespective of the choice made. The duality conditions guarantee associativity in the formation of the n-point amplitudes without any further assumptions. The reason is that the writing explicitly the expression for a particular bracketing of n-point function always leads to some bracketing of one particular 4-point function and if duality conditions hold true, the associativity holds true in general. To be precise, in quantum theory associativity must hold true only in projective sense, that is only modulo a phase factor.

(b) This framework encourages category theoretic approach. Besides different bracketing there are different permutations of the vertices of the triangle. These permutations can induce a phase factor to the amplitude so that braid group representations are enough. If one has representation for the basic braiding operation as a quantum phase $q = exp(i2\pi/N)$, the phase factors relating different bracketings reduce to a product of these phase factors since $(AB)C$ is obtained from $A(BC)$ by a cyclic permutation involving to permutations represented as a braiding. Yang-Baxter equations express the reduction of associator to braidings. In the general category theoretical setting associators and braidings correspond to natural isomorphisms leaving category theoretical structure invariant.

(c) By combining the duality rules with the condition that 4-point amplitude vanishes, when any two points co-incide, one obtains from $s_k = s_l$ and $s_m = s_n$ the condition stating that the sum (or integral in possibly existing continuum version) of $U^2(A_{klm})|f|^2(x_{kmr})$ over the third point s_r vanishes. This requires that the phase factor U is non-trivial so that Q must be non-vanishing if one accepts the identification of the phase factor as Bohm-Aharonov phase.

(d) Braiding operation gives naturally rise to a quantum phase. A good guess is that braiding operation maps triangle to its complement since only in this manner orientation is preserved so that area is A_{klm} is mapped to $A_{klm} - 4\pi$. If the f is proportional to the exponent $exp(-A_{klm}Q)$, braiding operation induces a complex phase factor $q = exp(-i4\pi Q)$.

(e) For half-integer values of Q the algebra is commutative. For $Q = M/N$, where M and N have no common factors, only braided commutativity holds true for $N \geq 3$ just as for quantum groups characterizing also Jones inclusions of HFFs. For $N = 4$ anti-commutativity and associativity hold true. Charge fractionization would correspond to non-trivial braiding and presumably to non-standard values of Planck constant and coverings of M^4 or CP_2 depending on whether S^2 corresponds to a sphere of light-cone boundary or homologically trivial geodesic sphere of CP_2.

17.5.4 Finite-Dimensional Version Of The Fusion Algebra

Algebraic discretization due to a finite measurement resolution is an essential part of quantum TGD. In this kind of situation the symplectic fields would be defined in a discrete set of N points of S^2: natural candidates are subsets of points of p-adic variants of S^2. Rational variant of S^2 has as its points points for which trigonometric functions of θ and ϕ have rational values and there exists an entire hierarchy of algebraic extensions. The interpretation for the resulting breaking of the rotational symmetry would be a geometric correlate for the choice of quantization axes in quantum measurement and the book like structure of the imbedding space would be direct correlate for this symmetry breaking. This approach gives strong support for the category theory inspired philosophy in which the symplectic triangles are dictated by fusion rules.

General observations about the finite-dimensional fusion algebra

(a) In this kind of situation one has an algebraic structure with a finite number of field values with integration over intermediate points in fusion rules replaced with a sum. The most natural option is that the sum is over all points involved. Associativity conditions reduce in this case to conditions for a finite set of structure constants vanishing when two indices are identical. The number $M(N)$ of non-vanishing structure constants is obtained from the recursion formula $M(N) = (N-1)M(N-1) + (N-2)M(N-2) + ... + 3M(3) = NM(N-1)$, $M(3) = 1$ given $M(4) = 4$, $M(5) = 20$, $M(6) = 120$, ... With a proper choice of the set of points associativity might be achieved. The structure constants are necessarily complex so that also the complex conjugate of the algebra makes sense.

(b) These algebras resemble nilpotent algebras ($x^n = 0$ for some n) and Grassmann algebras ($x^2 = 0$ always) in the sense that also the products of the generating elements satisfy $x^2 = 0$ as one can find by using duality conditions on the square of a product $x = yz$ of two generating elements. Also the products of more than N generating elements necessary vanish by braided commutativity so that nilpotency holds true. The interpretation in terms of measurement resolution is that partonic states and vertices can involve at most N fermions in this measurement resolution. Elements anti-commute for $q = -1$ and commute for $q = 1$ and the possibility to express the product of two generating elements as a sum of generating elements distinguishes these algebras from Grassman algebras. For $q = -1$ these algebras resemble Lie-algebras with the difference that associativity holds true in this particular case.

(c) I have not been able to find whether this kind of hierarchy of algebras corresponds to some well-known algebraic structure with commutativity and associativity possibly replaced with their braided counterparts. Certainly these algebras would be category theoretical generalization of ordinary algebras for which commutativity and associativity hold true in strict sense.

(d) One could forget the representation of structure constants in terms of triangles and think these algebras as abstract algebras. The defining equations are $x_i^2 = 0$ for generators plus braided commutativity and associativity. Probably there exists solutions to these conditions. One can also hope that one can construct braided algebras from commutative and associative algebras allowing matrix representations. Note that the solution the conditions allow scalings of form $C_{klm} \rightarrow \lambda_k \lambda_l \lambda_m C_{klm}$ as symmetries.

Formulation and explicit solution of duality conditions in terms of inner product

Duality conditions can be formulated in terms of an inner product in the function space associated with N points and this allows to find explicit solutions to the conditions.

(a) The idea is to interpret the structure constants C_{klm} as wave functions C_{kl} in a discrete space consisting of N points with the standard inner product

$$\langle C_{kl}, C_{mn}\rangle = \sum_r C_{klr}\overline{C}_{mnr} \quad . \tag{17.5.1}$$

(b) The associativity conditions for a trivial braiding can be written in terms of the inner product as

$$\langle C_{kl}, \overline{C}_{mn}\rangle = \langle C_{km}, \overline{C}_{ln}\rangle = \langle C_{kn}, \overline{C}_{ml}\rangle \quad . \tag{17.5.2}$$

(c) Irrespective of whether the braiding is trivial or not, one obtains for $k = m$ the orthogonality conditions

$$\langle C_{kl}, \overline{C}_{kn}\rangle = 0 \quad . \tag{17.5.3}$$

For each k one has basis of $N-1$ wave functions labeled by $l \neq k$, and the conditions state that the elements of basis and conjugate basis are orthogonal so that conjugate basis is the dual of the basis. The condition that complex conjugation maps basis to a dual basis is very special and is expected to determine the structure constants highly uniquely.

(d) One can also find explicit solutions to the conditions. The most obvious trial is based on orthogonality of function basis of circle providing representation for Z_{N-2} and is following:

$$C_{klm} = E_{klm} \times exp(i\phi_k + \phi_l + \phi_m) \quad , \quad \phi_m = \frac{n(m)2\pi}{N-2} \quad . \tag{17.5.4}$$

Here E_{klm} is non-vanishing only if the indices have different values. The ansatz reduces the conditions to the form

$$\sum_r E_{klr}E_{mnr}exp(i2\phi_r) = \sum_r E_{kmr}E_{lnr}exp(i2\phi_r) = \sum_r E_{knr}E_{mlr}exp(i2\phi_r) \tag{17.5.5}$$

In the case of braiding one can allow overall phase factors. Orthogonality conditions reduce to

$$\sum_r E_{klr}E_{knr}exp(i2\phi_r) = 0 \ . \tag{17.5.6}$$

If the integers $n(m)$, $m \neq k, l$ span the range $(0, N-3)$ ortogonality conditions are satisfied if one has $E_{klr} = 1$ when the indices are different. This guarantees also duality conditions since the inner products involving k, l, m, n reduce to the same expression

$$\sum_{r\neq k,l,m,n} exp(i2\phi_r) \ . \tag{17.5.7}$$

(e) For a more general choice of phases the coefficients E_{klm} must have values differing from unity and it is not clear whether the duality conditions can be satisfied in this case.

Do fusion algebras form little disk operad?

The improvement of measurement resolution means that one adds further points to an existing set of points defining a discrete fusion algebra so that a small disk surrounding a point is replaced with a little disk containing several points. Hence the hierarchy of fusion algebras might be regarded as a realization of a little disk operad [A11] and there would be a hierarchy of homomorphisms of fusion algebras induced by the fusion. The inclusion homomorphism should map the algebra elements of the added points to the algebra element at the center of the little disk.

A more precise prescription goes as follows.

(a) The replacement of a point with a collection of points in the little disk around it replaces the original algebra element ϕ_{k_0} by a number of new algebra elements ϕ_K besides already existing elements ϕ_k and brings in new structure constants C_{KLM}, C_{KLk} for $k \neq k_0$, and C_{Klm}.

(b) The notion of improved measurement resolution allows to conclude

$$C_{KLk} = 0 \ , \quad k \neq k_0 \ , \quad C_{Klm} = C_{k_0lm} \ . \tag{17.5.8}$$

(c) In the homomorphism of new algebra to the original one the new algebra elements and their products should be mapped as follows:

$$\begin{aligned} &\phi_K \rightarrow \phi_{k_0} \ , \\ &\phi_K\phi_L \rightarrow \phi_{k_0}^2 = 0 \ , \quad \phi_K\phi_l \rightarrow \phi_{k_0}\phi_l \ . \end{aligned} \tag{17.5.9}$$

Expressing the products in terms of structure constants gives the conditions

$$\sum_M C_{KLM} = 0 \ , \quad \sum_r C_{Klr} = \sum_r C_{k_0lr} = 0 \ . \tag{17.5.10}$$

The general ansatz for the structure constants based on roots of unity guarantees that the conditions hold true.

(d) Note that the resulting algebra is more general than that given by the basic ansatz since the improvement of the measurement resolution at a given point can correspond to different value of N as that for the original algebra given by the basic ansatz. Therefore the original ansatz gives only the basic building bricks of more general fusion algebras. By repeated local improvements of the measurement resolution one obtains an infinite hierarchy of algebras labeled by trees in which each improvement of measurement resolution means the splitting of the branch with arbitrary number N of branches. The number of improvements of the measurement resolution defining the height of the tree is one invariant of these algebras. The fusion algebra operad has a fractal structure since each point can be replaced by any fusion algebra.

How to construct geometric representation of the discrete fusion algebra?

Assuming that solutions to the fusion conditions are found, one could try to find whether they allow geometric representations. Here the category theoretical philosophy shows its power.

(a) Geometric representations for C_{klm} would result as functions $f(A_{klm})$ of the symplectic area for the symplectic triangles assignable to a set of N points of S^2.

(b) If the symplectic triangles can be chosen freely apart from the area constraint as the category theoretic philosophy implies, it should be relatively easy to check whether the fusion conditions can be satisfied. The phases of C_{klm} dictate the areas A_{klm} rather uniquely if one uses Bohm-Aharonov ansatz for a fixed the value of Q. The selection of the points s_k would be rather free for phases near unity since the area of the symplectic triangle associated with a given triplet of points can be made arbitrarily small. Only for the phases far from unity the points s_k cannot be too close to each other unless Q is very large. The freedom to chose the points rather freely conforms with the general view about the finite measurement resolution as the origin of discretization.

(c) The remaining conditions are on the moduli $|f(A_{klm})|$. In the discrete situation it is rather easy to satisfy the conditions just by fixing the values of f for the particular triangles involved: $|f(A_{klm})| = |C_{klm}|$. For the exact solution to the fusion conditions $|f(A_{klm})| = 1$ holds true.

(d) Constraints on the functional form of $|f(A_{klm})|$ for a fixed value of Q can be deduced from the correlation between the modulus and phase of C_{klm} without any reference to geometric representations. For the exact solution of fusion conditions there is no correlation.

(e) If the phase of C_{klm} has A_{klm} as its argument, the decomposition of the phase factor to a sum of phase factors means that the A_{klm} is sum of contributions labeled by the vertices. Also the symplectic area defined as a magnetic flux over the triangle is expressible as sum of the quantities $\int A_\mu dx^\mu$ associated with the edges of the triangle. These fluxes should correspond to the fluxes assigned to the vertices deduced from the phase factors of $\Psi(s_k)$. The fact that vertices are ordered suggest that the phase of $\Psi(s_j)$ fixes the value of $\int A_\mu dx^\mu$ for an edge of the triangle starting from s_k and ending to the next vertex in the ordering. One must find edges giving a closed triangle and this should be possible. The option for which edges correspond to geodesics or to solutions of equations of motion for a charged particle in magnetic field is not flexible enough to achieve this purpose.

(f) The quantization of the phase angles as multiples of $2\pi/(N-2)$ in the case of N-dimensional fusion algebra has a beautiful geometric correlate as a quantization of symplecto-magnetic fluxes identifiable as symplectic areas of triangles defining solid angles as multiples of $2\pi/(N-2)$. The generalization of the fusion algebra to p-adic case exists if one allows algebraic extensions containing the phase factors involved. This requires the allowance of phase factors $exp(i2\pi/p)$, p a prime dividing $N-2$. Only the exponents $exp(i\int A_\mu dx^\mu) = exp(in2\pi/(N-2))$ exist p-adically. The p-adic counterpart of the curve defining the edge of triangle exists if the curve can be defined purely algebraically (say as a solution of polynomial equations with rational coefficients) so that p-adic variant of the curve satisfies same equations.

Does a generalization to the continuous case exist?

The idea that a continuous fusion algebra could result as a limit of its discrete version does not seem plausible. The reason is that the spatial variation of the phase of the structure constants increases as the spatial resolution increases so that the phases $exp(i\phi(s)$ cannot be continuous at continuum limit. Also the condition $E_{klm} = 1$ for $k \neq l \neq m$ satisfied by the explicit solutions to fusion rules fails to have direct generalization to continuum case.

To see whether the continuous variant of fusion algebra can exist, one can consider an approximate generalization of the explicit construction for the discrete version of the fusion algebra by the effective replacement of points s_k with small disks which are not allowed to intersect. This would mean that the counterpart $E(s_k, s_l, s_m)$ vanishes whenever the distance between two arguments is below a cutoff a small radius d. Puncturing corresponds physically to the cutoff implied by the finite measurement resolution.

(a) The ansatz for C_{klm} is obtained by a direct generalization of the finite-dimensional ansatz:

$$C_{klm} = \kappa_{s_k,s_l,s_m}\Psi(s_k)\Psi(s_l)\Psi(s_m) \ . \quad (17.5.11)$$

where κ_{s_k,s_l,s_m} vanishes whenever the distance of any two arguments is below the cutoff distance and is otherwise equal to 1.

(b) Orthogonality conditions read as

$$\Psi(s_k)\Psi(s_l)\int \kappa_{s_k,s_l,s_r}\kappa_{s_k,s_n,s_r}\Psi^2(s_m)d\mu(s_r) = \Psi(s_k)\Psi(s_l)\int_{S^2(s_k,sl,s_n)}\Psi^2(s_r)d\mu(s_r) = 0 \quad (17.5.12)$$

The resulting condition reads as

$$\int_{S^2(s_k,sl,s_n)}\Psi^2(s_r)d\mu(s_r) = 0 \quad (17.5.13)$$

This condition holds true for any pair s_k, s_l and this might lead to difficulties.

(c) The general duality conditions are formally satisfied since the expression for all fusion products reduces to

$$\begin{aligned} & \Psi(s_k)\Psi(s_l)\Psi(s_m)\Psi(s_n)X \ , \\ X &= \int_{S^2}\kappa_{s_k,s_l,s_m,s_n}\Psi(s_r)d\mu(s_r) \\ &= \int_{S^2(s_k,s_l,s_m,s_n)}\Psi(s_m)d\mu(s_r) \\ &= -\int_{D^2(s_i)}\Psi^2(s_r)d\mu(s_r) \ , \ i=k,l,s,m \ . \end{aligned} \quad (17.5.14)$$

These conditions state that the integral of Ψ^2 any disk of fixed radius d is same: this result follows also from the orthogonality condition. This condition might be difficult to satisfy exactly and the notion of finite measurement resolution might be needed. For instance, it might be necessary to restrict the consideration to a discrete lattice of points which would lead back to a discretized version of algebra. Thus it seems that the continuum generalization of the proposed solution to fusion rules does not work.

17.6 Could Operads Allow The Formulation Of The Generalized Feynman Rules?

The previous discussion of symplectic fusion rules leaves open many questions.

(a) How to combine symplectic and conformal fields to what might be called symplecto-conformal fields?

(b) The previous discussion applies only in super-symplectic degrees of freedom and the question is how to generalize the discussion to super Kac-Moody degrees of freedom. One must of course also try to identify more precisely what Kac-Moody degrees of freedom are!

(c) How four-momentum and its conservation in the limits of measurement resolution enters this picture? Could the phase factors assocaited with the symplectic triangulation carry information about four-momentum?

(d) At least two operads related to measurement resolution seem to be present: the operads formed by the symplecto-conformal fields and by generalized Feynman diagrams. For generalized Feynman diagrams causal diamond (CD) is the basic object whereas disks of S^2 are the basic objects in the case of symplecto-conformal QFT with a finite measurement resolution. Could these two different views about finite measurement resolution be more or less equivalent and could one understand this equivalence at the level of details.

(e) Is it possible to formulate generalized Feynman diagrammatics and improved measurement resolution algebraically?

17.6.1 How To Combine Conformal Fields With Symplectic Fields?

The conformal fields of conformal field theory should be somehow combined with symplectic scalar field to form what might be called symplecto-conformal fields.

(a) The simplest thing to do is to multiply ordinary conformal fields by a symplectic scalar field so that the fields would be restricted to a discrete set of points for a given realization of N-dimensional fusion algebra. The products of these symplecto-conformal fields at different points would define a finite-dimensional algebra and the products of these fields at same point could be assumed to vanish.

(b) There is a continuum of geometric realizations of the symplectic fusion algebra since the edges of symplectic triangles can be selected rather freely. The integrations over the coordinates z_k (most naturally the complex coordinate of S^2 transforming linearly under rotations around quantization axes of angular momentum) restricted to the circle appearing in the definition of simplest stringy amplitudes would thus correspond to the integration over various geometric realizations of a given N-dimensional symplectic algebra.

Fusion algebra realizes the notion of finite measurement resolution. One implication is that all n-point functions vanish for $n > N$. Second implication could be that the points appearing in the geometric realizations of N-dimensional symplectic fusion algebra have some minimal distance. This would imply a cutoff to the multiple integrals over complex coordinates z_k varying along circle giving the analogs of stringy amplitudes. This cutoff is not absolutely necessary since the integrals defining stringy amplitudes are well-defined despite the singular behavior of n-point functions. One can also ask whether it is wise to introduce a cutoff that is not necessary and whether fusion algebra provides only a justification for the $1 + i\epsilon$ prescription to avoid poles used to obtain finite integrals.

The fixed values for the quantities $\int A_\mu dx^\mu$ along the edges of the symplectic triangles could indeed pose a lower limit on the distance between the vertices of symplectic triangles. Whether this occurs depends on what one precisely means with symplectic triangle.

(a) The conformally invariant condition that the angles between the edges at vertices are smaller than π for triangle and larger than π for its conjugate is not enough to exclude loopy edges and one would obtain ordinary stringy amplitudes multiplied by the symplectic phase factors. The outcome would be an integral over arguments $z_1, z_2, ..z_n$ for standard stringy n-point amplitude multiplied by a symplectic phase factor which is piecewise constant in the integration domain.

(b) The condition that the points at different edges of the symplectic triangle can be connected by a geodesic segment belonging to the interior of the triangle is much stronger and would induce a length scale cutoff since loops cannot be used to create large enough value of $\int A_\mu dx^\mu$ for a given side of triangle. Symplectic invariance would be obtained for small enough symplectic transformations. How to realize this cutoff at the level of calculations is not clear. One could argue that this problem need not have any nice solution and since finite measurement resolution requires only finite calculational resolution, the approximation allowing loopy edges is acceptable.

(c) The restriction of the edges of the symplectic triangle within a tubular neighborhood of a geodesic -more more generally an orbit of charged particle - with thickness determined by the length scale resolution in S^2 would also introduce the length scale cutoff with symplectic invariance within measurement resolution.

Symplecto-conformal should form an operad. This means that the improvement of measurement resolution should correspond also to an algebra homomorphism in which super-symplectic symplecto-conformal fields in the original resolution are mapped by algebra homomorphism into fields which contain sum over products of conformal fields at different points: for the symplectic parts of field the products reduces always to a sum over the values of field. For instance, if the field at point s is mapped to an average of fields at points s_k, nilpotency condition $x^2 = 0$ is satisfied.

17.6.2 Symplecto-Conformal Fields In Super-Kac-Moody Sector

The picture described above applies only in super-symplectic degrees of freedom. The vertices of generalized Feynman diagrams are absent from the description and CP_2 Kähler form induced to space-time surface which is absolutely essential part of quantum TGD is nowhere visible in the treatment.

How should one bring in Super Kac-Moody (SKM) algebra? The condition that the basic building bricks are same for the treatment of these degrees of freedom is a valuable guideline.

What does SKM algebra mean?

The first thing to consider is what SKM could mean. The recent view is that symplectic algebra corresponds to symplectic transformations for the boundary of causal diamond CD which looks locally like $\delta M^4_\pm \times CP_2$. For this super-algebra fermionic generators would be contractions of covariantly constant right-handed neutrino with the second quantized induced spinor field to which the contraction$j^k_A \Gamma_k$ of symplectic vector field with gamma matrices acts. For SKM algebra corresponding generators would be similar contractions of other spinor modes but involving only Killing vectors fields that is symplectic isometries.

The recent view about quantum criticality strongly suggests that the conformal symmetries act as almost gauge symmetries producing from a given preferred extremal new ones with same action and conserved charges. "Almost" means that sub-algebra of conformal algebra annihilates the physical states. The subalgebras in question form a fractal hierarchy and are isomorphic with the conformal algebra itself. They contain generators for which the conformal weight is multiple of integer n characterizing also the value of Planck constant given by $h_{eff} = n \times h$. n defines the number of conformal equivalence classes of space-time surfaces connecting fixed 3-surfaces at the boundaries of CD (see **Fig.** `http://tgdtheory.fi/appfigures/planckhierarchy.jpg` or **Fig. ??** in the appendix of this book).

Since Kähler action reduces for the general ansatz for the preferred extremals to 3-D Chern-Simons terms, the action of the conformal symmetries reduces also to the 3-D space-like surfaces where it is trivial by definition and to non-trivial action to the light-like 3-surfaces at which the signature of the induced metric changes: I have used to call this surface partonic orbits.

It must be however observed that one can consider also the possibility that SKM algebra corresponds to the isometries of $\delta M^4 \pm \times CP_2$ continued to the space-time surface by field equations. These isometries are conformal transformations of S^2 ($\delta M^4_\pm = S^2 \times R_+$) with conformal scaling compensated by the local scaling of the light-like radial coordinate r_M to guarantee that the metric reducing to that for S^2 apart from conformal scaling factor R^2_M remains invariant. If this is the case the SKM contains also other than symplectic isometries.

Attempt to formulate symplectic triangulation for SKM algebra

The analog of symplectic triangulation for SKM algebra obviously requires that SKM algebra corresponds to symplectic isometries rather than including all $\delta M^4_\pm = S^2 \times R_+$ isometries in one-one correspondence with conformal transformations of S^2.

(a) In the transition from super-symplectic to SKM degrees of freedom the light-cone boundary is naturally replaced with the light-like 3-surface X^3 representing the light-like random orbit of parton and serving as the basic dynamical object of quantum TGD. The sphere S^2 of light-cone boundary is in turn replaced with a partonic 2-surface X^2. This suggests how to proceed.

(b) In the case of SKM algebra the symplectic fusion algebra is represented geometrically as points of partonic 2-surface X^2 by replacing the symplectic form of S^2 with the induced CP_2 symplectic form at the partonic 2-surface and defining $U(1)$ gauge field. This gives similar hierarchy of symplecto-conformal fields as in the super-symplectic case. This also realizes the crucial aspects of the classical dynamics defined by Kähler action. In particular, for vacuum 2-surfaces symplectic fusion algebra trivializes since Kähler magnetic fluxes vanish identically and 2-surfaces near vacua require a large value of N for the dimension of the fusion algebra since the available Kähler magnetic fluxes are small.

(c) In super-symplectic case the projection along light-like ray allows to map the points at the light-cone boundaries of CD to points of same sphere S^2. In the case of light-like 3-surfaces light-like geodesics representing braid strands allow to map the points of the partonic two-surfaces at the future and past light-cone boundaries to the partonic 2-surface representing the vertex. The earlier proposal was that the ends of strands meet at the partonic 2-surface so that braids would replicate at vertices. The properties of symplectic fields would however force identical vanishing of the vertices if this were the case. There is actually no reason to assume this condition and with this assumption vertices involving total number N of incoming and outgoing strands correspond to symplecto-conformal N-point function as is indeed natural. Also now Kähler magnetic flux induces cutoff distance.

(d) SKM braids reside at light-like 3-surfaces representing lines of generalized Feynman diagrams. If super-symplectic braids are needed at all, they must be assigned to the two light-like boundaries of CD meeting each other at the sphere S^2 at which future and past directed light-cones meet.

17.6.3 The Treatment Of Four-Momentum

Four-momentum enjoys a special role in super-symplectic and SKM representations in that it does not correspond to a quantum number assignable to the generators of these algebras. It would be nice if the somewhat mysterious phase factors associated with the representation of the symplectic algebra could code for the four-momentum - or rather the analogs of plane waves representing eigenstates of four-momentum at the points associated with the geometric representation of the symplectic fusion algebra.

Also the vision about TGD as almost topological QFT suggests that the symplectic degrees of freedom added to the conformal degrees of freedom defining alone a mere topological QFt somehow code for the physical degrees of freedom should and also four-momentum. If so, the symplectic triangulation might somehow code for four-momentum.

The representation of longitudinal momentum in terms of phase factors

The following argument suggests that S^2 and X^2 triangulations cannot naturally represent four-momentum and that one needs extension into 3-D light-like triangulation to achieve this.

(a) The basic question is whether four-momentum could be coded in terms of non-integrable phase factors appearing in the representations of the symplectic fusion algebras.

(b) In the symplectic case S^2 triangulation suggests itself as a representation of angular momentum only: it would be kind of spin network. In the SKM case X^2 would suggest representation of color hyper charge and isospin in terms of phases since CP_2 symmetries act non-trivially in Chern-Simons action. Does this mean that symplectic and SKM triangulations must be extended so that they are 3-D and defined for space-like 3-surface and the light-like orbit of partonic 2-surface. This would give additional phase factors assignable to presumably light-like edges. Ligh-like momentum would be natural and the recent twistorial formulation of quantum TGD indeed assigns massless momenta to fermion lines.

Suppose that one has 3-D light-like triangulation eith at δCD or at light-like orbits of partonic 2-surface. Consider first coding of four-momentum assuming only Kähler gauge potential of CP_2 possibly having M^4 part which is pure gauge.

(a) Four different phase factors are needed if all components of four-momentum are to be coded. Both number theoretical vision about quantum TGD and the realization of the hierarchy of Planck constants assign to each point of space-time surface the same plane $M^2 \subset M^4$ having as the plane of non-physical polarizations. This condition allows to assign to a given light-like partonic 3-surface unique extremal of Kähler action defining the Kähler function as the value of Kähler action.

Also p-adic mass calculations support the view that the physical states correspond to eigen states for the components of longitudinal momentum only (also the parton model for hadrons assumes this). This encourages to think that only M^2 part of four-momentum is coded by the phase factors. Transversal momentum squared would be a well defined quantum number and determined from mass shell conditions for the representations of super-symplectic (or equivalently SKM) conformal algebra much like in string model.

(b) The phase factors associated with the 3-D symplectic fusion algebra in $S^2 \times R_+$ mean a deviation from conformal n-point functions, and the innocent question is whether these phase factors could be identified as plane-wave phase factors in S^2 could be associated with the transversal part of the four-momentum so that the n-point functions would be strictly analogous with stringy amplitudes. Alternative, and perhaps more natural, interpretation is in terms of spin and angular momentum.

(c) Suppose one allows a gauge transformation of Kähler gauge potential inducing a pure gauge M^4 component to the Kähler gauge potential expressible as scalar function of M^4 coordianates. This kind of term might allow to achieve the vanishing of $j^\alpha A_\alpha$ term of at least its integral over space-time surface in Kähler action implying reduction of Kähler action to Chern-Simons terms if weak form of electric magnetic duality holds true. The scalar function can be interpreted as integral of a position dependent momentum along curve defined by $S^2 \times R_+$ triangulation and gives hopes of coding four-momentum in terms of Kähler gauge potential.

In fact, the identification of the phase factors $exp(i\int A_\mu dx^\mu/\hbar)$ along a path as phase factors $exp(ip_{L,k}\Delta m^k)$ defined by the ends of the path and associated with the longitudinal part of four-momentum would correspond to an integral form of covariant constancy condition $\frac{dx^\mu}{ds}(\partial_\mu - iA_\mu)\Psi = 0$ along the edge of the symplectic triangle of more general path.

(d) For the SKM triangulation associated with the light-like orbit X^3_l of partonic 2-surface analogous phase factor would come from the integral along the (most naturally) light-like curve defining braid strand associated with the point in question. A geometric representation for the two projections of the four-momentum would thus result in SKM degrees of freedom and apart from the non-uniqueness related to the multiples of a 2π the components of M^2 momentum could be deduced from the phase factors. If one is satisfied with the projection of momentum in M^2, this is enough.

(e) Neither of these phase factors is able to code all components of four-momentum. One might however hope that together they could give enough information to deduce the four-momentum if it is assumed to correspond to the rest system.

(f) The phase factors assignable to the symplectic triangles in S^2 and X^2have nothing to do with momentum. Because the space-like phase factor $exp(iS_z\Delta\phi/\hbar)$ associated with the edge of the symplectic triangle is completely analogous to that for momentum, one can argue that the symplectic triangulation could define a kind of spin network utilized in discretized approaches to quantum gravity. The interpretation raises the question about the interpretation of the quantum numbers assignable to the Lorentz invariant phase factors defined by the CP_2 Kähler gauge potential.

The quantum numbers associated with phase factors for CP_2 parts of Kähler gauge potentials

Suppose that it is possible to assign two independent and different phase factors to the same geometric representation, in other words have two independent symplectic fields with the same geometric representation. The product of two symplectic fields indeed makes sense and satisfies the defining conditions. One can define prime symplectic algebras and decompose symplectic algebras to prime factors. Since one can allow permutations of elements in the products it becomes possible to detect the presence of product structure experimentally by detecting different combinations for products of phases caused by permutations realized as different combinations of quantum numbers assigned with the factors. The geometric representation for the product of n symplectic fields would correspond to the assignment of n edges to any pair of points. The question concerns the interpretation of the phase factors assignable to the CP_2 parts of Kähler gauge potentials of S^2 and CP_2 Kähler form.

(a) The natural interpretation for the two additional phase factors would be in terms of color quantum numbers. Color hyper charge and isospin are mathematically completely analogous to the components of four-momentum so that a possible identification of the phase factors is as a representation of these quantum numbers. The representation of plane waves as phase factors $exp(ip_k\Delta m^k/\hbar)$ generalizes to the representation $exp(iQ_A\Delta\Phi^A/\hbar)$, where Φ_A are the angle variables conjugate to the Hamiltonians representing color hyper charge and isospin. This representation depends on end points only so that the crucial symplectic invariance with respect to the symplectic transformations respecting the end points of the edge is not lost ($U(1)$ gauge transformation is induced by the scalar j^kA_k, where j^k is the symplectic vector field in question).

(b) One must be cautious with the interpretation of the phase factors as a representation for color hyper charge and isospin since a breaking of color gauge symmetry would result since the phase factors associated with different values of color isospin and hypercharge would be different and could not correspond to same edge of symplectic triangle. This is questionable since color group itself represents symplectic transformations. The construction of CP_2 as a coset space $SU(3)/U(2)$ identifies $U(2)$ as the holonomy group of spinor connection having interpretation as electro-weak group. Therefore also the interpretation of the phase factors in terms of em charge and weak charge can be considered. In TGD framework electro-weak gauge potential indeed suffer a non-trivial gauge transformation under color rotations so that the correlation between electro-weak quantum numbers and non-integrable phase factors in Cartan algebra of the color group could make sense. Electro-weak symmetry breaking would have a geometric correlate in the sense that different values of weak isospin cannot correspond to paths with same values of phase angles $\Delta\Phi^A$ between end points.

(c) If the phase factors associated with the M^4 and CP_2 are assumed to be identical, the existence of geometric representation is guaranteed. This however gives constraints between rest mass, spin, and color (or electro-weak) quantum numbers.

Some general comments

Some further comments about phase factors are in order.

(a) By number theoretical universality the plane wave factors associated with four-momentum must have values coming as roots of unity (just as for a particle in box consisting of discrete lattice of points). At light-like boundary the quantization conditions reduce to the condition that the value of light-like coordinate is rational of form m/N, if N: th roots of unity are allowed.

(b) In accordance with the finite measurement resolution of four-momentum, four-momentum conservation is replaced by a weaker condition stating that the products of phase factors representing incoming and outgoing four-momenta are identical. This means that positive and negative energy states at opposite boundaries of CD would correspond to complex conjugate representations of the fusion algebra. In particular, the product of phase factors in the decomposition of the conformal field to a product of conformal fields should correspond to the original field value. This would give constraints on the trees physically possible in the operad formed by the fusion algebras. Quite generally, the phases expressible as products of phases $exp(in\pi/p)$, where $p \leq N$ is prime must be allowed in a given resolution and this suggests that the hierarchy of p-adic primes is involved. At the limit of very large N exact momentum conservation should emerge.

(c) Super-conformal invariance gives rise to mass shell conditions relating longitudinal and transversal momentum squared. The massivation of massless particles by Higgs mechanism and p-adic thermodynamics pose additional constraints to these phase factors.

17.6.4 What Does The Improvement Of Measurement Resolution Really Mean?

To proceed one must give a more precise meaning for the notion of measurement resolution. Two different views about the improvement of measurement resolution emerge. The first one relies on the replacement of braid strands with braids applies in SKM degrees of freedom and the homomorphism maps symplectic fields into their products. The homomorphism based on the averaging of symplectic fields over added points consistent with the extension of fusion algebra described in previous section is very natural in super-symplectic degrees of freedom. The directions of these two algebra homomorphisms are different. The question is whether both can be involved with both super-symplectic and SKM case. Since the end points of SKM braid strands correspond to both super-symplectic and SKM degrees of freedom, it seems that division of labor is the only reasonable option.

(a) Quantum classical correspondence requires that measurement resolution has a purely geometric meaning. A purely geometric manner to interpret the increase of the measurement resolution is as a replacement of a braid strand with a braid in the improved resolution. If one assigns the phase factor assigned with the fusion algebra element with four-momentum, the conservation of the phase factor in the associated homomorphism is a natural constraint. The mapping of a fusion algebra element (strand) to a product of fusion algebra elements (braid) allows to realize this condition. Similar mapping of field value to a product of field values should hold true for conformal parts of the fields. There exists a large number equivalent geometric representations for a given symplectic field value so that one obtains automatically an averaging in conformal degrees of freedom. This interpretation for the improvement of measurement resolution looks especially natural for SKM degrees of freedom for which braids emerge naturally.

(b) One can also consider the replacement of symplecto-conformal field with an average over the points becoming visible in the improved resolution. In super-symplectic degrees of freedom this looks especially natural since the assignment of a braid with light-cone boundary is not so natural as with light-like 3-surface. This map does not conserve the phase factor but this could be interpreted as reflecting the fact that the values of the light-like radial coordinate are different for points involved. The proposed extension of the symplectic algebra proposed in the previous section conforms with this interpretation.

(c) In the super-symplectic case the improvement of measurement resolution means improvement of angular resolution at sphere S^2. In SKM sector it means improved resolution for the position at partonic 2-surface. This generalizes also to the 3-D symplectic triangulations. For SKM algebra the increase of the measurement resolution related to the braiding takes place inside light-like 3-surface. This operation corresponds naturally to an addition of sub-CD inside which braid strands are replaced with braids. This is like looking with a microscope a particular part of line of generalized Feynman graph inside CD and corresponds to a genuine physical process inside parton. In super-symplectic case the replacement of a braid strand with braid (at light-cone boundary) is induced by the replacement of the projection of a point of a partonic 2-surface to S^2 with a a collection of points coming from several partonic 2-surfaces. This replaces the point s of S^2 associated with CD with a set of points s_k of S^2 associated with sub-CD. Note that the solid angle spanned by these points can be rather larger so that zoom-up is in question.

(d) The improved measurement resolution means that a point of S^2 (X^2) at boundary of CD is replaced with a point set of S^2 (X^2) assignable to sub-CD. The task is to map the point set to a small disk around the point. Light-like geodesics along light-like X^3 defines this map naturally in both cases. In super-symplectic case this map means scaling down of the solid angle spanned by the points of S^2 associated with sub-CD.

17.6.5 How Do The Operads Formed By Generalized Feynman Diagrams And Symplecto-Conformal Fields Relate?

The discussion above leads to following overall view about the situation. The basic operation for both symplectic and Feynman graph operads corresponds to an improvement of measurement resolution. In the case of planar disk operad this means to a replacement of a white region of a map with smaller white regions. In the case of Feynman graph operad this means better space-time resolution leading to a replacement of generalized Feynman graph with a new one containing new sub-CD bringing new vertices into daylight. For braid operad the basic operation means looking a braid strand with a microscope so that it can resolve into a braid: braid becomes a braid of braids. The latter two views are equivalent if sub-CD contains the braid of braids.

The disks D^2 of the planar disk operad has natural counterparts in both super-symplectic and SKM sector.

(a) For the geometric representations of the symplectic algebra the image points vary in continuous regions of S^2 (X^2) since the symplectic area of the symplectic triangle is a highly flexible constraint. Posing the condition that any point at the edges of symplectic triangle can be connected to any another edge excludes symplectic triangles with loopy sides so that constraint becomes non-trivial. In fact, since two different elements of the symplectic algebra cannot correspond to the same point for a given geometric representation, each element must correspond to a connected region of S^2 (X^2). This allows a huge number of representations related by the symplectic transformations S^2 in super-symplectic case and by the symplectic transformations of CP_2 in SKM case. In the case of planar disk operad different representations are related by isotopies of plane.

This decomposition to disjoint regions naturally correspond to the decomposition of the disk to disjoint regions in the case of planar disk operad and Feynman graph operad (allowing zero energy insertions). Perhaps one might say that N-dimensional elementary symplectic algebra defines an N-coloring of S^2 (S^2) which is however not the same thing as the 2-coloring possible for the planar operad. TGD based view about Higgs mechanism leads to a decomposition of partonic 2-surface X^2 (its light-like orbit X^3) into conformal patches. Since also these decompositions correspond to effective discretizations of X^2 (X^3), these two decompositions would naturally correspond to each other.

(b) In SKM sector disk D^2 of the planar disk operad is replaced with the partonic 2-surface X^2 and since measurement resolution is a local notion, the topology of X^2 does not matter. The improvement of measurement resolution corresponds to the replacement of braid strand with braid and homomorphism is to the direction of improved spatial resolution.

(c) In super-symplectic case D^2 is replaced with the sphere S^2 of light-cone boundary. The improvement of measurement resolution corresponds to introducing points near the original point and the homomorphism maps field to its average. For the operad of generalized Feynman diagrams CD defined by future and past directed light-cones is the basic object. Given CD can be indeed mapped to sphere S^2 in a natural manner. The light-like boundaries of CDs are metrically spheres S^2. The points of light-cone boundaries can be projected to any sphere at light-cone boundary. Since the symplectic area of the sphere corresponds to solid angle, the choice of the representative for S^2 does not matter. The sphere defined by the intersection of future and past light-cones of CD however provides a natural identification of points associated with positive and negative energy parts of the state as points of the same sphere. The points of S^2 appearing in n-point function are replaced by point sets in a small disks around the n points.

(d) In both super-symplectic and SKM sectors light-like geodesic along X^3 mediate the analog of the map gluing smaller disk to a hole of a disk in the case of planar disk operad defining the decomposition of planar tangles. In super-symplectic sector the set of points at the sphere corresponding to a sub-CD is mapped by SKM braid to the larger CD and for a typical braid corresponds to a larger angular span at sub-CD. This corresponds to the gluing of D^2 along its boundaries to a hole in D^2 in disk operad. A scaling transformation allowed by the conformal invariance is in question. This scaling can have a non-trivial effect if the conformal fields have anomalous scaling dimensions.

(e) Homomorphisms between the algebraic structures assignable to the basic structures of the operad (say tangles in the case of planar tangle operad) are an essential part of the power of the operad. These homomorphisms associated with super-symplectic and SKM sector code for two views about improvement of measurement resolution and might lead to a highly unique construction of M-matrix elements.

The operad picture gives good hopes of understanding how M-matrices corresponding to a hierarchy of measurement resolutions can be constructed using only discrete data.

(a) In this process the n-point function defining M-matrix element is replaced with a superposition of n-point functions for which the number of points is larger: $n \to \sum_{k=1,...,m} n_k$. The numbers n_k vary in the superposition. The points are also obtained by downwards scaling from those of smaller S^2. Similar scaling accompanies the composition of tangles in the case of planar disk operad. Algebra homomorphism property gives constraints on the compositeness and should govern to a high degree how the improved measurement resolution affects the amplitude. In the lowest order approximation the M-matrix element is just an n-point function for conformal fields of positive and negative energy parts of the state at this sphere and one would obtain ordinary stringy amplitude in this approximation.

(b) Zero energy ontology means also that each addition in principle brings in a new zero energy insertion as the resolution is improved. Zero energy insertions describe actual physical processes in shorter scales in principle affecting the outcome of the experiment in longer time scales. Since zero energy states can interact with positive (negative) energy particles, zero energy insertions are not completely analogous to vacuum bubbles and cannot be neglected. In an idealized experiment these zero energy states can be assumed to be absent. The homomorphism property must hold true also in the presence of the zero energy insertions. Note that the Feynman graph operad reduces to planar disk operad in absence of zero energy insertions.

17.7 Possible Other Applications Of Category Theory

It is not difficult to imagine also other applications of category theory in TGD framework.

17.7.1 Categorification And Finite Measurement Resolution

I read a very stimulating article by John Baez with title "Categorification" (see http://tinyurl.com/ych6a8oa) [A65] about the basic ideas behind a process called categorification. The process starts from sets consisting of elements. In the following I describe the basic ideas and propose how categorification could be applied to realize the notion of finite measurement resolution in TGD framework.

What categorification is?

In categorification sets are replaced with categories and elements of sets are replaced with objects. Equations between elements are replaced with isomorphisms between objects: the right and left hand sides of equations are not the same thing but only related by an isomorphism so that they are not tautologies anymore. Functions between sets are replaced with functors between categories taking objects to objects and morphisms to morphisms and respecting the composition of morphisms. Equations between functions are replaced with natural isomorphisms between functors, which must satisfy certain coherence laws representable in terms of commuting diagrams expressing conditions such as commutativity and associativity.

The isomorphism between objects represents equation between elements of set replaces identity. What about isomorphisms themselves? Should also these be defined only up to an isomorphism of isomorphism? And what about functors? Should one continue this replacement ad infinitum to obtain a hierarchy of what might be called n-categories, for which the process stops after n: th level. This rather fuzzy buisiness is what mathematicians like John Baez are actually doing.

Why categorification?

There are good motivations for the categofication. Consider the fact that natural numbers. Mathematically oriented person would think number "3" in terms of an abstract set theoretic axiomatization of natural numbers. One could also identify numbers as a series of digits. In the real life the representations of three-ness are more concrete involving many kinds of associations. For a child "3" could correspond to three fingers. For a mystic it could correspond to holy trinity. For a Christian "faith, hope, love". All these representations are isomorphic representation of threeness but as real life objects three sheeps and three cows are not identical.

We have however performed what might be called decategorification: that is forgitten that the isomorphic objects are not equal. Decatecorification was of course a stroke of mathematical genius with enormous practical implications: our information society represents all kinds of things in terms of numbers and simulates successfully the real world using only bit sequences. The dark side is that treating people as mere numbers can lead to a rather cold society.

Equally brilliant stroke of mathematical genius is the realization that isomorphic objects are not equal. Decategorization means a loss of information. Categorification brings back this information by bringing in consistency conditions known as coherence laws and finding these laws is the hard part of categorization meaning discovery of new mathematics. For instance, for braid groups commutativity modulo isomorphisms defines a highly non-trivial coherence law leading to an extremely powerful notion of quantum group having among other things applications in topological quantum computation.

The so called associahedrons (see `http://tinyurl.com/ng2fqro`) [A36] emerging in n-category theory could replace space-time and space as fundamental objects. Associahedrons are polygons used to represent geometrically associativity or its weaker form modulo isomorphism for the products of n objects bracketed in all possible manners. The polygon defines a hierarchy containing sub-polygons as its edges containing.... Associativity states the isomorphy of these polygons. Associahedrons and related geometric representations of category theoretical arrow complexes in terms or simplexes allow a beautiful geometric realization of the coherence laws. One could perhaps say that categories as discrete structures are not enough: only by introducing the continuum allowing geometric representations of the coherence laws things become simple.

No-one would have proposed categorification unless it were demanded by practical needs of mathematics. In many mathematical applications it is obvious that isomorphism does not mean identity. For instance, in homotopy theory all paths deformable to each other in continuous manner are homotopy equivalent but not identical. Isomorphism is now homotopy. These paths can be connected and form a groupoid. The outcome of the groupoid operation is determined up to homotopy. The deformations of closed path starting from a given point modulo homotopies form homotopy group and one can interpret the elements of homotopy group as copies of the point which are isomorphic. The replacement of the space with its universal covering makes this distinction explicit. One can form homotopies of homotopies and continue this process ad infinitum and obtain in this manner homotopy groups as characterizes of the topology of the space.

Cateforification as a manner to describe finite measurement resolution?

In quantum physics gauge equivalence represents a standard example about equivalence modulo isomorphisms which are now gauge transformations. There is a practical strategy to treat the situation: perform a gauge choice by picking up one representative amongst infinitely many isomorphic objects. At the level of natural numbers a very convenient gauge fixing would correspond the representation of natural number as a sequence of decimal digits rather than image of three cows.

In TGD framework a excellent motivation for categorification is the need to find an elegant mathematical realization for the notion of finite measurement resolution. Finite measurement resolutions (or cognitive resolutions) at various levels of information transfer hierarchy imply accumulation of uncertainties. Consider as a concrete example uncertainty in the determination of basic parameters of a mathematical model. This uncertainty is reflected to final outcome as via a long sequence of mathematical maps and additional uncertainties are produced by the approximations at each step of this process.

How could onbe describe the finite measurement resolution elegantly in TGD Universe? Categorification suggests a natural method. The points equivalent with measurement resolution are isomorphic with each other. A natural guess inspired by gauge theories is that one should perform a gauge choice as an analog of decategorification. This allows also to avoid continuum of objects connected by arrows not n spirit with the discreteness of category theoretical approach.

(a) At space-time level gauge choice means discretization of partonic 2-surfaces replacing them with a discrete set points serving as representatives of equivalence classes of points equivalent under finite measurement resolution. An especially interesting choice of points is as rational points or algebraic numbers and emerges naturally in p-adicization process. One can also introduce what I have called symplectic triangulation of partonic 2-surfaces with the nodes of the triangulation representing the discretization and carrying quantum numbers of various kinds.

(b) At the level of "world classical worlds" (WCW) this means the replacement of the sub-group if the symplectic group of $\delta M^4 \times CP_2$ -call it G - permuting the points of the symplectic triangulation with its discrete subgroup obtained as a factor group G/H, where H is the normal subgroup of G leaving the points of the symplectic triangulation fixed. One can also consider subgroups of the permutation group for the points of the triangulation. One can also consider flows with these properties to get braided variant of G/H. It would seem that one cannot regard the points of triangulation as isomorphic in the category theoretical sense. This because, one can have quantum superpositions of states located at these points and the factor group acts as the analog of isometry group. One can also have many-particle states with quantum numbers at several points. The possibility to assign quantum numbers to a given point becomes the physical counterpart for the axiom of choice.

The finite measurement resolution leads to a replacement of the infinite-dimensional world of classical worlds with a discrete structure. Therefore operation like integration over entire "world of classical worlds" is replaced with a discrete sum.

(c) What suggests itself strongly is a hierarchy of n-categories as a proper description for the finite measurement resolution. The increase of measurement resolution means increase for the number of braid points. One has also braids of braids of braids structure implied by the possibility to map infinite primes, integers, and rationals to rational functions of several variables and the conjecture possibility to represent the hierarchy of Galois groups involved as symplectic flows. If so the hierarchy of n-categories would correspond to the hierarchy of infinite primes having also interpretation in terms of repeated second quantization of an arithmetic SUSY such that many particle states of previous level become single particle states of the next level.

The finite measurement resolution has also a representation in terms of inclusions of hyper-finite factors of type II_1 defined by the Clifford algebra generated by the gamma matrices of WCW [K87]

(a) The included algebra represents finite measurement resolution in the sense that its action generates states which are cannot be distinguished from each other within measurement resolution used. The natural conjecture is that this indistuinguishability corresponds to a gauge invariance for some gauge group and that TGD Universe is analogous to Turing machine in that almost any gauge group can be represented in terms of finite measurement resolution.

(b) Second natural conjecture inspired by the fact that symplectic groups have enormous representabive power is that these gauge symmetries allow representation as subgroups of the symplectic group of $\delta M^4 \times CP_2$. A nice article about universality of symplectic groups is the article "The symplectification of science" (see `http://tinyurl.com/y8us9sgw`) by Mark. J. Gotay [A23].

(c) An interesting question is whether there exists a finite-dimensional space, whose symplectomorphisms would allow a representation of any gauge group (or of all possible Galois groups as factor groups) and whether $\delta M^4 \times CP_2$ could be a space of this kind with the smallest possible dimension.

17.7.2 Inclusions Of HFFs And Planar Tangles

Finite index inclusions of HFFs are characterized by non-branched planar algebras for which only an even number of lines can emanate from a given disk. This makes possible a consistent coloring of the k-tangle by black and white by painting the regions separated by a curve using opposite colors. For more general algebras, also for possibly existing branched tangle algebras, the minimum number of colors is four by four-color theorem. For the description of zero energy states the 2-color assumption is not needed so that the necessity to have general branched planar algebras is internally consistent. The idea about the inclusion of positive energy state space into the space of negative energy states might be consistent with branched planar algebras and the requirement of four colors since this inclusion involves also conjugation and is thus not direct.

In [A17] if was proposed that planar operads are associated with conformal field theories at sphere possessing defect lines separating regions with different color. In TGD framework and for branched planar algebras these defect lines would correspond to light-like 3-surfaces. For fermions one has single wormhole throat associated with topologically condensed CP_2 type extremal and the signature of the induced metric changes at the throat. Bosons correspond to pairs of wormhole throats associated with wormhole contacts connecting two space-time sheets modellable as a piece of CP_2 type extremal. Each boson thus corresponds to 2 lines within CP_2 radius so that in purely bosonic case the planar algebra can correspond to that associated with an inclusion of HFFs.

17.7.3 2-Plectic Structures And TGD

Chris Rogers and Alex Hoffnung have demonstrated [A83] that the notion of symplectic structure generalizes to n-plectic structure and in $n = 2$ case leads to a categorification of Lie algebra to 2-Lie-algebra. In this case the generalization replaces the closed symplectic 2-form with a closed 3-form ω and assigns to a subset of one-forms defining generalized Hamiltonians vector fields leaving the 3-form invariant.

There are two equivalent definitions of the Poisson bracket in the sense that these Poisson brackets differ only by a gradient, which does not affect the vector field assignable to the Hamiltonian one-form. The first bracket is simply the Lie-derivate of Hamiltonian one form G with respect to vector field assigned to F. Second bracket is contraction of Hamiltonian one-forms with the three-form ω. For the first variant Jacobi identities hold true but Poisson bracket is antisymmetric only modulo gradient. For the second variant Jacobi identities hold true only modulo gradient but Poisson bracket is antisymmetric. This modulo property is in accordance with category theoretic thinking in which commutativity, associativity, antisymmetry, ... hold true only up to isomorphism.

For 3-dimensional manifolds n=2-plectic structure has the very nice property that *all* one-forms give rise to Hamiltonian vector field. In this case any 3-form is automatically closed so that a large variety of 2-plectic structures exists. In TGD framework the natural choice for the 3-form ω is as Chern-Simons 3-form defined by the projection of the Kähler gauge potential to the light-like 3-surface. Despite the fact the induced metric is degenerate, one can deduce the Hamiltonian vector field associated with the one-form using the general defining conditions

$$i_{v_F}\omega \quad = \quad dF \tag{17.7.1}$$

since the vanishing of the metric determinant appearing in the formal definition cancels out in the expression of the Hamiltonian vector field.

The explicit formula is obtained by writing ω as

$$\begin{aligned} &\omega = K\epsilon_{\alpha\beta\gamma} \times \epsilon^{\mu\nu\delta} A_\mu J_{\nu\delta}\sqrt{g} \ \ = \epsilon_{\alpha\beta\gamma} \times C - S \ \ , \\ &C - S = K E^{\alpha\beta\gamma} A_\alpha J_{\beta\gamma} \ \ . \end{aligned} \tag{17.7.2}$$

Here $E^{\alpha\beta\gamma} = \epsilon_{\alpha\beta\gamma}$ holds true numerically and metric determinant, which vanishes for light-like 3-surfaces, has disappeared.

The Hamiltonian vector field is the curl of F divided by the Chern-Simons action density $C - S$:

$$v_F^\alpha = \frac{1}{2}\times \frac{\epsilon^{\alpha\beta\gamma}(\partial_\beta F_\gamma - \partial_\gamma F_\beta)\sqrt{g}}{C-S\sqrt{g}} = \frac{1}{2}\times \frac{E^{\alpha\beta\gamma}(\partial_\beta F_\gamma - \partial_\gamma F_\beta)}{C-S} \quad . \tag{17.7.3}$$

The Hamiltonian vector field multiplied by the dual of 3-form multiplied by the metric determinant has a vanishing divergence and is analogous to a vector field generating volume preserving flow. and the value of Chern Simons 3-form defines the analog of the metric determinant for light-like 3-surfaces. The generalized Poisson bracket for Hamiltonian 1-forms defined in terms of the action of Hamiltonian vector field on Hamiltonian as $J_1^\beta D_\beta F_2\alpha - J_2^\beta D_\beta H_2\alpha$ is Hamiltonian 1-form. Here J_i denotes the Hamiltonian vector field associated with F_i. The bracked unique apart from gradient. The corresponding vector field is the commutator of the Hamiltonian vector fields.

The objection is that gauge invariance is broken since the expression for the vector field assigned to the Hamiltonian one-form depends on gauge. In TGD framework there is no need to worry since Kähler gauge potential has unique natural expression and the $U(1)$ gauge transformations of Kähler gauge potential induced by symplectic transformations of CP_2 are not genuine gauge transformations but dynamical symmetries since the induced metric changes and space-time surface is deformed. Another important point is that Kähler gauge potential for a given CD has M^4 part which is "pure gauge" constant Lorentz invariant vector and proportional to the inverse of gravitational constant G. Its ratio to CP_2 radius squared is determined from electron mass by p-adic mass calculations and mathematically by quantum criticality fixing also the value of Kähler coupling strength.

17.7.4 TGD Variant For The Category Ncob

John Baez has suggested that quantum field theories could be formulated as functors from the category of n-cobordisms to the category of Hilbert spaces [A71, A38]. In TGD framework light-like 3-surfaces containing the number theoretical braids define the analogs of 3-cobordisms and surface property brings in new structure. The motion of topological condensed 3-surfaces along 4-D space-time sheets brings in non-trivial topology analogous to braiding and not present in category nCob.

Intuitively it seems possible to speak about one-dimensional orbits of wormhole throats and -contacts (fermions and bosons) in background space-time (homological dimension). In this case linking or knotting are not possible since knotting is co-dimension 2 phenomenon and only objects whose homological dimensions sum up to $D-1$ can get linked in dimension D. String like objects could topologically condense along wormhole contact which is string like object. The orbits of closed string like objects are homologically co-dimension 2 objects and could get knotted if one does not allow space-time sheets describing un-knotting. The simplest examples are ordinary knots which are not allowed to evolve by forming self intersections. The orbits of point like wormhole contact and closed string like wormhole contact can get linked: a point particle moving through a closed string is basic dynamical example. There is no good reason preventing unknotting and unlinking in absolute sense.

17.7.5 Number Theoretical Universality And Category Theory

Category theory might be also a useful tool to formulate rigorously the idea of number theoretical universality and ideas about cognition. What comes into mind first are functors real to p-adic physics and vice versa. They would be obtained by composition of functors from real to rational physics and back to p-adic physics or vice versa. The functors from real to p-adic physics would provide cognitive representations and the reverse functors would correspond to the realization of intentional action. The functor mapping real 3-surface to p-adic 3-surfaces would be simple: interpret the equations of 3-surface in terms of rational functions with coefficients in some algebraic extension of rationals as equations in arbitrary number field. Whether this description applies or is needed for 4-D space-time surface is not clear.

At the Hilbert space level the realization of these functors would be quantum jump in which quantum state localized to p-adic sector tunnels to real sector or vice versa. In zero energy ontology this process is allowed by conservation laws even in the case that one cannot assign classical conserved quantities to p-adic states (their definition as integrals of conserved currents does not make sense since definite integral is not a well-defined concept in p-adic physics). The interpretation would be in terms of generalized M-matrix applying to cognition and intentionality. This M-matrix would have values in the field of rationals or some algebraic extension of rationals. Again a generalization of Connes tensor product is suggestive.

17.7.6 Category Theory And Fermionic Parts Of Zero Energy States As Logical Deductions

Category theory has natural applications to quantum and classical logic and theory of computation [A38]. In TGD framework these applications are very closely related to quantum TGD itself since it is possible to identify the positive and negative energy pieces of fermionic part of the zero energy state as a pair of Boolean statements connected by a logical deduction, or rather- quantum superposition of them. An alternative interpretation is as rules for the behavior of the Universe coded by the quantum state of Universe itself. A further interpretation is as structures analogous to quantum computation programs with internal lines of Feynman diagram would represent communication and vertices computational steps and replication of classical information coded by number theoretical braids.

17.7.7 Category Theory And Hierarchy Of Planck Constants

Category theory might help to characterize more precisely the proposed geometric realization of the hierarchy of Planck constants explaining dark matter as phases with non-standard value of Planck constant. The situation is topologically very similar to that encountered for generalized Feynman diagrams. Singular coverings and factor spaces of M^4 and CP_2 are glued together along 2-D manifolds playing the role of object and space-time sheets at different vertices could be interpreted as arrows going through this object.

Chapter 18

Could categories, tensor networks, and Yangians provide the tools for handling the complexity of TGD?

18.1 Introduction

The dynamics of TGD is extremely simple locally: space-times are surfaces of 8-D imbedding space so that only four field-like dynamical variables are present and preferred extremals satisfy strong form of holography (SH) meaning that almost 2-D data determine them. TGD Universe looks however also extremely complex. There is a hierarchy of space-times sheets, hierarchy of p-adic length scales, hierarchy of dark matters labelled by the values of Planck constant $h_{eff}/h = n$, hierarchy of extensions of rationals defining hierarchy of adeles in adelic physics view about TGD, hierarchy of infinite primes (and rationals), and also the hierarchy of conscious entities (quantum measurement theory in zero energy ontology can be seen as theory of consciousness [L39]).

During years it has become gradually clear that category theory could be the mathematical language of quantum TGD [K11, K10, K6]. Only category theory gives hopes about unifying various hierarchies making TGD Universe to look so horribly complex. Hierarchy formed by categories, categories of categories, ... could be the mathematics needed to keep book about this complexity and provide also otherwise unexpected constraints.

The arguments developed in the sequel suggest the following overall view.

(a) Positive and negative energy parts of zero energy states can be regarded as tensor networks [L23] identifiable as categories. The new element is that one does not have only particles (objects) replaced with partonic 2-surfaces but also strings connecting them (morphisms). Morphisms and functors provide a completely new element not present in the standard model. For instance, S-matrix would be a functor between categories. Various hierarchies of of TGD would in turn translate to hierarchies of categories.

(b) The recent view about generalized Feynman diagrams [L22, L24, L38] is inspired by two general ideas. First, the twistor lift of TGD replaces space-time surfaces with their twistor-spaces getting their twistor structure as induced twistor structure from the product of twistor spaces of M^4 and CP_2. Secondly, topological scattering diagrams are analogous to computations and can be reduced to minimal diagrams, which are tree diagrams with braiding. This picture fits very nicely with the picture provided by fusion categories. At fermionic level the basic interaction is 2+2 scattering of fermions occurring at the vertices identifiable as partonic 2-surface and re-distributes the fermion lines between partonic 2-surfaces. This interaction is highly analogous to what happens in braiding interaction defining basic gate in topological quantum computation [K85] but vertices expressed in terms of twistors depend on momenta of fermions.

(c) Braiding transformations for fermionic lines identified as boundaries of string world sheets can take place inside the light-like orbits of partonic 2-surfaces defining boundaries of space-time regions with Minkowskian and Euclidian signature of induced metric respectively. Braiding transformation is essentially a permutation for two braid strands mapping tensor product $A \otimes B$ to $B \otimes A$. R-matrix satisfying Yang-Baxter equation [B61] characterizes this operation algebraically.

(d) Reconnections of fermionic strings connecting partonic 2-surfaces are possible and suggest interpretation in terms of 2-braiding generalizing ordinary braiding. I have2-braiding in [K35]: string world sheets get knotted in 4-D space-time forming 2-knots and strings form 1-knots in 3-D space. I do not actually know whether my intuitive believe that 2-braiding reduces to reconnections is correct. Reconnection induces an exchange of braid strands defined by boundaries of the string world sheet and therefore exchange of fermion lines defining boundaries string world sheets. This requires a generalization of quantum algebras to include also algebraic representation for reconnection: this representation could reduce to a representation in terms of an analog of R-matrix.

Yangians [B30] seem to be especially natural quantum algebras from TGD point of view [K76, L38]. Quantum algebras are bi-algebras having co-product Δ, which in well-defined sense is the inverse of the product. This makes the algebra multi-local: this feature is very attractive as far as understanding of bound states is considered. Δ-iterates of single particle system would give many-particle systems with non-trivial interactions reducing to kinematics.

One should assign Yangian to various Super-Kac-Moody algebras (SKMAs) involved and even with super-symplectic algebra (SSA) [K15, K88, K110], which however reduces effectively to SKMA for finite-dimensional Lie group if the proposed gauge conditions meaning vanishing of Noether charges for some sub-algebra H of SSA isomorphic to it and for its commutator $[SSA, H]$ with the entire SSA. Strong form of holography (SH) implying almost 2-dimensionality motivates these gauge conditions. Each SKMA would define a direct summand with its own parameter defining coupling constant for the interaction in question. There is also extended SKMA associated with the light-like orbits of partonic 2-surfaces and it seems natural to identify appropriate sub-algebras of these two algebras as duals in Yangian sense.

There is also partonic super-Kac-Moody algebra (PSKMA) associated with partonic 2-surfaces extending ordinary SKMA. On old conjecture is that SSA and PSKMA are physically dual in the same sense as the conformal algebra and its dual in twistor Grassmannian approach and that this generalizes equivalence principle (EP) to all conserved charges.

The plan of the article is following.

(a) The basic notions and ideas about tensor networks as categories and about Yangians as multi-local symmetries and fundamental description of interactions are described.

(b) The questions related to the Yangianization in TGD framework are considered. Yangianization of four-momentum and mass squared operator are discussed as examples.

(c) The next section is devoted to category theory as tool of TGD: braided categories and fusion categories are briefly described and the notion of category with reconnection is considered.

(d) The last section tries to represent the "great vision" in more detail.

18.2 Basic vision

The existing vision about TGD is summarized first and followed by a proposal about tensor networks as categories and Yangians as a multi-local generalization of symmetries with partonic surfaces replacing point like particles.

18.2.1 Very concise summary about basic notions and ideas of TGD

Let us briefly summarize the basic notions and ideas of TGD.

(a) Space-times are regarded as 4-surfaces in $H = M^4 \times CP_2$, which is fixed uniquely by the condition that the factors of $H = M^4 \times S$ allow twistor space with Kähler structure [A63]. The twistor spaces of dynamically allowed space-time surfaces are assumed to be representable as 6-D surfaces in twistor space $T(H) = T(M^4) \times T(CP_2)$ getting their twistor structure by induction from that of $T(H)$. $T(M^4)$ is identified as its purely geometric variant $T(M^4) = M^4 \times CP_1$. At the level of momentum space the usual identification is more appropriate. It is also assumed that these space-time surfaces are obtained as extremals of 6-D Kähler action [K76, L24, L38]. At space-time level this gives rise to dimensionally reduced Kähler action equal to the sum of volume term and 4-D Kähler action. Either the entire action or volume term would correspond to vacuum energy parameterized by cosmological constant in standard cosmology. Planck length corresponds to the radius of twistor sphere of M^4.

(b) Strong form of holography (SH) implied by strong form of general coordinate invariance (SGCI) stating that light-like 3-surfaces defined by parton orbits and 3-D space-like ends of space-time surface at boundaries of CD separately code 3-D holography. SH states that 2-D data at string world sheets plus condition fixing the points of space-time surface with H-coordinates in extension of rationals fix the real space-time surface.

   i. SH strongly suggests that the preferred extremals of the dimensionally reduced action satisfy gauge conditions (vanishing Noether charges) for a subalgebra H of super-symplectic algebras (SSA) isomorphic to it and its commutator $[H, SSA]$ with SSA: this effectively reduces SSA to a finite-dimensional Kac-Moody algebra.

   ii. Similar dimensional reduction would take place in fermionic degrees of freedom, where super-conformal symmetry fixes 4-D Dirac action, when bosonic action is known [K88, K110]. This involves the new notion of modified gamma matrices determined in terms of canonical momentum currents associated with the action. Quantum classical correspondence (QCC) states that classical Cartan charges for SSA are equal to the eigenvalues of corresponding fermionic charges. This gives a correlation between space-time dynamics and quantum numbers of positive (negative) parts of zero energy states.

   iii. SH implies that fermions are effectively localized at string world sheets: in other words, the induced spinor fields Ψ_{int} in space-time interior are determined their values Ψ_{string} at string world sheets. There are two options: Ψ_{int} is either continuation of Ψ_{string} or Ψ_{string} serves as the source of Ψ_{int} [L29].

(c) At space-time level the dynamics is extremely simple locally since by general coordinate invariance (GCI) only 4 field-like variables are dynamical, and one has also SH by SGCI. Topologically the situation is rather complex: one has many-sheeted space-time having hierarchical structure. The GRT limit of TGD [K79] is obtained in long length scales by mapping the many-sheeted structure to a slightly curved piece of M^4 by demanding that the deformation of M^4 metric is sum of the deformation of he induced metrics of space-time surface from M^4 metric. Similar description implies to gauge potentials in terms of induced gauge potentials. The many-sheetedness is visible as anomalies of GRT and plays central role in quantum biology [K94].

(d) Zero energy ontology (ZEO) means that one consider space-time surfaces inside causal diamonds (CDs defined as intersections of future and past directed light-cones with points replaced with CP_2) forming a scale hierarchy. Zero energy states are tensor products of positive and negative energy parts at opposite boundaries of CD. Zero energy property means that the total conserved quantum numbers are opposite at the opposite boundaries of CD so that one has consistency with ordinary positive energy ontology. Zero energy states are analogous to physical events in the usual ontology but is much more flexible since given zero energy energy states is in principle creatable from vacuum.

(e) The "world of classical worlds" (WCW) [K34, K15, K110] generalizes the superspace of Wheeler. WCW decomposes to sub-WCWs assignable to CDs forming a scale hierarchy. Note that 3-surface in ZEO corresponds to a pair of disjoint collections 3-surfaces at opposite boundaries of CD- initial and final state in standard ontology. Super-symplectic symmetries (SCA) act as isometries of WCW. Zero energy states correspond to WCW spinor fields and the gamma matrices of WCW are expressible as linear combinations of fermionic oscillator operators for induced spinor fields. Besides SCA there is partonic super-Kac-Moody algebra (PSCA) acting on light-like orbits of partonic 2-surfaces and these algebras are suggested to be dual physically (generalized EP).

(f) One ends up with an extension of real physics to adelic physics [L34]. p-Adic physics for various primes are introduced as physical correlates of cognition and imagination: the original motivation come from p-adic mass calculations [K46]. p-Adic non-determinism (pseudo constants) [K45, K73] strongly suggests that one can always assign to 2-D holographic data a p-adic variant of space-time surface as a preferred extremal. In real case this need not be the case so that the space-time surface realized as preferred extremal is imaginable but not necessarily realizable.

   p-Adic physics and real physics are fused to adelic physics: space-time surface isa book-like structure with pages labelled by real number field and p-adic number fields in an extension induced by some extension of rationals. Planck constants $h_{eff} = n \times h$ corresponds to the dimension of the extension dividing the order of its Galois group and favored p-adic primes correspond to ramified primes for favored extensions. Evolution corresponds to increasing complexity of extension of rationals and favored extensions are the survivors in fight for number theoretic survival.

(g) Twistor lift of TGD leads to a proposal for the construction of scattering amplitudes assuming Yangian symmetry assignable to Kac-Moody algebras for imbedding space isometries, with electroweak gauge group, and for finite-D Lie dynamically generated Lie group selected by conditions on SSA algebra. 2+2 fermion vertex analogous to braiding interaction serves as the basic vertex in the formulation of [L38].

18.2.2 Tensor networks as categories

The challenge has been the identification of relevant categories and physical realization of them. One can imagine endless number of identifications but the identification of absolutely convincing candidate has been difficult. Quite recently an astonishingly simple proposal emerged.

(a) The notion of tensor network [B43] has emerged in condensed matter physics to describe strongly entangled systems and complexity associated with them. Holography is in an essential role in this framework. In TGD framework tensor network is realized physically at the level of the topology and geometry of many-sheeted space-time [L23]. Nodes would correspond to objects and links between them to morphisms. This structure would be realized as partonic 2-surfaces - objects - connected by fermionic strings - morphisms - assignable to magnetic flux tubes. Morphisms would be realized as Hilbert space isometries defined by entanglement. Physical state would be category or set of them!

   Functors are morphisms of categories mapping objects to objects and morphisms to morphisms and respecting the composition of morphisms so that the structure of the category is preserved. For instance, in zero energy ontology (ZEO) S-matrix for given space-time surface could be a unitary functor assigning to an initial category final category: they would be represented as quantum states at the opposite boundaries of causal diamond (CD). Also quantum states could be categories of categories of in accordance with various hierarchies.

(b) Skeptic could argue as follows. The passive part of zero energy states for which active part evolves by unitary time evolutions following by state function reductions inducing time localization in moduli space of CDs, could be category. But isn't the active path more naturally a quantum superposition of categories? Should one replace time evolution as a functor with its quantum counterpart, which generates a quantum superposition of categories? If so, then state function reduction to opposite boundary of CD would mean localization in the set of categories! This is quite an abstraction from simple localization in 3-space in wave mechanics.

(c) Categories form categories with functors between categories acting as morphisms. In principle one obtains an infinite hierarchy of categories identifiable as quantum states. This would fit nicely with various hierarchies associated with TGD, most of which are induced by the hierarchy of extensions of rationals.

(d) The language of categories fits like glove also to TGD inspired theory of consciousness. The fermionic strings and associated magnetic flux tubes would serve as correlates of attention. The associated morphism would define the direction of attention and also define sensory maps as morphisms. Conscious intelligence relies crucially on analogies and functors realize mathematically the notion of analogy. Categorification means basically classification and this is what cognition does all the time.

18.2.3 Yangian as a generalization of symmetries to multilocal symmetries

Mere networks of arrows are not enough. One needs also symmetry algebra associated with them giving flesh around the bones.

(a) Various quantum algebras, in particular Yangians are naturally related to physically interesting categories. The article of Jimbo [B61], one of the pioneers of quantum algebras, gives a nice summary of Yang-Baxter equation central in the construction of quantum algebras. R-matrix performs is an endomorphism permuting two tensor factors in quantal matter.

(b) One of the nice features of Yangian is that it gives hopes for a proper description of bound states problematic in quantum field theories (one can argue that QCD cannot really describe hadrons and already QED has problems with Bethe-Salpeter equation for hydrogen atom). The idea would be simple. Yangian would provide many-particle generalization of single particle symmetry algebra and give formulas for conserved charges of many-particle states containing also interaction terms. Interactions would reduce to kinematics. This - as I think - is a new idea.

The iteration of the co-product Δ would map single particle symmetry operator by homomorphism to operator acting in N-parton state space and one would obtain a hierarchy of algebra generators labelled by N and Yangian inariance would dictate the interaction terms completely (as it indeed does in $\mathcal{N}=4$ SUSY in twistor Grassmannian approach [B31]).

(c) There is however a delicacy involved. There is a mysterious looking doubling of the symmetry generators. One has besides ordinary local generators T_0^A generators T_1^A: in twistor Grassmann approach the latter correspond to dual conformal symmetries. For T_0^A the co-product is trivial: $\Delta(J_0^A) = J_0^A \otimes 1 + 1 \otimes J_0^A$, just like in non-interacting theory. This is true for all iterates of Δ.

For J_1^A one has $\Delta(J_1^A) = J_1^A \otimes 1 + 1 \otimes J_1^A + f^A_{BC} J_0^B \otimes J_0^C$. One has two representations and the duality suggests that the eigenvalues J_0^A and J_1^A are same (note that in Witten's approach [B30] $J_1^A = 0$ holds true so that it does not apply as such to TGD). The differences $T_0^A - T_1^A$ would give a precise meaning for "interaction charges" if the duality holds true, and more generally, to the perturbation theory formed by a pair of free and interacting theory. This picture raises hopes about first principle description of bound states: interactions described in wave mechanics in terms of phenomenological interaction Hamiltonians and interaction potentials would be reduced to kinematics.

For instance, for four-momentum $\Delta(P_1^k)$ would contain besides free particle term $P_0^k \otimes 1 + 1 \otimes P_0^k$ also the interaction term involving generators of - say - conformal group.

(d) What about the physical interpretation of the doubling? The most natural interpretation would be in terms of SSA and the extended super-conformal algebra assignable to the light-like orbits of partonic 2-surfaces. An attractive interpretation is in terms of a generalization of Equivalence Principle (EP) stating that inertial and gravitational charges are identical for the physical states.

(e) The tensor summands of Kac-Moody algebra would have different coupling constants k_i perhaps assignable to the 4 fundamental interactions and to the dynamical gauge group emerging from the SCA would give further coupling constant. This would give 5 tensor factors strongly suggested by p-adic mass calculations - p-adic masses depend only on the number of tensor factors [K46].

18.3 Some mathematical background about Yangians

In the following necessary mathematical background about Yangians are summarized.

18.3.1 Yang-Baxter equation (YBE)

Yang-Baxter equation (YBE) has been used for more than four decades in integrable models of statistical mechanics of condensed matter physics and of 2-D quantum field theories (QFTs) [A79]. It appears also in topological quantum field theories (TQFTs) used to classify braids and knots [B30] (see `http://tinyurl.com/mcvvcqp`) and in conformal field theories and models for anyons. Yangian symmetry appears also in twistor Grassmann approach to scattering amplitudes [B31, B39] and thus involves YBE. At the same time new invariants for links were discovered and new braid-type relation was found. YBEs emerged also in 2-D conformal field theories.

Yang-Baxter equation (YBE) has a long history described in the excellent introduction to YBE by Jimbo [B61] (see `http://tinyurl.com/l4z6zyr`, where one can also find a list of references). YBE was first discovered by McGuire (1964) and 3 years later by Yang in quantum mechanical many-body problem involving delta function potential $\sum_{i<j}\delta(x_i-x_j)$. Using Bethe's Ansatz for building wave functions they found that the scattering matrix factorized that it could be constructed using as building brick 2-particle scattering matrix - R-matrix. YBE emerged for R-matrix as a consistency condition for factorization. Baxter discovered 1972 solution of the eight vertex model in terms of YBE. Zamolodchikov pointed ot that the algebraic mechanism behind factorization of 2-D QFTs is same as in condensed matter models.

1978-1979 Faddeev, Sklyanin, and Takhtajan proposed quantum inverse scattering method as a unification of classical and quantum integrable models. Eventually the work with YBE led to the discovery of the notion of quantum group by Drinfeld. Quantum group can be regarded as a deformation $U_q(g)$ of the universal enveloping algebra $U(g)$ of Lie algebra. Drinfeld also introduced the universal R-matrix, which does not depend on the representation of algebra used.

R-matrix satisfying YBE is now the common aspect of all quantum algebras. I am not a specialist in YBE and can only list the basic points of Jimbo's article. Interested reader can look for details and references in the article of Jimbo.

In 2-D quantum field theories R-matrix $R(u)$ depends on one parameter u identifiable as hyperbolic angle characterizing the velocity of the particle. $R(u)$ characterizes the interaction experienced by two particles having delta function potential passing each other (see the figure of `http://tinyurl.com/kyw6xu6`). In 2-D quantum field theories and in models for basic gate in topological quantum computation (for early TGD vision see [K85] were also R-matrix is discussed in more detail) the R-matrix is unitary. One can interpret R-matrix as endomorphism mapping $V_1\otimes V_2$ to $V_2\otimes V_1$ representing permutation of the particles.

YBE

R-matrix satisfies Yang-Baxter equation (YBE)

$$R_{23}(u)R_{13}(u+v)R_{12}(v) = R_{12}(v)R_{13}(u+v)R_{23}(u) \tag{18.3.1}$$

having interpretation as associativity condition for quantum algebras.

At the limit $u, v \to \infty$ one obtains R-matrix characterizing braiding operation of braid strands. Replacement of permutation of the strands with braid operations replaces permutation group for n strands with its covering group. YBE states that the braided variants of identical permutations (23)(13)(12) and (12)(13)(23) are identical.

The equations represent n^6 equations for n^4 unknowns and are highly over-determined so that solving YBE is a difficult challenge. Equations have symmetries, which are obvious on basis of the topological interpretation. Scaling and automorphism induced by linear transformations of V act as symmetries, and the exchange of tensor factors in $V \otimes V$ and transposition are symmetries as also shift of all indices by a constant amount (using modulo N arithmetics).

One can pose to the R-matrix some boundary condition. For $V \otimes V$ the condition states that $R(0)$ is proportional to permutation matrix P for the factors.

General results about YBE

The following lists general results about YBE.

(a) Belavin and Drinfeld proved that the solutions of YBE can be continued meromorphic functions to complex plane and define with poles forming an Abelian group. R-matrices can be classified to rational, trigonometric, and elliptic R-matrices existing only for $sl(n)$. Rational and trigonometric solutions have pole at origin and elliptic solutions have a lattice of poles. In [B61] (see `http://tinyurl.com/l4z6zyr`) simplest examples about R-matrices for $V_1 = V_2 = C^2$ are discussed, one of each type.

(b) In [B61] it is described how the notions of R-matrix can be generalized to apply to a collection of vector spaces, which need not be identical. The interpretation is as commutation relations of abstract algebra with co-product Δ - say quantum algebra or Yangian algebra. YBE guarantees the associativity of the algebra.

(c) One can define quasi-classical R-matrices as R-matrices depending on Planck constant like parameter $\hbar$ (which need have anything to do with Planck constant) such that small values of u one has $R = constant \times (I + \hbar r(u) + O(\hbar^2))$. $r(u)$ is called classical r-matrix and satisfies CYBE conditions

$$[r_{12}(u), r_{13}(u+v)] + [r_{12}(u), r_{23}(v)] + [r_{13}(u+v), r_{23}(v)] = 0$$

obtained by linearizing YBE. $r(u)$ defines a deformation of Lie-algebra respecting Jacobi-identities. There are also non-quasi-classical solutions. The universal solution for r-matrix is formulated in terms of Lie-algebra so that the representation spaces V_i can be any representation spaces of the Lie-algebra.

(d) Drinfeld constructed quantum algebras $U_q(g)$ as quantized universal enveloping algebras $U_q(g)$ of Lie algebra g. One starts from a classical r-matrix r and Lie algebra g. The idea is to perform a "quantization" of the Lie-algebra as a deformation of the universal enveloping algebra $U_q(g)$ of $U(g)$ by r. Drinfeld introduces a universal R-matrix independent of the representation used. This construction will not be discussed here since it does not seem to be so interesting as Yangian: in this case co-product Δ does not seem to have a natural interpretation as a description of interaction. The quantum groups are characterized by parameter $q \in C$.

For a generic value the representation theory of q-groups does not differ from the ordinary one. For roots of unity situation changes due to degeneracy caused by the fact $q^N = 1$ for some N.

(e) The article of Jimbo discusses also fusion procedure initiated by Kulish, Restetikhin, and Sklyanin allowing to construct new R-matrices from existing one. Fusion generalizes the method used to construct group representation as powers of fundamental representation. Fusion procedure constructs R-matrix in $W \otimes V^2$, where one has $W = W_1 \otimes W_2 \subset V \otimes V^1$. Picking W is analogous to picking a subspace of tensor product representation $V \otimes V^1$.

18.3.2 Yangian

Yangian algebra $Y(g(u))$ is associative Hopf algebra (see `http://tinyurl.com/qfl8dwu`) that is bi-algebra consisting of associative algebra characterized by product μ: $A \otimes A \to A$ with unit element 1 satisfying $\mu(1, a) = a$ and co-associative co-algebra consisting of co-product $\Delta A \in A \otimes A$ and co-unit $\epsilon : A \to C$ satisfying $\epsilon \circ \Delta(a) = a$. Product and co-product are "time reversals" of each other. Besides this one has antipode S as algebra anti-homomorphism $S(ab) = S(b)S(a)$. YBE has interpretation as an associativity condition for co-algebra $(\Delta \otimes 1) \circ \Delta = (1 \otimes \Delta) \circ \Delta$. Also ϵ satisfies associativity condition $(\epsilon \otimes 1) \circ \Delta = (1 \otimes \epsilon) \circ \Delta$.

There are many alternative formulations for Yangian and twisted Yangian listed in the slides of Vidas Regelskis at `http://tinyurl.com/ms9q8u4`. Drinfeld has given two formulations and there is FRT formulation of Faddeev, Restetikhin and Takhtajan.

Drinfeld's formulation [B61] (see `http://tinyurl.com/qfl8dwu`) involves the notions of Lie bi-algebra and Manin triple, which corresponds to the triplet formed by half-loop algebras with positive and negative conformal weights, and full loop algebra. There is isomorphism mapping the generating elements of positive weight and negative weight loop algebra to the elements of loop algebra with conformal weights 0 and 1. The integer label n for positive half loop algebra corresponds in the formulation based on Manin triple to conformal weight. The alternative interpretation for $n+1$ would be as the number of factors in the tensor power of algebra and would in TGD framework correspond to the number of partonic 2-surfaces. In this interpretation the isomorphism becomes confusing.

In any case, one has two interpretations for $n + 1 \geq 1$: either as parton number or as occupation number for harmonic oscillator having interpretation as bosonic occupation number in quantum field theories. The relationship between Fock space description and classical description for n-particle states has remained somewhat mysterious and one can wonder whether these two interpretation improve the understanding of classical correspondence (QCC).

Witten's formulation of Yangian

The following summarizes my understanding about Witten's formulation of Yangian in $\mathcal{N} = 4$ SUSYs [B30], which does not mention explicitly the connection with half loop algebras and loop algebra and considers only the generators of Yangian and the relations between them. This formulation gives the explicit form of Δ and looks natural, when n corresponds to parton number. Also Witten's formulation for Super Yangian will be discussed.

It must be however emphasized that Witten's approach is not general enough for the purposes of TGD. Witten uses the identification $\Delta(J_1^A) = f^A_{BC} J_0^B \times J_0^C$ instead of the general expression $\Delta(J_1^A) = J_1^A \otimes 1 + 1 \times J_1^A + f^A_{BC} J_0^B \times J_0^C$ needed in TGD strongly suggested by the dual roles of the super-symplectic conformal algebra and super-conformal algebra associated with the light-like partonic orbits realizing generalized EP. There is also a nice analogy with the conformal symmetry and its dual twistor Grassmann approach.

