# On Hemiolia Principle and Ornamental Numbers 

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#### Abstract

In [4] we introduced hemiolia principle and considered ornamental alphabet discovered by Modris Tenisons [1]. Here we explain hemiolia principle and use it to define ornamental alphabet by use of ornamental numbers defined here.


## 1 Introduction

In [4] it was discussed a simple ornamental sign language that was discovered by the first author. The main role in the ornamental genesis of this language was played by two tools introduced, the notion of sieve (see page 3), its removal and its displacement, and ten asymmetric $3 \times 3$ matrices. On page 4 hemiolia principle was introduced and explained.

## 2 Hemiolia principle

Hemiolia principle is useful to explain how simplest ornamental signs and ornamental alphabet letters arise.

We describe ornamental signs, letters and ornamental tracery belts as binary matrices with convention that bit values are white or colored (red) checks, which allow these ornamental elements to visualize as they are to be imagined, thus allowing mathematical images be suitable for artistic aims too, at least what concerns simplest ornamental traceries we are dealing with.

In figure 7 we explain hemiolia principle and synthesis of ornamental alphabet letters, and all six pictures are to be imagined as $12 \times 12$ binary matrices with zeros as white checks and units as colored (red) checks.

Let us give a qualitative definition of the ornamental alphabet letters and then a precise definition.

Definition 1. Let $c$ be a hexadecimal number and $b=0$ stand for white and $b=1$ stand for colored (red), cb stand for two types of ornamental alphabet letters where an ornamental alphabet letter is a $12 \times 12$ binary matrix with following conditions holding:

1. $c$ is called the code of the letter and have the $2 \times 2$ binary matrix $C$ in correspondence, $c_{16}=c_{1} c_{2} c_{3} c_{4}$, where

$$
C=\left(\begin{array}{ll}
c_{1} & c_{2} \\
c_{4} & c_{3}
\end{array}\right)
$$

2. foreground board is a $12 \times 12$ binary matrix that holds the binary code according the code $C$ in its four corner $6 \times 6$ checks;

[^0]3. background is as foreground (checker coded) board but shifted diagonally by 3 rows and 3 columns with the white check in the center for a letter with $b=0$, and the colored (red) check in the center for a letter with $b=1$;
4. the $12 \times 12$ binary matrix corresponding for alphabet letter is received by

- shrinking of code checks of the foreground board by the factor 3/2 with keeping centering, i.e. as in c) of the figure 7;
- and screening background board by foreground board, receiving alphabet letter, as it is showed in the d) of the figure 7.

From the qualitative definition it is easy to see that we need only to specify precisely the $6 \times 6$ corner check of the $12 \times 12$ matrix, that corresponds to one of the code $h$ units.

$$
\begin{aligned}
& 0_{o}=\left(\begin{array}{llllll}
0 & 0 & 0 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 & 0
\end{array}\right) \\
& 1_{o}=\left(\begin{array}{llllll}
1 & 1 & 1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 1 & 1
\end{array}\right) \\
& 2_{o}=\left(\begin{array}{llllll}
0 & 0 & 0 & 1 & 1 & 1 \\
0 & 1 & 1 & 1 & 1 & 1 \\
0 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 0 \\
1 & 1 & 1 & 1 & 1 & 0 \\
1 & 1 & 1 & 0 & 0 & 0
\end{array}\right) \\
& 3_{o}=\left(\begin{array}{llllll}
1 & 1 & 1 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 1 & 0 \\
1 & 1 & 1 & 1 & 1 & 0 \\
0 & 1 & 1 & 1 & 1 & 1 \\
0 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 1 & 1 & 1
\end{array}\right)
\end{aligned}
$$

$$
c b=\left(\begin{array}{ll}
c_{1} b & c_{2} b \\
c_{4} b & c_{3} b
\end{array}\right)
$$

Definition 2. Ornamental alphabet letter cb is a matrix

$$
c b=\left(\begin{array}{ll}
c_{1} b & c_{2} b \\
c_{4} b & c_{3} b
\end{array}\right)
$$

for the given hexadecimal code $c=c_{1} c_{2} c_{3} c_{4}$ and the background bit $b$.
In the first definition we used the figure 7 to define ornamental alphabet letter and in the same explained hemiolia principle that is used in this definition. The shrinking by factor $3 / 2$, or, $11 / 2$, or, in Greek, hemiolia, $\eta \mu \iota o \lambda \iota \alpha$ allowed to turn $3 \times 3$ code checks to change to $2 \times 2$ code checks and come to $12 \times 12$ matrix for ornamental letters and tracery.


Figure 1: Hemiolia principle and ornamental alphabet letters explained: a) foreground with the binary code, in this example, for $\left.1011, B_{16} ; \mathrm{b}\right)$ background with checker board with the diagonal half check shift with respect to the code foreground; c) shrinking of the code by the factor $3 / 2 ; \mathrm{d}$ ) an ornament alphabet letter generated, here, $w B$, arisen from white center and a code $B_{16}$; e) an ornamental alphabet letter $c 4$, arisen from red (colored) center and a code $4_{16}$; f) ornamental alphabet letter $w 4$ that arises from $c 4$ by sieve displacement; one more sign alphabet letter possible, $c B$, left for reader to construct. The factor $3 / 2$, or, one and half, in Greek, hemiolia, gives the name to this principle of ornamental genesis.

## References

[1] Tenisons, Modris (2001) Intellectual game, patent of design pattern, patent Nr. D 10 577, LOC kl. 21-01, 20.05.2001.
[2] Tenisons, Modris; Strazds, Armands. The Anatomy of ZIME. Project Report,(2003), www.zime.lv, scireprints.lu.lv/61/


Figure 2: Binary code in ornamental sieve


Figure 3: Binary code in ornamental sieve
[3] Tenisons, Modris (2010) Ornaments and Symmetry: Language of Signs of Ornamental Tracery (interview with Modris Tenisons), scireprints.lu.lv/62/
[4] Tenisons, M; Zeps, D. Ornamental Sign Language In The First Order Tracery Belts, PSTJ, Vol 1, No 2 (2010), vixra.org/abs/1003.0252, 2010, 2013, 19pp.


Figure 4: Binary code in ornamental sieve


Figure 5: Binary code in ornamental sieve


Figure 6: Binary code in ornamental sieve


Figure 7: Binary code in ornamental sieve


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