The elements of Yangian algebra are labelled by non-negative integers so that there is a close analogy with the algebra spanned by the generators of Virasoro algebra with non-negative conformal weight. The Yangian symmetry algebra is defined by the following relations for the generators labeled by integers $n = 0$ and $n = 1$. The first half of these relations discussed in very clear manner in [B30] follows uniquely from the fact that adjoint representation of the Lie algebra is in question

$$\left[J^A, J^B\right] = f_C^{AB} J^C \quad , \quad \left[J^A, J^{(1)B}\right] = f_C^{AB} J^{(1)C} \quad . \tag{18.3.2}$$

Besides this Serre relations are satisfied. These have more complex form and read as

$$\begin{aligned} &\left[J^{(1)A}, \left[J^{(1)B}, J^C\right]\right] + \left[J^{(1)B}, \left[J^{(1)C}, J^A\right]\right] + \left[J^{(1)C}, \left[J^{(1)A}, J^B\right]\right] \\ &\qquad = \frac{1}{24} f^{ADK} f^{BEL} f^{CFM} f_{KLM} \{J_D, J_E, J_F\} \ , \\ &\left[\left[J^{(1)A}, J^{(1)B}\right], \left[J^C, J^{(1)D}\right]\right] + \left[\left[J^{(1)C}, J^{(1)D}\right], \left[J^A, J^{(1)B}\right]\right] \\ &\qquad = \frac{1}{24}(f^{AGL} f^{BEM} f_K^{CD} \\ &\qquad + f^{CGL} f^{DEM} f_K^{AB}) f^{KFN} f_{LMN} \{J_G, J_E, J_F\} \ . \end{aligned} \tag{18.3.3}$$

The indices of the Lie algebra generators are raised by invariant, non-degenerate metric tensor g_{AB} or g^{AB}. $\{A, B, C\}$ denotes the symmetrized product of three generators.

The right hand sides have often as a coefficient $\hbar^2$ instead of $1/24$. $\hbar$ need not have anything to do with Planck constant. The Serre relations give constraints on the commutation relations of $J^{(1)A}$. For $J^{(1)A=J^A}$ the first Serre relation reduces to Jacobi identity and second to antisymmetry of Lie bracket. The right hand sided involved completely symmetrized trilinears $\{J_D, J_E, J_F\}$ making sense in the universal covering of the Lie algebra defined by J^A.

Repeated commutators allow to generate the entire algebra whose elements are labeled by non-negative integer n. The generators obtain in this manner are n-local operators arising in $(n-1)$-commutator of $J^{(1)}$: s. For $SU(2)$ the Serre relations are trivial. For other cases the first Serre relation implies the second one so the relations are redundant. Why Witten includes it is for the purposed of demonstrating the conditions for the existence of Yangians associated with discrete one-dimensional lattices (Yangians exists also for continuum one-dimensional index).

Discrete one-dimensional lattice provides under certain consistency conditions a representation for the Yangian algebra. One assumes that each lattice point allows a representation R of J^A so that one has $J^A = \sum_i J_i^A$ acting on the infinite tensor power of the representation considered. The expressions for the generators J^{1A} in Witten's approach are given as

$$J^{(1)A} = f_{BC}^A \sum_{i<j} J_i^B J_j^C \ . \tag{18.3.4}$$

This formula gives the generators in the case of conformal algebra. This representation exists if the adjoint representation of G appears only one in the decomposition of $R \otimes R$. This is the case for SU(N) if R is the fundamental representation or is the representation of by k^{th} rank completely antisymmetric tensors.

This discussion does not apply as such to $\mathcal{N} = 4$ case the number of lattice points is finite and corresponds to the number of external particles so that cyclic boundary conditions are needed guarantee that the number of lattice points reduces effectively to a finite number. Note that the Yangian in color degrees of freedom does not exist for SU(N) SYM.

As noticed, Yangian algebra is a Hopf algebra and therefore allows co-product. The co-product Δ is given by

$$\begin{aligned} \Delta(J^A) &= J^A \otimes 1 + 1 \otimes J^A \ , \\ \Delta(J^{(1)A}) &= J^{(1)A} \otimes 1 + 1 \otimes J^{(1)A} + f^A_{BC} J^B \otimes J^C \end{aligned} \tag{18.3.5}$$

Δ allows to imbed Lie algebra to the tensor product in non-trivial manner and the non-triviality comes from the addition of the dual generator to the trivial co-product. In the case that the single spin representation of $J^{(1)A}$ is trivial, the co-product gives just the expression of the dual generator using the ordinary generators as a non-local generator. This is assumed in the recent case and also for the generators of the conformal Yangian.

Super-Yangian

Also the Yangian extensions of Lie super-algebras make sense. From the point of physics especially interesting Lie super-algebras are $SU(m|m)$ and $U(m|m)$. The reason is that $PSU(2,2|4)$ (P refers to "projective") acting as super-conformal symmetries of $\mathcal{N}=4$ SYM and this super group is a real form of $PSU(4|4)$. The main point of interest is whether this algebra allows Yangian representation and Witten demonstrated that this is indeed the case [B30].

These algebras are Z_2 graded and decompose to bosonic and fermionic parts which in general correspond to n- and m-dimensional representations of $U(n)$. The representation associated with the fermionic part dictates the commutation relations between bosonic and fermionic generators. The anti-commutator of fermionic generators can contain besides identity also bosonic generators if the symmetrized tensor product in question contains adjoint representation. This is the case if fermions are in the fundamental representation and its conjugate. For $SU(3)$ the symmetrize tensor product of adjoint representations contains adjoint (the completely symmetric structure constants d_{abc}) and this might have some relevance for the super $SU(3)$ symmetry.

The elements of these algebras in the matrix representation (no Grassmann parameters involved) can be written in the form

$$x = \left(\begin{array}{cc} a & b \\ c & d \end{array} \right) \ .$$

a and d representing the bosonic part of the algebra are $n\times n$ matrices and $m\times m$ matrices corresponding to the dimensions of bosonic and fermionic representations. b and c are fermionic matrices are $n\times m$ and $m\times n$ matrices, whose anti-commutator is the direct sum of $n\times n$ and $n\times n$ matrices. For $n=m$ bosonic generators transform like Lie algebra generators of $SU(n)\times SU(n)$ whereas fermionic generators transform like $n\otimes\overline{n}\oplus\overline{n}\otimes n$ under $SU(n)\times SU(n)$. Supertrace is defined as $Str(x)=Tr(a)-Tr(b)$. The vanishing of Str defines $SU(n|m)$. For $n\neq m$ the super trace condition removes identity matrix and $PU(n|m)$ and $SU(n|m)$ are same. That this does not happen for $n=m$ is an important delicacy since this case corresponds to $\mathcal{N}=4$ SYM. If any two matrices differing by an additive scalar are identified (projective scaling as now physical effect) one obtains $PSU(n|n)$ and this is what one is interested in.

Witten shows that the condition that adjoint is contained only once in the tensor product $R\otimes\overline{R}$ holds true for the physically interesting representations of $PSU(2,2|4)$ so that the generalization of the bilinear formula can be used to define the generators of $J^{(1)A}$ of super Yangian of $PU(2,2|4)$. The defining formula for the generators of the Super Yangian reads as

$$\begin{aligned} J_C^{(1)} &= g_{CC'} J^{(1)C'} = g_{CC'} f_{AB}^{C'} \sum_{i<j} J_i^A J_j^B \\ &= g_{CC'} f_{AB}^{C'} g^{AA'} g^{BB'} \sum_{i<j} J_{A'}^i J_{B'}^j \ . \end{aligned} \tag{18.3.6}$$

Here $g_{AB} = Str(J_A J_B)$ is the metric defined by super trace and distinguishes between $PSU(4|4)$ and $PSU(2,2|4)$. In this formula both generators and super generators appear.

18.4 Yangianization in TGD framework

Yangianization of quantum TGD is quite challenging. Super-conformal algebras are much larger than in say $\mathcal{N} = 4$ SUSY and even in superstring models and reconnection and 2-braiding are new topological elements.

18.4.1 Geometrization of super algebras in TGD framework

Super-conformal algebras allow a geometrization in TGD framework and this should be of considerable help in the Yangianization.

(a) The basic generators of various Super-algebras follow from modified Dirac action as Noether charges and their super counterparts obtained by replacing fermion field Ψ (its conjugate $\overline{\Psi}$) by a mode u_m ($\overline{u}_n$) of the induced spinor field [K88, K110]. The anti-commutators of these Noetherian super charges labelled by n define WCW gamma matrices. The replacement of both Ψ and $\overline{\Psi}$ with modes u_m and $\overline{u}_n$ gives a collection of conserved c-number currents and charges labelled by (n,m). These c-number charges define the anti-commutation relations for the induced spinor fields so that quantization reduces to dynamics thanks to the notion of modified gamma matrices forced by super-conformal symmetry.

(b) The natural generalization of Sugawara formula to the level of Yangian of SKMA starts from the Dirac operator for WCW defined like ordinary Dirac operator in terms of the contractions of WCW gamma matrices with the isometry generators (SCA) replacing the Super Virasoro generators G_r and WCW d'Alembert operator defined as its square replacing Virasoro generators L_n. Anti-commutators of WCW gamma matrices defined by super charges for super-symplectic generators define WCW Kähler metric [K88] for which action for preferred extremal would define Kähler function for WCW metric [K34].

(c) Quarks and leptons give rise to a doubling of WCW metric if associated with same space-time sheet that is with the same sector of WCW. The duplication of the super algebra generators - in particular WCW gamma matrices - does not seem to make sense. Do quarks and leptons therefore correspond to different sectors of WCW and live at different space-time surfaces? But what could distinguish between 3-surfaces associated with quarks and leptons?

    Could quarks be associated with homologically non-trivial partonic 2-surfaces with CP_2 homology charges 2,-1,-1 proportional to color hypercharges $2/3, -1/3, -1/3$ and leptons with partonic 2-surfaces with vanishing homology charges coming as multiples of 3? Vanishing of color hypercharge for color-confined states would topologize to a vanishing of total homology charge. Could spin/isospin half property of fundamental fermions topologize to 2-sheeted structure of the space-time surface representing elementary particle consisting of elementary fermions?

SSA acting as isometries of WCW is not the only super-conformal algebra involved.

(a) Partonic 2-surfaces are ends of light-like 3-surfaces- partonic orbits - and give rise to a generalization of SKMA of isometries of H so that they act as local isometries preserving the light-likeness property of the orbits. At the ends of the partonic 2-surface SKMA is associated with complex coordinate of partonic 2-surface. What is the role of this algebra, which is also extended SKMA (already christened PSCA) but with light-like coordinate parameterizing the SKMA generators?

Is it an additional symmetry combining with string world sheet symmetries to a symmetry involving complex coordinate and complex or hypercomplex coordinate? Or is it dual to the string world sheet symmetry? How do these symmetries relate to SSA? Does SGCI implying SH leave only SKMAs associated with isometries, holonomies of CP_2 (electroweak interactions) and dynamical SKMA remaining as remnant of SCA.

(b) I have earlier proposed that Equivalence Principle (EP) as identity of inertial and gravitational charges could reduce to the duality between these SSA assignable to strings and the partonic super-conformal algebra. This picture conforms with the expected form of the generators associated with these algebras. The dual generating elements T_0^A *resp.* T_1^A associated with generic Yangian could naturally correspond to isomorphic sub-algebras of super-conformal algebra associated with orbits of partonic 2-surfaces *resp.* super-symplectic algebra assignable to string world sheets.

18.4.2 Questions

There are many open questions to be answered.

Q1: What Yangianization could mean in TGD framework? The answer is not obvious and one can consider two options.

(a) Assuming that SH leads to an effective reduction of super-symplectic algebra to finite-D Kac-Moody algebra, assign to partonic 2-surfaces direct sum of Kac-Moody type algebras $L(g) = g(z, z^{-1})$ assigned with complex coordinate z of partonic 2-surface. One could perform Yangianization for this algebra meaning that these symmetries become multi-local with locus identified as partonic 2-surface.

In Drinfeld's approach this would mean Yangianization of $L(g)$ rather than g and would involve double loop algebra $L(L(g))$ and its positive and negative energy parts. In Minkowskian space-time regions the generators would be functions of complex coordinate z and hypercomplex coordinate u associated with string world sheet: in Euclidian space-time regions one would have 2 complex coordinates z and w. This would conform with holography. I do not know whether mathematicians have considered this generalization and whether it is possible. In the following this is assumed.

(b) Physical states at partonic 2-surfaces consist of pointlike fermions and one can ask whether this actually means that one can consider just the Lie algebra g so that in Drinfeld's approach one would have just string world sheets and $Y(g)$. Already this option requires the algebraization of reconnection mechanism as a new element. Whether this simpler approach make sense for fermions and by QQC for quantum TGD, is not clear.

Q2: Can one really follow the practice of Grassmannian twistor approach and say that T_1^A and TA^0 are dual?

One has $[T_0^A, T_1^B] = f_C^{AB} T_1^C$. Witten's definition $T_1^A = f_{BC}^A T^B \otimes T^C \equiv T_1^A = f_{BC}^A T^B T^C$ with T_1^A identified as total charges for lattice, identifies T_1^A as 2-particle generators of Yangian. One the other hand, in TGD T_0^A would correspond to partonic super-conformal algebra and T_1^A to bi-local super-symplectic algebra and the general definition to be used regards also T_1^A as single particle generators in Yangian sense and defines the generators at 2-particle level as $\Delta(T_0^A) = T_0^A \otimes 1 + 1 \otimes T_0^A$ and $\Delta(T_1^A) = T_1^A \otimes 1 + 1 \otimes T_1^A + f_{BC}^A T_0^B \otimes T_0^C$.

For the Witten's definition one cannot demand that T_0^A and T_1^A have same eigenvalues for the physical states. For the more general definition of Δ to be followed in the sequel it seems to be possible require that T_0^A and T_1^A obey the same commutation relations for appropriate sub-algebras at least, and that it is possible to diagonalize Cartan algebras simultaneously and even require same total Cartan charges. This issue is not however well-understood.

Q3: What algebras are Yangianized in TGD framework?

The Yangians of SKMAs associated with isometries of $M^4 \times CP_2$ and with the holonomy group $SU(2) \times U(1)$ of CP_2 appear as symmetries. M^4 should give SKMA in transversal degrees of freedom for fermionic string. CP_2 isometries would give SKMA associated with SU(3). $SU(2) \times U(1)$ would be assignable to electroweak symmetries. This gives 4 tensor factors.

Five of them are required by p-adic mass calculations [K46], whose outcome depends only on the number of tensor factors in Virasoro algebra. The estimates for the number of tensor factors has been a chronic head ache: in particular, do M^4 SKMA correspond to single tensor factor or two tensor factors assignable to 2 transversal degrees of freedom.

Supersymplectic algebra (SSA) is assumed to define maximal possible isometry group of WCW guaranteeing the existence of Kähler metric with a well-defined Riemann connection. The Yangian of SSA could be the ultimate symmetry group, which could realize the dream about the reduction of all interactions to mere kinematics. If SSA effectively reduces to a finite-D SKMA for fermionic strings, one would have 5 tensor factors.

Q4: What does SSA mean?

(a) SSA is associated with light-cone boundary $\delta M^4_\pm$ with one light-like direction. The generators (to be distinguished from generating elements) are products of Hamiltonians of symplectic transformations of CP_2 assignable to representations of color SU(3) and Hamiltonians for the symplectic transformations of light-cone boundary, which reduce to Hamiltonians for symplectic transformations of sphere S^2 depending parametrically on the light-like radial coordinate r. This algebra is generalized to analog of Kac-Moody algebra defined by finite-dimensional Lie algebra.

(b) The radial dependence of Hamiltonians of form r^h. The naive guess that conformal weights are integers for the bosonic generators of SSA is not correct. One must allow complex conformal weights of form $h = 1/2 + iy$: $1/2$ comes from the scaling invariant inner product for functions at $\delta M^4_\pm$ defined by integration measure dr/r [K15, K110].

(c) An attractive guess [L16] is that there is an infinite number of generating elements with radial conformal weights given by zeros of zeta. Conformal confinement must holds true meaning that the total conformal weights are real and thus half-odd integers. The operators creating physical states form a sub-algebra assignable by SH and QCC to fermionic string world sheets connecting partonic 2-surfaces.

(d) SH inspires the assumption that preferred extremal property requires that sub-algebra H of SSA isomorphic to itself (conformal weights are integer multiples of SSA) and its commutator SH with SH annihilate physical states and classical Noether charges vanish. This could reduce the symmetry algebra to SKMA for a finite-dimensional Lie group. SSA could be replaced also with the sub-algebra creating physical states having half-odd integer valued radial conformal weights.

    Similar conditions could make sense for the generalization of super-conformal KM algebra associated with light-like partonic orbits.

Q5: What is the precise meaning of SH in the fermionic sector?

Are string world sheets with their ends behaving like pointlike particles enough or are also partonic 2-surface needed. For the latter option a generalization of conformal field theory (CFT) would be needed assigning complex coordinate with partonic 2-surfaces and hyper-complex or complex coordinates with string world sheets. Elementary particle vacuum functionals depend on conformal moduli of partonic 2-surface [K12], which supports the latter option.

There could be however duality between partonic 2-surfaces and string world sheets so that either of them could be enough [L38]. There is also uncertainty about the relationship between induced spinor fields at string world sheets and space-time interior. Are 4-D induced spinor fields obtained by process analogous to analytic continuation in 2-complex dimensional space-time or do 2-D induced spinor fields serve as sources for 4-D induced spinor fields?

Quantum algebras are characterized by parameters such as complex parameter q characterizing R-matrices for quantum groups. Adelic physics [L34] demands number theoretical universality and in particular demands that the parameters - say q - of quantum algebraic structures involved are products $q = e^{m/n}xU$, where U is root of unity (note that e^p exists as ordinary p-adic number for Q_p) and x is real number in the extension. This guarantees that the induced extensions of p-adic numbers are finite-dimensional (the hypothesis is that the correlates of cognition are finite-D extensions of p-adic number fields) [K110].

In the recent view about twistorial scattering amplitudes [L38] the fundamental fermionic vertices are $2 \to 2$ vertices. There is no fermionic contact interaction in the sense of QFT but the fermions coming to the topological vertex defined by partonic 2-surface at which 3 partonic orbits meet (analogy for the 3-vertex for Feynman diagram) are re-distributed between partonic two surfaces. Also in integrable 2-D QFTs in M^2 the vertices are $2 \to 2$ vertices characterized by R-matrix. The twistorial vertex is however not topological.

18.4.3 Yangianization of four-momentum

The QFT picture about bound states is unsatisfactory. The basic question to be answered is whether one should approach the problem in terms of Lorentz invariant mass squared natural in conformal field theories or in terms of Poincare algebra. It is quite possible that the fundamental formulation allowing to understand binding energies is in terms of SCA and PSCA.

Twistor lift of TGD [L38] however suggests that Poincare and even finite-D conformal transformations associated with M^2 could play important role. These longitudinal degrees of freedom are non-dynamical in string dynamics. Maybe there is kind of sharing of labor between these degrees of freedom. In the following we consider two purely pedagogical examples about Yangianization of four-momentum in M^4 and in 8-D context regarding four-momentum as quaternionic 8-momentum in M^8.

Yangianization of four-momentum in conformal algebra of M^4

Consider as an example what the Yangianization for four-momentum P^k could mean. This is a pedagogical example.

(a) The first thing to notice is that the commutation relations between P_0^k and P_1^k are inherited from those between P_0^k and force P_1^k and P_0^k to commute. This holds true quite generally for Cartan algebra so that if the correspondence between T_0^A and T_1^A respects Cartan algebra property then Cartan algebras of T_0^A and T_1^A can be simultaneously diagonalized for the physical states. The Serre relations of Eq. 18.3.3 are identically satisfied for Cartan algebra and its image. This is consistent with the assumption that Cartan algebra is mapped to Cartan algebra but does not prove it.

(b) The formula $f^A_{BC}T_0^A \otimes T_0^C$ for the interaction term appearing in the expresion of Δ should be non-trivial also when T^A corresponds to four-momentum. Already the Poincare algebra gives this kind of term built from Lorentz generators and translation generators.

The extension of Poincare algebra extended to contain dilatation operator D can be considered as also M^4 conformal algebra with generators of special conformal transformations M^A included (see `http://tinyurl.com/nxlmfug`). One has doubling of all algebra generators. The interpretation as gravitational and inertial momenta is one possibility, and EP suggests that the two momenta have same values. In twistor Grassmannian approach the conformal algebras are regarded as dual and suggests the same. Hence one would have $P_0^k = P_1^k$ at the level of eigenvalues.

(c) For conformal group the proposed co-product for P_i^k would read as

$$
\begin{aligned}
\Delta(P_0^k) &= P_0^k \otimes 1 + 1 \otimes P_0^k \ , \\
\Delta(P_1^k) &= P_1^k \otimes 1 + 1 \otimes P_1^k + K f^k_{Al}(L_0^A \otimes P_0^l - P_0^k \otimes L_0^A) + K f^k_{Al}(M_0^A \otimes P_0^l - P_0^l \otimes M_0^A) \\
&+ K(D_0 \times P_0^k - P_0^k \times D_0) \ .
\end{aligned}
\tag{18.4.1}
$$

This condition could be combined with the condition for mass squared operator. For $K=0$ one would have additivity of mass squared requiring that P_1 and P_2 are parallel and light-like. For $K \neq 0$ it might be possible to have a simultaneous solution to the both conditions with massive total momentum.

The Δ-iterates of P_0^k contain no interaction terms. For P_1 one has interaction term. This holds true for all symmetry generators. Assume $P_0 = P_1$: does this mean that the interacting theory associated with P_1 is dual to free theory? The difference $\Delta P_0^k - \Delta(P_1^k)$ defines the analog interaction Hamilton, which would therefore be not due to a somewhat arbitrary decomposition of four-momentum to free and interaction parts. It should be possible to possible to measure this difference and its counterpart for other quantum numbers. One can only make questions about the interpretation for this duality applying to all quantum numbers.

(a) In Drinfeld's construction the negative and positive energy parts of loop algebra would be related by the duality. In ZEO it might be possible to relate them to positive and negative energy parts of zero energy states at the opposite boundaries of CD.

(b) If n is interpreted as number of partonic surfaces and the generators are interpreted as in Witten's construction then the duality could be seen as a geometric duality in plane mapping edges and vertices (partonic 2-surfaces ordered in sequence and string between them) to each other. In super-conformal algebra of twistor Grassmannian approach the generators T_0^A and T_1^A are associated with vertices and edges of the polygon defining the scattering diagram and this suggests that T_0^A corresponds to partonic 2-surfaces and T_1^A to the strings world sheets.

(c) Could the duality be a generalization of for Equivalence Principle identifying inertial and gravitational quantum numbers? This interpretation is encouraged by the presence of SSA action on space-like 3-surfaces at the ends of CDs and extended super-conformal algebra associated with the light-like orbits of partons: SGCI would suggest that these algebras or at least their appropriate sub-algebra are dual. This interpretation conforms also with the above geometric interpretation and twistor Grassmannian interpretation.

Consider for simplicity the situation in which only scaling generator D is present in the extension.

(a) Suppose that one has eigenstate of total momentum $\Delta(P_0^k)$ resp. $\Delta(P_1^k)$ with eigenvalue p_0^{tot} resp. p_1^{tot} and that

$$
p_0^{tot} = p_1^{tot}
\tag{18.4.2}
$$

holds true.

(b) Since D_0 and P_0^k do not commute, the action of D_0 must be realized as differential operator $D_0 = ip_0^k d/dp_0^k$ so that one has following eigenvalue equations

$$
\begin{aligned}
\Delta(P_0^k)\Psi &= (p_{0,1}^k + p_{0,2}^k)\Psi = p_0^{tot}\Psi \ , \\
\Delta(P_1^k)\Psi &= (p_{1,1}^k + p_{1,2}^k)\Psi + K(ip_{0,1}^k \otimes p_{0,2}^r \frac{d}{dp_{0,2}^r} - ip_{0,1}^r \frac{d}{dp_{0,1}^r} \otimes p_{0,2}^k)\Psi = p_1^{tot}\Psi \ .
\end{aligned}
\tag{18.4.3}
$$

Ψ must be a superposition of states $|p_{0,1}, p_{0,2}\rangle$. One has non-trivial interaction. Analogous interaction terms mixing states with different momenta emerge from the terms involving Lorentz generators and special conformal generators.

Four-momenta as quaternionic 8-momenta in octonionic 8-space

In octonionic approach to twistorial scattering amplitudes particles can be regarded as massless in 8-D sense [L38]. The light-like octonionic momenta are actually quaternionic and one would obtain massive states in 4-D sense. Different 4-D masses would correspond to discrete set of quaternionic momenta for 8-D massless particle. Could the above conditions generalize to this case?

(a) Suppose that the symmetries reduce to Poincare symmetry and to a number theoretic color symmetry acting as automorphisms of octonions. In this case the four-momentum for a given $M^4 \subset M^8$ decomposes to a sum of to a direct sum of M^2 invariant under SU(3) and E^2 invariant under $SU(2) \times U(1) \subset SU(3) \subset G_2$. ΔP_1 would be non-trivial for the transversal momentum and of form

$$\begin{aligned}
\Delta(P_0^{L,k})\Psi &= (p_{0,1}^{L,k} + p_{0,2}^{L,k})\Psi = p_0^{tot}\Psi \ , \\
\Delta(P_0^{T,k})\Psi &= (P_0^{T,k} \otimes 1 + 1 \otimes P_0^{T,k})\Psi \ , \\
\Delta(P_1^{L,k})\Psi &= (p_{1,1}^{L,k} + p_{1,2}^{L,k})\Psi = P_1^{L,tot}\Psi \ , \\
\Delta(P_1^{T,k})\Psi &= (P_1^{T,k} \otimes 1 + 1 \otimes P_1^{T,k} + K f_{Al}^{k}(ip_{0,1}^{l} \otimes t_{0,2}^{A} - i(ip_{0,2}^{l} \otimes t_{0,2}^{A})\Psi
\end{aligned} \tag{18.4.4}$$

Here P_0^L resp. P_0^T represents longitudinal resp. transversal momentum and T_0^b denotes $SU(2) \subset SU(3)$ generator representable as differential operator acting on complexified momentum and $p_0^T = p_0^{T,x} + ip_0^{T,y}$ and its conjugate.

(b) In transversal degrees of freedom the assumption about momentum eigenstates would be probably too strong. String model suggests Gaussian in transversal oscillator degrees of freedom. Hadronic physics suggests an eigenstate of transversal momentum squared. TGD based number theoretic considerations suggest that the transversal state is characterized by color quantum numbers.

Hence the conditions

$$p_0^{L,tot} = p_1^{L,tot} \quad , \quad (p_0^{T,tot})^2 = (p_1^{T,tot})^2 \tag{18.4.5}$$

are natural. It would be nice if the momenta p_{01} and p_{02} could be chosen to be on mass shell and satisfy stringy formula for mass squared where transverse momentum squared would correspond to stringy contribution.

One can also add to $\Delta(P)$ the terms coming from conformal group of M^4 or its subgroup. Since octonionic momentum is light-like M^2 momentum for a suitable choice of M^2, one must consider the possibility that the conformal group is that of $M^2 \subset M^4$. Twistorialization supports this view [L38]. The action of conformal generations would be on longitudinal momentum only.

One can wonder how gauge interactions and gravitational interaction do fit to this picture. Is the extension to super-conformal algebra and supersymplectic algebra the only manner to obtain gauge interactions and gravitation into the picture?

18.4.4 Yangianization for mass squared operator

It would be nice to have universal mass formulas as a generalization of mass squared formula for string models in terms of the conformal scaling generator $L_0 = zd/dz$. This operator should have besides single particle contributions also many particle contributions in bound states analogous to interaction Hamiltonian and interaction potential. Yangian as an algebra containing multi-local generators is a natural candidate in this respect.

One can consider Yangianization of Super Virasoro algebra (SVA). The Yangianization of various Super Kac-Moody algebras (SKMA) seems however more elegant if it induces the Yangianization of SVA. Consider first direct Yangianization of SVA. The commutation relations for SVA will be used in the sequel. They can be found in Wikipedia (see `http://tinyurl.com/klsgquz`) so that I do not bother to write them here. It must be emphasized that there might be delicate mathematical constraints on algebras which allow Yangianization as the article of Witten [B30] shows. The considerations here rely on physical intuition with unavoidable grain of wishful thinking.

What about the Yangian variant of mass squared operator m^2in terms of the conformal scaling generator $L_0 = zd/dz$? Consider first the definition of various Super algebras in TGD framework.

(a) In standard approach the basic condition at single particle level $L_0\Psi = h_{vac}\Psi$ giving the eigenvalues of m^2. Massless in generalize sense requires $h_{vac} = 0$. One would have $m^2_{op} = L_0^{vib} + h_{vac}Id$, where "vib" refers to vibrational degrees of freedom of Kac-Moody algebra (KMA). Sugawara construction [A61] allows to express the left-hand side of this formula in terms of Kac-Moody generators - one has sum over squares $T^a_n T_a^{-n}$. One can say that mass squared is Casimir operator vibrational degrees of freedom for KMA

(b) In absence of interactions - and always for $L_{0,0}$ - mass squared formula gives $m_1^2 + m_2^2 = L_0^{vib,1} + L_0^{vib,2}$ for vanishing vacuum weights. It is important to notice that this does *not* imply the additivity of mass squared since one does not have $(p_1 + p_2)^2 = m_1^2 + m_2^2$, which can hold true only for massless and parallel four-momenta. I have considered the possible additivity of mass mass squared for mesons [K47] but it of course fails for systems like hydrogen atom.

One can look what Yangianization of Super Virasoro algebra could mean.

(a) One would have doubling of the generators of SKMA and SVA: one possible explanation is in terms of generalized EP. The difference $\Delta(T_0^A) - \Delta(T_1^A)$ would define the analog of interaction Hamiltonian of the duality holds true.

One has $L_0 = G_0^2/2$. Quite generally, one has $\{G_r, G_{-r}\} = 2L_0$ apart from the central extension term. Generalization Yangian to Super Algebra suggests that one has

$$\begin{aligned} \Delta(L_{0,0}) &= L_{0,0} \otimes 1 + 1 \otimes L_{0,0} \ , \\ \Delta(L_{1,0}) &= L_{1,0} \otimes 1 + 1 \otimes L_{1,0} + K \sum_n G_{0,r} \otimes G_{0,-r} \end{aligned} \tag{18.4.6}$$

Both operators give the value of h_{vac} expected to vanish when acting on physical states and the eigenvalues of the interaction mass squared $K\sum_n G_2 \otimes G_{-r}/2$ would represent the difference $m^2_{0,1} + m^2_{0,2} - m^2_{2,1} - m^2_{2,2}$. By Lorentz invariance the interaction energy is expected to be proportional to the inner product $P_1 \cdot P_2$ and the interpretation in terms of gravitational interaction energy is attractive. The size scale of K would be determined by $l_P^2/R^2 \simeq 2^{-12}$, where l_P is Planck length and R is CP_2 radius gravitational constant [L24, L38].

(b) The action of $k\sum_n G_{0,n} \otimes G_{0,-n}/2$ on state $|p_1, p_2\rangle$ is analogous to the action of a tensor product of Dirac operators on tensor product of spinors. Since Dirac operator changes chirality, this suggests that the states are superpositions of eigenstates of chirality of form

$$\Psi = G_{0,0}\Psi_1 \otimes \Psi_2 + \epsilon \times \Psi_1 \otimes G_{0,0}\Psi_2 \ , \quad \epsilon = \pm 1 \ .$$

$L_{0,0}\Psi_i = 0$ and $\Delta(L_{0,0})\Psi = 0$ holds true. $\Delta(G_{0,0})$ and $\Delta(G_{1,0})$ are given by

$$\Delta(G_{0,0}) = G_{0,0}\otimes 1 - \epsilon\times 1\otimes G_{0,0} \ ,$$
$$\Delta(G_1,0) = G_{1,0}\otimes 1 - \epsilon\times 1\otimes G_{1,0} - \frac{3K}{2}\sum_r r(L_{0,r}\otimes G_{0,-r} - (G_{0,-r}\otimes L_{0,r}) \ , \tag{18.4.7}$$

and should annihilate Ψ. This is true if $L_{1,r}$ and $L_{0,r}$ annihilate the states.

(c) Perhaps the correct approach reduces to the Yangianization of SKMAs (including the dynamically generated SKM two which SSA effectively reduces by gauge conditions) provided that it induces Yangianization of SVA. Momentum components would be associated with KM generators for M^4 excitations of strings such that only transversal excitations are dynamical.

For fermionic and bosonic generators of SKMA one would have

$$\begin{aligned}
\Delta(F_0^a) &= F_0^a\otimes 1 + 1\times F_0^a \ ,\\
(F_1^a) &= F_1^a\otimes 1 + 1\times F_1^a + Kf_a^{Ab}(T_0^A\otimes F_0^b - F_0^b\otimes T_0^A) \ ,\\
\\
\Delta(T_0^A) &= T_0^A\otimes 1 + 1\otimes T_0^A \ ,\\
\Delta(T_1^A) &= T_1^A\otimes 1 + 1\otimes T_1^A + f_{BC}^A(T_0^B\otimes T_0^C \ .
\end{aligned} \tag{18.4.8}$$

Yangianization of SKMA would introduce interaction terms.

18.5 Category theory as a basic tool of TGD

I have already earlier developed ideas about the role of category theory in TGD [K11, K10, K6]. The hierarchy formed by categories, categories of categories, ... could allow to keep book about the complexity due to various hierarchies. WCW geometry with its huge symmetries combined with adelic physics; quantum states identified in ZEO as WCW spinor fields having topological interpretation as braided fusion categories with reconnection; the local symmetry algebras of quantum TGD extended to Yangians realizing elegantly the construction of interacting many-particle states in terms of iterated Δ operation assigning fundamental interactions to tensor summands of SKMAs: these could be the pillars of the basic vision.

18.5.1 Fusion categories

While refreshing my rather primitive physicist's understanding of categories, I found an excellent representation of fusion categories and braided categories [B14] introduced in topological condensed matter physics. The idea about product and co-product as fundamental vertices is not new in TGD [K6, K76, L38] but the physicist's view described in the article provided new insights.

Consider first fusion categories.

(a) In TGD framework scattering diagrams generalize Feynman diagrams in the sense that in 3-vertices the 2-D ends for orbits of 3 partonic 2-surfaces are glued together like the ends of lines in 3-vertex of Feynman diagram. One can say that particles fuse or decay. 3-vertex would be fundamental vertex since higher vertices are unstable against splitting to 3-vertices. Braiding and reconnection would bring in additional topological vertices. Note that reconnection represents basic vertex in closed string theory and appears also in open string theory.

Also fusions and splittings of 3-surfaces analogous to stringy trouser vertex appear as topological vertices but they do not represent particle decays but give rise to two paths along, which particles travel simultaneously: they appear in the TGD based description of double slit experiment. This is a profound departure from string models.

The key idea is that scattering diagrams are analogous to algebraic computations: the simplest computation corresponds to tree diagram apart from possible braiding and reconnections to be discussed below giving rise to purely topological dynamics. One has a generalization of the duality of the hadronic string model: one does not sum over all diagrams but takes only one of them, most naturally the simplest one. This is highly reminiscent to what happens for twistor Grassmann amplitudes.

One can eliminate all loops by moves and modify the tree diagram by moving lines along lines [?] Scattering diagrams would reduce to tree diagrams having in given vertex either product μ or its time reversal Δ plus propagator factors connecting them. The scattering amplitudes associated with tree diagrams related by these moves were earlier assumed to be identical. With better understanding of fusion categories I realized that the amplitudes corresponding to equivalent computations need not be numerically identical but only unitarily related and in this sense physically equivalent in ZEO.

(b) Fusion categories indeed realize algebraically in very simple form the idea that all scattering diagrams reduce to tree diagrams with 3-vertices as basic vertices. Fusion categories [B14] (the illustrations `http://tinyurl.com/l2jsrzc` are very helpful) involve typically tensor product $a \otimes b$ of irreducible representations a and b of an algebraic structure decomposed to irreducible representations c. This product is counterpart for the 3-parton vertex generalizing Feynmanian 3-vertex.

The article gives a graphical representation for various notions involved and these help enormously to concretize the notions. Fusion coefficients in $a \otimes b = N^c_{ab}c$ must satisfy consistency conditions coming from commutativity and associativity forcing the matrices $(N_a)_{bc} = N^c_{ab}$ to commute. One can diagonalize N_a simultaneously and their largest eigenvalues d_a are so called quantum dimensions. Fusion category contains also identity object and its presence leads to the identification of gauge invariants defining also topological invariants.

The fusion product $a \otimes b$ has decomposition $V^{c\alpha}_{ab}|c, \alpha\rangle$ for each c. Co-product is an analog of the decay of particle to two particles and product and co-product are inverses of each other in a well-defined sense expressed as an algebraic identities. This gives rise to completeness relations from the condition stating that states associated with various c form a complete basis for states for $a \otimes b$ and orthogonality relations for the states of associated with various c coefficients. Square roots of quantum dimensions d_a appear as normalization factors in the equations.

Diagrammatically the completeness relation means that scattering $ab \to c \to cd$ is trivial. This cannot be the case and the completeness relation must be more general. One would expect unitary S-matrix instead of identity matrix. The orthogonality relation says that loop diagram for $c \to ab \to c$ gives identity so that one can eliminate loops.

Further conditions come from the fact that the decay of particle to 3 particles can occur in two manners, which must give the same outcome apart from a unitary transformation denoted by matrix F (see Eq. (106) of `http://tinyurl.com/l2jsrzc`). Similar consistency conditions for decay to 4 particles give so called pentagon equation as a consistency condition (see Eq. (107) and Fig. 9 of `http://tinyurl.com/l2jsrzc`). These equations are all that is needed to get an internally consistent category.

In TGD framework the fusion algebra would be based on Super Yangian with super Variant of Lie-algebra commutator as product and Yangian co-product of form already discussed and determining the basic interaction vertices in amplitudes. Perhaps the scattering amplitude for a given space-time surface transforming two categories at boundaries of CD to each other could be seen as a diagrammatic representation of category defined by zero energy state.

18.5.2 Braided categories

Braided categories [B14] (see `http://tinyurl.com/l2jsrzc`) are fusion categories with braiding relevant in condensed matter physics and also in TGD.

(a) Braiding operation means exchange of braid strands defining particle world-lines at 3-D light-like orbits of partonic 2-surfaces (wormhole throats) defining the boundaries between Minkowskian and Euclidian regions of space-time surface. Braid operation is naturally realized in TGD for fermion lines at orbits of partonic 2-surfaces since braiding occurs in codimension 2.

(b) For quantum algebras braiding operation is algebraically realized as R-matrix satisfying YBE (see `http://tinyurl.com/l4z6zyr`). R-matrix is a representation for permutation of two objects represented quantally. Group theoretically the braid group for n-braid system is covering group of the ordinary permutation group.

In 2-D QFTs braiding operation defines the fundamental $2 \rightarrow 2$ scattering defining R-matrix as a building brick of S-matrix. This scattering matrix is trivial in the sense that the scattering involves only a phase lag but no exchange of quantum numbers: particles just pass by each other in the 2-particle scattering. This kind of S-matrix characterizes also topological quantum field theories used to deduce knot invariants as its quantum trace [A50, A25, A57]. I have considered knots from TGD point of view in [K35] [L8].

(c) For braided fusion categories one obtains additional conditions known as hexagon conditions since there are two manners to end up from $1 \rightarrow 3$ fusion diagram involving two 3-vertices and 2 braidings to an equivalent diagram using sliding of lines along lines and braiding operation (see Fig. 10 of `http://tinyurl.com/l2jsrzc`).

18.5.3 Categories with reconnections

Fusion and braiding are not enough to satisfy the needs of TGD.

(a) In TGD one does not have just objects - point like particles, whose world lines define braid strands in time direction. One has also the morphisms represented by the strings between the particles. Partonic 2-surfaces are connected by strings and these strings have topological interaction: they can reconnect or just go through each other. Reconnection is in key role in TGD inspired theory of consciousness and quantum biology [K94].

Reconnection is an additional topological reaction besides braiding and one must assign to it a generalization of R-matrix. Reconnection and going through each other are just the basic operations used to unknot ordinary knots in the construction of knot invariants in topological quantum field theories. Now topological time evolution would be a generalization of this process connecting the knotted and linked structures at boundaries of CD and allowing both knotting and un-knotting.

(b) Although 2-knots and braids are difficult to construct and visualize, it seems rather obvious (to me at least) that the reconnections correspond in 4-D space-time surface to basic operations giving rise to 2-knots [A46] - a generalization of ordinary knot that is 1-knot. 2-knots could be seen as a cobordism between 1-knots and this suggests a construction of 2-knot invariants as generalization of that for 1-knots [K35]. 2-knot would be the process transforming 1-knot by re-connections and "going through" the second 1-knot. The trace of the topological unitary S-matrix associated with it would give a knot invariant. If this view is correct, a generalization of TQFT for ordinary braids to include reconnection could give a TQFT for 2-braids with invariants as invariants of knot-cobordism. It must be however emphasized that the identification of 2-braids as knot-cobordisms is only an intuitive guess.

(c) From the point of view of braid strands at the ends of strings, reconnection means exchange of braid strands. Composite particles consisting of strands would exchange their building bricks - the analogy with a chemical reaction is obvious and various reactions could be interpreted as knot cobordisms. Since exchange is involved also now, one expects that the generalization of R-matrix to algebraically describe this process should obey the analog of YBE stating that the two braided versions of permutation $abc \rightarrow cba$ are identical.

If the strings are oriented, one could have YBEs separately for left and right ends such that braid operation would correspond to the exchange of braid between braid pairs. The topological interaction for strings AB and CD could correspond to a) trivial operation "going through" (AB + CD → AB+CD) visible in in the topological intersection matrix characterizing the union of string world sheets, exchanges of either left (AB+CD→ CB+AD) or right ends (AB+CD→ AD+CB), or exchange or right and left ends (AB+CD→ CD+AB) representable as composition of braid operation for string ends and exchange of right or left ends and giving rise to braiding operation for pairs AB and CD.

The following braiding operations would be involved.

i. Internal braiding operation $A \otimes B \to B \otimes A$ for string like object.
ii. Braiding operation $(A \otimes B) \otimes (C \otimes D) \to (C \otimes D) \otimes (A \otimes B)$ for two string like objects.
iii. Reconnection as braiding operation: $(A \otimes B) \otimes (C \otimes D) \to (A \otimes D) \otimes (C \otimes B)$ and $(A \otimes B) \otimes (C \otimes D) \to (C \otimes B) \otimes (A \otimes D)$.

I have not found by web search whether this generalization of YBE exists in mathematics literature or whether it indeed reduces to ordinary braiding for the exchanged braids for different options emerging in reconnection. One can ask whether the fusion procedure for R-matrices as an analog for the formation of tensor products already briefly discussed could allow to construct the R-matrix for the reconnection of 2 strings with braids as boundaries.

(d) The intersections of braid strands are stable against small perturbations unless one modifies the space-time surface itself (in TGD 2-braids are 2-surfaces inside 4-surfaces). Also the intersections of world lines in M^2 integrable theories are stable. Hence it would be natural to assign analog of R-matrix also to the intersections.

(e) Light-like 3-D partonic orbits can contain several fermion lines identifiable as boundaries of string world sheets so that reconnections could induce also more complex reactions in which partonic 2-surfaces exchange fermions. Quite generally one would have braid of braids able to braid and also exchange their constituent braids. This would give rise to a hierarchy of braids within braids and presumably to a hierarchy of categories. This might provide a first principle topological description of both hadronic, nuclear, and (bio-)chemical reactions. For instance, the mysterious looking ability of bio-molecules to find each other in dense molecular soup could rely on magnetic flux tubes (and associated strings) connecting them [K94].

(f) Reconnection requires a generalization of various quantum algebras, in particular Yangian, which seems to be especially relevant to TGD since it generalizes local symmetries to multi-local symmetries with locus identifiable as partonic 2-surface in TGD. Since braid strands are replaced with pairs of them, one might expect that the generalization of R-matrix involves two parameters instead of one.

18.6 Trying to imagine the great vision about categorification of TGD

The following tries to summarize the ideas described. This is mostly free play with the ideas in order to see what objects and arrows might be relevant physically and whether category theory might be of help in understanding poorly understood issues related to various hierarchies of TGD.

18.6.1 Different kind of categories

Category theory could be much more than mere book keeping device in TGD. Morphisms and functors could allow to see deep structural similarities between different levels of TGD remaining otherwise hidden.

Geometric and number theoretic categories

There are three geometric levels involved: space-time, CDs at imbedding space level, sectors of WCW assignable with CDs their subsectors characterized by a point for moduli space of CDs with second boundary fixed.

There are also number theoretic categories.

(a) Adelic physics would define a hierarchy of categories defined by extensions of rationals and identifiable as an evolutionary hierarchy in TGD inspired theory of consciousness. Inclusion of extensions parameterized by Galois group and ramified primes defining preferred p-adic primes would define a functor. The parameters of quantum algebras should be number theoretically universal and belong to the extension of rationals defining the adele in question. Powers or roots of e, roots of unity, and algebraic numbers would appear as building bricks. The larger the p-adic prime p the higher the dimension of extension containing e and possibly also some of its roots, the better the accuracy of the cognitive representation.

(b) These inclusions should relate closely to the inclusions of hyperfinite factors of type II_1 assignable to finite measurement resolution [K87]. The measurement resolution at space-time level would characterize the cognitive representation defined in terms of points with imbedding space coordinates in the extension of rationals defining the adele. The larger the extension, the larger the cognitive representation and the higher the accuracy of the representation.

   Should the points of cognitive representation be assigned

   i. only with partonic 2-surfaces (each point of representation is accompanied by fermion)
   ii. or also with the interior of space-time surface (it is not natural to assign fermion to the point unless the point belongs to string world sheet, even in this case this is questionable)?

   Many-fermion states define naturally a tensor product of quantum Boolean algebras at the opposite boundaries of CD in ZEO and the interpretation of time evolution as morphism of quantum Boolean algebras is natural. If cognition is always Boolean then the first option is more plausible.

(c) The hierarchy of Planck constants $h_{eff}/h = n$ with $n \leq ord(G)$ naturally the number of sheets and dividing the order $ord(G)$ of the Galois group G of the extension would relate closely to the hierarchy of extensions. n would be dimension of the covering of space-time surface defined by the action of Galois group to space-time sheet. Ramified primes for extensions are in special position for given extension. The conjecture is that p-adic primes near powers of two or more generally of small primes ramified primes for extensions, which are winners in number theoretic fight for survival [L34].

(d) The hierarchy of infinite primes [K72] might characterize many-sheeted space-time and leads to a generalization of number concept with infinitely complex number theoretic anatomy provided by infinite rationals, which correspond to real and p-adic units. The inclusion of lower level primes to the higher level primes would define morphism now. One can assign hierarchy of infinite primes with primes of any extension of rationals.

Consciousness and categories

Categories are especially natural from the point of view of cognition. Classification is the basic cognitive function and category is nothing but classification by defining objects as equivalence classes. Morphisms and functors serve as correlates for analogies and would provide the tool of understanding the power of analogies in conscious intelligence. Also attention could involve morphism and its direction would correlate with the direction of attention. Perhaps isomorphism corresponds to the state of consciousness in which the distinction between observer and observed is reported by meditators to cease. Cognitive representations would be provided by adelic physics at both space-time level, imbedding space level, and

WCW level (the preferred coordinates for WCW would be in extension of rationals defining the adele).

One would have a hierarchy of increasingly complex cognitive representations with inclusions as arrows and their sub-WCWs labelled by moduli of CDs and arrow of geometric time telling which boundary is affected in the sequence of state function reductions defining self as generalized Zeno effect [L39].

18.6.2 Geometric categories

Geometric categories appear at WCW level, imbedding space level, and space-time level.

WCW level

The hierarchies formed by the categories defined by the hierarchies of adeles, space-time sheets and hierarchy of CDs would be mapped also to the level of WCW. The preferred coordinates of WCW points would be in extension of rationals defining the adele and one would form inclusion hierarchy. The extension at the level of WCW would induce that at the level of imbedding space and space-time surface. Sub-CDs would correspond to sub-WCWs and the moduli space for given CD would correspond to moduli space for corresponding sub-WCWs. The different arrows of imbedding space time would correspond to sub-WCW and its time reflection. By the breaking of CP,T, and P the space-time surfaces within time reversed sub-WCWs would not be mere CP, T and P mirror images of each other [L37, L31].

Imbedding space level

ZEO emerges naturally at imbedding space level and CDs are key notion at this level. Consider next the categories that might be natural in ZEO [K91].

(a) Hierarchy of CDs could allow interpretation as hierarchy of categories. Overlapping CDs would define an analog of covering of manifold by open sets: one might speak of atlas with CDs defining conscious maps. Chart maps would be morphisms between different CDs assignable to common pieces of space-time surfaces. These morphisms would be also realized at the level of conscious experience. The sub-CD associated with CD would correspond to mental image defined by sub-self as image of the morphism.

(b) Quantum state of single space-time sheet at boundary of CD would define a geometric and topological representation for categories. States at partonic 2-surfaces would be the objects connected by fermionic strings and the associated flux tubes would serve as space-time correlates of attention in TGD inspired theory of consciousness. The arrows represented by fermionic strings would correspond to some morphisms, at least thre Hilbert space isometries defined by entanglement with coefficients in an extension of rationals. Unitary entanglement gives rise to a density matrix proportional to unitary matrix and maximal entanglement in both real and p-adic sense. Much more general entanglement gives rise to maximal entanglement in p-adic sense for some primes.

(c) Zero energy states the states at passive boundary would be naturally identifiable as categories. At active boundary quantum superpositions of categories could be in question. Maybe one should talk about quantum categories defined by the superposition of space-time sheets with category assigned with an equivalence class of space-time sheets satisfying the conditions for preferred extremal.

(d) One can imagine a hierarchy of zero energy states corresponding to the hierarchy of space-time sheets. One can build zero energy states also by adding zero energy states associated with smaller sub-CDs near the boundaries of CD to get an infinite hierarchy of zero energy states. The interpretation as a hierarchy of reflective levels of consciousness would be natural.

(e) Zero energy states would correspond to generalized Feynman diagrams interpreted as unitary functors between initial and final state categories. Scattering diagram would be seen as algebraic computation in a fusion category defined by Yangian. All diagrams would be reducible to braided tree diagrams with braidings and reconnections. The time evolution between boundaries could be seen as a topological evolution a of tensor net [L23].

Category theory would provide cognitive representations as morphisms. Morphisms would become the key element of physics completely discarded in the existing billiard ball view about Universe: Universe would be like Universal computer mimicking itself at all hierarchy levels. This extends dramatically the standard view about cognition where brain is seen as an isolated seat of cognition.

Space-time level

Many-sheeted space-time is the most obvious application for categorification.

(a) Smaller space-time sheets condensed at large space-time surface regarded as categories become objects at the level of larger space-time sheet. Functors between the categories defined by smaller space-time sheets define morphisms between them. Also now fermion lines and flux tubes connecting the condensed space-time sheets to each other via wormhole contacts with flux going along another space-time sheet could define functors. Closed loops involving larger space-time sheets and smaller space-time sheets are needed if monopole flux in question. The loop could visitat smaller space-time sheets.

(b) Interactions would reduce to product and co-product. Interaction term in Δ for generalized Yangian would characterize fundamental interactions with dynamically generated SKMAs assignable to SSA as additional interactions. The coupling parameters with Δ assigned to a direct sum of SKMAs would define coupling constants of fundamental interactions. Iteration of the co-product Δ would give rise to a hierarchy of many-particle states. The fact that morphism is in question would map the structure of single particle states to that of many-particle states.

SH would involve a functor mapping the category of string world sheets (and partonic 2-surfaces) to that of space-time surfaces having same points with coordinates in extension of rationals. In p-adic sectors this morphism presumably exists for all p-adic primes thanks to p-adic pseudo-constants. In real sector this need not be the case: all imaginations are not realizable.

The morphisms would be mediated by either continuation of strings world sheets (and partonic 2-surfaces) to space-time interiors (morphism would be analogous to a continuation of holomorphic functions of two complex coordinates from 2-D data at surfaces, where the functions are real). Possible quaternion analyticity [K76] encourages to consider even continuation of 1-D data to 4-D surfaces and twistor lift gives some support for this idea.

In the fermionic sector one must continue induced spinor fields at string world sheets to those at space-time surfaces. The 2-D induced spinor fields could also serve as sources for 4-D spinor fields.

Chapter 19

Are higher structures needed in the categorification of TGD?

19.1 Introduction

I encountered a very interesting work by Urs Schreiber related to so called higher structures and realized that these structures are part of the mathematical language for formulating quantum TGD in terms of Yangians and quantum algebras in a more general manner.

19.1.1 Higher structures and categorification of physics

What theoretical physicist Urs Screiber calls "higher structures" are closely related to the categorification program of physics. Baez, David Corfield and Urs Schreiber founded a group blog n-Category Cafe about higher category theory and its applications. John Baez is a mathematical physicists well-known from is pre-blog "This Week's Finds" (see `http://tinyurl.com/yddcabfl`) explaining notions of mathematical physics.

Higher structures or n-structures involve "higher" variants of various mathematical structures such as groups, algebras, homotopy theory, and also category theory (see `http://tinyurl.com/ydz9mbtp`. One can assign a higher structure to practically anything. Typically one loosens some conditions on the structure such as commutativity or associativity: a good example is the product for octonionic units which is associative only apart from sign factors [K74]. Braid groups and fusion algebras [L32], which seem to play crucial role in TGD can be seen as higher structures.

The key idea is simple: replace "=" with homotopy understood in much more general sense than in topology and identified as the procedure proving $A = B$! Physicist would call this operationalism. I would like a more concrete interpretation: "=" is replaced with "=" in a given measurement resolution. Even homotopies can be defined only modulo homotopies of homotopies - that is within measurement resolution - and one obtains a hierarchy of homotopies and at the highest level coherence conditions state that one has "=" almost in the good old sense. This kind of hierarchical structures are characteristic for TGD: hierarchy of space-time sheet, hierarchy of p-adic length scales, hierarchy of Planck constants and dark matters, hierarchy of inclusions of hyperfinite factors, hierarchy of extensions of rationals defining adels in adelic TGD, hierarchy of infinite primes, self hierarchy, etc...

19.1.2 Evolution of Schreiber's ideas

One of Schreiber's articles in Physics Forum articles has title "*Why higher category theory in physics?*" (see `http://tinyurl.com/ydcylrun`) telling his personal history concerning the notion of higher category theory. Supersymmetric quantum mechanics and string theory/M-theory are strongly involved with his story.

Wheeler's superspace and its deformations as starting point

Schreiber started with super variant of Wheeler's super-space. Intriguingly, also the "world of classical worlds" (WCW) of TGD [K34, K15, K110] emerged as a counterpart of superspace of Wheeler in which the generalization of super-symmetries is geometrized in terms of spinor structure of WCW expressible in terms of fermionic oscillator operators so that there is something common at least.

Screiber consider deformation theory of this structure. Deformations appear also in the construction of various quantum structures such as quantum groups and Yangians. Both quantum groups characterized by quantum phase, which is root of unity, and Yangians ideal for reduction of many-particle states and their interactions to kinematics seem to be the most important from the TGD point of view [L32].

These deformations are often called "quantizations" but this nomenclature is to my opinion misleading. In TGD framework the basic starting point is "*Do not quantize*" meaning the reduction of the entire quantum theory to classical physics at the level of WCW: modes of a formally classical WCW spinor fields correspond to the states of the Universe.

This does not however prevent the appearance of the deformations of basic structures also in TGD framework and they might be the needed mathematical tool to describe the notions of finite measurement resolution and cognitive resolution appearing in the adelic version of TGD. I proposed more than decade ago that inclusions of hyperfinite factors of II_1 (HFFs) [K87, K26] might provide a natural description of finite measurement resolution: the action of included factor would generate states equivalent under the measurement resolution used.

The description of non-point-like objects in terms of higher structures

Schreiber ends up with the notion of higher gauge field by considering the space of closed loops in 4-D target space [B73]. At the level of target space the loop space connection (1-form in loop space) corresponds to 2-form at the level of target space. At space-time level 1- form A defines gauge potentials in ordinary gauge theory and non-abelian 2-form B as its generalization with corresponding higher gauge field identified as 3-form $F = dB$.

The idea is that the values of 2-form B are defined for a string world sheet connecting two string configuration just like the values of 1-form are defined for a world-line connecting two positions of a point-like particle. The new element is that the ordinary curvature form does not anymore satisfy the usual Bianchi identities stating that magnetic monopole currents are vanishing (see `http://tinyurl.com/ya3ur2ad`).

It however turns out that one has $B = DA = F$ (D denotes covariant derivative) so that B is flat by the usual Bianchi-identities implying $dB = 0$ so that higher gauge field vanishes. B also turns out to be Abelian. In the Abelian case the value of 2-form would be magnetic flux depending only on the boundary of string world sheet. By $dB = 0$ gauge fields in loop space would vanish and only topology of field configurations would make itself manifest as for locally trivial gauge potentials in topological quantum field theories (TQFT): a generalization of Aharonov-Bohm effect would be in question. Schreiber calls this "*fake flatness condition*". This could be seen as an unsatisfactory outcome since dynamics would reduce to topological dynamics.

The assumption that loop space gauge fields reduce to those in target space could be argued to be non-realistic in TGD framework . For instance, high mass excitations of theories of extended structures like strings would be lost. In the case of loop spaces there is also problem with general coordinate invariance (GCI): one would like to have 2-D GCI assignable to string world sheets. In TGD the realization that one must have 4-D GCI for 3-D fundamental objects was a breakthrough, which occurred around 1990 about 12 years after the discovery of the basic idea of TGD and led to the discovery of WCW Kähler geometry and to "Do not quantize".

Understanding "fake flatness" condition

Schreiber tells how he encountered the article of John Baez titled "*Higher Yang-Mills Theory*" [B55] (see `http://tinyurl.com/yagkqsut`) based on the notion of 2-category and was surprised to find that also now the "*fake flatness condition*" emerged.

Schreiber concludes that the "*fake flatness condition*" results from "a kind of choice of coordinate composition": non-Abelian higher gauge field would reduce to Abelian gauge field over a background of ordinary non-Abelian gauge fields. Schreiber describes several string theory related examples involving branes and introduces connection with modern mathematics. Since branes in the stringy sense are not relevant to TGD and I do not know much about them, I will not discuss these here.

However, dimensional hierarchies formed by fermions located to points at partonic 2-surfaces, their world lines at 3-D light-like orbits of partons, strings and string world sheets as their orbits, and space-time surfaces as 4-D orbits of 3-surfaces definitely define a TGD analog for the brane hierarchy of string models. It is not yet completely clear whether strong form of holography (SH) implies that string world sheets and strings provide dual descriptions of 4-D physics or whether one could regard all levels of this hierarchy independent to some degree at least [L29].

Since the motion of measurement resolution is fundamental in TGD [K87, K26], it is interesting to see whether n-structures could emerge naturally also in TGD framework. There is also second aspect involved: various hierarchies appearing in TGD have basically the structure of abstraction hierarchy of statements about statements and higher structures seem to define just this kind of hierarchies. Of course, human mind - at least my mind - is in grave difficulties already with few lowest levels but here category theory and its computerization might come into a rescue.

19.1.3 What higher structures are?

Schreiber describes in very elegant and comprehensible manner the notion of higher structures (see `http://tinyurl.com/ydfspcld`). This description is a real gem for a physicists frustrated to the impenetrable formula jungle of the usual mathematical prose. Just the basic ideas and the reader can start to think using his/her own brains. The basic ideas ideas are very simple and general. Even if one were not enthusiastic about the notion of higher gauge field, the notion of higher structure is extremely attractive concerning the mathematical realization of the notion of finite measurement resolution.

(a) The idea is to reconsider the meaning of "=". Usually it is understood as equivalence: $A = B$ if A and B belong to same equivalence class defined by equivalence relation. The idea is to replace "=" with its operational definition, with the proof of equivalence. This could be seen as operationalism of physics applied to mathematics. Schreiber calls this proof homotopy identified as a generalization of a map f_t: $S \to X$ depending on parameter $t \in [0, 1]$ transforming two objects of a topological space X to each other in continuous manner: $f_0(S)$ is the initial object and $f_1(S)$ is the final object. Now homotopy would be much more general.

(b) One can also improve the precision of "=" meaning that equivalence classes decompose to smaller ones and equivalent homotopies decompose to subclasses of equivalent homotopies related by homotopies. One might say that "=" is deconstructed to more precise "=". Physicist would see this as a partial opening of a black box by improving the measurement resolution. This gives rise to n-variants of various algebraic structures.

(c) This hierarchy would have a finite number of levels. At highest level the accuracy would be maximal and "=" would have almost its usual meaning. This idea is formulated in terms of coherence conditions. Braiding involving R-matrix represents one example: permutations are replaced by braidings and permutation group is lifted to braid group but associativity still holds true for Yang-Baxter equation (YBE). Second example is 2-group for which associativity holds true only modulo homotopy so that $(x \circ y) \circ z$ is related to $x \circ (y \circ z)$ by homotopy $a_{x,y,z}$ depending on x, y, z and called an associator. For 2-group the composite homotopy $((w \circ x) \circ y) \circ z \to w \circ (x \circ (y \circ z))$ is however unique albeit non-trivial.

This gives rise to the so called pentagon identity encountered also in the theory of quantum groups and Yangians. The outcome is that all homotopies associated with rebracketings of an algebraic expression are identical. One can define in similar manner n-group and formally even infinity-group.

19.1.4 Possible applications of higher structures to TGD

Before listing some of the applications of higher structures imaginable in TGD framework, let us summarize the basic principles.

(a) Physics as WCW geometry [K62, K34, K15, K110] having super-symplectic algebra (SSA) and partonic super-conformal algebra (PSCA) as fundamental symmetries involving a generalization of ordinary conformal invariance to that for light-like 3-surfaces defined by the boundary of CD and by the light-like orbits of partonic 2-surfaces at which the signature of the induced metric changes from Minkowskian to Euclidian.

(b) Physics as generalized number theory [K71] [L34] leading to the notion of adelic physics with a hierarchy of adeles defined by the extensions of rationals.

(c) In adelic physics finite resolutions for sensory and cognitive representations (see the glossary of Appendix) could would characterize "=". Hierarchies of resolutions meaning hierarchies of n-structures rather than single n-structure would give inclusion hierarchies for HFFs, SSA, and PSCA, and extensions of rationals characterized by Galois groups with order identifiable as $h_{eff}/h = n$ and ramified primes of extension defining candidates for preferred p-adic primes.

   Finite measurement resolution defined by SSA and its isomorphic sub-algebra acting as pure gauge algebra would reduce SSA to finite-dimensional SKMA. WCW could become effectively a coset space of Kac-Moody group or of even Lie group associated with it. Same would take place for PSCA. This would give rise to n-structures. Quantum groups and Yangians would indeed represent examples of n-structures.

   In TGD the "conformal weight" of Yangian however corresponds to the number of partonic surfaces - parton number - whereas for quantum groups and Kac-Moody algebras it is analogous to harmonic oscillator quantum number n, which however has also interpretation as boson number. Maybe this co-incidence involves something much deeper and relates to quantum classical correspondence (QCC) remaining rather mysterious in quantum field theories (QFTs).

(d) An even more radical reduction of degrees of freedom can be imagined. Cognitive representations could replace space-time surfaces with discrete structures and points of WCW could have cognitive representations as disretized WCW coordinates.

(e) Categorification requires morphisms and homomorphisms mapping group to sub-group having normal sub-group defining the resolution as kernel would define "resolution morphisms". This normal sub-group principle would apply quite generally. One expects that the representations of the groups involved are those for quantum groups with quantum phase q equal to a root of unity.

Some examples helps to make this more concrete.

Scattering amplitudes as computations

The deterministic time devolution connecting two field patterns could define analog of homotopy in generalized sense. In TGD framework space-time surface (preferred extremals) having 3-D space-like surfaces at the opposite boundaries of causal diamond (CD) could therefore define analog of homotopy.

(a) Preferred extremal defines a topological scattering diagram in which 3-vertices of Feynman diagram are replaced with partonic 2-surfaces at which the ends of light-like orbits of partonic 2-surfaces meet and fermions moving along lines defined by string world sheets scatter classically, and are redistributed between partonic orbits [K76, L24, L38]. Also braidings and reconnections of strings are possible. It is important to notice that one does not sum over these topological diagrams. They are more like possible classical backgrounds.

 The conjecture is that scattering diagrams are analogous to algebraic computations so that one can find the shortest computation represented by a tree diagram. Homotopy in the roughest sense could mean identification of topological scattering diagrams connecting two states at boundaries of CD and differing by addition of topological loops. The functional integral in WCW is proposed to trivialize in the sense that loop corrections vanish as a manifestation of quantum criticality of Kähler coupling strength and one obtains an exponent of Kähler function which however cancels in scattering amplitudes if only single maximum of Kähler function contributes.

(b) In the optimal situation one could eliminate all loops of these diagrams and also move line ends along the lines of diagrams to get tree diagrams as representations of scattering diagrams. Similar conditions hold for fusion algebras. This might however hold true only in the minimal resolution. In an improved measurement resolution the diagrams could become more complex. For instance, one might obtain genuine topological loops.

(c) The diagrams and state spaces with different measurement resolutions could be related by Hilbert space *isometries* but would not be unitarily equivalent: Hilbert space isometries are also defined by entanglement in tensor nets [L23]. This would give an n-levelled hierarchy of higher structures (rather than single n-structure!) and at the highest level with best resolution one would have coherence rules. Generalized fusion algebras would partially realize this vision. In improved measurement resolution the diagrams would not be identical anymore and equivalence class would decompose to smaller equivalence classes. This brings in mind renormalization group equations with cutoff.

(d) Intuitively the improvement of the accuracy corresponds to addition of sub-CDs of CDs and smaller space-time sheets glued to the existing space-time sheets.

Zero energy ontology (ZEO)

In ZEO [K91] "=" could mean the equivalence of two zero energy states indistinguishable in given measurement resolution. Could one say that the 3-surfaces at the ends of space-time surface are equivalent in the sense that they are connected by preferred extremal and have thus same total Noether charges, or that entangled many-fermion states at the boundaries of CD correspond to quantal logical equivalences (fermionic oscillator algebra defines a quantum Boolean algebra)?

In the case of zero energy states "=" could tolerate a modification of zero energy state by zero energy state in smaller scale analogous to a quantum fluctuation in quantum field theories (QFTs). One could add to a zero energy state for given CD zero energy states associated with smaller CDs within it.

In TGD inspired theory of consciousness [L39] sub-CDs are correlates for the perceptive fields of conscious entities and the states associated with sub-CDs would correspond to sub-selves of self defining its mental images. Also this could give rise to hierarchies of n-structures with n characterizing the number of CDs with varying sizes. An interesting proposal is the distance between the tips of CD is integer multiple of CP_2 for number theoretic reasons. Primes and primes near powers of 2 are suggested by p-adic length scale hypothesis [K39, K42, K43] [L34].

"World of classical worlds" (WCW)

At the level of "world of classical worlds" (WCW) "=" could have both classical meaning and meaning in terms of quantum state defining the measurement resolution. At the level of WCW geometry n-levelled hierarchies formed by the isomorphic sub-algebras of SSA and PSCA are excellent candidates for n-structures. The sub-SCA or sub-PSCA would define the measurement resolution. The smaller the sub-SSA or sub-PSCA, the better the resolution.

This could correspond to a hierarchy of inclusions of HFFs [K87, K26] to which one can assign ADE SKMA by McKay correspondence or its generalization allowing also other Lie groups suggested by the hierarchy of extensions of rationals with Galois groups that are groups of Lie type. The conjecture generalizing McKay correspondence is that the Galois group Gal is representable as a subgroup of G in the case that it is of Lie type.

An attractive idea is that WCW is effectively reduced to a finite-dimensional coset space of the Kac-Moody group defined by the gauge conditions. Number theoretic universality requires that these parameters belong to the extension of rationals considered so that the Kac-Moody group G is discretized and also homotopies are discretized. SH raises the hope that it is enough to consider string world sheets with parameters (WCW coordinates) in the extension of rationals.

One can define quite concretely the action of elements of homotopy groups of Kac-Moody Lie groups G on space-time surfaces as induced action changing the parameters characterizing the space-time surface. $n+1$-dimensional homotopy would be 1-dimensional homotopy of n-dimensional homotopy. Also the spheres defining homotopies could be discretized so that the coordinates of its points would belong to the extension of rationals.

These kind of homotopy sequences could define analogs of Berry phases (see `http://tinyurl.com/yd4agwnt`) in Kac-Moody group. Could gauge theory for Kac-Moody group give an approximate description of the dynamical degrees of freedom besides the standard model degrees of freedom? This need not be a good idea. It is better to base the considerations of the physical picture provided by TGD. I have however discussed the TGD analog of the *fake flatness condition* in the Appendix.

Adelic physics

Also number theoretical meaning is possible for "=". It is good to start with an objection against adelic physics. The original belief was that adelic physics forces preferred coordinates. Indeed, the property of belonging to an extension of rationals does not conform with general coordinate invariance (GCI). Coordinate choice however matters cognitively as any mathematical physicist knows! One can therefore introduce preferred coordinates at the imbedding space level as cognitively optimal coordinates: they are dictated to a high degree by the isometries of H. One can use a sub-set of these coordinates also for space-time surfaces, string world sheets, and partonic 2-surfaces.

(a) Space-time surfaces can be regarded as multi-sheeted Galois coverings of a representative sheet [L34]. Minimal resolution means that quantum state is Galois singlet. Improving resolution means requiring that singlet property holds true only for normal sub-group H of Galois group Gal and states belong to the representations of Gal/H. Maximal resolution would mean that states are representations of the entire Gal. The hierarchy of normal sub-groups of Gal would define a resolution hierarchy and perhaps an analog of n-structure. $h_{eff}/h = n$ hypothesis suggests hierarchies of Galois groups with dimensions n_i dividing n_{i+1}. The number of extensions in the hierarchy would characterize n-structure.

(b) The increase of the complexity for the extension of rationals would bring new points in the *cognitive representations* defined by the points of the space-time surface with imbedding space coordinates in the extension of rationals used (see the glossary in Appendix). Also the size of the Gal would increase and higher-D representations would become possible. The value of $h_{eff}/h = n$ identifiable as dimension of Gal would increase. The cognitive representation would become more precise and the topology of the space-time surface would become more complex.

(c) In adelic TGD "=" could have meaning at the level of cognitive representations. One could go really radical and ask whether discrete cognitive representations replacing space-time surfaces with the set of points with H-coordinates in an extension of rationals (see the glossary in Appendix) defining the adele should provide the fundamental data and that all group representations involved should be realized as representations of Gal. This might apply in cognitive sector.

This would also replace space-time surfaces as points of WCW with their cognitive representations defining their WCW coordinates! All finite groups can appear as Galois groups for some number field. Whether this is case when one restricts the consideration to the extensions of rationals, is not known. Most finite groups are groups of Lie type and thus representable as rational points of some Lie group. Note that rational point can also mean rational point in extension of rationals as ratio of corresponding algebraic integers identifiable as roots of monic polynomials $P_n(x) = x^n + ...$ having rational coefficients.

(d) By SH space-time surface would in information theoretic sense effectively reduce to string world sheets and even discrete set of points with H-coordinates in extension of rationals. These points could even belong to the partonic 2-surface at the ends of strings at ends of CD carrying fermions and the partonic 2-surfaces defining topological vertices. If only this data is available, the WCW coordinates of space-time surface would reduce to these points of $H = M^4 \times CP_2$ and to the direction angles of strings emerging from these points and connecting them to the corresponding points at other partonic 2-surfaces besides Gal identifiable as sub-group of Lie group G of some Kac-Moody group! Not all pairs $Gal - G$ are possible.

(e) Could these data be enough to describe mathematically what one knows about space-time surface as point of WCW and the physics? One could indeed deduce $h_{eff}/h = n$ as the order of Gal and preferred p-adic primes as ramified primes of extension. The Galois representations acting on the covering defining space-time surface or string world sheets should be identifiable as representations of physical states. There is even number theoretical vision about coupling constant evolution relying on zeros of Riemann zeta [L16],

(f) This sounds fine but one must notice that there is also the global information about the conformal moduli of partonic 2-surfaces and the elementary particle vacuum functionals defined in this moduli space [K12] explain family replication phenomenon. There is also information about moduli of CDs. Also the excitations of SKMA representations with higher conformal weights are present and play a crucial role in p-adic thermodynamics predicting particle masses [K39]. It is far from clear whether the approach involving only cognitive representation is able to describe them.

To help the reader I have included a vocabulary at the end of the article and include here a list of the abbreviations used in the text.

General abbreviations: Quantum field theory (QFT); Topological quantum field theory (TQFT); Hyper-finite factor of type II_1 (HFF); General coordinate invariance (GCI); Equivalence Principle (EP).

TGD related abbreviations: Topological Geometrodynamics (TGD); General Relativity Theory (GRT); Zero energy ontology (ZEO); Strong form of holography (SH); Strong form of general coordinate invariance (SGCI); Quantum classical correspondence (QCC); Negentropy Maximization Principle (NMP); Negentropic entanglement (NE); Causal diamond (CD); Super-symplectic algebra (SSA); Partonic superconformal algebra (PSCA); Super Virasoro algebra (SVA); Kac-Moody algebra (KMA); Super-Kac-Moody algebra (SKMA);

19.2 TGD very briefly

TGD is a fusion of two approaches to physics. Physics as infinite-dimensional geometry based on the notion of "(" []WCW) [K62] and physics as generalized number theory [K71]. Here some aspects of the vision about physics as WCW geometry are discussed very briefly.

19.2.1 World of classical worlds (WCW)

TGD is a fusion of two approaches to physics. Physics as infinite-dimensional geometry based on the notion of "(" []WCW) [K62] and physics as generalized number theory [K71]. Here some aspects of the vision about physics as WCW geometry are discussed very briefly.

Construction of WCW geometry briefly

In the following the vision about physics in terms of classical physics of spinor fields of WCW is briefly summarized.

(a) The idea is to geometrize not only the classical physics in terms of geometry of space-time surfaces but also quantum physics in terms of WCW [K110]. Quantum states of the Universe would be modes of classical spinor fields in WCW and there would be no quantization. One must construct Kähler metric and Kähler form of WCW: in complex coordinates they differ by a multiplicative imaginary unit. Kähler geometry makes possible to geometrize hermitian conjugation fundamental for quantum theory.

(b) One manner to build WCW metric this is via the construction of gamma matrices of WCW in terms of second quantized oscillator operators for fermions described by induced spinor fields at space-time surfaces. By strong form of holography this would reduce to the construction of second quantized induced spinor fields at string world sheets. The anti-commutators of of WCW gamma matrices expressible in terms of oscillator operators would define WCW metric with maximal isometry group (SCA) [K88, K110].

(c) Second manner to achieve the geometrization is to construct Kähler metric and Kähler form directly [K34, K15, K110]. The idea is to induce WCW geometry from the Kähler form J of the imbedding space $H = M^4 \times CP_2$. The mere existence of the Riemann connection forces a maximal group of isometries. In fact, already in the case of loop space the Kähler geometry is essentially unique.

    The original construction used only the Kähler form of CP_2. The twistor lift of TGD [L38] forces to endow also M^4 with the Minkowskian analog of Kähler form involving complex and hypercomplex part and the sum of the two Kähler forms can be used to define what might be called flux Hamiltonians. They would define the isometries of WCW as symplectic transformations. What was surprising and also somewhat frustrating was that what I called almost 2-dimensionality of 3-surfaces emerges from the condition of general coordinate invariance and absence of dimensional parameters apart from the size scale of CP_2.

    In the recent formulation this corresponds to SH: 2-D string world sheets and 2-D partonic 2-surfaces would contain data allowing to construct space-time surfaces as preferred extremals. In adelic physics also the specification of points of space-time surface belonging to extension of rationals defining the adele would be needed. There are several options to consider but the general idea is clear.

    SH is analogous to a construction of analytic function of 2-complex from its real values at 2-D surface and the analogy at the level of twistor lift is holomorphy as generalization of holomorphy of solutions gauge fields in the twistor approach of Penrose. Also quaternionic analyticity [K76] is suggestive and might mean even stronger form of holography in which 1-D data allow to construct space-time surfaces as preferred extremals and quantum states.

    I have proposed formulas for the Kähler form of WCW in terms of flux Hamiltonians but the construction as anti-commutators of gamma matrices is the more convincing definition. Fermions and second quantize induced spinor fields could be an absolutely essential part of WCW geometry.

(d) WCW allows as infinitesimal isometries huge super-symplectic algebra (SSA) [K34, K15] acting on space-like 3-surfaces at the ends of space-time surfaces inside causal diamond (CD) and also generalization of Kac-Moody and conformal symmetries acting on the

3-D light-like orbits of partonic 2-surfaces (partonic super-conformal algebra (PSCA)). These symmetry algebras have a fractal structure containing a hierarchy of sub-algebras isomorphic to the full algebra. Even ordinary conformal algebra with non-negative conformal weights has similar fractal structure as also Yangian. In fact, quantum algebras are formulated in terms of these half algebras.

The proposal is that sub-algebra of SSA (with non-negative conformal weights) and isomorphic to entire SSA and its commutator with the full algebra annihilate the physical states. What remains seems to be finite-D Kac-Moody algebra as an effective "coset" algebra obtained. Note that the resulting normal sub-group is actually quantum group.

There is direct analogy with the decomposition of a group Gal to a product of sub-group and normal sub-group H. If the normal sub-group H acts trivially on the representation the representation of Gal reduces to that of the group Gal/H. Now one works at Lie algebra level: Gal is replaced with SSA and H with its sub-algebra with conformal weights multiples of those for SSA.

Super-symplectic conformal weights, zeros of Riemann zeta, and quantum phases?

In [L16] I have considered the possibility that the generators of super-symplectic algebra could correspond to zeros $h = 1/2 + iy$ of zeta. The hypothesis has several variants.

(a) The simplest variant is that the non-trivial zeros of zeta are labelling the generators of SSA associated with Hamiltonians proportional to the functions $f(r_M)$ of the light-like radial coordinate of light-cone boundary as $f(r_M) = (r_M/_0)^h \equiv exp(hu)$, $u = log(r_M/r_0)$, $h = -1/2+iy$. For infinitely large size of CD the plane waves are orthogonal but for finite-sized CD orthogonality is lost. Orthogonality requires periodic boundary conditions and these are simultanwously possible only for a finite number of zeros of zeta.

(b) One could modify the hypothesis by allowing superpositions of zeros of zeta but with a subtraction of half integer to make the real part of ih equal to $1/2$ so that one obtains an analog of plane-wave when using $u = log(r_M/r_0)$ as a radial coordinate. Equivalently, one can take dr_M/r_M out as integration measure and assume $h = iy$ plus the condition that the Riemannian plane waves are orthogonal and satisfy periodic boundary conditions for the allowed zeros $z = 1/2 + iy$.

(c) Periodic boundary conditions can be satisfied for given zero of zeta if the condition $r_{max}/r_{min} = p^n$ holds true and the additional conjecture that given non-trivial zeros of zeta correspond to prime $p(y)$ and p^{iy} is a root of unity. Given basis of $f(r_M)$ would correspond to n-ary p-adic length scales and also the size scales of CDs would correspond to powers of p-adic primes. This conjecture is rather attractive physically and I have not been able to prove it wrong.

One can associate to given zero $z = 1/2 + iy$ single and only single prime $p(y)$ by demanding that $p^{iy} = exp(i2\pi q)$, $q = m/n$ rational, implying $log(p)y = 2\pi q$. If there were two primes p_1 and p_2 of this kind, one one ends up with contradiction $p_1^m = p_2^n$ for some integers m and n.

One could however associate several zeros $y_i(p)$ to the same prime p as discussed in [L16]. If $N = \prod_i n_i$ is the smallest common denominator of q_i allowed conformal weights would be superpositions $ih = iN \sum n_i y_i(p)$ and conformal weights would form higher dimensional lattice rather than 1-D lattice as usually. If only single prime $p(y)$ can be associated to given y, then the original hypothesis identifying $h = 1/2+iy$ as conformal weight would be natural.

(d) The understanding of the p-adic length scale hypothesis is far from complete and one can ask whether preferred p-adic primes near powers of 2 and possibly also other small primes could be primes for which there are several roots $y_i(p)$.

19.2.2 Strong form of holography (SH)

There are several reasons why string world sheets and partonic 2-surfaces should code for physics. One reason for SH comes from $M^8 - H$ correspondence [K111]. Second motivation comes from the condition that spinor modes at string world sheets are eigenstates of em charge [K88]. The third reason could come the requirement that the notion of commutative quantum sub-manifold [A29] is equivalent with its number theoretic variant.

SH and $M^8 - H$ correspondence

The strongest form of $M^8 - H$ correspondence [K74, K111, L38] assumes that the 4-surfaces $X^4 \subset M^8$ have fixed $M^2 \subset M^4 \subset M^8$ as part of tangent space. A weaker form states that these 2-D subspaces M^2 define an integrable distribution and therefore 2-D surface in M^4. This condition guarantees that the quaternionic (associative) tangent space of X^4 is parameterized by a point of CP_2 so that the map of X^4 to a 4-surface in $M^4 \times CP_2$ is possible. One can consider also co-associative space-time surfaces having associative normal spaces. m Note that $M^8 - H$ [K74, K111] correspondence respects commutativity and quaternionic property by definition since it maps space-time surfaces having quaternionic tangent space having fixed M^2 as sub-set of tangent space.

What could be the relationship between SH and $M^8 - H$ correspondence? Number theoretic vision suggests rather obvious conjectures.

(a) Could the tangent spaces of string world sheets in H be commutative in the sense of complexified octonions and therefore be hyper-complex in Minkowskian regions. By $M^8 - H$ duality the commutative sub-manifolds would correspond to those of octonionic M^8 and finding of these could be the first challenge. The co-commutative manifolds in quaternionic X^4 would have commutative normal spaces. Could they correspond to partonic 2-surfaces?

(b) There is however a delicacy involved. Could world sheets and partonic 2-surfaces correspond to hyper-complex and co-hyper-complex sub-manifolds of space-time surface X^4 identifiable as quaternionic surface in octonionic M^8 mappable to similar surfaces in H. Or could their M^4 (CP_2) projections define hypercomplex (co-hypercomplex) 2-manifolds?

(c) Could co-commutativity condition for a foliation by partonic 2-surfaces select preferred string world sheets as normal spaces integrable to 2-surfaces identifiable as string world sheets? Note that induced gauge field on 2-surface is always Abelian so that QFT and number theory based views about commutativity co-incide.

    Preferred choices for these 2-surfaces would serve as natural representatives for the equivalence classes of string world sheets and partonic 2-surfaces with fermions at the boundaries of string world sheets serving as markers for the representatives? The end points of the string orbits would belong to extension of rationals or even correspond to singular points at which the different sheets co-incide and have rational coordinates: this possibility was considered in [L41].

Real curves correspond to the lowest level of the dimensional hierarchy of continuous surfaces. Could string world lines along light-like partonic orbits correspond to real sub-manifolds of octonionic M^8 mapped to $M^4 \times CP_2$ by $M^8 - H$ correspondence and carrying fermion number?

What about the set of points with coordinates in the extension of rationals? Do all these points carry fermion number? If so they must correspond to the edges of the boundaries of string world sheets at partonic 2-surfaces at the boundaries of CD or edges at the partonic 2-surfaces defining generalized vertices to which sub-CDs could be assigned.

Well-definedness of em charge forces 2-D fundamental objects

The proposal has been that the representative string world sheets should have vanishing induced W fields so that induced spinors could have well-defined em and Z^0 charges and partonic 2-surfaces would correspond to the ends of 3-D boundaries between Euclidian and Minkowskian space-time regions [K88, K110].

As a matter of fact, the projections of electroweak gauge fields to 2-D surfaces are always Abelian and by using a suitable $SU(2)_L \times U(1)$ rotation one can always find a gauge in which the induced W fields and even Z^0 field vanish. The highly non-trivial conclusion is that string world sheets as fundamental dynamical objects coding 4-D physics by SH would guarantee well-definedness of em charge as fermionic quantum number. Also the projections of all classical color gauge fields, whose components are proportional to $H^A J$, where H^A is color Hamiltonian and J is Kähler form of CP_2, are Abelian and in suitable gauge correspond to hypercharge and isospin.

One can imagine a foliation of space-time surfaces by string world sheets and partonic 2-surfaces. Could there be a $U(1)$ gauge invariance allowing to chose partonic 2-surfaces and string world sheets arbitrarily? If so, the assignment of the partonic 2-surfaces to the light-like boundaries between Minkowskian and Euclidian space-time regions would be only one - albeit very convenient - choice. I have proposed that this choice is equivalent with the choice of complex coordinates of WCW. The change of complex coordinates would introduce a $U(1)$ transformation of Kähler function of WCW adding to it a real part of holomorphic function and of Kähler gauge potential leaving the Kähler form and Kähler metric of WCW invariant.

String world sheets as sub-manifolds of quantum spaces for which commuting sub-set of coordinates are diagonalized?

The third notion of commutativity relates to the notion of non-commutative geometry. Unfortunately, I do not know much about non-commutative geometry.

(a) Should one follow Connes [A29] and replace string world sheets with non-commutative geometries with quantum dimension identifiable as fractal dimension. I must admit that I have felt aversion towards non-commutative geometries. For linear structures such as spinors the quantum Clifford algebra looks natural as a "coset space" obtained by taking the orbits of included factor as elements of quantum Clifford algebra. The application of this idea to string world sheets does not look attractive to me.

(b) The basic reason for my aversion is that non-commutative quantum coordinates lead to problems with general coordinate invariance (GCI). There is however a possible loophole here. One can approach the situation from two angles: number theoretically and from the point view of non-commutative space. Commutativity could mean two things: number theoretic commutativity and commutativity of quantum coordinates for H seen as observables. Could these two meanings be equivalent as quantum classical correspondence (QCC) encourages to think?

    Could the discreteness for cognitive representations correspond to a discretization of the eigenvalue spectrum of the coordinates as quantum operators? The choice of the coefficient number field for Hilbert space as extension of rationals would automatically imply this and resolve the problems related to continuous spectra.

    Quantum variant of string world sheet could correspond to a quantization using a sub-set of imbedding space coordinates as quantum commutative coordinates as coordinates for string world sheet. H-coordinates for string world sheet would correspond to eigenvalues of commuting quantum coordinates.

The above three views about SH suggests that Abelianity at the fundamental level is unavoidable because basic observable objects are 2-dimensional. This would correspond $A = J = -B = 0$ for non-Abelian gauge fields reducing to Abelian ones in Schreiber's approach. Also Schreiber finds that with suitable choice of coordinates this holds true always. In TGD this choice would correspond to gauge choice in which all induced gauge fields are Abelian (see Appendix).

Ordinary twistorialization maps points of M^4 to bi-spinors allowing quantum variants. Could twistorialization of M^4 and CP_2 allow something analogous?

19.3 The notion of finite measurement resolution

Finite measurement resolution [K87, K26] is central in TGD. It has several interpretations and the challenge is to unify the mutually consistent views.

19.3.1 Inclusions of HFFs, finite measurement resolution and quantum dimensions

Concerning measurement resolution the first proposal was that the inclusions of HFFs chacterize it.

(a) The key idea is simple. Yangians and/or quantum algebras associated with the dynamical SKMAs defined by pairs of SSA and its isomorphic sub-algebra acting as pure gauge transformations are characterized by quantum phases [L32] characterizing also inclusions of HFFs [K87, K26]. Quantum parameter would characterize the measurement resolution.

The Lie group characterizing SKMA would be replaced by its quantum counterpart. Quantum groups involve quantum parameter $q \in C$ involved also with n-structures. This parameter - in particular its phase- should belong to the extension of rationals considered. Notions like braiding making sense for 2-D structures are crucial. Remarkably, the representation theory for quantum groups with q different from a root of unity does not differ from that for ordinary groups. For the roots of unity the situation is different.

(b) The levels in the hierarchy of inclusions for HFFs [K87] are labelled by integer $n \in [3, \infty)$ or equivalenly by quantum phases $q = exp(i\pi/n)$ and quantum dimension is given by $d_q = 4cos^2(\pi/n)$. $n = 3$ gives $d = 2$ that is ideal SH with minimal measurement resolution. For instance, in extension of rationals only phases, which are powers of $exp(i\pi/3)$ are represented p-adically so that angle measurement is very imprecise. The hierarchy would correspond to an increasing measurement resolution and at the level $n \to \infty$ one would have $d_q \to 4$. Could the interpretation be that one sees space-time as 4-dimensional? This strongly suggests that the hierarchy of Lie groups characterizing SKMAs are characterized by the same quantum phase as inclusions of HFFs.

How does quantal dimension show itself at space-time level?

(a) Could SH reduce the 4-surfaces to effectively fractal objects with quantum dimension d_q? Could one speak of quantum variant of SH perhaps describe finite measurement resolution. In adelic picture this limit could correspond to an extension of rational consists of algebraic numbers extended by all rational powers of e. How much does this limit deviate from real numbers?

(b) McKay correspondence (see `http://tinyurl.com/z48d92t`) states that the hierarchy of finite sub-groups of $SU(2)$ corresponds to the hierarchy ADE Kac-Moody algebras in the following sense. The so called McKay graph codes for the information about the multiplicities of the tensor products of given representation of finite group (spin 1/2 doublet) - obviously one can assign McKay graph to any Galois group. McKay correspondence says that the McKay graph for the so called canonical representation of finite sub-group of $SU(2)$ co-incides with the Dynkin diagram for ADE type Kac-Moody algebra.

(c) A physically attractive idea is that these algebras correspond to a hierarchy of reduced SSAs and PSCAs defined by the gauge conditions of SSA and PSCA. The breaking of maximal effective gauge symmetry characterizing measurement resolution to isomorphic sub-algebra would bring in additional degrees of freedom increasing the quantum dimension of string world sheets from the minimal value $d_q = 2$.

My naive physical intuition suggests that McKay correspondence generalizes to a much wider class of Galois groups identifiable as finite groups of Lie type identifiable as subgroups of Lie groups (for the periodic table of finite groups see (see `http://tinyurl.com/y75r68hp`)). In general, the irreducible representation (irrep) of group is reducible representation of subgroup. The rule could be that the representations of the quantum Lie groups *allowed* as ground states of SKMA representations are *irreducible* also as representations of Galois group in case that it is Lie-type subgroup.

What about the concrete geometric interpretation of d_q? Two interpretations, which do not exclude each other, suggest themselves.

(a) A very naive idea is that string world sheets effectively fill the space-time surface as the measurement accuracy increases. The idea about fractal string world sheets does not however conform with the fact that preferred extremals must be rather smooth.

String world sheets could be however locally smooth if they define an analog of discretization for the space-time surface. At the limit $d_q \rightarrow 4$ string world sheets would fill space-time surface. Analogously, strings (string orbits) would fill the space-like 3-surfaces at the boundaries of CD (the light-like 3-surfaces connecting the partonic 2-surfaces at boundaries of CD). The number of fermions at partonic 2-surfaces would increase and lead to an increased measurement resolution at the level of physics. For anyonic systems [K55] one indeed would have have large number of fermions at 2-D surfaces.

(b) An alternative idea is that quantum dimension is temperature like parameter coding for the ignorance about the details of space-time surface and string world sheet due to finite cognitive resolution. Cognitive representation consists of a discrete set of points of H in an extension of rationals defining the adele and quantum dimension would represent this ignorance. A precise mathematical representation of ignorance can be extremely successful trick as ordinary thermodynamics and also p-adic thermodynamics for particle masses [K39] demonstrate!

19.3.2 Three options for the identification of quantum dimension

The quantum dimension would increase as the measurement accuracy increases but what quantum dimension of string world sheets could mean at space-time level? Identification of quantum dimension as fractal dimension could be the answer but how could one concretely define this notion? Could one find an elegant formulation for the fractality at space-time level.

Option I

One could argue that quantum dimension is temperature like parameter coding for the ignorance about the details of space-time surface and string world sheet due to finite cognitive resolution. Cognitive representation consists of a discrete set of points of H in an extension of rationals defining the adele and quantum dimension would represent this ignorance. One would give up the attempts to represent quantum superposition of space-time surfaces with single classical surface. This option would use only the discrete cognitive representations (see the glossary in Appendix).

(a) This would mean a radical simplification and could make sense for cognitive representations. String world sheet would be replaced by this discrete cognitive representation and one should be able to deduce its quantum dimension. *Gal* acts on this representation.

(b) Could one imagine q-variants of the representations of Gal defining also representations of the Lie group defining KMA? If one can imbed Gal to Lie-group as discrete sub-group then the q-representation of the Lie-group would define a q-representation of discrete group and one might be able to talk about q-Galois groups.

(c) On the other hand, the condition that these representations restricted to representations of Galois group remain irreducible poses similar condition. Are these two criteria equivalent? Could this allow to identify the value of root of unity associated with given Galois group and corresponding Lie group defining SKMA in case that it contains representations that remain irreps of Galois group? If so, the notion of quantum group would follow from adelic physics in a natural manner.

This would allow to assign quantum dimension to the discretized string world sheet without clumsy fractal constructions at space-time level involving a lot of redundant information. The really nice thing would be that one would use only the information defining the cognitive representations and the fact that one does not know about the rest. Just as in thermodynamics, things would become extremely simple!

(d) One might argue that giving just discrete points at partonic 2-surfaces gives very little information. If one however assumes that also the functions characterizing space-time surfaces as points of sub-WCW involved are constructed from rational polynomials with roots in the extension of rationals used, the situation improves dramatically.

Option II

A very naive idea is that string world sheets effectively fill the space-time surface as the measurement accuracy increases. Smooth strings would fill the space-like 3-surfaces at the boundaries of CD and light-like 3-surface connecting the partonic 2-surfaces at boundaries of CD. The number of fermions at partonic 2-surfaces would increase and lead to an increased measurement resolution. For anyonic systems one indeed would have have large number of fermions at 2-D surfaces.

This option would be based on fractal dimension of some kind. Most naturally the fractal dimension would be that of space-time surface discretized using string world sheets and possibly also partonic 2-surface instead of points. It is however difficult to imagine a practical realization for fractal dimension in this sense.

(a) Assume reference string world sheets in the minimal resolution defined by an extension of rationals with total area S_0. Study the total area S associated with string world sheets as function of the extension of rationals.

(b) As the size of the extension grows, new points of extension emerge at partonic 2-surfaces and therefore also new string world sheets and the total area of string worlds sheets increases. Twistor lift suggests that one can take the area S_1 defined by Planck length squared and the area S_2 of CP_2 geodesic sphere as units. Suppose that one has $S/S_0 = (S_1/S_2)^d$, where d depends on the extension and equals to $d = 0$ for rationals, holds true. Could $d + 2$ define the fractal dimension equal to d_q for Jones inclusions in the range $[2, 4)$? If the proposed notion of quantum Galois group makes sense this could be the case.

One must admit that the hopes of proving this picture works in practice are rather meager. Too much redundant information is involved.

Option III

One can also imagine an approach quantum dimension identifying quantum dimension as fractal dimension for space-time surface. If SH makes sense, one can consider the possibility that this dimension determined by the geometry of space-time surface as Riemann manifold has fractal dimension equal to the fractal dimension of string world sheets as sub-manifold.

(a) The spectral dimension of classical geometry is discussed in `http://tinyurl.com/yadcmjd6`). One considers heat equation describing essentially random walk in a given metric and constructs so called heat kernel as a solution of the heat equation. The Laplacian depends on metric only - now the induced metric. The trace of heat kernel characterizes the probability to return to the original position. The derivative of the logarithm of the heat trace with respect to the logarithm of fictive time coordinate gives time dependent spectral dimension, which for short times approaches to topological dimension and for flat space equals to it always. For long times the dimension is smaller than the topological dimension due to curvature effects and SH raises the hope that this dimension corresponds to the fractal dimension of string world sheets identified as quantum dimension.

(b) This approach can be criticized for the introduction of fictive time coordinate. Furthermore, Laplacian would be replaced with d'Alembertian in Minkowskian regions so that one cannot speak about diffusion anymore. Could one replace the heat equation with 4-D spinor d'Alembertian or modified Dirac operator so that also the induced gauge fields would appear in the equation? Artificial time coordinate would be replaced with some time coordinate for M^4 - light-cone proper time is the most natural choice. The probability would be defined as modulus squared for the fermionic propagator integrated over space-time surface.

The problem is that this approach is rather formal and might be of little practical value.

19.3.3 n-structures and adelic physics

TGD involves several concepts, which could relate to n-structures. The notion of finite measurement resolution realized in terms of HFFs is the oldest notion [K87, K26]. Adelic physics suggests that the measurement resolution could be realized in terms of a hierarchy of extensions of rationals [L34]. The parameters characterizing space-time surfaces and by SH the string world sheets would belong to the extension. Also the points of space-time surface in the extension would be data coding for the preferred extremals. The reconnection points and intersection points would belong to the extension [L32]. n-structures relate closely to the notion of non-commutative space and strings world sheets could be such. Also the role of classical number fields - in particular $M^8 - H$ correspondence suggest the same. The challenge is to develop a coherent view about all these structures.

(a) There should be also a connection with the adelic view. In this picture string world sheets and points of space-time surface with coordinates in the extension of rationals defining the adele code for the data for preferred extremals and quantum states. What these points are - could they correspond to points of partonic 2-surfaces carrying fermions or could the correspond also to the points in the interior of space-time surface is not clear. The larger the extension of rationals, the larger the number of these points, and the better the resolution and the larger the deviation of SH from ideal. The hierarchy of Galois groups of extension of rationals should relate closely to the inclusion hierarchies.

(b) Galois extension with given Galois group Gal allows hierarchy of intermediate extensions defining inclusion sequence for Galois groups. Besides inclusion homomorphisms there exists homomorphisms from Galois group Gal with order $h_{eff}/h = n$ to its sub-groups $H \subset Gal$ with order $h_{eff}/h = m < n$ dividing n. If it exists the sub-group mapped to identity element is normal sub-group H for which right and left cosets gH and Hg are identical. These homomorphisms to sub-groups identify the sheets of Galois covering of the space-time surface transformed to each other by H and thus define different number theoretical resolutions: measurement resolution would have precise geometric meaning. This would mean looking states with $h_{eff}/h = n$ in poorer resolution defined by $h_{eff}/h = m < n$.

   These arrows would define "resolution morphisms" in category theoretic description. Also the analogy with the homotopies of n-structures is obvious. There would be a finite number of normal sub-groups with order dividing n for given higher structure. Quantum phase equal to root of unity ($q = exp(i2\pi/k)$) could appear in these representations and distinguish them from ordinary group representations.

19.3.4 Could normal sub-groups of symplectic group and of Galois groups correspond to each other?

Measurement resolution realized in terms of various inclusion is the key principle of quantum TGD. There is an analogy between the hierarchies of Galois groups, of fractal sub-algebras of SSA, and of inclusions of HFFs. The inclusion hierarchies of isomorphic sub-algebras of SSA and of Galois groups for sequences of extensions of extensions should define hierarchies for measurement resolution. Also the inclusion hierarchies of HFFs are proposed to define hierarcies of measurement resolutions. How closely are these hierarchies related and could the notion of measurement resolution allow to gain new insights about these hierarchies and even about the mathematics needed to realize them?

(a) As noticed, SSA and its isomorphic sub-algebras are in a relation analogous to the between normal sub-group H of group Gal (analog of isomorphic sub-algebra) and the group G/H. One can assign to given Galois extension a hierarchy of intermediate extensions such that one proceeds from given number field (say rationals) to its extension step by step. The Galois groups H for given extension is normal sub-group of the Galois group of its extension. Hence Gal/H is a group. The physical interpretation is following. Finite measurement resolution defined by the condition that H acts trivially on the representations of Gal implies that they are representations of Gal/H. Thus Gal/H is completely analogous to the Kac-Moody type algebra conjecture to result from the analogous pair for SSA.

(b) How does this relate to McKay correspondence stating that inclusions of HFFs correspond to finite discrete sub-groups of $SU(2)$ acting as isometries of regular n-polygons and Platonic solids correspond to Dynkin diagrams of ADE type SKMAs determined by ADE Lie group G. Could one identify the discrete groups as Galois groups represented geometrically as sub-groups of $SU(2)$ and perhaps also those of corresponding Lie group? Could the representations of Galois group correspond to a sub-set of representations of G defining ground states of Kac-Moody representations. This might be possible. The sub-groups of $SU(2)$ can however correspond only to a very small fraction of Galois groups.

Can one imagine a generalization of ADE correspondence? What would be required that the representations of Galois groups relate in some natural manner to the representations as Kac-Moody groups.

Some basic facts about Galois groups and finite groups

Some basic facts about Galois groups mus be listed before continuing. Any finite group can appear as a Galois group for an extension of some number field. It is known whether this is true for rationals (see `http://tinyurl.com/hus4zso`).

Simple groups appear as building bricks of finite groups and are rather well understood. One can even speak about periodic table for simple finite groups (see `http://tinyurl.com/y75r68hp`). Finite groups can be regarded as a sub-group of permutation group S_n for some n. They can be classified to cyclic, alternating , and Lie type groups. Note that alternating group A_n is the subgroup of permutation group S_n that consists of even permutations. There are also 26 sporadic groups and Tits group.

Most simple finite groups are groups of Lie type that is rational sub-groups of Lie groups. Rational means ordinary rational numbers or their extension. The groups of Lie type (see `http://tinyurl.com/k4hrqr6`) can be characterized by the analogs of Dynkin diagrams characterizing Lie algebras. For finite groups of Lie type the McKay correspondence could generalize.

Representations of Lie groups defining Kac-Moody ground states as irreps of Galois group?

The goal is to generalize the McKay correspondence. Consider extension of rationals with Galois group Gal. The ground staes of KMA representations are irreps of the Lie group G defining KMA. Could the allow ground states for given Gal be irreps of also Gal?

This constraint would determine which group representations are possible as ground states of SKMA representations for a given Gal. The better the resolution the larger the dimensions of the allowed representations would be for given G. This would apply both to the representations of the SKMA associated with dynamical symmetries and maybe also those associated with the standard model symmetries. The idea would be quantum classical correspondence (QCC) space-time sheets as coverings would realize the ground states of SKMA representations assignable to the various SKMAs.

This option could also generalize the McKay correspondence since one can assign to finite groups of Lie type an analog of Dynkin diagram (see `http://tinyurl.com/k4hrqr6`). For Galois groups, which are discrete finite groups of $SU(2)$ the hypothesis would state that the Kac-Moody algebra has same Dynkin diagram as the finite group in question.

To get some perspective one can ask what kind of algebraic extensions one can assign to ADE groups appearing in the McKay correspondence? One can get some idea about this by studying the geometry of Platonic solids (see `http://tinyurl.com/p4rwc76`). Also the geometry of Dynkin diagrams telling about the geometry of root system gives some idea about the extension involved.

(a) Platonic solids have p vertices and q faces. One has $\{p,q\} \in \{\{3,3\},\{4,3\},\{3,4\},\{5,3\},\{3,5\}\}$. Tetrahedron is self-dual (see `http://tinyurl.com/qdl4sss` object whereas cube and octahedron and also dodecahedron and icosahedron are duals of each other. From the table of `http://tinyurl.com/p4rwc76` one finds that the cosines and sines for the angles between the vectors for the vertices of tetrahedron, cube, and octahedron are rational numbers. For icosahedron and dodecahedron the coordinates of vertices and the angle between these vectors involve Golden Mean $\phi = (1+\sqrt{5})/2$ so that algebraic extension must involve $\sqrt{5}$ at least.

The dihedral angle θ between the faces of Platonic solid $\{p,q\}$ is given by $sin(\theta/2) = cos(\pi/q)/sin(\pi/p)$. For tetrahedron, cube and octahedron $sin(\theta)$ and $cos(\theta)$ involve $\sqrt{3}$. For icosahedron dihedral angle is $tan(\theta/2) = \phi$. For instance, the geometry of tetrahedron involves both $\sqrt{2}$ and $\sqrt{3}$. For dodecahedron more complex algebraic numbers are involved.

(b) The rotation matrices for for the triangles of tetrahedron and icosahedron involve $cos(2\pi/3)$ and $sin(2\pi/3)$ associated with the quantum phase $q = exp(i2\pi/3)$ associated with it. The rotation matrices performing rotation for a pentagonal face of dodecahedron involves $cos(2\pi/5)$ and $sin(2\pi/5)$ and thus $q = exp(i2\pi/5)$ characterizing the extension. Both $q = exp(i2\pi/3)$ and $q = exp(i2\pi/5)$ are thus involved with icosahedral and dodecahedral rotation matrices. The rotation matrices for cube and for octahedron have rational matrix elements.

(c) The Dynkin diagrams characterize both the finite discrete groups of $SU(2)$ and those of ADE groups. The Dynkin diagrams of Lie groups reflecting the structure of corresponding Weyl groups involve only the angles $\pi/2, 2\pi/3, \pi-\pi/6, 2\pi-\pi/6$ between the roots. They would naturally relate to quadratic extensions.

For ADE Lie groups the diagram tells that the roots associated with the adjoint representation are either orthogonal or have mutual angle of $2\pi/3$ and have same length so that length ratios are equal to 1. One has $sin(2\pi/3) = \sqrt{3}/2$. This suggests that $\sqrt{3}$ belongs to the algebraic extension associated with ADE group always. For the non-simply laced Lie groups of type B, C, F, G the ratios of some root lengths can be $\sqrt{2}$ or $\sqrt{3}$.

For ADE groups assignable to n-polygons ($n > 5$) Galois group must involve the cyclic extension defined by $exp(i2\pi/n)$. The simplest option is that the extension corresponds to the roots of the polynomial $x^n = 1$.

19.3.5 A possible connection with number theoretic Langlands correspondence

I have discussed number theoretic version of Langlands correspondence in [K36, K117] trying to understand it using physical intuition provided by TGD (the only possible approach in my case). Concerning my unashamed intrusion to the territory of real mathematicians I have only one excuse: the number theoretic vision forces me to do this.

Number theoretic Langlands correspondence relates finite-dimensional representations of Galois groups and so called automorphic representations of reductive algebraic groups defined also for adeles, which are analogous to representations of Poincare group by fields. This is kind of relationship can exist follows from the fact that Galois group has natural action in algebraic reductive group defined by the extension in question.

The "Resiprocity conjecture" of Langlands states that so called Artin L-functions assignable to finite-dimensional representations of Galois group Gal are equal to L-functions arising from so called automorphic cuspidal representations of the algebraic reductive group G. One would have correspondence between finite number of representations of Galois group and finite number of cuspidal representations of G.

This is not far from what I am naively conjecturing on physical grounds: finite-D representations of Galois group are reductions of certain representations of G or of its subgroup defining the analog of spin for the automorphic forms in G (analogous to classical fields in Minkowski space). These representations could be seen as induced representations familiar for particle physicists dealing with Poincare invariance. McKay correspondence encourages the conjecture that the allowed spin representations are irreducible also with respect to Gal. For a childishly naive physicist knowing nothing about the complexities of the real mathematics this looks like an attractive starting point hypothesis.

In TGD framework Galois group could provide a geometric representation of "spin" (maybe even spin 1/2 property) as transformations permuting the sheets of the space-time surface identifiable as Galois covering. This geometrization of number theory in terms of cognitive representations analogous to the use of algebraic groups in Galois correspondence might provide a totally new geometric insights to Langlands correpondence. One could also think that Galois group represented in this manner could combine with the dynamical Kac-Moody group emerging from SSA to form its Langlands dual.

Skeptic physicist taking mathematics as high school arithmetics might argue that algebraic counterparts of reductive Lie groups are rather academic entities. In adelic physics the situation however changes completely. Evolution corresponds to a hierarchy of extensions of rationals reflected directly in the physics of dark matter in TGD sense: that is as phases of ordinary matter with $h_{eff}/h = n$ identifiable as divisor of the order of Galois group for an extension of rationals. Algebraic groups and their representations get physical meaning and also the huge generalization of their representation to adelic representations makes sense if TGD view about consciousness and cognition is accepted.

In attempts to understand what Langlands conjecture says one should understand first the rough meaning of many concepts. Consider first the Artin L-functions appearing at the number theoretic side. Consider first the Artin L-functions appearing at the number theoretic side.

(a) L-functions (see `http://tinyurl.com/y8dc4zv9`) are meromorphic functions on complex plane that can be assigned to number fields and are analogs of Riemann zeta function factorizing into products of contributions labelled by primes of the number field. The definition of L-function involves Direchlet characters: character is very general invariant of group representation defined as trace of the representation matrix invariant under conjugation of argument.

(b) In particular, there are Artin L-functions (see `http://tinyurl.com/y7thhodk`) assignable to the representations of *non-Abelian* Galois groups. One considers finite extension L/K of fields with Galois group G. The factors of Artin L-function are labelled by primes p of K. There are two cases: p is un-ramified or ramified depending on whether the number of primes of L to which p decomposes is maximal or not. The number of ramified primes is finite and in TGD framework they are excellent candidates for physical preferred p-adic primes for given extension of rationals.

These factors labelled by p analogous to the factors of Riemann zeta are identified as characteristic polynomials for a representation matrix associated with any element in a preferred conjugacy class of G. This preferred conjugacy class is known as Frobenius element $Frob(p)$ for a given prime ideal p , whose action on given algebraic integer in O_L is represented as its p:th power. For un-ramified p the characteristic polynomial is explicitly given as determinant $det[I - t\rho(Frob(p))]^{-1}$, where one has $t = N(p)^{-s}$ and $N(p)$ is the field norm of p in the extension L (see `http://tinyurl.com/o4saw2l`).

In the ramified case one must restrict the representation space to a sub-space invariant under inertia subgroup, which by definition leaves invariant integers of O_L/p that is the lowest part of integers in expansion of powers of p.

At the other side of the conjecture appear representations of algebraic counterparts of reductive Lie groups and their L-functions and the two number theoretic and automorphic L-functions would be identical.

(a) Automorphic form F generalizes the notion of plane wave invariant under discrete subgroup of the group of translations and satisfying Laplace equation defining Casimir operator for translation group. Automorphic representations can be seen as analogs for the modes of classical fields with given mass having spin characterized by a representation of subgroup of Lie group G ($SO(3)$ in case of Poincare group).

Automorphic functions as field modes are eigen modes of some Casimir operators assignable to G. Algebraic groups would in TGD framework relate to adeles defined by the hierarchy of extensions of rationals (also roots of e can be considered in extensions). Galois groups have natural action in algebraic groups.

(b) Automorphic form (see `http://tinyurl.com/create.php`) is a complex vector valued function F from topological group to some vector space V. F is an eigen function of certain Casimir operators of G. In the simplest situation these function are invariant under a discrete subgroup $\Gamma \subset G$ identifiable as the analog of the subgroup defining spin in the case of induced representations.

In general situation the automorphic form F transforms by a factor j of automorphy under Γ. The factor can also act in a finite-dimensional representation of group Γ, which would suggest that it reduces to a subgroup of Γ obtained by dividing with a normal subgroup. j satisfies 1-cocycle condition $j(g_1, g_2g_3) = j(g_1g_2, g_3)$ in group cohomology guaranteeing associativity (see `http://tinyurl.com/on7ffy9`). Cuspidality relates to the conditions on the growth of F at infinity.

(c) Elliptic functions in complex plane characterized by two complex periods are meromorphic functions of this kind. A less trivial situation corresponds to non-compact group $G = SL(2, R)$ and $\Gamma \subset SL(2, Q)$.

There are more groups involved: Langlands group L_F and Langlands dual group ^LG. A more technical formulation says that the automorphic representations of a reductive Lie group G correspond to homomorphisms from so called Langlands group L_F (see `http://tinyurl.com/ycnhkvm2`) at the number theoretic side to L-group ^LG or Langlands dual of algebraic G at group theory side (see `http://tinyurl.com/ycnk9ga5`). It is important to notice that ^LG is a complex Lie group. Note also that homomorphism is a representation of Langlands group L_F in L-group ^LG. In TGD this would be analogous to a homomorphism of Galois group defining it as subgroup of the group G defining Kac-Moody algebra.

(a) Langlands group L_F of number field is a speculative notion conjectured to be a extension of the Weil group of extension, which in turn is a modification of the absolute Galois group. Unfortunately, I was not able to really understand the Wikipedia definition of Weil group (`http://tinyurl.com/hk74sw7`). If E/F is finite extension as it is now, the Weil group would be $W_{E/F} = W_F/W_E^c$, W_E^c refers to the commutator subgroup W_E defining a normal subgroup, and the factor group is expected to be finite. This is not Galois group but should be closely related to it.

Only finite-D representations of Langlands group are allowed, which suggests that the representations are always trivial for some normal subgroup of L_F For Archimedean local fields L_F is Weil group, non-Archimedean local fields L_F is the product of Weil group of L and of SU(2). The first guess is that SU(2) relates to quaternions. For global fields the existence of L_F is still conjectural.

(b) I also failed to understand the formal Wikipedia definition of the L-group ${}^LG$ appearing at the group theory side. For a reductive Lie group one can construct its root datum $(X^*, \Delta, X_*, \Delta^c)$, where X^* is the lattice of characters of a maximal torus, X_* its dual, Δ the roots, and Δ^c the co-roots. Dual root datum is obtained by switching X^* and X_* and Δ and Δ^c. The root datum for G and ${}^LG$ are related by this switch.

For a reductive G the Dynkin diagram of ${}^LG$ is obtained from that of G by exchanging the components of type B_n with components of type C_n. For simple groups one has $B_n \leftrightarrow C_n$. Note that for ADE groups the root data are same for G and its dual and it is the Kac-Moody counterparts of ADE groups, which appear in McKay correspondence. Could this mean that only these are allowed physically?

(c) Consider now a reductive group over some field with a separable closure K (say k for rationals and K for algebraic numbers). Over K G as root datum with an action of Galois group of K/k. The full group ${}^LG$ is the semi-direct product ${}^LG^0 \rtimes Gal(K/k)$ of connected component as Galois group and Galois group. $Gal(K/k)$ is infinite (absolute group for rationals). This looks hopelessly complicated but it turns it that one can use the Galois group of a finite extension over which G is split. This is what gives the action of Galois group of extension (l/k) in ${}^LG$ having now finitely many components. The Galois group permutes the components. The action is easy to understand as automorphism on Gal elements of G.

Could TGD picture provide additional insights to Langlands duality or vice versa?

(a) In TGD framework the action of Gal on algebraic group G is analogous to the action of Gal on cognitive representation at space-time level permuting the sheets of the Galois covering, whose number in the general case is the order of Gal identifiable as $h_{eff}/h = n$. The connected component ${}^LG0$ would correspond to one sheet of the covering.

(b) What I do not understand is whether ${}^LG = G$ condition is actually forced by physical contraints for the dynamical Kac-Moody algebra and whether it relates to the notion of measurement resolution and inclusions of HFFs.

(c) The electric-magnetic duality in gauge theories suggests that gauge group action of G on electric charges corresponds in the dual phase to the action of ${}^LG$ on magnetic charges. In self-dual situation one would have $G =^L G$. Intriguingly, CP_2 geometry is self-dual (Kähler form is self-dual so that electric and magnetic fluxes are identical) but induced K̈ahler form is self-dual only at the orbits of partonic 2-surfaces if weak form of electric-magnetic duality holds true. Does this condition leads to ${}^LG = G$ for dynamical gauge groups? Or is it possible to distinguish between the two dynamical descriptions so that Langlands duality would correspond to electric-magnetic duality. Could this duality correspond to the proposed duality of two variants of SH: namely, the electric description provided by string world sheets and magnetic description provided by partonic 2-surfaces carrying monopole fluxes?

19.3.6 A formulation of adelic TGD in terms of cognitive representations?

The vision about p-adic physics as cognitive representations of real physics [L34] encourages to consider an amazingly simple formulation of TGD diametrically opposite to but perhaps consistent with the vision based on the notion of WCW and WCW spinor fields. Finiteness of cognitive and measurement resolutions would not be enemies of the theoretician but could make possible to deduce highly non-trivial predictions from the theory by getting rid of all irrelevant information and using only the most significant bits. Number theoretic physics need not of course cover the entire quantum physics and could be analogous to topological quantum field theories: even this might provide huge amounts of precise information about the quantum physics of TGD Universe.

Could the discrete variant of WCW geometry make sense?

The first thing that one can imagine is number theoretic discretization of WCW by assuming that WCW coordinates belong to an extension of rationals. Integration would reduce to a summation but the problem is that there are too many points in the extension so that sums do not make sense in real sense. In the case of space-time surfaces the problems are solved by the fact that space-time surfaces have dimension lower than the imbedding space and the number of points with coordinates in the extension is in typical case finite: exceptions are surfaces such as canonically imbedded M^4 or CP_2. This option does not work at the level of WCW.

Cognitive representations however carry information about the points with coordinates in the extension of rationals defining the adele and possibly about the directions of strings emanating from these points. The effective WCW is kind of coset space with most of degrees of freedom not visible in the cognitive representation. Cognitive representations would specify the points in the extension of rationals for space-time surface, string world sheets, or even for their intersection with partonic surfaces at the ends of CD carrying fermion number plus those at the ends of sub-CDs forming a hierarchy.

Could one use the points of cognitive representation as coordinates for this effective WCW so that everything including WCW integration would reduce to well-defined summations? This would solve the problem of too many points in sub-WCW associated with the extension. Could one formulate everything that one can know at given level of cognitive hierarchy defined by extensions?

This idea was already suggested by the interpretation of p-adic mass calculations.

(a) p-Adic mass calculations would correspond to cognitive representation of real physics [K12, K39]. For large p-adic primes p-adic thermodynamics converges extremely rapidly as powers $p^{-n/2}$ and the results from two lowest orders are practically exact.

(b) What is however required is a justification for the map of p-adic mass squared values to real numbers by canonical identification. Quite generally this map makes sense for group invariants - say Lorentz invariants defined by inner products of momenta. As a matter of fact, the construction of quantum algebras and Yangians demands p-adic topology for the antipode to exist mathematically so that this approach could be forced by mathematical consistency [B12].

Could scattering amplitudes be constructed in terms of cognitive representations?

The crazy looking idea that cognitive representations defined by common points of real and p-adic variants of space-time surfaces or even partonic 2-surfaces is at least worth of showing to be wrong. If the idea works, cognitive representations could code what can be known about classical and even quantum dynamics and reduce physics to number theory. Also WCW would be discretized with points of discretized space-time surface defining WCW coordinates. Functional integral over WCW would reduce to a converging sum over cognitive representations.

It is interesting to look what this could mean if scattering amplitudes correspond in some sense to algebraic computations in bi-algebra besides product also co-product as its time reversal and interpreted as 3-vertex physically.

(a) For the simplest option fermions would reside at the intersection points of partonic 2-surfaces and string world sheets. One possibility considered earlier is that at these points the Galois coverings are singular meaning that all sheets co-incide. This might be too strong condition and might be replacable by a weaker condition that Galois group at these points reduces to its sub-group and normal subgroup leaves amplitudes invariant. A reduction of measurement resolution would be in question.

(b) If the basic computational operation involves a fusion of representations of Galois group, fusion algebra could describe the situation [L32]. The Galois groups assignable to the incoming lines of 3-vertex must correspond to Galois groups, which define groups of 3-levelled hierarchy of extension of rationals allowing inclusion homomorphism. Therefore the values of Planck constant would be of from $h_{eff}/h \in \{n_1, n_1n_2, n_1n_2n_3\}$. The tensor product decomposition would tell the outcome of tensor product. One can consider also 2-vertices corresponding to a phase transition $n_1 \leftrightarrow n_1n_2$ changing the value of h_{eff}/h.

McKay graphs (see `http://tinyurl.com/z48d92t`) for Galois groups describe the decomposition of the tensor products of representations of Galois groups. In general the tensor products for corresponding KMAs restricted to Galois group are not irreducible. What could this mean? Are they allowed to occur? Are there general results allowing to conclude how do the analogs of McKay graphs for the tensor products of the irreps of the group defining Kac-Moody group relate to the McKay graphs for its finite discrete sub-groups?

Possible problems relate to the description of momenta and higher excitations of SKMAs. In topological QFTs one loses information about metric properties such as mass but what happens in number theoretic QFT? Could the Galois approach expanded to include also discrete variants of quaternions and octonions assigna ble to extensions of rationals allow also the number theoretic description of also momenta?

(a) Octonions and quaternions have $G(2)$ and $SO(3)$ as automorphisms groups (analogs of Galois groups). The octonionic automorphisms respecting chosen imaginary consist of $SU(3)$ rotations. These groups would be replaced with their dicrete variants with matrix elements in an extension of rationals.

The automorphism group Gal for the extension of rationals and automorphism group $Aut \in \{G_2, SU(3), SO(3)\}$ for octonions/for octonions with fixed unit/for quaternions form a semi-direct product $Gal \rtimes Aut$ with multiplication rule $(g_1, g_a) \circ (g_2, g_b) = (g_1g_2, g_2g_1(g_b))$, where $g_1(g_b)$ represents the element of Aut obtained by performing Gal automorphism g_1 for g_b. For rational elements g_b one has $(g_1, g_a) \circ (g_2, g_b) = (g_1g_2, g_ag_b)$ so that Gal Aut_Q commute. An interesting possibility is that the automorphisms of $Aut \in \{SU(3), SO(3)\}$ can be interpreted in terms of standard model symmetries whereas Gal would relate to the dynamical symmetries.

In M^8 picture one has naturally wave functions in the space of quaternionic light-like 8-momenta and it is natural to decompose quaternionic momenta to longitudinal M^2 piece and transversal E^2 piece. The physical interpretation of this condition has been discussed thoroughly in [L38]. One has thus more than mere analog of TQFT.

(b) If fermions propagate along the lines of the TGD analogs twistor graphs, one must have an analog of propagator. Twistor approach [L38] implies that the propagator is replaced with the inverse of the fermion propagator for quaternionic 8-momentum as a residue with sigma matrices representing the quaternionic units. This is non-vanishing only if the fermion chirality is "wrong". This has co-homological interpretation: for external lines the inverse of the propagator would annihilate the state (co-closedness) unlike for internal lines.

(c) Triality holds true for the octonionic vector representation assignable to momenta and octonionic spinors and their conjugates. All these should be quaternionic, in other words belong to some complexified quaternionic $M^4 \subset M^8$. The components of these spinors should belong to an extension of rational used with imaginary unit commuting with octonionic imaginary units.

(d) The condition that the amplitudes belong to an extension of rationals could be extremely powerful when combined with category theoretic view implying the Hilbert space isometries allowing to relate amplitudes at different levels of the hierarchy. This conditions should be true also for the twistors in terms which momenta can be expressed. Also the space $SU(3)/U(1) \times U(1)$ of CP_2 twistors would be replaced with a sub-space with points in an extension of rationals.

19.4 Could McKay correspondence generalize in TGD framework?

McKay correspondence is rather mysterious looking correspondence appearing in several fields. This correspondence is extremely interesting from point of view of adelic TGD [L35] [L34].

(a) McKay graphs code for the fusion algebra of irreducible representations (irreps) of finite groups (see http://tinyurl.com/z48d92t). For finite subgroups of $G \subset SU(2)$ McKay graphs are extended Dynkin diagrams for affine (Kac-Moody) algebras of ADE type coding the structure of the root diagram for these algebras. The correspondence looks mysterious since Dynkin diagrams have quite different geometric interpretation.

(b) McKay graphs for finite subgroups of $G \subset SU(2)$ characterize also the fusion rules of minimal conformal field theories (CFTs) having Kac-Moody algebra (KMA) of $SU(2)$ as symmetries (see http://tinyurl.com/y7doftpe). Fusion rules characterize the decomposition of the tensor products of primary fields in CFT. For minimal CFTs the primary fields belonging to the irreps of $SU(2)$ are in 1-1 correspondence with irreps of G, and the fusion rules for primary fields are same as for the irreps of G. The irreps of $SU(2)$ are also irreps of G.

    Could the ADE type affine algebra appear as dynamical symmetry algebra too? Could the primary fields for ADE defining extended ADE Cartan algebra be constructed as G-invariants formed from the irreps of G and be exponentiated using the standard free field construction using the roots of the ADE KMA a give ADE KMA acting as dynamical symmetries?

(c) McKay graphs for $G \subset SU(2)$ characterize also the double point singularities of algebraic surfaces of real dimension 4 in C^3 (or CP^3 , one variant of twistor space!) with real dimension 6 (see http://tinyurl.com/ydz93hle). The subgroup $G \subset SU(2)$ has a natural action in C^2 and it appears in the canonical representation of the singularity as orbifold C^2/G. This partially explains the appearance of the McKay graph of G. The resolved singularities are characterized by a set of projective lines CP_1 with intersection matrix in CP_2 characterized by McKay graph of G. Why the number of spheres is the number of irreps for G is not obvious to me.

    The double point singularities of $C^2 \subset C^3$ allow thus ADE classification. The number of added points corresponds to the dimension of Cartan algebra for ADE type affine algebra, whose Dynkin diagram codes for the finite subgroup $G \subset SU(2)$ leaving the algebraic surface looking locally like C^2 invariant and acting as isotropy group of the singularity.

These results are highly inspiring concerning adelic TGD.

(a) The appearance of Dynkin diagrams in the classification of minimal CFTs inspires the conjecture that in adelic physics Galois groups Gal or semi-direct products $G \triangleleft Gal$ of Gal with a discrete subgroup G of automorphism group $SO(3)$ (having $SU(2)$ as double covering!) classifies TGD generalizations of minimal CFTs. Also discrete subgroups of octonionic automorphism group can be considered. The fusion algebra of irreps of Gal would define also the fusion algebra for KMA for the counterparts of minimal fields. This would provide deep insights to the general structure of adelic physics.

(b) One cannot avoid the question whether the extended ADE diagram could code for a dynamical symmetry of a minimal CFT or its modification? If the *Gal* singlets formed from the primary fields of minimal model define primary fields in Cartan algebra of ADE type KMA, then standard free field construction would give the charged KMA generators. In TGD framework this conjecture generalizes.

(c) A further conjecture is that the singularities of space-time surface imbedded as 4-surface in its 6-D twistor bundle with twistor sphere as fiber could be classified by McKay graph of *Gal*. The singular intersection of the Euclidian and Minkowskian regions of space-time surface is especially interesting: the twistor spheres at the common points defining light-like partonic orbits need not be same but have intersections with intersection matrix given by McKay graph for *Gal*. The basic information about adelic CFT would be coded by the general character of singularities for the twistor bundle.

(d) In TGD also singularities in which the group *Gal* is reduced to its subgroup Gal/H, where H is normal group are possible and would correspond to phase transition reducing the value of Planck constant. What happens in these phase transitions to single particle states would be dictated by the decomposition of representations of *Gal* to those of Gal/H and transition matrix elements could be evaluated.

One can find from web excellent articles about the topics to be discussed in this article.

(a) The article "*Cartan matrices, finite groups of quaternions, and Kleinian singularities*" of John McKay [A68] (see `http://tinyurl.com/ydygjgge`) summarizes McKay correspondence.

(b) Miles Reid has written an article " *The Du Val singularities* A_n, D_n, E_6, E_7, E_8" [A73] (see `http://tinyurl.com/ydz93hle`). Also the article " *Chapters on algebraic surfaces*" [A74](see `http://tinyurl.com/yaty9rzy`) of Reid should be helpful. There is also an article "*Resolution of Singularities in Algebraic Varieties*" [A49] (see `http://tinyurl.com/yb7cuwkf`) of Emma Whitten about resolution of singularities.

(c) Andrea Cappelli and Jean-Benard Zuber have written an article "*A-D-E Classification of Conformal Field Theories*" [B28] about ADE classification of minimal CFT models (see `http://tinyurl.com/y7doftpe`).

(d) McKay correspondence appears also in M-theory, and the thesis "*On Algebraic Singularities, Finite Graphs and D-Brane Gauge Theories: A String Theoretic Perspective*" [B51] (see `http://tinyurl.com/ycmyjukn`) of Yang-Hui He might be helful for the reader. In this work the possible generalization of McKay correspondence so that it would apply form finite subgroups of $SU(n)$ is discussed. $SU(3)$ acting as subgroup of automorphism group G_2 of octonions is especially interesting in this respect. The idea is rather obvious: the fusion diagram for the theory in question would be the McKay graph for the finite group in question.

19.4.1 McKay graphs in mathematics and physics

McKay graphs for subgroups of $SU(2)$ reducing to Dynkin diagrams for affine Lie algebras of ADE type appear in several manners in mathematics and physics.

McKay graphs

McKay graphs [A68] (see `http://tinyurl.com/ydygjgge`) code for the fusion algebra of irrpes of finite groups G (for Wikipedia article see `http://tinyurl.com/z48d92t`). One considers the tensor products of irreps with the canonical representation (doublet representation for the finite sub-groups of $SU(2)$), call it V. The irreps V_i correspond to nodes and their number is equal to the number of irreps G.

Two nodes i and j are no connected if the decomposition of $V \otimes V_i$ to irreps does not contain V_j. There is arrow pointing from $i \rightarrow j$ in this case. The number $n_{ij} > 0$ or number of arrows tells how many times j is contained in $V \otimes V_j$. For $n_{ij} = n_{ji}$ there is no arrow.

One can characterize the fusion rules by matrix $A = d\delta_{ij} - n_{ij}$, where d is the dimension of the canonical representation. The eigenvalues of this matrix turn out to be given by $d - \xi_V(g)$, where $\xi_V(g)$ is the character of the canonical representation, which depends on the conjugacy class of g only. The number of eigenvalues is therefore equal to the number $n(class, G)$ of conjugacy classes. The components of eigenvectors in turn are given by the values $\chi_i(g)$ of characters of irreps.

MacKay graphs and Dynkin diagrams

The nodes of the Dynkin diagram (see `http://tinyurl.com/hpm5y9s`) are positive simple root vectors identified as vectors formed by the eigenvalues of the Cartan sub-algebra generators under adjoint action on Lie algebra. In the case of affine Lie algebra the Cartan algebra contains besides the Cartan algebra of the Lie group also scaling generator $L_0 = td/dt$ and the number of nodes increases by one.

The number of positive simple roots equals to the dimension of the root space. The number n_{ij} codes now for the angle between positive simple roots. The number of edges connecting root vectors is $n = 0, 1, 2, 3$ depending on whether the angle between root vectors is $\pi/2, 2\pi/3, 3\pi/4$, or $5\pi/6$. The ratios of lengths of connected roots can have values $\sqrt{n}$, $n \in \{1, 2, 3\}$, and the number n of edges corresponds to this ratio. The arrow is directed to the shorter root if present. For simply laced Lie groups (ADE groups) the roots have unit length so that only single undirected edge can connect the roots. Weyl group acts as symmetries of the root diagram as reflections in hyperplanes orthogonal to the roots.

The Dynkin diagrams of affine algebras are obtained by adding to the Cartan algebra a generator which corresponds to the scaling generator $L_0 = td/dt$ of affine algebra assumed to act via adjoint action to the Lie algebra. Depending on the position of the added node one obtains also twisted versions of the KMA.

For the finite subgroups of $SU(2)$ the McKay graphs reduce to Dynkin diagrams of affine Lie algebras of ADE type [A68] (see `http://tinyurl.com/ydygjgge`) so that one has either $n_{ij} = 0$ or $n_{ij} = 1$ for $i \neq j$. There are no self-loops ($n_{ii} \neq 0$). The result looks mysterious since the two diagrams describe quite different things. One can also raise the question whether ADE type affine algebra might somehow emerge in minimal CFT involving $SU(2)$ KMA for which ADE classification emerges.

In TGD framework the interpretation of finite groups $G \subset SU(2)$ in terms of quaternions is an attractive possibility since rotation group $SO(3)$ acts as automorphisms of quaternions and has $SU(2)$ as its covering group.

ADE diagrams and subfactors

ADE classification emerges also naturally for the inclusions of hyper-finite factors of type II_1 [K87, K26]. Subfactors with index smaller than four have so called principal graphs characterizing the sequence of inclusions equal to one of the A, D or E Coxeter-Dynkin diagrams: see the article "*In and around the origin of quantum groups*" of Vaughan Jones [A91] (see `http://tinyurl.com/ycbbbvpq`). As a matter of fact, only the D_{2n} and E_6 and E_8 do occur. It is also possible to construct $M : N = 4$ sub-factor such that the principle graph is that for any subgroup $G \subset SU(2)$. This suggests that the subfactors $M : N = 4cos^2(\pi/n) < 4$ correspond to quantum groups. The basic objects can be seen as quantum spinors so that again the appearance of subgroups of $SU(2)$ looks natural. One can still wonder whether ADE KMAs might be involved.

ADE classification for minimal CFTs

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CFTs on torus [B28] are characterized by modular invariant partition functions, which can be expressed in terms of characters of the scaling generator L_0 of Virasoro algebra (VA) given by

$$Z(\tau) = Tr(X) \ , \quad X = exp\{i2\pi \left[\tau(L_0 - c/24) - \overline{\tau}(\overline{L}_0 - c/24)\right]\} \ . \tag{19.4.1}$$

Modular invariance requires that $Z(\tau)$ is invariant under modular transformations leaving the conformal equivalence class of torus invariant. Modular group equals to $SL(2,Z)$ has as generators the transformations $T : \tau \to \tau + 1$ and $S : \tau \to -1/\tau$. The partition function can be expressed as

$$Z(\tau) = \sum N_{j\overline{j}} \chi_j(q) \chi_{\overline{j}}(\overline{q}) \ , \quad q = exp(i2\pi\tau) \ , \quad \overline{q} = exp(-i2\pi\overline{\tau}) \ . \tag{19.4.2}$$

Here χ_j corresponds to the trace of $L_0 - c/24$ for a representation of KMA inducing the VA representation. Modular invariance of the partition function requires $SNS^\dagger = N$ and $TNT^\dagger = N$.

The ADE classification for minimal conformal models summarized in [B28] (see `http://tinyurl.com/y7doftpe`) involves $SU(2)$ affine algebra with central extension parameter k. The central extension parameter for the VA is $c < 1$. The fusion algebra for primary fields in representations of $SU(2)$ KMA characterizes the CFT to a high degree.

The fusion rules characterized the decomposition of the tensor product of representation D_i with representation D_j as $i \otimes j = N_{ij}^k D_k$. Due to the properties of the tensor product the matrices $\mathcal{N}_i = N_{ij}^k$ form and associative and commutative algebra and one can diagonalize these matrices simultaneously. This algebra is known as Verlinde algebra and its elements can be expressed in terms of unitary modular matrix S_{ij} representing the transformation of characters in the modular transformation $\tau \to -1/\tau$.

The generator of the Verlinde algebra is fusion algebra for the 2-D representation of $SU(2)$ generating the fusion algebra (this corresponds to the fact that tensor powers of this representations give rise to all representations of $SU(2)$). It turns out that for minimal models with a finite number of primary fields (KMA representations) the fusion algebra of KMA reduces to that for a finite subgroup of $SU(2)$ and thus corresponds to ADE KMA. The natural interpretation is that the condition that the number of primary fields is finite is realized if the primary fields correspond also to the irreps of finite subgroup of SU(2).

Could the ADE type KMA actually correspond to a genuine dynamical symmetry of minimal CFT? For this conjecture makes sense, the roots of ADE type KMA should be in 1-1 correspondence with the irreps of $G \subset SU(2)$ assignable to primary fields. How could this be possible? In the free field construction of ADE type KMA generators one constructs charged KMA generators from free fields in Cartan algebra by exponentiating the quantities $\alpha \cdot \phi$, where α is the root and ϕ is a primary field corresponding to the element of Cartan algebra of KMA. Could $SU(2)$ invariants formed from the primary fields defined by each G-(equivalently $SU(2)$-) multiplet give rise to $SU(2)$ neutral multiplet of primary fields of ADE type Cartan algebra and could their exponentiation give rise to ADE type KMA acting as dynamical symmetries of a minimal CFT?

The resolution of singularities of algebraic surfaces and extended Dynkin diagrams of ADE type

The classification of singularities of algebraic surfaces leads also to extended Dynkin diagrams of ADE type.

Classification of singularities

In algebraic geometry the classification of singularities of algebraic varieties [A49] is a central task. The singularities of curves in plane represent simplest singularities (see `http://tinyurl.com/y8ub2c4s`). The resolution of singularities of complex curves in C^3 is less trivial task.

The resolution of singularity (http://tinyurl.com/y8veht3p) is a central concept and means elimination of singularity by modifying it locally. There is extremely general theorem by Hiroka stating that the resolution of singularities of algebraic varieties is always possible for fields with characteristic zero (reals and p-adic number fields included) using a sequence of birational transformations. For finite groups the situation is unclear for dimensions $d > 3$.

The articles of Reid [A73] and Whitten [A49] describe the resolution for algebraic surfaces (2-D surfaces with real dimension equal to four). The article of Reid describes how the resolutions of double-point singularities of $m = d_c =$ 2-D surfaces in $n = d_c =$ 3-D C^3 or CP_3 (d_c refers to complex dimension) are classified by ADE type extended Dynkin diagrams. Subgroups $G \subset SU(2)$ appear naturally because the surface has dimension $d_c = 2$. This is the simplest non-trivial situation since for Riemann surface with $(m, n) = (1, 2)$ the group would be discrete subgroup of $U(1)$.

Singularity and Jacobians

What does one mean with singularity and its resolution? Reid [A73] (see http://tinyurl.com/ydz93hle) discusses several examples. The first example is the singularity of the surface $P(x_1, x_2, x_3) = x_1^2 - x_2x_3 = 0$.

(a) One can look the situation from the point of view of imbedding of the 2-surface to C^3: one considers map from tangent space of the surface to the imbedding space C^3. The Jacobian of the imbedding map $(x_2, x_3) \to (x_1, x_2, x_3) = \pm\sqrt{x_2x_3}, x_2, x_3)$ becomes ill-defined at origin since the partial derivatives $\partial x_1/\partial x_2 = (\sqrt{x_3/x_2})/2$ and $\partial x_1/\partial x_3 = (\sqrt{x_2/x_3})/2$ have all possible limiting values at singularity. The resolution of singularity must as a coordinate transformation singular at the origin should make the Jacobian well-defined. Obviously this must mean addition of points corresponding to the directions of various lines of the surface through origin.

(b) A more elegant dual approach replaces parametric representation with representation in terms of conditions requiring function to be constant on the surface. Now the Jacobian of a map from C^3 to the 1-D normal space of the singularity having polynomial $P(x_1, x_2, x_3)$ as coordinate is considered. Singularity corresponds to the situation when the rank of the Jacobian defined by partial derivatives is less than maximal so that one has $\partial P/\partial x_i = 0$. The resolution of singularity means that the rank becomes maximal. Quite generally, for co-dimension m algebraic surface the vanishing of polynomials P_i, $i = 1, ..., m$ defines the surface. At the singularity the reduction of the rank for the matrix $\partial P_i/\partial x_n$ from its maximal value takes place.

Blowing up of singularity

Codimension one algebraic surface is defined by the condition $P(x_1, x_2, ..., x_n) = 0$, where $P(x_1, ..., x_n)$ is polynomial. For higher codimensions one needs more polynomials and the situation is not so neat anymore since so called complete intersection property need not hold anymore. Reid [A73] gives an easy-to- understand introduction to the blowing up of double-point singularities. Also the article "*Resolution of Singularities in Algebraic Varieties*" of Emma Whitten [A49] (see http://tinyurl.com/yb7cuwkf) is very helpful.

(a) Coordinates are chosen such that the singularity is at the origin $(x, y, z) = (0, 0, 0)$ of complex coordinates. The polynomial has vanishing linear terms at singularity and the first non-vanishing term is second power of some coordinate, say x_1, so that one has $x_1 = \pm\sqrt{P_1(x_1, x_2, x_3}$, where x_1 in P_1 appears in powers higher than 2. At the singularity the two roots co-incide. One can of course have also more complex singularities such as triple-points.

(b) The simplest example $P(x_1, x_2, x_3) = x_1^2 - x_2x_3 = 0$ has been already mentioned. This singularity has the structure of double cone since one as $x_1 = \pm\sqrt{x_2x_3}$. At $(0, 0, 0)$ the vertices of the two cones meet.

(c) One can look this particular situation from the perspective of projective geometry. Homogenous polynomials define a surface invariant under scalings of coordinates so that modulo scalings the surface can be regarded also as complex curve in CP_2. The conical surface can be indeed seen as a union of lines ($x_1 = k^2x_3, x_2 = kx_3$), where k is complex number. The ratio $x_1 : x_2 : x_3$ for the coordinates at given line is determined by $x_1 : x_2 = k$ and $x_2 : x_3 = k$ so that the surface can be parameterized by k and the coordinate along given line.

In this perspective the singularity decomposes to the directions of the lines going through it and the situation becomes non-singular. The replacement of the original view with this gives a geometric view idea about the resolution of singularity: the 2-surface is replaced by a bundle lines of surfaces going through the singularity and singularity is replaced with a union of directions for these lines.

Quite generally, in the resolution of singularity, origin is replaced by a set of points (x_1, x_2, x_3) with a well-defined ratio $(x_1 : x_2 : x_3)$. This interpretation applies also to more general singularities. One can say that origin is replaced with a projective sub-manifold of 2-D projective space CP_2 (very familiar to me)! This procedure is known as blowing up. Strictly speaking, one only replaces origin with the directions of lines in C^3.

Remark: In TGD the wormhole contacts connecting space-time sheets of many-sheeted space-time could be seen as outcomes of blowing up procedure.

Blowing up replaces the singular point with projective space CP_1 for which points with same value of $(x_1 : x_2 : x_3)$ are identified. Blowing up can be also seen as a process analogous to seeing the singularity such as self-intersection of curve as an illusion: the curve is actually a projection of a curve in higher dimensional space to which it is lifted so that the intersection disappears [A49] (see `http://tinyurl.com/yb7cuwkf`). Physicist can of course protest by saying that in space-time physics is is not allowed to introduce additional dimensions in this manner!

There is an analytic description for what happens at the singular point in blowing up process [A49] (see `http://tinyurl.com/yb7cuwkf`).

(a) In blowing up one lifts the surface in higher-dimensional space $C^3 \times CP_2$ (C^3 can be replaced by any affine space). The blowing up of the singularity would be the set of lines $\overline{q}$ of the surface S going through the singularity that is the set $B = \{(q, \overline{q}) | q \in S\}$. This set can be seen as a subset of $C^3 \times CP_2$ and one can represent it explicitly by using projective coordinates (y_1, y_2, y_3) for CP_2. Consider points of C^3 and CP_2 with coordinates $z = (x_1, x_2, x_3)$ and $y = (y_1, y_2, y_3)$. The coordinate vectors must be parallel x is to be at line y. This requires that all 2×2 sub-determinants of the matrix

$$\begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{bmatrix} \tag{19.4.3}$$

vanish: that is $x_iy_j - x_jy_i = 0$ for all pairs $i < j$. This description generalizes to the higher-dimensional case. The added CP_1s defined what is called exceptional divisor in the blown up surface. Recall that divisors (see `http://tinyurl.com/yc7x3ohx`) are by definition formal combinations of points of algebraic surface with integer coefficients. The principal divisors defined by functions are sums over their zeros and poles with integer weight equal to the order of zero (negative for pole).

The above example considers a surface $x_1^2 - x_2x_3 = 0$ which allows interpretation as a projective surface. The method however works also for more general case since the idea about replacing point with directions is applied only at origin.

(b) One can consider a more practical resolution of singularity by performing a bi-rational coordinate transformation becoming singular at the singular point. This can improve the singularity by blowing it up or make it worse by inducing blowing down. The idea is to perform a sequence of this kind of coordinate changes inducing blowing ups so that final outcome is free of singularities.

Since one considers polynomial equations both blowing up and its reversal must map polynomials to polynomials. Hence a bi-rational transformation b acting as a surjection from the modified surface to the original one must be in question (for bi-rational geometry see `http://tinyurl.com/yadoo3ot`). At the singularity b is many-to-one y so that at this point inverse image is multivalued and gives rise to the blowing up.

The equation $P(x_1, x_2, x_3) = 0$ combined with the equations $x_i y_j - x_j y_i = 0$ by putting $y_3 = 1$ (the coordinates are projective) leads to a parametric representation of S using y_1 and y_2 as coordinates instead of x_1 and x_2. Origin is replaced with CP_1. This representation is actually much more general. Whitten [A49] gives a systematic description of resolution of singularities using this representation. For instance, cusp singularity $P(x_1, x_2) = x_1^2 - x_2^3 = 0$ is discussed as a special case.

(c) Topologically the blow up process corresponds to the gluing of CP_2 to the algebraic surface $A : A \rightarrow A\#CP_2$ and clearly makes it more complex. One can say that gluing occurs along sphere CP_1 and since the process involves several steps several spheres are involved with the resolution of singularities.

ADE classification for resolutions of double point singularities of algebraic surfaces

ADE classification emerges for co-dimension one double point singularities of complex surfaces in C^3 known as Du Val singularities. The surface itself can be seen locally as C^2. These surfaces are 4-D in real sense can have self-intersections with real dimension 2. In the singular point the dimension of the intersection is reduced and the dimension of tangent space is reduced (the rank of Jacobian is not maximal). The vertices of cone and cusp are good examples of singularities.

The subgroup $G \subset SU(2)$ has a natural action in C^2 and it appears in the canonical representation of the singularity as orbifold C^2/G. This helps to understand the appearance of the McKay graph of G. The resolved singularities are characterized by a set of projective lines CP_1 with intersection matrix in CP_2 characterized by McKay graph of G. Why the number of projective lines equals to the number of irrepss of G appearing as nodes in McKay graph looks to me rather mysterious. Reid's article [A73] gives the characterization of groups G and canonical forms of the polynomials defining the singular surfaces.

The reason why Du Val singularities are so interesting from TGD point of view is that complex surfaces in Du Val theory have real dimension 4 and are surfaces in space of real dimension 6. The intersections of the branches of the 4-surfaces have real dimension $D = 2$ in the generic case. In TGD space-time surfaces as preferred extremals have real dimension 4 and assumed possess complex structure or its Minkowskian generalization that I have called Hamilton-Jacobi structure [K79].

19.4.2 Do McKay graphs of Galois groups give overall view about classical and quantum dynamics of quantum TGD?

McKay graphs for Galois groups are interesting from TGD view point for several reasons. Galois groups are conjectured to be the number theoretical symmetries for the hierarchy of extensions of rationals defining hierarchy of adelic physics [L35] [L34] and the notion of CFT is expected to generalize in TGD framework so that ADE classification for minimal CFTs might generalize to a classification of minimal number theoretic CFTS by Galois groups.

Vision

The arguments leading to the vision are roughly following.

(a) Adelic physics postulates a hierarchy of quantum physics with adeles at given level associated with extension of rationals characterized partially by Galois group and ramified primes of extension. The dimension of the extensions dividing the order of Galois group is excellent candidate for defining the value of Planck constant $h_{eff}/h = n$ and ramified primes could correspond to preferred p-adic primes. The discrete sets of points of space-time surface for which imbedding space coordinates are in the extension define what I have interpreted as cognitive representations and can be said to be in the intersection of all number fields involved forming kind of book like structure with pages intersecting at the points with coordinates in extension.

Galois groups would define a hierarchy of theories and the natural first guess is that Galois groups take the role of subgroups of $SU(2)$ in CFTs with $SU(2)$ KMA as symmetry. Could the MacKay graphs defining the fusion algebra of Galois group define the fusion algebra of corresponding minimal number theoretic QFTs in analogy with minimal conformal models? This would fix the primary fields of theories assignable to given level of adele hierarchy to be minimal representations of Gal perhaps having also interpretation as representations of KMAs or their generalization to TGD framework.

(b) The analogies between TGD and the theory of Du Val singularities is intriguing. Complex surfaces in Du Val theory have real dimension 4 and are surfaces in space of real dimension 6. The intersections of the branches of the 4-surfaces have real dimension $D = 2$ in the generic case. In TGD space-time surfaces have real dimension 4 and possess complex structure or its Minkowskian generalization that I have called Hamilton-Jacobi structure.

The twistor bundle of space-time surface has 2-sphere CP_1 as a fiber and space-time surface as base [L24, L38]. Space-time surfaces can be realized as sections in their own 6-D twistor bundle obtained by inducing twistor structure from the product $T(M^4) \times T(CP_2)$ of twistor bundles of M^4 and CP_2. Section is fixed only modulo gauge choice, which could correspond to the choice of the Kähler form defining twistor structure from quaternionic units represented as points of S^2. Even if this choice is made, $U(1)$ gauge transformations remain and could correspond to gauge transformations of WCW changing its Kähler gauge potential by gradient and adding to Kähler function a real part of holomorphic function of WCW coordinates.

If the imbedding of 4-D space-time surface as section can become singular in given gauge, it will have self-intersections with dimension 2 possibly assignable to partonic 2-surfaces and maybe also string world sheets playing a key role in strong form of holography (SH). Could SH mean that information about classical and quantum theory is coded by singularities of the imbedding of space-time surface to twistor bundle. This would be highly analogous to what happens in the case of complex functions and also in twistor Grassmann theory whether the amplitudes are determined by the data at singularities.

(c) Where would the intersections take place? Space-time regions with Minkowskian and Euclidian signature of metric have light-like orbits of partonic 2-surfaces as intersections. These surfaces are singular in the sense that the metric determinant vanishes and tangent space of space-time surface becomes effectively 3-D: this would correspond to the reduction of tangent space dimension of algebraic surface at singularity. It is attractive to think that the lifts of Minkowskian and Euclidian space-time sheets have twistor spheres, which only intersect and have intersection matrix represented by McKay graph of Gal.

What about string world sheets? Does it make sense to regard them as intersections of 4-D surfaces? This does not look plausible idea but there are also other characterizations of string world sheets. One can also ask about the interpretation of the boundaries of string world sheets, in particular the points at the partonic 2-surfaces. How could they relate to singularities? The points of cognitive representation at partonic 2-surfaces carrying fermion number should belong to cognitive representation with imbedding space coordinates belonging to an extension of rationals.

(d) In Du Val theory the resolution of singularity means that one adds additional points to a double singularity: the added points form projective sphere CP_1. The blowing up process is like lifting self-intersecting curve to a non-singular curve by imbedding it into 3-D space so that the original curve is its projection. Could singularity disappear as one looks at 6-D objects instead of 4-D object? Could the blowing up correspond in TGD to a transition to a new gauge in which the self intersection disappears or is shifted on new place? The intersections of 4-surfaces in 6-space analogous to roots of polynomial are topologically stable suggesting that they can be only shifted by a new choice of gauge.

Self-intersection be a genuine singularity if the spheres CP_1 defining the fibers of the twistor bundles of branches of the space-time surface do not co-incide in the 2-D intersection. In the generic case they would only intersect in the intersection. Could the McKay diagram of Galois group characterize the intersection matrix?

(e) The big vision could be following. Galois groups characterize the singularities at given level of the adelic hierarchy and code for the multiplets of primary fields and for the analogs of their fusion rules for TGD counterparts of minimal CFTs. Note that singularities themselves identified as partonic 2-surfaces and possibly also light partonic orbits and possibly even string world sheets are not restricted in any manner.

This idea need not be so far-fetched as it might look at first.

(a) One considers twistor lift and self-intersections indeed occur also in twistor theory. When the M^4 projections of two spheres of twistor space CP_3 (to which the geometric twistor space $T(M^4) = M^4 \times S^2$ has a projection) have light-like separation, they intersect. In twistor diagrams the intersection corresponds to an emission of massless particle.

(b) The physical expectation is that this kind of intersections could occur also for the twistor bundle associated with the space-time surface. Most naturally, they could occur along the light-like boundary of causal diamond (CD) for points with light-like separation. They could also occur along the partonic orbits which are light-like 3-surfaces defining the boundaries between Minkowskian and Euclidian space-time regions. The twistor spheres at the ends of light-like curve could intersect.

Why the number of intersecting twistor spheres should reduce to the number $n(irred, Gal)$ of irreducible representations (irreps) of Gal, which equals to $n(Gal)$ in Abelian case but is otherwise smaller? This question could be seen as a serious objection.

(a) Does it make sense to think that although there are $n(Gal)$ in the local fiber of twistor bundle, the part of Galois fiber associated with the twistor fiber CP_1 has only $n(irrep, Gal)$ CP_1:s and even that the spheres could correspond to irreps of Gal. I cannot invent any obvious objection against this. What would happen that Could this mean realization of quantum classical correspondence at space-time level.

(b) There are $n(irrep, G)$ irreps and $\sum_i n_i^2 = n(G)$. n_i^2 points at corresponding sheet labelled by irrep. The number of twistor spheres collapsing to single one would be n_i for n_i-D irrep so that instead of states of representations the twistor spheres would correspond to irrep. One would have analogy with the fractionization of quantum numbers. The points assignable to n_i-D representations would become effectively $1/n_i$-fractionized. At the level of base space this would not happen.

Phase transitions reducing h_{eff}/h

In TGD framework one can imagine also other kinds of singularities. The reduction of Gal to its subgroup Gal/H, where H is normal subgroup defining Galois group for the Gal as extension of Gal/H is one such singularity meaning that the H orbits of space-time sheets become trivial.

(a) The action of Gal could reduce locally to a normal subgroup H so that Gal would be replaced with Gal/H. In TGD framework this would correspond to a phase transition reducing the value of Planck constant $h_{eff}/h = n(Gal)$ labelling dark matter phases to $h_{eff}/h = n(Gal/H) = n(Gal)/n(H)$. The reduction to Gal/H would occur automatically for the points of cognitive representation belonging to a lower dimensional extension having Gal/H as Galois group. The singularity would occur for the cognitive points of both space-time surface and twistor sphere and would be analogous to $n(H)$-point singularity.

(b) A singularity of the discrete bundle defined by Galois group would be in question and is assumed to induce similar singularity of $n(Gal)$ -sheeted space-time surface and its twistor lift. Although the singularity would occur for the ends of strings it would induce reduction of the extension of rationals to Gal/H, which should also mean that string world sheets have representation with WCW coordinates in smaller extension of rationals.

(c) This would be visible as a reduction in the spectrum of primary fields of number theoretic variant of minimal model. I have considered the possibility that the points at partonic 2-surfaces carrying fermions located at the ends of string world sheets could correspond to singularities of this kind. Could string world sheets could correspond to this kind of bundle singularities? This singularity would not have anything to do with the above described self-interactions of the twistor spheres associated with the Minkowskian and Euclidian regions meeting at light-like orbits of partonic 2-surfaces.

(d) This provides a systematic procedure for constructing amplitudes for the phase transitions reducing $h_{eff}/h = n(Gal)$ to $h_{eff}/h = n(Gal/H)$. The representations of Gal would be simply decomposed to the representations of $Gal(G/H)$ in the vertex describing the phase transition. In the simplest 2-particle vertex the representation of Gal remains irreducible as representation of Gal/H. Transition amplitudes are given by overlap integrals of representation functions of group algebra representations of Gal restricted to Gal/H with those of Gal/H.

The description of transitions in which particles with different Galois groups arrive in same diagram would look like follows. The Galois groups must form an increasing sequence $... \subset Gal_i = Gal_{i+1}/H_{i+1} \subset ....$ The representations of the largest Galois group would be decomposed to the representations of smallest Galois group so that the scattering amplitudes could be constructed using the fusion algebra of the smallest Galois group. The decomposition to should be associative and commutative and could be carried in many manners giving the same outcome at the final step.

Also quaternionic and octonionic automorphisms might be important

What about the role of subgroups of $SU(2)$? What roles they could have? Could also they classify singularities in TGD framework?

(a) SU(2) is indeed realize as multiplication of quaternions. M^8-H correspondence suggests that space-time surfaces in M^8 can be regarded as associative or co-associative (normal space-is associative. Associative translates to quaternionic. Associativity makes sense also at the level of H although it is not necessary. This would mean that the tangent space of space-time surface has quaternionic structure and the multiplication by quaternions is makes sense.

(b) The Galois group of quaternions is $SO(3)$ and has discrete subgroups having discrete subgroups of $SU(2)$ as covering groups. Quaternions have action on the spinors from which twistors are formed as pairs of spinors. Could quaternionic automorphisms be lifted to a an $SU(2)$ action on these spinors by quaternion multiplication? Could one imagine that the representations formed as tensor powers of these representations give finite irreps of discrete subgroups of $SU(2)$ defining ground states of $SU(2)$ KMA a representations and define the primary fields of minimal models in this manner?

(c) Galois groups for extensions of rationals have automorphic action on $SO(3)$ and its algebraic subgroups replacing matrix elements with their automorphs: for subgroups represented by rational matrices the action is trivial. One would have analogs of representations of Lorentz group $SL(2,C)$ induced from spin representations of finite subgroups $G \subset SU(2)$ by Lorentz transformations realizing the representation in Lobatchevski space. Lorentz group would be replaced by Gal and the Lobatchevski spaces as orbit with the representation of Gal in its group algebra. An interesting question is whether the hierarchy of discrete subgroups of $SU(2)$ in McKay correspondence relates to quaternionicity.

G_2 acts as octonionic automorphisms and SU(3) appears as its subgroup leaving on octonionic imaginary unit invariant. Could these semi-direct products of *Gal* with these automorphism groups have some role in adelic physics?

About TGD variant of ADE classification for minimal models

I already considered the ADE classification of minimal models. The first question is whether the finite subgroups $G \subset SU(2)$ are replaced in TGD context with Galois groups or with their semi-direct products $G \triangleleft Gal$. Second question concerns the interpretation of the Dynkin diagram of affine ADE type Lie algebra. Does it correspond to a real dynamical symmetries.

(a) Could the MacKay correspondence and ADE classification generalize? Could fusion algebras of minimal models for KMA associated with general compact Lie group G be classified by the fusion algebras of the finite subgroups of G. This generalization seems to be discussed in [B51] (see `http://tinyurl.com/ycmyjukn`).

(b) Could the fusion algebra of Galois group*Gal* give rise to a generalization of the minimal model associated with a KMA of Lie group $G \supset Gal$. The fusion algebra of *Gal* would be identical with that for the primary fields of KMA for G. Galois groups could be also grouped to classes consisting of Galois groups appearing as a subgroup of a given Lie group G.

(c) In TGD one has a fractal hierarchy of isomorphic supersymplectic algebras (SSAs) (the conformal weights of sub-algebra are integer multiples of those of algebra) with gauge conditions stating that given sub-algebra of SSA and its commutator with the entire algebra annihilates the physical states. The remnant of the full SSA symmetry algebra would be naturally KMA.

    The pair formed by full SSA and sub-SSA would correspond to pair formed by group G and normal subgroup H and the dynamical KMA would correspond to the factor group G/H. This conjecture generalizes: one can replace G with Galois group and $SU(2)$ KMA with a KMA continuing *Gal* as subgroup. One the other hand, one has also hierarchies of extensions of rationals such that $i+1$:th extension of rationals is extension of i:th extension. G_i is a normal subgroup of G_{i+1} so that the group $Gal_{i+1,i} = Gal_{i+1}/Gal_i$ acts as the relative Galois group for $i+1$:th extension as extensions of i:th extension.

    This suggest the conjecture that the Galois groups Gal_i for extension hierarchies correspond to the inclusion hierarchies $SSA_i \supset SSA_{i+1}$ of fractal sub-algebras of SSA such that the gauge conditions for SSA_i define a hierarchy KMA_i of dynamical KMAs acting as dynamical symmetries of the theory. The fusion algebra of KMA_i theory would be characterized by Galois group Gal_i.

(d) I have considered the possibility that the McKay graphs for finite subgroups $G \subset SU(2)$ indeed code for root diagrams of ADE type KMAs acting as dynamical symmetries to be distinguished from $SU(2)$ KMA symmetry and from fundamental KMA symmetries assignable to the isometries and holonomies of $M^4 \times CP_2$.

    One can of course ask whether also the fundamental symmetries could have a representation in terms of *Gal* or its semi-direct product $G \triangleleft Gal$ with a finite sub-group automorphism group $SO(3)$ of quaternions lifting to finite subgroup $G \subset SU(2)$ acting on spinors. This is not necessary since *Gal* can form semidirect products with the algebraic subgroups of Lie groups of fundamental symmetries (Langlands program relies on this). In the generic case the algebraic subgroups spanned by given extension of rationals are infinite. When the finite subgroup $G \subset SU(2)$ is closed under *Gal* automorphism, the situation changes, and these extensions are expected to be in a special role physically.

    The number theoretic generalization of the idea that affine ADE group acts as symmetries would be roughly like following. The nodes of the McKay graph of $G \triangleleft Gal$ label its irreps, which should be in 1-1 correspondence with the Cartan algebra of the KMA. The KMA counterparts of the local bilinear *Gal* invariants associated with *Gal* irreps would give currents of dynamical KMA having unit conformal weight. The convolution of primary fields with respect to conformal weight would be completely analogous to that occurring in the expression of energy momentum tensor as local bilinears of KMA currents.

If the free field construction using the local invariants as Cartan algebra defined by the irreps of $G \triangleleft Gal$ works, it gives rise to charged primary fields for the dynamical KMA labelled by roots of the corresponding Lie algebra. For trivial Gal one would have ADE group acting as dynamical symmetries of minimal model associated with $G \subset SU(2)$.

(e) Number theoretic Langlands conjecture [L30] [K117] generalizes this to the semidirect product $G_0 \triangleleft Gal$ algebraic subgroup G_0 of the original KMA Lie group (p-adicization allows also powers of roots of e in extension). One can imagine a hierarchy of KMA type algebras KMA_n obtained by repeating the procedure for the $G_1 \triangleleft Gal$, where G_1 is discrete subgroup of the new KMA Lie group.

(f) In CFTs are also other manners to extend VA or SVA (Super-Virasoro algebra) to a larger algebra by discovering new dynamical symmetries. The hope is that symmetries would allow to solve the CFT in question. The general constraint is that the conformal weights of symmetry generators are integer or half-integer valued. For the energy momentum tensor defining VA the conformal weight is $h = 2$ whereas the conformal weights of primary fields for minimal models are rational numbers.

The simplest extension is SVA involving super generators with $h = 3/2$. Extension of (S)VA by (S)KMA so that (S)VA acts by semidirect product on (S)KMA means adding (S)KMA generators with with $h = 1$ (and 1/2). The generators of W_n-algebras (see `http://tinyurl.com/y93f6eoo`) have either integer or half integer conformal weights and the algebraic operations are defined as ordered products (an associative operation). These extensions are different from the proposed number theoretic extension for which the restriction to a discrete subgroup of KMA Lie group is essential.

19.5 Appendix

I have left the TGD counterpart of *fake flatness condition* in Appendix. Also a little TGD glossary is included.

19.5.1 What could be the counterpart of the fake flatness in TGD framework?

Schreiber considers the n-variant of gauge field concept with gauge potential A and gauge field $F = DA$ replaced with a hierarchy of gauge potential like entities $A^{k)}$, $k = 1,..,n$ with $DA^{n)} = 0$ and ends up in $n = 2$ case to what he calls *fake flatness condition* $DA^{1)} = A^{2)}$. This raises a chain of questions.

Could higher gauge fields of Schreiber and Baez [B73, B55] provide a proper description of the situation in finite measurement resolution? Could induction procedure make sense for them? Should one define the projections of the classical fields by replacing ordinary H-coordinates with their quantum counterparts? Could these reduce to c-numbers for number-theoretically commutative 2-surfaces with commutative tangent space? What about second fundamental form orthogonal to the string world sheet? Must its trace vanish so that one would have minimal 2-surface?

The proposal of Schreiber is a generalization of a massless gauge theory. My gut feeling is that the non-commutative counterpart of space-time surface is not promising in TGD framework. My feelings are however mixed.

(a) The effective reduction of SSA and PSCA to quantal variants of Kac-Moody algebras gives rise to a theory much more complex than gauge theory. On the other hand, the reduction to Galois groups by finiteness of measurement resolution would paradoxically reduce TGD to extremely simple theory.

(b) Analog of Yang Mills theory is not enough since it describes massless particles. Massless states in 4-D sense are only a very small portion of the spectrum of states in TGD. Stringy mass squared spectrum characterizes these theories rather than massless spectrum. On the other hand, in TGD particles are massless in 8-D sense and this is crucial for the success of generalized twistor approach.

(c) To define a generalization of gauge theory in WCW one needs homology and cohomology for differential forms and their duals. For infinite-dimensional WCW the notion of dual is difficult to define. The effective reduction of SSA and PSCA to SKMAs could however effectively replace WCW with a coset space of the Lie-group associated with SKMA and finite dimension would allow tod define dual.

(d) The idea about non-Abelian counterparts of Kähler gauge potential A and J in WCW does not look promising and the TGD counterpart of the *fake flatness condition* does not however encourage this.

Just for curiosity one could however ask whether one could generalize the Kähler structure of WCW to n-Kähler structure to describe finite measurement resolution at the level of WCW and whether also now something analogous to the *fake flatness condition* emerges. The "fake flatness" condition has interesting analogy in TGD framework starting from totally different geometric vision.

(a) SSA acts as dynamical symmetries on fermions at string world sheets. Gauge condition would make the situation effectively finite-dimensional and allow to define if the effectively finite-D variant of WCW n-structures using ordinary homotopies and homology and cohomology. Also n-gauge fields could be defined in this effectively finite-D WCW and they would allow a description in terms of string world sheets. The interpretation could be in terms of generalization of Bohm-Aharonov phase from space-time level to Berry phase in abstract configuration space defined now in reduced WCW.

(b) The Kähler form of $H = M^4 \times CP_2$ (involving also the analog of Kähler form for M^4) can be induced to space-time level. When limited to the string world sheet is both the curvature form of Kähler potential and the analog of flat 2-connection defining the 1-connection in the approaches of Schreiber's and Baez so that one would have $B = J$ and $dB = 0$.

(c) 2-form J as it is interpreted in Screiber's approach is hwoever not enough to construct WCW geometry. The generalization of the geometry of $M^4 \times CP_2$ (involving also the analog of Kähler form for M^4) to involve higher forms and its induction to the space-time level and level of WCW looks rather awkward idea and does not bring in anything new.

19.5.2 A little glossary

Topological Geometrodynamics (TGD): TGD can be regarded as a unified theory of fundamental interactions. In General Relativity space-time time is abstract pseudo-Riemannian manifold and the dynamical metric of the space-time describes gravitational interactions. In TGD space-time is a 4-dimensional surface of certain 8-dimensional space, which is non-dynamical and fixed by either physical or purely mathematical requirements. Hence space-time has shape besides metric properties. This identification solves the conceptual difficulties related to the definition of the energy-momentum in General Relativity. Even more, sub-manifold geometry, being considerably richer in structure than the abstract manifold geometry behind General Relativity, leads to a geometrization of known fundamental interactions and elementary particle quantum numbers.

Many-sheeted space-time, topological quantization, quantum classical correspondence (QCC): TGD forces the notion of many-sheeted space-time (see `http://tinyurl.com/mf99gpv`) with space-time sheets identified as geometric correlates of various physical objects (elementary particles, atoms, molecules, cells, ..., galaxies, ...). Quantum-classical correspondence (QCC) states that all quantum notions have topological correlates at the level of many-sheeted space-time.

Topological quantization: Topological field quantization is one of the basic distinctions between TGD and Maxwell's electrodynamics and GRT and means that various fields decompose to topological field quanta: radiation fields to "topological light rays"; magnetic fields to flux tube structures; and electric fields to electric flux quanta (electrets). Topological field quantization means that one can assign to every material system a field (magnetic) body, usually much larger than the material system itself, and providing a representation for various quantum aspects of the system.

Strong form of holography (SH): SH states that space-time surfaces as preferred extremals can be constructed from the data given at 2-D string world sheets and by a discrete set of points defining the cognitive representation of the space-time surface as points common to real and various p-adic variants of the space-time surface (intersection of realities and various p-adicities). Points of the cognitive representation have imbedding space coordinates in the extension of rationals defining the adele in question. Effective 2-dimensionality is a direct analogy for the continuation of 2-D data to analytic function of two complex variables.

Zero energy ontology (ZEO): In ZEO quantum states are replaced by pairs of positive and negative energy states having opposite total quantum numbers. The pair corresponds to the pair of initial and final state for a physical event in classical sense. The members of the pair are at opposite boundaries of causal diamond (CD) (see `http://tinyurl.com/mh9pbay`), which is intersection of future and past directed light-cones with each point replaced with CP_2. Given CD can be regarded as a correlate for the perceptive field of conscious entity.

p-Adic physics, adelic physics, hierarchy of Planck constants, p-adic length scale hypothesis: p-Adic physics is a generalization of real number based physics to p-adic number fields and interpreted as a correlate for cognitive representations and imagination. Adelic physics fuses real physics with various p-adic physics ($p = 2, 3, 5, ...$) to adelic physics. Adele is structure formed by reals and extensions of various p-adic number fields induced by extensions of rationals forming an evolutionary hierarchy. Hierarchy of Planck constants corresponds to the hierarchy of orders of Galois groups for these extensions. Preferred p-adic primes satisfying p-adic length scale hypothesis $p \simeq 2^k$, are so called ramified primes for certain extension of rationals appearing as winners in algebraic evolution.

Cognitive representation: Cognitive representation corresponds to the intersection of the sensory and cognitive worlds - realities and p-adicities - defined by real and p-adic space-time surfaces. The points of the cognitive representation have H-coordinates which belong to an extension of rationals defining the adele. The choice of H-coordinates is in principle free but symmetries of H define preferred coordinates especially suitable for cognitive representations. The Galois group of the extension of rationals has natural action in the cognitive representation, and one can decompose it into orbits, whose points correspond the sheets of space-time surface as Galois covering. The number n of sheets equals to the dimension of the Galois group in the general case and is identified as the value $h_{eff}/h = n$ of effective Planck constant characterizing levels in the dark matter hierarchy. One can also consider replacing space-time surfaces as points of WCW with their cognitive representations defined by the cognitive representation of the space-time surface and defining the natural coordinates of WCW point.

Quantum entanglement, negentropic entanglement (NE), Negentropy maximization principle (NMP): Quantum entanglement does not allow any concretization in terms of everyday concepts. Schrödinger cat is the classical popularization of the notion (see `http://tinyurl.com/lpjcjm9`): the quantum state, which is a superposition of the living cat + the open bottle of poison and the dead cat + the closed bottle of poison represents quantum entangled state. Schrödinger cat has clearly no self identity in this state.

In adelic physics one can assign to the same entanglement both real entropy and various p-adic negentropies identified as measures of conscious information. p-Adic negentropy - unlike real - can be positive, and one can speak of negentropic entanglement (NE). Negentropy Maximization Principle (NMP) states that it tends to increase. In the adelic formulation NMP holding true only in statistical sense is a consequence rather than separate postulate.

Self, subself, self hierarchy: In ZEO self is generalized Zeno effect. At the passive boundary nothing happens to the members of state pairs and the boundary remains unaffected. At active boundary members of state pairs change and boundary itself moves farther away from the passive boundary reduction by reduction inducing localization of the active boundary in the moduli space of CDs after unitary evolution inducing delocalization in it. Self dies as the first reduction takes place at opposite boundary. A self hierarchy extending from elementary particle level to the level of the entire Universe is predicted. Selves can have sub-selves which they experience as mental images. Sub-selves of two separate selves can quantum entangle and this gives rise to fusion of the mental images and the fused mental image is shared by both selves.

Sensory representations: The separation of data processing and its representation is highly desirable. In computers processing of the data is performed inside CPU and representation is realized outside it (monitor screen, printer,...). In standard neuroscience it is however believed that both data processing and representations are realized inside brain. TGD leads the separation of data processing and representations: the "manual" of the material body provided by field (or magnetic) body serves as the counterpart of the computer screen at which the sensory and other representations of the data processed in brain are realized. Various attributes of the objects of the perceptive field processed by brain are quantum entangled with simple "something exists" mental images at the MB. The topological rays of EEG serve are the electromagnetic bridges serving as the topological correlates for this entanglement.

Chapter 20

Is Non-associative Physics and Language Possible only in Many-Sheeted Space-time?

20.1 Introduction

In Thinking Allowed Original (see `https://www.facebook.com/groups/thinkallowed/`) there was very interesting link added by Ulla about the possibility of non-associative quantum mechanics (see `http://phys.org/news/2015-12-physicists-unusual-quantum-mechanics.html#jCp`).

Also I have been forced to consider this possibility.

(a) The 8-D imbedding space of TGD has octonionic tangent space structure and octonions are non-associative. Octonionic quantum theory however has serious mathematical difficulties since the operators of Hilbert space are by definition associative. The representation of say octonionic multiplication table by matrices is possible but is not faithful since it misses the associativity. More concretely, so called associators associated with triplets of representation matrices vanish. One should somehow transcend the standard quantum theory if one wants non-associative physics.

(b) Associativity seems to be fundamental in quantum theory as we understand it recently. Associativity is a fundamental and highly non-trivial constraint on the correlation functions of conformal field theories. It could be however broken in weak sense: as a matter of fact, Drinfeld's associator emerges in conformal field theory context. In TGD framework classical physics is an exact part of quantum theory so that quantum classical correspondence suggests that associativity could play a highly non-trivial role in classical TGD. The conjecture is that associativity requirement fixes the dynamics of space-time sheets - preferred extremals of Kähler action - more or less uniquely. One can endow the tangent space of 8-D imbedding $H = M^4 \times CP_2$ space at given point with octonionic structure: the 8 tangent vectors of the tangent space basis obey octonionic multiplication table.

Space-time realized as n-D surface in 8-D H must be either associative or co-associative: this depending on whether the tangent space basis or normal space basis is associative. The maximal dimension of space-time surface is predicted to be the observed dimension $D = 4$ and tangent space or normal space allows a quaternionic basis.

(c) There are also other conjectures [K76] about what the preferred extremals of Kähler action defining space-time surfaces are.

i. A very general conjecture states that strong form of holography allows to determine space-time surfaces from the knowledge of partonic 2-surfaces and 2-D string world sheets.

ii. Second conjecture involves quaternion analyticity and generalization of complex structure to quaternionic structure involving generalization of Cauchy-Riemann conditions.

iii. $M^8 - M^4 \times CP_2$ duality stating that space-time surfaces can be regarded as surfaces in either M^8 or $M^4 \times CP_2$ is a further conjecture.

iv. Twistorial considerations select $M^4 \times CP_2$ as a completely unique choice since M^4 and CP_2 are the only spaces allowing twistor space with Kähler structure. The conjecture is that preferred extremals can be identified as base spaces of 6-D sub-manifolds of the product $CP_3 \times SU(3)/U(1) \times U(1)$ of twistor spaces associated with M^4 and CP_2 having the property that it makes sense to speak about induced twistor structure.

The "super(optimistic)" conjecture is that all these conjectures are equivalent.

The motivation for what follows emerged from the observation that language is an essentially non-associative structure as the necessity to parse linguistic expressions essential also for computation using the hierarchy of brackets makes obvious. Hilbert space operators are however associative so that non-associative quantum physics does not seem plausible without an extension of what one means with physics. Associativity of the classical physics at the level of *single* space-time sheet in the sense that tangent or normal spaces of space-time sheets are associative as sub-spaces of the octonionic tangent space of 8-D imbedding space $M^4 \times CP_2$ is one of the key conjectures of TGD.

But what about many-sheeted space-time? The sheets of the many-sheeted space-time form hierarchies labelled by p-adic primes and values of Planck constants $h_{eff} = n \times h$. Could these hierarchies provide space-time correlates for the parsing hierarchies of language and music, which in TGD framework can be seen as kind of dual for the spoken language? For instance, could the braided flux tubes inside larger braided flux tubes inside... realize the parsing hierarchies of language, in particular topological quantum computer programs? And could the great differences between organisms at very different levels of evolution but having very similar genomes be understood in terms of widely different numbers of levels in the parsing hierarchy of braided flux tubes- that is in terms of magnetic bodies as indeed proposed. If the intronic portions of DNA connected by magnetic flux tubes to the lipids of lipid layers of nuclear and cellular membranes make them topological quantum computers, the parsing hierarchy could be realized at the level of braided magnetic bodies of DNA.

Fortunately the mathematics needed to describe the breaking of associativity at fundamental level seems to exist. The hierarchy of braid group algebras forming an operad combined with the notions of quasi-bialgebra and quasi-Hopf algebra discovered by Drinfeld are highly suggestive concerning the realization of weak breaking of associativity. With good luck this breaking of associativity is all that is needed. With not so good luck this breaking of associativity takes place already at the level of single space-time sheets and something else is needed in many-sheeted space-time.

20.2 Is Non-associative Physics Possible In Many-sheeted Space-time?

The key question in the sequel is whether non-associative physics could emerge in TGD via *many-sheeted* space-time as an outcome of many-sheetedness and therefore distinguishing TGD from GRT and various QFTs.

20.2.1 What Does Non-associativity Mean?

To answer this question one must first understand what non-associativity could mean.

(a) In non-associative situation brackets matter. $A(BC)$ is different from $(AB)C$. Here AB need not be restricted to a product or sum: it can be anything depending on A and B.

From schooldays or at least from the first year calculus course one recalls the algorithm: when calculating the expression involving brackets one first finds the innermost brackets and calculates what is inside them, then proceed to the next innermost brackets, etc... In computer programs the realization of the command sequences involving brackets is called parsing and compilers perform it. Parsing involves decomposition of program to modules calling modules calling.... Quite generally, the analysis of linguistic expressions involves parsing. Bells start to ring as one realizes that parsings form a hierarchy as also do the space-time sheets!

(b) More concretely, there is hierarchy of brackets and there is also a hierarchy of space-time sheets labelled by p-adic primes and perhaps also by Planck constants $h_{eff} = n \times h$. B and C inside brackets form (BC), something analogous to a bound state or chemical compound. In TGD this something could correspond to a "glueing" space-time sheets B and C at the same larger space-time sheet. More concretely, (BC) could correspond to braided pair of flux tubes B and C inside larger flux tube, whose presence is expressed as brackets (..). As one forms $A(BC)$ one puts flux tube A and flux tube (BC) containing braided flux tubes B and C inside larger flux tube. For $(AB)C$ flux one puts tube (AB) containing braided flux tubes A and B and tube C inside larger flux tube. The outcomes are obviously different.

(c) Non-associativity in this sense would be a key signature of many-sheeted space-time. It could show itself in say molecular chemistry, where putting on same sheet could mean formation of chemical compound AB from A and B. Another highly interesting possibility is hierarchy of braids formed from flux tubes: braids can form braids, which in turn can form braids,... Flux tubes inside flux tubes inside... Maybe this more refined breaking of associativity could underly the possible non-associativity of biochemistry: biomolecules looking exactly the same would differ in subtle manner.

(d) What about quantum theory level? Non-associativity at the level of quantum theory could correspond to the breaking of associativity for the correlation functions of n fields if the fields are not associated with the same space-time sheet but to space-time sheets labelled by different p-adic primes. At QFT limit of TGD giving standard model and GRT the sheets are lumped together to single piece of Minkowski space and all physical effects making possible non-associativity in the proposed sense are lost. Language would be thus possible only in TGD Universe!

20.2.2 Language And Many-sheeted Physics?

Non-associativity is an essentially linguistic phenomenon and relates therefore to cognition. p-Adic physics labelled by p-adic primes fusing with real physics to form adelic physics are identified as the physics of cognition in TGD framework.

(a) Could many-sheeted space-time of TGD provides the geometric realization of language like structures? Could sentences and more complex structures have many-sheeted space-time structures as geometrical correlates? p-Adic physics as physics of cognition would suggest that p-adic primes label the sheets in the parsing hierarchy. Could bio-chemistry with the hierarchy of magnetic flux tubes added, realize the parsing hierarchies?

(b) DNA is a language and might provide a key example about parsing hierarchy. The mystery is that human DNA and DNAs of most simplest creatures do not differ much. Our cousins have almost identical DNA with us. Why do we differ so much? Could the number of parsing levels be the reason- p-adic primes labelling space-time sheets? Could our DNA language be much more structured than that of our cousins. At the level of concrete language the linguistic expressions of our cousin are indeed simple signals rather than extremely complex sentences of old-fashioned German professor forming a single lecture each. Could these parsing hierarchies realize themselves as braiding hierarchies of magnetic flux tubes physically and - more abstractly - as analos of parsing hierarchies for social structures. Indeed, I have proposed that the presence of collective levels of consciousness having the hierarchy of magnetic bodies as a space-time correlates distinguishes us from our cousins so that this explanation is consistent with more quantitative one relying on language.

(c) I have also proposed that intronic portion of DNA is crucial for understanding why we differ so much from our cousins [K20, K80]. How does this view relate to the above proposal? In the simplest model for DNA as topological quantum computer introns would be connected by flux tubes to the lipids of nuclear and cell membranes. This would make possible topological quantum computations with the braiding of flux tubes defining the topological quantum computer program.

Ordinary computer programs rely on computer language. Same should be true about quantum computer programs realized as braidings. Now the hierarchical structure of parsings would correspond to that of braidings: one would have braids, braids of braids, etc... This kind of structure is also directly visible as the multiply coiled structure of DNA. The braids beginning from the intronic portion of DNA would form braided flux tubes inside larger braided flux tubes inside.... defining the parsing of the topological quantum computer program. The higher the number of parsing levels, the higher the position in the evolutionary hierarchy. Each braiding would define one particular fundamental program module and taking this kind of braided flux tubes and braiding them would give a program calling these programs as sub-programs.

(d) The phonemes of language have no meaning to us (at our level of self hierarchy) but the words formed by phonemes and involving at basic level the braiding of "phoneme flux tubes" would have. Sentences and their substructures would in turn involve braiding of "word flux tubes". Spoken language would correspond to temporal sequences of braidings of flux tubes at various hierarchy levels.

(e) The difference between us and our cousins (or other organisms) would not be at the level of visible DNA but at the level of magnetic body. Magnetic bodies would serve as correlates also for social structures and associated collective levels of consciousness. The degree of braiding would define the level in the evolutionary hierarchy. This is of course the basic vision of TGD inspired quantum biology and quantum bio-chemistry in which the double formed by organism and environment is completed to a triple by adding the magnetic body.

20.2.3 What About The Hierarchy Of Planck Constants?

p-Adic hierarchy is not the only hierarchy in TGD Universe: there is also the hierarchy of Planck constants $h_{eff} = n \times h$ giving rise to a hierarchy of intelligences. What is the relationship between these hierarchies?

(a) I have proposed that speech and music are fundamental aspects of conscious intelligence and that DNA realizes what I call bio-harmonies in quite concrete sense [L13] [K59]: DNA codons would correspond to 3-chords. DNA would both talk and sing. Both language and music are highly structured. Could the relation of h_{eff} hierarchy to language be same as the relation of music to speech?

(b) Are both musical and linguistic parsing hierarchies present? Are they somehow dual? What does parsing mean for music? How musical heard sounds could give rise to the analog of braided strands? Depending on the situation we hear music both as separate notes and as chords as separate notes fuse in our mind to a larger unit like phonemes fuse to a word. Could chords played by single instrument correspond to braidings of flux tubes at the same level? Could the duality between linguistic and musical intelligence (analogous to that between function and its Fourier transform) be very concrete and detailed and reflect itself also as the possibility to interpret DNA codons both as three letter words and as 3-chords [L13]?

20.3 Braiding Hierarchy Mathematically

More precise formulation of the braided flux tube hierarchy leads naturally to the notions of braid group and operad that I have considered earlier. They have a close relationship with quantum groups - more precisely, bialgebras and Hopf algebras and their generalizations quasi-bialgebras and quasi-Hopf algebras, which in turn allow to characterize what might be called minimal breaking of associativity in terms of Drinfeld associator. These notions are already familiar from conformal field theories and string theories them so that there are good hopes that no completely new mathematics is not needed.

It must be made clear that I am not a mathematician and the following is just a modest attempt to understand what the problem is. I try to identify the algebraic structure possibly allowing to realize the big vision and gather some results about these structures from Wikipedia: I confess that I do not understand the formulas at the deeper level and my goal is to find their physical interpretation in TGD framework.

20.3.1 How To Represent The Hierarchy Of Braids?

Before going to web to see how modern mathematics could help in the problem, try first to formulate the situation more concretely. One must consider a more detailed representation for braids and for their hierarchy.

Consider first rough physical geometric view about braids of braids represented in terms of flux tubes.

(a) Braid strands have two ends: one can label them as "lower" and "upper". Flux tubes can be labelled by p-adic prime p and $h_{eff} = n \times h$. Magnetic flux tubes can carry monopole flux and this could be crucial for the breaking of associativity - at least it is so in the proposed model (see `http://tinyurl.com/y7oom5kh`). The possibility of apparent magnetic monopoles in TGD framework indeed involves many-sheetedness in an essential manner: monopole flux flows from space-time sheet to another one through wormhole contact. This can be taken as one possible hint about the concrete physics involved.

(b) One can get more precise picture by using formulas. One has labelling of flux tubes by primes p and Planck constants h_{eff}: to be short call this label $a, b, c, ...$ Since the values of p and h_{eff} are graded one could also speak of grading. The states for given value of a assignable to braid strands are labelled by the quantum states $A, B, ...$ associated with them and analogous to algebra elements. One must however consider all possible situations so that has operators $A_a, B_a, ...$ analogous to algebra elements of a graded algebra about which Clifford algebras and super-algebras are familiar examples.

(c) Consider now the physical interpretation for the breaking of associativity. For ordinary associative algebra one considers $A(BC) = (AB)C$. This condition as such make sense if $A(BC)$ and $(AB)C$ are inside same flux tube and perhaps also that the strands A, B, C are not braids. In the general case one must must add the labels a, b, c, d and a, b_1, c_1, d_1 and one obtains $((A_dB_d)_cC_b)_a$ and $(A_{b_1}(B_{d_1}C_{d_1}))_{c_1})_a$. Obviously, these two states need not identical unless one has $a = b = c = d = b_1 = c_1 = d_1$, which is also possible and means that all strands are at the same flux tube labelled by a. The challenge is to combine various almost copies of algebraic structure defined by braidings and labelled by $a, b, ..$ to larger algebraic structure and formulate the breaking of associativity for this structure.

20.3.2 Braid Groups As Coverings Of Permutation Groups

Consider next the definition of braid group.

(a) The notion of braiding can be algebraized using the notion of braid group B_n of n strands, which is covering of the permutation group S_n. For ordinary permutations generating permutations are exchanges of P_i two neighboring elements in the ordered set $(a_1, ..., a_n)$: $(a_i, a_{i+1}) \rightarrow (a_{i+1}, a_i)$. Obviously one has P_i^2 so that permutation is analogous to reflection. For braid group permutation is replaced to twisting of neighboring braid strand. It looks like permutation if one looks at the ends of strands only. If one looks entire strands, there is no reason to have $P_i^2 = 1$ except possibly for the representation of braid group. For arbitrarily large n that one has $P_i^n \neq 1$. 2-D braid group B_n can be represented as a homotopies of 2-D plane with n punctures identifiable as ends of braid strands defined by their non-intersecting orbits.

(b) At the level of quantum description one must allow quantum superpositions of different braidings and must describe the quantum state of braid as wave function in braid group: one has element of group algebra of braid group. To each element of braid group one can assign unitary matrix representing the braiding and this unitary matrix would define a "topological time evolution" defined by braiding transforming the initial state at the lower end of braid to the state at upper end of braid. Hence it seems that braid group algebra is the proper mathematical notion. One has quantum superposition of topological time evolutions: something rather abstract.

20.3.3 Braid Having Braids As Strands

Many-sheeted space-time makes possible fractal hierarchy of braids. Braid group in above sense would act on flux tubes at the same space-time sheets or space-time of QFT and GRT. Braids can have as strands braids so that there is hierarchy of braiding levels. The hierarchy of coilings of DNA provides a simple example (very simple having not much to do with the hierarchy of braidings for flux tubes).

(a) Suppose that one has only two levels in the hierarchy. One has n braid strands/flux tubes altogether and there are k larger flux tubes containing n_i, $i = 1, .., k$ flux tubes so that one has $\sum_{i=1}^{k} n_i = n$. One can imagine a coloring of the braid strands inside given flux tube characterizing it. Only braid strands inside same flux tube - with the same color - can be braided. The full braid group B_n braiding freely all n braid strands is restricted to a subgbroup $B_{n_1} \times .... \times B_{n_2}$. This group can be regarded as subgroup of B_n so that permutations of B_{n_i} have a well-defined outcome, which seems however to be trivial classically. In quantum situation the exchange of the factors B_{n_i} however corresponds to braiding and for non-trivial quantum deformations its action is non-trivial. One has braided commutativity instead of commutativity.

(b) Besides this there are braidings for the k braids of braids and this gives braid group B_k acting at upper level of hierarchy. Clearly the higher level braids b_i, $i = 1, ..., k$ and lower level braids b_{ij}, $j = 1, ..., n_i$ form a two-levelled entity. The braid groups B_k and B_{n_i} form an algebraic entity such that B_k acts by permuting the entities. Same holds true for the braid group algebras. This structure generalizes to an entire hierarchy of braid groups and their group algebras.

The hierarchy of braid group algebras seems to closely relate to a very general notion known as operad (see `http://tinyurl.com/yavyhcsk`). The key motivation of the operad theory is to model the computational trees resulting from parsing. The action of permutations/braidings on the basic objects is central notion and one indeed has hierarchy of symmetric groups/braid groups such that the symmetric/braid group at $n + 1$:th level permutes/braids the objects at n:th level. Now the objects would be braids whose strands are braided. The braids can be strands of higher level braids and these strands can be braided. The action of braidings extends to that on braid group algebras defining candidates for wave functions.

20.4 General Formulation For The Breaking Of Associativity In The Case Of Operads

The formulas characterizing weak form of associativity by Drinfeld and others look rather mysterious without understanding of their origins. This understanding emerges from very simple but general basic arguments. Instead of studying given algebra one transcends to a higher abstraction level and studies - not the results of algebraic expressions - but the very process how the algebraic expression is evaluated and what kind of rules one can pose on it. The rules can be abstracted to what is called algebraic coherence.

The evaluation process - parsing - starts from inner most brackets and proceeds outwards so that eventually all brackets have disappeared and one has the value for the expression. This process can be regarded as a tree which starts from n inputs which are algebra elements, in the recent case they could be braid group algebra elements.

For instance, $(AB)C$ corresponds to an tree in which A, B, C are the branches. As one comes downwards, A and B fuse in the upper node and AB and C in the lower node. One manner to see this is as particle reaction proceeding backwards in time. For $A(BC)$ B and C fuse to BC in the upper node and A and BC at the lower node. Associativity says that the two trees give the same result. "Braided associativity" would say that these trees give results differing by an isomorphism just as braided commutativity says that AB and BA give results differing by isomorphism.

One can formulate this more concretely by denoting algebra decomposition $A \otimes B \in V \otimes V \rightarrow AB \in V$ by θ. In associativity condition one has 3 inputs so that 3-linear map $V \otimes V \otimes V \rightarrow V$ is in question. $(AB)C$ corresponds to $\theta \circ (\theta, 1)$ applied to $(A \otimes B \otimes C)$. Indeed, $(\theta, 1)$ gives $(AB, C) \in V \otimes V$. Second step $\theta \circ$ applied to this gives $(AB)C$. In the same manner, $A(BC)$ corresponds to $(\theta \circ (1, \theta)$ and associativity condition can be expressed as

$$\theta \circ (\theta, 1) = \theta \circ (1, \theta) \ .$$

An important delicacy should be mentioned. Although operations can be non-associative, the composition of operations is assumed to be associative. One can imagine obtaining $((ab)c)d$ either by $\theta \circ (\theta, 1) \circ (\theta, 1, 1))$ or by $(\theta \circ (\theta, 1)) \circ (\theta, 1, 1))$. The condition that these expressions are identical is completely analogous to the associativity for the composition of functions $f \circ (g \circ h) = (f \circ g) \circ h$ and this axiom looks obvious becomes one is used to *define* $f \circ g$ using this formula (starting from rightmost brackets). One could however imagine starting the evaluation of the composition of operators also from leftmost brackets. This makes sense if the composition can be done without the substitution of the value of argument.

20.4.1 How Associativity Could Be Broken?

How to obtain the breaking of associativity? The first thing is to get some idea about what (weak) breaking of associativity could mean.

Breaking of associativity at the level of algebras

Basic examples about breaking of associativity might help in the attempts to understand how many-sheetedness could induce the breaking of associativity. The intuitive feeling is that the effect is not large and disappears at QFT limit of TGD.

In the case of algebras one has bilinear map $V \otimes V \rightarrow V$. Now this map is from $V \otimes V \rightarrow V \otimes V$ so that the two situations need not have much common. Despite this one can look the situation in the case of algebras.

Lie-algebras and Jordan algebras represent key examples about non-associative algebras. Associative algebras, Lie-algebras, and Jordan algebras can be unified by weakning the associativity condition $A(BC) = (AB)C$ to a condition obtained by cyclically symmetrizing this condition to get the condition

$$A(BC) + B(CA) + C(AB) = (AB)C + (BC)A + (CA)B$$

plus the condition

$$(A^2B)A = A^2(BA)$$

defining together with commutativity condition $AB = BA$ Jordan algebra (`http://tinyurl.com/y8n9ol9p`). Note that Jordan algebra with multiplication $A \cdot B$ is realized in terms of associative algebra product as $A \cdot B = (AB + BA)/2$. A good guess is that the non-associative Malcev algebra formed by imaginary octonions with product $xy - yx$ satisfies these conditions.

Could the analog of the condition $A(BC) + B(CA) + C(AB) = (AB)C + (BC)A + (CA)B$ make sense also for the braiding group algebra assignable to quantum states of braids? The condition would say that cyclic symmetrization by superposing different braiding topologies gives a quantum state, which is in well-defined sense associative. Cyclic symmetry looks attractive because it plays also a key role in twistor Grassmannian approach.

Bi-algebras and Hopf algebras

One must start from bi-algebra $(B, \nabla, \eta, \Delta, \epsilon)$. One has product ∇ and co-product Δ analogous to replication of algebra element: particle physicists has tendency to see it as "time reversal" of product analogous to particle decay as reversal of particle fusion. The key idea is that co-multiplication is algebra homomorphism for multiplication and multiplication algebra homomorphism for co-multiplication. This leads to four commutative diagrams essentially expressing this property (see `http://tinyurl.com/y897z3es`).

Instead of giving the general definitions it is easier to consider concrete example of bi-algebra defined by group algebra. Bi-algebra has product $\nabla : H \otimes H \to H$ and co-product $\Delta : H \to H \otimes H$, which intuitively corresponds to inverse or time reversal of product. In the case of group algebra this holds true in very precise sense since one has $\Delta(g) = g \otimes g$: Δ is clearly analogous to replication. Besides this one has map $\epsilon : H \to K$ assigning to the algebra element a scalar and inverse map taking the unit 1 of the field to unit element of H, called also 1 in the following. For group algebra one has $\epsilon(g) = 1$. Bi-algebras are associative and co-associative. Commutativity is however only braided commutativity.

Hopf algebra $(H, \nabla, \eta, \Delta, \epsilon, S)$ is special case of bi-algebra and often loosely called quantum group. The additional building brick is algebra anti-homomorphism $S : H \to H$ known as antipode. S is analogous to mapping element of h to its inverse (it need not exist always). For group algebra one indeed has $S(g) = g^{-1}$. Besides the four commuting diagrams for bi-algebra one has commutative diagrams $\nabla(S, 1)\Delta = \eta\epsilon$ and $\nabla(1, S)\Delta = \eta\epsilon$, where ϵ is co-unit. The right hand side gives a scalar depending on h multiplied by unit element of H. For group algebra this gives unit at both sides. In the general case the situation $\Delta(h) = h \otimes h$ is true for group like element only and one has more complex formula $\Delta(h) = \sum_i a_i \otimes b_i$. One also defines primitive elements as elements satisfying $\Delta(h) = h \otimes 1 + 1 \otimes h$. Also Hopf algebras are associative and co-associative.

Quasi-bialgebras and quasi-Hopf algebras

Quasi-bi-algebras giving as special case quasi-Hopf algebras were discovered by Russian mathematician Drinfeld (for technical definition, which does not say much to non-specialist see `http://tinyurl.com/y7b6lpop` and `http://tinyurl.com/y89cs5oy`). They are non-associative or associative modulo isomoprhism.

Consider first quasi-bi-algebra $(B, \Delta, \epsilon, \Phi, l, r)$. Δ and ϵ are as for bi-algebra. Besides this one has invertible elements Φ (Drinfeld associator) and r, l called right and lef unit constraints. The conditions satisfied are following

- $$(1 \otimes \Delta) \circ \Delta(a) = \Phi[((\Delta \otimes 1) \circ \Delta(a)]\Phi^{-1} \ .$$

  For $\Phi = 1 \otimes 1 \otimes 1$ one obtains associativity.

- $$[(1 \otimes 1 \times \Delta)(\Phi)][(\Delta \otimes 1 \otimes 1)(\Phi)] = (1 \otimes \Phi)[1 \otimes \Delta \otimes 1)(\Phi)(\Phi \otimes 1) \ .$$

- $$(\epsilon \otimes 1)(\Delta(a)) = l^{-1}al \ , \ (1 \otimes \epsilon)(\Delta(a)) = r^{-1}ar \ .$$

- $$1 \otimes \epsilon \otimes 1)(\Phi) = 1 \otimes 1 \ .$$

These mysterious looking conditions express the fact that Drinfeld associator is a bialgebra co-cycle.

Quasi-bialgebra is braided if it has universal R-matrix which is invertible element in $B \otimes B$ such that the following conditions hold true.

$$(\Delta^{op})(a) = R\Delta(a)R^{-1} \ . \tag{20.4.1}$$

Note that for group algebra with $\Delta g = g \otimes g$ one has $\Delta^{op} = \Delta$ so that R must commute with Δ. Whether this forces R to be trivial is unclear to me. Certainly there are also other homomorphisms. A good candidate for a non-symmetric co-product is $\Delta g = g \times h(g)$ where h is a homomorpism of the braid group. This requires the replacement $S(g) \rightarrow S(h^{-1}g)$ in order to obtain unitarity for $\nabla(1, S)\Delta$ loop removing the braiding.

$$(1 \otimes \Delta)(R) = \Phi_{231}^{-1} R_{13} \Phi_{213} R_{12} \Phi_{213}^{-1} \ . \tag{20.4.2}$$

$$(\Delta \otimes 1)(R) = \Phi_{321}^{-1} R_{13} \Phi_{213}^{-1} R_{23} \Phi_{123} \ . \tag{20.4.3}$$

This and second condition imply for trivial R that also Φ is trivial.

For $\Phi = 1 \otimes 1 \otimes 1$ the conditions reduces to those for ordinary braiding. The universal R-matrix satisfies the non-associative version of Yang-Baxter equation

$$R_{12}\Phi_{321}R_{13}(\Phi_{132})^{-1}R_{23}\Phi_{123} = \Phi_{321}R_{23}(\Phi_{231})^{-1}R_{13}\Phi_{213}R_{12} \ . \tag{20.4.4}$$

Quasi-Hopf algebra is a special case of quasi-bialgebra. Also now one has product ∇, co-product Δ, antipode S not present in bialgebra, and maps ϵ and η. Besides this one has two special elements α and β of H such that the conditions $\nabla(S, \alpha) \cdot \Delta = \alpha$ and $\nabla(1, \beta S) \cdot \Delta = \alpha$. To my understanding these conditions generalize the conditions $\nabla(S, 1)\Delta = \eta\epsilon$ and $\nabla(1, S)\Delta = \eta\epsilon$.

Associativity holds but only modulo a morphism in the same way as commutativity becomes braided commutativity in the case of quantum groups. The braided commutativity is characterized by R-matrix. The morphism defining "braided associativity" is characterized by the product $\Phi = \sum_i X_i \otimes Y_i \otimes Z_i$ acting on triple tensor product $V \otimes V \otimes V$ and satisfying certain algebraic conditions. Φ has "inverse" $\Phi^{-1} = \sum_i P_i \otimes Q_i \otimes R_i$ The conditions $(1, \beta S, \alpha)\Phi = 1$ and $(S, \alpha, \beta S)\Phi = 1$. Here the action of S is that of algebra anti-homomorphism rather than algebra multiplication.

Drinfeld associator, which is a non-abelian bi-algebra 3-cocycle satisfying conditions analogous to the condition for weakened associativity holding true for Lie and Jordan algebras. These quasi-Hopf algebras are known in conformal field theory context and appear in Knizhnik-Zamolodchikov equations so that a lot of mathematical knowhow exists. According to Wikipedia, quasi-Hopf algebras are associated with finite-D irreps of quantum affine algebras in terms of F-matrices used to factorize R-matrix. The representations give rise to solutions of Quantum Yang-Baxter equation. The generalization of conformal invariance in TGD framework strongly suggests the relevance of Quasi-Hopf algebras in the realization of non-associativity in TGD framework.

Drinfeld double

Drinfeld double provides a concrete example about breaking of associativity. It can be formulated for finite groups as well as discrete groups. Drinfeld's approach is essentially algebraic: one works at the level of group algebra. In TGD framework the approach is geometric: algebraic constructs should emerge naturally from geometry. Braiding operations should induce algebras.

The basic notions involved are following.

1. One begins from a trivial tensor product of Hopf algebras and modified. In trivial case algebra product is tensor product of products, co-product is tensor product of co-products, antipode is tensor product of antipodes, map ϵ is product of the maps from the factors of the tensor product and delta maps unit element of field K to a product of unit elements. Drinfeld double represents a non-trivial tensor product of Hopf algebras.

2. One application of Drinfeld double construction is tensor product of group algebra and its dual. One can also interpret it as tensor product of braids as non-closed paths and closed braids (knots) as closed paths: in TGD framework this interpretation is suggestive and will be discussed later.

3. Drinfeld double allows breaking of associativity. It can be broken by introducing 3-cocycle (see `http://tinyurl.com/y9vcsmyg`) of group cohomology (see `http://tinyurl.com/y755gd36`). In the recent case group cohomology relies on homomorphism of group braid G to abelian group $U(1)$. n-cocycle is a map $G^n \rightarrow U(1)$ satisfying the condition that its derivation vanishes $d_nf = 0$. $d_n \circ d_{n-1} = 0$ holds true identically.

   The explicit definition of n-cocycle is in additive notion for U(1) product (usually multiplicative notation is used is) given by to illustrate that d_n acts like exterior derivative.

$$
\begin{aligned}
(d_nf)(g_1,g_2,g_n,g_{n+1}) = {} & g_1f(g_1,...g_n) - f(g_1g_2,g_2,...,g_{n+1}) + f(g_1,g_2g_3,...,g_{n+1}) \\
& -... + (-1)^n f(g_1,g_2...g_ng_{n+1}) + (-1)^{n+1}f(g_1,g_2...g_n) \ .
\end{aligned}
\tag{20.4.5}
$$

   This formula is easy to translate to multiplicative notion. The fact that group cohomology is universal concept strongly suggests that 3 co-cycle can be introduced quite generally to break associativity in the sense that different associations differ only by isomorphism.

The construction of quantum double of Hopf algebras is discussed in detail at `http://tinyurl.com/ybbvjaw5`. Here however non-associative option is not discussed. In `http://tinyurl.com/ya8n98o5` one finds explicit formula for Drinfeld double for the Drinfeld double formed by group algebra and its dual. Just to give some idea what is involved the following gives the formula for the product:

$$
(h,y)\circ(g,x) = \frac{\omega(h,g,x)\omega(hgx((hg)^{-1},h,g)}{\omega(h,gx(g)^{-1},h,g)}(hg,x) \ . \tag{20.4.6}
$$

Without background it does not tell much. What is essential however that the starting point is algebraic. The product is non-vanishing only between (g,x) and (h,gxg^{-1}). For gauge group like structure one would have x instead of $g^{-1}xg^{-1}$. ω is 3-cocycle: it it is non-trivial one as associativity modulo isomorphism.

I do not have any detailed understanding of quasi-Hopf algebras but to me they seem to provide a very promising approach in attempts to understand the character of non-associativity associated with the braiding hierarchy. The algebraic construction of Drinfeld double does not seem interesting from TGD point of view but the idea that group cocycle is behind the breaking of associativity is attractive. Also the generalization of construction of Drinfeld double to code what happens in braiding geometrically is attractive. One of the many difficult challenges is to understand the role of the varying parameters p, h_{eff}, q at the level of braid group algebras and their projective representations characterized by quantum phase q.

20.4.2 Construction Of Quantum Braid Algebra In TGD Framework

It seems that there is no hope that naive application of existing formulas makes sense. The variety of different variants of quantum algebras is huge and one should have huge mathematical knowledge and understanding in order to find the correct option if it exists at all. Therefore I bravely take the approach of physicists. I try to identify the physical picture and then look whether I can identify the algebraic structure satisfying the axioms of Hopf algebra. In the following I first list various inputs which help to identify constraints on the algebraic structure, which should be simple if it is to be fundamental.

Trying to map out the situation

Usually physicists have enough trouble when dealing with single algebraic structure: say group and its representations. Unfortunately, this does not seem to be possible now. It seems that one must deal with entire collection of algebraic structures defined by braid groups B_n with varying value of n forming a hierarchy in which braid groups act on lower level braid groups.

1. What is clear that the algebraic operation $(A\otimes B) \rightarrow AB$ is somehow related to the braiding of flux tubes or fermionic strings connecting partonic 2-surfaces. One can also consider strings connecting the ends of light-like 3-surfaces so that one has both space-like and time-like braiding. One has flux tubes inside flux tubes.

   The challenge is to identify the natural algebra. It seems best to work with the braiding operations themselves - analogs of linguistic expressions - than the states to which they act. Braiding operations form discrete group, braid group. One must deal with the quantum superpositions of braidings so that one has wave functions in braid group identifiable as elements of discrete group algebra of braid group B_n. One can multiply group algebra elements and include the group algebra of B_m to that of B_n m a factor of n so that the desired product structure is obtained. The group algebras associated with various braid numbers can be organized to operad.

   The operad formed by the braid group algebras has the desired hierarchical structure, and braid group algebra is one of the basic structures and quantum groups can be assigned with its projective representations.

2. For a given flux tube (and perhaps also for the fermionic string(s) assigned with it) one has degrees of freedom due different values of the quantum deformation parameter q for which roots of unity define preferred values in TGD framework. In TGD framework also hierarchy $h_{eff}/h = n$ of Planck constants brings in additional complexity. Also the p-adic prime p is expected to characterize the situation: preferred p-adic primes can be interpreted as so called ramified primes in the adelic vision about quantum TGD [K111] unifying real and various p-adic physics to a coherent whole. This brings in new elements. It is still unclear how closely n and $q = exp(i2\pi/m)$ are related and whether one might have $m = n$. Also the relationship of p to n is not well-understood. For instance, could p divide n.

3. Geometrically the association of braid strands means that they belong to the same flux tube. Moving the brackets in expression to transform say $(A(BC))$ to $((AB)C)$ means that strands are transferred from flux tube another one. Hence the breaking of associativity should take place at all hierarchy levels except the lowest one for which flux tube contains single irreducible braid strand - fermion line.

   The general mechanism for a weak breaking of associativity is describable in terms of Drinfeld's associator for quasi-bialgebras and known in some cases explicitly - in particular, shown by Drinfeld to exists when the number field used is rational numbers - is the first guess for the mechanism of the breaking of associativity. Drinfeld's associator is determined completely by group cohomology, which encourages to think that it can be used as such as as a multipler in the definition of product in suitable tensor product algebra. How the Drinfeld's associator depends on the p,n, and q is the basic question.

4. Besides the geometric action of braidings it is important to understand how the braidings act on the fundamental fermions. An attractive idea is that the representation is as holonomies

defined by the induced weak gauge potentials as non-integrable phase factors at the boundaries of string world sheets defining fermion lines. The vanishing of electroweak gauge fields at them implies that the non-Abelian part of holonomy is pure gauge as in topological gauge field theories for which the classical solutions have vanishing gauge field. The em part of the induce spinor curvature is however non-vanishing unless one poses the vanishing of electromagnetic field at the boundaries of string world sheets as boundary condition. This seems un-necessary. The outcome would be non-trivial holonomy and restriction to a particular representation of quantum group with quantum phase q coming as root of unity means conditions on the boundaries of string world sheets. Quantum phase would make itself visible also classically as properties of string world sheets which together with partonic 2-surfaces determined space-time surface by strong form of holography. An interesting question relates to the possibility of non-commutative statistics: it should come from the weak part of induced connection which is pure gauge and seems possible as it is possible also in topological QFTs based on Chern-Simons action.

Hints about the details of the braid structure

Concerning the details of the braid structure one has also strong hints.

1. There two are two basic types of braids: I have called them time-like and space-like braids. Time-like (or rather light-like) braids are associated with the 3-D light-like orbits of partonic 2-surfaces at which the signature of the induced metric changes signature from Minkowskian to Euclidian. Braid strands correspond to fermionic lines identifiable as parts of boundaries of string world sheets. Space-like braids are associated with the space-like 3-surfaces at the ends of causal diamond (CD). Also they consist of fermionic lines. These braids could be called fundamental.

   If these braids are associated with magnetic flux tubes carrying monopole flux, the flux tubes are closed. Typically they connect wormhole throats at first space-time sheet, go to the second space-time sheet and return. Hence two-sheeted objects are in question. The braids in question can closed to knots and could correspond to closed loops assigned with the Drinfeld quantum double. The tensor product of the groupoid algebra associated with time-like braids and group algebra associated with space-like braids is highly suggestive as the analog of Drinfeld double.

   Also magnetic flux tubes and light-like orbits of partonic 2-surfaces can become braided and one obtains the hierarchies of braids.

2. Since strong world sheets and partonic 2-surfaces have co-dimension 2 as sub-manifolds of space-time surface they can also get braided and knotted and give rise to 2-braids and 2-knots. This is something totally new. The unknotting of ordinary knots would take place via reconnections and the reconnections could correspond to the basic vertices for 2-knots analogous to the crossing of the plane projections of ordinary knot. Reconnections actually correspond to string vertices. A fascinating mathematical challenge is to generalize existing theories so that they apply to 2-braids and 2-knots.

3. Dance metaphor emerged in the model for DNA-lipid membrane system as topological quantum computer [K20, K80]. Dancers whose feet are connected to wall by threads define time-like braiding and also space-like braiding through the resulting entanglement of threads. The assumption was that DNA codons or nucleotides are connected by space-like flux tubes to the lipids of lipid layer of cell membrane or nuclear membrane.

   If they carry monopolo flux they make closed loops at the structure formed by two space-time sheets. The lipid layer of cell membrane is 2-dimensional and can be in liquid crystal state. The 2-D liquid flow of lipids induces braiding of both space-like braids if the DNA end is fixed and of time-like braids. This leads to the dance metaphor: the liquid flow is stored at space-time level to the topology of space-time as a space-like braiding of flux tubes induced by it. Space-like braiding would be like written text. Time-like braiding would be like spoken language.

4. If the space-like braids are closed, they form knots and the flow caused at the second end of braid by liquid flow must be compensated at the parallel flux tube by its reversal since braid strands cannot be cut. The isotopy equivalence class of knot remains unchanged since knots get gg^{-1} piece which can be deformed away. Second interpretation is that the braid X transforms to gXg^{-1}. This kind of transformation appears also in Drinfeld construction. This suggests that the purely algebraic tensor product of braid algebra and its dual corresponds in TGD framework semi-direct tensor product of the groupoid of time-like braids and space-like braids associated with closed knots. The semi-direct tensor product would define the fundamental topological interaction between braids.

5. One can also consider sequence of n tensor factors each consisting of time-like and space-like braids. This require a generalization of the product of two tensor factors to $2n$ tensor factors. Dance metaphor suggests that a kind of chain reaction occurs.

What the structure of the algebra could be?

With this background one can try to guess what the structure of the algebra in question is. Certainly the algebra is semi-direct product of above defined braid group algebras. The multiplication rule would have purely geometric interpretation.

1. The multiplication rule inspired by dance metaphor for 2 tensor factors would be

$$(a_1, a_2) \circ (b_1, b_2) = (a_1 a_2 b_1 a_2^{-1}, a_2 b_2) \ . \tag{20.4.7}$$

   Here a_1, b_1 correspond label elements of time-like braid groupoid and a_2, b_2 the elements of braid group associated with the space-like braid. This would replace the trivial product rule $(a_1, a_2)(b_1 g_)) = (a_1 b_1, a_2 b_2)$ for the trivial tensor product. The structure is same as for Poincare group as semi-direct product of Lorentz group and translation group: $(\Lambda_1, T_1)(\Lambda_2, T_2) = (\Lambda_1\Lambda_2, T_1 + \Lambda_1(T_2))$.

   It is easy to check that this product is associative. One can however add exactly the same 3-cocycle factor

$$(h, y) \circ (g, x) = \frac{\omega(h, g, x)\omega(hgx((hg)^{-1}, h, g)}{\omega(h, gx(g)^{-1}, h, g)}(hg, x) \ . \tag{20.4.8}$$

   Here (h, y) corresponds to (a_1, a_2) and (g, x) to (b_1, b_2). This should give breaking of non-associativity and third group cohomology of braid group B_n would characterize the non-equivalent associators.

2. The product rule generalizes to n factors. This generalization could be relevant for the understanding of braid hierarchy.

$$(a_1, a_2, ...a_n) \circ (b_1, b_2, ...b_n) \equiv (c_1, ..., c_n) \ , \tag{20.4.9}$$

   where one has

$$\begin{array}{lll} c_n = a_n b_n \ , & c_{n-1} = a_{n-1} Ad_{a_n}(b_{n-1}) \ , & c_{n-2} = a_{n-2} Ad_{a_{n-1}a_n}(b_{n-2}) \ , \\ c_{n-3} = a_{n-3} Ad_{a_{n-2}a_{n-1}a_n}(b_{n-3}) & , ... & c_1 = a_1 Ad_{a_2.....a_n}(b_1) \ . \\ Ad_x(y) = xyx^{-1} \ . & & \end{array} \tag{20.4.10}$$

In this case a good guess for the breaking of associativity is that the associator is defined in terms of n-cocyle in group cohomology.

What is remarkable that this formula guarantees without any further assumptions the condition

$$\begin{gathered}\nabla_{1\otimes 2}(\Delta_1(a),\Delta_2(b)) = \nabla_1(\Delta_1(a))\nabla_2(\Delta_2(b)) = \sum_{(a)} a_1a_2 \sum_{(b)} b_1b_2 \ , \\ \Delta_1(a) = \sum_{(a)} a_1\otimes a_2 \quad , \quad \Delta_2(b) = \sum_{(b)} b_1\otimes b_2\end{gathered} \tag{20.4.11}$$

as a little calculation shows. For group algebra one has $\Delta(a) = g\otimes g$. $\nabla_{1\otimes 2}$ refers to the product defined above.

3. The formula for $\Delta_{1\otimes 2}$ is also needed. The simplest guess is that it corresponds to replication for both factors. This would mean $\Delta^{op} = \Delta$: non-symmetric form guaranteeing non-trivial braiding is however desirable. A candidate satisfying this condition in $n = 2$ case is asymmetric replication:

$$\begin{array}{l}\Delta_{1\otimes 2}(bab^{-1},b)\otimes(a,b) \\ \Delta^{op}_{1\otimes 2}(a,b)\otimes(bab^{-1},b)\ .\end{array} \tag{20.4.12}$$

4. In $n = 2$ case the formula for antipode would read as

$$S(a_1,a_2) = (a_2^{-1}a_1^{-1}a_2, a_2^{-1}) \tag{20.4.13}$$

instead of $S(a_1,a_2) = (a_1^{-1},a_2^{-1})$. Again the semi-direct structure would be involved. One can check that the formula

$$\nabla_{1\otimes 2}(1,S)\Delta_{1\otimes 2} = 1\otimes 1 \tag{20.4.14}$$

holds true.

20.4.3 Should One Quantize Complex Numbers?

The TGD inspired proposal for the concrete realization of quantum groups might help in attempts to understand the situation. The approach relies on what might be regarded as quantization of complex numbers appearing as matrix elements of ordinary matrices.

1. Quantum matrices are obtained by replacing complex number valued of matrix elements of ordinary matrices with operators. They are are products of hermitian non-negative matrix P analogous to modulus of complex number and unitary matrix S analogous to its phase. One can also consider the condition $[P,S] = iS$ inspired by the idea that radial momentum and phase angle define analog of phase space.

2. The notions of eigenvalue and eigenstate are generalized. Hermitian operator or equivalently the spectrum of its eigenvalues replaces real number. The condition that eigenvalue problem generalizes, demands that the symmetric functions formed from the elements of quantum matrix commute and can be diagonalized simultaneously. The commutativity of symmetric functions holds also for unitary matrices. These conditions is highly non-trivial, and consistent with quantum group conditions if quantum phases are roots of unity. In this framework also Planck constant is replaced by a hermitian operator having $h_{eff} = n \times h$ as its spectrum. Also $q = exp(in2\pi/m)$ generalizes to a unitary operator with these eigenvalues.

3. This leads to a possible concrete representation of quantum group in TGD framework allowing to realize the hierarchy of inclusions of hyperfinite factors obtained by repeatedly replacing the operators appearing as matrix elements with quantum matrices.

4. This procedure can be repeated. One might speak of a fractal quantization. At the first step one obtains what might be called 1-hermitian operators with eigenvalues replaced with hermitian operators. For 1-unitary matrices eigenvalues, which are phases are replaced with unitary operators. At the next step one considers what might be called 2-hermitian and 2-unitary operators. An abstraction hierarchy in which instance (localization to a point as member of class) is replaced with wave function in the class. This hierarchy is analogous to that formed by infinite primes and by the sheets of the many-sheeted space-time. Also braids of braids of ... form this kind of abstraction hierarchy as also the parsing hierarchy for linguistic expressions.

I have proposed that generalized Feynman diagrams or rather - TGD analogs of twistor diagrams - should have interpretation as sequences of arithmetic operators with each vertex representing product or co-product and having interpretation as time reversal of the product operation.

1. The arithmetic operations could be induced by the algebraic operations for Yangian algebra [A27] [B39, B30, B31] assignable to the super-symplectic algebra. I have also proposed that there TGD allows a very powerful symmetry generalizing the duality symmetry of old-fashioned string models relating s- and t-channel exchanges. This symmetry would state that one can freely move the ends of the propagator lines around the diagrams and that one can remove loops by transforming the loop to tadpole and snipping it away. This symmetry would allow to consider only tree diagrams as shortest representations for computations: this would reduce enormously the calculational complexity. The TGD view about coupling constant evolution allows still to have discrete coupling constant evolution induced by the spectrum of critical values of Kähler coupling strength: an attractive conjecture is that the critical values can be expressed in terms of zeros of Riemann zeta [L16].

2. One can represent the tree representing a sequence of computations in algebra as an analog of twistor diagram and the proposed symmetry implies associativity since moving the line ends induces motion of brackets. If co-algebra operations are allowed also loops become possible and can be eliminated by this symmetry provided the loop acts as identity transformation. This would suggest strong form of associativity at the level of single sheet and weaker form at the level of many-sheeted space-time. One could however still hope that loops can be cancelled so that one would still have only tree diagrams in the simplest description. One would have however sum over amplitudes with different association structures.

3. Co-product could be associated with the basic vertices of TGD, which correspond to a fusion of light-like parton orbits along their ends having no counterpart in super-string models (tensor product vertex) or the decay of light-like parton orbit analogous to a splitting of closed string (direct sum vertex). For the direct sum vertex one has direct sum (unlike string models): one can say that the particle propagates along two path in the sense of superposition as photons in double slit experiment. For the tensor product vertex $D(g) = \Delta(g) = g \times g$ is the first guess. $D(g) = (1, S)\Delta(g) = g \otimes Sg$ or $D(g) = Sg \otimes g$ or their sum suitably normalized is natural second guess. Unitarity allows only the latter option since $\nabla\Delta$ does not conserve probability for probability amplitudes unlike $\nabla(1, S)\Delta$ although it does so for probability distributions. For the direct sum vertex $\Delta(g) = 1 \otimes g \oplus g \otimes 1$ suitably normalized is the natural first guess.

4. Co-product Δ might allow interpretation as annihilation vertex in particle physics context. Co-product might also allow interpretation in terms of replication - at least at the level of topological dynamics of braiding. The possible application of co-product to the replication occurring biology assumed to be induce by replication of magnetic flux tubes in TGD based vision is highly suggestive idea. Is the identification of co-product as replication consistent with its identification as particle annihilation?

   Second question relates to the antipode S, which is anti-homomorphism and brings in mind time reversal. Could one interpret also S as an operation, which should be included to the braid group algebra in the same way as the inclusion of complex conjugation to the algebra of complex numbers produces quaternions? Could one interpret the identity $\nabla(1\otimes S)\Delta(g) = \eta\epsilon(g) = 1$ by saying that the annihilation to $g\otimes S(g)$ followed by fusion produces braid wave function concentrated on trivial braiding and destroying the information associated with braiding completely. The fusion would produce non-braided particle rather than destroying particles altogether.

5. The condition that loop involving product and annihilation does not affect braid group wave function would require that it takes g to g. For the standard realization of co-product Δ of group algebra $g \rightarrow g\otimes g \rightarrow g^2$ so that this is not the case. The condition defining Δ is not easy to modify since one loses homomorphism property of Δ. The repetitions of loops would give sequence of powers g^{2n}. For wave function $\sum D(g)g$ this would give the sequence $\sum D(g)g \rightarrow \sum D(g)g^2 \rightarrow .... \rightarrow \sum D(g)g^{2n}$: since given group element has typically several roots one expects that eventually the wave function becomes concentrated to unity with coefficient $\sum D(g)$! For wave functions one has $\sum D(g) = 0$ if they are orthogonal to $D(g) = constant$ as is natural to require. Almost all wave functions would approach to zero so that unitary would be lost. For probability distributions the evolution would make sense since the normalization condition would be respected.

   Also the irreversible behaviour looks strange from particle physics perspective unless $D(g)$ is concentrated on identity so that braiding is trivial. Topological dissipation might take care that this is the case. For elementary particles partonic 2-surfaces carry in the first approximation only single fermion so that braid group would be trivial. Braiding effects become interesting only for strand number larger than 2. The situations in which partonic surface carries large number of fermion lines would be more interesting. Anyonic systems to which TGD based model assigns large h_{eff} and parton surfaces of nanoscopic size could represent a condensed matter example of this situation.

6. Does the behavior of Δ force to regard generalized Feynman diagrams representing computations with different numbers of self-energy loops non-equivalent and to sum over self-energy loops in the construction of scattering amplitudes? The time evolution implied by topological self energy loops is not unitary which suggest that one must perform the sum. There are hopes that the sum converges since the contributions approaches to $\sum D(g) = 0$. This does not however look elegant and is in conflict with the general vision.

   Particle physics intuition tells that in pair annihilation second line has opposite time direction. Should one therefore identify annihilation $g \rightarrow g\otimes S(g)$. Antiparticles would differ from particles by conjugation in braid group. The self energy loop would give trivial braiding with coefficient $\sum D(g)D(g^{-1}) = \sum D(g)D(g)^* = 1$ so that unitarity would be respected and higher self energy loops would be trivial. The conservation of fermion number at fundamental level could also prevent the decays $g \rightarrow g\otimes g$.

One could also take biological replication as a guide line.

1. In biological scales replication by $g \rightarrow g\otimes g$ vertex might not be prevented by fermion number conservation but probability conservation favors $g \rightarrow g\otimes Sg$. Braid replication might be perhaps said to provide replicas of information: whether this conforms with no-cloning theorem remains to be seen. Braid replication followed by fusion means topological dissipation by a loss of braiding and loss of information. Could the fusion of reproduction cells corresponds to product and that replication to co-product possibly involving the action of S one the second line. Fusion followed by replication would lead to a loss of braiding: for

$g \rightarrow g \otimes g$ perhaps making sense in probabilistic description gradually and for $g \rightarrow g \otimes Sg$ instantaneously: a reset for memory? Could these mechanisms serve as basic mechanisms of evolution?

2. There might be also a connection with the p-adic length scale hypothesis. The naive expectation is that $g \rightarrow g^2$ in fusion followed by Δ means the increase of the length of braid by factor 2 - kind of ageing? Could the appearance of powers of two for the length of braid relate to the p-adic length scale hypothesis stating that primes p near powers of 2 are of special importance?

To summarize, the proposed framework gives hopes about description of braids of braids of .... Abstraction would mean transition from classical to quantum: from localized state to a de-localized one: from configuration space to the space of complex valued wave functions in configuration space. Now the configuration space would involve different braidings and corresponding evolutions, and various values of p, h_{eff} and q. If this general framework is to be useful it should be able to tell how the braiding matrices depend on p and h_{eff}: note that p and h_{eff} would be fixed only at the highest abstraction level - the largest flux tubes. This indeterminacy could be interpreted in terms of finite measurement resolution and inclusions of HFFs should help to describe the situation. Indeterminacy could also be interpreted in terms of abstraction in a manner similar to the interpretation of negentropically entangled state as a rule for which the state pairs in the superposition represent instances of the rule.

Part V

MISCELLANEOUS TOPICS

Chapter 21

Does the QFT Limit of TGD Have Space-Time Super-Symmetry?

21.1 Introduction

Contrary to the original expectations, TGD seems to allow the analog of the space-time super-symmetry. This became clear with the increased understanding of both Kähler action and Kähler-Dirac action [K88, K14]. It is however far from clear whether SUSY type QFT can define the QFT limit of TGD and whether this kind of formulation is the optimal one.

21.1.1 Is The Analog Of Space-Time SUSY Possible In TGD?

The basic question is whether the huge algebras with super-conformal structure acting as symmetries of quantum TGD give rise to a SUSY algebra at space-time level (meaning super-Poincare symmetry). A more technical question is whether the QFT limit of TGD could be formulated as a generalization of SUSY QFT or whether one must generalize this approach just as it seems necessary to generalize the notion of twistor by replacing masslessness in 4-D sense with masslessness in 8-D sense.

1. From the beginning it was clear that super-conformal symmetry is realized in TGD but differs in many respects from the more standard realizations such as $\mathcal{N} = 1$ SUSY realized in MSSM [B11] involving Majorana spinors in an essential manner.

   Note that the belief that Majorana spinors are somehow an intrinsic aspect of super-symmetry can be used as an objection against TGD. Besides Majorana spinors Weyl spinors meaning complex theta parameters are also possible. Theta parameters can also carry fermion number meaning only the supercharges carry fermion number and are non-hermitian. The general classification of super-symmetric theories indeed demonstrates that for $D = 8$ Weyl spinors and complex and non-hermitian super-charges are possible. The original motivation for Majorana spinors might come from MSSM assuming that right handed neutrino does not exist. This belief might have also led to string theories in D=10 and D=11 as the only possible candidates for TOE after it turned out that chiral anomalies cancel.

2. In TGD framework the covariantly constant right-handed neutrino generates the super-symmetry at the level of CP_2 geometry. The original idea was that the construction of super-partners would be more or less equivalent with the addition of *covariantly constant* right-handed neutrino and antineutrinos to the state. It was however not clear whether space-time supersymmetry is realized at all since one could argue that that by covariant constancy these states are just gauge degrees of freedom or that SUSY is only realized for the spinor harmonics of imbedding space with 8-D notion of masslessness. Much later it became clear that covariantly constant right handed neutrino indeed represents gauge degree of freedom at *space-time level*.

3. A more general general SUSY algebra is generated by the modes of the Kähler-Dirac operator at partonic 2-surface being also Clifford algebra. This algebra can be associated with the ends of the boundaries of string world sheets and each string defines its own sub-algebra of oscillator operators.

   (a) At first it would seem that the value of $\mathcal{N}$ can be very large - even infinite as the fact that fermionic oscillator operators are labelled by conformal weight. It is however the number of *massless states* in M^4 sense, which determines the value of $\mathcal{N}$ for SUSY in M^4: for the full theory the analog of SUSY in H $\mathcal{N} = \infty$ could make sense. Indeed, super-symplectic generators bring in the analog of wave function of fermion at partonic 2-surfaces and constant wave functions and therefore massless states are expected to be favored by Uncertainty Principle. The dimension of SUSY algebra is expected to just the number of spinor components of the imbedding space spinor possessing physical imbedding space helicity.

   A more general situation is that the conformal gauge algebra is its sub-algebra isomorphic to the entire algebra having conformal weights coming as n-ples of those for the full algebra. The conformal gauge symmetry would be broken so that only the super-symplectic generators for which the conformal weight is proportional to fixed integer $n \in \{1, 2, ...\}$ annihilate the physical states. This increases the value of $\mathcal{N}$ and a possible interpretation is in terms of improved measurement resolution. N would also correspond to the value of Planck constant $h_{eff}/n = N$ and N would label phases of dark matter and also a hierarchy of criticalities. As N increases, super-conformal gauge degrees of freedom are transformed to physical ones. This kind of situation might be possible for quantum deformations of the oscillator operator algebra characterized by quantum phase as $q = exp(i2\pi/N)$ and possible by the 2-dimensionality of string world sheets.

   An alternative manner to see the situation is as a fractionization of conformal weights due to the emergence of N-fold coverings of space-time surfaces analogous to coverings of complex plane defined by analytic function $z^{1/N}$. Only the states with integer conformal weights would be annihilated by the original conformal algebra and quantum group would describe the situation.

   The SUSY in standard sense is expected to be broken. First, the notion of masslessness is generalized: fermions associated with the boundaries of string world sheets have light-like 8-momentum and therefore can be massive in 4-D sense: this allows to generalize twistor description to massive case [K76]. The ordinary 4-D SUSY is expected to emerge only as an approximate description in massless sector (as it also appears in dimensional reduction). Secondly, standard SUSY characterizes the QFT description obtained by replacing many-sheeted space-time time with a slightly curved region of Minkowski space.

   (b) SUSY algebra is replaced with Clifford algebra at the level of partonic 2-surfaces and the generators can be identified as fermionic oscillator operatiors at the end points of fermionic lines, which are light-like geodesics. Light-like four-momenta in anti-commutation relations are replaced with 8-D light-like momenta demanding a generalization of twistor approach. The octonionic realization of twistors is a very attractive possibility in this framework and quaternionicity condition guaranteeing associativity leads to twistors which are almost equivalent with ordinary 4-D twistors.

   The space-time super-symmetry means addition of fermion to the state assign to a partonic surface and since the number of spinor modes is larger states with large spin and fermion numbers are obtained. This picture does not fit to the standard view about super-symmetry. In particular, the identification of theta parameters as Majorana spinors and super-charges as Hermitian operators is not possible. The non-hermitian character of super conformal generator $G \neq G^\dagger$ made impossible the naive generalization of stringy rules to TGD framework since they involve G as the analog of fermionic propagator. This problem disappears in the twistor Yangian approach [K76].

   (c) The notion of super-field does not seem natural in the full TGD framework but would be replaced with a Yangian of the super-symplectic algebra and related conformal algebras with generators identified as Noether charges assignable to strings connecting partonic 2-surfaces. Multi-locality coded by Yangian in the scale of partonic surfaces is a new element. There is also the hierarchy of Planck constants interpreted in terms of dark matter and Zero Energy Ontology.

21.1.2 What Happens When Many-Sheeted Space-Time Is Approximated With Minkowski Space?

The question is what happens when one replaces many-sheeted space-time with a region of Minkowski space and identifies gauge potentials as sum of the induced gauge potentials?

1. It is plausible that gauge theory like description is a good approximation. But what happens to the SUSY? Can one replace 8-D light-likeness with 4-D light-likeness and describe massivation in terms of Higgs mechanism and analogous - not very successful - mechanisms for 4-D SUSY? It is quite possible that this is not possible: 4-D QFT approximation taken partonic 2-surfaces to points might miss too much of physics and too much elegance.

2. Should one try to find a generalization of ordinary 4-D SUSY allowing the description of massive particles in terms of 8-D light-likeness? This would allow also to understand baryons and lepton number conservation as 8-D chiral symmetry, to avoid Majorana spinors, and would force a new view about QCD color. Maybe the attempt to describe things by QFT or even ordinary string model is like an attempt to describe quantum physics using classical mechanics. To my opinion generalization of twistor approach from 4-D to 8-D context based on the notion of super-symplectic Yangian is a more promising approach than sticking to effective field theory thinking [K76].

The first guess - much before the understanding of the Kähler-Dirac equation and the role of right-handed neutrino - was that it might be possible to formulate even quantum TGD proper in terms of super-field defined in the world of classical worlds (WCW). Super-fields could provide in this framework an elegant book-keeping apparatus for the elements of local Clifford algebra of WCW extended to fields in the $M^4 \times CP_2$, whose points label the positions of the tips of the causal diamonds CDs). At this moment I feel skeptic about this approach.

21.1.3 What SUSY QFT Limit Could Mean?

What the actual construction of SUSY QFT limit means depends on how strong approximations one wants to make.

1. The minimal approach to SUSY QFT limit is based on an approximation assuming only the super-multiplets generated from fundamental fermions by right-handed neutrino or both right-handed neutrino and its antineutrino.

2. Elementary are particles are composed of fundamental fermions so that the super-multiplets are more complex for them. One of the key predictions of TGD is that elementary particles can be regarded as bound states of fermions and anti-fermions located at the throats of two wormhole contacts. As a special case this implies bosonic emergence meaning that it QFT limit can be defined in terms of Dirac action.

21.1.4 Scattering Amplitudes As Sequences Of Algebraic Operations

The attempts to generalize twistor Grassmannian approach in TGD framework led to a revival an old idea about scattering amplitudes as representations of sequences of algebraic operations connecting two sets of algebraic objects. Any two sequences connecting same sets would give rise to same scattering amplitudes. One might say that instead of mathematics representing physics physics represents mathematics.

1. In Yangian approach fundamental vertices correspond to product and co-product for the generators of Yangian of super-symplectic algebra with charges identified in terms of Noether charges assignable to strings connecting partonic 2-surfaces [K76]. Scattering amplitudes

are obtained by the analog of Wick contraction procedure in which fermion lines connecting different vertices would be obtained. This also allows creation of fermion pairs from vacuum with members at opposite throats of wormhole contact defining the fundamental boson propagators. This picture about bosonic emergence is similar to the earlier one.

2. Yangian approach has huge symmetries since the duality symmetry of string models generalizes in the sense that one can freely move the ends of the lines and snip off loops in this manner. The fact that all diagram representing computation connecting same initial and final states are equivalent implies huge number of constraints and it is clear that ordinary Feynman diagrammatics cannot satisfy these constraints. Twistor diagrammatics could however do so since it has turned out that twistor diagrams indeed have symmetries analogous to this kind of symmetry. It seems however that one must generalized 4-D twistors to 8-D ones so that the twistor Yangian approach looks like the most promising approach at this moment: if of course applies to full theory rather than only in massless sector of the theory.

The plan of the chapter reflects partially my own needs. I had to learn space-time supersymmetry at the level of the basic formalism and the best manner to do it was to write it out. As the vision about fermions in TGD crystallized it became also clear that SUSY QFT in Feynman graph formulation does not catch the simplicity of what I identify as fundamental formulation of TGD. Therefore I dropped a lot of material in the original chapter.

1. The chapter begins with a brief summary of the basic concepts of SUSYs without doubt revealing my rather fragmentary knowledge about these theories. The original belief was that super-field formalism could be generalized to TGD framework. At this moment I however believe that Yangian approach is more realistic one for reasons already mentioned. Therefore I have dropped the section about the formalism proposed earlier. I have also dropped material about various attempts to understand the role right-handed neutrinos. The chapter in its recent form is about whether SUSY limit could emerge from TGD. Just general conditions are formulated since I do not have the expertise to formulate the theory in detail.

2. The Clifford algebra of fermionic oscillator operators assignable to the ends of strings connecting partonic 2-surfaces replaces SUSY algebra, and anti-commutation relations realize the analog of super Poincare symmetry. Since the number of conformal weights is infinite, one would naively expect $\mathcal{N} = \infty$ SUSY. States are however created by super-symplectic generators bringing in the analog of wave function of fermion at partonic 2-surface rather fermionic oscillator operators. Also conformal gauge invariance conditions are satisfied, and this is expected to change the situation. For ideal measurement resolution only the fermionic oscillator operators with vanishing conformal weight are expected to remain effective. The description of finite measurement resolution in terms of quantum variant of fermionic anti-commutation relations is expected to increase the number of conformal weights so that $\mathcal{N}$ increases for dark matter. Right-handed neutrino and its antineutrino would define the least broken sub-algebra of SUSY.

3. Twistors have become a part of the calculational arsenal of SUSY gauge theories, and TGD leads to a proposal how to avoid the problems caused by massive particles by using the notion of masslessness in 8-D sense and the notion of induced octo-twistor [K76]. The equivalence of octonionic spinor structure with the ordinary one leads also to the localization of spinors to string world sheets and fermions at light-like geodesics at their boundaries at partonic 2-surfaces. Already the fundamental formulation keeps just the knowledge that particle moves along light-like geodesic of $M^4 \times CP_2$ and strings connect partonic 2-surfaces. Could QFT limit could be formulated as SUSY in $M^4 \times S^1$ allowing to describe massive particles as massless particles in $M^4 \times S^1$? Or could simplified string model type description in $M^4 \times S^1$ make sense?

4. With the improved understanding of Kähler-Dirac equation one can develop arguments that $\mathcal{N} = 2$ or $\mathcal{N} = 4$ SUSY generated by right-handed neutrino emerges naturally in TGD framework and corresponds to the addition of a collinear right-handed neutrino and and antineutrino to the state representing massless particle.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found at `http://tgdtheory.fi/cmaphtml.html` [L11]. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [L12].

21.2 SUSY Briefly

The Tasi 2008 lectures by Yuri Shirman [B74] provide a modern introduction to 4-dimensional $\mathcal{N}=1$ super-symmetry and super-symmetry breaking. In TGD framework the super-symmetry is 8-dimensional super-symmetry induced to 4-D space-time surface and one $\mathcal{N}=2N$ can be large so that this introduction is quite not enough for the recent purposes. This section provides only a brief summary of the basic concepts related to SUSY algebras and SUSY QFTs and the breaking of super-symmetry is mentioned only by passign. I have also listed the crucial basic facts about $\mathcal{N}>1$ super-symmetry [B2, B10] with emphasis in demonstrating that for 8-D super-gravity with one time-dimension super-charges are non-Hermitian and that Majorana spinors are absent as required by quantum TGD.

21.2.1 Weyl Fermions

Gamma matrices in chiral basis.

$$\begin{array}{llll}
\gamma^\mu = \left(\begin{array}{cc} 0 & \sigma^\mu \\ \overline{\sigma}^\mu & 0 \end{array}\right) \ , & \gamma_5 = \left(\begin{array}{cc} \sigma_0 & 0 \\ 0 & -\sigma_0 \end{array}\right) \ , & & \\
\sigma^0 = \left(\begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array}\right) \ , & \sigma^1 = \left(\begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array}\right) \ , & \sigma^2 = \left(\begin{array}{cc} 0 & -i \\ i & 0 \end{array}\right) \ , & \sigma^3 = \left(\begin{array}{cc} 1 & 0 \\ 0 & -1 \end{array}\right) \ , \\
\overline{\sigma}^0 = \sigma^0 \ , & \overline{\sigma}^i = -\sigma^i \ . & &
\end{array} \tag{21.2.1}$$

Note that Pauli sigma matrices can be interpreted as matrix representation for hyper-quaternion units.

Dirac spinors can be expressed in terms of Weyl spinors as

$$\Psi = \left(\begin{array}{c} \eta^\alpha \\ \overline{\chi}^*_{\dot{\alpha}} \end{array}\right) \ . \tag{21.2.2}$$

Note that does not denote complex conjugation and that complex conjugation transforms non-dotted and dotted indices to each other. η and $\overline{\chi}$ are both left handed Weyl spinors and transform according to complex conjugate representations of Lorentz group and one can interpret $\overline{\chi}$ as representing that charge conjugate of right handed Dirac fermion.

Spinor indices can be lowered and raised using antisymmetric tensors $\epsilon^{\alpha\beta}$ and $\epsilon_{\dot{\alpha}\dot{\beta}}$ and one has

$$\begin{array}{ll}
\eta^\alpha\eta_\alpha = 0 \ , & \overline{\chi}^*_{\dot{\alpha}}\overline{\chi}^*_{\dot{\alpha}} = 0 \ per, \\
\eta\overline{\chi} = \overline{\chi}\eta = \epsilon^{\alpha\beta}\eta_\alpha\overline{\chi}_\beta \ , & \eta^*\overline{\chi}^* = \overline{\chi}^*\eta^* = \epsilon^{\alpha\beta}\eta^*_\alpha\overline{chi}^*_\beta \ .
\end{array} \tag{21.2.3}$$

Left-handed and right handed spinors can be combined to Lorentz vectors as

$$\eta^*_{\dot{\alpha}}\sigma^{\mu\dot{\alpha}\alpha}\eta_\alpha = -\eta^{*\alpha}\sigma^\mu_{\alpha\dot{\alpha}}\eta^{*\dot{\alpha}} \ . \tag{21.2.4}$$

The SUSY algebra at QFT limit differs from the SUSY algebra defining the fundamental anti-commutators of the fermionic oscillator operators for the induced spinor fields since the Kähler-Dirac gamma matrices defined by the Kähler action are replaced with ordinary gamma matrices. This is quite a dramatic difference and raises two questions.

The Dirac action

$$L = i\overline{\Psi}\partial_\mu\gamma^\mu\Psi - m\overline{\Psi}\Psi \quad (21.2.5)$$

for a massive particle reads in Weyl representation as

$$L = i\eta^*\partial_\mu\sigma^\mu\eta + i\overline{\chi}^*\partial_\mu\overline{\sigma}^\mu\overline{\chi} - m\overline{\chi}\eta - m\overline{\chi}^*\eta^* \ . \quad (21.2.6)$$

21.2.2 SUSY Algebras

In the following 4-D SUSY algebras are discussed first following the representation of [B74]. After that basic results about higher-dimensional SUSY algebras are listed with emphasis on 8-D case.

$D = 4$ SUSY algebras

Poincare SUSY algebra contains as super-generators transforming as Weyl spinors transforming in complex conjugate representations of Lorentz group. The basic anti-commutation relations of Poincare SUSY algebra in Weyl fermion basis can be expressed as

$$\begin{aligned}
\{Q_\alpha, Q_{\dot\beta}\} &= 2\sigma^\mu_{\alpha\dot\beta}P_\mu \ , \\
\{Q_\alpha, Q_\beta\} &= \{Q_{\dot\alpha}, Q_{\dot\beta}\} = 0 \ , \\
[Q_\alpha, P_\mu] &= [Q_{\dot\alpha}, P_\mu] = 0 \ .
\end{aligned} \quad (21.2.7)$$

By taking a trace over spinor indices one obtains expression for energy as $P^0 = \sum_i Q_i\overline{Q}_i + \overline{Q}_iQ_i$. Since super-generators must annihilated super-symmetric ground states, the energy must vanish for them.

This algebra corresponds to simplest $\mathcal{N} = 1$ SUSY in which only left-handed fermion appears. For $\mathcal{N} = 1$ SUSY the super-charges are are hermitian whereas in TGD framework super-charges carry fermion number. This implies that super-charges come in pairs of super charge so that $\mathcal{N} = 2N$ must hold true and its hermitian conjugate and only the second half of super-charges can annihilate vacuum state. Weyl spinors must also come as pairs of right- and left-handed spinors.

The construction generalizes in a straightforward manner to allow arbitrary number of fermionic generators. The most general anti-commutation relations in this case are

$$\begin{aligned}
\{Q_{i\alpha}, Q^j_{\dot\beta}\} &= 2\delta_i^j\sigma^\mu_{\alpha\dot\beta}P_\mu \ , \\
\{Q_{i\alpha}, Q_{j\beta}\} &= \epsilon_{\alpha\beta}Z_{ij}, \\
\{Q_{\dot\alpha}, Q_{\dot\beta}\} &= \epsilon^{\dot\alpha\dot\beta}Z^*_{ij} \ .
\end{aligned} \quad (21.2.8)$$

The complex constants are called central charges because they commute with all generators of the super-Poincare group.

Higher-dimensional SUSY algebras

The character of supersymmetry is sensitive to the dimension D of space-time and to the signature of the space-time metric higher dimensions [B2]. The available spinor representations depend on k; the maximal compact subgroup of the little group of the Lorentz that preserves the momentum of a massless particle is $Spin(d-1)\times Spin(D-d-1)$, where d is the number of spatial dimensions $D-d$ is the number time dimensions and k is defined as $k = 2d - D$. Due to the mod 8 Bott periodicity of the homotopy groups of the Lorentz group, really we only need to consider $k = 2d - D$ modulo 8. In TGD framework one has $D = 8$, $d = 7$ and $k = 6$.

For any value of k there is a Dirac representation, which is always of real dimension $N = [2^{1+[(2d-k)/2]}$ where $[x]$ is the greatest integer less than or equal to x. For TGD this of course gives $2^5 = 32$ corresponding to complex 8-component quark and lepton like spinors. For $-2 \leq k \leq 2$ not realized in TGD there is a real Majorana spinor representation, whose dimension is $N/2$. When k

is even (TGD) there is a Weyl spinor representation, whose real dimension is $N/2$. For $k \ mod\ 8 = 0$ (say in super-string models) there is a Majorana-Weyl spinor, whose real dimension is $N/4$. For $3 \leq k \leq 5$ so called symplectic Majorana spinor with dimension $D/2$ and for $k = 4$ symplectic Weyl-Majorana spinors with dimension $D/4$ is possible. The matrix Γ_{D+1} defined as the product of all gamma matrices has eigenvalues $\pm(-1)^{-k/2}$. The eigenvalue of Γ_{D+1} is the chirality of the spinor. CPT theorem implies that the for $D \ mod\ 4 = 0$ the numbers of left and right handed super-charges are same. For $D \ mod\ 4 = 2$ the numbers of left and right handed chiralities can be different and corresponding SUSYs are classified by $\mathcal{N} = (\mathcal{N}_L, \mathcal{N}_R)$, where $\mathcal{N}_L$ and $\mathcal{N}_R$ are the numbers of left and right handed super charges. Note that in TGD the chiralities are ± 1 and correspond to quark and leptons like spinors.

TGD does not allow super-symmetry with Majorana particles. It is indeed possible to have non-hermitian super-charges [B10] in dimension $D = 8$. In $D = 8$ SUGRA with one time dimension super-charges ar non-hermitian and Majorana particles are absent. Also in $D = 4$ SUGRA predicts super-charges are non-hermitian super-charges but Majorana particles are present.

1. $D = 8$ super-gravity corresponds to $\mathcal{N} = 2$ and allows complex super-charges $Q^i_\alpha \in 8$ and their hermitian conjugates $\overline{Q}^i_\alpha \in \overline{8}$. The group of R symmetries is $U(2)$. Bosonic fields consists the metric g_{mn}, seven real scalars, six vectors, three 2-form fields and one 3-form field. Fermionic fields consist of two Weyl (left) gravitini $\psi^{\alpha i}$, six Weyl (right) spinors plus their hermitian conjugates of opposite chirality. There are no Majorana fermions.

2. $D = 4, \mathcal{N} = 8$ SUGRA is second example allowing complex non-hermitian super-charges. The supercharges $Q^i_\alpha \in 2$ and their hermitian conjugates $\overline{Q}^i_{\dot\alpha} \in \overline{2}$. R-symmetry group is $U(8)$. Bosonic fields are metric g_{mn}, 70 real scalars and 28 vectors. Fermionic fields are 8 Majorana gravitini $\Psi^{a;i}_m$ and 56 Majorana spinors.

For $\mathcal{N} = 2N$ and at least $D = 8$ with one time dimension the super charges can be assumed to come in hermitian conjugate pairs and the non-vanishing anti-commutators can be expressed as

$$\begin{aligned} \{Q^\dagger_{i\alpha}, Q^j_{\dot\beta}\} &= 2\delta^j_i \sigma^\mu_{\alpha\dot\beta} P_\mu \ , \\ \{Q^\dagger_{i\alpha}, Q_{j\beta}\} &= \epsilon_{\alpha\beta} Z_{ij} , \\ \{Q^\dagger_{\dot\alpha}, Q_{\dot\beta}\} &= \epsilon^{\dot\alpha \ dot\beta} Z^*_{ij} \ . \end{aligned} \tag{21.2.9}$$

In this case Z_{ij} is anti-hermitian matrix. 8-D chiral invariance (separate conservation of lepton and quark numbers) suggests strongly that that the condition $Z_{ij} = 0$ must hold holds true. A given pair of super-charges is analogous to creation and annihilation operators for a given fermionic chirality. In TGD framework opposite chiralities correspond to quark and lepton like spinors.

Representations of SUSY algebras in dimension $D = 4$

The physical components of super-fields correspond to states in the irreducible representations of SUSY algebras. The representations can be constructed by using the basic anti-commutation relations for $Q_{i\alpha}$ and $Q_{j\dot\alpha}$, $i,j \in \{1,...,\mathcal{N}\}$, $\alpha, \dot\alpha \in \{1,2\}$. The representations can be classified to massive and massless ones. Also the presence of central charges affects the situation. A given irreducible representation is characterized by its ground state and R-parity assignments distinguish between representations with the same spin content, say fermion and its scalar super-partner and Higss with its fermionic super-partner.

1. In the massive case one obtains in the rest system just fermionic creation operators and $2^{\mathcal{N}}$ annihilation operators. The number of states created from a vacuum state with spin s_0 is $2\mathcal{N}$ and maximum spin is $s_0 + \mathcal{N}/2$. For instance, for $\mathcal{N} = 1$ and $s_0 = 0$ one obtains for 4 states with spins $J \leq 1/2$. Renormalizability requires massive matter to have $s \leq 1/2$ so that only $\mathcal{N} = 1$ is possible in this case. For particles massless at fundamental level and getting their masses by symmetry breaking this kind of restriction does not apply.

2. In the massless case only one half of fermionic oscillator operators have vanishing anti-commutators corresponding to the fact that for massless state only the second helicity is physical. This implies that the number of states is only $2^{\mathcal{N}}$ and the helicities vary from λ_0 to $\lambda_0 + \mathcal{N}/2$. For $\mathcal{N} = 1$ the representation is 2-dimensional.

3. In the presence of central charges $Z_{ij} = -Z_{ji}$ the representations are in general massive (Z_{ij} has dimensions of mass), $U(N)$ acts as symmetries of Z, and since Z^2 is symmetric its diagonalizability implies that Z matrix can be cast by a unitary transformation into a direct sum of 2-D antisymmetric real matrices multiplied by constants Z_i. Therefore the super-algebra can be cast in diagonal form with anti-commutators proportional to $M \pm Z_m$ with $M - Z_m \geq 0$ by unitarity. This implies the celebrated Bogomol'nyi bound $M \geq max\{Z_n\}$. For this value of varying mass parameter it is possible to have reduction of the dimension of the representation by one half. If the eigenvalues Z_n are identical the number of states is reduced to that for a massless representation. This multiplet is known as short BPS multiplet. Although BPS multiplets are massive (mass is expressible in terms of Higgs expectation value) they form multiplets shorter than the usual massive SUSY multiplets.

21.2.3 Super-Space

The heuristic view about super-space [B9] is as a manifold with D local bosonic coordinates x^μ and $\mathcal{N}D/2$ complex anti-commuting spinor coordinates θ_i^α and their complex conjugates $\overline{\theta}_{\dot{\alpha}}^i = (\theta_i^\alpha)^*$. For $\mathcal{N} = 1$, which is relevant to minimally super-symmetric standard model (MSSM), the spinors θ can also chosen to be real that is Majorana spinors, so that one has 4 bosonic and four real coordinates. In TGD framework one must however use Weyl spinors.

The anti-commutation relations for the super-coordinates are

$$\{\theta_\alpha, \theta_\beta\} = \{\theta_{\dot{\alpha}}, \theta_{\dot{\beta}}\} = \{\theta_\alpha, \theta_{\dot{\beta}}\} = 0 \ . \tag{21.2.10}$$

The integrals over super-space in 4-D $\mathcal{N} = 1$ case are defined by the following formal rules which actually state that super-integration is formally analogous to derivation.

$$\begin{gathered}
\int d\theta = \int d\bar{\theta} = \int d\theta\bar{\theta} = \int d\bar{\theta}\theta = 0 \ , \\
\int d\theta^\alpha d\theta_\beta = \delta^\alpha_\beta \ , \quad \int d\overline{\theta}_{\dot{\alpha}} d\overline{\theta}_{\dot{\beta}} = \delta^{\dot{\beta}}_{\dot{\alpha}} \ , \\
\int d^2\theta\theta^2 = \int d^2\overline{\theta}\overline{\theta}^2 \ , \quad \int d^4\theta\theta^2\overline{\theta}^2 = 1 \ .
\end{gathered} \tag{21.2.11}$$

Here the shorthand notations

$$\begin{gathered}
d^2\theta \equiv -\frac{1}{4}\epsilon_{\alpha\beta} d\theta^\alpha d\theta^\beta \ , \\
d^2\overline{\theta} \equiv -\frac{1}{4}\epsilon^{\dot{\alpha}\dot{\beta}} d\theta_{\dot{\alpha}} d\ theta_{\dot{\beta}} \ , \\
d^4\theta \equiv d^2\theta d^2\overline{\theta} \ .
\end{gathered} \tag{21.2.12}$$

are used.

The generalization of the formulas to $D > 4$ and $\mathcal{N} > 1$ cases is trivial. In infinite-dimensional case relevant for the super-symmetrization of the WCW geometry in terms of local Clifford algebra of WCW to be proposed later the infinite number of complex theta parameters poses technical problems unless one defines super-space functions properly.

Chiral super-fields

Super-multiplets can be expressed as single super-field define in super-space. Super-field can be expanded as a Taylor series with respect to the theta parameters. In 4-dimensional $\mathcal{N} = 1$ case one has

$$\Phi(x^\mu, \theta, \overline{\theta}) = \phi(x^\mu) + \theta\eta(x^\mu) + \overline{\theta}\eta^\dagger(x^\mu) + \overline{\overline{\theta}\ sigma}^\alpha\theta V_\alpha(x^\mu) + \theta^2 F(x^\mu) + \overline{\theta^2}\overline{F}(x^\mu) ... + \theta^2\overline{\theta}^2 D(x^\mu)\ . \tag{21.2.13}$$

The action of super-symmetries on super-fields can be expressed in terms of super-covariant derivatives defined as

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} - i\sigma^\mu_{\alpha\dot{\alpha}}\overline{\theta}^{\ dot\alpha}\frac{\partial}{\partial_\mu}\ \ ,\ \ \overline{D}_{\dot{\alpha}} = -\frac{\partial}{\partial\overline{\theta}^{\dot{\alpha}}} + i\theta^\alpha\sigma^\mu_{\alpha\ dot\alpha}\frac{\partial}{\partial_\mu}\ \ . \tag{21.2.14}$$

This allows very concise realization of super-symmetries.

General super-field defines a reducible representation of super-symmetry. One can construct irreducible representations of super-fields a pair of chiral and antichiral super-fields by posing the condition

$$\overline{D}_{\dot{\alpha}}\Phi = 0\ \ ,\ \ D_\alpha\Phi^\dagger = 0\ \ . \tag{21.2.15}$$

The hermitian conjugate of chiral super-field is anti-chiral.

Chiral super-fields can be expressed in the form

$$\Phi = \Phi(\theta, y^\mu)\ \ ,\ \ y^\mu = x^\mu + i\overline{\theta}\sigma^\mu\theta\ \ , y^{\mu\dagger} = x^\mu - i\overline{\theta}\sigma^\mu\theta\ \ . \tag{21.2.16}$$

These formulas generalize in a rather straightforward manner to $D > 4$ and $\mathcal{N} > 1$ case.

It is easy to check that any analytic function of a chiral super-field, call it $W(\Phi)$, is a chiral super-field. In super-symmetries its θ^2 component transforms by a total derivative so that the action defined by the super-space integral of $W(\phi)$ is invariant under super-symmetries. This allows to construct super-symmetric actions using $W(\Phi)$ and $W(\Phi^\dagger)$. The so called super-potential is defined using the sum of $W(\Phi) + W(\Phi^\dagger)$.

Analytic functions of does not give rise to kinetic terms in the action. The observation $\theta^2\overline{\theta}^2$ component of a real function of chiral super-fields transforms also as total derivative under super-symmetries allows to circumvent this problem by introducing the notion of Kähler potential $K(\Phi, \Phi^\dagger)$ as a real function of chiral super-field and its conjugate. In he simplest case one has

$$K = \sum_i \Phi_i^\dagger\Phi_i\ \ . \tag{21.2.17}$$

$L_K = \int K d^4\theta$ gives rise to simples super-symmetric action for left-handed fermion and its scalar super-partner.

Kähler potential allows an interpretation as a Kähler function defining the Kähler metric for the manifold defined by the scalars ϕ_i. This Kähler metric depends in the general case on ϕ_i and appears in the kinetic term of the super-symmetric action. Super-potential in turn can be interpreted as a counterpart of real part of a complex function which can be added to the Kähler function without affect the Kähler metric. This geometric interpretation suggests that in TGD framework every complex coordinate ϕ_i of WCW defines a chiral super-field whose bosonic part.

Wess-Zumino model as simple example

Wess-Zumino model without interaction term serves as a simple illustration of above formal considerations. The action density of Wess-Zumino Witten model can be deduced by integration Kähler potential $K = \Phi^\dagger\Phi$ for chiral super fields over theta parameters. The result is

$$L = \partial_u\phi^*\partial^\mu\phi + i\eta^*\partial^\mu\eta + F^*F \ . \tag{21.2.18}$$

The action of super-symmetry

$$\delta\Phi = \epsilon^\alpha D_\alpha\Phi \ , \quad \delta\Phi^\dagger = \overline{\epsilon}^{\dot{\alpha}}\overline{D}_{\dot{\alpha}}\Phi \ , \quad \epsilon_{\dot{\alpha}} = \epsilon^{*\alpha} \tag{21.2.19}$$

gives the transformation formulas

$$\delta\phi = \epsilon^\alpha\eta_\alpha \ , \quad \delta\eta = -i\eta^{*\dot{\alpha}}\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu\phi + \epsilon_\alpha F \ , \quad \delta F = -i\epsilon_{\dot{\alpha}}\overline{\sigma}^{\mu\dot{\alpha}\alpha} \ partial_\mu\eta_\alpha \tag{21.2.20}$$

plus their hermitian conjugates. The corresponding Noether current is indeed hermitian since the transformation parameters ϵ^α and $\overline{\epsilon}_{\dot{\alpha}} = \epsilon^{*\alpha}$ appear in it and cannot be divided away. This conserved current has as such no meaning and the statement that ground state is annihilated by the corresponding super-charge means that vacuum field configuration rather than Fock vacuum remains invariant under supersymmetries. Rather, the breaking of super-symmetry by adding a super-potential implies that F develops vacuum expectation and the vacuum solution ($\phi = 0, \eta = 0, F = constant$) of field equations is not anymore invariant super super-symmetries.

The non-hermitian parts of the super current corresponding to different fermion numbers are separately conserved and corresponding super-charges are non-Hermitian and together with other charges define a super-algebra which to my best understanding is not equivalent with the super-algebra defined by allowing the presence of anti-commuting parameters ϵ. The situation is similar in TGD where one class of non-hermitian super-currents correspond to the modes of the induced spinor fields contracted with $\overline{\Psi}$ and their conjugates. The octonionic solution ansatz for the induced spinor field allows to express the solutions in terms of two complex scalar functions so that the super-currents in question would be analogous to those of $\mathcal{N} = 2$ SUSY and one might see the super-symmetry of quantum TGD extended super-symmetry obtained from the fundamental $\mathcal{N} = 2$ super-symmetry.

Vector super-fields and supersymmetric variant of YM action

Chiral super-fields allow only the super-symmetrization of Dirac action. The super-symmetrization of YM action requires the notion of a hermitian vector super field $V = V^\dagger$, whose components correspond to vector bosons, their super-counterparts and additional degrees of freedom which cannot be dynamical. These degrees of freedom correspond gauge degrees of freedom.

In the Abelian case the gauge symmetries are realized as $V \to V + \Lambda + \Lambda^\dagger$, where Λ is a chiral super-field. These symmetries induce gauge transformations of the vector potential. Their action on chiral super-fields is $\Phi \to exp(-q\Lambda)\Phi$, $\Phi^\dagger \to \Phi^\dagger exp(-\Lambda^\dagger)$. In non-Abelian case the realization is as $exp(V) \to exp(-\Lambda^\dagger)exp(V)exp(\Lambda)$ so that the modified Kähler potential $K(\Phi^\dagger, exp(qV)\Phi)$ remains invariant.

One can assign to V a gauge invariant chiral spinor super-field as

$$\begin{aligned} W_\alpha &= -\frac{1}{4}\overline{D}^2(e^V D_\alpha e^{-V}) \ , \\ \overline{D}^2 &= \epsilon^{\dot{\alpha}\dot{\beta}}\overline{D}_{\dot{alpha}}\overline{D}_{\dot{\beta}} \end{aligned} \tag{21.2.21}$$

defining the analog of gauge field. $\overline{D}^2$ eliminates all terms the exponent of $\overline{\theta}$ is higher than that of θ since these would spoil the chiral super-field property (the anti-commutativity of super-covariant derivatives $\overline{D}_{\dot{\alpha}}$ makes this obvious). D_α in turn eliminates from the resulting scalar part so that one indeed has chiral spinor super-field. In higher dimensions and for larger value of $\mathcal{N}$ the definition of W_α must be modified in order to achieve this: what is needed is the product of all derivatives $\overline{D}_{i\dot{\alpha}}$.

The analytic functions of chiral spinor super-fields are chiral super-fields and θ^2 component of $W^\alpha W_\alpha$ transforms as a total derivatives. The super-symmetric Lagrangian of U(1) theory can be written as

$$L = \frac{1}{4g^2}\left(\int d^2\theta W^\alpha W_\alpha + \int d^2\overline{\theta} W^\dagger_{\dot{\alpha}} W^\dagger_{\dot{\alpha}}\right) . \quad (21.2.22)$$

Note that in standard form of YM action $1/2g^2$ appears.

R-symmetry

R-symmetry is an important concomitant of super-symmetry. In $\mathcal{N}=1$ case R-symmetry performs a phase rotation $\theta \to e^{i\alpha}\theta$ for the super-space coordinate θ and an opposite phase rotation for the differential $d\theta$. For $\mathcal{N}>1$ R-symmetries are $U(N)$ rotations. R-symmetry is an additional symmetry of the Lagrangian terms due to Kähler potential since both $d^4\theta$ (and its generalization) as well as Kähler potential are real. Also super-symmetric YM action is R-invariant. R-symmetry is a symmetry of if super-potential W only if it has super-charge $Q_R=2$ ($Q_R=2\mathcal{N}$) in order to compensate the super-charge of $d^{2\mathcal{N}}\theta$.

21.2.4 Non-Renormalization Theorems

Super-symmetry gives powerful constraints on the super-symmetric Lagrangians and leads to non-renormalization theorems.

The following general results about renormalization of supersymmetric gauge theories hold true (see [B74], where a heuristic justification of the non-renormalization theorems and explicit formulas are discussed).

1. Super-potential is not affected by the renormalization.
2. Kähler potential is subject to wave function renormalization in all orders. The renormalization depends on the parameters with dimensions of mass. In particular, quadratic divergences to masses cancel.
3. Gauge coupling suffers renormalization only by a constant which corresponds to one-loop renormalization. Any renormalization beyond one loop is due to wave function renormalization of the Kähler potential and it is possible to calculate the beta function exactly.

It is interesting to try to see these result from TGD perspective.

1. In TGD framework super-potential interpreted as defining the modification of WCW Kähler function, which does not affect Kähler metric and would reflect measurement interaction. The non-renormalization of W would mean that the measurement interaction is not subject to renormalization. The interpretation is in terms of quantum criticality which does not allow renormalization of the coefficients appearing in the measurement interaction term since otherwise Kähler metric of WCW would be affected.
2. The wave function renormalization of Kähler potential would correspond in TGD framework scaling of the WCW Kähler metric. Quantum criticality requires that Kähler function remains invariant. Also since no parameters with dimensions of mass are available, there is temptation to conclude that wave function renormalization is trivial.
3. Only the gauge coupling would be suffer renormalization. If one however believes in the generalization of bosonic emergence it is Kähler function which defines the SUSY QFT limit of TGD so that gauge couplings follow as predictions and their renormalization is a secondary -albeit real- effect having interpretation in terms of the dependence of the gauge coupling on the p-adic length scale. The conclusion would be that at the fundamental level the quantum TGD is RG invariant.

21.3 Does TGD Allow The Counterpart Of Space-Time Supersymmetry?

The question whether TGD allows space-time super-symmetry or something akin to it has been a longstanding problem. A considerable progress in the respect became possible with the better understanding of the Kähler-Dirac equation.

21.3.1 Kähler-Dirac Equation

Before continuing one must briefly summarize the recent view about Kähler-Dirac equation.

1. The localization of the induced spinor fields to 2-D string world sheets is crucial. It is demanded both by the well-definedness of em charge and by number theoretical constraints. Induced W boson fields must vanish, and the Frobenius integrability conditions guaranteeing that the K-D operator involves no covariant derivatives in directions normal to the string world sheet must be satisfied.

2. The Kähler-Dirac equation (or Kähler Dirac equation) reads as

$$D_K\Psi \quad = \quad 0 \ . \tag{21.3.1}$$

   in the interior of space-time surface. The boundary variation of K-D equation gives the term

$$\Gamma^n\Psi = 0 \tag{21.3.2}$$

   at the light-like orbits of partonic 2-surfaces. Clearly, Kähler-Dirac gamma matrix Γ^n in normal direction must be light-like or vanish.

3. To the boundaries of string world sheets at the orbits of partonic 2-surfaces one assigns 1-D Dirac action in induced metric line with length as bosonic counterpart. By field equations both actions vanish, and one obtains light-like geodesic carrying light-like 8-momentum. Algebraic variant of massless 8-D Dirac equation is satisfied for the 8-momentum parallel to 8-velocity.

   The boundaries of the string world sheets are thus pieces of light-like M^8 geodesics and different fermion lines should have more or less parallel M^4 momenta for the partonic 2-surface to preserve its size. This suggests strongly a connection with quantum field theory and an 8-D generalization of twistor Grassmannian approach encourages also by the very special twistorial properties of M^4 and CP_2.

   One can wonder how this relates to braiding which is one of the key ingredients of TGD. Is the braiding possible unless it is induced by particle exchanges so that the 8-momentum changes its direction and partonic 2-surface replicates. In principle it should be possible to construct the orbits of partonic 2-surfaces in such a manner that braiding occurs. Situation is the reverse of the usual in which one has fixed 3-manifold in which one constructs braid.

4. One can construct preferred extremals by starting from string world sheets satisfying the vanishing of normal components of canonical momentum currents as analogs of boundary conditions. One can also fix 3-D space-like surfaces and partonic orbits and pose the vanishing of super-symplectic charges for a sub-algebra with conformal weights coming as multiples of fixed integer n as conditions selecting preferred extremals.

5. The quantum numbers characterizing zero energy states couple directly to space-time geometry via the measurement interaction terms in Kähler action expressing the equality of classical conserved charges in Cartan algebra with their quantal counterparts for space-time surfaces in quantum superposition. This makes sense if classical charges parametrize zero modes. The localization in zero modes in state function reduction would be the WCW counterpart of state function collapse. Thermodynamics would naturally couple to the space-time geometry via the thermodynamical or quantum averages of the quantum numbers.

21.3.2 Development Of Ideas About Space-Time SUSY

Let us first summarize the recent overall view about space-time super-symmetry for TGD discussed in detail in chapter " WCW spinor structure" and also in [K88].

1. Right-handed covariantly constant neutrino spinor ν_R defines a super-symmetry in CP_2 degrees of freedom in the sense that CP_2 Dirac equation is satisfied by covariant constancy and there is no need for the usual ansatz $\Psi = D\Psi_0$ giving $D^2\Psi = 0$. This super-symmetry allows to construct solutions of Dirac equation in CP_2 [A53, A62, A43, A58].

2. In $M^4 \times CP_2$ this means the existence of massless modes $\Psi = \not{p}\Psi_0$, where Ψ_0 is the tensor product of M^4 and CP_2 spinors. For these solutions M^4 chiralities are not mixed unlike for all other modes which are massive and carry color quantum numbers depending on the CP_2 chirality and charge. As matter fact, massless right-handed neutrino covariantly constant in CP_2 spinor mode is the only color singlet. The mechanism leading to non-colored states for fermions is based on super-conformal representations for which the color is neutralized [K39, K46]. The negative conformal weight of the vacuum also cancels the enormous contribution to mass squared coming from mass in CP_2 degrees of freedom.

3. All spinor modes define conserved fermion super-currents and also the super-symplectic algebra has a fermion representation as Noether currents at string world sheets. WCW metric can be constructed as anti-commutators of super-symplectic Noether currents and one obtains a generalization of AdS/CFT duality to TGD framework from the possibility to express Kähler also in terms of Kähler function (and thus Kähler action). The fact that that super-Poincare anti-commutator vanishes for oscillator operators associated with covariantly constant right-handed neutrino and anti-neutrino implies that it corresponds to a pure gauge degree of freedom.

4. The natural conjecture is that the TGD analog space-time SUSY is generated by the Clifford algebra of the second quantized fermionic oscillator operators at string world sheets. This algebra in turn generalizes to Yangian. The oscillator operators indeed allow the 8-D analog of super-Poincare anti-commutation relations at the ends of 1-D light-like geodesics defined by the boundaries of string world sheets belonging to the orbits of partonic 2-surfaces and carrying 8-D light-like momentum.

   For incoming on mass shell particles one can identify the M^4 part of 8-momentum as gravitational for momentum equal to the inertial four-momentum assignable to imbedding space spinor harmonic for incoming on mass shell state. The square of E^4 momentum giving mass squared corresponds to the eigenvalue of CP_2 d'Alembertian.

   8-D light-like momentum forces an 8-D generalization of the twistor approach and M^4 and CP_2 are indeed unique in that they allow twistor space with Kähler structure [A63]. The conjecture is that integration over virtual momenta restricts virtual momenta to 8-D light-like momenta but the polarizations of virtual fermions are non-physical.

5. The 8-D generalization of SUSY describes also massive states and one has $\mathcal{N} = \infty$. Ordinary 4-D SUSY is obtained by restricting the states to the massless sector of the theory. The value of $\mathcal{N}$ is finite in this case and corresponds to the value of massless modes for fundamental fermions. Quark and lepton type spinor components with physical helicity for fermions and anti-fermions define the basis of the SUSY algebra as Clifford algebra of oscillator operators with anti-commutators analogous to those associated with super Poincare algebra. Therefore the generators of SUSY correspond to the $4+4$ components of imbedding space spinor modes (quarks and leptons) with vanishing conformal weight so that analogs of $\mathcal{N} = 4$ SUSY are obtained in quark and lepton sectors.

   The SUSY is broken due to the electro-weak and color interactions between the fundamental fermions. For right-handed neutrinos these interactions are not present but the mixing with left handed neutrino due to the mixing of M^4 and CP_2 gamma matrices in Kähler-Dirac gamma matrices at string world sheets implies SUSY breaking also now: also R-parity is broken.

Basically a small mixing with the states with CP_2 mass is responsible for the generation of mass and breaking of SUSY. p-Adic thermodynamics describes this mixing. SUSY is broken at QFT limit also due the replacement of the many-sheeted space-time with single slightly curved region of M^4.

6. The SUSY in question is not the conventional $\mathcal{N} = 1$ SUSY. Space-time (in the sense of Minkowski space M^4) $\mathcal{N} = 1$ SUSY in the conventional sense of the word is impossible in TGD framework since it would require require Majorana spinors. In 8-D space-time with Minkowski signature of metric Majorana spinors are definitely ruled out by the standard argument leading to super string model. Majorana spinors would also break the separate conservation of lepton and baryon numbers in TGD framework. What is remarkable is that in 8-D space-time one obtains naturally SUSY with Dirac spinors.

21.3.3 Summary About TGD Counterpart Of Space-Time SUSY

This picture allows to define more precisely what one means with the approximate super-symmetries in TGD framework.

1. One can in principle construct many-fermion states containing both fermions and anti-fermions at fermion lines located at given light-like parton orbit. The four-momenta of states related by super-symmetry need not be same. Super-symmetry breaking is present and has as the space-time correlate the deviation of the Kähler-Dirac gamma matrices from the ordinary M^4 gamma matrices. In particular, the fact that $\hat{\Gamma}^\alpha$ possesses CP_2 part in general means that different M^4 chiralities are mixed: a space-time correlate for the massivation of the elementary particles.

2. For right-handed neutrino super-symmetry breaking is expected to be smallest but also in the case of the right-handed neutrino mode mixing of M^4 chiralities takes place and breaks the TGD counterpart of super-symmetry. Maybe the correct manner to interpret the situation is to speak about 8-D massless states for which the counterpart of SUSY would not be broken but mass splittings are possible.

3. The fact that all helicities in the state are physical for a given light-like 3-surface has important implications. For instance, the addition of a right-handed antineutrino to right-handed (left-handed) electron state gives scalar (spin 1) state. Also states with fermion number two are obtained from fermions. For instance, for e_R one obtains the states $\{e_R, e_R\nu_R\overline{\nu}_R, e_R\overline{\nu}_R, e_R\nu_R\}$ with lepton numbers $(1,1,0,2)$ and spins $(1/2,1/2,0,1)$. For e_L one obtains the states $\{e_L, e_L\nu_R\overline{\nu}_R, e_L\overline{\nu}_R, e_L\nu_R\}$ with lepton numbers $(1,1,0,2)$ and spins $(1/2,1/2,1,0)$. In the case of gauge boson and Higgs type particles -allowed by TGD but not required by p-adic mass calculations- gauge boson has 15 super partners with fermion numbers $[2,1,0,-1,-2]$.

The cautious conclusion is that the recent view about quantum TGD allows the analog of super-symmetry, which is necessary broken and for which the multiplets are much more general than for the ordinary super-symmetry. Right-handed neutrinos might however define something resembling ordinary super-symmetry to a high extent. The question is how strong prediction one can deduce using quantum TGD and proposed super-symmetry.

1. For a minimal breaking of super-symmetry only the p-adic length scale characterizing the super-partner differs from that for partner but the mass of the state is same. This would allow only a discrete set of masses for various super-partners coming as half octaves of the mass of the particle in question. A highly predictive model results.

2. The quantum field theoretic description could be based on QFT limit of TGD, which I have formulated in terms of bosonic emergence. The idea was that his formulation allows to calculate the propagators of the super-partners in terms of fermionic loops. Similar description of exchanged boson as fermionic loop emerges also in the proposed identification of scattering amplitudes as representations of algebraic computations in Yangian using product and co-product as fundamental vertices assignable to partonic 2-surfaces at which 3-surfaces replicate.

3. This TGD variant of space-time super-symmetry resembles ordinary super-symmetry in the sense that selection rules due to the right-handed neutrino number conservation and analogous to the conservation of R-parity hold true (the mixing of right-handed neutrino with the left-handed one breaks R-parity). The states inside super-multiplets have identical electro-weak and color quantum numbers but their p-adic mass scales can be different. It should be possible to estimate reaction reaction rates using rules very similar to those of super-symmetric gauge theories.

4. It might be even possible to find some simple generalization of standard super-symmetric gauge theory to get rough estimates for the reaction rates. There are however problems. The fact that spins $J = 0, 1, 2, 3/2, 2$ are possible for super-partners of gauge bosons forces to ask whether these additional states define an analog of non-stringy strong gravitation. Note that graviton in TGD framework corresponds to a pair of wormhole throats connected by flux tube (counterpart of string) and for gravitons one obtains 2^8-fold degeneracy.

21.3.4 SUSY Algebra Of Fermionic Oscillator Operators And WCW Local Clifford Algebra Elements As Super-fields

Whether TGD allows space-time supersymmetry has been a long-standing question. Majorana spinors appear in $N = 1$ super-symmetric QFTs- in particular minimally super-symmetric standard model (MSSM). Majorana-Weyl spinors appear in M-theory and super string models. An undesirable consequence is chiral anomaly in the case that the numbers of left and right handed spinors are not same. For $D = 11$ and $D = 10$ these anomalies cancel, which led to the breakthrough of string models and later to M-theory. The probable reason for considering these dimensions is that standard model does not predict right-handed neutrino (although neutrino mass suggests that right handed neutrino exists) so that the numbers of left and right handed Weyl-spinors are not the same.

In TGD framework the situation is different. Covariantly constant right-handed neutrino spinor acts as a super-symmetry in CP_2. One might think that right-handed neutrino in a well-defined sense disappears from the spectrum as a zero mode so that the number of right and left handed chiralities in $M^4 \times CP_2$ would not be same. For light-like 3-surfaces covariantly constant right-handed neutrino does not however solve the counterpart of Dirac equation for a non-vanishing four-momentum and color quantum numbers of the physical state. Therefore it does not disappear from the spectrum anymore and one expects the same number of right and left handed chiralities.

In TGD framework the separate conservation of baryon and lepton numbers excludes Majorana spinors and also the the Minkowski signature of $M^4 \times CP_2$ makes them impossible. The conclusion that TGD does not allow super-symmetry is however wrong. For $\mathcal{N} = 2N$ Weyl spinors are indeed possible and if the number of right and left handed Weyl spinors is same super-symmetry is possible. In 8-D context right and left-handed fermions correspond to quarks and leptons and since color in TGD framework corresponds to CP_2 partial waves rather than spin like quantum number, also the numbers of quark and lepton-like spinors are same.

The physical picture suggest a new kind of approach to super-symmetry in the sense that the anti-commutations of fermionic oscillator operators associated with the modes of the induced spinor fields define a structure analogous to SUSY algebra in 8-D sense. Massless modes of spinors in 1-1 corresponds with imbedding space spinors with physical helicity are in 1-1 correspondence with the generators of SUSY at space-time level giving $\mathcal{N} = 4 + 4$. Right handed neutrino modes define a sub-algebra for which the SUSY is only slightly broken by the absence of weak interactions and one could also consider a theory containing a large number of $\mathcal{N} = 2$ super-multiplets corresponding to the addition of right-handed neutrinos and antineutrinos at the wormhole throat.

Masslessness condition is essential if super-symmetric quantum field theories and at the fundamental level it can be generalized to masslessness in 8-D sense in terms of Kähler-Dirac gamma matrices using octonionic representation and assuming that they span local quaternionic sub-algebra at each point of the space-time sheet. SUSY algebra has standard interpretation with respect to spin and isospin indices only at the partonic 2-surfaces so that the basic algebra should be formulated at these surfaces: in fact, out that the formulation is needed only at the ends of fermion lines. Effective 2-dimensionality would require that partonic 2-surfaces can be taken to be ends of any light-like 3-surface Y_l^3 in the slicing of the region surrounding a given wormhole throat.

Super-algebra associated with the Kähler-Dirac action

Anti-commutation relations for fermionic oscillator operators associated with the induced spinor fields are naturally formulated in terms of the Kähler-Dirac gamma matrices. The canonical anti-commutation relations for the fermionic oscillator operators at light-like 3-surfaces or at their ends can be formulated as anti-commutation relations for SUSY algebra. The algebra creating physical states is super-symplectic algebra whose generators are expressed as Noether charges assignable to strings connecting partonic 2-surfaces.

Lepton and quark like spinors are now the counterparts of right and left handed Weyl spinors. Spinors with dotted and un-dotted indices correspond to conjugate representations of $SO(3,1) \times SU(4)_L \times SU(2)_R$. The anti-commutation relations make sense for sigma matrices identified as 6-dimensional matrices $1_6, \gamma_7, \gamma_1, ...\gamma_6$.

Consider first induced spinor fields at the boundaries of string world sheets at the orbits of wormhole throats. Dirac action for induced spinor fields and its bosonic counterpart defined by line-length are required by the condition that one obtains fermionic propagators massless in 8-D sense.

1. The localization of induced spinor fields to string world sheets and the addition of 1-D Dirac action at the boundaries of string world sheets at the orbits of partonic 2-surfaces reduces the quantization to that at the end of the fermion line at partonic 2-surface located at the boundary of CD. Therefore the situation reduces to that for point particle.

2. The boundary is by the extremization of line length a geodesic line of imbedding space, which can be characterized by conserved four-momentum and conserved angular momentum like charge - call it hypercharge Y. The square of 8-velocity vanishes: $v_4^2 - (v^\phi)^2 = 0$ and one can choose $v_4^2 = 1$. 8-momentum is proportional to 8-velocity expressible as (v^k, v^ϕ).

3. Dirac equation gives $\Gamma^t \partial_t \Psi = (\gamma_k v^k + \gamma_\phi) v^\phi) \partial_t \Psi = 0$. The non-trivial solution corresponds to $\partial_t \Psi = i\omega\Psi$ and the light-likeness condition. The value of parameter ω defines the mass scale and quantum classical correspondences suggests that ω^2 gives the mass squared identifiable as the eigenvalue of CP_2 Laplacian for spinor modes.

4. Anti-commutation relations must be fixed at either end of fermion line for the oscillator operators associated with the modes of induced spinor field at string world sheet labelled by integer value conformal weight and spin and weak isospin for the H-spinor involved. These anti-commutation relations must be consistent with standard canonical quantization allowing in turn to assign Noether charges to super-symplectic algebra defined as integrals over string world sheet. The identification of WCW gamma matrices as these charges allows to calculate WCW metric as their anti-commutators.

5. The oscillator operators for the modes with different values of conformal weight vanish. Standard anti-commutation relations in massive case are completely fixed and correspond to just Kronecker delta for conformal weights, spin, and isospin.

Space-time supersymmetry and the need to generalize 4-D twistors to 8-D ones suggest the anti-commutation relations obeyed by 8-D analogs of massless Weyl spinors and thus proportional to $p_8^k \sigma_k$, where p_8^k is the 8-momentum associated with the end of the fermion line and σ_k are the 8-D analogs of 2×2 sigma matrices.

1. This requires the introduction of octonionic spinor structure with gamma matrices represented in terms of octonionic units and introducing octonionic gamma matrices. The natural condition is that the octonionic gamma matrices are equivalent with the ordinary one. This is true if fermions are localied at time-like or light-like geodesic lines of imbedding space since they represent- not only quaternionic, but even hypercomplex sub-manifolds of imbedding space. This allows ordinary matrix representations for the gamma matrices at fermion lines.

2. One can avoid the problems with the non-associativity also at string world sheets possible caused by the Kähler Dirac gamma matrices if the two Kähler Dirac gamma matrices span commutative subspace of complexified octonions. The sigma matrices appearing in induced gauge potentials could be second source of non-associativity. By assuming that the solutions are holomorphic spinors (just as in string models) and that in the gauge chosen only holomorphic or anti-holomorphic components of gauge boson fields are non-vanishing, one avoids these problems.

3. It must be admitted that the constraints on string world sheets are strong: vanishing W induced gauge fields, Frobenius integrability conditions, and the condition that K-D gamma matrices span a commutative sub-space of complexified octonions, and I have not really proven that they can be satisfied.

The super-generators of space-time SUSY are proportional to fermionic oscillator operators obeying the canonical anti-commutation relations. It is not quite clear to me whether the proportionality constant can be taken to be equal to one although intuition suggests this strongly. The anti-commutations can contain only the light-like 8-velocity at the right hand side carrying information about the direction of the fermion line.

One can wonder in how strong sense the strong form of holography is realized.

1. Is the only information about the presence of strings at the level of scattering amplitudes the information coded by the anti-commutation relations at their end points? This would be the case if the fermion super-conformal charges vanish or create zero norm states for non-vanishing conformal weights. It could however happen that also the super-conformal generators associated with a sub-algebra of conformal algebra with weights coming as integer multiples of the entire algebra do this. At least this should be the case for the super-symplectic algebra.

2. Certainly one must assume that the 8-velocities associated with the ends of the fermionic string are independent so that strings would imply bi-locality of the dynamics.

Summing up the anti-commutation relations

In leptonic sector one would have the anti-commutation relations

$$\begin{aligned} \{a^{\dagger}_{m\dot{\alpha}}, a^{n}_{\beta}\} &= 2\delta^n_m D_{\dot{\alpha}\beta} \ , \\ D &= (p_\mu + \sum_a Q^a_\mu)\sigma^\mu \ . \end{aligned} \tag{21.3.3}$$

In quark sector σ^μ is replaced with $\overline{\sigma}^\mu$ obtained by changing the signs of space-like sigma matrices in leptonic sector. p_μ and Q^a_μ are the projections of momentum and color charges in Cartan algebra to the space-time surface and their values correspond to those assignable to the fermion line and related by quantum classical correspondence to those associated with incoming spinor harmonic.

The anti-commutation relations define a generalization of the ordinary equal-time anti-commutation relations for fermionic oscillator operators to a manifestly covariant form. Extended SUSY algebra suggest that the anti-commutators could contain additional central charge term proportional to $\delta_{\alpha\beta}$ but the 8-D chiral invariance excludes this term.

In the octonionic representation of the sigma matrices matrix indices cannot be present at the right handed side without additional conditions. Octonionic units however allow a representation as matrices defined by the structure constants failing only when products of more than two octonions are considered. For the quaternionic sub-algebra this does not occur. Both spinor modes and and gamma matrices must belong to the local hyper-quaternionic sub-algebra and do trivially so for fermion lines and string. Octonionic representation reduces $SO(7,1)$ so G_2 as a tangent space group. Similar reduction for 7-dimensional compact space takes place also M-theory.

In standard SUSY local super-fields having values in the Grassmann algebra generated by theta parameters appear. In TGD framework this would mean allowance of many-fermion states at single space-time point and this is perhaps too heavy an idealization since partonic 2-surfaces are the fundamental objects. Multi-stringy generators in the extension of super-symplectic algebra to Yangian is a more natural concept in TGD framework since one expects that partonic 2-surfaces involve several strings connecting them to other partonic 2-surfaces. Super-symplectic charges would be Noether charges assignable to these strings and quantum states would be created by these charges from vacuum. Scattering amplitudes would be defined in terms of Yangian algebra [K76]. Only at QFT limit one can hope that super-field formalism works.

21.4 Understanding Of The Role Of Right-Handed Neutrino In Supersymmetry

The development of the TGD view about space-time SUSY has been like a sequence of questions loves -doesn't love- loves.... From the beginning it was clear that right-handed neutrino could generate super-conformal symmetry of some kind, and the natural question was whether it generates also space-time SUSY. Later it became clear that all fermion oscillator operators can be interpreted as super generators for the analog of space-time SUSY. After that the challenge was to understand whether all spin-isospin states of fermions correspond super generators.

$\mathcal{N}=1$ SUSY was excluded by separate conservation of B and L but $\mathcal{N}=2$ variant of this symmetry could be considered and could be generated by massless right-handed neutrino and antineutrino mode.

The new element in the picture was the physical realization of the SUSY by adding fermions - in special case right-handed neutrino - to the state associated with the orbit of partonic 2-surface. An important realization was the necessity to localized spinors to string world sheet and the assignment of fernionic oscillator operator with boundaries of string world sheets at them. Variational principles implies that the fermions have light-like 8-momenta and that the fermion lines are light-like geodesics in 8-D sense. This leads to a precise view about the quantization of induced spinor fields. Fermionic oscillator operator algebra would generate Clifford algebra replacing the SUSY algebra and one would obtain the analog of super Poincare algebra from anti-commutation relations.

21.4.1 Basic Vision

As already explained, the precise meaning of SUSY in TGD framework has been a long-standing head ache. In TGD framework SUSY is inherited from super-conformal symmetry at the level of WCW [K15, K14]. The SUSY differs from $\mathcal{N}=1$ SUSY of the MSSM and from the SUSY predicted by its generalization and by string models. Allowing only right-handed neutrinos as SUSY generators, one obtains the analog of the $\mathcal{N}=4$ SUSY in bosonic sector but there are profound differences in the physical interpretation. The most general view is that all fermion modes with vanishing conformal weights define super charges.

1. One could understand SUSY in very general sense as an algebra of fermionic oscillator operators acting on vacuum states at partonic 2-surfaces. Oscillator operators are assignable to braids ends and generate fermionic many particle states. SUSY in this sense is badly broken and the algebra corresponds to rather large $\mathcal{N}$. The restriction to covariantly constant right-handed neutrinos (in CP_2 degrees of freedom) gives rise to the counterpart of ordinary SUSY, which is more physically interesting at this moment.

2. Right handed neutrino and antineutrino are not Majorana fermions. This is necessary for separate conservation of lepton and baryon numbers. For fermions one obtains the analog $\mathcal{N}=2$ SUSY.

3. Bosonic emergence means the construction of bosons as bound states of fermions and antifermions at opposite throats of wormhole contact. Later it became clear that all elementary particles emerge as bound states of fundamental fermions located at the wormhole throats of a pair of wormhole contacts. Two wormhole contacts are required by the assumption wormhole contacts carry monopole magnetic flux stabilizing them.

   This reduces TGD SUSY to that for fundamental fermions. This difference is fundamental and means deviation from the $\mathcal{N}=4$ SUSY, where SUSY acts on gauge boson states. Bosonic representations are obtained as tensor products of representations assigned to the opposite throats of wormhole contacts. One can also have several fermion lines at given throat but these states are expected to be exotic.

Further tensor products with representations associated with the wormhole ends of magnetic flux tubes are needed to construct physical particles. This represents a crucial difference with respect to standard approach, where one introduces at the fundamental level both fermions and bosons or gauge bosons as in $\mathcal{N} = 4$ SUSY. Fermionic $\mathcal{N} = 2$ representations are analogous to "short" $\mathcal{N} = 4$ representations for which one half of super-generators annihilates the states.

4. If stringy super-conformal symmetries act as gauge transformations, the analog of $\mathcal{N} = 4$ SUSY is obtained in both quark and lepton sector. This extends to $\mathcal{N} = 8$ SUSY if parton orbits can carry both quarks and leptons. Lepto-quark is the simplest state of this kind.

5. The introduction of both fermions and gauge bosons as fundamental particles leads in quantum gravity theories and string models to $d = 10$ condition for the target space, spontaneous compactification, and eventually to the landscape catastrophe.

   For a supersymmetric gauge theory (SYM) in d-dimensional Minkowski space the condition that the number of transversal polarization for gauge bosons given by $d - 2$ equals to the number of fermionic states made of Majorana fermions gives $d - 2 = 2^k$, since the number of fermionic spinor components is always power of 2.

   This allows only $d = 3, 4, 6, 10, 16, ...$ Also the dimensions $d + 1$ are actually possible since the number of spinor components for d and $d + 1$ is same for d even. This is the standard argument leading to super-string models and M-theory. It it lost - or better to say, one gets rid of it - if the basic fields include only fermion fields and bosonic states are constructed as the tensor products of fermionic states. This is indeed the case in TGD, where spontaneous compactification plays no role and bosons are emergent.

6. Spontaneous compactification leads in string model picture from $\mathcal{N} = 1$ SUSY in say $d = 10$ to $\mathcal{N} > 1$ SUSY in $d = 4$ since the fermionic multiplet reduces to a direct sum of fermionic multiplets in $d = 4$. In TGD imbedding space is not dynamical but fixed by internal consistency requirements, and also by the condition that the theory is consistent with the standard model symmetries. The identification of space-time as 4-surface makes the induced spinor field dynamical and the notion of many-sheeted space-time allows to circumvent the objections related to the fact that only 4 field like degrees of freedom are present.

21.4.2 What Is The Role Of The Right-Handed Neutrino?

Whether right-handed neutrinos generate a supersymmetry in TGD has been a long standing open question. $\mathcal{N} = 1$ SUSY is certainly excluded by fermion number conservation but already $\mathcal{N} = 2$ defining a "complexification" of $\mathcal{N} = 1$ SUSY is possible and could generate right-handed neutrino and its antiparticle. Right-handed neutrinos should however possess a non-vanishing light-like momentum since the fully covariantly constant right-handed neutrino generates zero norm states.

The general view about the preferred extremals of Kähler action and application of the conservation of em charge to the Kähler-Dirac equation have led to a rather detailed view about classical and TGD and allowed to build a bridge between general vision about super-conformal symmetries in TGD Universe and field equations. This vision is discussed in detail in [K88].

1. Many-sheeted space-time means that single space-time sheet need not be a good approximation for astrophysical systems. The GRT limit of TGD can be interpreted as obtained by lumping many-sheeted space-time time to Minkowski space with effective metric defined as sum M^4 metric and sum of deviations from M^4 metric for various space-time sheets involved [K79]. This effective metric should correspond to that of General Relativity and Einstein's equations would reflect the underlying Poincare invariance. Gravitational and cosmological constants follow as predictions and EP is satisfied.

2. The general structure of super-conformal representations can be understood: super-symplectic algebra is responsible for the non-perturbative aspects of QCD and determines also the ground states of elementary particles determining their quantum numbers. The hierarchy of breakings of conformal symmetry as gauge gauge symmetry would explain dark matter. The sub-algebra for which super-conformal symmetry remains gauge symmetry would be isomorphic to the original algebra and generated by generators for which conformal weight is multiple of integer $n = h_{eff}/h$. This would would be true for super-symplectic algebra at least and possible for all other conformal algebras involved.

3. Super-Kac-Moody algebras associated with isometries and holonomies dictate standard model quantum numbers and lead to a massivation by p-adic thermodynamics: the crucial condition that the number of tensor factors in Super-Virasoro represention is 5 is satisfied.

4. One can understand how the Super-Kac-Moody currents assignable to stringy world sheets emerging naturally from the conservation of em charge defined as their string world sheet Hodge duals gauge potentials for standard model gauge group and also their analogs for gravitons. Also the conjecture Yangian algebra generated by Super-Kac-Moody charges emerges naturally.

5. One also finds that right handed neutrino is in a very special role because of its lacking couplings in electroweak sector and its role as a generator of the least broken SUSY. The most feasible option is that all modes of the induced spinor field are restricted to 2-D string world sheets. If covariantly constant right-handed neutrino could be de-localized completely it cannot generate ordinary kind of gauge super-symmetry. It is not yet completely clear whether the modes of the induced spinor field are localized at string world sheets also inside the Euclidian wormhole contacts defining the lines of the generalized Feynman diagrams.

   Intermediate gauge boson decay widths require that sparticles are either heavy enough or dark in the sense of having non-standard value of Planck constant. Darkness would provide an elegant explanation for their non-observability. It should be emphasized that TGD predicts that all fermions act as generators of badly broken super-symmetries at partonic 2-surfaces but these super-symmetries could correspond to much higher mass scale as that associated with the de-localized right-handed neutrino. The following piece of text summarizes the argument.

6. Ordinary SUSY means that apart from kinematical spin factors sparticles and particles behave identically with respect to standard model interactions. These spin factors would allow to distinguish between particles and sparticles. This requires strong correlations between fermion and right-handed neutrino: in fact, they should be at rest with respect to each other. Right-handed neutrinos have vanishing color and electro-weak quantum numbers. How it is possible to have sparticles as bound states with ordinary particle and right-handed neutrino?

   The localization of induced spinor fields to string world sheets suggests a solution to the problem.

   (a) The localization forces the fermions to move in parallel although they have no interactions. The 8-momenta and 8-velocities of fermion are light-like and they move along light-like 8-geodesics. Since the size of the partonic 2-surface should not change much. If all fundamental fermions involved are massive one can assume that they are at rest and in this manner geometrically stable state.

   (b) If one has massive fermion and massless right-handed neutrino, they should be at rest with respect to each other. What looks paradoxical that one cannot reduce the velocity to exactly zero in any coordinate system since covariantly constant right-handed neutrino represents a pure gauge degree of freedom. It is of course possible to assume that the relative velocity is some sufficiently low velocity. One can also argue that sparticles are unstable and that this is basically due to a geometric instability implied by the non-parallel 3-momenta of fundamental fermions.

(c) If one assumes that the 4-momentum squared corresponds to that associated with the imbedding space spinor harmonics, one can to estimate the mass of the sparticle once the energy of the right-handed neutrino is fixed. This argument applies also to n-fermion states at associated with the wormhole contact pairs.

(d) p-Adic mass calculations however give to mass squared also other contributions that that coming from the spinor harmonic, in particular negative ground state contribution and that the mass squared of the fundamental fermion vanishes for lowest states which would therefore have vanishing CP_2 velocity. Why the light-like four-momentum of the resulting state should not characterize the fermion line? In this picture p-adic thermal excitations would make the state unstable. One could in fact turn this argument to an explanation for why the stable physical particles must parallel 4-momenta.

(e) What is still not well-understood is the tachyonic contribution to four-momentum. One possibility is that wormhole contact gives imaginary contribution to four-momentum. Second possibility is that the generating super-symplectic conformal weights are the negatives for the zeros of zeta. For non-trivial zeros the real part of the conformal would be -1/2.

So called massless extremals (MEs) define massless represent classical field pattern moving with light velocity and preserving its shape. This suggests that particle represented as a magnetic flux tube structure carrying monopole flux with two wormhole contacts and sliced between two MEs could serve as a starting point in attempts to understand the role of right handed neutrinos and how $\mathcal{N}=2$ or $\mathcal{N}=4$ type SYM emerges at the level of space-time geometry.

21.4.3 The Impact From LHC And Evolution Of TGD Itself

The missing energy predicted standard SUSY seems to be absent at LHC. The easy explanation would be that the mass scale of SUSY is unexpectedly high, of order 1-10 TeV. This would however destroy the original motivations for SUSY. The arguments developed in the following manner.

1. One must distinguish between imbedding space spinor harmonics and the modes of the induced spinor field. Right-handed neutrino with vanishing color quantum numbers and thus covariantly constant in CP_2 is massless. All other modes of the induced spinor field are massive and in according to the p-adic mass calculations negative conformal weight of the ground state and the presence of Kac-Moody and super-symplectic generators make possible massless states having thermal excitations giving to the state a thermal mass. Right-handed neutrino can mix with left-handed neutrino ad can get mass. One can assign to any fermion a super-multiplet with 4 members.

   One cannot assign full super-4-plet also to non-colored right handed neutrino itself: the multiplet would contain only 3 states. The most natural possibility is that the ground state is now a color excitation of right-handed neutrino and massless non-colored right-handed neutrinos give rise to the 4-plet. The colored spinor mode at imbedding space level is however a mixture or left- and right handed neutrinos.

2. In TGD framework the natural first guess is that right-handed neutrinos carrying four-momentum can give rise to missing energy. The assumption that fermions correspond to color partial waves in H implies that color excitations of the right handed neutrino that would appear in asymptotic states are necessarily colored. It could happen that these excitations are color neutralized by super-conformal generators. If this is not the case, these neutrinos would be like quarks and color confinement would explain why they cannot be observed as asymptotic states in macroscopic scales.

   Second possibility is that SUSY itself is generated by color partial waves of right-handed neutrino, octet most naturally. This option is however not consistent with the above model for one-fermion states and their super-partners.

21.4.4 Supersymmetry In Crisis

Supersymmetry is very beautiful generalization of the ordinary symmetry concept by generalizing Lie-algebra by allowing grading such that ordinary Lie algebra generators are accompanied by super-generators transforming in some representation of the Lie algebra for which Lie-algebra commutators are replaced with anti-commutators. In the case of Poincare group the super-generators would transform like spinors. Clifford algebras are actually super-algebras. Gamma matrices anti-commute to metric tensor and transform like vectors under the vielbein group (SO(n) in Euclidian signature). In supersymmetric gauge theories one introduced super translations anti-commuting to ordinary translations.

Supersymmetry algebras defined in this manner are characterized by the number of super-generators and in the simplest situation their number is one: one speaks about $\mathcal{N}=1$ SUSY and minimal super-symmetric extension of standard model (MSSM) in this case. These models are most studied because they are the simplest ones. They have however the strange property that the spinors generating SUSY are Majorana spinors- real in well-defined sense unlike Dirac spinors. This implies that fermion number is conserved only modulo two: this has not been observed experimentally. A second problem is that the proposed mechanisms for the breaking of SUSY do not look feasible.

LHC results suggest MSSM does not become visible at LHC energies. This does not exclude more complex scenarios hiding simplest $\mathcal{N}=1$ to higher energies but the number of real believers is decreasing. Something is definitely wrong and one must be ready to consider more complex options or totally new view abot SUSY.

What is the analog of SUSY in TGD framework? I must admit that I am still fighting to gain understanding of SUSY in TGD framework [K95]. That I can still imagine several scenarios shows that I have not yet completely understood the problem but I am working hardly to avoid falling to the sin of sloppying myself.

At the basic level one has super-conformal invariance generated in the fermion sector by the super-conformal charges assignable to the strings emanating from partonic 2-surfaces and connecting them to each other. For elementary particles one has 2 wormhole contacts and 4 wormhole throats. If the number of strings is just one, one has symplectic super-conformal symmetry, which is already huge. Several strings must be allowed and this leads to the Yangian variant of super-conformal symmetry, which is multi-local (multi-stringy).

One can also say that fermionic oscillator operators generate infinite-D super-algebra. One can restrict the consideration to lowest conformal weights if spinorial super-conformal invariance acts as gauge symmetry so that one obtains a finite-D algebra with generators labelled by electro-weak quantum numbers of quarks and leptons. This super-symmetry is badly broken but contains the algebra generated by right-handed neutrino and its conjugate as sub-algebra.

The basic question is whether covariantly constant right handed neutrino generators $\mathcal{N}=\in$ SUSY or whether the SUSY is generated as approximate symmetry by adding massless right-handed neutrino to the state thus changing its four-momentum. The problem with the first option is that it the standard norm of the state is naturally proportional to four-momentum and vanishes at the limit of vanishing four-momentum: is it possible to circumvent this problem somehow? In the following I summarize the situation as it seems just now.

1. In TGD framework $\mathcal{N}=1$ SUSY is excluded since B and L and conserved separately and imbedding space spinors are not Majorana spinors. The possible analog of space-time SUSY should be a remnant of a much larger super-conformal symmetry in which the Clifford algebra generated by fermionic oscillator operators giving also rise to the Clifford algebra generated by the gamma matrices of the "world of classical worlds" (WCW) and assignable with string world sheets. This algebra is indeed part of infinite-D super-conformal algebra behind quantum TGD. One can construct explicitly the conserved super conformal charges accompanying ordinary charges and one obtains something analogous to $\mathcal{N}=\infty$ super algebra. This SUSY is however badly broken by electroweak interactions.

2. The localization of induced spinors to string world sheets emerges from the condition that electromagnetic charge is well-defined for the modes of induced spinor fields. There is however an exception: covariantly constant right handed neutrino spinor ν_R: it can be de-localized along entire space-time surface. Right-handed neutrino has no couplings to electroweak

fields. It couples however to left handed neutrino by induced gamma matrices except when it is covariantly constant. Note that standard model does not predict ν_R but its existence is necessary if neutrinos develop Dirac mass. ν_R is indeed something which must be considered carefully in any generalization of standard model.

Could covariantly constant right handed neutrinos generate SUSY?

Could covariantly constant right-handed spinors generate exact $\mathcal{N} = 2$ SUSY? There are two spin directions for them meaning the analog $\mathcal{N} = 2$ Poincare SUSY. Could these spin directions correspond to right-handed neutrino and antineutrino. This SUSY would not look like Poincare SUSY for which anti-commutator of super generators would be proportional to four-momentum. The problem is that four-momentum vanishes for covariantly constant spinors! Does this mean that the sparticles generated by covariantly constant ν_R are zero norm states and represent super gauge degrees of freedom? This might well be the case although I have considered also alternative scenarios.

What about non-covariantly constant right-handed neutrinos?

Both imbedding space spinor harmonics and the Kähler-Dirac equation have also right-handed neutrino spinor modes not constant in M^4 and localized to the partonic orbits. If these are responsible for SUSY then SUSY is broken.

1. Consider first the situation at space-time level. Both induced gamma matrices and their generalizations to Kähler-Dirac gamma matrices defined as contractions of imbedding space gamma matrices with the canonical momentum currents for Kähler action are superpositions of M^4 and CP_2 parts. This gives rise to the mixing of right-handed and left-handed neutrinos. Note that non-covariantly constant right-handed neutrinos must be localized at string world sheets.

   This in turn leads neutrino massivation and SUSY breaking. Given particle would be accompanied by sparticles containing varying number of right-handed neutrinos and antineutrinos localized at partonic 2-surfaces.

2. One an consider also the SUSY breaking at imbedding space level. The ground states of the representations of extended conformal algebras are constructed in terms of spinor harmonics of the imbedding space and form the addition of right-handed neutrino with non-vanishing four-momentum would make sense. But the non-vanishing four-momentum means that the members of the super-multiplet cannot have same masses. This is one manner to state what SUSY breaking is.

What one can say about the masses of sparticles?

The simplest form of massivation would be that all members of the super-multiplet obey the same mass formula but that the p-adic length scales associated with them are different. This could allow very heavy sparticles. What fixes the p-adic mass scales of sparticles? If this scale is CP_2 mass scale SUSY would be experimentally unreachable. The estimate below does not support this option.

One can consider the possibility that SUSY breaking makes sparticles unstable against phase transition to their dark variants with $h_{eff} = n \times h$. Sparticles could have same mass but be non-observable as dark matter not appearing in same vertices as ordinary matter! Geometrically the addition of right-handed neutrino to the state would induce many-sheeted covering in this case with right handed neutrino perhaps associated with different space-time sheet of the covering.

This idea need not be so outlandish at it looks first.

1. The generation of many-sheeted covering has interpretation in terms of breaking of conformal invariance. The sub-algebra for which conformal weights are n-tuples of integers becomes the algebra of conformal transformations and the remaining conformal generators do note represent gauge degrees of freedom anymore. They could however represent conserved conformal charges still.

2. This generalization of conformal symmetry breaking gives rise to infinite number of fractal hierarchies formed by sub-algebras of conformal algebra and is also something new and a fruit of an attempt to avoid sloppy thinking. The breaking of conformal symmetry is indeed expected in massivation related to the SUSY breaking.

The following poor man's estimate supports the idea about dark sfermions and the view that sfermions cannot be very heavy.

1. Neutrino mixing rate should correspond to the mass scale of neutrinos known to be in eV range for ordinary value of Planck constant. For $h_{eff}/h = n$ it is reduced by factor $1/n$, when mass kept constant. Hence sfermions could be stabilized by making them dark.

2. A very rough order of magnitude estimate for sfermion mass scale is obtained from Uncertainty Principle: particle mass should be higher than its decay rate. Therefore an estimate for the decay rate of sfermion could give a lower bound for its mass scale.

3. Assume the transformation $\nu_R \rightarrow \nu_L$ makes sfermion unstable against the decay to fermion and ordinary neutrino. If so, the decay rate would be dictated by the mixing rate and therefore to neutrino mass scale for the ordinary value of Planck constant. Particles and sparticles would have the same p-adic mass scale. Large h_{eff} could however make sfermion dark, stable, and non-observable.

A rough model for the neutrino mixing in TGD framework

The mixing of neutrinos would be the basic mechanism in the decays of sfermions. The following argument tries to capture what is essential in this process.

1. Conformal invariance requires that the string ends at which fermions are localized at wormhole throats are light-like curves. In fact, light-likeness gives rise to Virasosoro conditions.

2. Mixing is described by a vertex residing at partonic surface at which two partonic orbits join. Localization of fermions to string boundaries reduces the problem to a problem completely analogous to the coupling of point particle coupled to external gauge field. What is new that orbit of the particle has edge at partonic 2-surface. Edge breaks conformal invariance since one cannot say that curve is light-like at the edge. At edge neutrino transforms from right-handed to left handed one.

3. In complete analogy with $\overline{\Psi}\gamma^t A_t\Psi$ vertex for the point-like particle with spin in external field, the amplitude describing $nu_R - \nu_L$ transition involves matrix elements of form $\overline{\nu}_R\Gamma^t(CP_2)Z_t\nu_L$ at the vertex of the CP_2 part of the Kähler-Dirac gamma matrix and classical Z^0 field.

   How Γ^t is identified? The Kähler-Dirac gamma matrices associated with the interior need not be well-defined at the light-like surface and light-like curve. One basis of weak form of electric magnetic duality the Kähler-Dirac gamma matrix corresponds to the canonical momentum density associated with the Chern-Simons term for Kähler action. This gamma matrix contains only the CP_2 part.

The following provides as more detailed view.

1. Let us denote by $\Gamma^t_{CP_2}(in/out)$ the CP_2 part of the Kähler-Dirac gamma matrix at string at at partonic 2-surface and by Z^0_t the value of Z^0 gauge potential along boundary of string world sheet. The direction of string line in imbedding space changes at the partonic 2-surface. The question is what happens to the Kähler-Dirac action at the vertex.

2. For incoming and outgoing lines the equation

$$D(in/out)\Psi(in/out) = p^k(in, out)\gamma_k\Psi(in/out) \ ,$$

   where the Kähler-Dirac operator is $D(in/out) = \Gamma^t(in/out)D_t$, is assumed. ν_R corresponds to "in" and ν_R to "out". It implies that lines corresponds to massless M^4 Dirac propagator and one obtains something resembling ordinary perturbation theory.

It also implies that the residue integration over fermionic internal momenta gives as a residue massless fermion lines with non-physical helicities as one can expect in twistor approach. For physical particles the four-momenta are massless but in complex sense and the imaginary part comes classical from four-momenta assignable to the lines of generalized Feynman diagram possessing Euclidian signature of induced metric so that the square root of the metric determinant differs by imaginary unit from that in Minkowskian regions.

3. In the vertex $D(in/out)$ could act in $\Psi(out/in)$ and the natural idea is that $\nu_R - \nu_L$ mixing is due to this so that it would be described the classical weak current couplings $\overline{\nu}_R \Gamma^t_{CP_2}(out) Z^0_t(in)\nu_L$ and $\overline{\nu}_R \Gamma^t_{CP_2}(out) Z^0_t(in)\nu_L$.

To get some idea about orders of magnitude assume that the CP_2 projection of string boundary is geodesic circle thus describable as $\Phi = \omega t$, where Φ is angle coordinate for the circle and t is Minkowski time coordinate. The contribution of CP_2 to the induced metric g_{tt} is $\Delta g_{tt} = -R^2\omega^2$.

1. In the first approximation string end is a light-like curve in Minkowski space meaning that CP_2 contribution to the induced metric vanishes. Neutrino mixing vanishes at this limit.

2. For a non-vanishing value of ωR the mixing and the order of magnitude for mixing rate and neutrino mass is expected to be $R \sim \omega$ and $m \sim \omega/h$. p-Adic length scale hypothesis and the experimental value of neutrino mass allows to estimate m to correspond to p-adic mass to be of order eV so that the corresponding p-adic prime p could be $p \simeq 2^{167}$. Note that $k = 127$ defines largest of the four Gaussian Mersennes $M_{G,k} = (1+i)^k - 1$ appearing in the length scale range 10 nm -2.5 μm. Hence the decay rate for ordinary Planck constant would be of order $R \sim 10^{14}$/s but large value of Planck constant could reduced it dramatically. In living matter reductions by a factor 10^{-12} can be considered.

To sum up, the space-time SUSY in TGD sense would differ crucially from SUSY in the standard sense. There would no Majorana spinors and sparticles could correspond to dark phase of matter with non-standard value of Planck constant. The signatures of the standard SUSY do not apply to TGD. Of course, a lot of professional work would be needed to derive the signatures of TGD SUSY.

21.4.5 Right-Handed Neutrino As Inert Neutrino?

There is a very interesting posting by Jester in Resonaances with title "How many neutrinos in the sky?" (see `http://tinyurl.com/y8scxzqr`) [C1]. Jester tells about the recent 9 years WMAP data [C3] and compares it with earlier 7 years data. In the earlier data the effective number of neutrino types was $N_{eff} = 4.34 \pm 0.87$ and in the recent data it is $N_{eff} = 3.26 \pm 0.35$. WMAP alone would give $N_{eff} = 3.89 \pm 0.67$ also in the recent data but also other data are used to pose constraints on N_{eff}.

To be precise, N_{eff} could include instead of fourth neutrino species also some other weakly interacting particle. The only criterion for contributing to N_{eff} is that the particle is in thermal equilibrium with other massless particles and thus contributes to the density of matter considerably during the radiation dominated epoch.

Jester also refers to the constraints on N_{eff} from nucleosynthesis (see `http://tinyurl.com/y8fkfn5y`) , which show that $N_{eff} \sim 4$ us slightly favored although the entire range $[3, 5]$ is consistent with data.

It seems that the effective number of neutrinos could be 4 instead of 3 although latest WMAP data combined with some other measurements favor 3. Later a corrected version e `http://tinyurl.com/y9er8szf`) of the eprint appeared [C3] telling that the original estimate of N_{eff} contained a mistake and the correct estimate is $N_{eff} = 3.84 \pm 0.40$.

An interesting question is what $N_{eff} = 4$ could mean in TGD framework?

1. One poses to the modes of the Kähler-Dirac equation the following condition: electric charge is conserved in the sense that the time evolution by Kähler-Dirac equation does not mix a mode with a well-defined em charge with those with different em charge. The implication is that all modes except pure right handed neutrino are restricted at string world sheets. The

first guess is that string world sheets are minimal surfaces of space-time surface (rather than those of imbedding space). One can also consider minimal surfaces of imbedding space but with effective metric defined by the anti-commutators of the Kähler-Dirac gamma matrices. This would give a direct physical meaning for this somewhat mysterious effective metric.

For the neutrino modes localized at string world sheets mixing of left and right handed modes takes place and they become massive. If only 3 lowest genera for partonic 2-surfaces are light, one has 3 neutrinos of this kind. The same applies to all other fermion species. The argument for why this could be the case relies on simple observation [K12]: the genera g=0, 1, 2 have the property that they allow for all values of conformal moduli Z_2 as a conformal symmetry (hyper-ellipticity). For $g > 2$ this is not the case. The guess is that this additional conformal symmetry is the reason for lightness of the three lowest genera.

2. Only purely right-handed neutrino is completely de-localized in 4-volume so that one cannot assign to it genus of the partonic 2-surfaces as a topological quantum number and it effectively gives rise to a fourth neutrino very much analogous to what is called sterile neutrino. De-localized right-handed neutrinos couple only to gravitation and in case of massless extremals this forces them to have four-momentum parallel to that of ME: only massless modes are possible. Very probably this holds true for all preferred extremals to which one can assign massless longitudinal momentum direction which can vary with spatial position.

3. The coupling of ν_R is to gravitation alone and all electroweak and color couplings are absent. According to standard wisdom de-localized right-handed neutrinos cannot be in thermal equilibrium with other particles. This according to standard wisdom. But what about TGD?

   One should be very careful here: de-localized right-handed neutrinos is proposed to give rise to SUSY (not $\mathcal{N} = 1$ requiring Majorana fermions) and their dynamics is that of passive spectator who follows the leader. The simplest guess is that the dynamics of right handed neutrinos at the level of amplitudes is completely trivial and thus trivially supersymmetric. There are however correlations between four-momenta.

   (a) The four-momentum of ν_R is parallel to the light-like momentum direction assignable to the massless extremal (or more general preferred extremal). This direct coupling to the geometry is a special feature of the Kähler-Dirac operator and thus of sub-manifold gravity.

   (b) On the other hand, the sum of massless four-momenta of two parallel pieces of preferred extremals is the - in general massive - four-momentum of the elementary particle defined by the wormhole contact structure connecting the space-time sheets (which are glued along their boundaries together since this is seems to be the only manner to get rid of boundary conditions requiring vacuum extremal property near the boundary). Could this direct coupling of the four-momentum direction of right-handed neutrino to geometry and four-momentum directions of other fermions be enough for the right handed neutrinos to be counted as a fourth neutrino species in thermal equilibrium? This might be the case!

   One cannot of course exclude the coupling of 2-D neutrino at string world sheets to 4-D purely right handed neutrinos analogous to the coupling inducing a mixing of sterile neutrino with ordinary neutrinos. Also this could help to achieve the thermal equilibrium with 2-D neutrino species.

21.4.6 Experimental Evidence For Sterile Neutrino?

Many physicists are somewhat disappointed to the results from LHC: the expected discovery of Higgs has been seen as the main achievement of LHC hitherto. Much more was expected. To my opinion there is no reason for disappointment. The exclusion of the standard SUSY at expected energy scale is very far reaching negative result. Also the fact that Higgs mass is too small to be stable without fine tuning is of great theoretical importance. The negative results concerning heavy dark matter candidates are precious guidelines for theoreticians. The non-QCD like behavior in heavy ion collisions and proton-ion collisions is bypassed my mentioning something about AdS/CFT correspondence and non-perturbative QCD effects. I tend to see these effects as direct evidence for M_{89} hadron physics [K42].

In any case, something interesting has emerged quite recently. Resonaances tells that the recent analysis (see `http://tinyurl.com/ycf4vbkq`) [C2] of X-ray spectrum of galactic clusters claims the presence of monochromatic 3.5 keV photon line. The proposed interpretation is as a decay product of sterile 7 keV neutrino transforming first to a left-handed neutrino and then decaying to photon and neutrino via a loop involving W boson and electron. This is of course only one of the many interpretations. Even the existence of line is highly questionable.

One of the poorly understood aspects of TGD is right-handed neutrino, which is obviously the TGD counterpart of the inert neutrino.

1. The old idea is that covariantly constant right handed neutrino could generate $\mathcal{N}=2$ supersymmetry in TGD Universe. In fact, all modes of induced spinor field would generate superconformal symmetries but electroweak interactions would break these symmetries for the modes carrying non-vanishing electroweak quantum numbers: they vanish for ν_R. This picture is now well-established at the level of WCW geometry [K110]: super-conformal generators are labelled angular momentum and color representations plus two conformal weights: the conformal weight assignable to the light-like radial coordinate of light-cone boundary and the conformal weight assignable to string coordinate. It seems that these conformal weights are independent. The third integer labelling the states would label genuinely Yangian generators: it would tell the poly-locality of the generator with locus defined by partonic 2-surface: generators acting on single partonic 2-surface, 2 partonic 2-surfaces, ...

2. It would seem that even the SUSY generated by ν_R must be badly broken unless one is able to invent dramatically different interpretation of SUSY. The scale of SUSY breaking and thus the value of the mass of right-handed neutrino remains open also in TGD. In lack of better one could of course argue that the mass scale must be CP_2 mass scale because right-handed neutrino mixes considerably with the left-handed neutrino (and thus becomes massive) only in this scale. But why this argument does not apply also to left handed neutrino which must also mix with the right-handed one!

3. One can of course criticize the proposed notion of SUSY: wonder whether fermion + extremely weakly interacting ν_R at same wormhole throat (or interior of 3-surface) can behave as single coherent entity as far spin is considered [K95] ?

4. The condition that the modes of induced spinor field have a well-defined electromagnetic charge eigenvalue [K88] requires that they are localized at 2-D string world sheets or partonic 2-surfaces: without this condition classical W boson fields would mix the em charged and neutral modes with each other. Right-handed neutrino is an exception since it has no electroweak couplings. Unless right-handed neutrino is covariantly constant, the Kähler-Dirac gamma matrices can however mix the right-handed neutrino with the left handed one and this can induce transformation to charged mode. This does not happen if each Kähler-Dirac gamma matrix can be written as a linear combination of either M^4 or CP_2 gamma matrices and Kähler-Dirac equation is satisfied separately by M^4 and CP_2 parts of the Kähler-Dirac equation.

5. Is the localization of the modes other than covariantly constant neutrino to string world sheets a consequence of dynamics or should one assume this as a separate condition? If one wants similar localization in space-time regions of Euclidian signature - for which CP_2 type vacuum extremal is a good representative - one must assume it as a separate condition. In number theoretic formulation string world sheets/partonic 2-surfaces would be commutative/co-commutative sub-manifolds of space-time surfaces which in turn would be associative or co-associative sub-manifolds of imbedding space possessing (hyper-)octonionic tangent space structure. For this option also right-handed neutrino would be localized to string world sheets. Right-handed neutrino would be covariantly constant only in 2-D sense.

One can consider the possibility that ν_R is de-localized to the entire 4-D space-time sheet. This would certainly modify the interpretation of SUSY since the number of degrees of freedom would be reduced for ν_R.

6. Non-covariantly constant right-handed neutrinos could mix with left-handed neutrinos but not with charged leptons if the localization to string world sheets is assumed for modes carrying non-vanishing electroweak quantum numbers. This would make possible the decay of right-handed to neutrino plus photon, and one cannot exclude the possibility that ν_R has mass 7 keV.

   Could this imply that particles and their spartners differ by this mass only? Could it be possible that practically unbroken SUSY could be there and we would not have observed it? Could one imagine that sfermions have annihilated leaving only states consisting of fundamental fermions? But shouldn't the total rate for the annihilation of photons to hadrons be two times the observed one? This option does not sound plausible.

   What if one assumes that given sparticle is charactrized by the same p-adic prime as corresponding particle but is dark in the sense that it corresponds to non-standard value of Planck constant. In this case sfermions would not appear in the same vertex with fermions and one could escape the most obvious contradictions with experimental facts. This leads to the notion of shadron: shadrons would be [K95] obtained by replacing quarks with dark squarks with nearly identical masses. I have asked whether so called X and Y bosons having no natural place in standard model of hadron could be this kind of creatures.

The interpretation of 3.5 keV photons as decay products of right-handed neutrinos is of course totally ad hoc. Another TGD inspired interpretation would be as photons resulting from the decays of excited nuclei to their ground state.

1. Nuclear string model [L3] predicts that nuclei are string like objects formed from nucleons connected by color magnetic flux tubes having quark and antiquark at their ends. These flux tubes are long and define the "magnetic body" of nucleus. Quark and antiquark have opposite em charges for ordinary nuclei. When they have different charges one obtains exotic state: this predicts entire spectrum of exotic nuclei for which statistic is different from what proton and neutron numbers deduced from em charge and atomic weight would suggest. Exotic nuclei and large values of Planck constant could make also possible cold fusion [K19].

2. What the mass difference between these states is, is not of course obvious. There is however an experimental finding [C4] (see *Analysis of Gamma Radiation from a Radon Source: Indications of a Solar Influence* at `http://tinyurl.com/d9ymwm3`) that nuclear decay rates oscillate with a period of year and the rates correlate with the distance from Sun. A possible explanation is that the gamma rays from Sun in few keV range excite the exotic nuclear states with different decay rate so that the average decay rate oscillates [L3]. Note that nuclear excitation energies in keV range would also make possible interaction of nuclei with atoms and molecules.

3. This allows to consider the possibility that the decays of exotic nuclei in galactic clusters generates 3.5 keV photons. The obvious question is why the spectrum would be concentrated at 3.5 keV in this case (second question is whether the energy is really concentrated at 3.5 keV: a lot of theory is involved with the analysis of the experiments). Do the energies of excited states depend on the color bond only so that they would be essentially same for all nuclei? Or does single excitation dominate in the spectrum? Or is this due to the fact that the thermal radiation leaking from the core of stars excites predominantly single state? Could $E = 3.5$ keV correspond to the maximum intensity for thermal radiation in stellar core? If so, the temperature of the exciting radiation would be about $T \simeq E/3 \simeq 1.2 \times 10^7$ K. This in the temperature around which formation of Helium by nuclear fusion has begun: the temperature at solar core is around 1.57×10^7 K.

21.4.7 Delicacies of the induced spinor structure and SUSY mystery

The discussion of induced spinor structure leads to a modification of an earlier idea (one of the many) about how SUSY could be realized in TGD in such a manner that experiments at LHC energies could not discover it and one should perform experiments at the other end of energy spectrum at energies which correspond to the thermal energy about .025 eV at room temperature. I have the feeling that this observation could be of crucial importance for understanding of SUSY.

Induced spinor structure

The notion of induced spinor field deserves a more detailed discussion. Consider first induced spinor structures.

1. Induced spinor field are spinors of $M^4 \times CP_2$ for which modes are characterized by chirality (quark or lepton like) and em charge and weak isospin.

2. Induced spinor spinor structure involves the projection of gamma matrices defining induced gamma matrices. This gives rise to superconformal symmetry if the action contains only volume term.

   When Kähler action is present, superconformal symmetry requires that the modified gamma matrices are contractions of canonical momentum currents with imbedding space gamma matrices. Modified gammas appear in the modified Dirac equation and action, whose solution at string world sheets trivializes by super-conformal invariance to same procedure as in the case of string models.

3. Induced spinor fields correspond to two chiralities carrying quark number and lepton number. Quark chirality does not carry color as spin-like quantum number but it corresponds to a color partial wave in CP_2 degrees of freedom: color is analogous to angular momentum. This reduces to spinor harmonics of CP_2 describing the ground states of the representations of super-symplectic algebra.

   The harmonics do not satisfy correct correlation between color and electroweak quantum numbers although the triality t=0 for leptonic waves and t=1 for quark waves. There are two manners to solve the problem.

   (a) Super-symplectic generators applied to the ground state to get vanishing ground states weight instead of the tachyonic one carry color and would give for the physical states correct correlation: leptons/quarks correspond to the same triality zero(one partial wave irrespective of charge state. This option is assumed in p-adic mass calculations [K39].

   (b) Since in TGD elementary particles correspond to pairs of wormhole contacts with weak isospin vanishing for the entire pair, one must have pair of left and right-handed neutrinos at the second wormhole throat. It is possible that the anomalous color quantum numbers for the entire state vanish and one obtains the experimental correlation between color and weak quantum numbers. This option is less plausible since the cancellation of anomalous color is not local as assume in p-adic mass calculations.

The understanding of the details of the fermionic and actually also geometric dynamics has taken a long time. Super-conformal symmetry assigning to the geometric action of an object with given dimension an analog of Dirac action allows however to fix the dynamics uniquely and there is indeed dimensional hierarchy resembling brane hierarchy.

1. The basic observation was following. The condition that the spinor modes have well-defined em charge implies that they are localized to 2-D string world sheets with vanishing W boson gauge fields which would mix different charge states. At string boundaries classical induced W boson gauge potentials guarantee this. Super-conformal symmetry requires that this 2-surface gives rise to 2-D action which is area term plus topological term defined by the flux of Kähler form.

2. The most plausible assumption is that induced spinor fields have also interior component but that the contribution from these 2-surfaces gives additional delta function like contribution: this would be analogous to the situation for branes. Fermionic action would be accompanied by an area term by supersymmetry fixing modified Dirac action completely once the bosonic actions for geometric object is known. This is nothing but super-conformal symmetry.

One would actually have the analog of brane-hierarchy consisting of surfaces with dimension D= 4,3,2,1 carrying induced spinor fields which can be regarded as independent dynamical variables and characterized by geometric action which is D-dimensional analog of the action for Kähler charged point particle. This fermionic hierarchy would accompany the hierarchy of geometric objects with these dimensions and the modified Dirac action would be uniquely determined by the corresponding geometric action principle (Kähler charged point like particle, string world sheet with area term plus Kähler flux, light-like 3-surface with Chern-Simons term, 4-D space-time surface with Kähler action).

3. This hierarchy of dynamics is consistent with SH only if the dynamics for higher dimensional objects is induced from that for lower dimensional objects - string world sheets or maybe even their boundaries orbits of point like fermions. Number theoretic vision [K111] suggests that this induction relies algebraic continuation for preferred extremals. Note that quaternion analyticity [L22] means that quaternion analytic function is determined by its values at 1-D curves.

4. Quantum-classical correspondences (QCI) requires that the classical Noether charges are equal to the eigenvalues of the fermionic charges for surfaces of dimension $D = 0, 1, 2, 3$ at the ends of the CDs. These charges would not be separately conserved. Charges could flow between objects of dimension $D + 1$ and D - from interior to boundary and vice versa. Four-momenta and also other charges would be complex as in twistor approach: could complex values relate somehow to the finite life-time of the state?

   If quantum theory is square root of thermodynamics as zero energy ontology suggests, the idea that particle state would carry information also about its life-time or the time scale of CD to which is associated could make sense. For complex values of α_K there would be also flow of canonical and super-canonical momentum currents between Euclidian and Minkowskian regions crucial for understand gravitational interaction as momentum exchange at imbedding space level.

5. What could be the physical interpretation of the bosonic and fermionic charges associated with objects of given dimension? Condensed matter physicists assign routinely physical states to objects of various dimensions: is this assignment much more than a practical approximation or could condensed matter physics already be probing many-sheeted physics?

SUSY and TGD

From this one ends up to the possibility of identifying the counterpart of SUSY in TGD framework [K95, K24].

1. In TGD the generalization of much larger super-conformal symmetry emerges from the super-symplectic symmetries of WCW. The mathematically questionable notion of super-space is not needed: only the realization of super-algebra in terms of WCW gamma matrices defining super-symplectic generators is necessary to construct quantum states. As a matter of fact, also in QFT approach one could use only the Clifford algebra structure for super-multiplets. No Majorana condition on fermions is needed as for $\mathcal{N} = 1$ space-time SUSY and one avoids problems with fermion number non-conservation.

2. In TGD the construction of sparticles means quite concretely adding fermions to the state. In QFT it corresponds to transformation of states of integer and half-odd integer spin to each other. This difference comes from the fact that in TGD particles are replaced with point like particles.

3. The analog of $\mathcal{N} = 2$ space-time SUSY could be generated by covariantly constant right handed neutrino and antineutrino. Quite generally the mixing of fermionic chiralities implied by the mixing of M^4 and CP_2 gamma matrices implies SUSY breaking at the level of particle masses (particles are massless in 8-D sense). This breaking is purely geometrical unlike the analog of Higgs mechanism proposed in standard SUSY.

There are several options to consider.

1. The analog of brane hierarchy is realized also in TGD. Geometric action has parts assignable to 4-surface, 3-D light like regions between Minkowskian and Euclidian regions, 2-D string world sheets, and their 1-D boundaries. They are fixed uniquely. Also their fermionic counterparts - analogs of Dirac action - are fixed by super-conformal symmetry. Elementary particles reduce so composites consisting of point-like fermions at boundaries of wormhole throats of a pair of wormhole contacts.

   This forces to consider 3 kinds of SUSYs! The SUSYs associated with string world sheets and space-time interiors would certainly be broken since there is a mixing between M^4 chiralities in the modified Dirac action. The mass scale of the broken SUSY would correspond to the length scale of these geometric objects and one might argue that the decoupling between the degrees of freedom considered occurs at high energies and explains why no evidence for SUSY has been observed at LHC. Also the fact that the addition of massive fermions at these dimensions can be interpreted differently. 3-D light-like 3-surfaces could be however an exception.

2. For 3-D light-like surfaces the modified Dirac action associated with the Chern-Simons term does not mix M^4 chiralities (signature of massivation) at all since modified gamma matrices have only CP_2 part in this case. All fermions can have well-defined chirality. Even more: the modified gamma matrices have no M^4 part in this case so that these modes carry no four-momentum - only electroweak quantum numbers and spin. Obviously, the excitation of these fermionic modes would be an ideal manner to create spartners of ordinary particles consting of fermion at the fermion lines. SUSY would be present if the spin of these excitations couples - to various interactions and would be exact.

   What would be these excitations? Chern-Simons action and its fermionic counterpart are non-vanishing only if the CP_2 projection is 3-D so that one can use CP_2 coordinates. This strongly suggests that the modified Dirac equation demands that the spinor modes are covariantly constant and correspond to covariantly constant right-handed neutrino providing only spin.

   If the spin of the right-handed neutrino adds to the spin of the particle and the net spin couples to dynamics, $\mathcal{N} = 2$ SUSY is in question. One would have just action with unbroken SUSY at QFT limit? But why also right-handed neutrino spin would couple to dynamics if only CP_2 gamma matrices appear in Chern-Simons-Dirac action? It would seem that it is independent degree of freedom having no electroweak and color nor even gravitational couplings by its covariant constancy. I have ended up with just the same SUSY-or-no-SUSY that I have had earlier.

3. Can the geometric action for light-like 3-surfaces contain Chern-Simons term?

   (a) Since the volume term vanishes identically in this case, one could indeed argue that also the counterpart of Kähler action is excluded. Moreover, for so called massless extremals of Kähler action reduces to Chern-Simons terms in Minkowskian regions and this could happen quite generally: TGD with only Kähler action would be almost topological QFT as I have proposed. Volume term however changes the situation via the cosmological constant. Kähler-Dirac action in the interior does not reduce to its Chern-Simons analog at light-like 3-surface.

   (b) The problem is that the Chern-Simons term at the two sides of the light-like 3-surface differs by factor $\sqrt{-1}$ coming from the ratio of $\sqrt{g_4}$ factors which themselves approach to zero: oOne would have the analog of dipole layer. This strongly suggests that one should not include Chern-Simons term at all.

   Suppose however that Chern-Simons terms are present at the two sides and α_K is real so that nothing goes through the horizon forming the analog of dipole layer. Both bosonic and fermionic degrees of freedom for Euclidian and Minkowskian regions would decouple completely but currents would flow to the analog of dipole layer. This is not physically attractive.

   The canonical momentum current and its super counterpart would give fermionic source term $\Gamma^n \Psi_{int,\pm}$ in the modified Dirac equation defined by Chern-Simons term at given side $\pm$: $\pm$ refers to Minkowskian/Euclidian part of the interior. The source term is proportional to $\Gamma^n \Psi_{int,\pm}$ and Γ^n is in principle mixture of M^4 and CP_2 gamma matrices and therefore induces mixing of M^4 chiralities and therefore also 3-D SUSY breaking. It must be however emphasized that Γ^n is singular and one must be consider the limit carefully also in the case that one has only continuity conditions. The limit is not completely understood.

(c) If α_K is complex there is coupling between the two regions and the simplest assumption has been that there is no Chern-Simons term as action and one has just continuity conditions for canonical momentum current and hits super counterpart.

The cautious conclusion is that 3-D Chern-Simons term and its fermionic counterpart are absent.

4. What about the addition of fermions at string world sheets and interior of space-time surface ($D = 2$ and $D = 4$). For instance, in the case of hadrons $D = 2$ excitations could correspond to addition of quark in the interior of hadronic string implying additional states besides the states obtained assuming only quarks at string ends. Let us consider the interior ($D = 4$). For instance, inn the case of hadrons $D = 2$ excitations could correspond to addition of quark in the interior of hadronic string implying additional states besides the states obtained assuming only quarks at string ends. The smallness of cosmological constant implies that the contribution to the four-momentum from interior should be rather small so that an interpretation in terms of broken SUSY might make sense. There would be mass $m \sim .03$ eV per volume with size defined by the Compton scale $\hbar/m$. Note however that cosmological constant has spectrum coming as inverse powers of prime so that also higher mass scales are possible.

   This interpretation might allow to understand the failure to find SUSY at LHC. Sparticles could be obtained by adding interior right-handed neutrinos and antineutrinos to the particle state. They could be also associated with the magnetic body of the particle. Since they do not have color and weak interactions, SUSY is not badly broken. If the mass difference between particle and sparticle is of order $m = .03$ eV characterizing dark energy density ρ_{vac}, particle and sparticle could not be distinguished in higher energy physics at LHC since it probes much shorter scales and sees only the particle. I have already earlier proposed a variant of this mechanism but without SUSY breaking.

   To discover SUSY one should do very low energy physics in the energy range $m \sim .03$ eV having same order of magnitude as thermal energy $kT = 2.6 \times 10^{-2}$ eV at room temperature 25 °C. One should be able to demonstrate experimentally the existence of sparticle with mass differing by about $m \sim .03$ eV from the mass of the particle (one cannot exclude higher mass scales since Λ is expected to have spectrum). An interesting question is whether the sfermions associated with standard fermions could give rise to Bose-Einstein condensates whose existence in the length scale of large neutron is strongly suggested by TGD view about living matter.

21.4.8 Conclusions

The conclusion that the standard SUSY ($\mathcal{N} = 1$ SUSY with Majorana spinors) is absent in TGD Universe and also in the real one looks rather feasible in light of various arguments discussed in this chapter and also conforms with the LHC data. A more general SUSY with baryon and lepton conservation and Dirac spinors is however possible in TGD framework.

During the attempts to understand SUSY several ideas have emerged and the original discussions are retained as such in this chapter. It is interesting to see that their fate is if standard SUSY has no TGD counterpart.

1. One of the craziest ideas was that spartners indeed exists and even with the same p-adic mass scale but might be realized as dark matter. Same mass scale is indeed a natural prediction if right-handed neutrino and particle have same mass scale. Therefore even the mesons of ordinary hadron physics would be accompanied by smesons - pairs of squark and anti-squark. In fact, this is what the most recent form of the theory predicts: unfortunately there is no

manner to experimentally distinguish between fermion and pseudo-sfermion if ν_R is zero momentum state lacking even gravitational interactions.

2. There are indications that charmonium as exotic states christened as X and Y mesons and the question was that they could correspond to mesons built either from colored excitations of charged quark and antiquark or from squark and anti-squark. The recent view leaves only the option based on colored excitations alive. The states in question would be analogous to pairs of color excitations of leptons introduced to explain various anomalies in leptonic sector [K78]. The question was whether lepto-hadrons could correspond to bound states of colored sleptons and have same p-adic mass scale as leptons have [K78]. The original form of lepto-hadron hypothesis remains intact.

3. Evidence that pion and also other hadrons have what could be called infrared Regge trajectories has been reported, and one could ask whether these trajectories could include spion identified as a bound state of squarks. Also this identification is excluded and the proposed identification in terms of stringy states assignable to long color magnetic flux tubes accompanying hadron remains under consideration. IR Regge trajectories would serve as a signature for the non-perturbative aspects of hadron physics.

4. The latest idea along these lines is that spartners are obtained by adding right-handed neutrinos to the interior of space-time surface assignable to the particle. SUSY would not be detectable at high energies, which would explain the negative findings at LHC. Spartners could be discovered at low energy physics perhaps assignable to the magnetic bodies of particles: the mass scale could be as low .03 eV determined by cosmological constant in the scale of cosmology. Note however that cosmological constant has spectrum coming as inverse powers of prime.

21.5 SUSY Algebra At QFT Limit

The first expectation is that QFT limit TGD corresponds to a situation in which given space-time surface is representable as a graph for some map $M^4 \rightarrow CP_2$. This assumption is essential for the understanding of how the QFT limit of TGD emerges when many-sheeted space-time is replaced with a piece of Minkowski space in macroscopic scales and how gauge potentials of standard model relate to the induced gauge potentials. Already at elementary particle scales this assumption fails if they are regarded as pairs of wormhole contacts at distance characterized by Compton length: two sheetedness is involved in an essential manner.

This assumption is not actually needed in zero energy ontology if M^4 is assumed to label the positions of either tip of CD rather than points of the space-time sheet. The position of the other tip of CD relative to the first one could be interpreted in terms of Robertson-Walker coordinates for quantum cosmology [K67].

An intuitively plausible idea is that particle space-time sheets with Euclidian signature of the induced metric are replaced with world-lines. Fermions can be said to propagate along the boundaries of string world sheets so that this approximation would force all fermion lines of the parton orbit to form single line. Intuitively this might correspond to the replacement of multi-stringy Yangian [K76] with a super-field.

Strings bring in bi-locality at fundamental level and the hierarchy of Planck constants implies this non-locality in arbitrarily long length scales. The formation of gravitational bound states would involve gigantic values of Planck constant $h_{eff} = n \times h$ and macroscopic quantum coherence in astrophysical scales [K22, K106, K66]. This requires a generalization of quantum theory itself and of course challenges the idea that SUSY limit of TGD could make sense except in special situations.

What is essential for QFT limit is that only perturbations around single maximum of Kähler function are considered. If several maxima are important, one must include a weighting defined by the values of the exponent of Kähler function. The huge symmetries of WCW geometry are expected to make the functional integral over perturbations calculable.

21.5.1 Minimum Information About Space-Time Sheet And Particle Quantum Numbers Needed To Formulate SUSY Algebra

The basic problem is how to feed just the essential information about quantum states and space-time surfaces to the definition of the QFT limit.

1. The information about quantum numbers of particles must be fed also to the QFT approximation. It is natural to start from the classical description of point like fermions in H in terms of light-like geodesics of H at the light-like parton orbits carrying light-like 8-momentum: action principle indeed leads to this picture. Momentum and color charges serve as natural quantum numbers besides electroweak quantum numbers. The conserved color charges associated with CP_2 geodesics need not correspond to the usual color charges since they correspond to center of mass rotational motion in CP_2 degrees of freedom. Ordinary color charges correspond to the spinorial partial wavs assignable to CP_2 type extremals.

   The propagators of fundamental fermions massless in 8-D sense are the basic building bricks of the scattering amplitudes in the fundamental formulation of TGD. Elementary particles emerge as bound states of fundamental fermions, and one might of hope that the scattering amplitudes might allow also at the QFT limit a formulation involving only fundamental fermions. The basic vertices would correspond to product and co-product for super-symplectic Yangian and these 3-vertices should correspond to gauge theory vertices. The basic building brick of gauge boson would be wormhole contact with throats carrying fermion and antifermion. It might be that the QFT limit requires the introduction of boson fields. Both fermions and bosons consist of at least two wormhole contacts.

2. Should one interpret QFT limit as a QFT in X^4 representable as a graph for a map $M^4 \to CP_2$, or in M^4, or perhaps in $M^4 \times CP_2$? In zero energy ontology the proper interpretation is in terms of QFT in M^4 defining the coordinates of the M^4 projection of space-time point. Minimal Kaluza-Klein type extension to $M^4 \times S^1$ might be required in order to take into account the geodesic motion of fundamental fermions in CP_2 degrees of freedom.

3. What information about space-time surface is needed?

   (a) One can in principle feed all information about space-time sheet without losing Poincare invariance since momentum operators do not act on space-time coordinates. The description becomes however in-practical even if one restricts the consideration to the maxima of Kähler function.

   (b) Partonic two-surfaces X^2 are identified as intersections of 3-D light-like wormhole throats with the boundary of CD characterizes basic building bricks of elementary particles and elementary particle itself corresponds to space-like 3-surface at the boundary of CD. The minimal approach would use only cm degrees of freedom for the 3-surface characterizing the particle. A better accuracy would be obtained by using cm coordinates for the partonic 2-surfaces. Even better approximation would be obtained by using the positions fermions associated with given partonic 2-surface.

   (c) The ends of fermion lines defined by the boundaries of string world sheets represent necessary information but correspond to single point of M^4 in QFT approximation. The conformal moduli of the partonic 2-surface are very relevant and the elementary particle vacuum functional in the moduli space [K12] depending on the genus of the partonic 2-surface codes for a relevant information. This information could be compressed to genus its genus characterizing fermion generations plus a rule stating that the particles in the same 3-vertex have same genus and that bosons are superpositions over different genera. Only the three lowest genera have been observed and this can be understood in terms of hyper-ellipticity [K12].

   (d) Some information about zero modes characterized by the induced Kähler form invariant under quantum fluctuations assignable to Hamiltonians of $\delta M^4_\pm \times CP_2$ at boundaries of CD is certainly needed: here the identification of Kähler potential as the Kähler function of WCW is highly attractive hypothesis.

21.5.2 The Physical Picture Behind The Realization Of SUSY Algebra At Point Like Limit

The challenge is to deduce SUSY algebra in the approximation that particle like 3-surfaces are replaced by points. The basic physical constraint on the realization of the SUSY algebra come from the condition that one must be able to describe also massive particles as members of SUSY multiplets. This should make possible also 8-D counterpart of twistorialization in terms of octonionic gamma matrices reducing to quaternionic ones using representation of octonion units in terms of the structure constants of the octonionic algebra. The general structure of Kähler-Dirac action suggests how to proceed. $p^k\gamma_k$ should be replaced with a simplified version of its 8-D variant in $M^4\times CP_2$ and the CP_2 part of this operator should describe the massivation.

1. Fermion lines correspond to light-like geodesics of imbedding space. For particles which are massless in M^4, the geodesic circle defining CP_2 projection must contract to a point.

2. The generalization of the Dirac operator appearing in commutation relations reads as

$$
\begin{aligned}
p^k\gamma_k \quad &\rightarrow \quad D = p^k\gamma_k + Q\gamma_k\frac{ds^k}{ds} \quad ,\\
s_{kl}\frac{ds^k}{dt}\frac{ds^l}{dt} &= 1 \quad .
\end{aligned}
\tag{21.5.1}
$$

   Mass shell condition fixes the value of Q

$$
Q \quad = \quad \pm m \quad . \tag{21.5.2}
$$

   For geodesic circle the angle coordinate to be angle parameterizing the geodesic circle is the natural variable and the gamma matrices can be taken to be just single constant gamma matrix along the geodesic circle.

3. Imbedding space spinors have anomalous color charge equal to -1 unit for lepton and 1/3 units for quarks. Mass shell condition is satisfied if Q is proportional to anomalous hypercharge and mass of the particle in turn determined by p-adic thermodynamics. Quantum classical correspondence suggests that the square of CP_2 part of 8-momentum equals to the eigenvalue of CP_2 spinor Laplacian given the mass square of the spinor mode for an incoming particle.

4. Particle mass m should relate closely to the frequencies characterizing general extremals. Quite generally, one can write in cylindrical coordinates the general expressions of CP_2 angle variables Ψ and Φ as $(\Psi,\Phi)=(\omega_1 t+k_1z+n_1\phi...,\omega_2 t+k_2z+n_2\phi...)$. Here... denotes Fourier expansion [L2], [L2]: this corresponds to Cartan algebra of Poincare group with energy, one momentum component and angular momentum defining the quantum numbers. One can say that the frequencies define a warping of M^4 for $(\Psi,\Phi)=(\omega_1 t,\omega_2 t)$. The frequencies characterizing the warping of the canonically imbedded M^4 should closely relate to the mass of the particle. This raises the question whether the replacement of S^1 with $S^1\times S^1$ is appropriate.

5. Twistor description is also required. Generalization of ordinary twistors to octotwistor with quaternionicity condition as constraint allows to describe massive particles using almost-twistors. For massive particle the unit octonion corresponding to momentum in rest frame, the octonion defined by the polarization vector $\epsilon_k\gamma_k$, and the tangent vector $\gamma_k ds^k/ds$ (analog of polarization vector in CP_2) generate quaternionic sub-algebra. For massless particle momentum and polarization generate quaternionic sub-algebra as M^4 tangent space.

The SUSY algebra at QFT limit differs from the SUSY algebra defining the fundamental anti-commutators of the fermionic oscillator operators for the induced spinor fields since the Kähler-Dirac gamma matrices defined by the Kähler action are replaced with ordinary gamma matrices. The canonical commutation relations are however those between Ψ and its canonical momentum density $\overline{\Psi}\Gamma^t_{K-D}$ with the same right-hand side as usually (for quantum variant quantum phase appears in the anti-commutation relations). Hence the general form of anti-commutation relations are not changed in the transition and SUSY character is preserved if present in the fundamental formulation.

21.5.3 Explicit Form Of The SUSY Algebra At QFT Limit

The explicit form of the SUSY algebra follows from the proposed picture.

1. Spinor modes at X^2 correspond to the generators of the algebra. Effective 2-D property implies that spinor modes at partonic 2-surface can be assumed to have well-defined weak isospin and spin and be proportional to constant spinors.

2. The anti-commutators of oscillator operators define SUSY algebra. In leptonic sector one has

$$\begin{aligned} \{a^{\dagger}_{m\dot{\alpha}}, a^{n}_{\beta}\} &= \delta^n_m D_{\dot{\alpha}\beta} \ , \\ D &= (p^k\sigma_k + Q^a\sigma_a) \ . \end{aligned} \tag{21.5.3}$$

   Q^a denote color charges. The notions are same as in the case of WCW Clifford algebra. In quark sector one has opposite chirality and σ is replaced with $\hat{\sigma}$. Both the ordinary and octonionic representations of sigma matrices are possible.

21.5.4 How The Representations Of SUSY In TGD Differ From The Standard Representations?

The minimal super-sub-algebra generated by right-handed neutrino and antineutrino are the most interesting at low energies, and it is interesting to compare the naturally emerging representations of SUSY to the standard representations appearing in super-symmetric YM theories.

The basic new element is that it is possible to have short representations of SUSY algebra for massive states since particles are massless in 8-D sense. The mechanism causing the massivation remains open and p-adic thermodynamics can be responsible for it. Higgs mechanism could however induce small corrections to the masses.

The SUSY representations of SYM theories are constructed from $J = 0$ ground state (chiral multiplet for $\mathcal{N} = 1$ hyper-multiplet for $\mathcal{N} = 2$: more logical naming convention would be just scalar multiplet) and $J = 1/2$ ground state for vector multiplet in both cases. $\mathcal{N} = 2$ multiplet decomposes to vector and chiral multiplets of $\mathcal{N} = 1$ SUSY. Hyper-multiplet decomposes into two chiral multiplets which are hermitian conjugates of each other. The group of R-symmetries is $SU(2)_R \times U(1)_R$. In TGD framework the situation is different for two reasons.

1. The counterparts of ordinary fermions are constructed from $J = 1/2$ ground state with standard electro-weak quantum numbers associated with wormhole throat rather than $J = 0$ ground state.

2. The counterparts of ordinary bosons are constructed from $J = 0$ and $J = 1$ ground states assigned to wormhole contacts with the electroweak quantum numbers of Higgs and electroweak gauge bosons. If one poses no restrictions on bound states, the value of $\mathcal{N}$ is effectively doubled from that for representation associated with single wormhole throat.

These differences are allowed by general SUSY symmetry which allow the ground state to have arbitrary quantum numbers. Standard SYM theories however correspond to different representations so that the formalism used does not apply as such.

Consider first the states associated with single wormhole throat. The addition of right-handed neutrinos and their antineutrinos to a state with the constraint that $p^k\gamma^k$ annihilates the state at partonic 2-surface X^2 would mean that the helicities of the two super-symmetry generators are opposite. In this respect the situation is same as in the case of ordinary SUSY.

1. If one starts from $J = 0$ ground state, which could correspond to a bosonic state generated by WCW Hamiltonian and carrying $SO(2) \times SU(3)_c$ quantum numbers one obtains the counterparts of chiral/hyper- multiplets. These states have however vanishing electro-weak quantum numbers and do not couple to ordinary quarks neither.

2. If one starts $J = 1/2$ ground state one obtains the analog of the vector multiplet as in SYM but but belonging to a fundamental representation of rotation group and weak isospin group rather than to adjoint representation. For $\mathcal{N} = 1$ one obtains the analog of vector chiral multiplet but containing spins $J = 1/2$ and $J = 1$. For $\mathcal{N} = 2$ on obtains two chiral multiplets with $(J, F, R) = (1, 2, 1)$ and $(J, F, R) = (1/2, 1, 0)$ and $(J, F, R) = (0, 0, -1)$ and $(-1/2, 1, 0) = (0, 0, 0)$.

3. It is possible to have standard SUSY multiplet if one assumes that the added neutrino has always fermionic number opposite that the fermion in question. In this case on obtains $\mathcal{N} = 1$ scalar multiplet. This option could be defended by stability arguments and by the fact that it does not put right-handed neutrino itself to a special role.

For the states associated with wormhole contact zero energy ontology allows to consider two non-equivalent options. The following argument supports the view that gauge bosons are obtained as wormhole throats only if the throats correspond to different signs of energy.

1. For the first option the both throats correspond to positive energies so that spin 1 bosons are obtained only if the fermion and anti-fermion associated with throats have opposite M^4 chirality in the case that they are massless (this is important!). This looks somewhat strange but reflects the fact that $J = 1$ states constructed from fermion and anti-fermion with same chirality and parallel 4-momenta have longitudinal polarization. If the ground state has longitudinal polarization the spin of the state is due to right-handed neutrinos alone: in this case however spin 1 states would have fermion number 2 and -2.

2. If the throats correspond to positive and negative energies the momenta are related by time reflection and physical polarizations for the negative energy anti-fermion corespond to non-physical polarizations of positive energy anti-fermion. In this case physical polarizations are obtained.

If one assumes that the signs of the energy are opposite for the wormhole throats, the following picture emerges.

1. If fermion and anti-fermion correspond to $\mathcal{N} = 2$-dimensional representation of super-symmetry, one expects $2\mathcal{N} = 4$ gauge boson states obtained as a tensor product of two hyper-multiplets if bound states with all possible quantum number combinations are possible. Taking seriously the idea that only the bound states of fermion and anti-fermion are possible, one is led to consider the idea that the wormhole throats carry representations of $\mathcal{N} = 1$ super-symmetry generated by M^4 Weyl spinors with opposite chiralities at the two wormhole throats (right-handed neutrino and its antineutrino). This would give rise to a vector representation and eliminate a large number of exotic quantum number combinations such as the states with fermion number equal to two and also spin two states. This idea makes sense a also for a general value of $\mathcal{N}$. Bosonic representation could be also seen as the analog of short representation for $\mathcal{N} = 2N$ super-algebra reducing to a long representation $\mathcal{N} = N$. Short representations occur quite generally for the massive representations of SUSY and super-conformal algebras when 2^r generators annihilate the states [B67].

   Note that in TGD framework the fermionic states of vector and hyper multiplets related by $U(2)_R$ R-symmetry differ by a $\nu_R\overline{\nu}_R$ pair whose members are located at the opposite throats of the wormhole contact.

2. If no restrictions on the quantum numbers of the boson like representation are posed, zero energy ontology allows to consider also an alternative interpretation. $\mathcal{N}=4$ (or more generally, $\mathcal{N}=2N$-) super-algebra could be interpreted as a direct sum of positive and negative energy super-algebras assigned to the opposite wormhole throats. Boson like multiplets could be interpreted as a long representation of the full algebra and fermionic representations as short representations with states annihilated either by the positive or negative energy part of the super-algebra. The central charges Z_{ij} must vanish in order to have a trivial representations with $p^k=0$. This is expected since the representations are massless in the generalized sense.

3. Standard $\mathcal{N}=2$ multiplets are obtained if one assume that right-handed neutrino has always opposite fermion number than the fermion at the throat. The arguments in favor of this option have been already given.

Chapter 22

Coupling Constant Evolution in Quantum TGD

22.1 Introduction

In quantum TGD two kinds of discrete coupling constant evolutions emerge. p-Adic coupling constant evolution is with respect to the discrete hierarchy of p-adic length scales and p-adic length scale hypothesis suggests that only the length scales coming as half octaves of a fundamental length scale are relevant here. Second coupling constant evolution corresponds to hierarchy of Planck constants requiring a generalization of the notion of imbedding space. One can assign this evolution with angle resolution in number theoretic approach. It is now clear that the two evolutions can be understood as different aspects of number theoretic evolution defined by a hierarchy of algebraic extensions of rationals.

This picture is inspired by quantum criticality of TGD Universe realized concretely as a hierarchy of supersymplectic symmetry breakings with sub-algebra of the entire super-symplectic algebra with conformal weights coming as n-multiples of those of the entire algebras acting as conformal gauge symmetries. Number theoretic coupling constant evolution is discrete: various coupling constant parameters depend on algebraic extension but are RG invariant for a given extension. Phase transitions between extensions give rise to number theoretic RG evolution. It should be possible to express number theoretic coupling constant evolution in terms of the parameters of extension: such as ramified prime defining p-adic primes (and p-adic length scales) and the degree of the polynomial defining the extension and defining angle resolution.

The continuous coupling constant evolution of quantum field theories follows at GRT limit when many-sheeted space-time is approximated by GRT space-time by replacing sheets with single slightly curved region of Minkowski space with gravitational and gauge fields identified as sums of those for the sheets.

The notion of zero energy ontology allows to justify p-adic length scale hypothesis and formulate the discrete coupling constant evolution at fundamental level. WCW would consists of sectors associated with causal diamonds (CDs) identified as intersections of future and past directed light-cones. If the sizes of CDs come in powers of 2^n, p-adic length scale hypothesis emerges, and coupling constant evolution is discrete provided RG invariance holds true inside CDs for space-time evolution of coupling constants defined in some sense to be defined It is however clear that all integer scalings of CDs are allowed and p-adic length scale hypothesis is prediction rather than input. In this chapter arguments supporting this conclusion are given by starting from a detailed vision about the basic properties of preferred extremals of Kähler action.

22.1.1 New Ingredients Helping To Understand Coupling Constant Evolution

How to calculate or at least “understand” the correlation functions and coupling constant evolution has remained a basic unresolved challenge. Basically the in-ability to calculate is of course due to the lack of understanding.

ZEO, the construction of M-matrix as time like entanglement coefficients defining Connes tensor product characterizing finite measurement resolution in terms of inclusion of hyper-finite factors of type II_1, the realization that symplectic invariance of N-point functions providing a detailed mechanism eliminating UV divergences, and the understanding of the relationship between super-symplectic and super Kac-Moody symmetries. As already mentioned, continuous coupling constant evolution is replaced by a discrete number theoretical coupling constant evolution.

These ideas were seen as the most important pieces of the puzzle. Their combination was thought to make possible a rather concrete vision about coupling constant evolution in TGD Universe and one can even speak about rudimentary form of generalized Feynman rules.

This was the picture behind previous updating. Several steps of progress have however occurred since then.

1. A crucial step in progress has been the understanding of how GRT space-time emerges from the many-sheeted space-time of TGD. At classical level Equivalence Principle (EP) follows from the interpretation of GRT space-time as effective space-time obtained by replacing many-sheeted space-time with Minkowski space with effective metric determined as a sum of Minkowski metric and sum over the deviations of the induced metrics of space-time sheets from Minkowski metric. Poincare invariance suggests strongly classical EP for the GRT limit in long length scales at least. One can consider also other kinds of limits such as the analog of GRT limit for Euclidian space-time regions assignable to elementary particles. In this case deformations of CP_2 metric define a natural starting point and CP_2 indeed defines a gravitational instanton with very large cosmological constant in Einstein-Maxwell theory. Also gauge potentials of standard model correspond classically to superpositions of induced gauge potentials over space-time sheets.

   The coupling constant evolution in QFT sense is in this framework an approximate notion emerging when TGD space-time is replaced with GRT space-time.

2. Second powerful (possibly too strong) idea is quantum classical correspondence in statistical sense stating that the statistical properties of a preferred extremal in quantum superposition of them are same as those of the zero energy state in question. This principle would be quantum generalization of ergodic theorem stating that the time evolution of a single member of ensemble represents the ensemble statistically. This principle would allow to deduce correlation functions and S-matrix from the statistical properties of single preferred extremal alone using classical intuition. Also coupling constant evolution would be coded by the statistical properties of the representative preferred extremal.

   This idea can be formulated more convincingly in terms of a generalization of the AdS/CFT duality to TGD framework motivated by the generalization of conformal symmetry. In full generality this principle would state that all predictions of the theory can be expressed either in terms of classical fields in the interior of the space-time surface or in terms of scattering amplitudes formulated in terms of fundamental fermions defining the building bricks of elementary particles. The implication would that correlation functions can be also identified as those for classical induce gauge and gravitational fields.

3. The third ingredient is ZEO leading to a rather concrete picture about the architecture of scattering amplitudes. The basic notions are U-, M, and S-matrix. M-matrix is defined between positive and negative energy parts of zero energy states and essential for the definition of zero energy states. M-matrix is a product of hermitian square root of density matrix and of unitary S-matrix, whose powers corresponds to the standard S-matrix with positive integer exponent taking the role of discretized time. U-matrix is realized betweeen zero energy states and is the analog of unitary time evolution operator acting in the moduli space of CDs and associated zero energy states.

   U-matrix is expressible in M-matrices [K91] so that the basic matrix to be constructed is S-matrix. S-matrix should be constructible by a generalization the twistorial approach possible only for $M^4 \times CP_2$, whose Cartesian factors are the only 4-D manifolds for which twistor spaces are Kähler manifolds [K76]. The huge symmetries and the close analogies with ordinary Grassmann twistorial program raise hopes about quite concrete construction.

4. Fourth powerful vision inspired by the notion of preferred extremal - I gave up the vision for years as too crazy - is that scattering amplitudes correspond to sequences of computations and that all computations connecting collections of algebraic objects produce same scattering amplitudes [K76, K6]. All scattering amplitudes could be reduced to minimal tree diagrams by moving the ends of the lines and snipping away the loops: this means a huge generalization of the duality symmetry of hadronic string models. The 8-D generalization of twistor approach to TGD allows to identify the arithmetics as that of super-symplectic Yangian and basic vertices in the construction correspond to product and co-product in Yangian.

5. The fifth new ingredient is the dramatic increase in the number theoretical understanding of the hierarchy of Planck constants $h_{eff} = n \times h$. The hierarchy corresponds to hierarchy of quantum criticalities at which the sub-algebra of super-symplectic algebra with natural conformal structure changes. Sub-algebras are labelled by integer n: the conformal weights of the sub-algebra come as multiples of n. One has infinite number of hierarchies $n_{i+1} = \prod_{k<i+1} m_k$ which relate naturally to the hierarchies of inclusions of hyper-finite factors. The sub-algebra acts as gauge symmetries whereas the other generators of the full algebra fail to do so. Therefore the increase of n means that gauge degrees of freedom become physical ones. One can assign coupling constant evolution also with these hierarchies and the natural conjecture is that coupling constants for given value of n are renormalization group invariances.

   Especially interesting are the implications for the understanding of gravitational binding assuming that strings connecting partonic 2-surfaces are responsible for the formation of bound states. This leads together with the generalization of AdS/CFT corresponds and localization of fermions to string world sheets to a prediction that Kähler action is expressible as string area in the effective metric defined by the anti-commutators of Kähler-Dirac gamma matrices. This predicts that the size scale of bound states scales as h_{eff} and it is possible to obtain bound states of macroscopic size unlike for ordinary string area action for which their sizes would be given by Planck length.

6. The original picture was that there are two separate evolutions: one associated with p-adic length scale hierarchy and second associated with angle resolution. It is now clear that these two evolutions can be unified to a number theoretic evolution in terms of increasing complexity of an algebraic extension of rational numbers inducing also the extensions of p-adic number fields. Space-time and quantum physics become adelic. The algebraic extensions are associated with the parameters characterizing partonic 2-surfaces and string world sheets, which by strong form of holography determine space-time surfaces as preferred extremals of Kähler action. In this framework the crucial number theoretical universality necessary for adelization is almost trivially realized by algebraic continuation from the intersection of realities and p-adicities defined by the 2-surfaces with parameters in algebraic extensions of rationals.

   The existence of preferred p-adic primes can be understood in this picture: they correspond to the so called ramified primes of the algebraic extension. One can also deduce a generalization of p-adic length scale hypothesis in terms of Negentropy Maximization Principle (NMP) [K41]. One might hope that all basic building bricks have been identified.

22.1.2 A Sketch For The Coupling Constant Evolution

The following summarizes the basic vision about coupling constant evolution. Needless to say that it involved a lot of guesses and should be taken only as a sketch.

p-Adic evolution in phase resolution and the spectrum of values for Planck constants

The quantization of Planck constant has been the basic theme of TGD. The basic idea was that different values of Planck constant could relate to the evolution in angular resolution in p-adic context characterized by quantum phase $q = exp(i\pi/n)$ characterizing Jones inclusion is. The higher the value of n, the better the angular resolution since the number of different complex phases in extension of p-adic numbers increases with n.

The breakthrough became with the realization that standard type Jones inclusions lead to a detailed understanding of what is involved and predict very simple spectrum for Planck constants associated with M^4 and CP_2 degrees of freedom. This picture allows to understand also gravitational Planck constant and coupling constant evolution and leads also to the understanding of ADE correspondences (index $\beta \leq 4$ and $\beta = 4$) from the point of view of Jones inclusions.

The most recent view about coupling constant evolution

In classical TGD only Kähler coupling constant appears explicitly but does not affect the classical dynamics. Other gauge couplings do not appear at all in classical dynamics since the the definition of classical fields absorbs them as normalization constants. Hence the notion of continuous coupling constant evolution at space-time level is not needed nor makes sense in quantum TGD proper and emerges only at the QFT limit when space-time is replaced with general relativistic effective space-time.

Discrete p-adic coupling constant evolution replacing in TGD the ordinary continuous coupling constant evolution emerges only when space-time sheets are lumped together to define GRT space-time. This evolution would have as parameters the p-adic length scale characterizing the causal diamond (CD) associated with particle and the phase factors characterizing the algebraic extension of p-adic numbers involved.

The p-adic prime and therefore also the length scale and coupling constants characterizing the dynamics for given CD would vary wildly as function of integer characterizing CD size scale. This could mean that the CDs whose size scales are related by multiplication of small integer are close to each other. They would be near to each other in logarithmic sense and logarithms indeed appear in running coupling constants. This “prediction” is of course subject to criticism.

Zero energy ontology, the construction of M-matrix as time like entanglement coefficients defining Connes tensor product characterizing finite measurement resolution in terms of inclusion of hyper-finite factors of type II_1, the realization that symplectic invariance of N-point functions provides a detailed mechanism eliminating UV divergences, and the understanding of the relationship between super-symplectic and super Kac-Moody symmetries: these are the pieces of the puzzle whose combination might make possible a concrete vision about coupling constant evolution in TGD Universe and one can even speak about rudimentary form of generalized Feynman rules.

The work during 2016-2017 with adelic TGD [L22, L24, L38] has led to a purely number theoretic view about coupling constant evolution. Coupling constant evolution is discrete and the phase transitions changing the values of coupling parameters correspond to changes for the extension of rationals inducing the extensions of p-adic number fields defining together with reals the adele. $h_{eff}/h = n$ can be identified as the dimension of the extension dividind the order of its Galois group.

The earlier proposal discussed also below is that gravitational coupling could be understood in terms of Kähler coupling strength and p-adic length scale hypothesis. The twistor lift of TGD however introduces Planck length as fundamental length scale assignable to the twistor sphere of twistor bundle $M^4 \times S^2$ of M^4. The value of the ratio $l_P^2/R^2(CP_2)$ remains to be predicted and could follow from quantum criticality.

p-Adic length scale evolution of gauge couplings

Understanding the dependence of gauge couplings constants on p-adic prime is one of the basic challenges of quantum TGD. The problem has been poorly understood even at the conceptual level to say nothing about concrete calculations. The generalization of the motion of S-matrix to that of M-matrix changed however the situation [K13] . M-matrix is always defined with respect to measurement resolution characterized in terms of an inclusion of von Neumann algebra. Coupling constant evolution reduces to a discrete evolution involving only octaves of $T(k) = 2^k T_0$ of the fundamental time scale $T_0 = R$, where R CP_2 scale. p-Adic length scale $L(k)$ is related to $T(k)$ by $L^2(k) = T(k)T_0$. p-Adic length scale hypothesis $p \simeq 2^k$, k integer, is automatic prediction of the theory. There is also a close connection with the description of coupling constant evolution in terms of radiative corrections.

If RG invariance at given space-time sheet holds true, the question arises whether it is possible to understand p-adic coupling constant evolution at space-time level and why certain p-adic primes are favored.

1. Simple considerations lead to the idea that M^4 scalings of the intersections of 3-surfaces defined by the intersections of space-time surfaces with light-cone boundary induce transformations of space-time surface identifiable as RG transformations. If sufficiently small they leave gauge charges invariant: this seems to be the case for known extremals which form scaling invariant families. When the scaling corresponds to a ratio p_2/p_1, $p_2 > p_1$, bifurcation would become possible replacing p_1-adic effective topology with p_2-adic one.

2. Stability considerations determine whether p_2-adic topology is actually realized and could explain why primes near powers of 2 are favored. The renormalization of coupling constant would be dictated by the requirement that Q_i/g_i^2 remains invariant.

The chapter decomposes into sections. In the first part basic notions are introduced and a general vision about coupling constant evolution is introduced. After that a general formulation of coupling constant evolution at space-time level and related interpretational issues are considered. In the second part quantitative predictions involving some far from rigorous arguments, which I however dare to take half-seriously, are discussed. It must be emphasized that this chapter like many others is more like a still continuing story about development of ideas - not a brief summary about a solution of a precisely defined problem. What I take very seriously is the general vision discussed above, addition of details to end up with formulas tends to lead to all kinds of fuzziness. There are many ad hoc ideas and conflicting views. These books are just lab note books - nothing more.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [L12].

22.2 Summary Of Basic Ideas Of Quantum TGD

22.2.1 General Ideas Of Quantum TGD

TGD relies heavily on geometric ideas, which have gradually generalized during the years. Symmetries play a key role as one might expect on basis of general definition of geometry as a structure characterized by a given symmetry.

Physics as infinite-dimensional Kähler geometry

1. The basic idea is that it is possible to reduce quantum theory to configuration space geometry and spinor structure. The geometrization of loop spaces inspires the idea that the mere existence of Riemann connection fixes configuration space Kähler geometry uniquely. Accordingly, configuration space can be regarded as a union of infinite-dimensional symmetric spaces labeled by zero modes labeling classical non-quantum fluctuating degrees of freedom.

   The huge symmetries of the configuration space geometry deriving from the light-likeness of 3-surfaces and from the special conformal properties of the boundary of 4-D light-cone would guarantee the maximal isometry group necessary for the symmetric space property. Quantum criticality is the fundamental hypothesis allowing to fix the Kähler function and thus dynamics of TGD uniquely. Quantum criticality leads to surprisingly strong predictions about the evolution of coupling constants.

2. WCW spinors correspond to Fock states and anti-commutation relations for fermionic oscillator operators correspond to anti-commutation relations for the gamma matrices of the configuration space. WCW gamma matrices contracted with Killing vector fields give rise to a super-algebra which together with Hamiltonians of the configuration space forms what I have used to called super-symplectic algebra.

   Super-symplectic degrees of freedom represent completely new degrees of freedom and have no electroweak couplings. In the case of hadrons super-symplectic quanta correspond to what has been identified as non-perturbative sector of QCDthey define TGD correlate for the degrees of freedom assignable to hadronic strings. They are responsible for the most of the mass of hadron and resolve spin puzzle of proton.

Besides super-symplectic symmetries there are Super-Kac Moody symmetries assignable to light-like 3-surfaces and together these algebras extend the conformal symmetries of string models to dynamical conformal symmetries instead of mere gauge symmetries. The construction of the representations of these symmetries is one of the main challenges of quantum TGD. The assumption that the commutator algebra of these super-symplectic and super Kac-Moody algebras annihilates physical states gives rise to Super Virasoro conditions which could be regarded as analogs of configuration space Dirac equation.

Modular invariance is one aspect of conformal symmetries and plays a key role in the understanding of elementary particle vacuum functionals and the description of family replication phenomenon in terms of the topology of partonic 2-surfaces.

3. WCW spinors define a von Neumann algebra known as hyper-finite factor of type II_1 (HFFs). This realization has led also to a profound generalization of quantum TGD through a generalization of the notion of imbedding space to characterize quantum criticality. The resulting space has a book like structure with various almost-copies of imbedding space representing the pages of the book meeting at quantum critical sub-manifolds.

p-Adic physics as physics of cognition

p-Adic mass calculations relying on p-adic length scale hypothesis led to an understanding of elementary particle masses using only super-conformal symmetries and p-adic thermodynamics. The need to fuse real physics and various p-adic physics to single coherent whole led to a generalization of the notion of number obtained by gluing together reals and p-adics together along common rationals and algebraics. The interpretation of p-adic space-time sheets is as correlates for cognition. p-Adic and real space-time sheets intersect along common rationals and algebraics and the subset of these points defines what I call number theoretic braid in terms of which both configuration space geometry and S-matrix elements should be expressible. Thus one would obtain number theoretical discretization which involves no ad hoc elements and is inherent to the physics of TGD.

Perhaps the most dramatic implication relates to the fact that points, which are p-adically infinitesimally close to each other, are infinitely distant in the real sense (recall that real and p-adic imbedding spaces are glued together along rational imbedding space points). This means that any open set of p-adic space-time sheet is discrete and of infinite extension in the real sense. This means that cognition is a cosmic phenomenon and involves always discretization from the point of view of the real topology. The testable physical implication of effective p-adic topology of real space-time sheets is p-adic fractality meaning characteristic long range correlations combined with short range chaos.

Also a given real space-time sheets should correspond to a well-defined prime or possibly several of them. The classical non-determinism of Kähler action should correspond to p-adic non-determinism for some prime(s) p in the sense that the effective topology of the real space-time sheet is p-adic in some length scale range. p-Adic space-time sheets with same prime should have many common rational points with the real space-time and be easily transformable to the real space-time sheet in quantum jump representing intention-to-action transformation. The concrete model for the transformation of intention to action leads to a series of highly non-trivial number theoretical conjectures assuming that the extensions of p-adics involved are finite-dimensional and can contain also transcendentals.

An ideal realization of the space-time sheet as a cognitive representation results if the CP_2 coordinates as functions of M^4_+ coordinates have the same functional form for reals and various p-adic number fields and that these surfaces have discrete subset of rational numbers with upper and lower length scale cutoffs as common. The hierarchical structure of cognition inspires the idea that S-matrices form a hierarchy labeled by primes p and the dimensions of algebraic extensions.

The number-theoretic hierarchy of extensions of rationals appears also at the level of configuration space spinor fields and allows to replace the notion of entanglement entropy based on Shannon entropy with its number theoretic counterpart having also negative values in which case one can speak about genuine information. In this case case entanglement is stable against Negentropy Maximization Principle stating that entanglement entropy is minimized in the self measurement and can be regarded as bound state entanglement. Bound state entanglement makes possible macro-temporal quantum coherence. One can say that rationals and their finite-dimensional extensions define islands of order in the chaos of continua and that life and intelligence correspond to these islands.

TGD inspired theory of consciousness and number theoretic considerations inspired for years ago the notion of infinite primes [K72]. It came as a surprise, that this notion might have direct relevance for the understanding of mathematical cognition. The ideas is very simple. There is infinite hierarchy of infinite rationals having real norm one but different but finite p-adic norms. Thus single real number (complex number, (hyper-)quaternion, (hyper-)octonion) corresponds to an algebraically infinite-dimensional space of numbers equivalent in the sense of real topology. Space-time and imbedding space points ((hyper-)quaternions, (hyper-)octonions) become infinitely structured and single space-time point would represent the Platonia of mathematical ideas. This structure would be completely invisible at the level of real physics but would be crucial for mathematical cognition and explain why we are able to imagine also those mathematical structures which do not exist physically. Space-time could be also regarded as an algebraic hologram. The connection with Brahman=Atman idea is also obvious.

Hierarchy of Planck constants and dark matter hierarchy

The identification of dark matter as phases having large value of Planck constant [K66, K22, K17] led to a vigorous evolution of ideas. Entire dark matter hierarchy with levels labelled by increasing values of Planck constant is predicted, and in principle TGD predicts the values of Planck constant if physics as a generalized number theory vision is accepted [K22].

The original vision was that the hierarchy of Planck constants demands a generalization of quantum TGD. This would have required a generalization of the causal diamond $CD \times CP_2$, where CD is defined as an intersection of the future and past directed light-cones of 4-D Minkowski space M^4. It however became clear that the hierarchy of Planck constants labels a hierarchy of quantum criticalities characterized by sub-algebras of super-symplectic algebras possessing a natural conformal structure. The sub-algebra for which the conformal weights come as n-ples of those for the entire algebra is isomorphic to the full algebra and acts as a conformal gauge algebra at given level of criticality.

In particular, the classical symplectic Noether charges for preferred extremals connecting 3-surfaces at the ends of CD vanish. and this defines preferred extremal property. There would be n conformal gauge equivalence classes of preferred extremals which would correspond to n sheets of a covering of the space-time surface serving as base space. There is very close similarity with the Riemann surfaces. Therefore coverings would be generated dynamically and there is no need for actual coverings of the imbedding space.

The gauge degeneracy corresponds to the non-determinism associated with the criticality having interpretation in terms of non-determinism of Kähler action and with strong form of holography. The extremely strong super-symplectic gauge conditions would guarantee that the continuation of string world sheets and partonic 2-surface to preferred extremals is possible at least for somes value of p-adic prime. A good guess is that this is the case for the so called ramified primes associated with the algebraic extension in question at least. These ramified primes would characterize physical system and the weak form of NMP would allow to understand how p-adic length scale hypothesis follows [K111]. The continuation could be possible for all p-adic primes due to the possibility of p-adic pseudo-constants having vanishing derivative.

The continuation could fail for most configurations of partonic 2-surfaces and string world sheets in the real sector: the interpretation would be that some space-time surfaces can be imagined but not realized [K48]. For certain extensions the number of realizable imaginations could be exceptionally large. These extensions would be winners in the number theoretic fight for survivalandcorresponding ramified primes would be preferred p-adic primes.

A further strong prediction is that the phase transitions increasing h_{eff} and thus reducing criticality (TGD Universe is like hill at the top of the hill at....) occur spontaneously [K106]. This conforms with NMP and suggests that evolution occurs spontaneously. The state function reduction increasing h_{eff} means however the death of a sub-self so that selves are fighting to stay at the criticality. The metabolic energy bringing inNE allows to satisfy the needs of NMP so that the system survives and provides a garden in which subselves can are born and die and gradually generate negentropic entanglement. Living systems are thus negentropy gatherers and each death and re-incarnation generates new negentropy.

All particles in the vertices of Feynman diagrams have the same value of Planck constant so that the particles at different pages cannot have local interactions. Thus one can speak about relative darkness in the sense that only the interactions mediated by the exchange of particles and by classical fields are possible between different pages. Dark matter in this sense can be observed, say through the classical gravitational and electromagnetic interactions. It is in principle possible to photograph dark matter by the exchange of photons which leak to another page of book, reflect, and leak back. This leakage corresponds to h_{eff} changing phase transition occurring at quantum criticality and living matter is expected carry out these phase transitions routinely in bio-control. This picture leads to no obvious contradictions with what is really known about dark matter and to my opinion the basic difficulty in understanding of dark matter (and living matter) is the blind belief in standard quantum theory. These observations motivate the tentative identification of the macroscopic quantum phases in terms of dark matter and also of dark energy with gigantic "gravitational" Planck constant.

What is especially remarkable is that the construction gives also the 4-D space-time sheets associated with the light-like orbits of the partonic 2-surfaces: it remains to be shown whether they correspond to preferred extremals of Kähler action. It is clear that the hierarchy of Planck constants has become an essential part of the construction of quantum TGD and of mathematical realization of the notion of quantum criticality rather than a possible generalization of TGD.

Identification of symplectic and Kac-Moody symmetries

The basic symmetries are isometries of "world of classical worlds" (WCW) proposed to be realized as symplectic transformations of the boundaries of causal diamonds (CD) locally identifiable as $\delta M^4_\pm \times CP_2$. These symplectic symmetries contains as algebra symplectic isometries which are expected to be of special importance. These transformations are expected to have continuation to deformations of the entire preferred extremal. They cannot be symmetries of Kähler action.

The Kac-Moody algebra of symmetries acting as symmetries respecting the light-likeness of 3-surfaces plays a crucial role in the identification of quantum fluctuating configuration space degrees of freedom contributing to the metric. These symmetries would act as gauge symmetries and related to quantum criticality due to the non-determinism of Kähler action in turn giving rise to the hierarchy of Planck constants explaining dark matter. The recent vision looks like follows.

1. The recent interpretation is that these gauge symmetries are due to the non-determinism of Kähler action and transform to each other preferred extremals with same space-like surfaces as their ends at the boundaries of causal diamond. These space-time surfaces have same Kähler action and possess same conserved quantities.

2. The sub-algebra of conformal symmetries acts as gauge transformations of these infinite set of degenerate preferred extremals and there is finite number n of gauge equivalence classes. n corresponds to the effective (or real depending on interpretation) value of Planck constant $h_{eff} = n \times h$. The further conjecture is that the sub-algebra of conformal algebra for which conformal weights are integers divisible by n act as genuine gauge symmetries. If Kähler action reduces to a sum of 3-D Chern-Simons terms for preferred extremals, it is enough to consider the action on light-like 3-surfaces. For gauge part of algebra the algebra acts trivially at space-like 3-surfaces.

3. A good guess is that the Kac-Moody type algebra corresponds to the sub-algebra of symplectic isometries of $\delta M^4_\pm \times CP_2$ acting on light-like 3-surfaces and having continuation to the interior.

   A stronger assumption is that isometries are in question. For CP_2 nothing would change but light-cone boundary $\delta M^4_\pm = S^2 \times R_+$ has conformal transformations of S^2 as isometries. The conformal scaling is compensated by S^2-local scaling of the light like radial coordinate of R_+.

4. This super-conformal algebra realized in terms of spinor modes and second quantized induced spinor fields would define the Super Kac-Moody algebra. The generators of this Kac-Moody

type algebra have continuation from the light-like boundaries to deformations of preferred extremals and at least the generators of sub-algebra act trivially at space-like 3-surfaces.

Zero energy ontology

Zero energy ontology motivated originally by TGD inspired cosmology means that physical states have vanishing conserved net quantum numbers and are decomposable to positive and negative energy parts separated by a temporal distance characterizing the system as a space-time sheet of finite size in time direction. The particle physics interpretation is as initial and final states of a particle reaction. Obviously a profound modification of existing views about realization of symmetries is in question.

S-matrix and density matrix are unified to the notion of M-matrix defining time-like entanglement and expressible as a product of square root of density matrix and of unitary S-matrix. Thermodynamics becomes therefore a part of quantum theory. One must distinguish M-matrices identifiable as products of orthonormal hermitian square roots of density matrices and universal S-matrix from U-matrix defined between zero energy states and analogous to S-matrix and characterizing the unitary process associated with quantum jump. The detailed description of U- and M-matrices is considered in [K91].

Quantum TGD as almost topological QFT

Light-likeness of the basic fundamental objects suggests that TGD is almost topological QFT so that the formulation in terms of category theoretical notions is expected to work. The original proposal that Chern-Simons action for light-like 3-surfaces defined by the regions at which the signature of the induced metric changes its sign however failed and one must use Kähler action and corresponding Kähler-Dirac action with measurement term to define the fundamental theory. At the limit when the momenta of particles vanish, the theory reduces to topological QFT defined by Kähler action and corresponding modified Dirac action. The imaginary exponent of the instanton term associated with the induced Kähler form defines the counterpart of Chern-Simons action as a phase of the vacuum functional and contributes also to Kähler-Dirac equation.

M-matrices form in a natural manner a functor from the category of cobordisms to the category of pairs of Hilbert spaces and this gives additional strong constraints on the theory. Super-conformal symmetries implied by the light-likeness pose very strong constraints on both state construction and on M-matrix and U-matrix. The notions of n-category and n-groupoid which represents a generalization of the notion of group could be very relevant to this view about M-matrix.

Quantum measurement theory with finite measurement resolution

The notion of measurement resolution represented in terms of inclusions $\mathcal{N} \subset \mathcal{M}$ of HFFs is an essential element of the picture. Measurement resolution corresponds to the action of the included sub-algebra creating zero energy states in time scales shorter than the cutoff scale. This means that complex rays of state space are effectively replaced with $\mathcal{N}$ rays. The condition that the action of $\mathcal{N}$ commutes with the M-matrix is a powerful symmetry and implies that the time-like entanglement characterized by M-matrix is consistent with Connes tensor product. This does not fix the M-matrix as was the original belief but only realizes mathematically the notion of finite measurement resolution. Together with super-conformal symmetries this constraint should fix possible M-matrices to a very high degree if one assumes the existence of universal M-matrix from which M-matrices with finite measurement resolution are obtained.

The notion of number theoretical braid realizes the notion of finite measurement resolution at space-time level and gives a direct connection to topological QFTs describing braids. The connection with quantum groups is highly suggestive since already the inclusions of HFFs involve these groups. Effective non-commutative geometry for the quantum critical sub-manifolds $M^2 \subset M^4$ and $S^2 \subset CP_2$ might provide an alternative notion for the reduction of stringy anti-commutation relations for induced spinor fields to anti-commutations at the points of braids.

22.2.2 The Construction Of U, M-, And S-Matrices

The general architecture of matrices is now rather well-understood and described in chapter [K91]. A brief summary is also given in the introduction. The key matrix is U-matrix acting in the space of zero states but leaving the states at the second boundary of CD invariant. M-matrix acts between positive and negative energy parts of given zero energy state being the product of a hermitian square root of density matrix and of a unitary S-matrix. The hermitian matrices involved would naturally form a representation of super-symplectic algebra or its sub-algebra and their "moduli squared" define a density matrix characterizing the second part of zero energy state. An open question is whether this density matrix relates to thermodynamics only formally or whether there is a deeper connection.

The recipe reduces the decisive step to a construction of S-matrix for a given CD and of a unitary time evolution operator in the moduli space of CDs providing unitary representation for a discrete subgroup of Lorentz group. The S-matrix for a given CD is n:th power of fundamental S-matrix S^n for CD whose size is n times the minimal size of CD characterized by the CP_2 time scale.

The construction of S-matrix involves several ideas that have emerged during last years and involve symmetries in an essential manner.

Emergence of particles as bound state of fundamental fermions, extended space-time supersymmetry, and generalized twistors

During year 2009 several new ideas emerged and give hopes about a concrete construction of M-matrix.

1. The notion of bosonic emergence [K54] follows from the fact that gauge bosons are identifiable as pairs of fermion and anti-fermion at opposite light-like throats of wormhole contact. As a consequence, bosonic propagators and vertices are generated radiatively from a fundamental action for fermions and their super partners. At QFT limit without super-symmetry this means that Dirac action coupled to gauge bosons is the fundamental action and the counterpart of YM action is generated radiatively. All coupling constants follow as predictions as they indeed must do on basis of the general structure of quantum TGD.

2. Whether the counterparts of space-time supersymmetries are possible in TGD Universe has remained a long-standing open question and my cautious belief has been that the super partners do not exist. The resolution of the problem came with the increased understanading of the dynamics of the Kähler-Dirac action [K23, K24]. In particular, the localization of the electroweakly charged modes at 2-D surfaces - string world sheets and possibly also partonic 2-surfaces- meant an enormous simplification since the solutions of the Kähler-Dirac equation are conformal spinor modes.

   The oscillator operators associated with the modes of the induced spinor field satisfy the anti-commutation relations defining the generalization of space-time super-symmetry algebra and these oscillator operators serve as the building blocks of various super-conformal algebras. The number of super-symmetry generators is very large, perhaps even infinite. This forces a generalization of the standard super field concept. The action for chiral super-fields emerges as a generalization of the Dirac action to include all possible super-partners. The huge super-symmetry gives excellent hopes about cancelation of UV divergences. The counterpart of super-symmetric YM action emerges radiatively. This formalism works at the QFT limit. The generalization of the formalism to quantum TGD proper is yet to be carried out.

3. Twistor program has become one of the most promising approaches to gauge theories. This inspired the question whether TGD could allow twistorialization [K86]. Massive states -both real and virtual- are the basic problem of twistor approach. In TGD framework the obvious idea is that massive on mass shell states can be interpreted as massless states in 8-D sense. Massive off-mass shell states in turn could be regarded as pairs of positive and negative on mass shell states. This means opening of the black box of virtual state attempted already in the model for bosonic propagators inspired by the bosonic emergence , and one can even hope that individual loop integrals are finite and that Wick rotation is not needed. The

third observation is that 8-dimensional gamma matrices allow a representation in terms of octonions (matrices are not in question anymore). If the Kähler-Dirac gamma “matrices” associated with space-time surface define a quaternionic sub-algebra of the complexified octonion algebra, they allow a matrix representation defined by octonionic structure constants. This holds true for are hyper-quaternionic space-time surfaces so that a connection with number theoretic vision emerges. This would more or less reduce the notion of twistor to its 4-dimensional counterpart.

Generalization of Feynman diagrams

An essential difference between TGD and string models is the replacement of stringy diagrams with generalized Feynman diagrams obtained by gluing 3-D light-like surfaces (instead of lines) together at their ends represented as partonic 2-surfaces. This makes the construction of vertices very simple. The notion of number theoretic braid in turn implies discretization having also interpretation in terms of non-commutativity due to finite measurement resolution replacing anti-commutativity along stringy curves with anti-commutativity at points of braids. Braids can replicate at vertices which suggests an interpretation in terms of topological quantum computation combined with non-faithful copying and communication of information. The analogs of stringy diagrams have quite different interpretation in TGD for instance, photons traveling via two different paths in double slit experiment are represented in terms of stringy branching of the photonic 2-surface.

Scattering amplitudes as computations in Yangian arithmetics?

One of the old TGD inspired really crazy ideas about scattering amplitudes is that Universe is doing some sort of arithmetics so that scattering amplitude are representations for computational sequences of minimum length and that all diagrams connecting the same states at the boundaries of CD produce the same scattering amplitude. This would mean enormous calculational simplification.

The idea is so crazy that I have even given up its original form, which led to an attempt to assimilate the basic ideas about bi-algebras, quantum groups [K6], Yangians [K76], and related exotic things. The work with twistor Grassmannian approach inspired a reconsideration of the original idea seriously with the idea that super-symplectic Yangian could define the arithmetics.

The identification of universal 3-vertex as a product or co-product in Yangian looks highly promising approach to the construction of the scattering amplitude. The Nother charges of the super-symplectic Yangian are associated with strings and are either linear or bilinear in the fermion field. The fermion fields associated with the partonic 2-surface defining the vertex are contracted with fermion fields associated with other partonic 2-surfaces using the same rule as in Wick expansion in quantum field theories. The contraction gives fermion propagator for each leg pair associated with two vertices. Vertex factor is proportional to the contraction of spinor modes with the operators defining the Noether charge or super charge and essentially Kähler-Dirac gamma matrix and the representation of the action of the symplectic generator on fermion realizable in terms of sigma matrices.

This resembles strongly the corresponding expression in gauge theories but with gauge algebra replaced with symplectic algebra. The possibility of contractions of creation and annihilation operator for fermion lines associated with opposite wormhole throats at the same partonic 2-surface (for Noether charge bilinear in fermion field) gives bosonic exchanges as lines in which the fermion lines turns in time direction: otherwise only regroupings of fermions would take place.

Could correlation functions, S-matrix, and coupling constant evolution be coded the statistical properties of preferred extremals?

How to calculate the correlation functions and coupling constant evolution has remained a basic unresolved challenge. Generalized Feynman diagrams provide a powerful vision which however does not help in practical calculations. Some big idea has been lacking.

Quantum classical correspondence states that all aspects of quantum states should have correlates in the geometry of preferred extremals. In particular, various elementary particle propagators should have a representation as properties of preferred extremals. This would allow to realize the old dream about being able to say something interesting about coupling constant evolution although it is not yet possible to calculate the M-matrices and U-matrix. The general structure of U-matrix is however understood [K91]. Hitherto everything that has been said about coupling constant evolution has been rather speculative arguments except for the general vision that it reduces to a discrete evolution defined by p-adic length scales. General first principle definitions are however much more valuable than ad hoc guesses even if the latter give rise to explicit formulas.

In quantum TGD and also at its QFT limit various correlation functions in given quantum state should code for its properties. By quantum classical correspondence these correlation functions should have counterparts in the geometry of preferred extremals. Even more: these classical counterparts for a given preferred extremal ought to be identical with the quantum correlation functions for the superposition of preferred extremals. This correspondence could be called quantum ergodicity by its analogy with ordinary ergodicity stating that the member of ensemble becomes representative of ensemble.

This principle would be a quantum generalization of ergodic theorem stating that the time evolution of a single member of ensemble represents the ensemble statistically. This symmetry principle analogous to holography might allow to fix S-matrix uniquely even in the case that the hermitian square root of the density matrix appearing in the M-matrix would lead to a breaking of quantum ergodicity as also 4-D spin glass degeneracy suggests.

This principle would allow to deduce correlation functions from the statistical properties of single preferred extremal alone using just classical intuition. Also coupling constant evolution would be coded by the statistical properties of preferred extremals. Quantum ergodicity would mean an enormous simplification since one could avoid the horrible conceptual complexities involved with the functional integrals over WCW .

This might of course be too optimistic guess. If a sub-algebra of symplectic algebra acts as gauge symmmetries of the preferred extremals in the sense that corresponding Noether charges vanish, it can quite well be that correlations functions correspond to averages for extremals belonging to single conformal equivalence class.

1. The marvellous implication of quantum ergodicity would be that one could calculate everything solely classically using the classical intuition - the only intuition that we have. Quantum ergodicity would also solve the paradox raised by the quantum classical correspondence for momentum eigenstates. Any preferred extremal in their superposition defining momentum eigenstate should code for the momentum characterizing the superposition itself. This is indeed possible if every extremal in the superposition codes the momentum to the properties of classical correlation functions which are identical for all of them.

2. The only manner to possibly achieve quantum ergodicity is in terms of the statistical properties of the preferred extremals. It should be possible to generalize the ergodic theorem stating that the properties of statistical ensemble are represented by single space-time evolution in the ensemble of time evolutions. Quantum superposition of classical worlds would effectively reduce to single classical world as far as classical correlation functions are considered. The notion of finite measurement resolution suggests that one must state this more precisely by adding that classical correlation functions are calculated in a given UV and IR resolutions meaning UV cutoff defined by the smallest CD and IR cutoff defined by the largest CD present.

3. The skeptic inside me immediately argues that TGD Universe is 4-D spin glass so that this quantum ergodic theorem must be broken. In the case of the ordinary spin classes one has not only statistical average for a fixed Hamiltonian but a statistical average over Hamiltonians. There is a probability distribution over the coupling parameters appearing in the Hamiltonian. Maybe the quantum counterpart of this is needed to predict the physically measurable correlation functions.

   Could this average be an ordinary classical statistical average over quantum states with different classical correlation functions? This kind of average is indeed taken in density matrix formalism. Or could it be that the square root of thermodynamics defined by ZEO actually gives automatically rise to this average? The eigenvalues of the "hermitian square root" of the density matrix would code for components of the state characterized by different classical correlation functions. One could assign these contributions to different "phases".

4. Quantum classical correspondence in statistical sense would be very much like holography (now individual classical state represents the entire quantum state). Quantum ergodicity would pose a rather strong constraint on quantum states. This symmetry principle could actually fix the spectrum of zero energy states to a high degree and fix therefore the M-matrices given by the product of hermitian square root of density matrix and unitary S-matrix and unitary U-matrix constructible as inner products of M-matrices associated with CDs with various size scales [K91].

5. In TGD inspired theory of consciousness the counterpart of quantum ergodicity is the postulate that the space-time geometry provides a symbolic representation for the quantum states and also for the contents of consciousness assignable to quantum jumps between quantum states. Quantum ergodicity would realize this strongly self-referential looking condition. The positive and negative energy parts of zero energy state would be analogous to the initial and final states of quantum jump and the classical correlation functions would code for the contents of consciousness like written formulas code for the thoughts of mathematician and provide a sensory feedback.

How classical correlation functions should be defined?

1. General Coordinate Invariance and Lorentz invariance are the basic constraints on the definition. These are achieved for the space-time regions with Minkowskian signature and 4-D M^4 projection if linear Minkowski coordinates are used. This is equivalent with the contraction of the indices of tensor fields with the space-time projections of M^4 Killing vector fields representing translations. Accepting ths generalization, there is no need to restrict oneself to 4-D M^4 projection and one can also consider also Euclidian regions identifiable as lines of generalized Feynman diagrams.

   Quantum ergodicity very probably however forces to restrict the consideration to Minkowskian and Euclidian space-time regions and various phases associated with them. Also CP_2 Killing vector fields can be projected to space-time surface and give a representation for classical gluon fields. These in turn can be contracted with M^4 Killing vectors giving rise to gluon fields as analogs of graviton fields but with second polarization index replaced with color index.

2. The standard definition for the correlation functions associated with classical time evolution is the appropriate starting point. The correlation function $G_{XY}(\tau)$ for two dynamical variables $X(t)$ and $Y(t)$ is defined as the average $G_{XY}(\tau) = \int_T X(t)Y(t+\tau)dt/T$ over an interval of length T, and one can also consider the limit $T \to \infty$. In the recent case one would replace τ with the difference $m_1 - m_2 = m$ of M^4 coordinates of two points at the preferred extremal and integrate over the points of the extremal to get the average. The finite time interval T is replaced with the volume of causal diamond in a given length scale. Zero energy state with given quantum numbers for positive and negative energy parts of the state defines the initial and final states between which the fields appearing in the correlation functions are defined.

3. What correlation functions should be considered? Certainly one could calculate correlation functions for the induced spinor connection given electro-weak propagators and correlation functions for CP_2 Killing vector fields giving correlation functions for gluon fields using the description in terms of Killing vector fields. If one can uniquely separate from the Fourier transform uniquely a term of form $Z/(p^2 - m^2)$ by its momentum dependence, the coefficient Z can be identified as coupling constant squared for the corresponding gauge potential component and one can in principle deduce coupling constant evolution purely classically. One can imagine of calculating spinorial propagators for string world sheets in the same manner. Note that also the dependence on color quantum numbers would be present so that in principle all that is needed could be calculated for a single preferred extremal without the need to construct QFT limit and to introduce color quantum numbers of fermions as spin like quantum numbers (color quantum numbers corresponds to CP_2 partial wave for the tip of the CD assigned with the particle).

Many detailed speculations about coupling constant evolution to be discussed in the sections below must be taken as innovative guesses doomed to have the eventual fate of guesses. The notion of quantum ergodicity could however be one of the really deep ideas about coupling constant evolution comparable to the notion of p-adic coupling constant evolution. Quantum Ergodicity (briefly QE) would also state something extremely non-trivial also about the construction of correlation functions and S-matrix. Because this principle is so new, the rest of the chapter does not yet contain any applications of QE. This should not lead the reader to under-estimate the potential power of QE.

22.2.3 Are Both Symplectic And Conformal Field Theories Needed In TGD Framework?

Before one can say anything quantitative about coupling constant evolution, one must have a formulation for its TGD counterpart and thus also a more detailed formulation for how to calculate M-matrix elements. There is also the question about infinities. By very general arguments infinities of quantum field theories are predicted to cancel in TGD Universe - basically by the non-locality of Kähler function as a functional of 3-surface and by the general properties of the vacuum functional identified as the exponent of Kähler function. The precise mechanism leading to the cancellation of infinities of local quantum field theories has remained unspecified. Only the realization that the symplectic invariance of quantum TGD provides a mechanism regulating the short distance behavior of N-point functions changed the situation in this respect. This also leads to one possible concrete view about the generalized Feynman diagrams giving M-matrix elements and at least a resemblance with ordinary Feynman diagrammatics.

It must be of course admitted that there are several apparentely competing visions. Twistorial vision [K76] and the vision about scattering amplitudes as representations for sequences of algebraic operations in super-symplectic Yangian [A27] [B39, B30, B31] seem to be consistent views. Symplectic approach seems to be suitable to understand the integration over WCW zero mode degrees of freedom not included in the other approaches.

Symplectic invariance

Symplectic symmetries of $\delta M^4_+ \times CP_2$ (light-cone boundary briefly) act as isometries of the "world of classical worlds". One can see these symmetries as analogs of Kac-Moody type symmetries with symplectic transformations of $S^2 \times CP_2$, where S^2 is $r_M = constant$ sphere of light-cone boundary, made local with respect to the light-like radial coordinate r_M taking the role of complex coordinate. Thus finite-dimensional Lie group G is replaced with infinite-dimensional group of symplectic transformations. This inspires the question whether a symplectic analog of conformal field theory at $\delta M^4_+ \times CP_2$ could be relevant for the construction of n-point functions in quantum TGD and what general properties these n-point functions would have. This section appears already in the previous chapter about symmetries of quantum TGD [K14] but because the results of the section provide the first concrete construction recipe of M-matrix in zero energy ontology, it is included also in this chapter.

Symplectic QFT at sphere

Actually the notion of symplectic QFT emerged as I tried to understand the properties of cosmic microwave background which comes from the sphere of last scattering which corresponds roughly to the age of 5×10^5 years [K53]. In this situation vacuum extremals of Kähler action around almost unique critical Robertson-Walker cosmology imbeddable in $M^4 \times S^2$, where there is homologically trivial geodesic sphere of CP_2. Vacuum extremal property is satisfied for any space-time surface which is surface in $M^4 \times Y^2$, Y^2 a Lagrangian sub-manifold of CP_2 with vanishing induced Kähler form. Symplectic transformations of CP_2 and general coordinate transformations of M^4 are dynamical symmetries of the vacuum extremals so that the idea of symplectic QFT emerges natural. Therefore I shall consider first symplectic QFT at the sphere S^2 of last scattering with temperature fluctution $\Delta T/T$ proportional to the fluctuation of the metric component g_{aa} in Robertson-Walker coordinates.

1. In quantum TGD the symplectic transformation of the light-cone boundary would induce action in the "world of classical worlds" (light-like 3-surfaces). In the recent situation it is convenient to regard perturbations of CP_2 coordinates as fields at the sphere of last scattering

(call it S^2) so that symplectic transformations of CP_2 would act in the field space whereas those of S^2 would act in the coordinate space just like conformal transformations. The deformation of the metric would be a symplectic field in S^2. The symplectic dimension would be induced by the tensor properties of R-W metric in R-W coordinates: every S^2 coordinate index would correspond to one unit of symplectic dimension. The symplectic invariance in CP_2 degrees of freedom is guaranteed if the integration measure over the vacuum deformations is symplectic invariant. This symmetry does not play any role in the sequel.

2. For a symplectic scalar field $n \geq 3$-point functions with a vanishing anomalous dimension would be functions of the symplectic invariants defined by the areas of geodesic polygons defined by subsets of the arguments as points of S^2. Since n-polygon can be constructed from 3-polygons these invariants can be expressed as sums of the areas of 3-polygons expressible in terms of symplectic form. n-point functions would be constant if arguments are along geodesic circle since the areas of all sub-polygons would vanish in this case. The decomposition of n-polygon to 3-polygons brings in mind the decomposition of the n-point function of conformal field theory to products of 2-point functions by using the fusion algebra of conformal fields (very symbolically $\Phi_k\Phi_l = c_{kl}^m\Phi_m$). This intuition seems to be correct.

3. Fusion rules stating the associativity of the products of fields at different points should generalize. In the recent case it is natural to assume a non-local form of fusion rules given in the case of symplectic scalars by the equation

$$\Phi_k(s_1)\Phi_l(s_2) = \int c_{kl}^m f(A(s_1,s_2,s_3))\Phi_m(s)d\mu_s \quad . \tag{22.2.1}$$

Here the coefficients c_{kl}^m are constants and $A(s_1,s_2,s_3)$ is the area of the geodesic triangle of S^2 defined by the sympletic measure and integration is over S^2 with symplectically invariant measure $d\mu_s$ defined by symplectic form of S^2. Fusion rules pose powerful conditions on n-point functions and one can hope that the coefficients are fixed completely.

4. The application of fusion rules gives at the last step an expectation value of 1-point function of the product of the fields involves unit operator term $\int c_{kl}f(A(s_1,s_2,s))Idd\mu_s$ so that one has

$$\langle\Phi_k(s_1)\Phi_l(s_2)\rangle = \int c_{kl}f(A(s_1,s_2,s))d\mu_s \quad . \tag{22.2.2}$$

Hence 2-point function is average of a 3-point function over the third argument. The absence of non-trivial symplectic invariants for 1-point function means that $n = 1$- an are constant, most naturally vanishing, unless some kind of spontaneous symmetry breaking occurs. Since the function $f(A(s_1,s_2,s_3))$ is arbitrary, 2-point correlation function can have both signs. 2-point correlation function is invariant under rotations and reflections.

Symplectic QFT with spontaneous breaking of rotational and reflection symmetries

CMB data suggest breaking of rotational and reflection symmetries of S^2. A possible mechanism of spontaneous symmetry breaking is based on the observation that in TGD framework the hierarchy of Planck constants assigns to each sector of the generalized imbedding space a preferred quantization axes. The selection of the quantization axis is coded also to the geometry of "world of classical worlds", and to the quantum fluctuations of the metric in particular. Clearly, symplectic QFT with spontaneous symmetry breaking would provide the sought-for really deep reason for the quantization of Planck constant in the proposed manner.

1. The coding of angular momentum quantization axis to the generalized imbedding space geometry allows to select South and North poles as preferred points of S^2. To the three arguments s_1, s_2, s_3 of the 3-point function one can assign two squares with the added point being either North or South pole. The difference

$$\Delta A(s_1, s_2, s_3) \equiv A(s_1, s_2, s_3, N) - A(s_1, s_2, s_3, S) \tag{22.2.3}$$

   of the corresponding areas defines a simple symplectic invariant breaking the reflection symmetry with respect to the equatorial plane. Note that ΔA vanishes if arguments lie along a geodesic line or if any two arguments co-incide. Quite generally, symplectic QFT differs from conformal QFT in that correlation functions do not possess singularities.

2. The reduction to 2-point correlation function gives a consistency conditions on the 3-point functions

$$\langle(\Phi_k(s_1)\Phi_l(s_2))\Phi_m(s_3)\rangle = c^r_{kl}\int f(\Delta A(s_1, s_2, s))\langle\Phi_r(s)\Phi_m(s_3)\rangle d\mu_s$$
$$= \tag{22.2.4}$$
$$c^r_{kl}c_{rm}\int f(\Delta A(s_1, s_2, s))f(\Delta A(s, s_3, t))d\mu_s d\mu_t \ . \tag{22.2.5}$$

   Associativity requires that this expression equals to $\langle\Phi_k(s_1)(\Phi_l(s_2)\Phi_m(s_3))\rangle$ and this gives additional conditions. Associativity conditions apply to $f(\Delta A)$ and could fix it highly uniquely.

3. 2-point correlation function would be given by

$$\langle\Phi_k(s_1)\Phi_l(s_2)\rangle = c_{kl}\int f(\Delta A(s_1, s_2, s))d\mu_s \tag{22.2.6}$$

4. There is a clear difference between $n > 3$ and $n = 3$ cases: for $n > 3$ also non-convex polygons are possible: this means that the interior angle associated with some vertices of the polygon is larger than π. $n = 4$ theory is certainly well-defined, but one can argue that so are also $n > 4$ theories and skeptic would argue that this leads to an inflation of theories. TGD however allows only finite number of preferred points and fusion rules could eliminate the hierarchy of theories.

5. To sum up, the general predictions are following. Quite generally, for $f(0) = 0$ n-point correlation functions vanish if any two arguments co-incide which conforms with the spectrum of temperature fluctuations. It also implies that symplectic QFT is free of the usual singularities. For symmetry breaking scenario 3-point functions and thus also 2-point functions vanish also if s_1 and s_2 are at equator. All these are testable predictions using ensemble of CMB spectra.

Generalization to quantum TGD

(Number theoretic) braids are identifiable as boundaries of string world sheets at which the modes of induced spinor fields are localized in the generic case in Minkowskian space-time regions. Fundamental fermions can be assigned to these lines. Braids are the basic objects of quantum TGD, one can hope that the n-point functions assignable to them could code the properties of ground states and that one could separate from n-point functions the parts which correspond to the symplectic degrees of freedom acting as symmetries of vacuum extremals and isometries of the "world of classical worlds".

1. This approach indeed seems to generalize also to quantum TGD proper and the n-point functions associated with partonic 2-surfaces can be decomposed in such a manner that one obtains coefficients which are symplectic invariants associated with both S^2 and CP_2 Kähler form.

2. Fusion rules imply that the gauge fluxes of respective Kähler forms over geodesic triangles associated with the S^2 and CP_2 projections of the arguments of 3-point function serve basic building blocks of the correlation functions. The North and South poles of S^2 and three poles of CP_2 can be used to construct symmetry breaking n-point functions as symplectic invariants. Non-trivial 1-point functions vanish also now.

3. The important implication is that n-point functions vanish when some of the arguments co-incide. This might play a crucial role in taming of the singularities: the basic general prediction of TGD is that standard infinities of local field theories should be absent and this mechanism might realize this expectation.

Next some more technical but elementary first guesses about what might be involved.

1. It is natural to introduce the moduli space for n-tuples of points of the symplectic manifold as the space of symplectic equivalence classes of n-tuples. In the case of sphere S^2 convex n-polygon allows $n+1$ 3-sub-polygons and the areas of these provide symplectically invariant coordinates for the moduli space of symplectic equivalence classes of n-polygons (2^n-D space of polygons is reduced to $n+1$-D space). For non-convex polygons the number of 3-sub-polygons is reduced so that they seem to correspond to lower-dimensional sub-space. In the case of CP_2 n-polygon allows besides the areas of 3-polygons also 4-volumes of 5-polygons as fundamental symplectic invariants. The number of independent 5-polygons for n-polygon can be obtained by using induction: once the numbers $N(k,n)$ of independent $k \leq n$-simplices are known for n-simplex, the numbers of $k \leq n+1$-simplices for $n+1$-polygon are obtained by adding one vertex so that by little visual gymnastics the numbers $N(k,n+1)$ are given by $N(k,n+1) = N(k-1,n)+N(k,n)$. In the case of CP_2 the allowance of 3 analogs $\{N,S,T\}$ of North and South poles of S^2 means that besides the areas of polygons (s_1,s_2,s_3), (s_1,s_2,s_3,X), (s_1,s_2,s_3,X,Y), and (s_1,s_2,s_3,N,S,T) also the 4-volumes of 5-polygons (s_1,s_2,s_3,X,Y), and of 6-polygon (s_1,s_2,s_3,N,S,T), $X,Y \in \{N,S,T\}$ can appear as additional arguments in the definition of 3-point function.

2. What one really means with symplectic tensor is not clear since the naive first guess for the n-point function of tensor fields is not manifestly general coordinate invariant. For instance, in the model of CMB, the components of the metric deformation involving S^2 indices would be symplectic tensors. Tensorial n-point functions could be reduced to those for scalars obtained as inner products of tensors with Killing vector fields of $SO(3)$ at S^2. Again a preferred choice of quantization axis would be introduced and special points would correspond to the singularities of the Killing vector fields.

   The decomposition of Hamiltonians of the "world of classical worlds" expressible in terms of Hamiltonians of $S^2 \times CP_2$ to irreps of $SO(3)$ and $SU(3)$ could define the notion of symplectic tensor as the analog of spherical harmonic at the level of WCW . Spin and gluon color would have natural interpretation as symplectic spin and color. The infinitesimal action of various Hamiltonians on n-point functions defined by Hamiltonians and their super counterparts is well-defined and group theoretical arguments allow to deduce general form of n-point functions in terms of symplectic invariants.

3. The need to unify p-adic and real physics by requiring them to be completions of rational physics, and the notion of finite measurement resolution suggest that discretization of also fusion algebra is necessary. The set of points appearing as arguments of n-point functions could be finite in a given resolution so that the p-adically troublesome integrals in the formulas for the fusion rules would be replaced with sums. Perhaps rational/algebraic variants of $S^2 \times CP_2 = SO(3)/SO(2) \times SU(3)/U(2)$ obtained by replacing these groups with their rational/algebraic variants are involved. Tedrahedra, octahedra, and dodecahedra suggest themselves as simplest candidates for these discretized spaces. Also the symplectic moduli

space would be discretized to contain only n-tuples for which the symplectic invariants are numbers in the allowed algebraic extension of rationals. This would provide an abstract looking but actually very concrete operational approach to the discretization involving only areas of n-tuples as internal coordinates of symplectic equivalence classes of n-tuples. The best that one could achieve would be a formulation involving nothing below measurement resolution.

4. This picture based on elementary geometry might make sense also in the case of conformal symmetries. The angles associated with the vertices of the S^2 projection of n-polygon could define conformal invariants appearing in n-point functions and the algebraization of the corresponding phases would be an operational manner to introduce the space-time correlates for the roots of unity introduced at quantum level. In CP_2 degrees of freedom the projections of n-tuples to the homologically trivial geodesic sphere S^2 associated with the particular sector of CH would allow to define similar conformal invariants. This framework gives dimensionless areas (unit sphere is considered). p-Adic length scale hypothesis and hierarchy of Planck constants would bring in the fundamental units of length and time in terms of CP_2 length.

The recent view about M-matrix described in [K13] is something almost unique determined by Connes tensor product providing a formal realization for the statement that complex rays of state space are replaced with $\mathcal{N}$ rays where $\mathcal{N}$ defines the hyper-finite sub-factor of type II_1 defining the measurement resolution. M-matrix defines time-like entanglement coefficients between positive and negative energy parts of the zero energy state and need not be unitary. It is identified as square root of density matrix with real expressible as product of of real and positive square root and unitary S-matrix. This S-matrix is what is measured in laboratory. There is also a general vision about how vertices are realized: they correspond to light-like partonic 3-surfaces obtained by gluing incoming and outgoing partonic 3-surfaces along their ends together just like lines of Feynman diagrams. Note that in string models string world sheets are non-singular as 2-manifolds whereas 1-dimensional vertices are singular as 1-manifolds. These ingredients we should be able to fuse together. So we try once again!

1. *Iteration* starting from vertices and propagators is the basic approach in the construction of n-point function in standard QFT. This approach does not work in quantum TGD. Symplectic and conformal field theories suggest that *recursion* replaces iteration in the construction. One starts from an n-point function and reduces it step by step to a vacuum expectation value of a 2-point function using fusion rules. Associativity becomes the fundamental dynamical principle in this process. Associativity in the sense of classical number fields has already shown its power and led to a hyper-octoninic formulation of quantum TGD promising a unification of various visions about quantum TGD [K74].

2. Let us start from the representation of a zero energy state in terms of a causal diamond defined by future and past directed light-cones. Zero energy state corresponds to a quantum superposition of light-like partonic 3-surfaces each of them representing possible particle reaction. These 3-surfaces are very much like generalized Feynman diagrams with lines replaced by light-like 3-surfaces coming from the upper and lower light-cone boundaries and glued together along their ends at smooth 2-dimensional surfaces defining the generalized vertices.

3. It must be emphasized that the generalization of ordinary Feynman diagrammatics arises and conformal and symplectic QFTs appear only in the calculation of single generalized Feynman diagram. Therefore one could still worry about loop corrections. The fact that no integration over loop momenta is involved and there is always finite cutoff due to discretization together with recursive instead of iterative approach gives however good hopes that everything works. Note that this picture is in conflict with one of the earlier approaches based on positive energy ontology in which the hope was that only single generalized Feynman diagram could define the U-matrix thought to correspond directly to physical S-matrix at that time.

4. One can actually simplify things by identifying generalized Feynman diagrams as maxima of Kähler function with functional integration carried over perturbations around it. Thus

one would have conformal field theory in both fermionic and WCW degrees of freedom. The light-like time coordinate along light-like 3-surface is analogous to the complex coordinate of conformal field theories restricted to some curve. If it is possible continue the light-like time coordinate to a hyper-complex coordinate in the interior of 4-D space-time sheet, the correspondence with conformal field theories becomes rather concrete. Same applies to the light-like radial coordinates associated with the light-cone boundaries. At light-cone boundaries one can apply fusion rules of a symplectic QFT to the remaining coordinates. Conformal fusion rules are applied only to point pairs which are at different ends of the partonic surface and there are no conformal singularities since arguments of n-point functions do not co-incide. By applying the conformal and symplectic fusion rules one can eventually reduce the n-point function defined by the various fermionic and bosonic operators appearing at the ends of the generalized Feynman diagram to something calculable.

5. Finite measurement resolution defining the Connes tensor product is realized by the discretization applied to the choice of the arguments of n-point functions so that discretion is not only a space-time correlate of finite resolution but actually defines it. No explicit realization of the measurement resolution algebra $\mathcal{N}$ seems to be needed. Everything should boil down to the fusion rules and integration measure over different 3-surfaces defined by exponent of Kähler function and by imaginary exponent of Chern-Simons action. The continuation of WCW Clifford algebra for 3-surfaces with cm degrees of freedom fixed to a hyper-octonionic variant of gamma matrix field of super-string models defined in M^8 (hyper-octonionic space) and $M^8 \leftrightarrow M^4 \times CP_2$ duality leads to a unique choice of the points, which can contribute to n-point functions as intersection of M^4 subspace of M^8 with the counterparts of partonic 2-surfaces at the boundaries of light-cones of M^8. Therefore there are hopes that the resulting theory is highly unique. Symplectic fusion algebra reduces to a finite algebra for each space-time surface if this picture is correct.

6. Consider next some of the details of how the light-like 3-surface codes for the fusion rules associated with it. The intermediate partonic 2- surfaces must be involved since otherwise the construction would carry no information about the properties of the light-like 3-surface, and one would not obtain perturbation series in terms of the relevant coupling constants. The natural assumption is that partonic 2-surfaces belong to future/past directed light-cone boundary depending on whether they are on lower/upper half of the causal diamond. Hyper-octonionic conformal field approach fixes the n_{int} points at intermediate partonic two-sphere for a given light-like 3-surface representing generalized Feynman diagram, and this means that the contribution is just N-point function with $N = n_{out} + n_{int} + n_{in}$ calculable by the basic fusion rules. Coupling constant strengths would emerge through the fusion coefficients, and at least in the case of gauge interactions they must be proportional to Kähler coupling strength since n-point functions are obtained by averaging over small deformations with vacuum functional given by the exponent of Kähler function. The first guess is that one can identify the spheres $S^2 \subset \delta M^4_\pm$ associated with initial, final and, and intermediate states so that symplectic n-points functions could be calculated using single sphere.

These findings raise the hope that quantum TGD is indeed a solvable theory. The coupling constant evolution is based on the same mechanism as in QFT and symplectic invariance replaces ad hoc UV cutoff with a genuine dynamical regulation mechanism. Causal diamond itself defines the physical IR cutoff. p-Adic and real coupling constant evolutions reflect the underlying evolution in powers of two for the temporal distance between the tips of the light-cones of the causal diamond and the association of macroscopic time scale as secondary p-adic time scale to elementary particles (.1 seconds for electron) serves as a first test for the picture. Even if one is not willing to swallow any bit of TGD, the classification of the symplectic QFTs remains a fascinating mathematical challenge in itself. A further challenge is the fusion of conformal QFT and symplectic QFT in the construction of n-point functions. One might hope that conformal and symplectic fusion rules could be treated independently.

More detailed view about the construction of M-matrix elements

After three decades there are excellent hopes of building an explicit recipe for constructing M-matrix elements but the devil is in the details.

1. Elimination of infinities and coupling constant evolution

The elimination of infinities could follow from the symplectic QFT part of the theory. The symplectic contribution to n-point functions vanishes when two arguments co-incide. The UV cancellation mechanism has nothing to do with the finite measurement resolution which corresponds to the size of the causal diamonds inside which the space-time sheets representing radiative corrections are. There is also IR cutoff due to the presence of largest causal diamond.

On can decompose the radiative corrections two two types. First kind of corrections appear both at the level of positive/and negative energy parts of zero energy states. Second kind of corrections appear at the level of interactions between them. This decomposition is standard in quantum field theories and corresponds to the renormalization constants of fields *resp.* renormalization of coupling constants. The corrections due to the increase of measurement resolution in time comes as very specific corrections to positive and negative energy states involving gluing of smaller causal diamonds to the upper and lower boundaries of causal diamonds along any radial light-like ray. The radiative corresponds to the interactions correspond to the addition of smaller causal diamonds in the interior of the larger causal diamond. Scales for the corrections come as scalings in powers of 2 rather than as continuous scaling of measurement resolution.

UV finiteness is suggested also by the generalized Feynman rules providing a phenomenological view about what TGD predicts. According to these rules fundamental fermions propagate like massless particles. In twistor Grassmann approach residue integration is expected to reduce internal fermion lines to on mass shell propagation with non-physical helicity. The fundamental 4-fermion interaction is assignable to wormhole contact and corresponds to stringy exchange of four-momentum with propagator being defined by the inverse of super-conformal scaling generator $1/L_0$. Wormhole contacts carrying fermion and antifermion at their throats behave like fundamental bosons. Stringy propagators at wormhole contacts make TGD rules a hybrid of Feynmann and stringy rules. Stringy propagators are necessary in order to avoid logarithmic divergences. Higher mass excitations crucial for finiteness belong to the representations of super-conformal algebra and can be regarded as bound states of massless fermions. Massivation of external particles allows to avoid infrared divergences. Not only physical bosons but also physical fermions emerge from fundamental massless fermions.

2. Conformal symmetries

The basic questions are the following ones. How hyper-octonionic/-quaternionic/-complex super-conformal symmetry relates to the super-symplectic conformal symmetry at the imbedding space level and the super Kac-Moody symmetry associated with the light-like 3-surfaces? How do the dual $HO = M^8$ and $H = M^4 \times CP_2$ descriptions (number theoretic compactifcation) relate?

Concerning the understanding of these issues, the earlier construction of physical states poses strong constraints [K14].

1. The state construction utilizes both super-symplectic and super Kac-Moody algebras. super-symplectic algebra has negative conformal weights and creates tachyonic ground states from which Super Kac-Moody algebra generates states with non-negative conformal weight determining the mass squared value of the state. The commutator of these two algebras annihilates the physical states. This requires that both super conformal algebras must allow continuation to hyper-octonionic algebras, which are independent.

2. The light-like radial coordinate at $\delta M^4_\pm$ can be continued to a hyper-complex coordinate in $M^2_\pm$ defined the preferred commutative plane of non-physical polarizations, and also to a hyper-quaternionic coordinate in $M^4_\pm$. Hence it would seem that super-symplectic algebra can be continued to an algebra in $M^2_\pm$ or perhaps in the entire $M^4_\pm$. This would allow to continue also the operators G, L and other super-symplectic operators to operators in hyper-quaternionic $M^4_\pm$ needed in stringy perturbation theory.

3. Also the super KM algebra associated with the light-like 3-surfaces should be continueable to hyper-quaternionic $M^4_\pm$. Here $HO-H$ duality comes in rescue. It requires that the preferred hyper-complex plane M^2 is contained in the tangent plane of the space-time sheet at each point, in particular at light-like 3-surfaces. We already know that this allows to assign a unique space-time surface to a given collection of light-like 3-surfaces as hyper-quaternionic

4-surface of HO hypothesized to correspond to (an obviously preferred) extremal of Kähler action. An equally important implication is that the light-like coordinate of X^3 can be continued to hyper-complex coordinate M^2 coordinate and thus also to hyperquaternionic M^4 coordinate.

4. The four-momentum appears in super generators G_n and L_n. It seems that the formal Fourier transform of four-momentum components to gradient operators to $M^4_\pm$ is needed and defines these operators as particular elements of the WCW Clifford algebra elements extended to fields in imbedding space.

3. What about stringy perturbation theory?

The analog of stringy perturbation theory does not seems only a highly attractive but also an unavoidable outcome since a generalization of massless fermionic propagator is needed. The inverse for the sum of super Kac-Moody and super-symplectic super-Virasoro generators G (L) extended to an operator acting on the difference of the M^4 coordinates of the end points of the propagator line connecting two partonic 2-surfaces should appear as fermionic (bosonic) propagator in stringy perturbation theory. Virasoro conditions imply that only G_0 and L_0 appear as propagators. Momentum eigenstates are not strictly speaking possible since since discretization is present due to the finite measurement resolution. One can however represent these states using Fourier transform as a superposition of momentum eigenstates so that standard formalism can be applied.

Symplectic QFT gives an additional multiplicative contribution to n-point functions and there would be also braiding S-matrices involved with the propagator lines in the case that partonic 2-surface carriers more than 1 point. This leaves still modular degrees of freedom of the partonic 2-surfaces describable in terms of elementary particle vacuum functionals and the proper treatment of these degrees of freedom remains a challenge.

4. What about non-hermiticity of the WCW super-generators carrying fermion number?

TGD represents also a rather special challenge, which actually represents the fundamental difference between quantum TGD and super string models. The assignment of fermion number to WCW gamma matrices and thus also to the super-generator G is unavoidable. Also M^4 and H gamma matrices carry fermion number. This has been a long-standing interpretational problem in quantum TGD and I have been even ready to give up the interpretation of four-momentum operator appearing in G_n and L_n as actual four-momenta. The manner to get rid of this problem would be the assumption of Majorana property but this would force to give up the interpretation of different imbedding space chiralities in terms of conserved lepton and quark numbers and would also lead to super-string theory with critical dimension 10 or 11. A further problem is how to obtain amplitudes which respect fermion number conservation using string perturbation theory if $1/G = G^\dagger/L_0$ carries fermion number.

The recent picture does not leave many choices so that I was forced to face the truth and see how everything falls down to this single nasty detail! It became as a total surprise that gamma matrices carrying fermion number do not cause any difficulties in zero energy ontology and make sense even in the ordinary Feynman diagrammatics.

1. Non-hermiticity of G means that the center of mass terms CH gamma matrices must be distinguished from their Hermitian conjugates. In particular, one has $\gamma_0 \neq \gamma_0^{dagger}$. One can interpret the fermion number carrying M^4 gamma matrices of the complexified quaternion space.

2. One might think that $M^4 \times CP_2$ gamma matrices carrying fermion number is a catastrophe but this is not the case in massless theory. Massless momentum eigen states can be created by the operator $p^k\gamma_k^\dagger$ from a vacuum annihilated by gamma matrices and satisfying massless Dirac equation. The conserved fermion number defined by the integral of $\overline{\Psi}\gamma^0\Psi$ over 3-space gives just its standard value. A further experimentation shows that Feynman diagrams with non-hermitian gamma matrices give just the standard results since ordinary fermionic propagator and boson-emission vertices at the ends of the line containing WCW gamma matrix and its conjugate give compensating fermion numbers [K76].

3. If the theory would contain massive fermions or a coupling to a scalar Higgs, a catastrophe would result. Hence ordinary Higgs mechanism is not possible in this framework. Of course, also the quantization of fermions is totally different. In TGD fermion mass is not a scalar in H. Part of it is given by CP_2 Dirac operator, part by p-adic thermodynamics for L_0, and part by Higgs field which behaves like vector field in CP_2 degrees of freedom, so that the catastrophe is avoided.

4. In zero energy ontology zero energy states are characterized by M-matrix elements constructed by applying the combination of stringy and symplectic Feynman rules and fermionic propagator is replaced with its super-conformal generalization reducing to an ordinary fermionic propagator for massless states. The norm of a single fermion state is given by a propagator connecting positive energy state and its conjugate with the propagator G_0/L_0 and the standard value of the norm is obtained by using Dirac equation and the fact that Dirac operator appears also in G_0.

5. The hermiticity of super-generators G would require Majorana property and one would end up with superstring theory with critical dimension $D = 10$ or $D = 11$ for the imbedding space. Hence the new interpretation of gamma matrices, proposed already years ago, has very profound consequences and convincingly demonstrates that TGD approach is indeed internally consistent.

In this framework coupling constant evolution would correspond evolution as a function of the scale of CD. It might have interpretation also in terms of addition of intermediate zero energy states corresponding to the generalized Feynman diagrams obtained by the insertion of causal diamonds with a new shorter time scale $T = T_{prev}/2$ to the previous Feynman diagram as the size of CD is increased. p-Adic length scale hypothesis follows naturally. A very close correspondence with ordinary Feynman diagrammatics arises and and ordinary vision about coupling constant evolutions arises. The absence of infinities follows from the symplectic invariance which is genuinely new element. p-Adic and real coupling constant evolutions can be seen as completions of coupling constant evolutions for physics based on rationals and their algebraic extensions.

22.3 General Vision About Real And P-Adic Coupling Constant Evolution

Many new pieces of understanding have emerge since the last updating of the views about coupling constant evolution. It is now understood how GRT space-time and QFT gauge theory limit emerge from many-sheeted space-time in long length scales. Quantum classical correspondence (QCC) suggests that classical correlation functions correspond to those for elementary particles. What is new that the generalization of AdS/CFT correspondence strongly suggested by the extension of super-conformal symmetries and the possibility to express WCW Kähler metric in two manners provides support for QCC. It is now understood how the hierarchy of Planck constants relates to a hierarchy of symmetry breakings for super-symplectic algebra and to a hierarchy of quantum criticalities. The vision about scattering amplitudes as representations of sequences of arithmetic operations in the Yangian of super-sympelectic algebra [A27] [B39, B30, B31] gives hopes about the computation of scattering amplitudes and already now gives vision about their general structure and what is the counterpart for the coupling constant evolution at the fundamental level.

The older approach was rather phenomenological and based mostly on p-adic considerations and the future challenge is to combine the new ingredients with p-adic picture. Perhaps the most important questions about p-adic coupling constant evolution relate to the basic hypothesis about preferred role of primes $p \simeq 2^k$, k an integer. Why integer values of k are favored, why prime values are even more preferred, and why Mersenne primes $M_n = 2^n - 1$ and Gaussian Mersennes seem to be at the top of the hierarchy?

Second bundle of questions relates to the color coupling constant evolution. Do Mersenne primes really define a hierarchy of fixed points of color coupling constant evolution for a hierarchy of asymptotically non-free QCD type theories both in quark and lepton sector of the theory? How the transitions $M_n \rightarrow M_{n(next)}$ occur? What are the space-time correlates for the coupling constant evolution and for for these transitions and how space-time description relates to the usual description in terms of parton loops? How the condition that p-adic coupling constant evolution reflects the real coupling constant evolution can be satisfied and how strong conditions it poses on the coupling constant evolution?

22.3.1 A General View About Coupling Constant Evolution

The following general vision about coupling constant evolution summarizes the recent understanding. The details of the picture are of course bound to fluctuate.

Einstein's equations, Equivalence Principle, and GRT and QFT limits of TGD

Coupling constant evolution makes sense in quantum field theory defined in fixed background space-time, say Minkowski space-time. In TGD framework imbedding space replaces this fixed space-time and in ZEO the hierarchy of causal diamonds replaces imbedding space. It is not at all clear whether at the level of basic TGD coupling constant evolution makes sense at all whereas it should make sense at QFT limit of TGD. This requires understanding of QFT and GRT limits of TGD including also Equivalence Principle.

At quantum level Equivalence Principle (EP) can be reduced to quantum classical correspondence: the conserved four-momentum associated with Kähler action equals to the eigenvalue of conserved quantal four-momentum assignable to Kähler-Dirac equation [K88]. This quantal four-momentum in turn can be associated with string world sheets which emerge naturally from Kähler-Doirac equation.

Einstein's equation give a purely local meaning for EP. How Einstein's equations and General Relativity in long length scales emerges from TGD has been a long-standing interpretational problem of TGD, whose resolution came from the realization that GRT is only an effective theory obtained by endowing M^4 with effective metric.

1. The replacement of superposition of fields with superposition of their effects means replacing superposition of fields with the set-theoretic union of space-time surfaces. Particle experiences sum of the effects caused by the classical fields at the space-time sheets (see **Fig.** `http://tgdtheory.fi/appfigures/fieldsuperpose.jpg` or **Fig. ??** in the appendix of this book). .

2. This is true also for the classical gravitational field defined by the deviation from flat Minkowski metric instandard M^4 coordinates for the space-time sheets. One can define effective metric as sum of M^4 metric and deviations. This effective metric would correspond to that of General Relativity. This resolves long standing issues relating to the interpretation of TGD. Similar description applies to induced electroweak gauge potentials and color gauge potentials: the sum of these gauge potentials over space-time sheets should define the classical gauge fields of QFT limit of TGD.

3. Einstein's equations could hold true for the effective metric. They are motivated by the underlying Poincare invariance which cannot be realized as global conservation laws for the effective metric. The conjecture vanishing of divergence of Khler energy momentum tensor can be seen as the microscopic justification for the claim that Einstein's equations hold true for the effective space-time.

4. The breaking of Poincare invariance could have interpretation as effective breaking in zero energy ontology (ZEO), in which various conserved charges are length dependent and defined separately for each causal diamond (CD).

What coupling constant evolution could mean in TGD framework? Kähler action and Kähler-Dirac action do not contain any fundamental couplings affecting to the dynamics. Kähler coupling strength does not affect classical dynamics and is analogous to critical temperature, and therefore invariant under renormalization group if defined in TGD framework. This suggests that the analog of renormalization group equations at space-time level does not look feasible. Continuous coupling constant evolution might be useful notion only at the QFT limit.

The natural length scale hierarchy associated with coupling constant evolution would be the hierarchy of length scales assignable to CDs. The minimal sizes of CDs assumed to be equal to secondary p-adic length scales in the case of elementary particles. More generally, number theoretical arguments suggest that the scales of CDs come as integer multiples of CP_2 radius. What is new that coupling constant evolution would be discretized, being labelled by integers. Primes and primes near powers of 2 could correspond to physically favored minimal size scales for CDs: kind of survivors in fight for survival. Discrete coupling constant evolution as evolution of various M-matrix elements as function of the size-scale of CD would look like a reasonable TGD counterpart of coupling constant evolution. For single CD one might say that system is quantum critical, and coupling constants do not evolve.

Does the finiteness of measurement resolution dictate the laws of physics?

The hypothesis that the mere finiteness of measurement resolution could determine the laws of quantum physics [K13] completely belongs to the category of not at all obvious first principles. The basic observation is that the Clifford algebra spanned by the gamma matrices of the "world of classical worlds" represents a von Neumann algebra [A67] known as hyperfinite factor of type II_1 (HFF) [K13, K87, K22]. HFF [A44, A55] is an algebraic fractal having infinite hierarchy of included subalgebras isomorphic to the algebra itself [A1]. The structure of HFF is closely related to several notions of modern theoretical physics such as integrable statistical physical systems [A88], anyons [D4] , quantum groups and conformal field theories [A91], and knots and topological quantum field theories [A86, A50].

Zero energy ontology is second key element. In zero energy ontology these inclusions allow an interpretation in terms of a finite measurement resolution: in the standard positive energy ontology this interpretation is not possible. Inclusion hierarchy defines in a natural manner the notion of coupling constant evolution and p-adic length scale hypothesis follows as a prediction. In this framework the extremely heavy machinery of renormalized quantum field theory involving the elimination of infinities is replaced by a precisely defined mathematical framework. More concretely, the included algebra creates states which are equivalent in the measurement resolution used. Zero energy states are associated with causal diamond formed by a pair of future and past directed light-cones having positive and negative energy parts of state at their boundaries. Zero energy state can be modified in a time scale shorter than the time scale of the zero energy state itself.

On can imagine two kinds of measurement resolutions. The element of the included algebra can leave the quantum numbers of the positive and negative energy parts of the state invariant, which means that the action of subalgebra leaves M-matrix invariant. The action of the included algebra can also modify the quantum numbers of the positive and negative energy parts of the state such that the zero energy property is respected. In this case the Hermitian operators subalgebra must commute with M-matrix.

The temporal distance between the tips of light-cones corresponds to the secondary p-adic time scale $T_{p,2} = \sqrt{p}T_p$ by a simple argument based on the observation that light-like randomness of light-like 3-surface is analogous to Brownian motion. This gives the relationship $T_p = L_p^2/Rc$, where R is CP_2 size. The action of the included algebra corresponds to an addition of zero energy parts to either positive or negative energy part of the state and is like addition of quantum fluctuation below the time scale of the measurement resolution. The natural hierarchy of time scales is obtained as $T_n = 2^{-n}T$ since these insertions must belong to either upper or lower half of the causal diamond. This implies that preferred p-adic primes are near powers of 2. For electron the time scale in question is.1 seconds defining the fundamental biorhythm of 10 Hz.

M-matrix representing a generalization of S-matrix and expressible as a product of a positive square root of the density matrix and unitary S-matrix would define the dynamics of quantum theory [K13]. The notion of thermodynamical state would cease to be a theoretical fiction and in a well-defined sense quantum theory could be regarded as a square root of thermodynamics. The original hope was that Connes tensor product realizing mathematical the finite measurement resolution could fix M-matrix to high degree turned out be too optimistic.

How do p-adic coupling constant evolution and p-adic length scale hypothesis emerge?

In zero energy ontology zero energy states have as imbedding space correlates causal diamonds for which the distance between the tips of the intersecting future and past directed light-cones comes as integer multiples of a fundamental time scale: $T_n = n \times T_0$. p-Adic length scale hypothesis allows to consider a stronger hypothesis $T_n = 2^n T_0$ and its generalization a slightly more general hypothesis $T_n = p^n T_0$, p prime. It however seems that these scales are dynamically favored but that also other scales are possible.

Could the coupling constant evolution in powers of 2 implying time scale hierarchy $T_n = 2^n T_0$ induce p-adic coupling constant evolution and explain why p-adic length scales correspond to $L_p \propto \sqrt{p}R$, $p \simeq 2^k$, R CP_2 length scale? This looks attractive but there is a problem. p-Adic length scales come as powers of $\sqrt{2}$ rather than 2 and the strongly favored values of k are primes and thus odd so that $n = k/2$ would be half odd integer. This problem can be solved.

1. The observation that the distance traveled by a Brownian particle during time t satisfies $r^2 = Dt$ suggests a solution to the problem. p-Adic thermodynamics applies because the partonic 3-surfaces X^2 are as 2-D dynamical systems random apart from light-likeness of their orbit. For CP_2 type vacuum extremals the situation reduces to that for a one-dimensional random light-like curve in M^4. The orbits of Brownian particle would now correspond to light-like geodesics γ_3 at X^3. The projection of γ_3 to a time=constant section $X^2 \subset X^3$ would define the 2-D path γ_2 of the Brownian particle. The M^4 distance r between the end points of γ_2 would be given $r^2 = Dt$. The favored values of t would correspond to $T_n = 2^n T_0$ (the full light-like geodesic). p-Adic length scales would result as $L^2(k) = DT(k) = D2^k T_0$ for $D = R^2/T_0$. Since only CP_2 scale is available as a fundamental scale, one would have $T_0 = R$ and $D = R$ and $L^2(k) = T(k)R$.

2. p-Adic primes near powers of 2 would be in preferred position. p-Adic time scale would not relate to the p-adic length scale via $T_p = L_p/c$ as assumed implicitly earlier but via $T_p = L_p^2/R_0 = \sqrt{p}L_p$, which corresponds to secondary p-adic length scale. For instance, in the case of electron with $p = M_{127}$ one would have secondary Compton length Electron's secondary Compton time $T_e(127) = \sqrt{5}T_2(127) = .1$ seconds defines a fundamental biological rhythm. A deep connection between elementary particle physics and biology becomes highly suggestive.

3. In the proposed picture the p-adic prime $p \simeq 2^k$ would characterize the thermodynamics of the random motion of light-like geodesics of X^3 so that p-adic prime p would indeed be an inherent property of X^3.

Could symplectic variant of QFT allow to understand coupling constant evolution in zero modes?

Symplectic variant of conformal field theories might be a further key element in the concrete construction of n-point functions and M-matrix in zero energy ontology. Although I have known super-symplectic (super-symplectic) symmetries to be fundamental symmetries of quantum TGD for almost two decades, I failed for some reason to realize the existence of symplectic QFT, and discovered it while trying to understand quite different problem - the fluctuations of cosmic microwave background! The symplectic contribution to the n-point function satisfies fusion rules and involves only variables which are symplectic invariants constructed using geodesic polygons assignable to the sub-polygons of n-polygon defined by the arguments of n-point function. Fusion rules lead to a concrete recursive formula for n-point functions and M-matrix in contrast to the iterative construction of n-point functions used in perturbative QFT.

Symplectic QFT might allow to calculate the coupling constant evolution in zero modes which do not contribute to the line element of sub- WCW expect as contribute a conformal factor depending on zero modes invariant under symplectic transformations.

22.3.2 Number Theoretical Vision About Coupling Constant Evolution

The recent progress in the understanding of TGD has led to a rather abstract number theoretical vision about coupling constant evolution.

Coupling constant evolution as increase in computational precision in Yangian arithmetics

One should relate the picture of the usual perturbation theory to the picture in which one identifies scattering amplitudes as sequences of arithmetical manipulations in super-symplectic Yangian [A27] [B39, B30, B31]. How does one obtain a perturbation theory in powers of coupling constant, what does running coupling constant mean, etc...? I have already discussed how the superposition of diagrams could be understood in the new picture [K76].

1. The QFT picture with running coupling constant is expected at QFT limit, when many-sheeted space-time is replaced with a slightly curved region of M^4 and gravitational field and gauge potentials are identified as sums of the deviations of induced metric from M^4 metric and classical induced gauge potentials associated with the sheets of the many-sheeted space-time. The running coupling constant would be due to the dependence of the size scale of CD, and p-adic coupling constant evolution would be behind the continuous one. A good first guess is that secondary p-adic length scales proportional to p define preferred size scales for CD among integer multiples of CP_2 scale. For electron the scale corresponds to time scale of .1 second defining a fundamental bio-rhythm.

2. The notion of running coupling constant is very physical concept and should have a description also at the fundamental level and be due to a finite computational resolution, which indeed has very concrete description in terms of Noether charges of super-symplectic Yangian creating the states at the ends of space-time surface at the boundaries of CD. The space-time surface and the diagram associated with a given pair of 3-surfaces and stringy Noether charges associated with them can be characterized by a complexity measured in terms of the number of vertices (3-surface at which three 3-surfaces meet).

   For instance, 3-particle scattering can be possible only by using the simplest 3-vertex defined by product or co-product for pairs of 3-surfaces. In the generic case one has more complex diagram and what looks first 3-particle vertex has complex substructure rather than being simple product or co-product.

3. Complexity seems to have two separate aspects: the complexities of the positive and negative parts of zero energy state as many-fermion states and the complexity of associated 3-surfaces. The generalization of AdS/CFT however suggests that once the string world sheets and partonic 2-surfaces appearing in the diagram have been fixed, the space-time surface itself is fixed. The principle also suggests that the fixing partonic 2-surface and the strings connecting them at the boundaries of CD fixes the 3-surface apart from the action of sub-algebra of Yangian acting as gauge algebra (vanishing classical Noether charges). If one can determine the minimal sequence of allowed algebraic operation of Yangian connecting initial and final fermion states, one knows the minimum number of vertices and therefore the topological structure of the connecting minimal space-time surface.

4. In QFT spirit one could describe the finite measurement resolution by introducing effective 3-point vertex, which is need not be product/co-produce anymore. 3-point scattering amplitudes in general involve microscopic algebraic structure involving several vertices. One can however give up the nice algebraic interpretation and just talk about effective 3-vertex for practical purposes. Just as the QFT vertex described by running coupling constant decomposes to sum of diagrams, product/co-product in TGD could be replaced with effective product/co-product expressible as a longer computation. This would imply coupling constant evolution.

   Fermion lines could however remain as such since they are massless in 8-D sense and mass renormalization does not make sense.

   Similar practical simplification could be done the initial and final states to get rid of superposition of the Yangian generators with different numbers of strings ("cloud of virtual particles"). This would correspond to wave function renormalization.

The number of vertices and wormhole contact orbits serves as a measure for the complexity of the diagram.

1. Since fermion lines are associated with wormhole throats assignable with wormhole contacts identifiable as deformations CP_2 type vacuum extremals, one expects that the exponent of the Kähler function defining vacuum functional is in the first approximation the total CP_2 volume of wormhole contacts giving a measure for the importance of the contribution in functional integral. If it converges very rapidly only Gaussian approximation around maximum is needed.

2. Convergence depends on how large the fraction of volume of CP_2 is associated with a given wormhole contact. The volume is proportional to the length of the wormhole contact orbit. One expects exponential convergence with the number of fermion lines and their lengths for long lines. For short distances the exponential damping is small so that diagrams with microscopic structure of diagrams are needed and are possible. This looks like adding small scale details to the algebraic manipulations.

3. One must be of course be very cautious in making conclusions. For instance, the presence of $1/\alpha_K \propto h_{eff}$ in the exponent of Kähler function would suggest that for large values of h_{eff} only the 3-surfaces with smallest possible number of wormhole contact orbits contribute. On the other hand, the generalization of AdS/CFT duality suggests that Kähler action reducible to area of string world sheet in the effective metric defined by canonical momentum currents of Kähler action behaves as $\alpha_K^2 \propto 1/h_{eff}^2$. How $1/h_{eff}^2$ proportionality might be understood is discussed in [K106] in terms electric-magnetic duality.

Renormalization group flow would have two meanings.

1. RG flow can correspond to the increase of resolution in the sense that the number of fermionic strings per partonic 2-surfaces increases This would mean increase of the resolution and replace computational sequences with more complex ones involving more vertices. Both length scale resolution and angular resolution can increase and these resolutions should relate to the algebraic resolution.

2. Another meaning is as flow defined by the hierarchy of quantum criticalities and having interpretation in terms of hierarchy of h_{eff}. This process transforms gauge degrees of freedom to physical degrees of freedom and there is temptation to interpret this as discrete evolution such that each h_{eff} defines a plateau in the evolution somewhat like in fractional quantum Hall effect for which I have indeed proposed an explanation in terms of hierarchy of Planck constants.

Coupling constant evolution and ramified primes in algebraic extensions of rationals

The recent adelic vision about TGD allows totally new insights about p-adic coupling constant evolution. In particular, the origin of preferred p-adic primes can be understood and the realization of number theoretical universality by algebraic continuations becomes almost trivial.

1. The recent general picture about coupling constant evolution relies on the hierarchy of algebraic extensions of rational in which the parameters characterizing string world sheets and partonic 2-surfaces. They define the intersection of reality and various p-adicities, and strong holography is assumed to allow a continuation of these 2-surfaces to preferred extremals of Kähler action [K111].

2. By strong form of holography also scattering amplitudes are determined by the data in the intersection so that coupling constant evolution should reduce to the evolution of complexity for the hierarchy of algebraic extensions of rationals. The basic parameters characterizing the extensions are the ramified primes - the primes containing higher powers in the product expansion using the primes of the extension (to be precise, one should talk about prime ideals).

3. The product of ramified primes is a basic parameter characterizing extension. Preferred p-adic primes would naturally correspond to the ramified primes. p-Adic continuations identifiable as imaginations would be due to the existence of p-adic pseudo-constants. The continuation could fail for most configurations of partonic 2-surfaces and string world sheets in

the real sector: the interpretation would be that some space-time surfaces can be imagined but not realized [K48]. For certain extensions the number of realizable imaginations could be exceptionally large. These extensions would be winners in the number theoretic fight for survivalandcorresponding ramified primes would be preferred p-adic primes.

NMP [K41] in turn could allow to understand why the p-adic primes near but below powers of primes are favored: $p \simeq p_1^k$. The original form of the p-adic length scale hypothesis corresponds to $p_1 = 2$.

This justifies the basic picture implied by p-adic mass calculations and allows to generalize canonical identification as a map taking real values of various group invariants (inner products of four-momenta, etc..) to their p-adic counterparts as one algebraically continues scattering amplitudes from the intersection to various number fields.

4. The reasonable expectation is that coupling constant evolution reduces to the evolution of complexity for the algebraic extensions of rationals and is thus discretized and that the preferred primes serve as parameters defining p-adic length scales appearing in the p-adic length scale evolution as varying parameters.

5. Infinite primes at the lowest level representing bound states label can be mapped to polynomials, and parametrize irreducible extensions of rationals so that coupling constant evolution corresponds to evolution at the level of infinite primes too. An interesting question concerns the meaning of higher level infinite primes mappable to polynomials of several variables.

To sum up, the number theoretical vision has become rather concrete.

22.3.3 Could Correlation Functions, S-Matrix, And Coupling Constant Evolution Be Coded The Statistical Properties Of Preferred Extremals?

How to calculate the correlation functions and coupling constant evolution has remained a basic unresolved challenge. Generalized Feynman diagrams provide a powerful vision which however does not help in practical calculations. Some big idea has been lacking.

Quantum classical correspondence states that all aspects of quantum states should have correlates in the geometry of preferred extremals. In particular, various elementary particle propagators should have a representation as properties of preferred extremals. This would allow to realize the old dream about being able to say something interesting about coupling constant evolution although it is not yet possible to calculate the M-matrices and U-matrix. The general structure of U-matrix is however understood [K91]. Hitherto everything that has been said about coupling constant evolution has been rather speculative arguments except for the general vision that it reduces to a discrete evolution defined by p-adic length scales. General first principle definitions are however much more valuable than ad hoc guesses even if the latter give rise to explicit formulas.

In quantum TGD and also at its QFT limit various correlation functions in given quantum state should code for its properties. By quantum classical correspondence these correlation functions should have counterparts in the geometry of preferred extremals. Even more: these classical counterparts for a given preferred extremal ought to be identical with the quantum correlation functions for the superposition of preferred extremals. This correspondence could be called quantum ergodicity by its analogy with ordinary ergodicity stating that the member of ensemble becomes representative of ensemble.

This principle would be a quantum generalization of ergodic theorem stating that the time evolution of a single member of ensemble represents the ensemble statistically. This symmetry principle analogous to holography might allow to fix S-matrix uniquely even in the case that the hermitian square root of the density matrix appearing in the M-matrix would lead to a breaking of quantum ergodicity as also 4-D spin glass degeneracy suggests.

This principle would allow to deduce correlation functions from the statistical properties of single preferred extremal alone using just classical intuition. Also coupling constant evolution would be coded by the statistical properties of preferred extremals. Quantum ergodicity would mean an enormous simplification since one could avoid the horrible conceptual complexities involved with the functional integrals over WCW .

This might of course be too optimistic guess. If a sub-algebra of symplectic algebra acts as gauge symmmetries of the preferred extremals in the sense that corresponding Noether charges vanish, it can quite well be that correlations functions correspond to averages for extremals belonging to single conformal equivalence class.

1. The marvellous implication of quantum ergodicity would be that one could calculate everything solely classically using the classical intuition - the only intuition that we have. Quantum ergodicity would also solve the paradox raised by the quantum classical correspondence for momentum eigenstates. Any preferred extremal in their superposition defining momentum eigenstate should code for the momentum characterizing the superposition itself. This is indeed possible if every extremal in the superposition codes the momentum to the properties of classical correlation functions which are identical for all of them.

2. The only manner to possibly achieve quantum ergodicity is in terms of the statistical properties of the preferred extremals. It should be possible to generalize the ergodic theorem stating that the properties of statistical ensemble are represented by single space-time evolution in the ensemble of time evolutions. Quantum superposition of classical worlds would effectively reduce to single classical world as far as classical correlation functions are considered. The notion of finite measurement resolution suggests that one must state this more precisely by adding that classical correlation functions are calculated in a given UV and IR resolutions meaning UV cutoff defined by the smallest CD and IR cutoff defined by the largest CD present.

3. The skeptic inside me immediately argues that TGD Universe is 4-D spin glass so that this quantum ergodic theorem must be broken. In the case of the ordinary spin classes one has not only statistical average for a fixed Hamiltonian but a statistical average over Hamiltonians. There is a probability distribution over the coupling parameters appearing in the Hamiltonian. Maybe the quantum counterpart of this is needed to predict the physically measurable correlation functions.

   Could this average be an ordinary classical statistical average over quantum states with different classical correlation functions? This kind of average is indeed taken in density matrix formalism. Or could it be that the square root of thermodynamics defined by ZEO actually gives automatically rise to this average? The eigenvalues of the "hermitian square root" of the density matrix would code for components of the state characterized by different classical correlation functions. One could assign these contributions to different "phases".

4. Quantum classical correspondence in statistical sense would be very much like holography (now individual classical state represents the entire quantum state). Quantum ergodicity would pose a rather strong constraint on quantum states. This symmetry principle could actually fix the spectrum of zero energy states to a high degree and fix therefore the M-matrices given by the product of hermitian square root of density matrix and unitary S-matrix and unitary U-matrix constructible as inner products of M-matrices associated with CDs with various size scales [K91].

5. In TGD inspired theory of consciousness the counterpart of quantum ergodicity is the postulate that the space-time geometry provides a symbolic representation for the quantum states and also for the contents of consciousness assignable to quantum jumps between quantum states. Quantum ergodicity would realize this strongly self-referential looking condition. The positive and negative energy parts of zero energy state would be analogous to the initial and final states of quantum jump and the classical correlation functions would code for the contents of consciousness like written formulas code for the thoughts of mathematician and provide a sensory feedback.

How classical correlation functions should be defined?

1. General Coordinate Invariance and Lorentz invariance are the basic constraints on the definition. These are achieved for the space-time regions with Minkowskian signature and 4-D M^4 projection if linear Minkowski coordinates are used. This is equivalent with the contraction of the indices of tensor fields with the space-time projections of M^4 Killing vector fields

representing translations. Accepting ths generalization, there is no need to restrict oneself to 4-D M^4 projection and one can also consider also Euclidian regions identifiable as lines of generalized Feynman diagrams.

Quantum ergodicity very probably however forces to restrict the consideration to Minkowskian and Euclidian space-time regions and various phases associated with them. Also CP_2 Killing vector fields can be projected to space-time surface and give a representation for classical gluon fields. These in turn can be contracted with M^4 Killing vectors giving rise to gluon fields as analogs of graviton fields but with second polarization index replaced with color index.

2. The standard definition for the correlation functions associated with classical time evolution is the appropriate starting point. The correlation function $G_{XY}(\tau)$ for two dynamical variables $X(t)$ and $Y(t)$ is defined as the average $G_{XY}(\tau) = \int_T X(t)Y(t+\tau)dt/T$ over an interval of length T, and one can also consider the limit $T \rightarrow \infty$. In the recent case one would replace τ with the difference $m_1 - m_2 = m$ of M^4 coordinates of two points at the preferred extremal and integrate over the points of the extremal to get the average. The finite time interval T is replaced with the volume of causal diamond in a given length scale. Zero energy state with given quantum numbers for positive and negative energy parts of the state defines the initial and final states between which the fields appearing in the correlation functions are defined.

3. What correlation functions should be considered? Certainly one could calculate correlation functions for the induced spinor connection given electro-weak propagators and correlation functions for CP_2 Killing vector fields giving correlation functions for gluon fields using the description in terms of Killing vector fields. If one can uniquely separate from the Fourier transform uniquely a term of form $Z/(p^2 - m^2)$ by its momentum dependence, the coefficient Z can be identified as coupling constant squared for the corresponding gauge potential component and one can in principle deduce coupling constant evolution purely classically. One can imagine of calculating spinorial propagators for string world sheets in the same manner. Note that also the dependence on color quantum numbers would be present so that in principle all that is needed could be calculated for a single preferred extremal without the need to construct QFT limit and to introduce color quantum numbers of fermions as spin like quantum numbers (color quantum numbers corresponds to CP_2 partial wave for the tip of the CD assigned with the particle).

Many detailed speculations about coupling constant evolution to be discussed in the sections below must be taken as innovative guesses doomed to have the eventual fate of guesses. The notion of quantum ergodicity could however be one of the really deep ideas about coupling constant evolution comparable to the notion of p-adic coupling constant evolution. Quantum Ergodicity (briefly QE) would also state something extremely non-trivial also about the construction of correlation functions and S-matrix. Because this principle is so new, the rest of the chapter does not yet contain any applications of QE. This should not lead the reader to under-estimate the potential power of QE.

22.4 P-Adic Coupling Constant Evolution

p-Adic coupling constant evolution is one of the genuinely new elements of quantum TGD. In the following some aspects of the evolution will be discussed. The discussion is a little bit obsolete as far as the role of canonical identification is considered. The most recent view about p-adic coupling constant evolution is discussed at the end of the section.

22.4.1 General Considerations

One of the basic challenges of quantum TGD is to understand whether the notion of p-adic coupling constant evolution is something related to the basic TGD or whether it emerges at GRT and QFT limits only.

1. Since neither classical field equations for Kähler action nor Kähler-Dirac action depend on coupling constants except as overall multiplicative normalization factor, one expects that at

the level of TGD space-time the notion of coupling constant evolution is not well-defined or at least fails to be a fundamental notion. Coupling constant evolution would characterize GRT and QFT limits of TGD and since causal diamond (CD) is the basic unit, the scale of CD would serve as a fundamental scale.

What would give rise to the ordinary continuous coupling constant evolution at long length scales, would be the replacement of many-sheeted space-time with GRT space-time containing gauge potentials which are sums of induced gauge potentials associated with various space-time sheets. The increase in the size of CD would induce the scaling of the size of the space-time sheet. Hence the geometric correlate for coupling constant evolution would be the scaling of CD size. The original belief was that it would be scaling of the size of the space-time sheet.

2. The original view was that there are two separate coupling constant evolutions: one associated with p-adic length scale hierarchy and second associated with angle resolution and characterized by the hierarchy of Planck constants. In the recent view these evolutions are unified to a number theoretic evolution in terms of increasing complexity of an algebraic extension of rational numbers inducing also the extensions of p-adic number fields. Space-time and quantum physics become adelic. The algebraic extensions are associated with the parameters characterizing partonic 2-surfaces and string world sheets, which by strong form of holography determine space-time surfaces as preferred extremals of Kähler action. The crucial number theoretical universality necessary for the adelization is almost trivially realized by algebraic continuation from the intersection of realities and p-adicities defined by the 2-surfaces with parameters in algebraic extensions of rationals.

   Preferred p-adic primes emerge naturally in the number theoretic vision as so called ramified primes of the algebraic extension. One can also deduce a generalization of p-adic length scale hypothesis in terms of Negentropy Maximization Principle (NMP) [K41]. Hence

3. At the fundamental level this evolution is discrete by p-adic length scale hypothesis justified by zero energy ontology, where CD sizes are assumed to come as integer multiples of CP_2 mass: the discretization is for number theoretical reasons and gives hopes of number theoretical universality. The most general option is that the CD sizes come as integer multiples of CP_2 size. Discreteness means that continuous mass scale is replaced by mass scales coming square root prime multiples of CP_2 mass. Obviously continuous evolution is an excellent approximation in elementary particle p-adic mass scales. p-Adic length scale hypothesis allows only half octaves of CP_2 mass. Kähler coupling strength α_K or gravitational coupling constant is assumed to remain invariant under p-adic coupling constant evolution. The basic problem is to understand the value of α_K and here p-adic mass calculations give strong constraints.

4. The realization that well-definedness of em charge requires the localization of the modes of the induced spinor field to string world sheets or partonic 2-surfaces was an important step in the process of understanding super-symplectic and other symmetries, and has led to the recent realization that strong form of holography is realized for string world sheets and partonic 2-surfaces by continuing them to preferred extremals of Kähler action.

   Coupling constant evolution should allow a reduction to string model type description. The ordinary AdS/CFT correspondence between $n-1$-D conformal field theory in $AdS^n \times S^5$ is modified. Super-symplectic generalization of conformal field theory is associated with string world sheets and partonic 2-surfaces and 10-D bulk containing strings is replaced with space-time surface contained in $M^4 \times CP_2$. This holography looks very much like old-fashioned holography.

   This framework has powerful physical implications. Gravitationally bound states correspond to partonic 2-surfaces connected by fermionic strings. The ordinary string theory picture with string tension defined by Planck length would not allow gravitationally bound states above Planck length scale. The string tension is however dynamical since effective stringy action can be assigned to the effective metric defined by Kähler-Dirac gamma matrices appearing in the K-D equation. The value of string tension must characterize the value of Kähler action by strong form of holography, and it decreases in long length scales by its $1/h_{eff}^2$-proportionality. How $1/h_{eff}^2$ proportionality might be understood is discussed in [K106] in terms electric-magnetic duality.

Quantum gravitational coherence is present in astrophysical scales and assignable to the large values of $h_{eff} = h_{gr}$, where $\hbar_{gr} = GMm/v_0$, $v_0/c < 1$, with v_0 having dimensions of velocity, is gravitational Planck constant characterizing magnetic flux tubes mediating gravitational interaction between masses M and m [K66]. v_0 is of the order of magnitude for a typical rotational velocity in the system. Hadronic string tension can be also understand as this kind of string tension.

Gravitational quantum coherence is predicted in astrophysical scales, and dark matter is macroscopically quantum coherent. In this framework superstring models give quantum gravitation in Planck length scale but the proposed QFT based vision about long length scale limit must be seen as a figment of bad imagination leading to the landscape catastrophe and total loss of predictive power.

5. If weak form of electric magnetic duality and $j \cdot A = 0$ condition for Kähler current and gauge potential in the interior of space-time sheets are satisfied, Kähler action reduces to Chern-Simons terms at light-like partonic orbits and space-like 3-surfaces at the ends of space-time surface. Induced metric would apparently disappear from the action in accordance with the idea about TGD as almost topological QFT.

22.4.2 How P-Adic And Real Coupling Scattering Amplitudes Are Related To Each Other?

p-Adic and real scattering amplitudes would be obtained from the amplitudes having values in the intersection of reality and p-adicities corresponding to a particular algebraic extension of rationals inducing those of p-adic number fields.

1. The algebraic intersection of reality and various p-adicities would be defined by string world sheets and partonic 2-surfaces. The amplitudes in the intersection are algebraically universal amplitudes having the same values as numbers of algebraic extension in any number field. Unitary is satisfied and symmetries respected in their discrete versions. This requires that various quantum numbers such as momenta belong to the algebraic extension of rationals. One might say that scattering amplitudes in algebraic extension are kind of genes for scattering amplitudes in various number fields and define adelic S-matrix as the analog of finite-dimensional adelic matrices appearing in Langlands program.

2. This kind of direct identification of real and p-adic amplitudes cannot be continuous if one allows all possible values of the parameters - say various Lorentz invariants appearing in the amplitude. The possibility of p-adic pseudo-constants allows the identification via common algebraics if one poses a cutoff and performs the algebraic identification only for the discrete set of parameters in the extension defined by the cutoff. The values of amplitudes for arbitrary reals and p-adics are obtained by algebraic continuation of these amplitudes to real and p-adic sectors. Unitarity and symmetries should fix the continuation highly uniquely.

   Both UV and IR cutoffs are necessary: otherwise arbitrarily large/small algebraic numbers in p-adic topology (proportional to negative/positive power of p) could correspond to arbitrarily small/large algebraic numbers in real topology.

3. This is not the only possible definition of the scattering amplitudes in the intersection. The definition of intersection of real and p-adic variants of WCWs does not introduce discretization at space-time level but at the level of parameter space characterizing the space of string world sheets and partonic 2-surfaces. Also in the case of scattering amplitudes a more abstract manner to define the intersection would use (say) Lorentz invariant rational functions of four-momenta and of other parameters with coefficients belonging to the algebraic extension of rationals. The algebraic continuation would be reduced to that for the parameters whereas the arguments of these function could be taken to belong the appropriate number field. This is certainly the more elegant option.

Is there any need for canonical identification or some of its variants with IR and UV cutoffs mapping real momenta/Lorentz invariants formed from them to p-adic ones and vice versa?

1. Canonical identification does not seem to be absolutely necessary. The correlates of matter and mind are independent below the resolution outside the set of the parameter values belonging to the algebraic intersection with cutoffs: dynamics for cognition would not reduce to that for matter.

   p-Adic mass calculations [K100] however suggest that canonical identification maps p-adic mass squared to its real counterpart. This map would assign to real mass scale its p-adic counterpart and would be essential for the physical interpretation of p-adic mass calculations. p-Adic/real mass squared can be generalized to Lorentz invariants defined by 4-momenta. The modification of the canonical identification would co-incide with the direct identification as algebraic numbers inside the range of parameters defining the intersection and map the powers of p above a upper cutoff to their inverses to achieve continuity.

2. The minimal assumption would be that some variant of canonical identification with cutoffs applies only to Lorentz invariants (for the variants of canonical identification see [K71].

   (a) This kind of variant would map the coefficients of powers of p^M to itself but invert the powers. It could also map rationals m/n to $I(m)/I(n)$. This kind of map would allow to assign to real scattering amplitudes p-adic scattering amplitudes with the same real values of Lorentz invariants defined by canonical identification.

   (b) A stronger correspondence would be obtained by mapping also the resulting p-adic scattering amplitudes to real ones by some variant of the canonical identification. The deviations between the two amplitudes should not be large if p-adic physics allows precise cognitive representations. p-Adic description mapped to real context might mean huge simplification as it indeed does in the case of p-adic mass calculations.

3. There is no need to apply canonical identification at space-time level. It has indeed become clear that canonical identification is ugly at space-time level although I was for some time enthusiastic about it [K104]. Strong form of holography however allows to realize the correspondence between real and p-adic space-time surfaces as a non-local correspondence since string world sheets and partonic 2-surfaces serve as "genes of space-time" and the correspondence between p-adicities and realities identified as space-time surfaces becomes non-local.

A further objection against the continuation from the algebraic intersection is based on the fact that non-algebraic transcendentals like π appear in the real scattering amplitudes. Could the counterpart of 2π in algebraic extension be defined as $N \times sin(2\pi/N)$, where $sin(2\pi/N)$ define the smallest angle in the abelian algebraic extension containing roots of unity? In the real sector the limit could be taken to give $2/pi$. One can wonder whether the positivity of the real numbers as Cartesian factor of ideles (invertible adeles) could somehow relate to the positive Grassmannians encountered in the twistor approach.

How to achieve consistency with the unitarity of topological mixing matrices and of CKM matrix?

It is easy to invent an objection against the proposed relationship between p-adic and real coupling constants. Topological mixing matrices U, D and CKM matrix $V = U^\dagger D$ define an important part of the electro-weak coupling constant structure and appear also in coupling constants. The problem is that canonical identification does not respect unitarity and does not commute with the matrix multiplication in the general case unlike gluing along common rationals. Even if matrices U and D which contain only ratios of integers smaller than p are constructed, the construction of V might be problematic since the products of two rationals can give a rational $q = r/s$ for which r or s or both are larger than p.

One might hope that the objection could be circumvented if the ratios of the integers of the algebraic extension defining the matrix elements of CKM matrix are such that the integer components of algebraic integers are smaller than p in U and D and even the products of integers in $U^\dagger D$ satisfy this condition so that modulo p arithmetics is avoided.

In the standard parameterization all matrix elements of the unitarity matrix can be expressed in terms of real and imaginary parts of complex phases ($p\ mod\ 4 = 3$ guarantees that $\sqrt{-1}$ is not an ordinary p-adic number involving infinite expansion in powers of p). These phases are expressible as products of Pythagorean phases and phases in some algebraic extension of rationals.

1. Pythagorean phases defined as complex rationals $[r^2 - s^2 + i2rs]/(r^2 + s^2)$ are an obvious source of potential trouble. However, if the products of complex integers appearing in the numerators and denominators of the phases have real and imaginary parts smaller than p it seems to be possible to avoid difficulties in the definition of $V = U^\dagger D$.

2. Pythagorean phases are not periodic phases. Algebraic extensions allow to introduce periodic phases of type $exp(i\pi m/n)$ expressible in terms of p-adic numbers in a finite-dimensional algebraic extension involving various roots of rationals. Also in this case the product $U^\dagger D$ poses conditions on the size of integers appearing in the numerators and denominators of the rationals involved.

If the expectation that topological mixing matrices and CKM matrix characterize the dynamics at the level $p \simeq 2^k$, $k = 107$, is correct, number theoretical constraints are not expected to bring much new to what is already predicted. Situation changes if these matrices appear already at the level k. For $k = 89$ hadron physics the restrictions would be even stronger and might force much simpler U, D and CKM matrices.

k-adicity constraint would have even stronger implications for S-matrix and could give very powerful constraints to the S-matrix of color interactions. Quite generally, the constraints would imply a p-adic hierarchy of increasingly complex S-matrices: kind of a physical realization for number theoretic emergence. The work with CKM matrix has shown how powerful the number theoretical constraints are, and there are no reasons to doubt that this could not be the case also more generally since in the lowest order the construction would be carried out in finite (Galois) fields $G(p, k)$.

22.4.3 How Could P-Adic Coupling Constant Evolution And P-Adic Length Scale Hypothesis Emerge From Quantum TGD Proper?

What p-adic coupling constant evolution really means has remained for a long time more or less open. The progress made in the understanding of the S-matrix of theory has however changed the situation dramatically.

M-matrix and coupling constant evolution

A breakthrough in the understanding of p-adic coupling constant evolution came through the understanding of S-matrix, or actually M-matrix defining entanglement coefficients between positive and negative energy parts of zero energy states in ZEO [K13] . M-matrix has interpretation as a "complex square root" of density matrix and thus provides a unification of thermodynamics and quantum theory. S-matrix is analogous to the phase of Schrödinger amplitude multiplying positive and real square root of density matrix analogous to modulus of Schrödinger amplitude.

The notion of finite measurement resolution realized in terms of inclusions of von Neumann algebras allows to demonstrate that the irreducible components of M-matrix are unique and possesses huge symmetries in the sense that the hermitian elements of included factor $\mathcal{N} \subset \mathcal{M}$ defining the measurement resolution act as symmetries of M-matrix, which suggests a connection with integrable quantum field theories.

As discussed in [K62] and in the earlier chapter about number theoretical vision, it is also possible to understand coupling constant evolution as a discretized evolution associated with time scales T_n, which come as integer multiples of a fundamental time scale: $T_n = n \times T_0$. p-Adic length scale hypothesis allows to consider a stronger hypothesis $T_n = 2^n T_0$ and a slightly more general hypothesis $T_n = p^n T_0$, p prime. It seems that these scales are dynamically favored but that also other scales are possible.

Number theoretic universality requires that renormalized coupling constants are rational or at most algebraic numbers and this is achieved by this discretization since the logarithms of discretized mass scale appearing in the expressions of renormalized coupling constants reduce to the form $log(2^n) = nlog(2)$ and with a proper choice of the coefficient of logarithm log(2) dependence disappears so that rational number results. Recall that also the weaker condition $T_p = pT_0$, p prime, would assign secondary p-adic time scales to the size scale hierarchy of CDs: $p \simeq 2^n$ would result as an outcome of some kind of "natural selection" for this option. The highly satisfactory feature would be that p-adic time scales would reflect directly the geometry of imbedding space and WCW.

The origin of the preferred p-adic length scales

This question was posed already for two decades ago has but remained without a convincing answer. Quite recently however the number theoretical vision allowed to understand both the origin of preferred p-adic number fields and the emergence of p-adic length scale hypothesis in a generalized form. Preferred primes are near but below powers prime which can be also larger that $p = 2$.

The preferred primes could correspond to so called ramified rational primes, which split in to products of the primes of the extension. If some prime appears as higher than first power, one has ramification. The number of ramified primes is finite.

In strong form of holography p-adic continuations of 2-surfaces to preferred extremals identifiable as imaginations would be easy due to the existence of p-adic pseudo-constants. The continuation could fail for most configurations of partonic 2-surfaces and string world sheets in the real sector: the interpretation would be that some space-time surfaces can be imagined but not realized [K48]. For certain extensions the number of realizable imaginations could be exceptionally large. These extensions would be winners in the number theoretic fight for survivalandcorresponding ramified primes would be preferred p-adic primes.

Weak form of NMP allows to understand the emergence of preferred p-adic length scales. NMP favors ramified primes, for which the integer n is power of single prime p. If n is a prime slightly below $n_{max} = p^n$ defining the dimension of the sub-space corresponding to maximal negentropy gain, weak form of NMP favors its selection since the p-adic topology is farthest from the discrete topology assignable to formal p-adic topology characterized by $p = 1$ [K111].

22.5 Quantitative Guesses For The Values Of Coupling Constants

This focus of attention in this section is in quantitative for the p-adic evolution of couplings constants obtained by combining information coming from p-adic mass calculations with number theoretic constraints and general formula for gravitational constant inspired by a simple physical picture.

Only educated (if even this) guesses are in question since real understanding of coupling constant evolution has begun to emerge only rather recently (2014) as the relationship between TGD and GRT and QFT was finally clarified.

The quite recent powerful results following from strong form of holography, adelic vision about space-time allowing to realize number theoretical universality, and from the vision that scattering amplitudes can be seen as representations for computational sequences in in Yangian [A27] [B39, B30, B31] of super-symplectic algebra have not been taken account at all.

The most recent idea is that exponent of Kähler action is number theoretically universal for preferred extremals and that also coupling constants are number theoretically universal [K111]. These conditions are extremely powerful and force to modify the earlier ad hoc ideas about number theoretic anatomy of Kähler coupling strength. Therefore this section is not a summary of final results but summary of various approaches to a difficult problem.

The view about coupling constant evolution has changed radically during 2016-2017 [L16, L22, L24, L38] as the number theoretic vision about TGD as adelic physics and the vision about twistor lift of TGD have co-evolved. Number theoretic vision has extremely powerful consequences and has led to amazingly simple proposals for the scattering amplitudes and coupling constant evolution. The following arguments trying to guess values of coupling constants are in the light of the new vision simply wrong so that this section can be regarded more or less as a curiosity.

22.5.1 A Revised View About Coupling Constant Evolution

The development of the ideas related to number theoretic aspects has been rather tortuous and based on guess work since basic theory has been lacking.

1. The original hypothesis was that Kähler coupling strength is invariant under p-adic coupling constant evolution. Later I gave up this hypothesis and replaced it with the invariance of gravitational coupling since otherwise the prediction would have been that gravitational coupling strength is proportional to p-adic length scale squared. Second first guess was that Kähler coupling strength equals to the value of fine structure constant at electron length scale corresponding to Mersenne prime M_{127}. Later I replaced fine structure constant with electro-weak U(1) coupling strength at this length scale. The recent discussion returns back to the roots in both aspects.

2. The recent discussion relies on the progress made in the understanding of quantum TGD at partonic level [K88]. What comes out is an explicit formula for Kähler couplings strength in terms of Dirac determinant involving only a finite number of eigenvalues of the Kähler-Dirac operator. This formula dictates the number theoretical anatomy of g_K^2 and also of other coupling constants: the most general option is that α_K is a root of rational. The requirement that the rationals involved are simple combined with simple experimental inputs leads to very powerful predictions for the coupling parameters.

3. A further simplification is due to the discreteness of p-adic coupling constant evolution allowing to consider only length scales coming as powers of $\sqrt{2}$. This kind of discretization is necessary also number theoretically since logarithms can be replaced with 2-adic logarithms for powers of 2 giving integers. This raises the question whether $p \simeq 2^k$ should be replaced with 2^k in all formulas as the recent view about quantum TGD suggests.

4. The prediction is that Kähler coupling strength α_K is invariant under p-adic coupling constant evolution and from the constraint coming from electron and top quark masses very near to fine structure constant so that the identification as fine structure constant is natural. Gravitational constant is predicted to be proportional to p-adic length scale squared and corresponds to the largest Mersenne prime (M_{127}), which does not correspond to a completely super-astronomical p-adic length scale. For the parameter R^2/G p-adicization program allows to consider two options: either this constant is of form e^q or 2^q: in both cases q is rational number. $R^2/G = exp(q)$ allows only M_{127} gravitons if number theory is taken completely seriously. $R^2/G = 2^q$ allows all p-adic length scales for gravitons and thus both strong and weak variants of ordinary gravitation.

5. A relationship between electromagnetic and color coupling constant evolutions based on the formula $1/\alpha_{em} + 1/\alpha_s = 1/\alpha_K$ is suggested by the induced gauge field concept, and would mean that the otherwise hard-to-calculate evolution of color coupling strength is fixed completely. The predicted value of α_s at intermediate boson length scale is correct.

It seems fair to conclude that the attempts to understand the implications of p-adicization for coupling constant evolution have begun to bear fruits.

General formula for the Kähler coupling strength

The identification of exponent of Kähler function as Dirac determinant leads to a formula relating Kähler action for the preferred extremal to the Dirac determinant. The eigenvalues are proportional to $1/\alpha_K$ since the matrices $\hat{\Gamma}^\alpha$ have this proportionality. This gives the formula

$$exp(\frac{S_{K,R}(X^4(X^3))}{2g_K^2}) = \prod_i \lambda_i = \frac{\prod_i \lambda_{0,i}}{(g_K)^{2N}} \ . \quad (22.5.1)$$

Here $\lambda_{0,i}$ by definition corresponds to $g_K^2 = 4\pi\alpha_K = 1$. $S_{K,R} = \int J^*J$ is the reduced Kähler action.

For $S_{K,R} = 0$, which might correspond to so called massless extremals [K7] one obtains the formula

$$g_K^2 = (\prod_i \lambda_{0,i})^{1/N} \ . \quad (22.5.2)$$

Thus for $S_{K,R} = 0$ extremals one has an explicit formula for g_K^2 having interpretation as the geometric mean of the eigenvalues $\lambda_{0,i}$. Several values of α_K are in principle possible.

p-Adicization suggests that $\lambda_{0,i}$ are rational or at most algebraic numbers. This would mean that g_K^2 is N: th root of this kind of number. $S_{K,R}$ in turn would be

$$S_{K,R} = 2g_K^2 log(\frac{\prod_i \lambda_{0,i}}{g_K^{2N}}) \ . \quad (22.5.3)$$

so that the reduced Kähler action $S_{K,R}$ would be expressible as a product N: th root of rational, and logarithm of rational. This result would provide a general answer to the question about number theoretical anatomy of Kähler coupling strength and S_K.

For CP_2 type vacuum extremal one would have $S_{K,R} = \frac{\pi^2}{2}$ in apparent conflict with the above result. The conflict is of course only apparent since topological condensation of CP_2 type vacuum extremal generates a hole in CP_2 having light-like wormhole throat as boundary so that the value of the action is modified.

Identifications of Kähler coupling strength and gravitational coupling strength

To construct an expression for gravitational constant one can use the following ingredients.

1. The exponent $exp(S_K(CP_2))$ defining vacuum functional and thus the value of Kähler function in terms of the Kähler action $S_K(CP_2)$ of CP_2 type extremal representing elementary particle expressible as

$$S_K(CP_2) = \frac{S_{K,R}(CP_2)}{8\pi\alpha_K} = \frac{\pi}{8\alpha_K} \ . \quad (22.5.4)$$

   Since CP_2 type extremals suffer topological condensation, one expects that the action is modified:

$$S_K(CP_2) \rightarrow a \times S_K(CP_2) \ . \quad (22.5.5)$$

   $a < 1$ conforms with the idea that a piece of CP_2 type extremal defining a wormhole contact is in question. One must however keep mind open in this respect.

2. The p-adic length scale L_p assignable to the space-time sheet along which gravitational interactions are mediated. Since Mersenne primes seem to characterized elementary bosons and since the Mersenne prime $M_{127} = 2^{127}-1$ defining electron length scale is the largest non-super-astronomical length scale it is natural to guess that M_{127} characterizes these space-time sheets.

1. The formula for the gravitational constant

A long standing basic conjecture has been that gravitational constant satisfies the following formula

$$\begin{aligned} \hbar G &\equiv r\hbar_0 G = L_p^2 \times exp(-2aS_K(CP_2)) \ , \\ L_p &= \sqrt{p}R \ . \end{aligned} \quad (22.5.6)$$

Here R is CP_2 radius defined by the length $2\pi R$ of the geodesic circle. What was noticed before is that this relationship allows even constant value of G if a has appropriate dependence on p.

This formula seems to be correct but the argument leading to it was based on two erratic assumptions compensating each other.

1. I assumed that modulus squared for vacuum functional is in question: hence the factor $2a$ in the exponent. The interpretation of zero energy state as a generalized Feynman diagram requires the use of vacuum functional so that the replacement $2a \rightarrow a$ is necessary.
2. Second wrong assumption was that graviton corresponds to CP_2 type vacuum extremal- that is wormhole contact in the recent picture. This does allow graviton to have spin 2. Rather, two wormhole contacts represented by CP_2 vacuum extremals and connected by fluxes associated with various charges at their throats are needed so that graviton is string like object. This saves the factor $2a$ in the exponent.

The highly non-trivial implication to be discussed later is that ordinary coupling constant strengths should be proportional to $exp(-aS_K(CP_2))$.

The basic constraint to the coupling constant evolution comes for the invariance of g_K^2 in p-adic coupling constant evolution:

$$\begin{aligned} g_K^2 &= \frac{a(p,r)\pi^2}{log(pK)} \ , \\ K &= \frac{R^2}{\hbar G(p)} = \frac{1}{r}\frac{R^2}{\hbar_0 G(p)} \equiv \frac{K_0(p)}{r} \ . \end{aligned} \tag{22.5.7}$$

2. How to guarantee that g_K^2 is RG invariant and N: th root of rational?

Suppose that g_K^2 is N: th root of rational number and invariant under p-adic coupling constant evolution.

1. The most general manner to guarantee the expressibility of g_K^2 as N: th root of rational is guaranteed for both options by the condition

   $$a(p,r) = \frac{g_K^2}{\pi^2} log(\frac{pK_0}{r}) \ . \tag{22.5.8}$$

   That a would depend logarithmically on p and $r = \hbar/\hbar_0$ looks rather natural. Even the invariance of G under p-adic coupling constant evolution can be considered.
2. The condition

   $$\frac{r}{p} < K_0(p) \ . \tag{22.5.9}$$

   must hold true to guarantee the condition $a > 0$. Since the value of gravitational Planck constant is very large, also the value of corresponding p-adic prime must very large to guarantee this condition. The condition $a < 1$ is guaranteed by the condition

   $$\frac{r}{p} > exp(-\frac{\pi^2}{g_K^2}) \times K_0(p) \ . \tag{22.5.10}$$

   The condition implies that for very large values of p the value of Planck constant must be larger than $\hbar_0$.

3. The two conditions are summarized by the formula

$$K_0(p) \times exp(-\frac{\pi^2}{g_K^2}) < \frac{r}{p} < K_0(p) \tag{22.5.11}$$

characterizing the allowed interval for r/p. If G does not depend on p, the minimum value for r/p is constant. The factor $exp(-\frac{\pi^2}{g_K^2})$ equals to 1.8×10^{-47} for $\alpha_K = \alpha_{em}$ so that $r > 1$ is required for $p \geq 4.2 \times 10^{-40}$. $M_{127} \sim 10^{38}$ is near the upper bound for p allowing $r = 1$. The constraint on r would be roughly $r \geq 2^{k-131}$ and $p \simeq 2^{131}$ is the first p-adic prime for which $\hbar > 1$ is necessarily. The corresponding p-adic length scale is.1 Angstroms.

This conclusion need not apply to elementary particles such as neutrinos but only to the space-time sheets mediating gravitational interaction so that in the minimal scenario it would be gravitons which must become dark above this scale. This would bring a new aspect to vision about the role of gravitation in quantum biology and consciousness.

The upper bound for r behaves roughly as $r < 2.3 \times 10^7 p$. This condition becomes relevant for gravitational Planck constant GM_1M_2/v_0 having gigantic values. For Earth-Sun system and for $v_0 = 2^{-11}$ the condition gives the rough estimate $p > 6 \times 10^{63}$. The corresponding p-adic length scale would be of around $L(215) \sim 40$ meters.

4. p-Adic mass calculations predict the mass of electron as $m_e^2 = (5 + Y_e)2^{-127}/R^2$ where $Y_e \in [0, 1)$ parameterizes the not completely known second order contribution. Top quark mass favors a small value of Y_e (the original experimental estimates for m_t were above the range allowed by TGD but the recent estimates are consistent with small value Y_e [K47]). The range $[0, 1)$ for Y_e restricts $K_0 = R^2/\hbar_0 G$ to the range $[2.3683, 2.5262] \times 10^7$.

5. The best value for the inverse of the fine structure constant is $1/\alpha_{em} = 137.035999070(98)$ and would correspond to $1/g_K^2 = 10.9050$ and to the range $(0.9757, 0.9763)$ for a for $\hbar = \hbar_0$ and $p = M_{127}$. Hence one can seriously consider the possibility that $\alpha_K = \alpha_{em}(M_{127}$ holds true. As a matter fact, this was the original hypothesis but was replaced later with the hypothesis that α_K corresponds to electro-weak $U(1)$ coupling strength in this length scale. The fact that M_{127} defines the largest Mersenne prime, which does not correspond to super-astrophysical length scale might relate to this co-incidence.

To sum up, this view about coupling constant evolution differs strongly from previous much more speculative scenarios. It implies that g_K^2 is root of rational number, possibly even rational, and can be assumed to be equal to e^2. Also $R^2/\hbar G$ could be rational. The new element is that G need not be proportional to p and can be even invariant under coupling constant evolution since the parameter a can depend on both p and r. An unexpected constraint relating p and r for space-time sheets mediating gravitation emerges.

Algebraic universality and the value of Kähler coupling strength

With the development of the vision about number theoretically universal view about functional integration in WCW [K111], a concrete vision about the exponent of Kähler action in Euclidian and Minkowskian space-time regions. The basic requirement is that exponent of Kähler action belongs to an algebraic extension of rationals and therefore to that of p-adic numbers and does not depend on the ordinary p-adic numbers at all - this at least for sufficiently large primes p. Functional integral would reduce in Euclidian regions to a sum over maxima since the troublesome Gaussian determinants that could spoil number theoretic universality are cancelled by the metric determinant for WCW.

The adelically exceptional properties of Neper number e, Kähler metric of WCW, and strong form of holography posing extremely strong constraints on preferred extremals, could make this possible. In Minkowskian regions the exponent of imaginary Kähler action would be root of unity. In Euclidian space-time regions expressible as power of some root of e which is is unique in sense that e^p is ordinary p-adic number so that e is p-adically an algebraic number - p:th root of e^p.

These conditions give conditions on Kähler coupling strength $\alpha_K = g_K^2/4\pi$ ($\hbar = 1$)) identifiable as an analog of critical temperature. Quantum criticality of TGD would thus make possible number theoretical universality (or vice versa).

1. In Euclidian regions the natural starting point is CP_2 vacuum extremal for which the maximum value of Kähler action is

$$S_K = \frac{\pi^2}{2g_K^2} = \frac{\pi}{8\alpha_K} \quad .$$

   The condition reads $S_K = q$ if one allows roots of e in the extension. If one requires minimal extension of involving only e and its powers one would have $S_K = n$. One obtains

$$\frac{1}{\alpha_K} = \frac{8q}{\pi} \quad ,$$

   where the rational $q = m/n$ can also reduce to integer. One cannot exclude the possibiity that q depends on the algebraic extension of rationals defining the adele in question [K111].

   For CP_2 type extremals the value of p-adic prime should be larger than $p_{min} = 53$. One can consider a situation in which large number of CP_2 type vacuum extremals contribute and in this case the condition would be more stringent. The condition that the action for CP_2 extremal is smaller than 2 gives

$$\frac{1}{\alpha_K} \leq \frac{16}{\pi} \simeq 5.09 \quad .$$

   It seems there is lower bound for the p-adic prime assignable to a given space-time surface inside CD suggesting that p-adic prime is larger than $53 \times N$, where N is particle number.

   This bound has not practical significance. In condensed matter particle number is proportional to $(L/a)^3$ - the volume divided by atomic volume. On basis p-adic mass calculations [K39] p-Adic prime can be estimated to be of order $(L/R)^2$. Here a is atomic size of about 10 Angstroms and R CP_2 "radius". Using $R \simeq 10^4 L_{Planck}$ this gives as upper bound for the size L of condensed matter blob a completely super-astronomical distance $L \leq a^3/R^2 \sim 10^{25}$ ly to be compared with the distance of about 10^{10} ly travelled by light during the lifetime of the Universe. For a blackhole of radius $r_S = 2GM$ with $p \sim (2GM/R)^2$ and consisting of particles with mass above $M \simeq \hbar/R$ one would obtain the rough estimate $M > (27/2) \times 10^{-12} m_{Planck} \sim 13.5 \times 10^3$ TeV trivially satisfied.

2. The physically motivated expectation from earlier arguments - not necessarily consistent with the recent ones - is that the value α_K is quite near to fine structure constant at electron length scale: $\alpha_K \simeq \alpha_{em} \simeq 137.035999074(44)$.

   The latter condition gives $n = 54 = 2 \times 3^3$ and $1/\alpha_K \simeq 137.51$. The deviation from the fine structure constant is $\Delta\alpha/\alpha = 3 \times 10^{-3} - .3$ per cent. For $n = 53$ one obtains $1/\alpha_K = 134.96$ with error of 1.5 per cent. For $n = 55$ one obtains $1/\alpha_K = 150.06$ with error of 2.2 per cent. Is the relatively good prediction could be a mere accident or there is something deeper involved?

What about Minkowskian regions? It is difficult to say anything definite. For cosmic string like objects the action is non-vanishing but proportional to the area A of the string like object and the conditions would give quantization of the area. The area of geodesic sphere of CP_2 is proportional to π. If the value of g_K is same for Minkowskian and Euclidian regions, $g_K^2 \propto \pi^2$ implies $S_K \propto A/R^2\pi$ so that $A/R^2 \propto \pi^2$ is required.

This approach leads to different algebraic structure of α_K than the earlier arguments.

1. α_K is rational multiple of π so that g_K^2 is proportional to π^2. At the level of quantum TGD the theory is completely integrable by the definition of WCW integration(!) [K111] and there are no radiative corrections in WCW integration. Hence α_K does not appear in vertices and therefore does not produce any problems in p-adic sectors.

2. This approach is consistent with the proposed formula relating gravitational constant and p-adic length scale. G/L_p^2 for $p = M_{127}$ would be rational power of e now and number theoretically universally. A good guess is that G does not depend on p. As found this could be achieved also if the volume of CP_2 type extremal depends on p so that the formula holds for all primes. α_K could also depend on algebraic extension of rationals to guarantee the independence of G on p. Note that preferred p-adic primes correspond to ramified primes of the extension so that extensions are labelled by collections of ramified primes, and the ramimified prime corresponding to gravitonic space-time sheets should appear in the formula for G/L_p^2.

3. Also the speculative scenario for coupling constant evolution could remain as such. Could the p-adic coupling constant evolution for the gauge coupling strengths be due to the breaking of number theoretical universality bringing in dependence on p? This would require mapping of p-adic coupling strength to their real counterparts and the variant of canonical identification used is not unique.

4. A more attractive possibility is that coupling constants are algebraically universal (no dependence on number field). Even the value of α_K, although number theoretically universal, could depend on the algebraic extension of rationals defining the adele. In this case coupling constant evolution would reflect the evolution assignable to the increasing complexity of algebraic extension of rationals. The dependence of coupling constants on p-adic prime would be induced by the fact that so called ramified primes are physically favored and characterize the algebraic extension of rationals used.

5. One must also remember that the running coupling constants are associated with QFT limit of TGD obtained by lumping the sheets of many-sheeted space-time to single region of Minkowski space. Coupling constant evolution would emerge at this limit. Whether this evolution reflects number theoretical evolution as function of algebraic extension of rationals, is an interesting question.

Are the color and electromagnetic coupling constant evolutions related?

Classical theory should be also able to say something non-trivial about color coupling strength α_s too at the general level. The basic observations are following.

1. Both classical color YM action and electro-weak U(1) action reduce to Kähler action.

2. Classical color holonomy is Abelian which is consistent also with the fact that the only signature of color that induced spinor fields carry is anomalous color hyper charge identifiable as an electro-weak hyper charge.

Suppose that α_K is a strict RG invariant. One can consider two options.

1. The original idea was that the sum of classical color action and electro-weak U(1) action is RG invariant and thus equals to its asymptotic value obtained for $\alpha_{U(1)} = \alpha_s = 2\alpha_K$. Asymptotically the couplings would approach to a fixed point defined by $2\alpha_K$ rather than to zero as in asymptotically free gauge theories.

   Thus one would have

$$\frac{1}{\alpha_{U(1)}} + \frac{1}{\alpha_s} = \frac{1}{\alpha_K} \ . \tag{22.5.12}$$

   The relationship between $U(1)$ and em coupling strengths is

$$\begin{aligned}
\alpha_{U(1)} &= \frac{\alpha_{em}}{cos^2(\theta_W)} \simeq \frac{1}{104.1867} \ , \\
sin^2(\theta_W)_{|10\ MeV} &\simeq 0.2397(13) \ , \\
\alpha_{em}(M_{127}) &= 0.00729735253327 \ .
\end{aligned} \tag{22.5.13}$$

Here Weinberg angle corresponds to 10 MeV energy is reasonably near to the value at electron mass scale. The value $sin^2(\theta_W) = 0.2397(13)$ corresponding to 10 MeV mass scale [E2] is used. Note however that the previous argument implying $\alpha_K = \alpha_{em}(M_{127})$ excludes $\alpha = \alpha_{U(1)}(M_{127})$ option.

2. Second option is obtained by replacing $U(1)$ with electromagnetic gauge $U(1)_{em}$.

$$\frac{1}{\alpha_{em}} + \frac{1}{\alpha_s} = \frac{1}{\alpha_K}. \quad (22.5.14)$$

Possible justifications for this assumption are following. The notion of induced gauge field makes it possible to characterize the dynamics of classical electro-weak gauge fields using only the Kähler part of electro-weak action, and the induced Kähler form appears only in the electromagnetic part of the induced classical gauge field. A further justification is that em and color interactions correspond to unbroken gauge symmetries.

The following arguments are consistent with this conclusion.

1. In TGD framework coupling constant is discrete and comes as powers of $\sqrt{2}$ corresponding to p-adic primes $p \simeq 2^k$. Number theoretic considerations suggest that coupling constants g_i^2 are algebraic or perhaps even rational numbers, and that the logarithm of mass scale appearing as argument of the renormalized coupling constant is replaced with 2-based logarithm of the p-adic length scale so that one would have $g_i^2 = g_i^2(k)$. g_K^2 is predicted to be N: th root of rational but could also reduce to a rational. This would allow rational values for other coupling strengths too. This is possible if $sin(\theta_W)$ and $cos(\theta_W)$ are rational numbers which would mean that Weinberg angle corresponds to a Pythagorean triangle as proposed already earlier. This would mean the formulas $sin(\theta_W) = (r^2 - s^2)/(r^2 + s^2)$ and $cos(\theta_W) = 2rs(r^2 + s^2)$.

2. A very strong prediction is that the beta functions for color and $U(1)$ degrees of freedom are apart from sign identical and the increase of $U(1)$ coupling compensates the decrease of the color coupling. This allows to predict the hard-to-calculate evolution of QCD coupling constant strength completely.

3. $\alpha(M_{127}) = \alpha_K$ implies that M_{127} defines the confinement length scale in which the sign of α_s becomes negative. TGD predicts that also M_{127} copy of QCD should exist and that M_{127} quarks should play a key role in nuclear physics [K70, L3], [L3]. Hence one can argue that color coupling strength indeed diverges at M_{127} (the largest not completely super-astrophysical Mersenne prime) so that one would have $\alpha_K = \alpha(M_{127})$. Therefore the precise knowledge of $\alpha(M_{127})$ in principle fixes the value of parameter $K = R^2/G$ and thus also the second order contribution to the mass of electron.

4. $\alpha_s(M_{89})$ is predicted to be $1/\alpha_s(M_{89}) = 1/\alpha_K - 1/\alpha(M_{89})$. $sin^2(\theta_W) = .23120$, $\alpha_{em}(M_{89}) \simeq 1/127$, and $\alpha_{U(1)} = \alpha_{em}/cos^2(\theta_W)$ give $1/\alpha_{U(1)}(M_{89}) = 97.6374$. $\alpha = \alpha_{em}$ option gives $1/\alpha_s(M_{89}) \simeq 10$, which is consistent with experimental facts. $\alpha = \alpha_{U(1)}$ option gives $\alpha_s(M_{89}) = 0.1572$, which is larger than QCD value. Hence $\alpha = \alpha_{em}$ option is favored.

To sum up, the proposed formula would dictate the evolution of α_s from the evolution of the electro-weak parameters without any need for perturbative computations. Although the formula of proposed kind is encouraged by the strong constraints between classical gauge fields in TGD framework, it should be deduced in a rigorous manner from the basic assumptions of TGD before it can be taken seriously.

Can one deduce formulae for gauge couplings?

The improved physical picture behind gravitational constant allows also to consider a general formula for gauge couplings.

1. The natural guess for the general formula would be as

$$g^2(p,r) = kg_K^2 \times exp[-a_g(p,r) \times S_K(CP_2)] \ . \tag{22.5.15}$$

   here k is a numerical constant.

2. The condition
   $g_K^2 = e^2(M_{127})$ fixes the value of k if it's value does not depend on the character of gauge interaction:

$$k = exp[a_{gr}(M_{127}, r = 1) \times S_K(CP_2)] \ . \tag{22.5.16}$$

   Hence the general formula reads as

$$g^2(p,r) = g_K^2 \times exp[(-a_g(p,r) + a_{gr}(M_{127}), r = 1)) \times S_K(CP_2)] \ . \tag{22.5.17}$$

   The value of $a(M_{127}, r = 1)$ is near to its maximum value so that the exponential factor tends to increase the value of g^2 from e^2. The formula can reproduce α_s and various electro-weak couplings although it is quite possibile that Weinberg angle corresponds to a group theoretic factor not representable in terms of $a_g(p,r)$. The volume of the CP_2 type vacuum extremal would characterize gauge bosons. Analogous formula should apply also in the case of Higgs.

3. α_{em} in very long length scales would correspond to

$$e^2(p \to \infty, r = 1) = e^2 \times exp[(-1 + a(M_{127}), r = 1)) \times S_K(CP_2)] = e^2 x \ , \tag{22.5.18}$$

   where x is in the range $[0.6549, 0.6609]$.

Formula relating v_0 to α_K and $R^2/\hbar G$

The parameter $v_0 = 2^{-11}$ plays a key role in the formula for gravitational Planck constant and can be also seen as a fundamental constant in TGD framework. As a matter, factor v_0 has interpretation as velocity parameter and is dimensionless when $c = 1$ is used.

If v_0 is identified as the rotation velocity of distant stars in galactic plane, one can use the Newtonian model for the motion of mass in the gravitational field of long straight string giving $v_0 = \sqrt{TG}$. String tension T can be expressed in terms of Kähler coupling strength as

$$T = \frac{b}{2\alpha_K R^2} \ ,$$

where R is the radius of geodesic circle. The factor $b \leq 1$ would explain reduction of string tension in topological condensation caused by the fact that not entire geodesic sphere contributes to the action.

This gives

$$\begin{aligned} v_0 &= \frac{b}{2\sqrt{\alpha_K K}} \ , \\ \alpha_K(p) &= \frac{a\pi}{4log(pK)} \ , \\ K &= \frac{R^2}{\hbar G} \ . \end{aligned} \tag{22.5.19}$$

The condition that α_K has the desired value for $p = M_{127} = 2^{127} - 1$ defining the p-adic length scale of electron fixes the value of b for given value of a. The value of b should be smaller than 1 corresponding to the reduction of string tension in topological condensation.

The condition 22.5.19 for $v_0 = 2^{-m}$, say $m = 11$, allows to deduce the value of a/b as

$$\frac{a}{b} = \frac{4 * log(pK)}{\pi} \frac{2^{2m-1}}{K} . \tag{22.5.20}$$

For both $K = e^q$ with $q = 17$ and $K = 2^q$ option with $q = 24 + 1/2$ $m = 10$ is the smallest integer giving $b < 1$. $K = e^q$ option gives $b = .3302$ $(.0826)$ and $K = 2^q$ option gives $b = .3362$ $(.0841)$ for $m = 10$ $(m = 11)$.

$m = 10$ corresponds to one third of the action of free cosmic string. $m = 11$ corresponds to much smaller action smaller by a factor rather near $1/12$. The interpretation would be that as m increases the action of the topologically condensed cosmic string decreases. This would correspond to a gradual transformation of the cosmic string to a magnetic flux tube.

To sum up, the resulting overall vision seems to be internally consistent and is consistent with generalized Feynman graphics, predicts exactly the spectrum of α_K, suggests the identification of the inverse of p-adic temperature with k, allows to understand the differences between fermionic and bosonic massivation. One might hope that the additional objections (to be found sooner or later!) could allow to develop a more detailed picture.

22.5.2 Why Gravitation Is So Weak As Compared To Gauge Interactions?

The weakness of gravitational interaction in contrast to other gauge interactions is definitely a fundamental test for the proposed picture. The heuristic argument allowing to understand the value of gravitational constant is based on the assumption that graviton exchange corresponds to the exchange of CP_2 type extremal for which vacuum functional implies huge reduction of the gravitational constant from the value $\sim L_p^2$ implied by dimensional considerations based on p-adic length scale hypothesis to a value $G = exp(-2S_K)L_p^2$ which for $p = M_{127}$ gives gravitational constant for $\alpha_K = \pi a/log(M_{127} \times K)$, where a is near unity and $K = 2 \times 3 \times 5.... \times 23$ is a choice motivated by number theoretical arguments. The value of K is fixed rather precisely from electron mass scale and the proposed scenario for coupling constant evolution fixes both α_K and K completely in terms of electron mass (using p-adic mass calculations) and electro-magnetic coupling at electron length scale $L_{M_{127}}$ by the formula $\alpha_K = \alpha_{em}$ [K22]. The interpretation would be that gravitational masses are measured using p-adic mass scale $M_p = \pi/L_p$ as a natural unit.

Why gravitational interaction is weak?

The first problem is that CP_2 type extremal cannot represent the lowest order contribution to the interaction since otherwise the normalization of WCW vacuum functional would give $exp[-2S_K(CP_2)]$ factor cancelling the exponential in the propagator so that one would have $G = L_p^2$. The following observations allow to understand the solution of the problem.

1. As already found, the key feature of CP_2 type vacuum extremals distinguishing them from other 3-surfaces is their non-deterministic behavior allowing them to carry off mass shell four-momenta. Other 3-surfaces can give rise only to scattering involving exchange of on mass shell particles and for space-like momentum exchanges there is no contribution.

2. All possible light-like 3-surfaces must be allowed as propagator portions of surfaces X^3_V but in absence of non-determinism they can give rise to massless exchanges which are typically non-allowed.

3. The contributions of CP_2 type vacuum extremals are suppressed by $exp[-2NS_K(CP_2)]$ factor in presence of N CP_2 type extremals with maximal action. CP_2 type extremals are vacuum extremals and interact with surrounding world only via the topological condensation generating 3-D CP_2 projection near the throat of the wormhole contact. This motivates the assumption that the sector of the WCW containing N CP_2 type extremals has the

approximate structure $CH(N) = CH(0) \times CP^N$, where $CH(0)$ corresponds to the situation without CP_2 type extremals and CP to the degrees of freedom associated with single CP_2 type extremal. With this assumption the functional integral gives a result of form $X \times exp(-2NS_K(CP_2)$ for N CP_2 type extremals. This factorization allows to forget all the complexities of the world of classical worlds which on the first sight seem to destroy all hopes about calculating something and the normalization factor is in lowest order equal to $X(0)$ whereas single CP_2 type extremal gives $exp[-2S_K(CP_2)]$ factor. This argument generalizes also to the case when CP_2 type extremals are allowed to have varying value of action (the distance travelled by the virtual particle can vary).

Massless extremals (MEs) define a natural candidate for the lowest order contribution since for them Kähler action vanishes. MEs describes a dispersion free on-mass shell propagation of massless modes of both induced gauge fields and metric. Hence they can describe only on mass shell massless exchanges of bosons and gravitons which typically vanishes for kinematical reasons except for collinear scattering in the case of massless particles so that CP_2 type extremals would give the leading contribution to the S-matrix element.

There are however exceptional situations in which exchange of ordinary CP_2 type extremals makes kinematically possible the emission of MEs as brehmstrahlung in turn giving rise to exchange of light-like momentum. Since MEs carry also classical gravitational fields, one can wonder whether this kind of exchanges could make possible strong on mass shell gravitation made kinematically possible by ordinary gauge boson exchanges inside interacting systems.

If one takes absolutely seriously the number theoretic argument based on $R^2/G = exp(q)$ ansatz then M_{127} is selected uniquely as the space-time sheet of gravitons and the predicted gravitational coupling strength is indeed weak.

What differentiates between gravitons and gauge bosons?

The simplest explanation for the difference between gauge bosons and gravitons is that for virtual gauge bosons the volume of CP_2 type extremals is reduced dramatically from its maximal value so that $exp(-2S_K)$ brings in only a small reduction factor. The reason would be that for virtual gauge bosons the length of a typical CP_2 type extremal is far from the value giving rise to the saturation of the Kähler action. For gravitational interactions in astrophysical length scales CP_2 type extremals must indeed be very long.

Gravitational interaction should become strong sufficiently below the saturation length scale with gravitational constant approaching its stringy value L_p^2. According to the argument discussed in [K22], this length scale corresponds to the Mersenne prime M_{127} characterizing gravitonic space-time sheets so that gravitation should become strong below electron's Compton length. This suggests a connection with stringy description of graviton. M_{127} quarks connected by the corresponding strings are indeed a basic element of TGD based model of nuclei [K70]. TGD suggests also the existence of lepto-hadrons as bound state of color excited leptons in length scale M_{127} [K78]. Also gravitons corresponding to smaller Mersenne primes are possible but corresponding forces are much weaker than ordinary gravitation. On the other hand, M_{127} is the largest Mersenne prime which does not give rise to super-astronomical p-adic length scale so that stronger gravitational forces are not be predicted in experimentally accessible length scales.

More generally, the saturation length scale should relate very closely to the p-adic length scale L_p characterizing the particle. The amount of zitterbewegung determines the amount dS_K/dl of Kähler action per unit length along the orbit of virtual particle. L_p would naturally define the length scale below which the particle moves in a good approximation along M^4 geodesic. The shorter this length scale is, the larger the value of dS_K/dl is.

If the Kähler action of CP_2 type extremal increases linearly with the distance (in a statistical sense at least), an exponential Yukawa screening results at distances much shorter than saturation length. Therefore CP_2 extremals would provide a fundamental description of particle massivation at space-time level. p-Adic thermodynamics would characterize what happens for a topologically condensed CP_2 type extremal carrying given quantum numbers at the resulting light-like CD. Besides p-adic length scale also the quantized value $T_p = 1/n$ of the p-adic temperature would be decisive. For weak bosons Mersenne prime M_{89} would define the saturation length scale. For photons the p-adic length scale defining the Yukawa screening should be rather long. An n-ary p-adic length scale $L_{M_{89}}(n) = p^{(n-1)/2}L_{M_{89}}$ would most naturally be in question so that the p-adic temperature associated with photon would be $T_p = 1/n$, $n > 1$ [K39]. In the case of gluons confinement length scale should be much shorter than the scale at which the Yukawa screening becomes visible. If also gluons correspond to $n > 1$ this is certainly the case.

All gauge interactions would give rise to ultra-weak long ranged interactions, which are extremely weak compared to the gravitational interaction: the ratio for the strengths of these interactions would be of order $\alpha Q_1 Q_2 m_e^2/M_1 M_2$ and very small for particles whose masses are above electron mass. Note however that MEs give rise to arbitrarily unscreened long ranged weak and color interactions restricted to light-like momentum transfers and these interactions play a key role in the TGD based model of living matter [K17, K18]. This prediction is in principle testable.

22.5.3 Super-Symplectic Gluons And Non-Perturbative Aspects Of Hadron Physics

What happens mathematically in the transition to non-perturbative QCD has remained more or less a mystery. The number theoretical considerations of [K85] inspired the idea that Planck constant is dynamical and has a spectrum given as $\hbar(n) = n\hbar_0$, where n characterizes the quantum phase $q = exp(i2\pi/n)$ associated with Jones inclusion. The strange finding that the orbits of planets seem to obey Bohr quantization rules with a gigantic value of Planck constant inspired the hypothesis that the increase of Planck constant provides a unique mechanism allowing strongly interacting system to stay in perturbative phase [K66, K22]. The resulting model allows to understand dark matter as a macroscopic quantum phase in astrophysical length and time scales, and strongly suggest a connection with dark matter and biology.

The phase transition increasing Planck constant could provide a model for the transition to confining phase in QCD. When combined with the recent ideas about value spectrum of Kähler coupling strength one ends up with a rather explicit model about non-perturbative aspects of hadron physics already successfully applied in hadron mass calculations [K47].

According to the model of hadron masses [K47], in the case of light pseudo-scalar mesons the contribution of quark masses to the mass squared of meson dominates whereas spin 1 mesons contain a large contribution identified as color interaction conformal weight (color magnetic spin-spin interaction conformal weight and color Coulombic conformal weight). This conformal weight cannot however correspond to the ordinary color interactions alone and is negative for pseudo-scalars and compensated by some unknown contribution in the case of pion in order to avoid tachyonic mass. Quite generally this realizes the idea about light pseudo-scalar mesons as Goldstone bosons. Analogous mass formulas hold for baryons but in this case the additional contribution which dominates.

The unknown contribution can be assigned to the $k = 107$ hadronic space-time sheet and must correspond to the non-perturbative aspects of QCD and the failure of the quantum field theory approach at low energies. In TGD the failure of QFT picture corresponds to the presence of WCW degrees of freedom ("world of classical worlds") in which super-symplectic algebra acts. The failure of the approximation assuming single fixed background space-time is in question.

The purely bosonic generators carry color and spin quantum numbers: spin has however the character of orbital angular momentum. The only electro-weak quantum numbers of super-generators are those of right-handed neutrino. If the super-generators degrees carry the quark spin at high energies, a solution of proton spin puzzle emerges.

The presence of these degrees of freedom means that there are two contributions to color interaction energies corresponding to the ordinary gluon exchanges and exchanges of super-symplectic gluons. It turns out the model assuming same topological mixing of super-symplectic bosons identical to that experienced by U type quarks leads to excellent understanding of hadron masses assuming that hadron spin correlates with the super-symplectic particle content of the hadronic space-time sheet.

According to the argument already discussed, at the hadronic $k = 107$ space electro-weak interactions would be absent and classical $U(1)$ action should vanish. This is guaranteed if $\alpha_{U(1)}$ diverges. This would give

$$\alpha_s = \alpha_K = \frac{1}{4} \ .$$

This would give also a quantitative articulation for the statement that strong interactions are charge independent.

This α_s would correspond to the interaction via super-symplectic colored gluons and would lead to the failure of perturbation theory. By the general criterion stating that the failure of perturbation theory leads to a phase transition increasing the value of Planck constant one expects that the value of $\hbar$ increases [K22]. The value leaving the value of α_K invariant would be $\hbar \rightarrow 26\hbar$ and would mean that p-adic length scale L_{107} is replaced with length scale $26L_{107} = 46$ fm, the size of large nucleus so that also the basic length scale nuclear physics would be implicitly coded into the structure of hadrons.

22.5.4 Why Mersenne Primes Should Label A Fractal Hierarchy Of Physics?

There are motivations for the working hypothesis stating that there is fractal hierarchy of copies of standard model physics, and that Mersenne primes label both hadronic space-time sheets and gauge bosons. The reason for this is not yet well understood and I have considered several speculative explanations.

First picture

The first thing to come in mind is that Mersenne primes correspond to fixed points of the discrete p-adic coupling constant evolution, most naturally to the maxima of the color coupling constant strength. This would mean that gluons are emitted with higher probability than in other p-adic length scales.

There is however an objection against this idea. If one accepts the new vision about non-perturbative aspects of QCD, it would seem that super-symplectic bosons or the interaction between super-symplectic bosons and quarks for some reason favors Mersenne primes. However, if color coupling strength corresponds to $\alpha_K = \alpha_s = 1/4$ scaled down by the increase of the Planck constant, the evolution of super-symplectic color coupling strength does not seem to play any role. What becomes large should be a geometric "form factor", when the boson in the vertex corresponds to Mersenne prime rather than "bare" coupling.

The resolution of the problem could be that boson emission vertices $g(p_1, p_2, p_3)$ are functions of p-adic primes labeling the particles of the vertices so that actually three p-adic length scales are involved instead of single length scale as in the ordinary coupling constant evolution. Hence one can imagine that the interaction between particles corresponding to primes near powers of 2 and Mersenne primes is especially strong and analogous to a resonant interaction. The geometric resonance due to the fact that the length scales involved are related by a fractal scaling by a power of 2 would make the form factors $F(p_1 \simeq 2^{k_1}, p_2 \simeq 2^{k_2}, M_n)$ large. The selection of primes near powers of two and Mersenne bosons would be analogous to evolutionary selection of a population consisting of species able to interact strongly.

Since $k = 113$ quarks are possible for $k = 107$ hadron physics, it seems that quarks can have flux tubes directed to M_n space-times with $n < k$. This suggests that neighboring Mersenne primes compete for flux tubes of quarks. For instance, when the p-adic length scale characterizing quark of M_{107} hadron physics begins to approach M_{89} quarks tend to feed their gauge flux to M_{89} space-time sheet and M_{89} hadron physics takes over and color coupling strength begins to increase. This would be the space-time correlate for the loss of asymptotic freedom.

Second picture

Preferred values of Planck constants could play a key role in the selection of Mersenne primes. Ruler-and-compass hypothesis predicts that Planck constants, which correspond to ratios of ruler and compass integers proportional to a product of distinct Fermat primes (four of them are known) and any power of two are favored. As a special case one obtains ruler and compass integers. As a consequence, p-adic length scales have satellites obtained by multiplying them with ruler-and-compass integers, and entire fractal hierarchy of power-of-two multiples of a given p-adic length scale results.

Mersenne length scales would be special since their satellites would form a subset of satellites of shorter Mersenne length scales. The copies of standard model physics associated with Mersenne primes would define a kind of resonating subset of physics since corresponding wavelengths and frequencies would coincide. This would also explain why fermions labeled by primes near power of two couple strongly with Mersenne primes.

22.5.5 The Formula For The Hadronic String Tension

It is far from clear whether the strong gravitational coupling constant has same relation to the parameter $M_0^2 = 16m_0^2 = 1/\alpha' = 2\pi T$ as it would have in string model.

1. One could estimate the strong gravitational constant from the fundamental formula for the gravitational constant expressed in terms of exponent of Kähler action in the case that one has $\alpha_K = 1/4$. The formula reads as

$$\frac{L_p^2}{G_p} = exp(2aS_K(CP_2)) = exp(\pi/4\alpha_K) = e^{\pi} \ . \tag{22.5.21}$$

   a is a parameter telling which fraction the action of wormhole contact is about the full action for CP_2 type vacuum extremal and $a \sim 1/2$ holds true. The presence of a can take care that the exponent is rational number. For $a = 1$ The number at the right hand side is Gelfond constant and one obtains

$$G_p = exp(-\pi) \times L_p^2 \ . \tag{22.5.22}$$

2. One could relate the value of the strong gravitational constant to the parameter $M_0^2(k) = 16m(k)^2$, $p \simeq 2^k$ also assuming that string model formula generalizes as such. The basic formulas can be written in terms of gravitational constant G, string tension T, and $M_0^2(k)$ as

$$\frac{1}{8\pi G(k)} = \frac{1}{\alpha'} = 2\pi T(k) = \frac{1}{M_0^2(k)} = \frac{1}{16m(k)^2} \ . \tag{22.5.23}$$

   This allows to express G in terms of the hadronic length scale $L(k) = 2\pi/m(k)$ as

$$G(k) = \frac{1}{16^2\pi^2}L(k)^2 \simeq 3.9 \times 10^{-4}L(k)^2 \ . \tag{22.5.24}$$

   The value of gravitational coupling would be by two orders of magnitude smaller than for the first option.

22.5.6 Large Values Of Planck Constant And Electro-Weak And Strong Coupling Constant Evolution

Kähler coupling constant is the only coupling parameter in TGD. The original great vision is that Kähler coupling constant is analogous to critical temperature and thus uniquely determined. Later I concluded that Kähler coupling strength could depend on the p-adic length scale. The reason was that the prediction for the gravitational coupling strength was otherwise non-sensible. This motivated the assumption that gravitational coupling is RG invariant in the p-adic sense.

The expression of the basic parameter $v_0 = 2^{-11}$ appearing in the formula of $\hbar_{gr} = GMm/v_0$ in terms of basic parameters of TGD leads to the unexpected conclusion that α_K in electron length scale can be identified as electro-weak $U(1)$ coupling strength $\alpha_{U(1)}$. This identification is what group theory suggests but I had given it up since the resulting evolution for gravitational coupling was $G \propto L_p^2$ and thus completely un-physical. However, if gravitational interactions are mediated by space-time sheets characterized by Mersenne prime, the situation changes completely since M_{127} is the largest non-super-astrophysical p-adic length scale.

The second key observation is that all classical gauge fields and gravitational field are expressible using only CP_2 coordinates and classical color action and U(1) action both reduce to Kähler action. Furthermore, electroweak group U(2) can be regarded as a subgroup of color SU(3) in a well-defined sense and color holonomy is abelian. Hence one expects a simple formula relating various coupling constants. Let us take α_K as a p-adic renormalization group invariant in strong sense that it does not depend on the p-adic length scale at all.

The relationship for the couplings must involve $\alpha_{U(1)}$, α_s and α_K. The formula $1/\alpha_{U(1)} + 1/\alpha_s = 1/\alpha_K$ states that the sum of $U(1)$ and color actions equals to Kähler action and is consistent with the decrease of the color coupling and the increase of the U(1) coupling with energy and implies a common asymptotic value $2\alpha_K$ for both. The hypothesis is consistent with the known facts about color and electroweak evolution and predicts correctly the confinement length scale as p-adic length scale assignable to gluons. The hypothesis reduces the evolution of α_s to the calculable evolution of electro-weak couplings: the importance of this result is difficult to over-estimate.

22.6 Appendix: Identification Of The Electro-Weak Couplings

The delicacies of the spinor structure of CP_2 make it a unique candidate for space S. First, the coupling of the spinors to the $U(1)$ gauge potential defined by the Kähler structure provides the missing $U(1)$ factor in the gauge group. Secondly, it is possible to couple different H-chiralities independently to a half odd multiple of the Kähler potential. Thus the hopes of obtaining a correct spectrum for the electromagnetic charge are considerable. In the following it will be demonstrated that the couplings of the induced spinor connection are indeed those of the GWS model [B58] and in particular that the right handed neutrinos decouple completely from the electro-weak interactions.

To begin with, recall that the space H allows to define three different chiralities for spinors. Spinors with fixed H-chirality $e = \pm 1$, CP_2-chirality l, r and M^4-chirality L, R are defined by the condition

$$\begin{aligned} \Gamma\Psi &= e\Psi \ , \\ e &= \pm 1 \ , \end{aligned} \tag{22.6.1}$$

where Γ denotes the matrix $\Gamma_9 = \gamma_5 \times \gamma_5$, $1 \times \gamma_5$ and $\gamma_5 \times 1$ respectively. Clearly, for a fixed H-chirality CP_2- and M^4-chiralities are correlated.

The spinors with H-chirality $e = \pm 1$ can be identified as quark and lepton like spinors respectively. The separate conservation of baryon and lepton numbers can be understood as a consequence of generalized chiral invariance if this identification is accepted. For the spinors with a definite H-chirality one can identify the vielbein group of CP_2 as the electro-weak group: $SO(4) = SU(2)_L \times SU(2)_R$.

The covariant derivatives are defined by the spinorial connection

$$A = V + \frac{B}{2}(n_+ 1_+ + n_- 1_-) \ . \tag{22.6.2}$$

Here V and B denote the projections of the vielbein and Kähler gauge potentials respectively and $1_{+(-)}$ projects to the spinor H-chirality $+(-)$. The integers $n_\pm$ are odd from the requirement of a respectable spinor structure.

The explicit representation of the vielbein connection V and of B are given by the equations

$$\begin{array}{rclcrcl} V_{01} & = & -\frac{e^1}{r} \ , & & V_{23} & = & \frac{e^1}{r} \ , \\ V_{02} & = & -\frac{e^2}{r} \ , & & V_{31} & = & \frac{e^2}{r} \ , \\ V_{03} & = & (r-\frac{1}{r})e^3 \ , & & V_{12} & = & (2r+\frac{1}{r})e^3 \ , \end{array} \tag{22.6.3}$$

and

$$B = 2re^3 \ , \tag{22.6.4}$$

respectively. The explicit representation of the vielbein is not needed here.

Let us first show that the charged part of the spinor connection couples purely left handedly. Identifying Σ_3^0 and Σ_2^1 as the diagonal (neutral) Lie-algebra generators of $SO(4)$, one finds that the charged part of the spinor connection is given by

$$A_{ch} = 2V_{23}I_L^1 + 2V_{13}I_L^2 \ , \tag{22.6.5}$$

where one have defined

$$\begin{array}{rcl} I_L^1 & = & \frac{(\Sigma_{01}-\Sigma_{23})}{2} \ , \\ I_L^2 & = & \frac{(\Sigma_{02}-\Sigma_{13})}{2} \ . \end{array} \tag{22.6.6}$$

A_{ch} is clearly left handed so that one can perform the identification

$$W^{\pm} = \frac{2(e^1 \pm ie^2)}{r} \ , \tag{22.6.7}$$

where $W^{\pm}$ denotes the charged intermediate vector boson.

Consider next the identification of the neutral gauge bosons γ and Z^0 as appropriate linear combinations of the two functionally independent quantities

$$\begin{array}{rcl} X & = & re^3 \ , \\ Y & = & \frac{e^3}{r} \ , \end{array} \tag{22.6.8}$$

appearing in the neutral part of the spinor connection. We show first that the mere requirement that photon couples vectorially implies the basic coupling structure of the GWS model leaving only the value of Weinberg angle undetermined.

To begin with let us define

$$\begin{array}{rcl} \bar{\gamma} & = & aX + bY \ , \\ \bar{Z}^0 & = & cX + dY \ , \end{array} \tag{22.6.9}$$

where the normalization condition

$$ad - bc = 1 \ ,$$

is satisfied. The physical fields γ and Z^0 are related to $\bar{\gamma}$ and $\bar{Z}^0$ by simple normalization factors.

Expressing the neutral part of the spinor connection in term of these fields one obtains

$$\begin{array}{rcl} A_{nc} & = & [(c+d)2\Sigma_{03} + (2d-c)2\Sigma_{12} + d(n_+1_+ + n_-1_-)]\bar{\gamma} \\ & + & [(a-b)2\Sigma_{03} + (a-2b)2\Sigma_{12} - b(n_+1_+ + n_-1_-)]\bar{Z}^0 \ . \end{array} \tag{22.6.10}$$

Identifying Σ_{12} and $\Sigma_{03} = 1 \times \gamma_5 \Sigma_{12}$ as vectorial and axial Lie-algebra generators, respectively, the requirement that γ couples vectorially leads to the condition

$$c = -d \ . \tag{22.6.11}$$

Using this result plus previous equations, one obtains for the neutral part of the connection the expression

$$A_{nc} = \gamma Q_{em} + Z^0(I_L^3 - sin^2\theta_W Q_{em}) \ . \tag{22.6.12}$$

Here the electromagnetic charge Q_{em} and the weak isospin are defined by

$$\begin{aligned} Q_{em} &= \Sigma^{12} + \frac{(n_+ 1_+ + n_- 1_-)}{6} \ , \\ I_L^3 &= \frac{(\Sigma^{12} - \Sigma^{03})}{2} \ . \end{aligned} \tag{22.6.13}$$

The fields γ and Z^0 are defined via the relations

$$\begin{aligned} \gamma &= 6d\bar{\gamma} = \frac{6}{(a+b)}(aX + bY) \ , \\ Z^0 &= 4(a+b)\bar{Z}^0 = 4(X - Y) \ . \end{aligned} \tag{22.6.14}$$

The value of the Weinberg angle is given by

$$sin^2\theta_W = \frac{3b}{2(a+b)} \ , \tag{22.6.15}$$

and is not fixed completely. Observe that right handed neutrinos decouple completely from the electro-weak interactions.

The determination of the value of Weinberg angle is a dynamical problem. The angle is completely fixed once the YM action is fixed by requiring that action contains no cross term of type γZ^0. Pure symmetry non-broken electro-weak YM action leads to a definite value for the Weinberg angle. One can however add a symmetry breaking term proportional to Kähler action and this changes the value of the Weinberg angle.

To evaluate the value of the Weinberg angle one can express the neutral part F_{nc} of the induced gauge field as

$$F_{nc} = 2R_{03}\Sigma^{03} + 2R_{12}\Sigma^{12} + J(n_+ 1_+ + n_- 1_-) \ , \tag{22.6.16}$$

where one has

$$\begin{aligned} R_{03} &= 2(2e^0 \wedge e^3 + e^1 \wedge e^2) \ , \\ R_{12} &= 2(e^0 \wedge e^3 + 2e^1 \wedge e^2) \ , \\ J &= 2(e^0 \wedge e^3 + e^1 \wedge e^2) \ , \end{aligned} \tag{22.6.17}$$

in terms of the fields γ and Z^0 (photon and Z- boson)

$$F_{nc} = \gamma Q_{em} + Z^0(I_L^3 - sin^2\theta_W Q_{em}) \ . \tag{22.6.18}$$

Evaluating the expressions above one obtains for γ and Z^0 the expressions

$$\begin{aligned} \gamma &= 3J - sin^2\theta_W R_{03} \ , \\ Z^0 &= 2R_{03} \ . \end{aligned} \tag{22.6.19}$$

For the Kähler field one obtains

$$J = \frac{1}{3}(\gamma + sin^2\theta_W Z^0) \ . \tag{22.6.20}$$

Chapter i

Appendix

Originally this appendix was meant to be a purely technical summary of basic facts but in its recent form it tries to briefly summarize those basic visions about TGD which I dare to regarded stabilized. I have added illustrations making it easier to build mental images about what is involved and represented briefly the key arguments. This chapter is hoped to help the reader to get fast grasp about the concepts of TGD.

The basic properties of imbedding space and related spaces are discussed and the relationship of CP_2 to standard model is summarized. The notions of induction of metric and spinor connection, and of spinor structure are discussed. Many-sheeted space-time and related notions such as topological field quantization and the relationship many-sheeted space-time to that of GRT space-time are discussed as well as the recent view about induced spinor fields and the emergence of fermionic strings. Various topics related to p-adic numbers are summarized with a brief definition of p-adic manifold and the idea about generalization of the number concept by gluing real and p-adic number fields to a larger book like structure. Hierarchy of Planck constants can be now understood in terms of the non-determinism of Kähler action and the recent vision about connections to other key ideas is summarized.

A-1 Imbedding Space $M^4 \times CP_2$ And Related Notions

Space-times are regarded as 4-surfaces in $H = M^4 \times CP_2$ the Cartesian product of empty Minkowski space - the space-time of special relativity - and compact 4-D space CP_2 with size scale of order 10^4 Planck lengths. One can say that imbedding space is obtained by replacing each point m of empty Minkowski space with 4-D tiny CP_2. The space-time of general relativity is replaced by a 4-D surface in H which has very complex topology. The notion of many-sheeted space-time gives an idea about what is involved.

Fig. 1. Imbedding space $H = M^4 \times CP_2$ as Cartesian product of Minkowski space M^4 and complex projective space CP_2. `http://tgdtheory.fi/appfigures/Hoo.jpg`

Denote by M^4_+ and M^4_- the future and past directed lightcones of M^4. Denote their intersection, which is not unique, by CD. In zero energy ontology (ZEO) causal diamond (CD) is defined as cartesian product $CD \times CP_2$. Often I use CD to refer just to $CD \times CP_2$ since CP_2 factor is relevant from the point of view of ZEO.

Fig. 2. Future and past light-cones M^4_+ and M^4_-. Causal diamonds (CD) are defined as their intersections. `http://tgdtheory.fi/appfigures/futurepast.jpg`

Fig. 3. Causal diamond (CD) is highly analogous to Penrose diagram but simpler. `http://tgdtheory.fi/appfigures/penrose.jpg`

A rather recent discovery was that CP_2 is the only compact 4-manifold with Euclidian signature of metric allowing twistor space with Kähler structure. M^4 is in turn is the only 4-D space with Minkowskian signature of metric allowing twistor space with Kähler structure so that $H = M^4 \times CP_2$ is twistorially unique.

One can loosely say that quantum states in a given sector of "world of classical worlds" (WCW) are superpositions of space-time surfaces inside CDs and that positive and negative energy parts of zero energy states are localized and past and future boundaries of CDs. CDs form a hierarchy. One can have CDs within CDs and CDs can also overlap. The size of CD is characterized by the proper time distance between its two tips. One can perform both translations and also Lorentz boosts of CD leaving either boundary invariant. Therefore one can assign to CDs a moduli space and speak about wave function in this moduli space.

In number theoretic approach it is natural to restrict the allowed Lorentz boosts to some discrete subgroup of Lorentz group and also the distances between the tips of CDs to multiples of CP_2 radius defined by the length of its geodesic. Therefore the moduli space of CDs discretizes. The quantization of cosmic recession velocities for which there are indications, could relate to this quantization.

A-2 Basic Facts About CP_2

CP_2 as a four-manifold is very special. The following arguments demonstrates that it codes for the symmetries of standard models via its isometries and holonomies.

A-2.1 CP_2 As A Manifold

CP_2, the complex projective space of two complex dimensions, is obtained by identifying the points of complex 3-space C^3 under the projective equivalence

$$(z^1, z^2, z^3) \equiv \lambda(z^1, z^2, z^3) \ . \tag{A-2.1}$$

Here λ is any non-zero complex number. Note that CP_2 can be also regarded as the coset space $SU(3)/U(2)$. The pair z^i/z^j for fixed j and $z^i \neq 0$ defines a complex coordinate chart for CP_2. As j runs from 1 to 3 one obtains an atlas of three coordinate charts covering CP_2, the charts being holomorphically related to each other (e.g. CP_2 is a complex manifold). The points $z^3 \neq 0$ form a subset of CP_2 homoeomorphic to R^4 and the points with $z^3 = 0$ a set homeomorphic to S^2. Therefore CP_2 is obtained by "adding the 2-sphere at infinity to R^4".

Besides the standard complex coordinates $\xi^i = z^i/z^3$, $i = 1, 2$ the coordinates of Eguchi and Freund [A53] will be used and their relation to the complex coordinates is given by

$$\begin{aligned} \xi^1 &= z + it \ , \\ \xi^2 &= x + iy \ . \end{aligned} \tag{A-2.2}$$

These are related to the "spherical coordinates" via the equations

$$\begin{aligned} \xi^1 &= rexp(i\frac{(\Psi + \Phi)}{2})cos(\frac{\Theta}{2}) \ , \\ \xi^2 &= rexp(i\frac{(\Psi - \Phi)}{2})sin(\frac{\Theta}{2}) \ . \end{aligned} \tag{A-2.3}$$

The ranges of the variables r, Θ, Φ, Ψ are $[0, \infty], [0, \pi], [0, 4\pi], [0, 2\pi]$ respectively.

Considered as a real four-manifold CP_2 is compact and simply connected, with Euler number Euler number 3, Pontryagin number 3 and second $b = 1$.

Fig. 4. CP_2 as manifold. `http://tgdtheory.fi/appfigures/cp2.jpg`

A-2.2 Metric And Kähler Structure Of CP_2

In order to obtain a natural metric for CP_2, observe that CP_2 can be thought of as a set of the orbits of the isometries $z^i \rightarrow exp(i\alpha)z^i$ on the sphere S^5: $\sum z^i\bar{z}^i = R^2$. The metric of CP_2 is obtained by projecting the metric of S^5 orthogonally to the orbits of the isometries. Therefore the distance between the points of CP_2 is that between the representative orbits on S^5.

The line element has the following form in the complex coordinates

$$ds^2 = g_{a\bar{b}}d\xi^a d\bar{\xi}^b \ , \tag{A-2.4}$$

where the Hermitian, in fact Kähler metric $g_{a\bar{b}}$ is defined by

$$g_{a\bar{b}} = R^2\partial_a\partial_{\bar{b}}K \ , \tag{A-2.5}$$

where the function K, Kähler function, is defined as

$$\begin{aligned} K &= log(F) \ , \\ F &= 1+r^2 \ . \end{aligned} \tag{A-2.6}$$

The Kähler function for S^2 has the same form. It gives the S^2 metric $dzd\bar{z}/(1+r^2)^2$ related to its standard form in spherical coordinates by the coordinate transformation $(r,\phi) = (tan(\theta/2),\phi)$.

The representation of the CP_2 metric is deducible from S^5 metric is obtained by putting the angle coordinate of a geodesic sphere constant in it and is given

$$\frac{ds^2}{R^2} = \frac{(dr^2+r^2\sigma_3^2)}{F^2} + \frac{r^2(\sigma_1^2+\sigma_2^2)}{F} \ , \tag{A-2.7}$$

where the quantities σ_i are defined as

$$\begin{aligned} r^2\sigma_1 &= Im(\xi^1 d\xi^2 - \xi^2 d\xi^1) \ , \\ r^2\sigma_2 &= -Re(\xi^1 d\xi^2 - \xi^2 d\xi^1) \ , \\ r^2\sigma_3 &= -Im(\xi^1 d\bar{\xi}^1 + \xi^2 d\bar{\xi}^2) \ . \end{aligned} \tag{A-2.8}$$

R denotes the radius of the geodesic circle of CP_2. The vierbein forms, which satisfy the defining relation

$$s_{kl} = R^2\sum_A e^A_k e^A_l \ , \tag{A-2.9}$$

are given by

$$\begin{aligned} e^0 &= \tfrac{dr}{F} \ , & e^1 &= \tfrac{r\sigma_1}{\sqrt{F}} \ , \\ e^2 &= \tfrac{r\sigma_2}{\sqrt{F}} \ , & e^3 &= \tfrac{r\sigma_3}{F} \ . \end{aligned} \tag{A-2.10}$$

The explicit representations of vierbein vectors are given by

$$\begin{aligned} e^0 &= \tfrac{dr}{F} \ , & e^1 &= \tfrac{r(sin\Theta cos\Psi d\Phi + sin\Psi d\Theta)}{2\sqrt{F}} \ , \\ e^2 &= \tfrac{r(sin\Theta sin\Psi d\Phi - cos\Psi d\Theta)}{2\sqrt{F}} \ , & e^3 &= \tfrac{r(d\Psi + cos\Theta d\Phi)}{2F} \ . \end{aligned} \tag{A-2.11}$$

The explicit representation of the line element is given by the expression

$$ds^2/R^2 = \frac{dr^2}{F^2} + \frac{r^2}{4F^2}(d\Psi + cos\Theta d\Phi)^2 + \frac{r^2}{4F}(d\Theta^2 + sin^2\Theta d\Phi^2) \ . \tag{A-2.12}$$

The vierbein connection satisfying the defining relation

$$de^A = -V^A_B \wedge e^B \ , \tag{A-2.13}$$

is given by

$$\begin{array}{lllllll} V_{01} & = & -\frac{e^1}{r} & , & V_{23} & = & \frac{e^1}{r} \ , \\ V_{02} & = & -\frac{e^2}{r} & , & V_{31} & = & \frac{e^2}{r} \ , \\ V_{03} & = & (r-\frac{1}{r})e^3 & , & V_{12} & = & (2r+\frac{1}{r})e^3 \ . \end{array} \tag{A-2.14}$$

The representation of the covariantly constant curvature tensor is given by

$$\begin{array}{lllllll} R_{01} & = & e^0\wedge e^1 - e^2\wedge e^3 & , & R_{23} & = & e^0\wedge e^1 - e^2\wedge e^3 \ , \\ R_{02} & = & e^0\wedge e^2 - e^3\wedge e^1 & , & R_{31} & = & -e^0\wedge e^2 + e^3\wedge e^1 \ , \\ R_{03} & = & 4e^0\wedge e^3 + 2e^1\wedge e^2 & , & R_{12} & = & 2e^0\wedge e^3 + 4e^1\wedge e^2 \ . \end{array} \tag{A-2.15}$$

Metric defines a real, covariantly constant, and therefore closed 2-form J

$$J = -ig_{a\bar{b}}d\xi^a d\bar{\xi}^b \ , \tag{A-2.16}$$

the so called Kähler form. Kähler form J defines in CP_2 a symplectic structure because it satisfies the condition

$$J^k_{\ r}J^{rl} = -s^{kl} \ . \tag{A-2.17}$$

The form J is integer valued and by its covariant constancy satisfies free Maxwell equations. Hence it can be regarded as a curvature form of a $U(1)$ gauge potential B carrying a magnetic charge of unit $1/2g$ (g denotes the gauge coupling). Locally one has therefore

$$J = dB \ , \tag{A-2.18}$$

where B is the so called Kähler potential, which is not defined globally since J describes homological magnetic monopole.

It should be noticed that the magnetic flux of J through a 2-surface in CP_2 is proportional to its homology equivalence class, which is integer valued. The explicit representations of J and B are given by

$$\begin{array}{lll} B & = & 2re^3 \ , \\ J & = & 2(e^0\wedge e^3 + e^1\wedge e^2) = \frac{r}{F^2}dr\wedge(d\Psi + cos\Theta d\Phi) + \frac{r^2}{2F}sin\Theta d\Theta d\Phi \ . \end{array} \tag{A-2.19}$$

The vierbein curvature form and Kähler form are covariantly constant and have in the complex coordinates only components of type (1, 1).

Useful coordinates for CP_2 are the so called canonical coordinates in which Kähler potential and Kähler form have very simple expressions

$$\begin{aligned} B &= \sum_{k=1,2} P_k dQ_k \ , \\ J &= \sum_{k=1,2} dP_k \wedge dQ_k \ . \end{aligned} \tag{A-2.20}$$

The relationship of the canonical coordinates to the "spherical" coordinates is given by the equations

$$\begin{aligned} P_1 &= -\frac{1}{1+r^2} \ , \\ P_2 &= \frac{r^2 cos\Theta}{2(1+r^2)} \ , \\ Q_1 &= \Psi \ , \\ Q_2 &= \Phi \ . \end{aligned} \tag{A-2.21}$$

A-2.3 Spinors In CP_2

CP_2 doesn't allow spinor structure in the conventional sense [A43]. However, the coupling of the spinors to a half odd multiple of the Kähler potential leads to a respectable spinor structure. Because the delicacies associated with the spinor structure of CP_2 play a fundamental role in TGD, the arguments of Hawking are repeated here.

To see how the space can fail to have an ordinary spinor structure consider the parallel transport of the vierbein in a simply connected space M. The parallel propagation around a closed curve with a base point x leads to a rotated vierbein at x: $e^A = R^A_B e^B$ and one can associate to each closed path an element of $SO(4)$.

Consider now a one-parameter family of closed curves $\gamma(v) : v \in (0, 1)$ with the same base point x and $\gamma(0)$ and $\gamma(1)$ trivial paths. Clearly these paths define a sphere S^2 in M and the element $R^A_B(v)$ defines a closed path in $SO(4)$. When the sphere S^2 is contractible to a point e.g., homologically trivial, the path in $SO(4)$ is also contractible to a point and therefore represents a trivial element of the homotopy group $\Pi_1(SO(4)) = Z_2$.

For a homologically nontrivial 2-surface S^2 the associated path in $SO(4)$ can be homotopically nontrivial and therefore corresponds to a nonclosed path in the covering group Spin(4) (leading from the matrix 1 to -1 in the matrix representation). Assume this is the case.

Assume now that the space allows spinor structure. Then one can parallel propagate also spinors and by the above construction associate a closed path of Spin(4) to the surface S^2. Now, however this path corresponds to a lift of the corresponding $SO(4)$ path and cannot be closed. Thus one ends up with a contradiction.

From the preceding argument it is clear that one could compensate the non-allowed -1-factor associated with the parallel transport of the spinor around the sphere S^2 by coupling it to a gauge potential in such a way that in the parallel transport the gauge potential introduces a compensating -1-factor. For a $U(1)$ gauge potential this factor is given by the exponential $exp(i2\Phi)$, where Φ is the magnetic flux through the surface. This factor has the value -1 provided the $U(1)$ potential carries half odd multiple of Dirac charge $1/2g$. In case of CP_2 the required gauge potential is half odd multiple of the Kähler potential B defined previously. In the case of $M^4 \times CP_2$ one can in addition couple the spinor components with different chiralities independently to an odd multiple of $B/2$.

A-2.4 Geodesic Sub-Manifolds Of CP_2

Geodesic sub-manifolds are defined as sub-manifolds having common geodesic lines with the imbedding space. As a consequence the second fundamental form of the geodesic manifold vanishes, which means that the tangent vectors h^k_α (understood as vectors of H) are covariantly constant quantities with respect to the covariant derivative taking into account that the tangent vectors are vectors both with respect to H and X^4.

In [A85] a general characterization of the geodesic sub-manifolds for an arbitrary symmetric space G/H is given. Geodesic sub-manifolds are in 1-1-correspondence with the so called Lie triple systems of the Lie-algebra g of the group G. The Lie triple system t is defined as a subspace of g characterized by the closedness property with respect to double commutation

$$[X,[Y,Z]] \in t \ \text{ for } \ X,Y,Z \in t \ . \tag{A-2.22}$$

$SU(3)$ allows, besides geodesic lines, two nonequivalent (not isometry related) geodesic spheres. This is understood by observing that $SU(3)$ allows two nonequivalent $SU(2)$ algebras corresponding to subgroups $SO(3)$ (orthogonal 3×3 matrices) and the usual isospin group $SU(2)$. By taking any subset of two generators from these algebras, one obtains a Lie triple system and by exponentiating this system, one obtains a 2-dimensional geodesic sub-manifold of CP_2.

Standard representatives for the geodesic spheres of CP_2 are given by the equations

$$S^2_I \ : \ \xi^1 = \bar{\xi}^2 \ \text{ or equivalently } \ (\Theta = \pi/2, \Psi = 0) \ ,$$

$$S^2_{II} \ : \ \xi^1 = \xi^2 \ \text{ or equivalently } \ (\Theta = \pi/2, \Phi = 0) \ .$$

The non-equivalence of these sub-manifolds is clear from the fact that isometries act as holomorphic transformations in CP_2. The vanishing of the second fundamental form is also easy to verify. The first geodesic manifold is homologically trivial: in fact, the induced Kähler form vanishes identically for S^2_I. S^2_{II} is homologically nontrivial and the flux of the Kähler form gives its homology equivalence class.

A-3 CP_2 Geometry And Standard Model Symmetries

A-3.1 Identification Of The Electro-Weak Couplings

The delicacies of the spinor structure of CP_2 make it a unique candidate for space S. First, the coupling of the spinors to the $U(1)$ gauge potential defined by the Kähler structure provides the missing $U(1)$ factor in the gauge group. Secondly, it is possible to couple different H-chiralities independently to a half odd multiple of the Kähler potential. Thus the hopes of obtaining a correct spectrum for the electromagnetic charge are considerable. In the following it will be demonstrated that the couplings of the induced spinor connection are indeed those of the GWS model [B58] and in particular that the right handed neutrinos decouple completely from the electro-weak interactions.

To begin with, recall that the space H allows to define three different chiralities for spinors. Spinors with fixed H-chirality $e = \pm 1$, CP_2-chirality l, r and M^4-chirality L, R are defined by the condition

$$\begin{aligned} \Gamma\Psi &= e\Psi \ , \\ e &= \pm 1 \ , \end{aligned} \tag{A-3.1}$$

where Γ denotes the matrix $\Gamma_9 = \gamma_5 \times \gamma_5$, $1 \times \gamma_5$ and $\gamma_5 \times 1$ respectively. Clearly, for a fixed H-chirality CP_2- and M^4-chiralities are correlated.

The spinors with H-chirality $e = \pm 1$ can be identified as quark and lepton like spinors respectively. The separate conservation of baryon and lepton numbers can be understood as a consequence of generalized chiral invariance if this identification is accepted. For the spinors with a definite H-chirality one can identify the vielbein group of CP_2 as the electro-weak group: $SO(4) = SU(2)_L \times SU(2)_R$.

The covariant derivatives are defined by the spinorial connection

$$A = V + \frac{B}{2}(n_+ 1_+ + n_- 1_-) \ . \tag{A-3.2}$$

Here V and B denote the projections of the vielbein and Kähler gauge potentials respectively and $1_{+(-)}$ projects to the spinor H-chirality $+(-)$. The integers $n_\pm$ are odd from the requirement of a respectable spinor structure.

The explicit representation of the vielbein connection V and of B are given by the equations

$$\begin{array}{lllllll} V_{01} & = & -\frac{e^1}{r} \ , & V_{23} & = & \frac{e^1}{r} \ , \\ V_{02} & = & -\frac{e^2}{r} \ , & V_{31} & = & \frac{e^2}{r} \ , \\ V_{03} & = & (r-\frac{1}{r})e^3 \ , & V_{12} & = & (2r+\frac{1}{r})e^3 \ , \end{array} \qquad \text{(A-3.3)}$$

and

$$B = 2re^3 \ , \qquad \text{(A-3.4)}$$

respectively. The explicit representation of the vielbein is not needed here.

Let us first show that the charged part of the spinor connection couples purely left handedly. Identifying Σ_3^0 and Σ_2^1 as the diagonal (neutral) Lie-algebra generators of $SO(4)$, one finds that the charged part of the spinor connection is given by

$$A_{ch} = 2V_{23}I_L^1 + 2V_{13}I_L^2 \ , \qquad \text{(A-3.5)}$$

where one have defined

$$\begin{array}{lll} I_L^1 & = & \frac{(\Sigma_{01}-\Sigma_{23})}{2} \ , \\ I_L^2 & = & \frac{(\Sigma_{02}-\Sigma_{13})}{2} \ . \end{array} \qquad \text{(A-3.6)}$$

A_{ch} is clearly left handed so that one can perform the identification

$$W^\pm = \frac{2(e^1 \pm ie^2)}{r} \ , \qquad \text{(A-3.7)}$$

where $W^\pm$ denotes the charged intermediate vector boson.

Consider next the identification of the neutral gauge bosons γ and Z^0 as appropriate linear combinations of the two functionally independent quantities

$$\begin{array}{lll} X & = & re^3 \ , \\ Y & = & \frac{e^3}{r} \ , \end{array} \qquad \text{(A-3.8)}$$

appearing in the neutral part of the spinor connection. We show first that the mere requirement that photon couples vectorially implies the basic coupling structure of the GWS model leaving only the value of Weinberg angle undetermined.

To begin with let us define

$$\begin{array}{lll} \bar{\gamma} & = & aX + bY \ , \\ \bar{Z}^0 & = & cX + dY \ , \end{array} \qquad \text{(A-3.9)}$$

where the normalization condition

$$ad - bc = 1 \ ,$$

is satisfied. The physical fields γ and Z^0 are related to $\bar{\gamma}$ and $\bar{Z}^0$ by simple normalization factors.

Expressing the neutral part of the spinor connection in term of these fields one obtains

$$\begin{aligned} A_{nc} &= [(c+d)2\Sigma_{03} + (2d-c)2\Sigma_{12} + d(n_+1_+ + n_-1_-)]\bar{\gamma} \\ &+ [(a-b)2\Sigma_{03} + (a-2b)2\Sigma_{12} - b(n_+1_+ + n_-1_-)]\bar{Z}^0 \ . \end{aligned} \tag{A-3.10}$$

Identifying Σ_{12} and $\Sigma_{03} = 1 \times \gamma_5\Sigma_{12}$ as vectorial and axial Lie-algebra generators, respectively, the requirement that γ couples vectorially leads to the condition

$$c = -d \ . \tag{A-3.11}$$

Using this result plus previous equations, one obtains for the neutral part of the connection the expression

$$A_{nc} = \gamma Q_{em} + Z^0(I_L^3 - sin^2\theta_W Q_{em}) \ . \tag{A-3.12}$$

Here the electromagnetic charge Q_{em} and the weak isospin are defined by

$$\begin{aligned} Q_{em} &= \Sigma^{12} + \frac{(n_+1_+ + n_-1_-)}{6} \ , \\ I_L^3 &= \frac{(\Sigma^{12} - \Sigma^{03})}{2} \ . \end{aligned} \tag{A-3.13}$$

The fields γ and Z^0 are defined via the relations

$$\begin{aligned} \gamma &= 6d\bar{\gamma} = \frac{6}{(a+b)}(aX + bY) \ , \\ Z^0 &= 4(a+b)\bar{Z}^0 = 4(X - Y) \ . \end{aligned} \tag{A-3.14}$$

The value of the Weinberg angle is given by

$$sin^2\theta_W = \frac{3b}{2(a+b)} \ , \tag{A-3.15}$$

and is not fixed completely. Observe that right handed neutrinos decouple completely from the electro-weak interactions.

The determination of the value of Weinberg angle is a dynamical problem. The angle is completely fixed once the YM action is fixed by requiring that action contains no cross term of type γZ^0. Pure symmetry non-broken electro-weak YM action leads to a definite value for the Weinberg angle. One can however add a symmetry breaking term proportional to Kähler action and this changes the value of the Weinberg angle.

To evaluate the value of the Weinberg angle one can express the neutral part F_{nc} of the induced gauge field as

$$F_{nc} = 2R_{03}\Sigma^{03} + 2R_{12}\Sigma^{12} + J(n_+1_+ + n_-1_-) \ , \tag{A-3.16}$$

where one has

$$\begin{aligned} R_{03} &= 2(2e^0 \wedge e^3 + e^1 \wedge e^2) \ , \\ R_{12} &= 2(e^0 \wedge e^3 + 2e^1 \wedge e^2) \ , \\ J &= 2(e^0 \wedge e^3 + e^1 \wedge e^2) \ , \end{aligned} \tag{A-3.17}$$

in terms of the fields γ and Z^0 (photon and Z- boson)

$$F_{nc} = \gamma Q_{em} + Z^0(I_L^3 - sin^2\theta_W Q_{em}) \ . \tag{A-3.18}$$

Evaluating the expressions above one obtains for γ and Z^0 the expressions

$$\begin{aligned} \gamma &= 3J - sin^2\theta_W R_{03} \ , \\ Z^0 &= 2R_{03} \ . \end{aligned} \tag{A-3.19}$$

For the Kähler field one obtains

$$J = \frac{1}{3}(\gamma + sin^2\theta_W Z^0) \ . \tag{A-3.20}$$

Expressing the neutral part of the symmetry broken YM action

$$\begin{aligned} L_{ew} &= L_{sym} + fJ^{\alpha\beta}J_{\alpha\beta} \ , \\ L_{sym} &= \frac{1}{4g^2}Tr(F^{\alpha\beta}F_{\alpha\beta}) \ , \end{aligned} \tag{A-3.21}$$

where the trace is taken in spinor representation, in terms of γ and Z^0 one obtains for the coefficient X of the γZ^0 cross term (this coefficient must vanish) the expression

$$\begin{aligned} X &= -\frac{K}{2g^2} + \frac{fp}{18} \ , \\ K &= Tr\left[Q_{em}(I_L^3 - sin^2\theta_W Q_{em})\right] \ , \end{aligned} \tag{A-3.22}$$

In the general case the value of the coefficient K is given by

$$K = \sum_i \left[-\frac{(18+2n_i^2)sin^2\theta_W}{9}\right] \ , \tag{A-3.23}$$

where the sum is over the spinor chiralities, which appear as elementary fermions and n_i is the integer describing the coupling of the spinor field to the Kähler potential. The cross term vanishes provided the value of the Weinberg angle is given by

$$sin^2\theta_W = \frac{9\sum_i 1}{(fg^2 + 2\sum_i(18+n_i^2))} \ . \tag{A-3.24}$$

In the scenario where both leptons and quarks are elementary fermions the value of the Weinberg angle is given by

$$sin^2\theta_W = \frac{9}{(\frac{fg^2}{2} + 28)} \ . \tag{A-3.25}$$

The bare value of the Weinberg angle is 9/28 in this scenario, which is quite close to the typical value 9/24 of GUTs [B17] .

A-3.2 Discrete Symmetries

The treatment of discrete symmetries C, P, and T is based on the following requirements:

1. Symmetries must be realized as purely geometric transformations.
2. Transformation properties of the field variables should be essentially the same as in the conventional quantum field theories [B24] .

The action of the reflection P on spinors of is given by

$$\Psi \rightarrow P\Psi = \gamma^0 \otimes \gamma^0 \Psi \ . \tag{A-3.26}$$

in the representation of the gamma matrices for which γ^0 is diagonal. It should be noticed that W and Z^0 bosons break parity symmetry as they should since their charge matrices do not commute with the matrix of P.

The guess that a complex conjugation in CP_2 is associated with T transformation of the physicist turns out to be correct. One can verify by a direct calculation that pure Dirac action is invariant under T realized according to

$$\begin{aligned} m^k &\rightarrow T(M^k) \ , \\ \xi^k &\rightarrow \bar{\xi}^k \ , \\ \Psi &\rightarrow \gamma^1\gamma^3 \otimes 1\Psi \ . \end{aligned} \tag{A-3.27}$$

The operation bearing closest resemblance to the ordinary charge conjugation corresponds geometrically to complex conjugation in CP_2:

$$\begin{aligned} \xi^k &\rightarrow \bar{\xi}^k \ , \\ \Psi &\rightarrow \Psi^\dagger\gamma^2\gamma^0 \otimes 1 \ . \end{aligned} \tag{A-3.28}$$

As one might have expected symmetries CP and T are exact symmetries of the pure Dirac action.

A-4 The Relationship Of TGD To QFT And String Models

TGD could be seen as a generalization of quantum field theory (string models) obtained by replacing pointlike particles (strings) as fundamental objects with 3-surfaces.

Fig. 5. TGD replaces point-like particles with 3-surfaces. `http://tgdtheory.fi/appfigures/particletgd.jpg`

The fact that light-like 3-surfaces are effectively metrically 2-dimensional and thus possess generalization of 2-dimensional conformal symmetries with light-like radial coordinate defining the analog of second complex coordinate suggests that this generalization could work and extend the super-conformal symmetries to their 4-D analogs.

The boundary $\delta M^4_+ = S^2 \times R_+$- of 4-D light-cone M^4_+ is also metrically 2-dimensional and allows extended conformal invariance. Also the group of isometries of light-cone boundary and of light-like 3-surfaces is infinite-dimensional since the conformal scalings of S^2 can be compensated by S^2-local scaling of the light-like radial coordinate of R_+. These simple facts mean that 4-dimensional Minkowski space and 4-dimensional space-time surfaces are in completely unique position as far as symmetries are considered.

String like objects obtained as deformations of cosmic strings $X^2 \times Y^2$, where X^2 is minimal surface in M^4 and Y^2 a holomorphic surface of CP_2 are fundamental extremals of Kähler action having string world sheet as M^4 projections. Cosmic strings dominate the primordial cosmology of TGD Universe and inflationary period corresponds to the transition to radiation dominated cosmology for which space-time sheets with 4-D M^4 projection dominate.

Also genuine string like objects emerge from TGD. The conditions that the em charge of modes of induces spinor fields is well-defined requires in the generic case the localization of the modes at 2-D surfaces -string world sheets and possibly also partonic 2-surfaces. This in Minkowskian space-time regions.

Fig. 6. Well-definedness of em charge forces the localization of induced spinor modes to 2-D surfaces in generic situtation in Minkowskian regions of space-time surface. `http://tgdtheory.fi/appfigures/fermistring.jpg`

TGD based view about elementary particles has two aspects.

1. The space-time correlates of elementary particles are identified as pairs of wormhole contacts with Euclidian signature of metric and having 4-D CP_2 projection. Their throats behave effectively as Kähler magnetic monopoles so that wormhole throats must be connected by Kähler magnetic flux tubes with monopole flux so that closed flux tubes are obtained.

2. Fermion number is carried by the modes of the induced spinor field. In Minkowskian space-time regions the modes are localized at string world sheets connecting the wormhole contacts.

Fig. 7. TGD view about elementary particles. a) Particle corresponds 4-D generalization of world line or b) with its light-like 3-D boundary (holography). c) Particle world lines have Euclidian signature of the induced metric. d) They can be identified as wormhole contacts. e) The throats of wormhole contacts carry effective Kähler magnetic charges so that wormhole contacts must appear as pairs in order to obtain closed flux tubes. f) Wormhole contacts are accompnied by fermionic strings connecting the throats at same sheet: the strings do not extend inside the wormhole contacts. `http://tgdtheory.fi/appfigures/elparticletgd.jpg`

Particle interactions involve both stringy and QFT aspects.

1. The boundaries of string world sheets correspond to fundamental fermions. This gives rise to massless propagator lines in generalized Feynman diagrammatics. One can speak of "long" string connecting wormhole contacts and having hadronic string as physical counterpart. Long strings should be distinguished from wormhole contacts which due to their super-conformal invariance behave like "short" strings with length scale given by CP_2 size, which is 10^4 times longer than Planck scale characterizing strings in string models.

2. Wormhole contact defines basic stringy interaction vertex for fermion-fermion scattering. The propagator is essentially the inverse of the superconformal scaling generator L_0. Wormhole contacts containing fermion and antifermion at its opposite throats beheave like virtual bosons so that one has BFF type vertices typically.

3. In topological sense one has 3-vertices serving as generalizations of 3-vertices of Feynman diagrams. In these vertices 4-D "lines" of generalized Feynman diagrams meet along their 3-D ends. One obtains also the analogs of stringy diagrams but stringy vertices do not have the usual interpretation in terms of particle decays but in terms of propagation of particle along two different routes.

Fig. 8. a) TGD analogs of Feynman and string diagrammatics at the level of space-time topology. b) The 4-D analogs of both string diagrams and QFT diagrams appear but the interpretation of the analogs stringy diagrams is different. `http://tgdtheory.fi/appfigures/tgdgraphs.jpg`

A-5 Induction Procedure And Many-Sheeted Space-Time

Since the classical gauge fields are closely related in TGD framework, it is not possible to have space-time sheets carrying only single kind of gauge field. For instance, em fields are accompanied by Z^0 fields for extremals of Kähler action.

Classical em fields are always accompanied by Z^0 field and some components of color gauge field. For extremals having homologically non-trivial sphere as a CP_2 projection em and Z^0 fields are the only non-vanishing electroweak gauge fields. For homologically trivial sphere only W fields are non-vanishing. Color rotations does not affect the situation.

For vacuum extremals all electro-weak gauge fields are in general non-vanishing although the net gauge field has U(1) holonomy by 2-dimensionality of the CP_2 projection. Color gauge field has $U(1)$ holonomy for all space-time surfaces and quantum classical correspondence suggest a weak form of color confinement meaning that physical states correspond to color neutral members of color multiplets.

Induction procedure for gauge fields and spinor connection

Induction procedure for gauge potentials and spinor structure is a standard procedure of bundle theory. If one has imbedding of some manifold to the base space of a bundle, the bundle structure can be induced so that it has as a base space the imbedded manifold, whose points have as fiber the fiber if imbedding space at their image points. In the recent case the imbedding of space-time surface to imbedding space defines the induction procedure. The induced gauge potentials and gauge fields are projections of the spinor connection of the imbedding space to the space-time surface (see **Fig. ??**).

Induction procedure makes sense also for the spinor fields of imbedding space and one obtains geometrization of both electroweak gauge potentials and of spinors. The new element is induction of gamma matrices which gives their projections at space-time surface.

As a matter fact, the induced gamma matrices cannot appear in the counterpart of massless Dirac equation. To achieve super-symmetry, Dirac action must be replaced with Kähler-Dirac action for which gamma matrices are contractions of the canonical momentum currents of Kähler action with imbedding space gamma matrices. Induced gamma matrices in Dirac action would correspond to 4-volume as action.

Fig. 9. Induction of spinor connection and metric as projection to the space-time surface. `http://tgdtheory.fi/appfigures/induct.jpg`

Induced gauge fields for space-times for which CP_2 projection is a geodesic sphere

If one requires that space-time surface is an extremal of Kähler action and has a 2-dimensional CP_2 projection, only vacuum extremals and space-time surfaces for which CP_2 projection is a geodesic sphere, are allowed. Homologically non-trivial geodesic sphere correspond to vanishing W fields and homologically non-trivial sphere to non-vanishing W fields but vanishing γ and Z^0. This can be verified by explicit examples.

$r = \infty$ surface gives rise to a homologically non-trivial geodesic sphere for which e_0 and e_3 vanish imply the vanishing of W field. For space-time sheets for which CP_2 projection is $r = \infty$ homologically non-trivial geodesic sphere of CP_2 one has

$$\gamma = (\frac{3}{4} - \frac{sin^2(\theta_W)}{2})Z^0 \simeq \frac{5Z^0}{8} \quad .$$

The induced W fields vanish in this case and they vanish also for all geodesic sphere obtained by $SU(3)$ rotation.

$Im(\xi^1) = Im(\xi^2) = 0$ corresponds to homologically trivial geodesic sphere. A more general representative is obtained by using for the phase angles of standard complex CP_2 coordinates constant values. In this case e^1 and e^3 vanish so that the induced em, Z^0, and Kähler fields vanish but induced W fields are non-vanishing. This holds also for surfaces obtained by color rotation. Hence one can say that for non-vacuum extremals with 2-D CP_2 projection color rotations and weak symmetries commute.

A-5.1 Many-Sheeted Space-Time

TGD space-time is many-sheeted: in other words, there are in general several space-sheets which have projection to the same M^4 region. Second manner to say this is that CP_2 coordinates are many-valued functions of M^4 coordinates. The original physical interpretation of many-sheeted space-time time was not correct: it was assumed that single sheet corresponds to GRT space-time and this obviously leads to difficulties since the induced gauge fields are expressible in terms of only four imbedding space coordinates.

Fig. 10. Illustration of many-sheeted space-time of TGD. `http://tgdtheory.fi/appfigures/manysheeted.jpg`

Superposition of effects instead of superposition of fields

The first objection against TGD is that superposition is not possible for induced gauge fields and induced metric. The resolution of the problem is that it is effects which need to superpose, not the fields.

Test particle topologically condenses simultaneously to all space-time sheets having a projection to same region of M^4 (that is touches them). The superposition of effects of fields at various space-time sheets replaces the superposition of fields.This is crucial for the understanding also how GRT space-time relates to TGD space-time, which is also in the appendix of this book).

Wormhole contacts

Wormhole contacts are key element of many-sheeted space-time. One does not expect them to be stable unless there is non-trivial Kähler magnetic flux flowing through then so that the throats look like Kähler magnetic monopoles.

Fig. 11. Wormhole contact. `http://tgdtheory.fi/appfigures/wormholecontact.jpg`

Since the flow lines of Kähler magnetic field must be closed this requires the presence of another wormhole contact so that one obtains closed monopole flux tube decomposing to two Minkowskian pieces at the two space-time sheets involved and two wormhole contacts with Euclidian signature of the induced metric. These objects are identified as space-time correlates of elementary particles and are clearly analogous to string like objects.

The relationship between the many-sheeted space-time of TGD and of GRT space-time

The space-time of general relativity is single-sheeted and there is no need to regard it as surface in H although the assumption about representability as vacuum extremal gives very powerful constraints in cosmology and astrophysics and might make sense in simple situations.

The space-time of GRT can be regarded as a long length scale approximation obtained by lumping together the sheets of the many-sheeted space-time to a region of M^4 and providing it with an effective metric obtained as sum of M^4 metric and deviations of the induced metrics of various space-time sheets from M^4 metric. Also induced gauge potentials sum up in the similar manner so that also the gauge fields of gauge theories would not be fundamental fields.

Fig. 12. The superposition of fields is replaced with the superposition of their effects in many-sheeted space-time. `http://tgdtheory.fi/appfigures/fieldsuperpose.jpg`

Space-time surfaces of TGD are considerably simpler objects that the space-times of general relativity and relate to GRT space-time like elementary particles to systems of condensed matter physics. Same can be said about fields since all fields are expressible in terms of imbedding space coordinates and their gradients, and general coordinate invariance means that the number of bosonic field degrees is reduced locally to 4. TGD space-time can be said to be a microscopic description whereas GRT space-time a macroscopic description. In TGD complexity of space-time topology replaces the complexity due to large number of fields in quantum field theory.

Topological field quantization and the notion of magnetic body

Topological field quantization also TGD from Maxwell's theory. TGD predicts topological light rays ("massless extremals (MEs)") as space-time sheets carrying waves or arbitrary shape propagating with maximal signal velocity in single direction only and analogous to laser beams and carrying light-like gauge currents in the generi case. There are also magnetic flux quanta and electric flux quanta. The deformations of cosmic strings with 2-D string orbit as M^4 projection gives rise to magnetic flux tubes carrying monopole flux made possible by CP_2 topology allowing homological Kähler magnetic monopoles.

Fig. 13. Topological quantization for magnetic fields replaces magnetic fields with bundles of them defining flux tubes as topological field quanta. `http://tgdtheory.fi/appfigures/field.jpg`

The imbeddability condition for say magnetic field means that the region containing constant magnetic field splits into flux quanta, say tubes and sheets carrying constant magnetic field. Unless one assumes a separate boundary term in Kähler action, boundaries in the usual sense are forbidden except as ends of space-time surfaces at the boundaries of causal diamonds. One obtains typically pairs of sheets glued together along their boundaries giving rise to flux tubes with closed cross section possibly carrying monopole flux.

These kind of flux tubes might make possible magnetic fields in cosmic scales already during primordial period of cosmology since no currents are needed to generate these magnetic fields: cosmic string would be indeed this kind of objects and would dominated during the primordial period. Even superconductors and maybe even ferromagnets could involve this kind of monopole flux tubes.

A-5.2 Imbedding Space Spinors And Induced Spinors

One can geometrize also fermionic degrees of freedom by inducing the spinor structure of $M^4 \times CP_2$.

CP_2 does not allow spinor structure in the ordinary sense but one can couple the opposite H-chiralities of H-spinors to an $n = 1$ ($n = 3$) integer multiple of Kähler gauge potential to obtain a respectable modified spinor structure. The em charges of resulting spinors are fractional (integer valued) and the interpretation as quarks (leptons) makes sense since the couplings to the induced spinor connection having interpretation in terms electro-weak gauge potential are identical to those assumed in standard model.

The notion of quark color differs from that of standard model.

1. Spinors do not couple to color gauge potential although the identification of color gauge potential as projection of $SU(3)$ Killing vector fields is possible. This coupling must emerge only at the effective gauge theory limit of TGD.

2. Spinor harmonics of imbedding space correspond to triality $t = 1$ ($t = 0$) partial waves. The detailed correspondence between color and electroweak quantum numbers is however not correct as such and the interpretation of spinor harmonics of imbedding space is as representations for ground states of super-conformal representations. The wormhole pairs associated with physical quarks and leptons must carry also neutrino pair to neutralize weak quantum numbers above the length scale of flux tube (weak scale or Compton length). The total color quantum numbers or these states must be those of standard model. For instance, the color quantum numbers of fundamental left-hand neutrino and lepton can compensate each other for the physical lepton. For fundamental quark-lepton pair they could sum up to those of physical quark.

The well-definedness of em charge is crucial condition.

1. Although the imbedding space spinor connection carries W gauge potentials one can say that the imbedding space spinor modes have well-defined em charge. One expects that this is true for induced spinor fields inside wormhole contacts with 4-D CP_2 projection and Euclidian signature of the induced metric.

2. The situation is not the same for the modes of induced spinor fields inside Minkowskian region and one must require that the CP_2 projection of the regions carrying induced spinor field is such that the induced W fields and above weak scale also the induced Z^0 fields vanish in order to avoid large parity breaking effects. This condition forces the CP_2 projection to be 2-dimensional. For a generic Minkowskian space-time region this is achieved only if the spinor modes are localized at 2-D surfaces of space-time surface - string world sheets and possibly also partonic 2-surfaces.

3. Also the Kähler-Dirac gamma matrices appearing in the modified Dirac equation must vanish in the directions normal to the 2-D surface in order that Kähler-Dirac equation can be satisfied. This does not seem plausible for space-time regions with 4-D CP_2 projection.

4. One can thus say that strings emerge from TGD in Minkowskian space-time regions. In particular, elementary particles are accompanied by a pair of fermionic strings at the opposite space-time sheets and connecting wormhole contacts. Quite generally, fundamental fermions would propagate at the boundaries of string world sheets as massless particles and wormhole contacts would define the stringy vertices of generalized Feynman diagrams. One obtains geometrized diagrammatics, which brings looks like a combination of stringy and Feynman diagrammatics.

5. This is what happens in the the generic situation. Cosmic strings could serve as examples about surfaces with 2-D CP_2 projection and carrying only em fields and allowing delocalization of spinor modes to the entire space-time surfaces.

A-5.3 Space-Time Surfaces With Vanishing Em, Z^0, Or Kähler Fields

In the following the induced gauge fields are studied for general space-time surface without assuming the extremal property. In fact, extremal property reduces the study to the study of vacuum extremals and surfaces having geodesic sphere as a CP_2 projection and in this sense the following arguments are somewhat obsolete in their generality.

Space-times with vanishing em, Z^0, or Kähler fields

The following considerations apply to a more general situation in which the homologically trivial geodesic sphere and extremal property are not assumed. It must be emphasized that this case is possible in TGD framework only for a vanishing Kähler field.

Using spherical coordinates (r, Θ, Ψ, Φ) for CP_2, the expression of Kähler form reads as

$$\begin{aligned} J &= \frac{r}{F^2}dr \wedge (d\Psi + cos(\Theta)d\Phi) + \frac{r^2}{2F}sin(\Theta)d\Theta \wedge d\Phi \ , \\ F &= 1 + r^2 \ . \end{aligned} \tag{A-5.1}$$

The general expression of electromagnetic field reads as

$$\begin{aligned} F_{em} &= (3+2p)\frac{r}{F^2}dr \wedge (d\Psi + cos(\Theta)d\Phi) + (3+p)\frac{r^2}{2F}sin(\Theta)d\Theta \wedge d\Phi \ , \\ p &= sin^2(\Theta_W) \ , \end{aligned} \tag{A-5.2}$$

where Θ_W denotes Weinberg angle.

1. The vanishing of the electromagnetic fields is guaranteed, when the conditions

$$\begin{aligned} \Psi &= k\Phi \ , \\ (3+2p)\frac{1}{r^2F}(d(r^2)/d\Theta)(k + cos(\Theta)) &+ (3+p)sin(\Theta) = 0 \ , \end{aligned} \tag{A-5.3}$$

hold true. The conditions imply that CP_2 projection of the electromagnetically neutral space-time is 2-dimensional. Solving the differential equation one obtains

$$\begin{aligned} r &= \sqrt{\frac{X}{1-X}} \ , \\ X &= D\left[|\frac{(k+u}{C}|\right]^{\epsilon} \ , \\ u &\equiv cos(\Theta) \ , \ C = k + cos(\Theta_0) \ , \ D = \frac{r_0^2}{1+r_0^2} \ , \ \epsilon = \frac{3+p}{3+2p} \ , \end{aligned} \tag{A-5.4}$$

where C and D are integration constants. $0 \le X \le 1$ is required by the reality of r. $r = 0$ would correspond to $X = 0$ giving $u = -k$ achieved only for $|k| \le 1$ and $r = \infty$ to $X = 1$ giving $|u + k| = [(1 + r_0^2)/r_0^2)]^{(3+2p)/(3+p)}$ achieved only for

$$sign(u+k) \times [\frac{1+r_0^2}{r_0^2}]^{\frac{3+2p}{3+p}} \le k + 1 \ ,$$

where $sign(x)$ denotes the sign of x.

The expressions for Kähler form and Z^0 field are given by

$$\begin{aligned} J &= -\frac{p}{3+2p} X du \wedge d\Phi \ , \\ Z^0 &= -\frac{6}{p} J \ . \end{aligned} \tag{A-5.5}$$

The components of the electromagnetic field generated by varying vacuum parameters are proportional to the components of the Kähler field: in particular, the magnetic field is parallel to the Kähler magnetic field. The generation of a long range Z^0 vacuum field is a purely TGD based feature not encountered in the standard gauge theories.

2. The vanishing of Z^0 fields is achieved by the replacement of the parameter ϵ with $\epsilon = 1/2$ as becomes clear by considering the condition stating that Z^0 field vanishes identically. Also the relationship $F_{em} = 3J = -\frac{3}{4}\frac{r^2}{F} du \wedge d\Phi$ is useful.

3. The vanishing Kähler field corresponds to $\epsilon = 1, p = 0$ in the formula for em neutral space-times. In this case classical em and Z^0 fields are proportional to each other:

$$\begin{aligned} Z^0 &= 2e^0 \wedge e^3 = \frac{r}{F^2}(k+u)\frac{\partial r}{\partial u} du \wedge d\Phi = (k+u) du \wedge d\Phi \ , \\ r &= \sqrt{\frac{X}{1-X}} \ , \ X = D|k+u| \ , \\ \gamma &= -\frac{p}{2} Z^0 \ . \end{aligned} \tag{A-5.6}$$

For a vanishing value of Weinberg angle ($p = 0$) em field vanishes and only Z^0 field remains as a long range gauge field. Vacuum extremals for which long range Z^0 field vanishes but em field is non-vanishing are not possible.

The effective form of CP_2 metric for surfaces with 2-dimensional CP_2 projection

The effective form of the CP_2 metric for a space-time having vanishing em,Z^0, or Kähler field is of practical value in the case of vacuum extremals and is given by

$$\begin{aligned} ds^2_{eff} &= (s_{rr}(\frac{dr}{d\Theta})^2 + s_{\Theta\Theta}) d\Theta^2 + (s_{\Phi\Phi} + 2k s_{\Phi\Psi}) d\Phi^2 = \frac{R^2}{4}[s^{eff}_{\Theta\Theta} d\Theta^2 + s^{eff}_{\Phi\Phi} d\Phi^2] \ , \\ s^{eff}_{\Theta\Theta} &= X \times \left[\frac{\epsilon^2(1-u^2)}{(k+u)^2} \times \frac{1}{1-X} + 1 - X\right] \ , \\ s^{eff}_{\Phi\Phi} &= X \times \left[(1-X)(k+u)^2 + 1 - u^2\right] \ , \end{aligned} \tag{A-5.7}$$

and is useful in the construction of vacuum imbedding of, say Schwartchild metric.

Topological quantum numbers

Space-times for which either em, Z^0, or Kähler field vanishes decompose into regions characterized by six vacuum parameters: two of these quantum numbers (ω_1 and ω_2) are frequency type parameters, two (k_1 and k_2) are wave vector like quantum numbers, two of the quantum numbers (n_1 and n_2) are integers. The parameters ω_i and n_i will be referred as electric and magnetic quantum numbers. The existence of these quantum numbers is not a feature of these solutions alone but represents a much more general phenomenon differentiating in a clear cut manner between TGD and Maxwell's electrodynamics.

The simplest manner to avoid surface Kähler charges and discontinuities or infinities in the derivatives of CP_2 coordinates on the common boundary of two neighboring regions with different vacuum quantum numbers is topological field quantization, 3-space decomposes into disjoint topological field quanta, 3-surfaces having outer boundaries with possibly macroscopic size.

Under rather general conditions the coordinates Ψ and Φ can be written in the form

$$\begin{aligned} \Psi &= \omega_2 m^0 + k_2 m^3 + n_2\phi + \text{Fourier expansion} \ , \\ \Phi &= \omega_1 m^0 + k_1 m^3 + n_1\phi + \text{Fourier expansion} \ . \end{aligned} \tag{A-5.8}$$

m^0,m^3 and ϕ denote the coordinate variables of the cylindrical M^4 coordinates) so that one has $k = \omega_2/\omega_1 = n_2/n_1 = k_2/k_1$. The regions of the space-time surface with given values of the vacuum parameters ω_i,k_i and n_i and m and C are bounded by the surfaces at which space-time surface becomes ill-defined, say by $r > 0$ or $r < \infty$ surfaces.

The space-time surface decomposes into regions characterized by different values of the vacuum parameters r_0 and Θ_0. At $r = \infty$ surfaces n_2,ω_2 and m can change since all values of Ψ correspond to the same point of CP_2: at $r = 0$ surfaces also n_1 and ω_1 can change since all values of Φ correspond to same point of CP_2, too. If $r = 0$ or $r = \infty$ is not in the allowed range space-time surface develops a boundary.

This implies what might be called topological quantization since in general it is not possible to find a smooth global imbedding for, say a constant magnetic field. Although global imbedding exists it decomposes into regions with different values of the vacuum parameters and the coordinate u in general possesses discontinuous derivative at $r = 0$ and $r = \infty$ surfaces. A possible manner to avoid edges of space-time is to allow field quantization so that 3-space (and field) decomposes into disjoint quanta, which can be regarded as structurally stable units a 3-space (and of the gauge field). This doesn't exclude partial join along boundaries for neighboring field quanta provided some additional conditions guaranteeing the absence of edges are satisfied.

For instance, the vanishing of the electromagnetic fields implies that the condition

$$\Omega \equiv \frac{\omega_2}{n_2} - \frac{\omega_1}{n_1} = 0 \ , \tag{A-5.9}$$

is satisfied. In particular, the ratio ω_2/ω_1 is rational number for the electromagnetically neutral regions of space-time surface. The change of the parameter n_1 and n_2 (ω_1 and ω_2) in general generates magnetic field and therefore these integers will be referred to as magnetic (electric) quantum numbers.

A-6 P-Adic Numbers And TGD

A-6.1 P-Adic Number Fields

p-Adic numbers (p is prime: 2, 3, 5, ...) can be regarded as a completion of the rational numbers using a norm, which is different from the ordinary norm of real numbers [A40]. p-Adic numbers are representable as power expansion of the prime number p of form

$$x = \sum_{k\geq k_0} x(k)p^k, \ x(k) = 0, ..., p-1 \ . \tag{A-6.1}$$

The norm of a p-adic number is given by

$$|x| = p^{-k_0(x)} \ . \tag{A-6.2}$$

Here $k_0(x)$ is the lowest power in the expansion of the p-adic number. The norm differs drastically from the norm of the ordinary real numbers since it depends on the lowest pinary digit of the p-adic number only. Arbitrarily high powers in the expansion are possible since the norm of the p-adic number is finite also for numbers, which are infinite with respect to the ordinary norm. A convenient representation for p-adic numbers is in the form

$$x = p^{k_0}\varepsilon(x) \ , \tag{A-6.3}$$

where $\varepsilon(x) = k + ...$ with $0 < k < p$, is p-adic number with unit norm and analogous to the phase factor $exp(i\phi)$ of a complex number.

The distance function $d(x,y) = |x-y|_p$ defined by the p-adic norm possesses a very general property called ultra-metricity:

$$d(x,z) \leq max\{d(x,y), d(y,z)\} \ . \tag{A-6.4}$$

The properties of the distance function make it possible to decompose R_p into a union of disjoint sets using the criterion that x and y belong to same class if the distance between x and y satisfies the condition

$$d(x,y) \leq D \ . \tag{A-6.5}$$

This division of the metric space into classes has following properties:

1. Distances between the members of two different classes X and Y do not depend on the choice of points x and y inside classes. One can therefore speak about distance function between classes.

2. Distances of points x and y inside single class are smaller than distances between different classes.

3. Classes form a hierarchical tree.

Notice that the concept of the ultra-metricity emerged in physics from the models for spin glasses and is believed to have also applications in biology [B46]. The emergence of p-adic topology as the topology of the effective space-time would make ultra-metricity property basic feature of physics.

A-6.2 Canonical Correspondence Between P-Adic And Real Numbers

The basic challenge encountered by p-adic physicist is how to map the predictions of the p-adic physics to real numbers. p-Adic probabilities provide a basic example in this respect. Identification via common rationals and canonical identification and its variants have turned out to play a key role in this respect.

Basic form of canonical identification

There exists a natural continuous map $I: R_p \rightarrow R_+$ from p-adic numbers to non-negative real numbers given by the "pinary" expansion of the real number for $x \in R$ and $y \in R_p$ this correspondence reads

$$\begin{aligned} y &= \sum_{k>N} y_k p^k \rightarrow x = \sum_{k<N} y_k p^{-k} \ , \\ y_k &\in \{0,1,..,p-1\} \ . \end{aligned} \tag{A-6.6}$$

This map is continuous as one easily finds out. There is however a little difficulty associated with the definition of the inverse map since the pinary expansion like also decimal expansion is not unique ($1 = 0.999...$) for the real numbers x, which allow pinary expansion with finite number of pinary digits

$$\begin{aligned}
x &= \sum_{k=N_0}^{N} x_k p^{-k} \ , \\
x &= \sum_{k=N_0}^{N-1} x_k p^{-k} + (x_N - 1)p^{-N} + (p-1)p^{-N-1}\sum_{k=0,..} p^{-k} \ .
\end{aligned} \tag{A-6.7}$$

The p-adic images associated with these expansions are different

$$\begin{aligned}
y_1 &= \sum_{k=N_0}^{N} x_k p^{k} \ , \\
y_2 &= \sum_{k=N_0}^{N-1} x_k p^{k} + (x_N - 1)p^{N} + (p-1)p^{N+1}\sum_{k=0,..} p^{k} \\
&= y_1 + (x_N-1)p^N - p^{N+1} \ ,
\end{aligned} \tag{A-6.8}$$

so that the inverse map is either two-valued for p-adic numbers having expansion with finite pinary digits or single valued and discontinuous and non-surjective if one makes pinary expansion unique by choosing the one with finite pinary digits. The finite pinary digit expansion is a natural choice since in the numerical work one always must use a pinary cutoff on the real axis.

The topology induced by canonical identification

The topology induced by the canonical identification in the set of positive real numbers differs from the ordinary topology. The difference is easily understood by interpreting the p-adic norm as a norm in the set of the real numbers. The norm is constant in each interval $[p^k, p^{k+1})$ (see **Fig. A-6.2**) and is equal to the usual real norm at the points $x = p^k$: the usual linear norm is replaced with a piecewise constant norm. This means that p-adic topology is coarser than the usual real topology and the higher the value of p is, the coarser the resulting topology is above a given length scale. This hierarchical ordering of the p-adic topologies will be a central feature as far as the proposed applications of the p-adic numbers are considered.

Ordinary continuity implies p-adic continuity since the norm induced from the p-adic topology is rougher than the ordinary norm. p-Adic continuity implies ordinary continuity from right as is clear already from the properties of the p-adic norm (the graph of the norm is indeed continuous from right). This feature is one clear signature of the p-adic topology.

Fig. 14. The real norm induced by canonical identification from 2-adic norm. `http://tgdtheory.fi/appfigures/norm.png`

The linear structure of the p-adic numbers induces a corresponding structure in the set of the non-negative real numbers and p-adic linearity in general differs from the ordinary concept of linearity. For example, p-adic sum is equal to real sum only provided the summands have no common pinary digits. Furthermore, the condition $x +_p y < max\{x, y\}$ holds in general for the p-adic sum of the real numbers. p-Adic multiplication is equivalent with the ordinary multiplication only provided that either of the members of the product is power of p. Moreover one has $x \times_p y < x \times y$ in general. The p-Adic negative -1_p associated with p-adic unit 1 is given by $(-1)_p = \sum_k (p-1)p^k$ and defines p-adic negative for each real number x. An interesting possibility is that p-adic linearity might replace the ordinary linearity in some strongly nonlinear systems so these systems would look simple in the p-adic topology.

These results suggest that canonical identification is involved with some deeper mathematical structure. The following inequalities hold true:

$$\begin{aligned}
(x+y)_R &\leq x_R + y_R \ , \\
|x|_p |y|_R \leq (xy)_R &\leq x_R y_R \ ,
\end{aligned} \tag{A-6.9}$$

where $|x|_p$ denotes p-adic norm. These inequalities can be generalized to the case of $(R_p)^n$ (a linear vector space over the p-adic numbers).

$$
\begin{aligned}
(x+y)_R &\leq x_R + y_R \ , \\
|\lambda|_p |y|_R \leq (\lambda y)_R &\leq \lambda_R y_R \ ,
\end{aligned}
\tag{A-6.10}
$$

where the norm of the vector $x \in T_p^n$ is defined in some manner. The case of Euclidian space suggests the definition

$$
(x_R)^2 = (\sum_n x_n^2)_R \ . \tag{A-6.11}
$$

These inequalities resemble those satisfied by the vector norm. The only difference is the failure of linearity in the sense that the norm of a scaled vector is not obtained by scaling the norm of the original vector. Ordinary situation prevails only if the scaling corresponds to a power of p.

These observations suggests that the concept of a normed space or Banach space might have a generalization and physically the generalization might apply to the description of some non-linear systems. The nonlinearity would be concentrated in the nonlinear behavior of the norm under scaling.

Modified form of the canonical identification

The original form of the canonical identification is continuous but does not respect symmetries even approximately. This led to a search of variants which would do better in this respect. The modification of the canonical identification applying to rationals only and given by

$$
I_Q(q = p^k \times \frac{r}{s}) = p^k \times \frac{I(r)}{I(s)} \tag{A-6.12}
$$

is uniquely defined for rationals, maps rationals to rationals, has also a symmetry under exchange of target and domain. This map reduces to a direct identification of rationals for $0 \leq r < p$ and $0 \leq s < p$. It has turned out that it is this map which most naturally appears in the applications. The map is obviously continuous locally since p-adically small modifications of r and s mean small modifications of the real counterparts.

Canonical identification is in a key role in the successful predictions of the elementary particle masses. The predictions for the light elementary particle masses are within extreme accuracy same for I and I_Q but I_Q is theoretically preferred since the real probabilities obtained from p-adic ones by I_Q sum up to one in p-adic thermodynamics.

Generalization of number concept and notion of imbedding space

TGD forces an extension of number concept: roughly a fusion of reals and various p-adic number fields along common rationals is in question. This induces a similar fusion of real and p-adic imbedding spaces. Since finite p-adic numbers correspond always to non-negative reals n-dimensional space R^n must be covered by 2^n copies of the p-adic variant R_p^n of R^n each of which projects to a copy of R_+^n (four quadrants in the case of plane). The common points of p-adic and real imbedding spaces are rational points and most p-adic points are at real infinity.

Real numbers and various algebraic extensions of p-adic number fields are thus glued together along common rationals and also numbers in algebraic extension of rationals whose number belong to the algebraic extension of p-adic numbers. This gives rise to a book like structure with rationals and various algebraic extensions of rationals taking the role of the back of the book. Note that Neper number is exceptional in the sense that it is algebraic number in p-adic number field Q_p satisfying $e^p \ mod \ p = 1$.

Fig. 15. Various number fields combine to form a book like structure. `http://tgdtheory.fi/appfigures/book.jpg`

For a given p-adic space-time sheet most points are literally infinite as real points and the projection to the real imbedding space consists of a discrete set of rational points: the interpretation in terms of the unavoidable discreteness of the physical representations of cognition is natural. Purely local p-adic physics implies real p-adic fractality and thus long range correlations for the real space-time surfaces having enough common points with this projection.

p-Adic fractality means that M^4 projections for the rational points of space-time surface X^4 are related by a direct identification whereas CP_2 coordinates of X^4 at these points are related by I, I_Q or some of its variants implying long range correlates for CP_2 coordinates. Since only a discrete set of points are related in this manner, both real and p-adic field equations can be satisfied and there are no problems with symmetries. p-Adic effective topology is expected to be a good approximation only within some length scale range which means infrared and UV cutoffs. Also multi-p-fractality is possible.

A-6.3 The Notion Of P-Adic Manifold

The notion of p-adic manifold is needed in order to fuse real physics and various p-adic physics to a larger structure which suggests that real and p-adic number fields should be glued together along common rationals bringing in mind adeles. The notion is problematic because p-adic topology is totally disconnected implying that p-adic balls are either disjoint or nested so that ordinary definition of manifold using p-adic chart maps fails. A cure is suggested to be based on chart maps from p-adics to reals rather than to p-adics (see the appendix of the book)

The chart maps are interpreted as cognitive maps, "thought bubbles".

Fig. 16. The basic idea between p-adic manifold. `http://tgdtheory.fi/appfigures/padmanifold.jpg`

There are some problems.

1. Canonical identification does not respect symmetries since it does not commute with second pinary cutoff so that only a discrete set of rational points is mapped to their real counterparts by chart map arithmetic operations which requires pinary cutoff below which chart map takes rationals to rationals so that commutativity with arithmetics and symmetries is achieved in finite resolution: above the cutoff canonical identification is used

2. Canonical identification is continuous but does not map smooth p-adic surfaces to smooth real surfaces requiring second pinary cutoff so that only a discrete set of rational points is mapped to their real counterparts by chart map requiring completion of the image to smooth preferred extremal of Kähler action so that chart map is not unique in accordance with finite measurement resolution

3. Canonical identification vreaks general coordinate invariance of chart map: (cognition-induced symmetry breaking) minimized if p-adic manifold structure is induced from that for p-adic imbedding space with chart maps to real imbedding space and assuming preferred coordinates made possible by isometries of imbedding space: one however obtains several inequivalent p-adic manifold structures depending on the choice of coordinates: these cognitive representations are not equivalent.

A-7 Hierarchy Of Planck Constants And Dark Matter Hierarchy

Hierarchy of Planck constants was motivated by the "impossible" quantal effects of ELF em fields on vertebrate cyclotron energies $E = hf = \hbar \times eB/m$ are above thermal energy is possible only if $\hbar$ has value much larger than its standard value. Also Nottale's finding that planetary orbits migh be understood as Bohr orbits for a gigantic gravitational Planck constant.

Hierachy of Planck constant would mean that the values of Planck constant come as integer multiples of ordinary Planck constant: $h_{eff} = n \times h$. The particles at magnetic flux tubes characterized by h_{eff} would correspond to dark matter which would be invisible in the sense that only particle with same value of h_{eff} appear in the same vertex of Feynman diagram.

Hierarchy of Planck constants would be due to the non-determism of the Kähler action predicting huge vacuum degeneracy allowing all space-time surfaces which are sub-manfolds of any $M^4 \times Y^2$, where Y^2 is Lagrangian sub-manifold of CP_2. For agiven Y^2 one obtains new manifolds Y^2 by applying symplectic transformations of CP_2.

Non-determinism would mean that the 3-surface at the ends of causal diamond (CD) can be connected by several space-time surfaces carrying same conserved Kähler charges and having same values of Kähler action. Conformal symmetries defined by Kac-Moody algebra associated with the imbedding space isometries could act as gauge transformations and respect the light-likeness property of partonic orbits at which the signature of the induced metric changes from Minkowskian to Euclidian (Minkowskianb space-time region transforms to wormhole contact say). The number of conformal equivalence classes of these surfaces could be finite number n and define discrete physical degree of freedom and one would have $h_{eff} = n \times h$. This degeneracy would mean "second quantization" for the sheets of n-furcation: not only one but several sheets can be realized.

This relates also to quantum criticality postulated to be the basic characteristics of the dynamics of quantum TGD. Quantum criticalities would correspond to an infinite fractal hierarchy of broken conformal symmetries defined by sub-algebras of conformal algebra with conformal weights coming as integer multiples of n. This leads also to connections with quantum criticality and hierarchy of broken conformal symmetries, p-adicity, and negentropic entanglement which by consistency with standard quantum measurement theory would be described in terms of density matrix proportional $n \times n$ identity matrix and being due to unitary entanglement coefficients (typical for quantum computing systems).

Formally the situation could be described by regarding space-time surfaces as surfaces in singular n-fold singular coverings of imbedding space. A stronger assumption would be that they are expressible as as products of n_1 -fold covering of M^4 and n_2-fold covering of CP_2 meaning analogy with multi-sheeted Riemann surfaces and that M^4 coordinates are n_1-valued functions and CP_2 coordinates n_2 -valued functions of space-time coordinates for $n = n_1 \times n_2$. These singular coverings of imbedding space form a book like structure with singularities of the coverings localizable at the boundaries of causal diamonds defining the back of the book like structure.

Fig. 17. Hierarchy of Planck constants. `http://tgdtheory.fi/appfigures/planckhierarchy.jpg`

A-8 Some Notions Relevant To TGD Inspired Consciousness And Quantum Biology

Below some notions relevant to TGD inspired theory of consciousness and quantum biology.

A-8.1 The Notion Of Magnetic Body

Topological field quantization inspires the notion of field body about which magnetic body is especially important example and plays key role in TGD inspired quantum biology and consciousness theory. This is a crucial departure fromt the Maxwellian view. Magnetic body brings in third level to the description of living system as a system interacting strongly with environment. Magnetic body would serve as an intentional agent using biological body as a motor instrument and sensory receptor. EEG would communicated the information from biological body to magnetic body and Libet's findings from time delays of consciousness support this view.

The following pictures illustrate the notion of magnetic body and its dynamics relevant for quantum biology in TGD Universe.

Fig. 18. Magnetic body associated with dipole field. `http://tgdtheory.fi/appfigures/fluxquant.jpg`

Fig. 19. Illustration of the reconnection by magnetic flux loops. `http://tgdtheory.fi/appfigures/reconnect1.jpg`

Fig. 20. Illustration of the reconnection by flux tubes connecting pairs of molecules. `http://tgdtheory.fi/appfigures/reconect2.jpg`

Fig. 21. Flux tube dynamics. a) Reconnection making possible magnetic body to "recognize" the presence of another magnetic body, b) braiding, knotting and linking of flux tubes making possible topological quantum computation, c) contraction of flux tube in phase transition reducing the value of h_{eff} allowing two molecules to find each other in dense molecular soup. `http://tgdtheory.fi/appfigures/fluxtubedynamics.jpg`

A-8.2 Number Theoretic Entropy And Negentropic Entanglement

TGD inspired theory of consciousness relies heavily p-Adic norm allows an to define the notion of Shannon entropy for rational probabilities (and even those in algebraic extension of rationals) by replacing the argument of logarithm of probability with its p-adic norm. The resulting entropy can be negative and the interpretation is that number theoretic entanglement entropy defined by this formula for the p-adic prime minimizing its value serves as a measure for conscious information. This negentropy characterizes two-particle system and has nothing to do with the formal negative negentropy assignable to thermodynamic entropy characterizing single particle. Negentropy Maximization Principle (NMP) implies that number theoretic negentropy increases during evolution by quantum jumps. The condition that NMP is consistent with the standard quantum measurement theory requires that negentropic entanglement has a density matrix proportional to unit matrix so that in 2-particle case the entanglement matrix is unitary.

Fig. 22. Schrödinger cat is neither dead or alive. For negentropic entanglement this state would be stable. `http://tgdtheory.fi/appfigures/cat.jpg`

A-8.3 Life As Something Residing In The Intersection Of Reality And P-Adicities

In TGD inspired theory of consciousness p-adic space-time sheets correspond to space-time correlates for thoughts and intentions. The intersections of real and p-adic preferred extremals consist of points whose coordinates are rational or belong to some extension of rational numbers in preferred imbedding space coordinates. They would correspond to the intersection of reality and various p-adicities representing the "mind stuff" of Descartes. There is temptation to assign life to the intersection of realities and p-adicities. The discretization of the chart map assigning to real space-time surface its p-adic counterpart would reflect finite cognitive resolution.

At the level of "world of classical worlds" (WCW) the intersection of reality and various p-adicities would correspond to space-time surfaces (or possibly partonic 2-surfaces) representable in terms of rational functions with polynomial coefficients with are rational or belong to algebraic extension of rationals.

The quantum jump replacing real space-time sheet with p-adic one (vice versa) would correspond to a buildup of cognitive representation (realization of intentional action).

Fig. 23. The quantum jump replacing real space-time surface with corresponding p-adic manifold can be interpreted as formation of though, cognitive representation. Its reversal would correspond to a transformation of intention to action. `http://tgdtheory.fi/appfigures/padictoreal.jpg`

A-8.4 Sharing Of Mental Images

The 3-surfaces serving as correlates for sub-selves can topologically condense to disjoint large space-time sheets representing selves. These 3-surfaces can also have flux tube connections and this makes possible entanglement of sub-selves, which unentangled in the resolution defined by the size of sub-selves. The interpretation for this negentropic entanglement would be in terms of sharing of mental images. This would mean that contents of consciousness are not completely private as assumed in neuroscience.

Fig. 24. Sharing of mental images by entanglement of subselves made possible by flux tube connections between topologically condensed space-time sheets associated with mental images. http://tgdtheory.fi/appfigures/sharing.jpg

A-8.5 Time Mirror Mechanism

Zero energy ontology (ZEO) is crucial part of both TGD and TGD inspired consciousness and leads to the understanding of the relationship between geometric time and experience time and how the arrow of psychological time emerges. One of the basic predictions is the possibiity of negative energy signals propagating backwards in geometric time and having the property that entropy basically associated with subjective time grows in reversed direction of geometric time. Negative energy signals inspire time mirror mechanism (see **Fig.** http://tgdtheory.fi/appfigures/timemirror.jpg or **Fig.** 24 in the appendix of this book) providing mechanisms of both memory recall, realization of intentational action initiating action already in geometric past, and remote metabolism. What happens that negative energy signal travels to past and is reflected as positive energy signal and returns to the sender. This process works also in the reverse time direction.

Fig. 25. Zero energy ontology allows time mirror mechanism as a mechanism of memory recall. Essentially "seeing" in time direction is in question. http://tgdtheory.fi/appfigures/timemirror.jpg

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